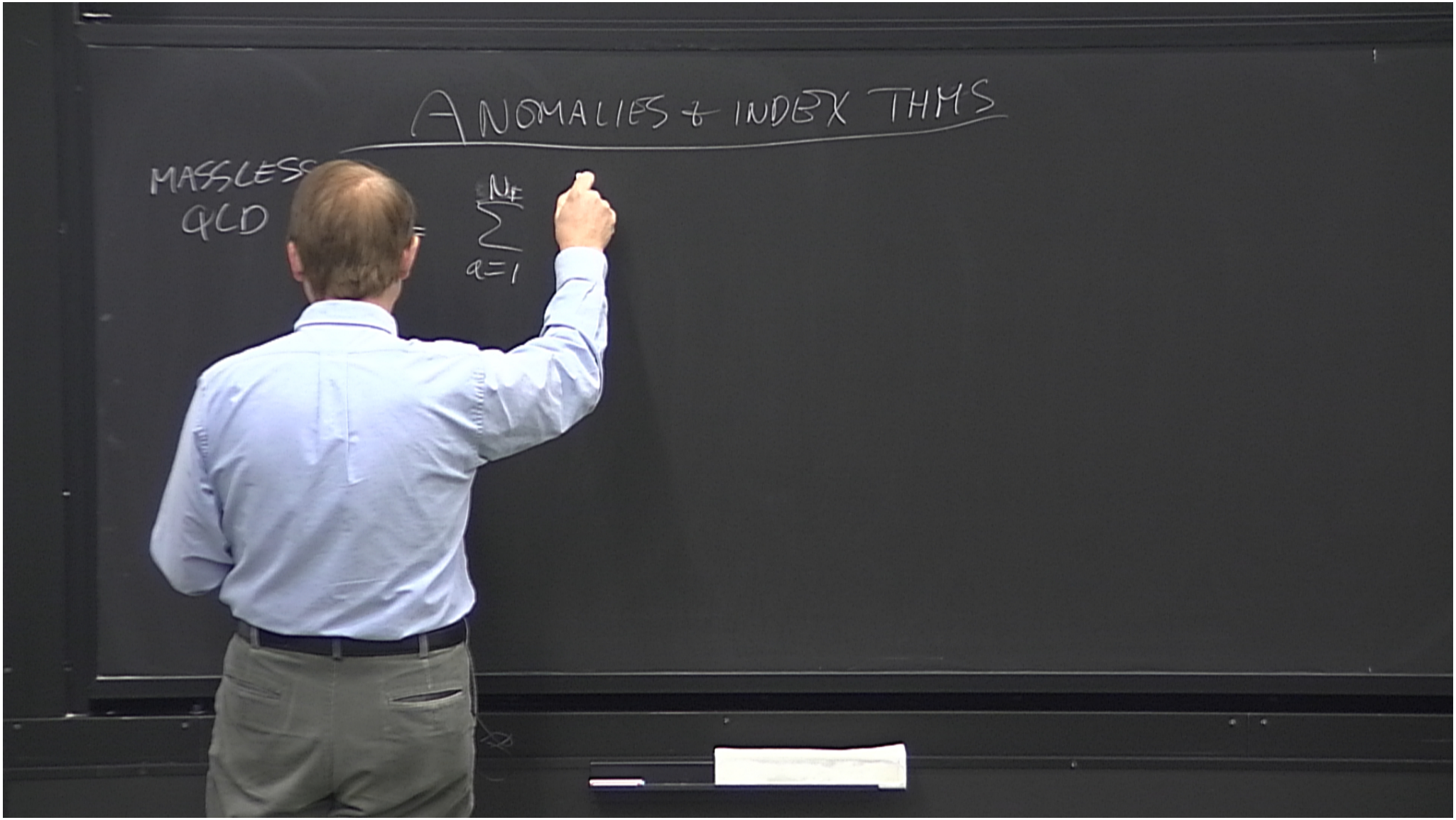
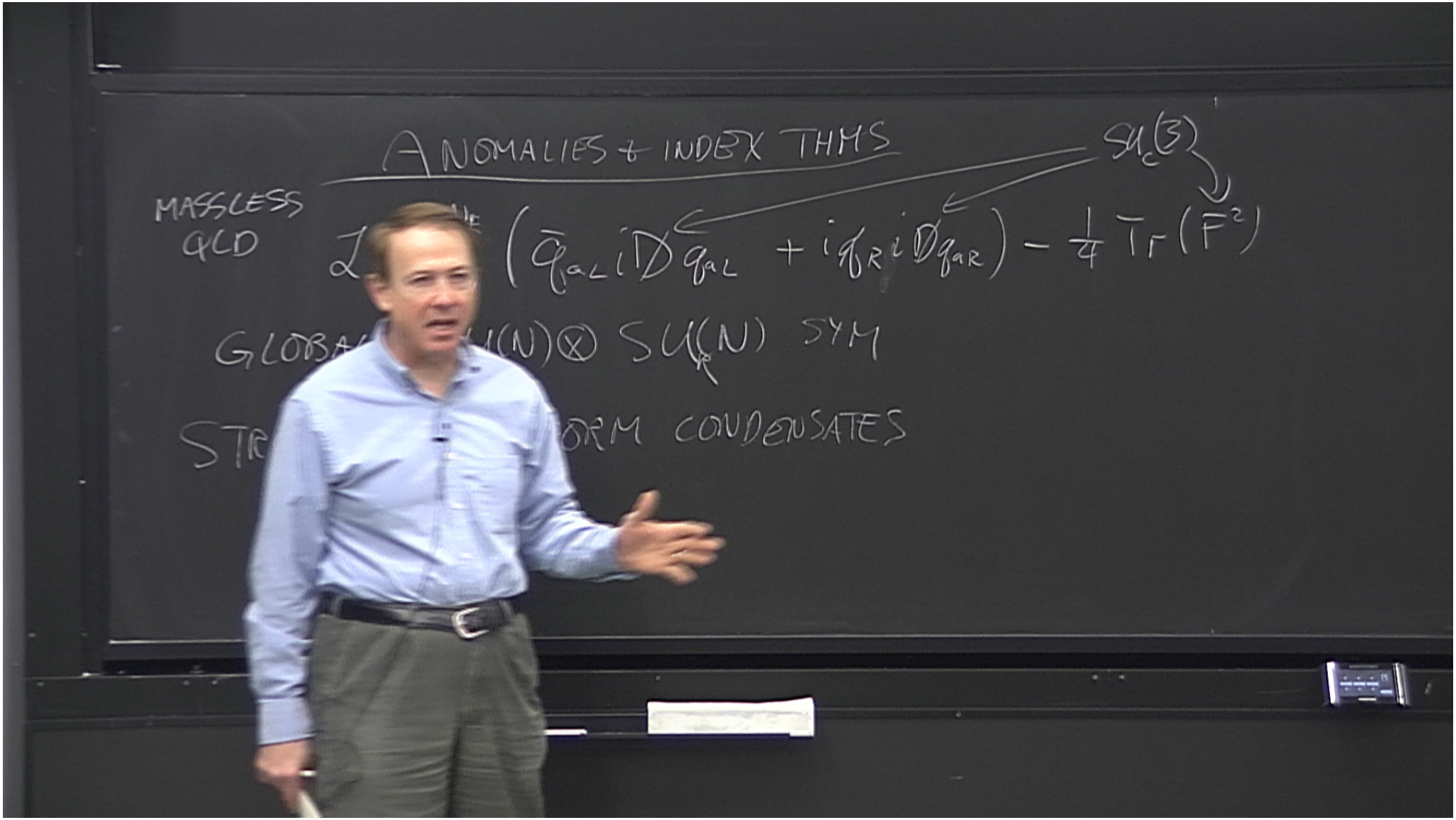
Title: 13/14 PSI - Beyond the Standard Model - Lecture 8

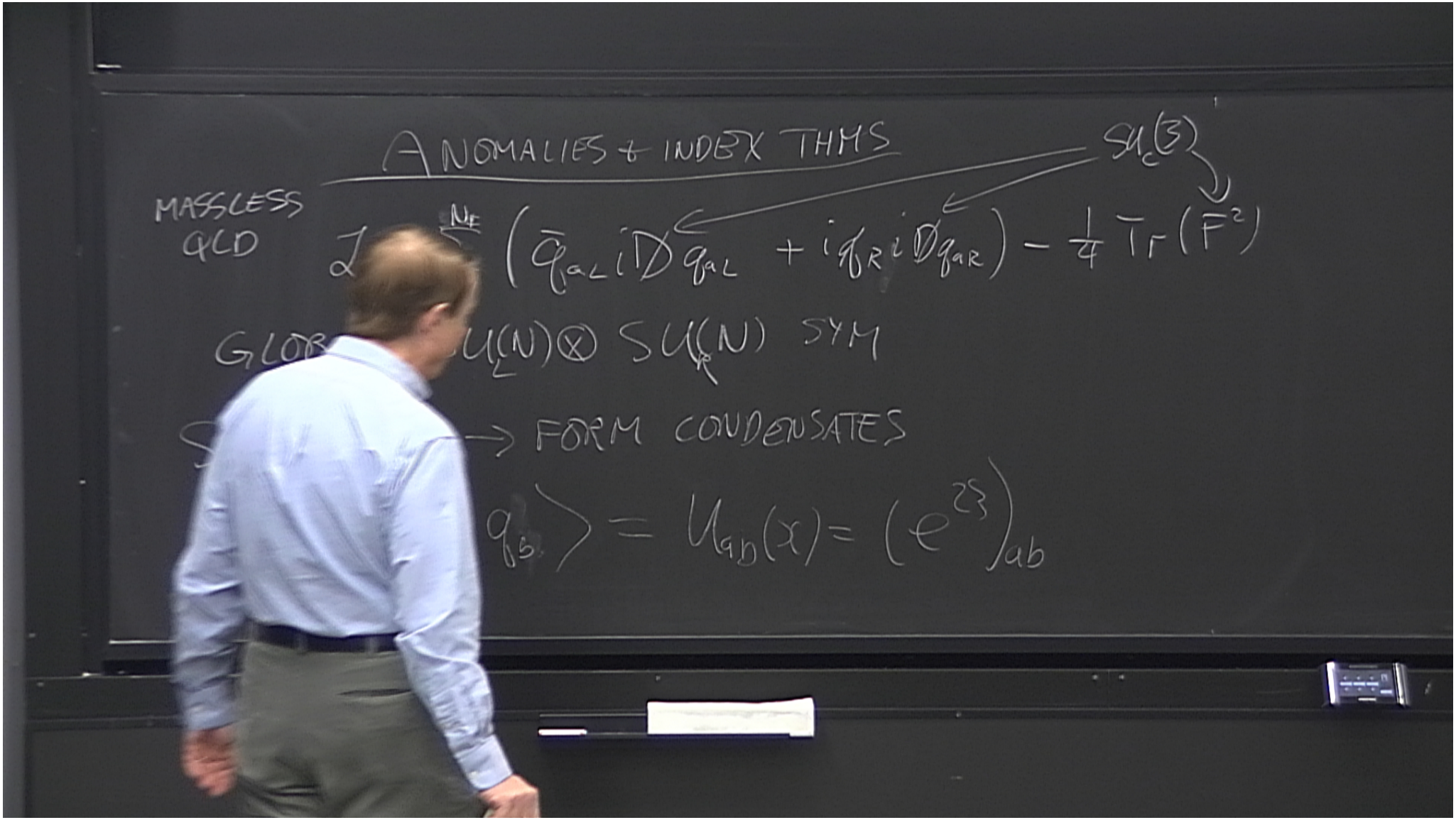
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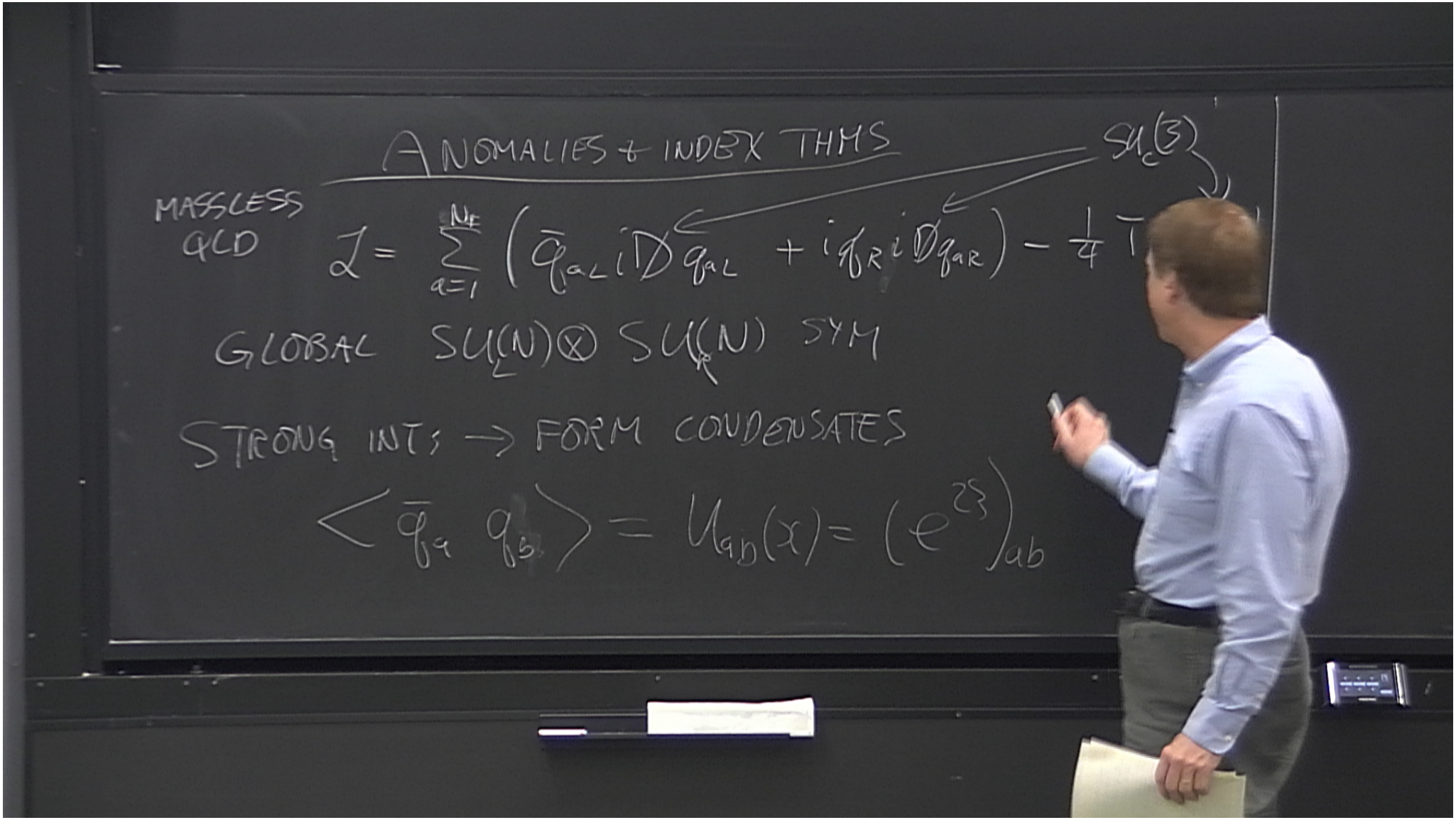
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Abstract:


ANOMALIES & INDEX THMS
MASSLESS
QCD
$\sum_{q=1}^{N_F}$


ANOMALIES & INDEX THMS
MASSLESS QCD
$\mathcal{L}$
$(\bar{q}_{aL} i D q_{aL} + i q_{aR} i D q_{aR}) - \frac{1}{4} \mathrm{Tr}(F^2)$
$SU_c(3)$
GLOBA
$(N) \otimes SU_R(N)$ SYM
STR
ORM CONDENSATES


ANOMALIES & INDEX THMS
MASSLESS QCD
$\mathcal{L} \overset{N_F}{=} (\bar{q}_{aL} i D q_{aL} + i q_{aR} i D q_{aR}) - \frac{1}{4} \mathrm{Tr}(F^2)$
$SU_c(3)$
GLOB $U_L(N) \otimes SU_R(N)$ SYM
→ FORM CONDENSATES
$\langle q_b \rangle = U_{ab}(x) = (e^{2\pi})_{ab}$


ANOMALIES & INDEX THMS
$SU_c(3)$
MASSLESS QCD
$\mathcal{L} = \sum_{a=1}^{N_F} \left(\bar{q}_{aL} i D q_{aL} + i q_{aR} i D q_{aR} \right) - \frac{1}{4} T$
GLOBAL $SU_L(N) \otimes SU_R(N)$ SYM
STRONG INTS $\rightarrow$ FORM CONDENSATES
$\langle \bar{q}_a q_b \rangle = U_{ab}(x) = (e^{2\beta})_{ab}$

ANOMALIES & INDEX THMS

MASSLESS QCD

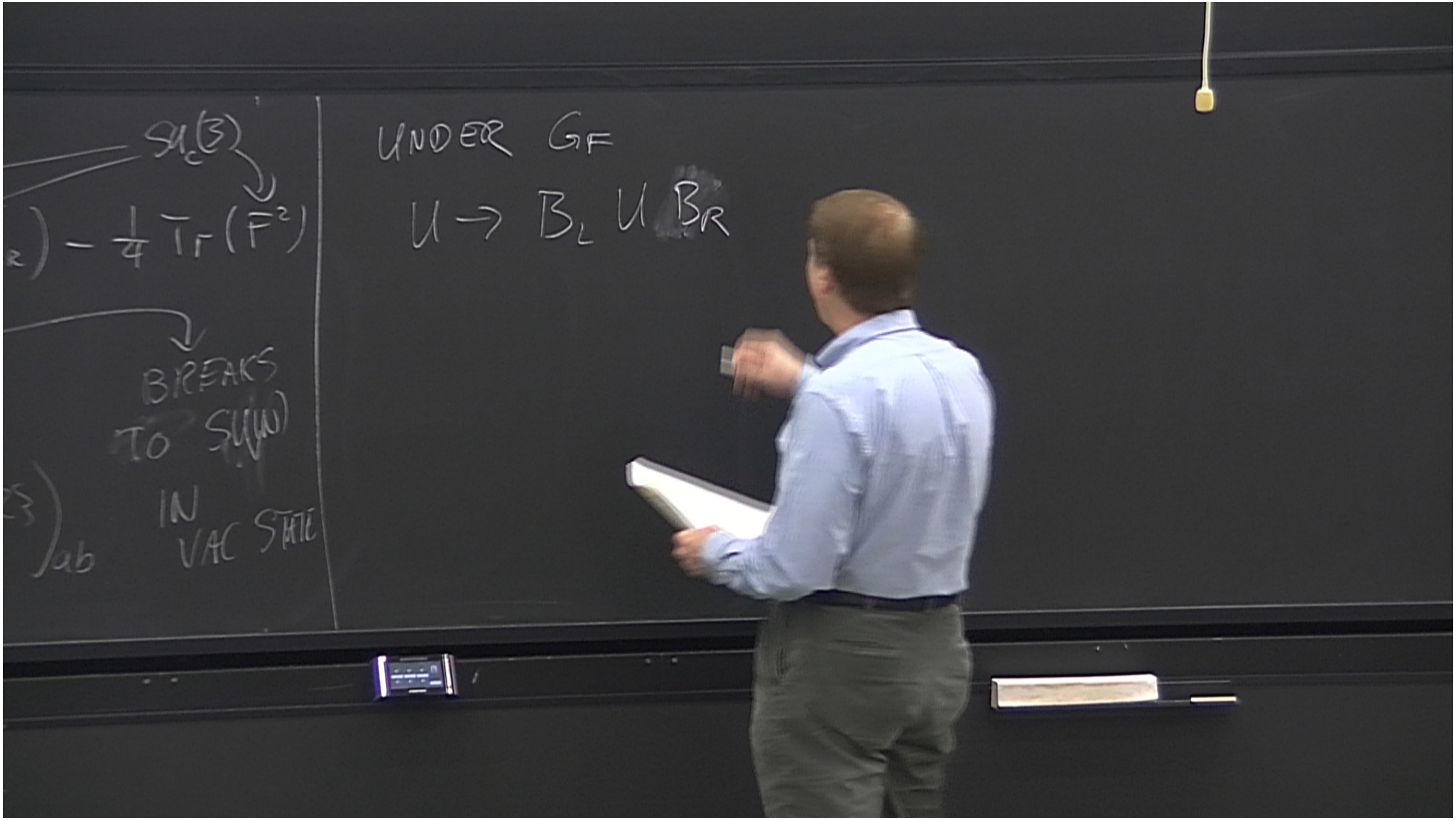
$$\mathcal{L} = \sum_{a=1}^{N_F} \left(\bar{q}_{aL} i D q_{aL} + i q_{aR} i D q_{aR} \right) - \frac{1}{4} \mathrm{Tr}(F^2)$$

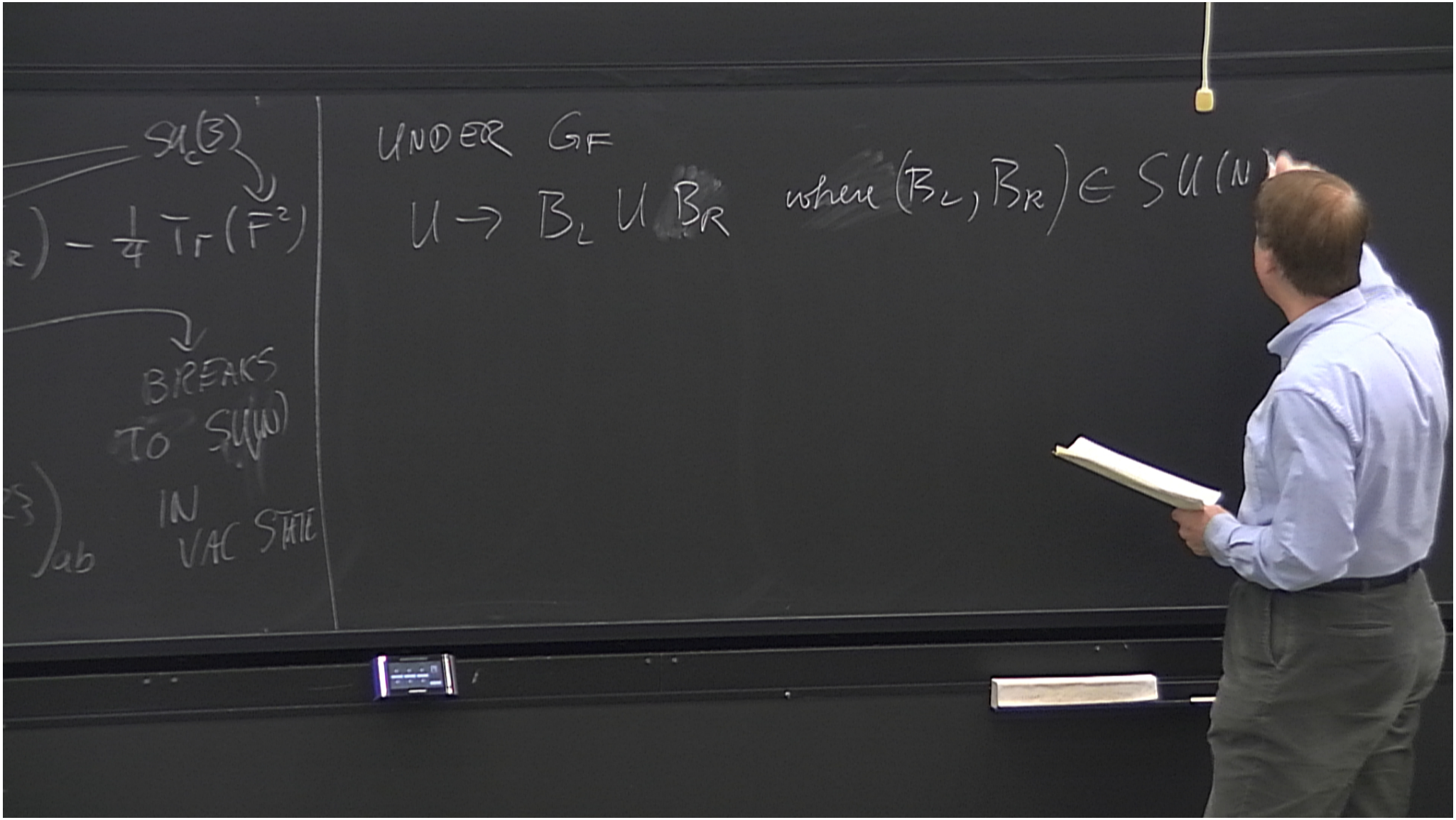
$SU_C(3)$

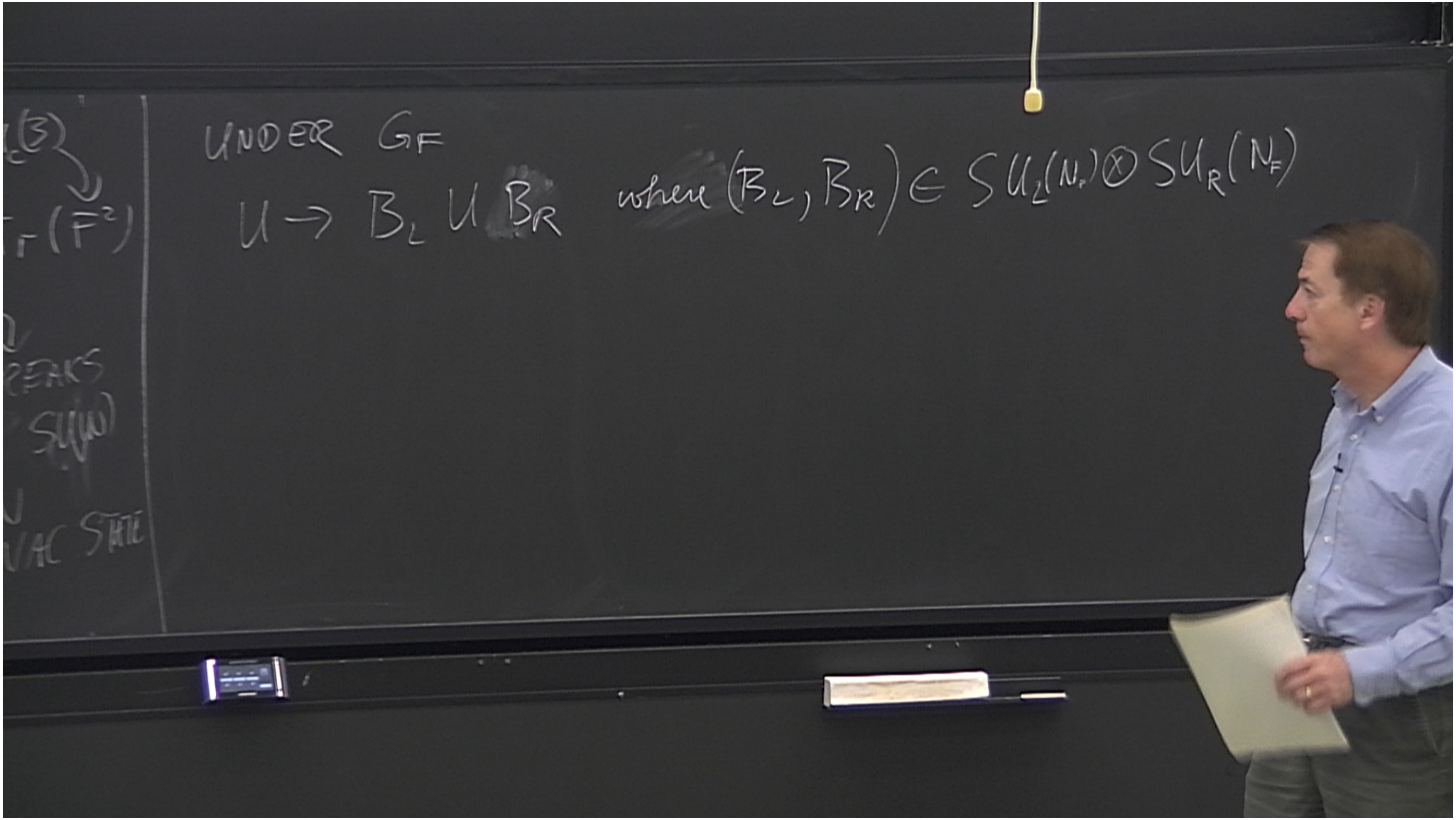
GLOBAL $SU_L(N) \otimes SU_R(N)$ SYM → BREAKS TO $SU(N)$

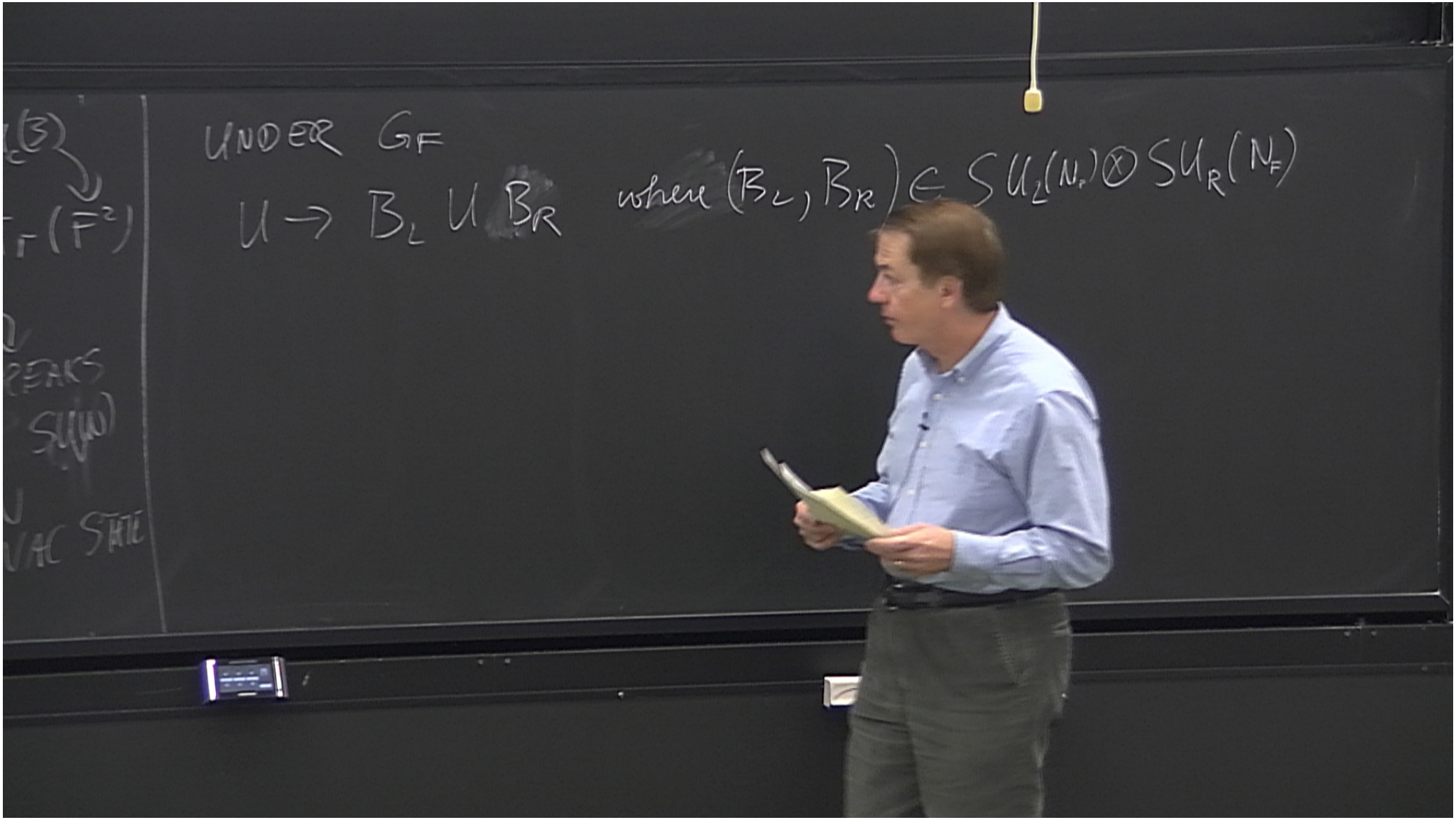
STRONG INTS → FORM CONDENSATES IN VAC STATE

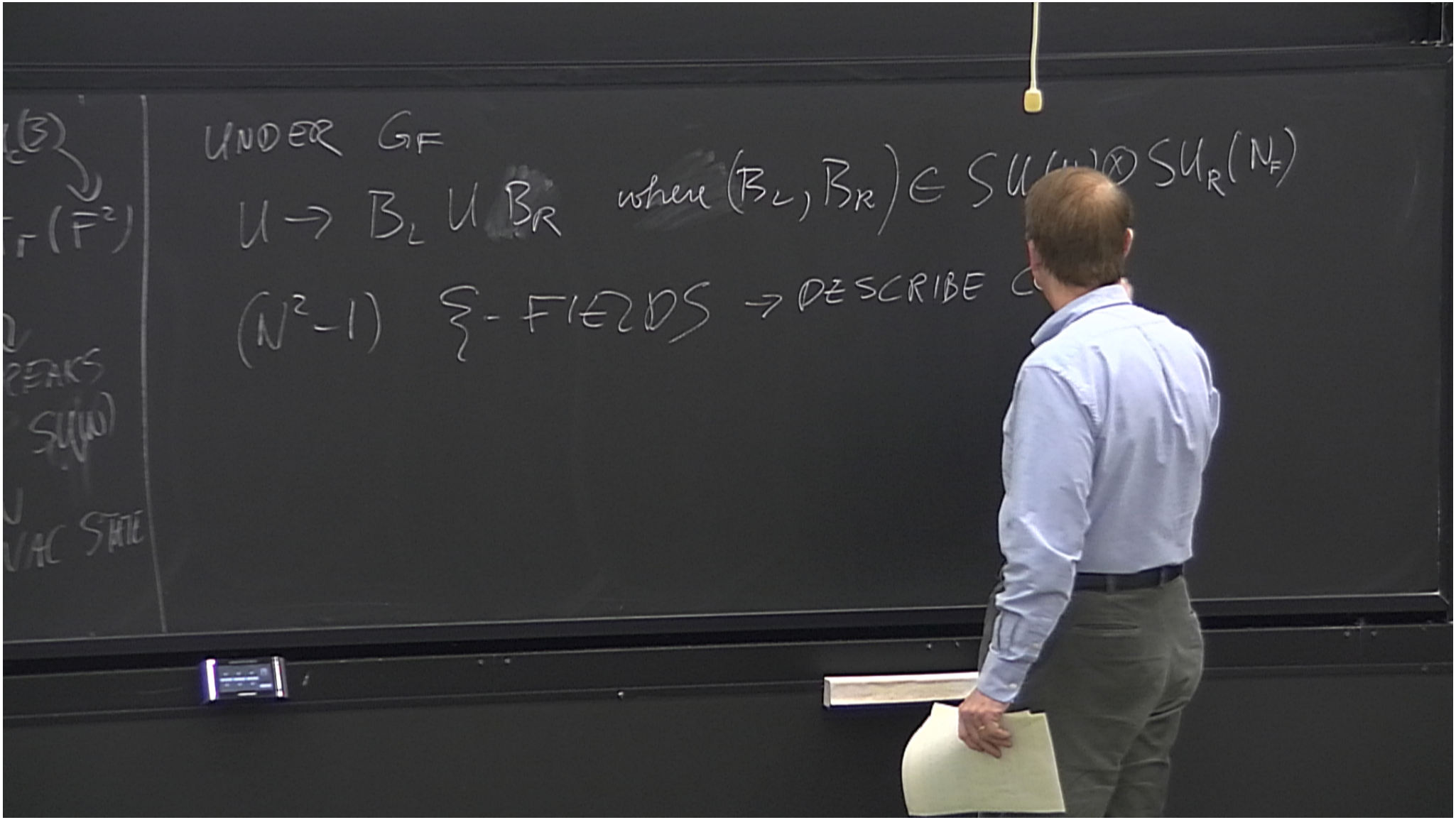
$$\langle \bar{q}_a q_b \rangle = U_{ab}(x) = (e^{i\beta})_{ab}$$

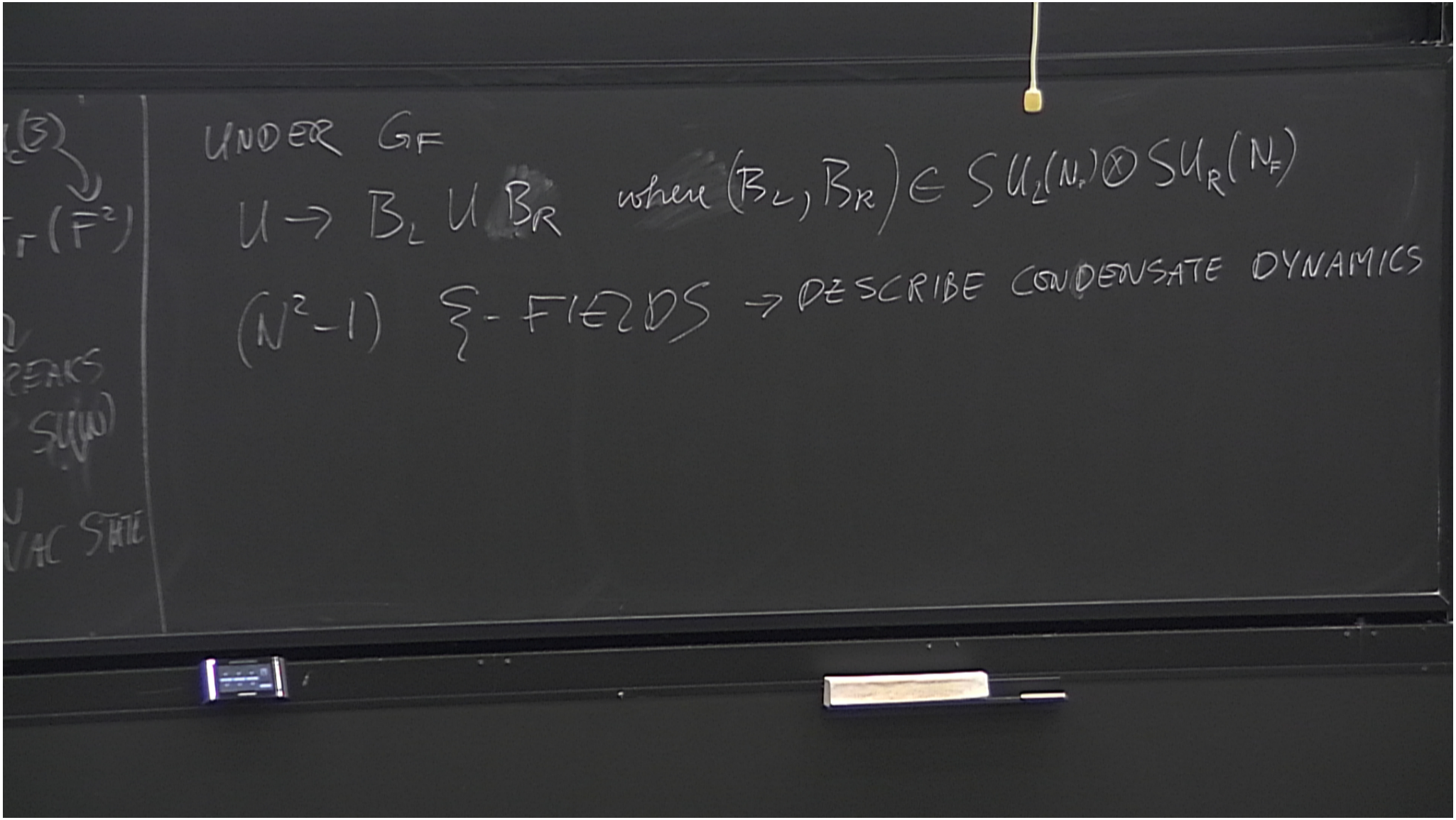

$SU_c(3)$
$-\frac{1}{4} T_r(F^2)$
BREAKS
TO $SU(N)$
IN
VAC STATE
UNDER GF
$U \to B_L U B_R$

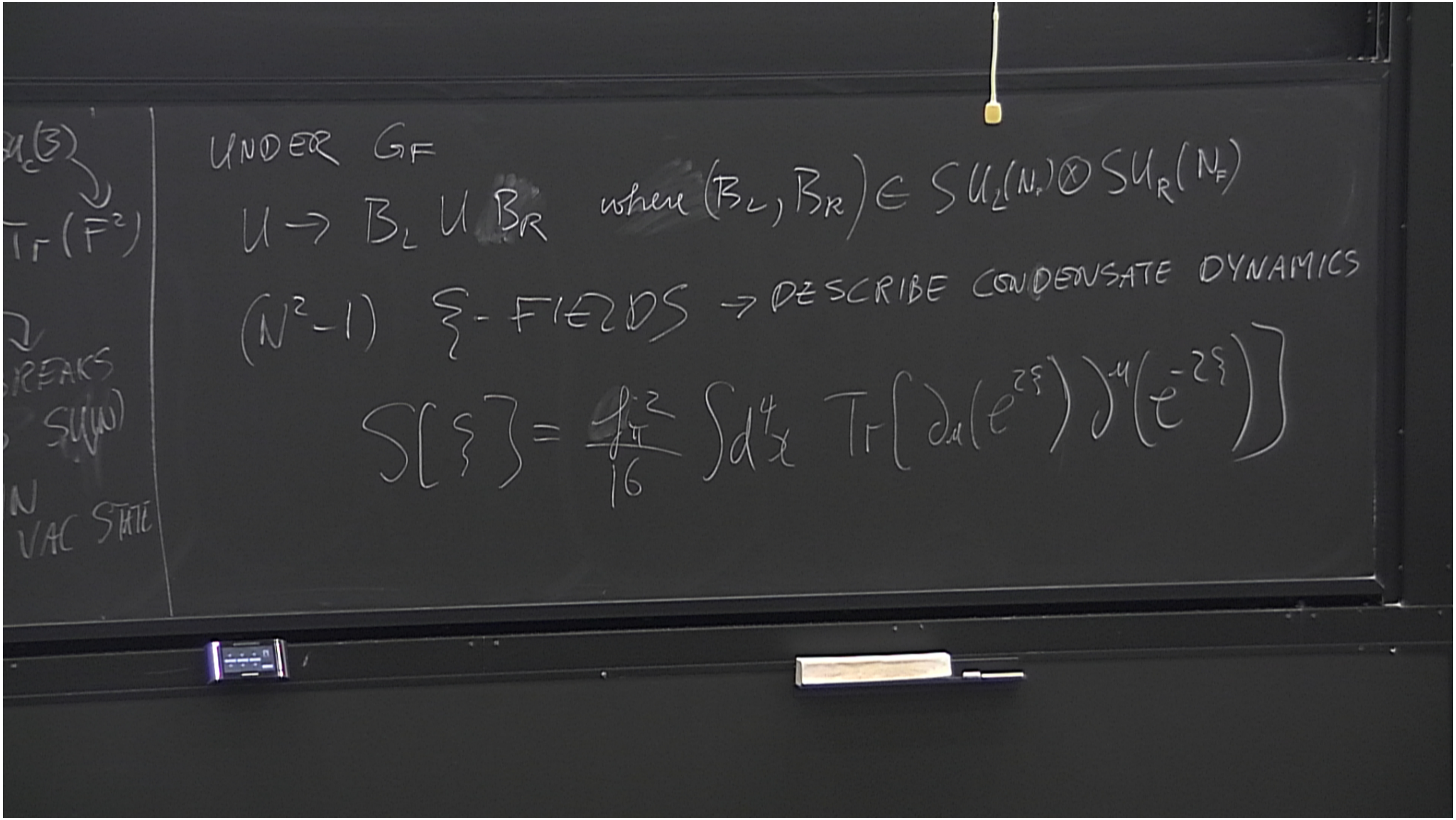

UNDER GF
$U \to B_L U B_R$ where $(B_L, B_R) \in SU(N)$
BREAKS TO SU(N)
IN VAC STATE

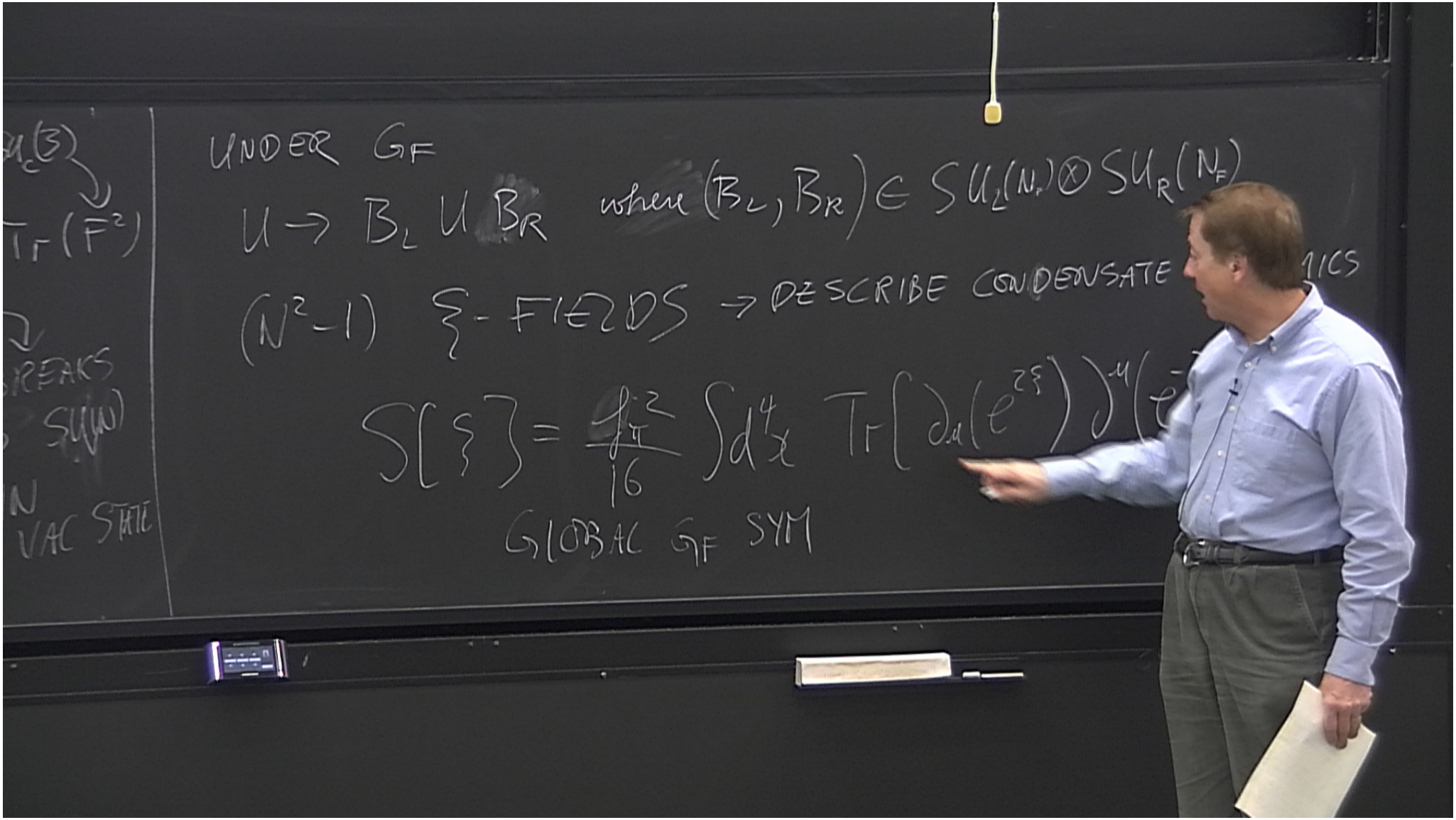

UNDER G_F
$U \to B_L U B_R$ where $(B_L, B_R) \in SU_L(N_F) \otimes SU_R(N_F)$

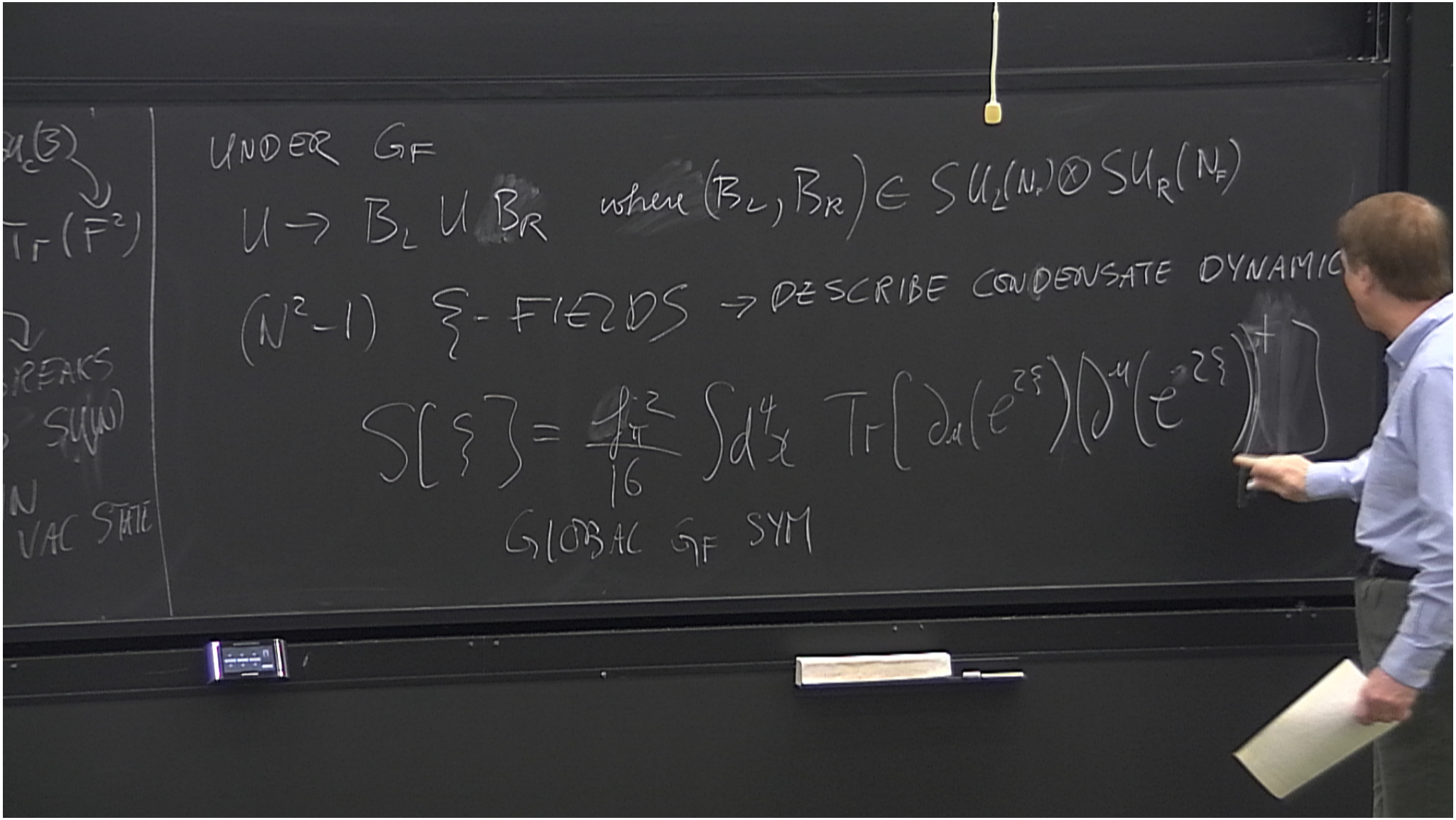

UNDER G_F
$U \to B_L U B_R^\dagger$ where $(B_L, B_R) \in SU_L(N_F) \otimes SU_R(N_F)$

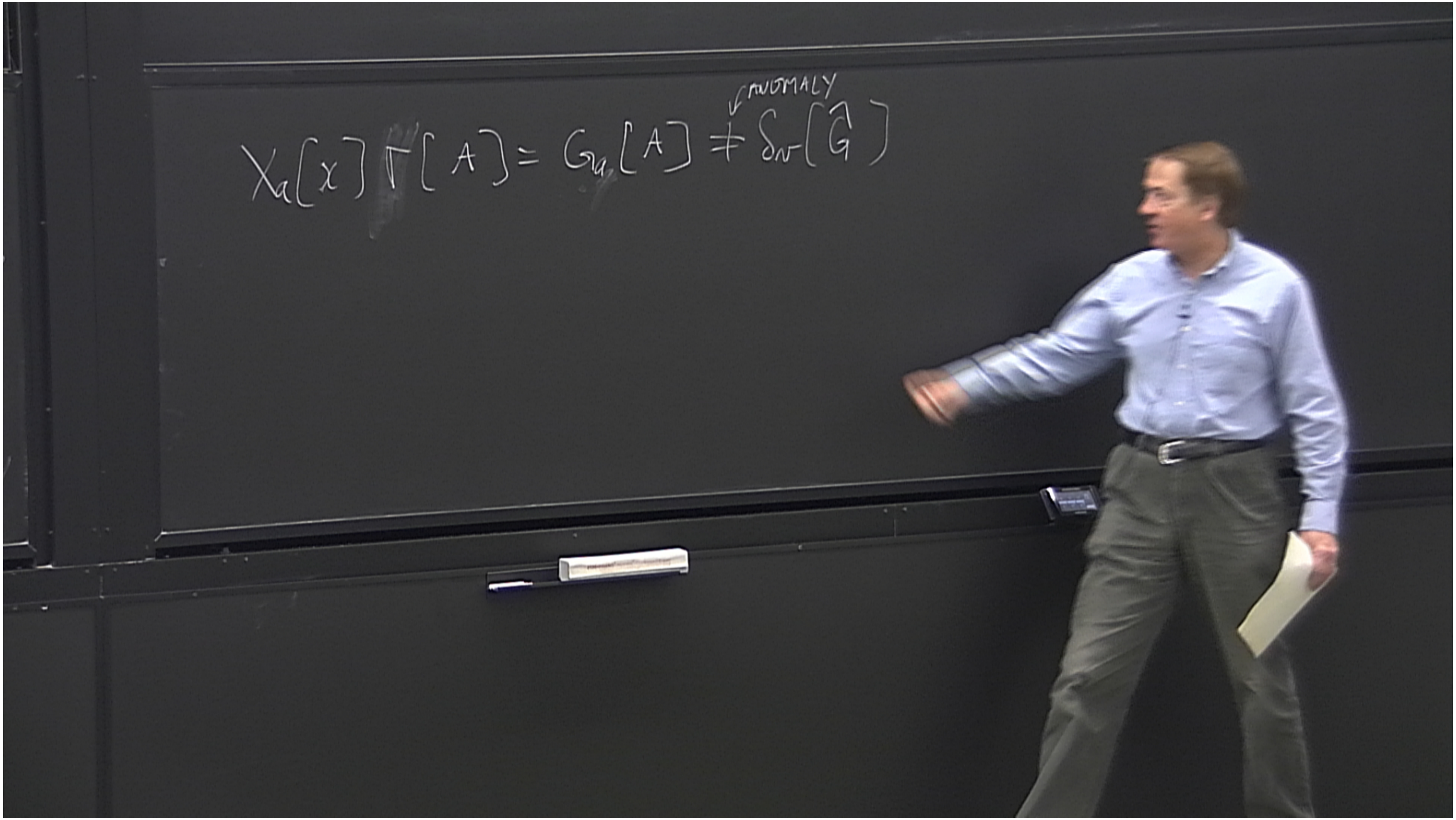

UNDER GF
$U \to B_L U B_R$ where $(B_L, B_R) \in SU$
(N^2-1) ξ-FIELDS → DESCRIBE
PEAKS
SU(N)
VAC STATE

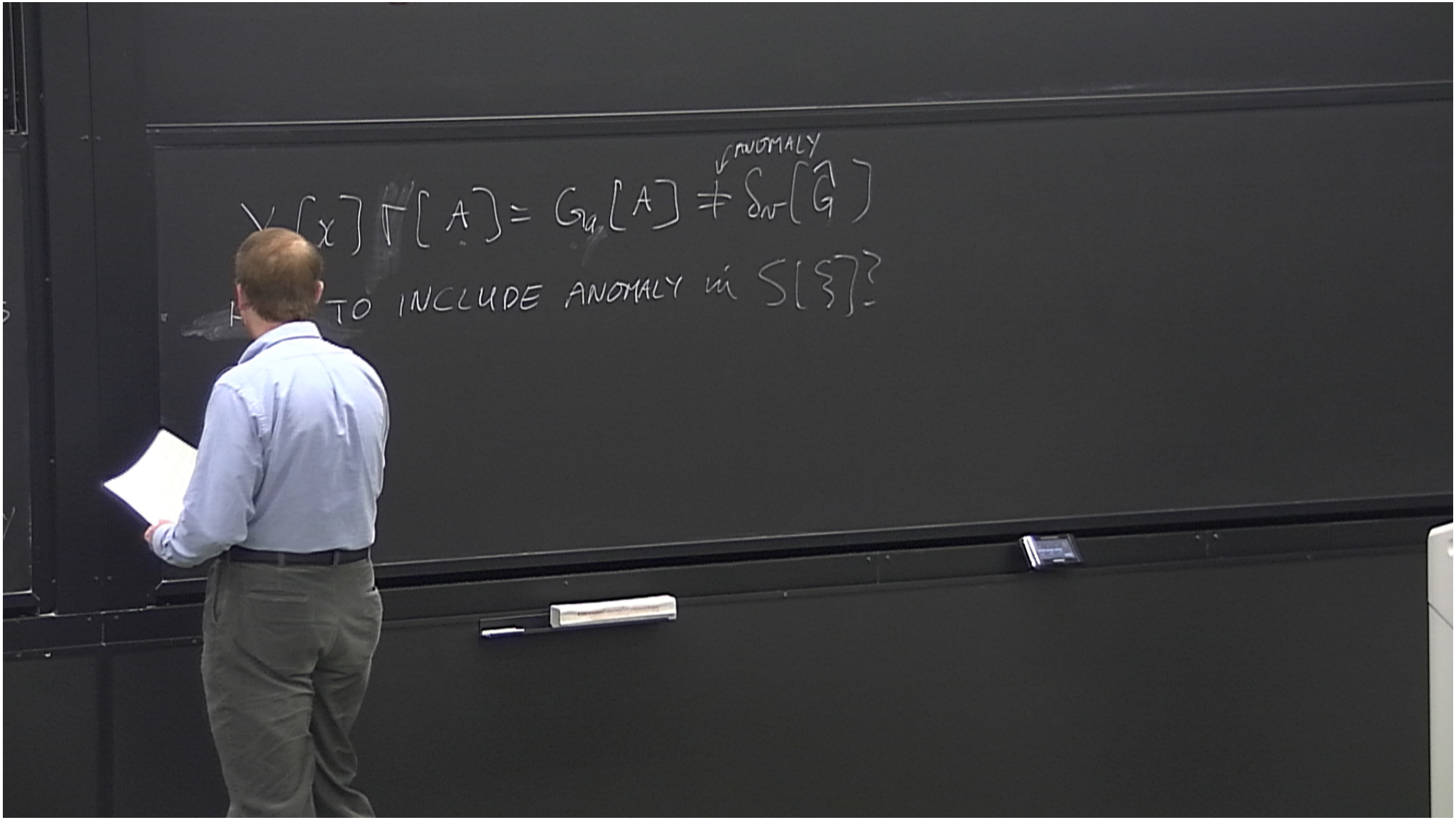

UNDER G_F
$U \to B_L U B_R^\dagger$ where $(B_L, B_R) \in SU_L(N_F) \otimes SU_R(N_F)$
(N^2-1) ξ-FIELDS → DESCRIBE CONDENSATE DYNAMICS

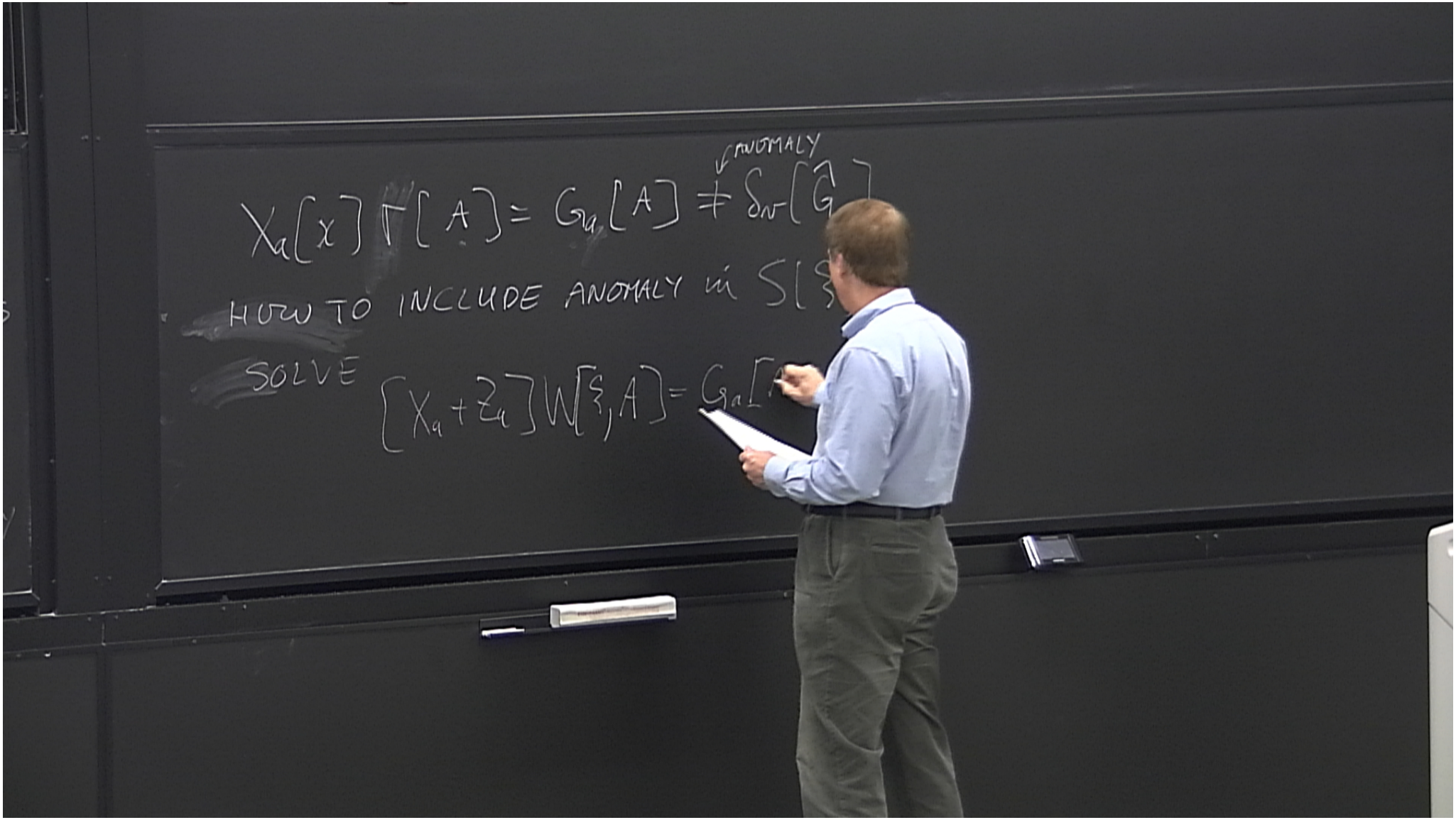

UNDER GF
$U \to B_L U B_R$ where $(B_L, B_R) \in SU_L(N_F) \otimes SU_R(N_F)$
(N^2-1) ξ-FIELDS → DESCRIBE CONDENSATE DYNAMICS
$S[\xi] = \frac{f_\pi^2}{16} \int d^4x \, Tr\left[\partial_\mu (e^{2\xi}) \partial^\mu (e^{-2\xi})\right]$

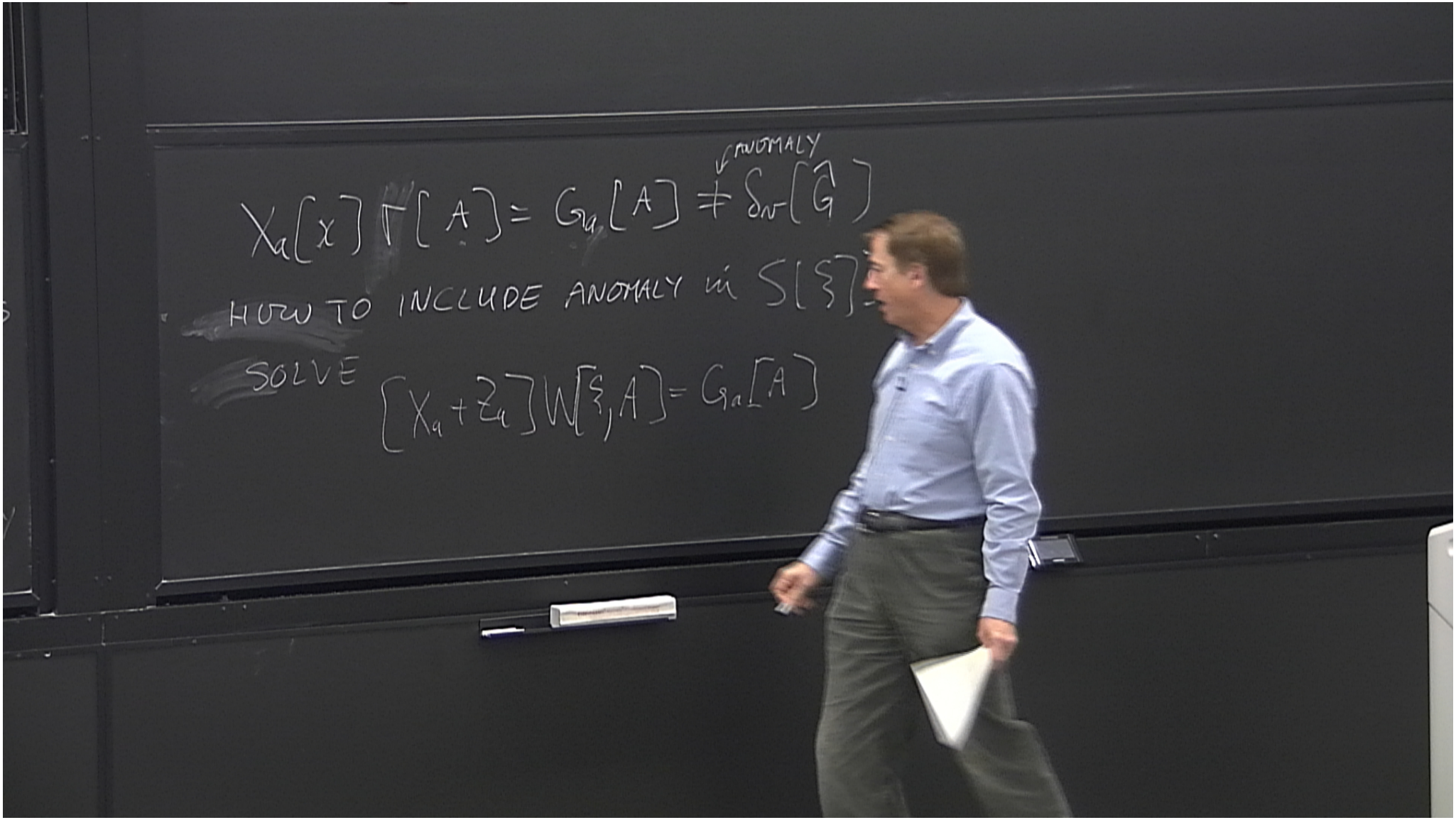

UNDER G_F
$U \to B_L U B_R$ where $(B_L, B_R) \in SU_L(N_F) \otimes SU_R(N_F)$
(N^2-1) ξ-FIELDS → DESCRIBE CONDENSATE
$S[\xi] = \frac{f_\pi^2}{16} \int d^4x \, \mathrm{Tr}(\partial_\mu (e^{2\xi}))^4 (e^{-}$
GLOBAL G_F SYM
$SU_c(3)$
$\mathrm{Tr}(F^2)$
BREAKS
$SU(N)$
VAC STATE

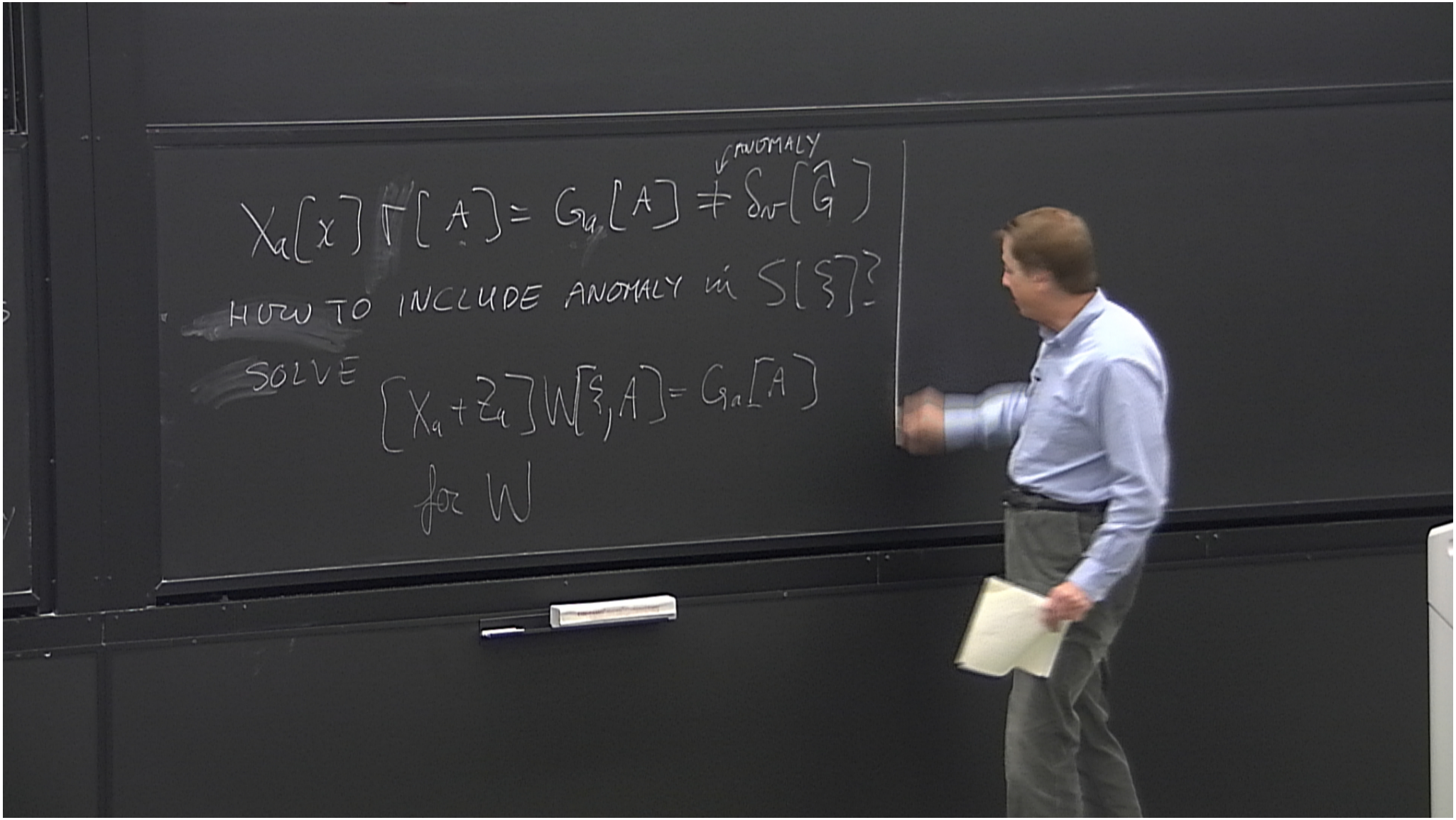

UNDER G_F
$U \to B_L U B_R$ where $(B_L, B_R) \in SU_L(N_F) \otimes SU_R(N_F)$
(N^2-1) ξ-FIELDS → DESCRIBE CONDENSATE DYNAMIC
$S[\xi] = \frac{f_\pi^2}{16} \int d^4x \, Tr\left[\partial_\mu(e^{2\xi})(\partial^\mu(e^{2\xi}))^\dagger\right]$
GLOBAL G_F SYM

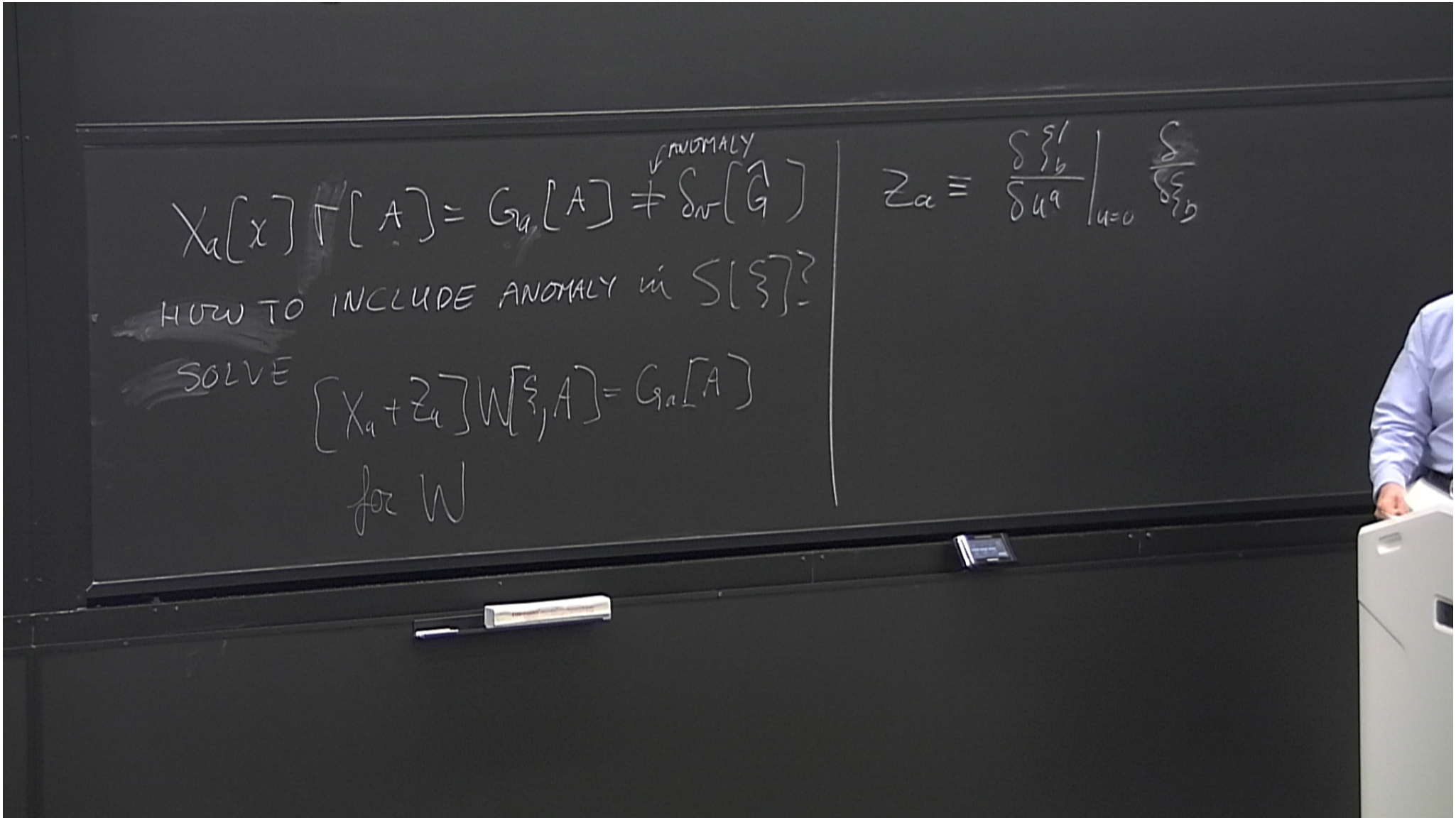

$X_a[x]\,\Gamma[A] = G_a[A] \neq S_{NI}[\Gamma]$
ANOMALY

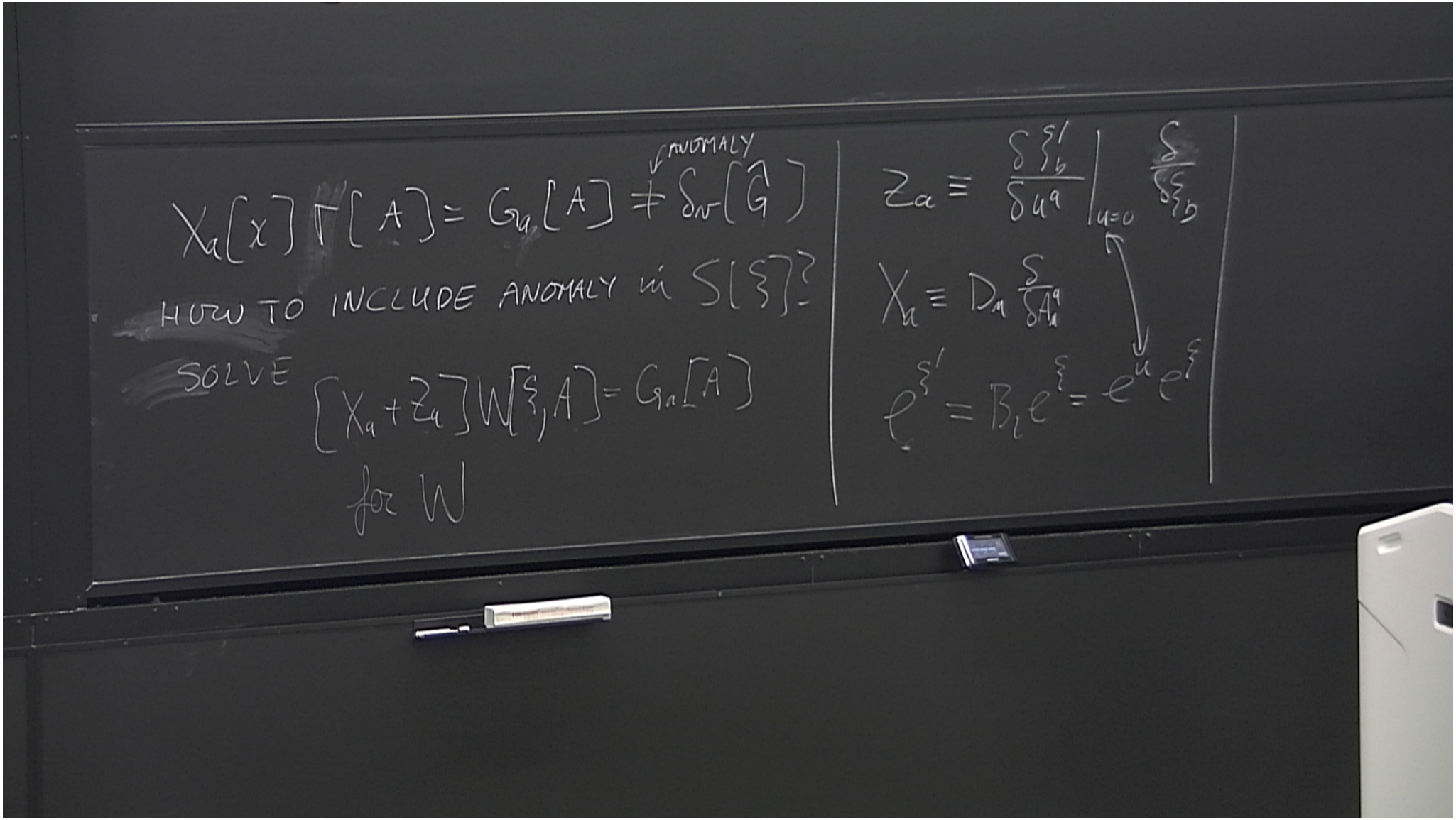

ANOMALY
TO INCLUDE ANOMALY in S[S]?

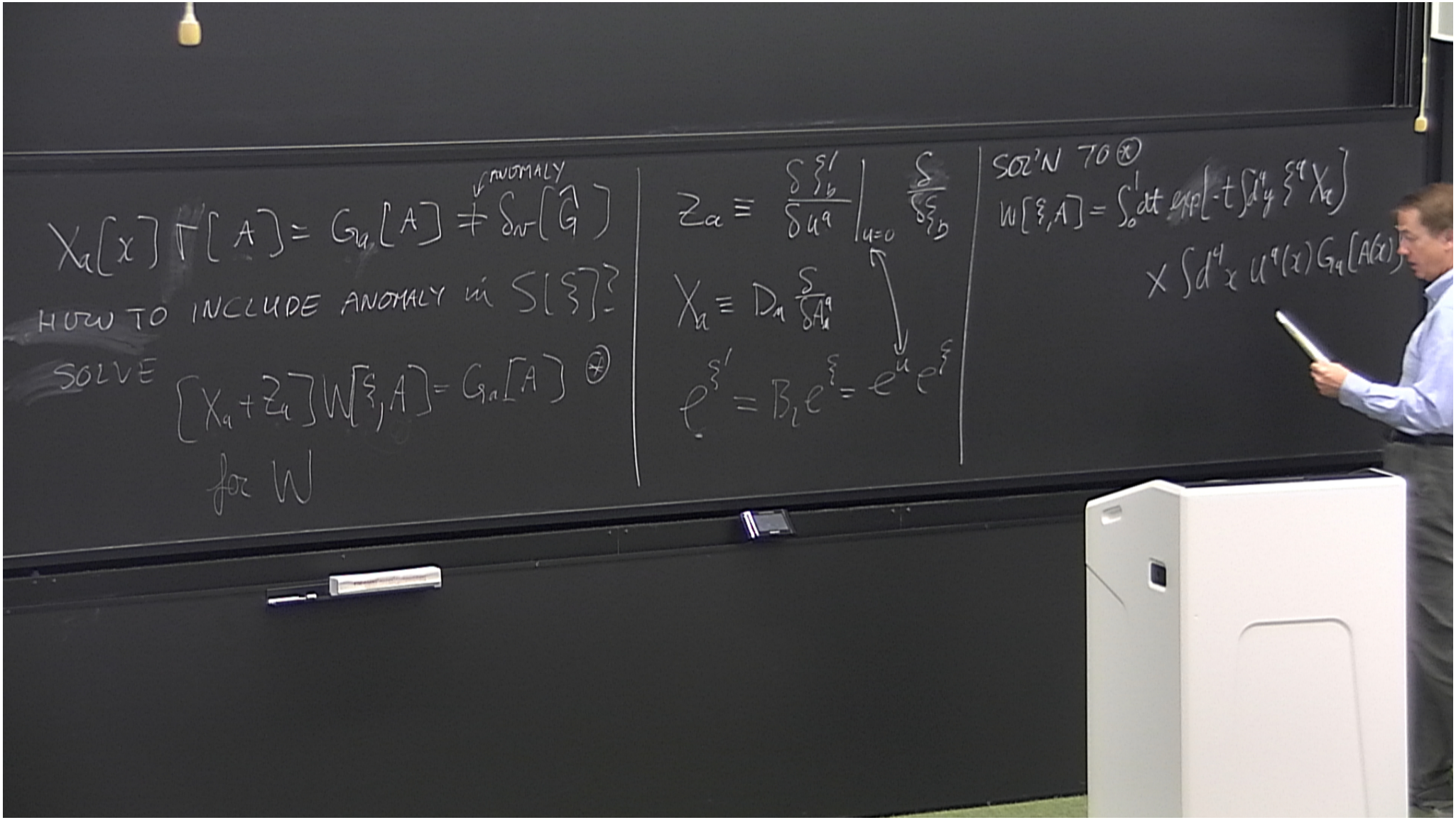

$X_a[x]\,\Gamma[A] = G_a[A] \neq \delta_W[\hat{G}]$
ANOMALY
HOW TO INCLUDE ANOMALY in S[
SOLVE
$[X_a + Z_a]\,W[\xi, A] = G_a[$

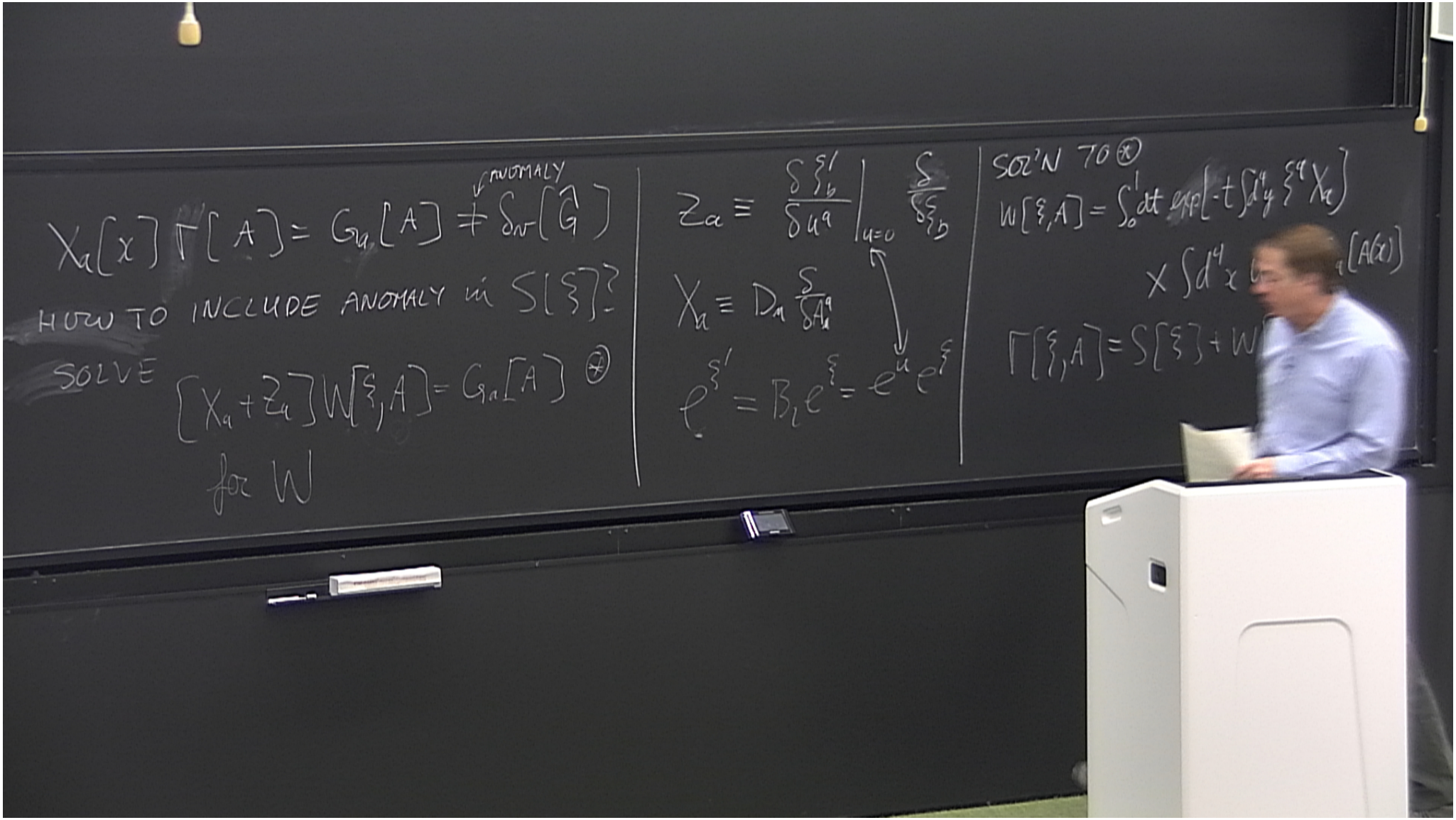

$X_a[x]\,\Gamma[A] = G_a[A] \neq \delta_W[\hat{G}]$
ANOMALY
HOW TO INCLUDE ANOMALY in $S[\xi]$
SOLVE
$[X_a + Z_a]\,W[\xi, A] = G_a[A]$

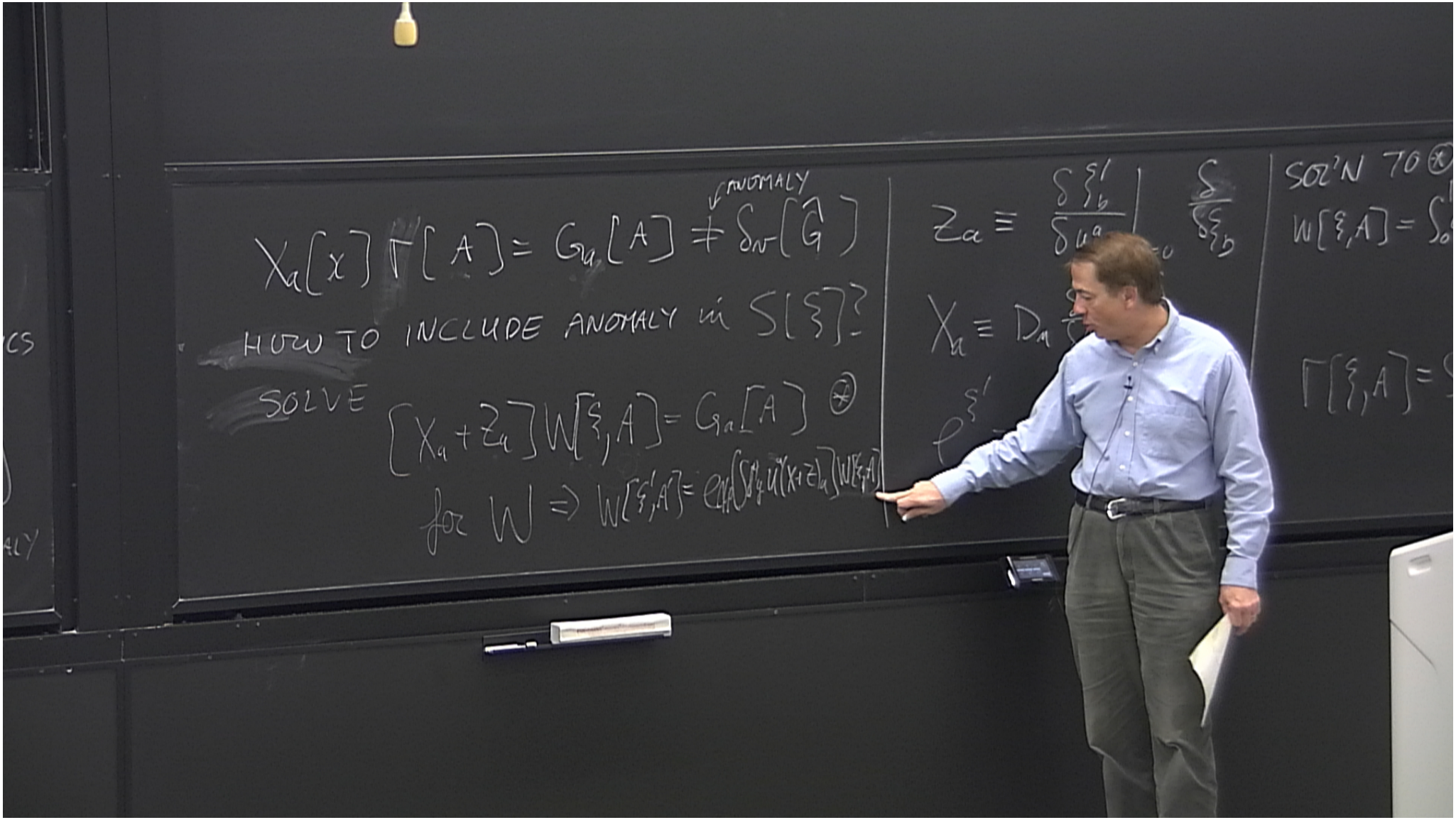

$X_a[x]\,\Gamma[A] = G_a[A] \neq \delta_W[\hat{G}]$
ANOMALY
HOW TO INCLUDE ANOMALY in $S[\xi]$?
SOLVE
$[X_a + Z_a]\,W[\xi, A] = G_a[A]$
for W

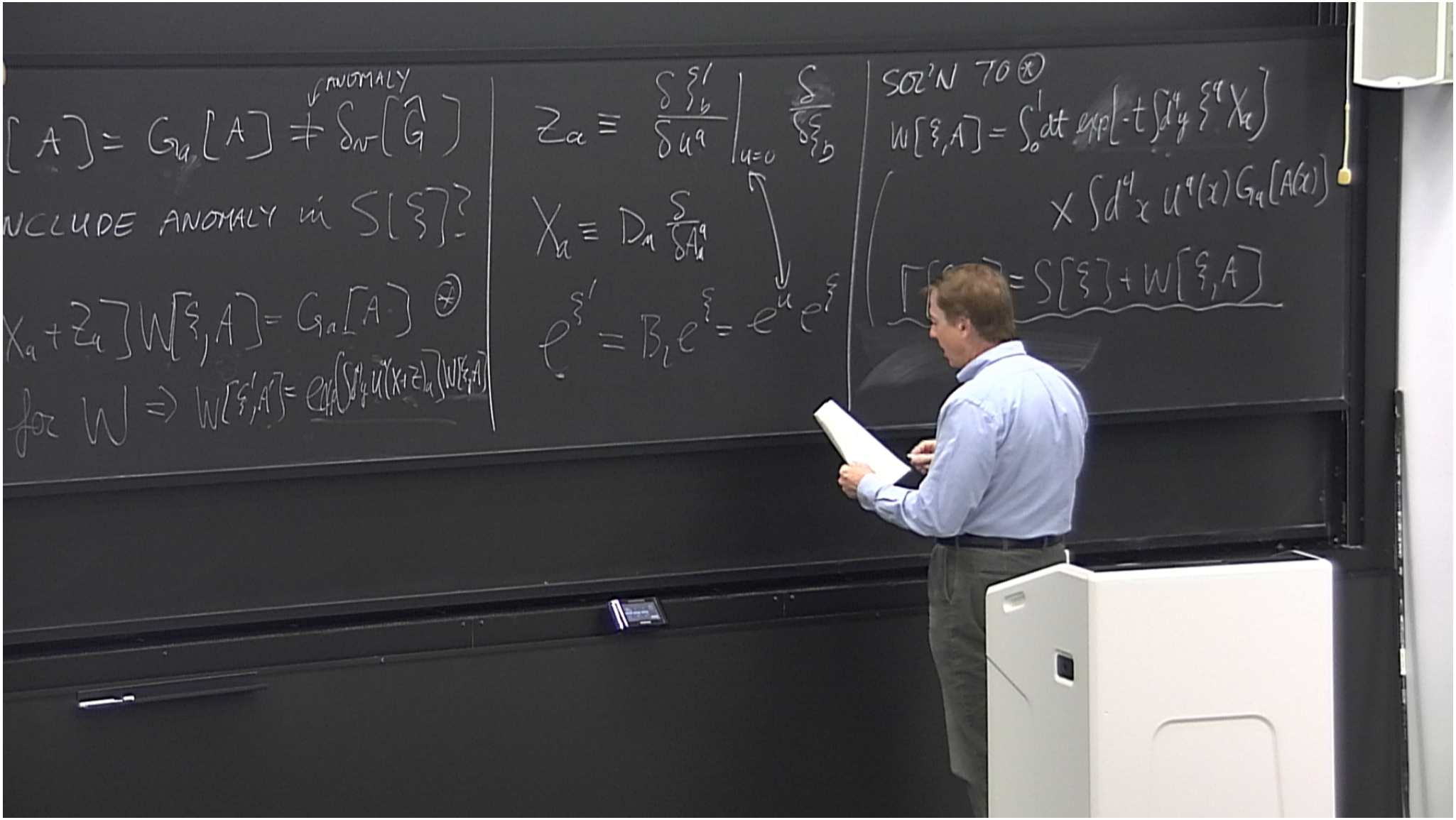

$X_a[x]\,\Gamma[A] = G_a[A] \neq \delta_{NV}[\hat{G}]$
ANOMALY
$Z_a \equiv \left.\frac{\delta \xi'_b}{\delta u^a}\right|_{u=0} \frac{\delta}{\delta \xi_b}$
HOW TO INCLUDE ANOMALY in $S[\xi]$?
SOLVE $[X_a + Z_a]\,W[\xi, A] = G_a[A]$
for W

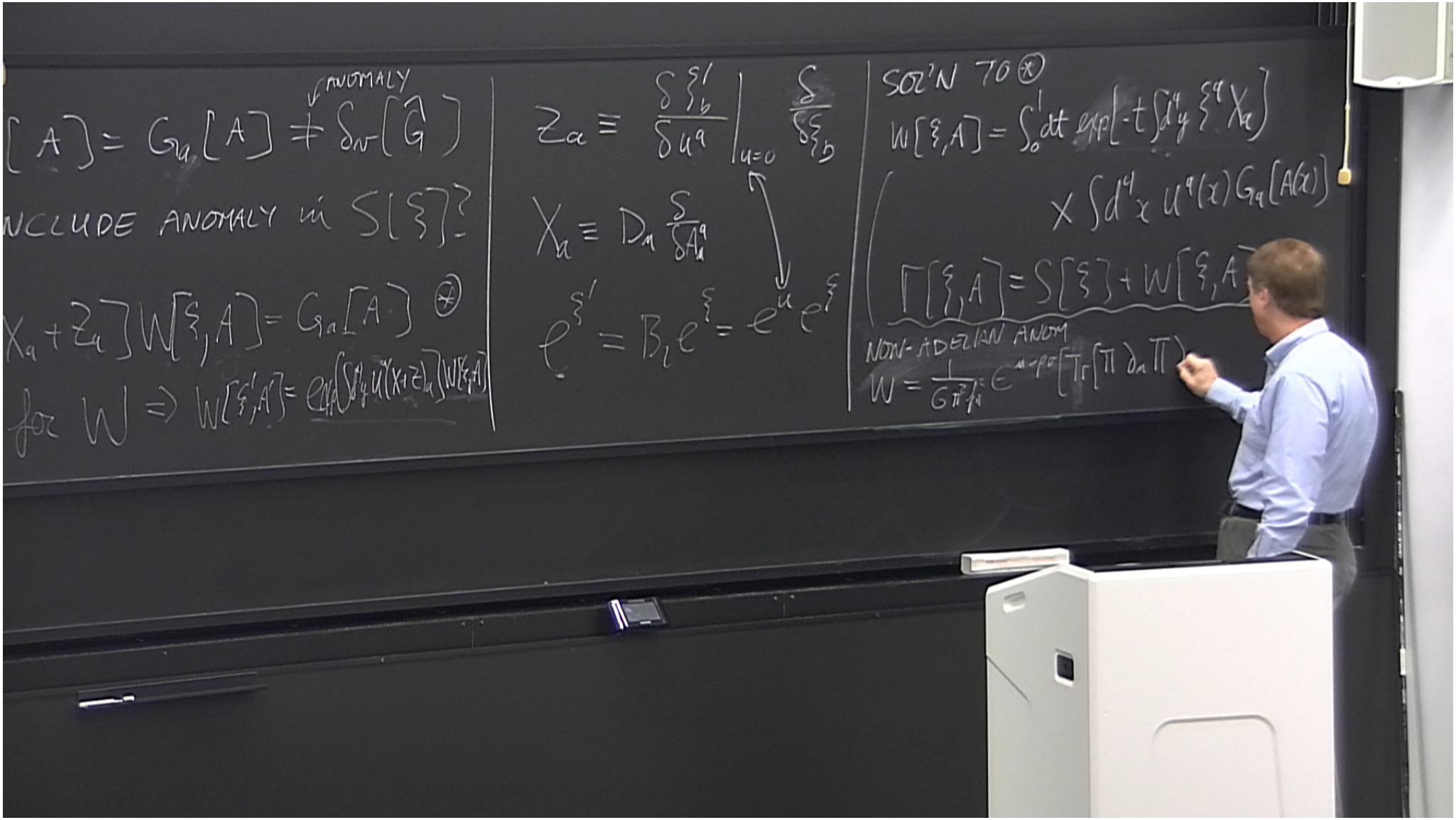

$X_a[x]\,\Gamma[A] = G_a[A] \neq \delta_{NV}[\hat{G}]$
ANOMALY
HOW TO INCLUDE ANOMALY in $S[\xi]$?
SOLVE
$[X_a + Z_a]\,W[\xi, A] = G_a[A]$
for W
$Z_a \equiv \frac{\delta \xi'_b}{\delta u^a}\Big|_{u=0} \frac{\delta}{\delta \xi_b}$
$X_a \equiv D_a \frac{\delta}{\delta A^a_a}$
$e^{\xi'} = B_\xi e^{\xi} = e^{u} e^{\xi}$

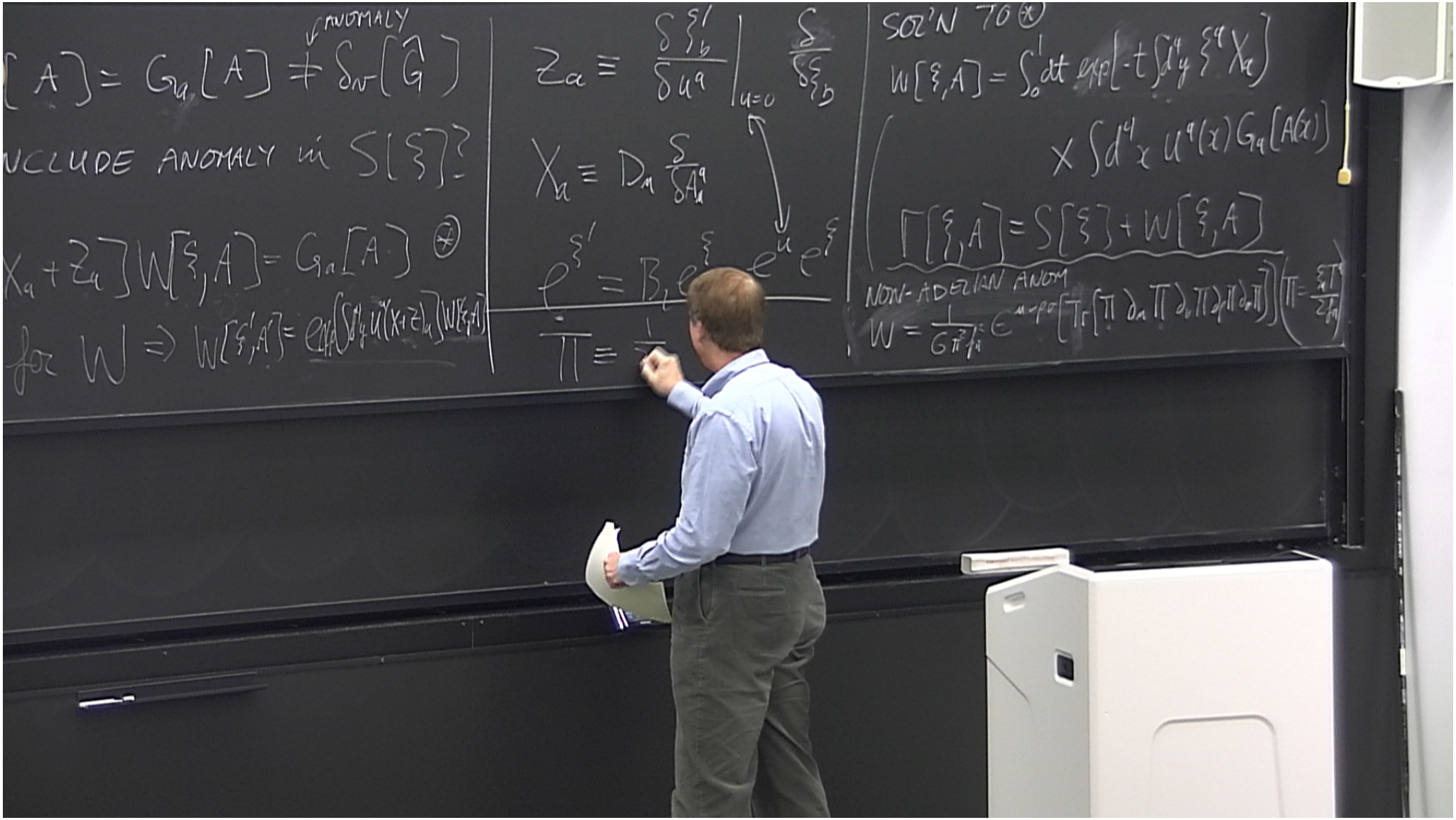

$X_a[x]\,\Gamma[A] = G_a[A] \neq \delta_{NI}[\hat{G}]$
ANOMALY
HOW TO INCLUDE ANOMALY in $S[\xi]$?
SOLVE
$[X_a + Z_a]\,W[\xi, A] = G_a[A]$ ⊛
for W
$Z_a \equiv \frac{\delta \xi'_b}{\delta u^a}\Big|_{u=0} \frac{\delta}{\delta \xi_b}$
$X_a \equiv D_m \frac{\delta}{\delta A^a_m}$
$e^{\xi'} = B_\xi e^{\xi} = e^{u} e^{\xi}$
SOL'N TO ⊛
$W[\xi, A] = \int_0^1 dt \exp\left[-t \int d^4y\, \xi^a X_a\right]$
$\times \int d^4x\, u^a(x)\, G_a[A(x)]$

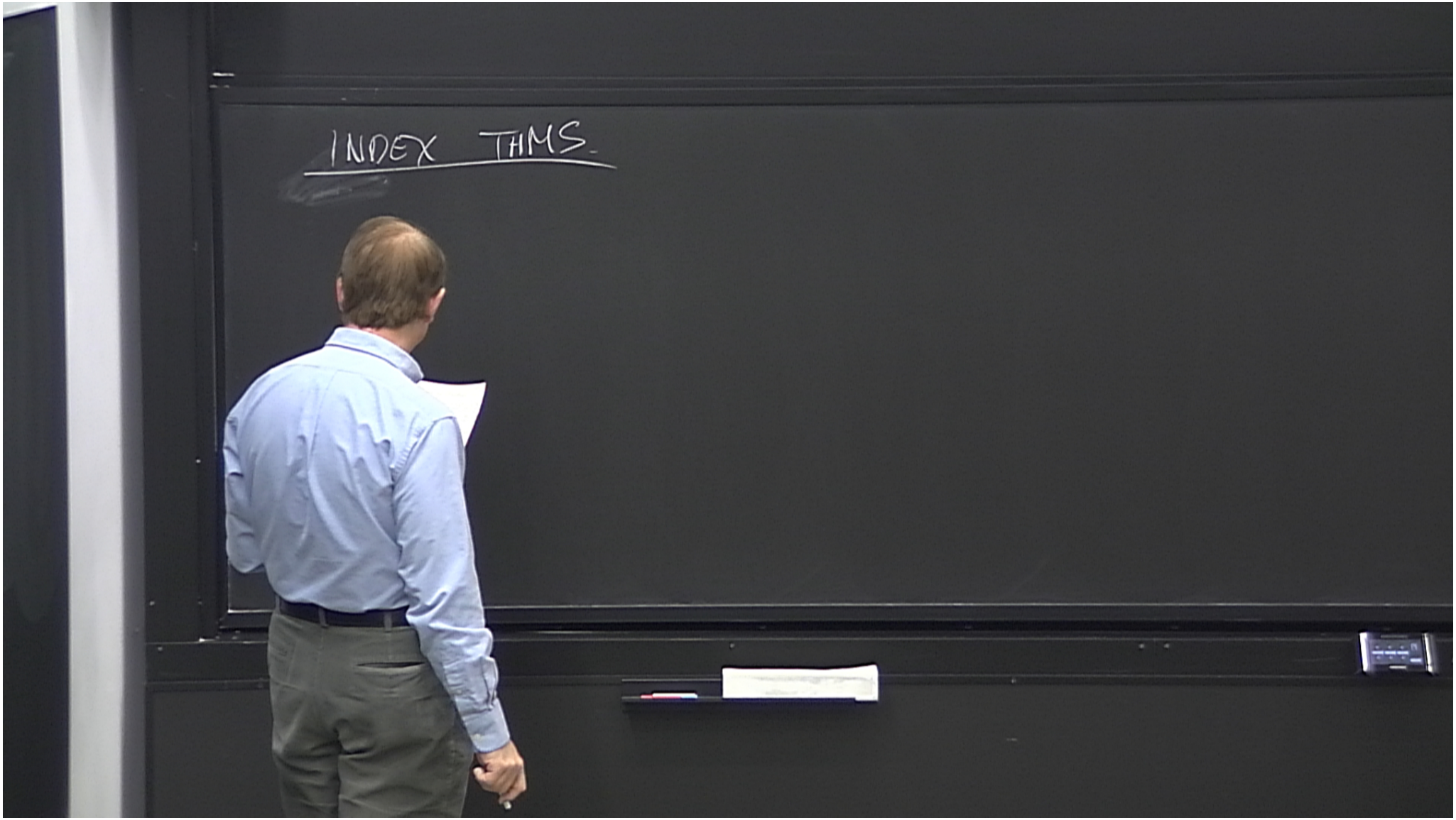

$X_a[x]\,\Gamma[A] = G_a[A] \neq \delta_{NV}[\hat{G}]$
ANOMALY
HOW TO INCLUDE ANOMALY in $S[\xi]$?
SOLVE
$[X_a + Z_a]\,W[\xi, A] = G_a[A]$ ⊛
for W
$Z_a \equiv \frac{\delta \xi'_b}{\delta u^a}\Big|_{u=0} \frac{\delta}{\delta \xi_b}$
$X_a \equiv D_m \frac{\delta}{\delta A^a_m}$
$e^{\xi'} = B_\xi e^{\xi} = e^{u} e^{\xi}$
SOL'N TO ⊛
$W[\xi, A] = \int_0^1 dt \exp[-t \int d^4y\, \xi^a X_a]$
$\times \int d^4x$
$\Gamma[\xi, A] = S[\xi] + W$

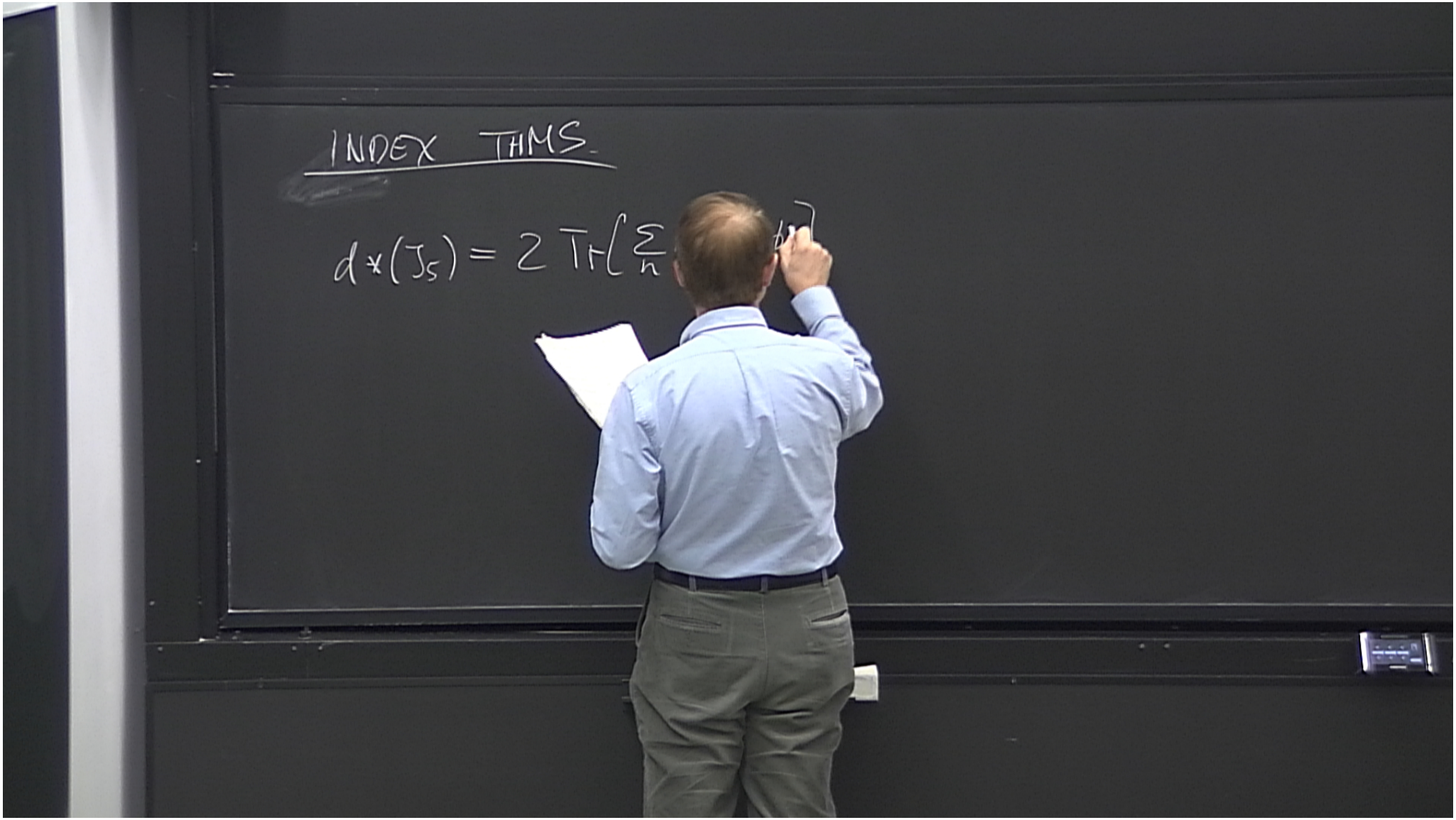

ANOMALY
HOW TO INCLUDE ANOMALY in S[ξ]?
SOLVE
for W ⇒
SO'N 70

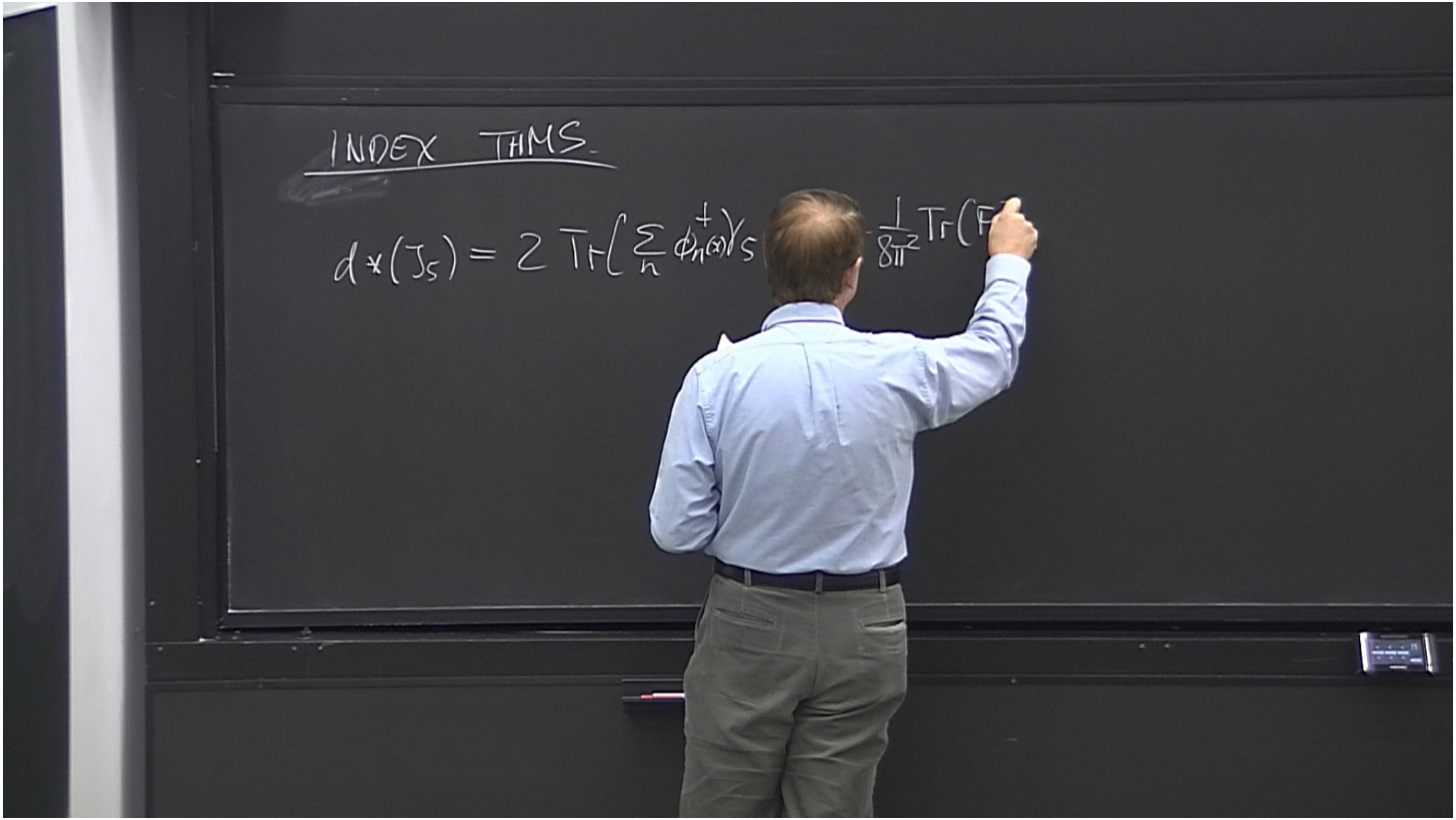

ANOMALY
SOL'N TO
INCLUDE ANOMALY in S[ξ]?
for W ⇒

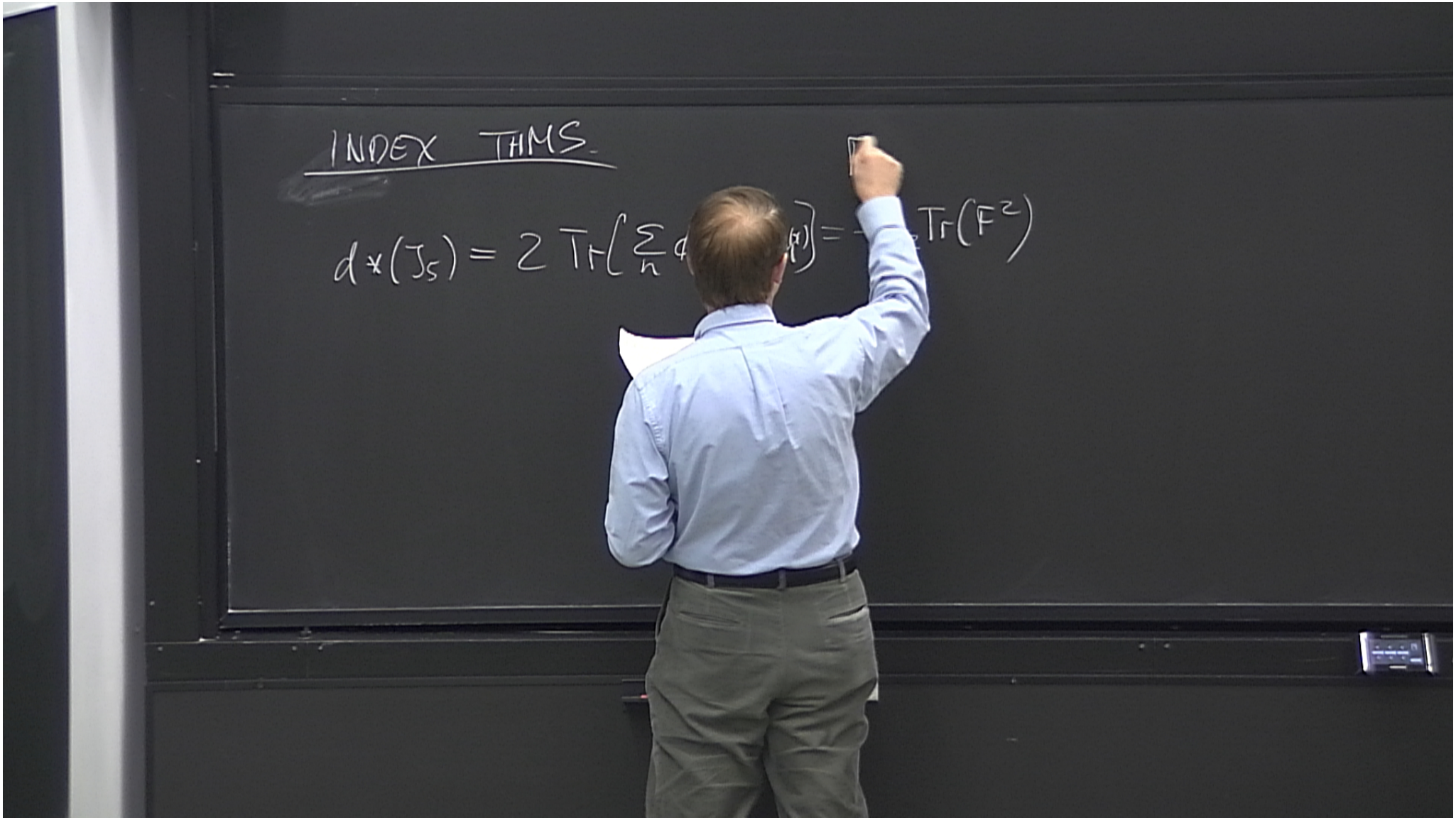

[A] = G_a[A] ≠ δ_NA[Ĝ]
ANOMALY
INCLUDE ANOMALY in S[ξ]?
Z_a ≡ δξ'_b/δu^a |_{u=0} δ/δξ_b
X_a = D_a δ/δA^a_a
e^{ξ'} = B_L e^{ξ} = e^{u} e^{ξ}
SOL'N TO ⊛
W[ξ,A] = ∫_0^1 dt exp[-t ∫d^4y ξ^a X_a]
× ∫d^4x u^a(x) G_a[A(x)]
Γ[ξ,A] = S[ξ] + W[ξ,A]
NON-ABELIAN ANOM
W = 1/(G π^2 f_π) ε^{μνρσ} [Tr[Π ∂_μ Π]

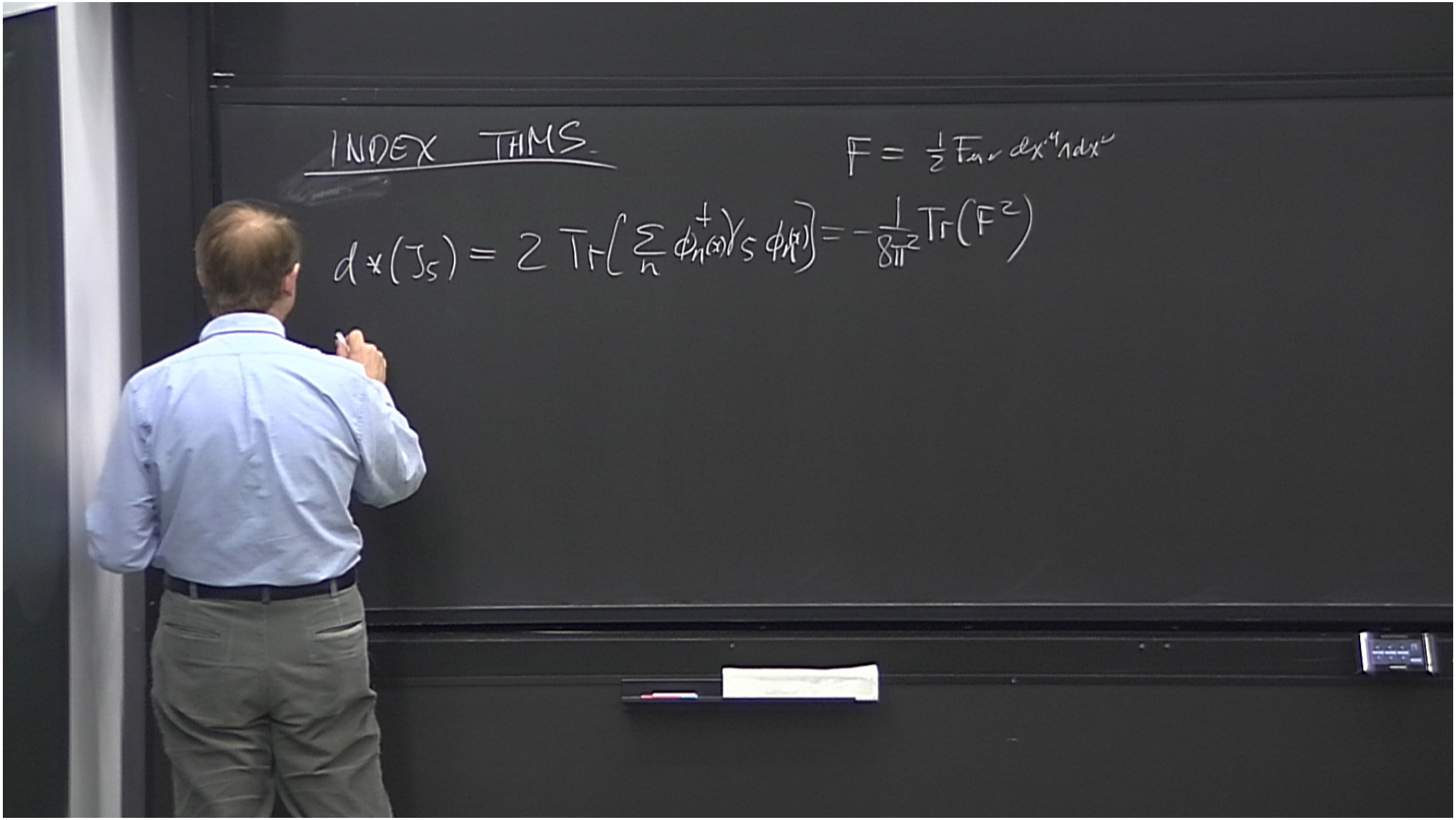

ANOMALY
NCLUDE ANOMALY in
for W
SOL'N TO
NON-ABELIAN ANOM

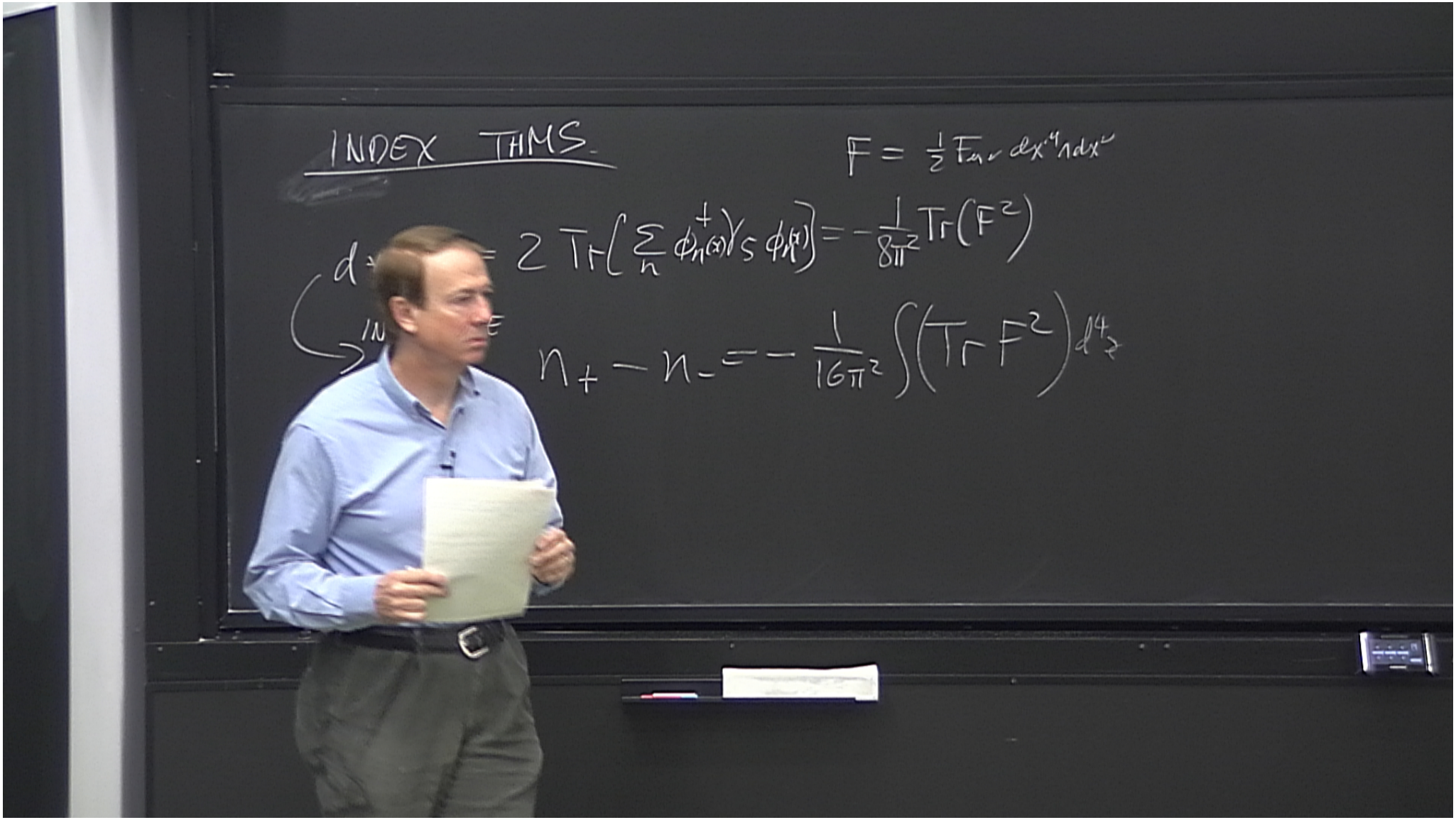

INDEX THMS.

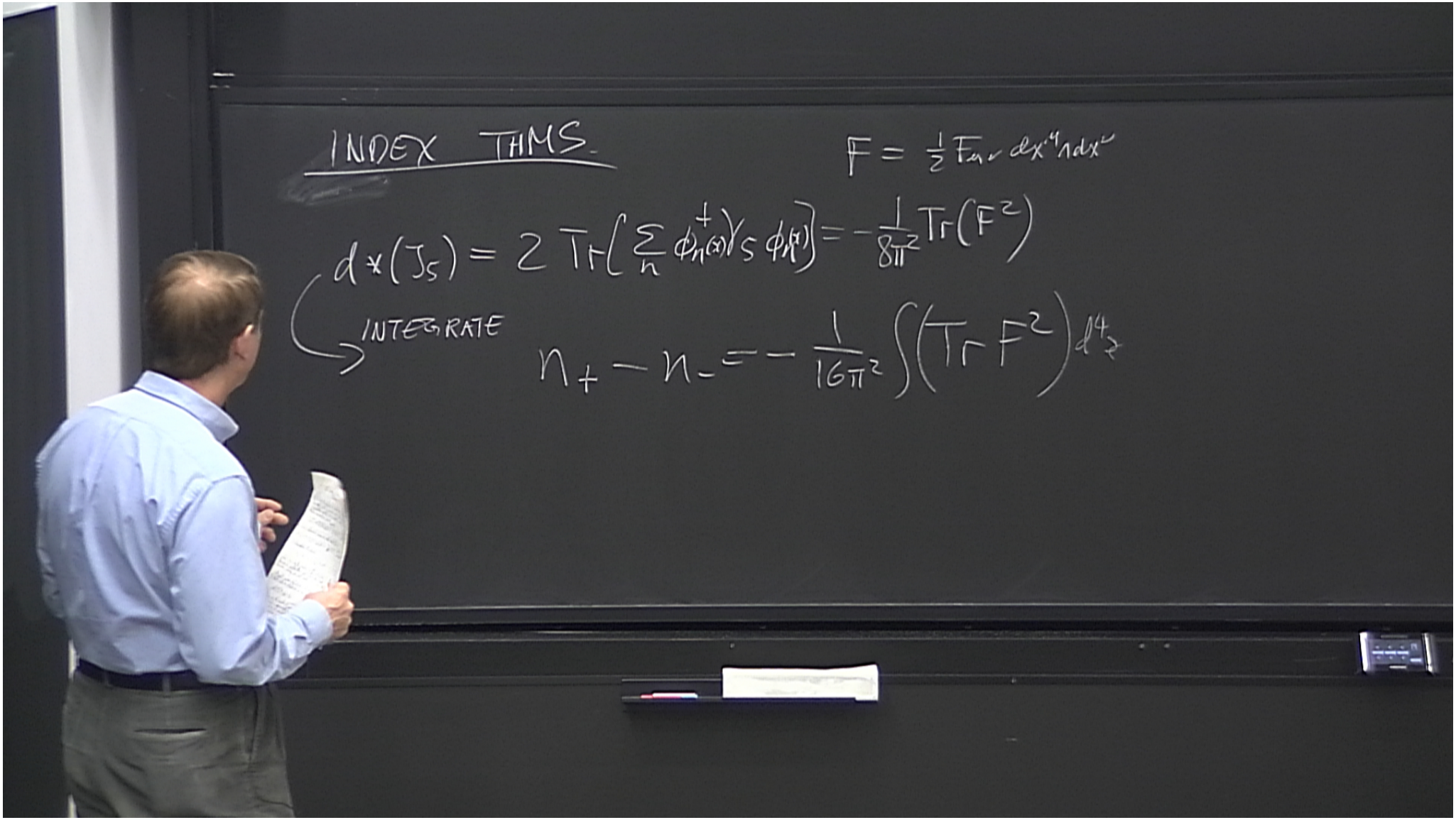

INDEX THMS.
$d*(J_S) = 2\,\mathrm{Tr}\left(\sum_n$

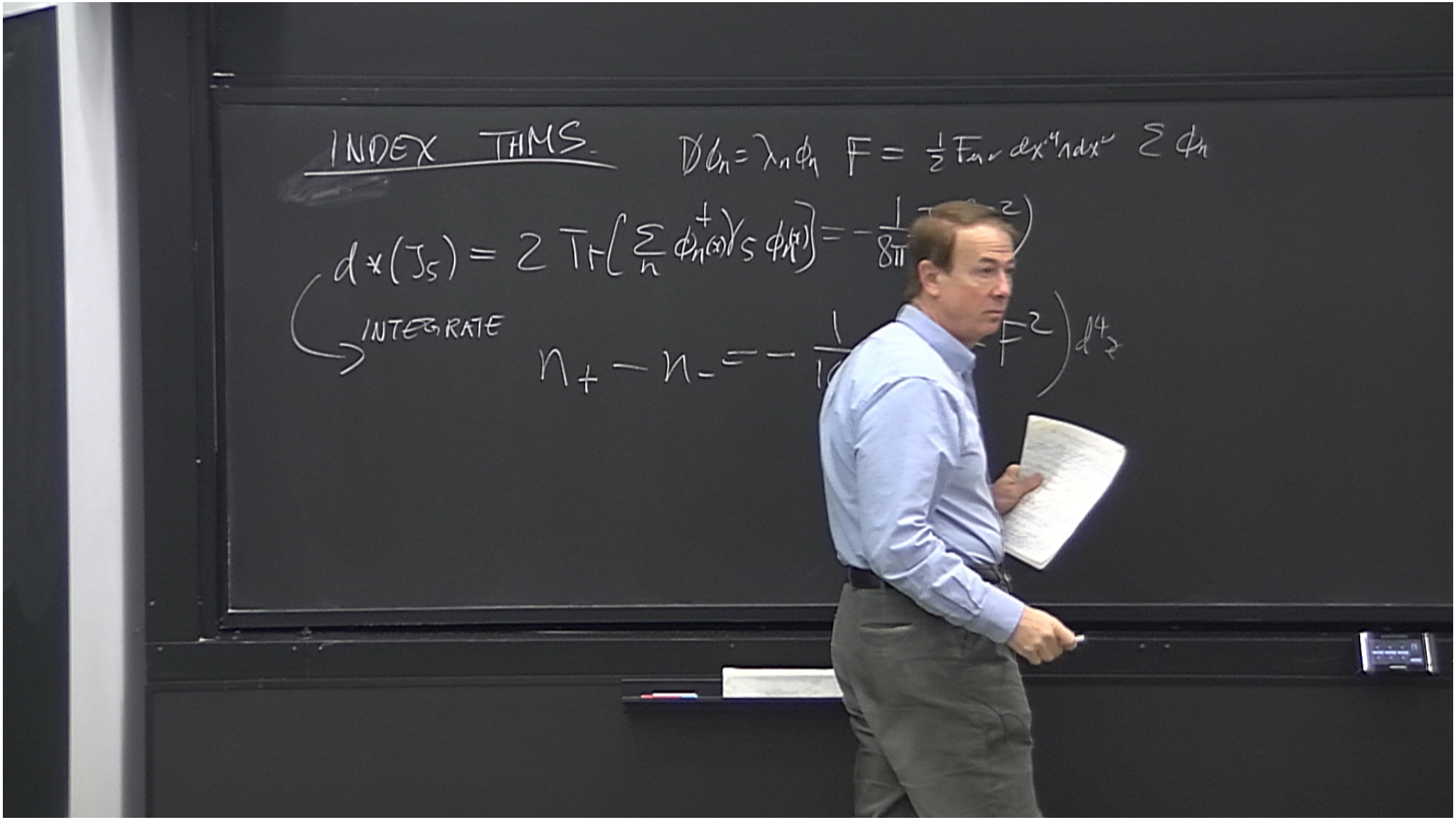

INDEX THMS.
$d*(J_5) = 2\,\mathrm{Tr}\left(\sum_n \phi_n^{\dagger}(x)\gamma_5\right) - \frac{1}{8\pi^2}\mathrm{Tr}(F$

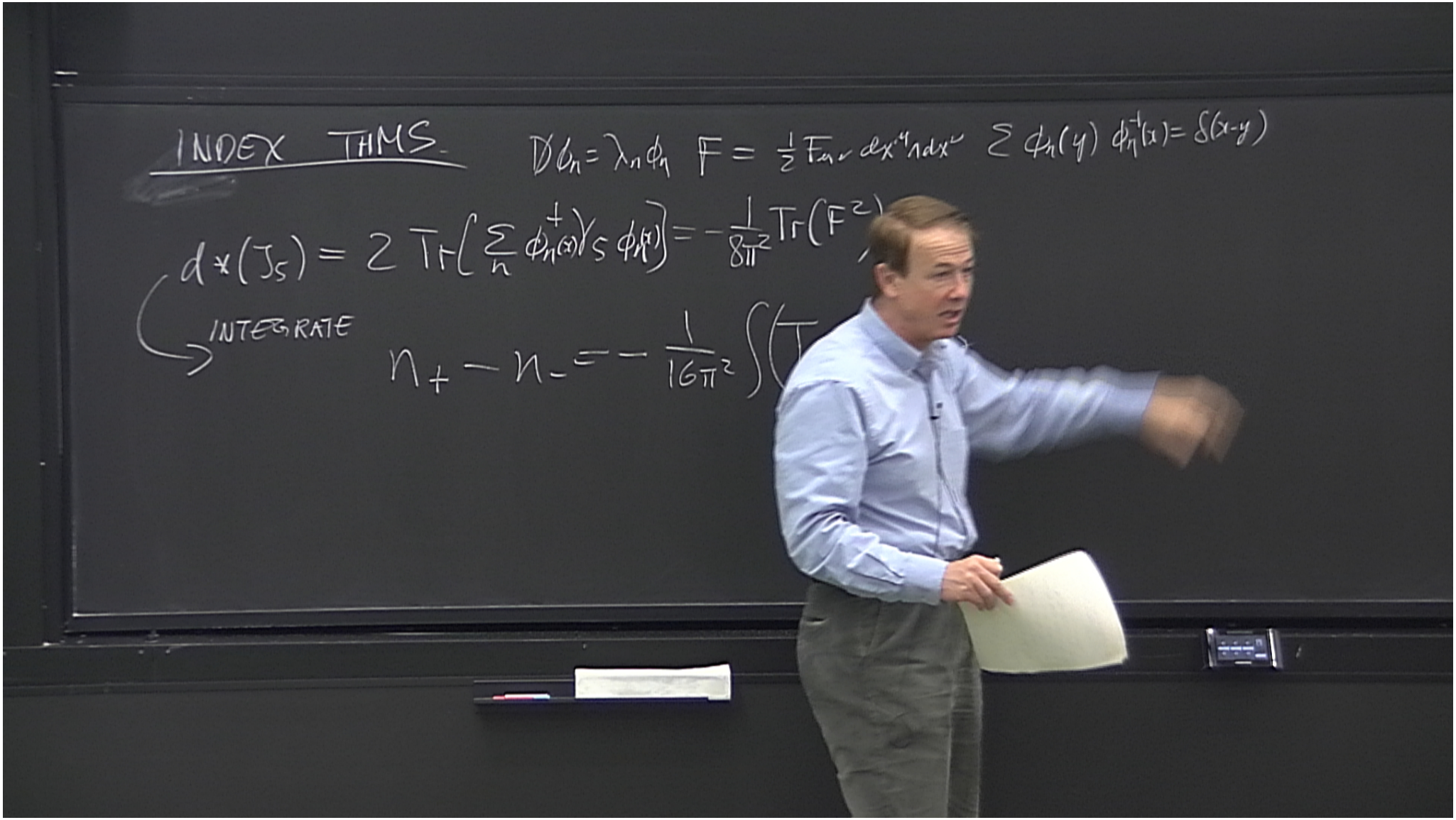

INDEX THMS.
$d*(J_5) = 2\,\mathrm{Tr}\left(\sum_n q$
$\mathrm{Tr}(F^2)$

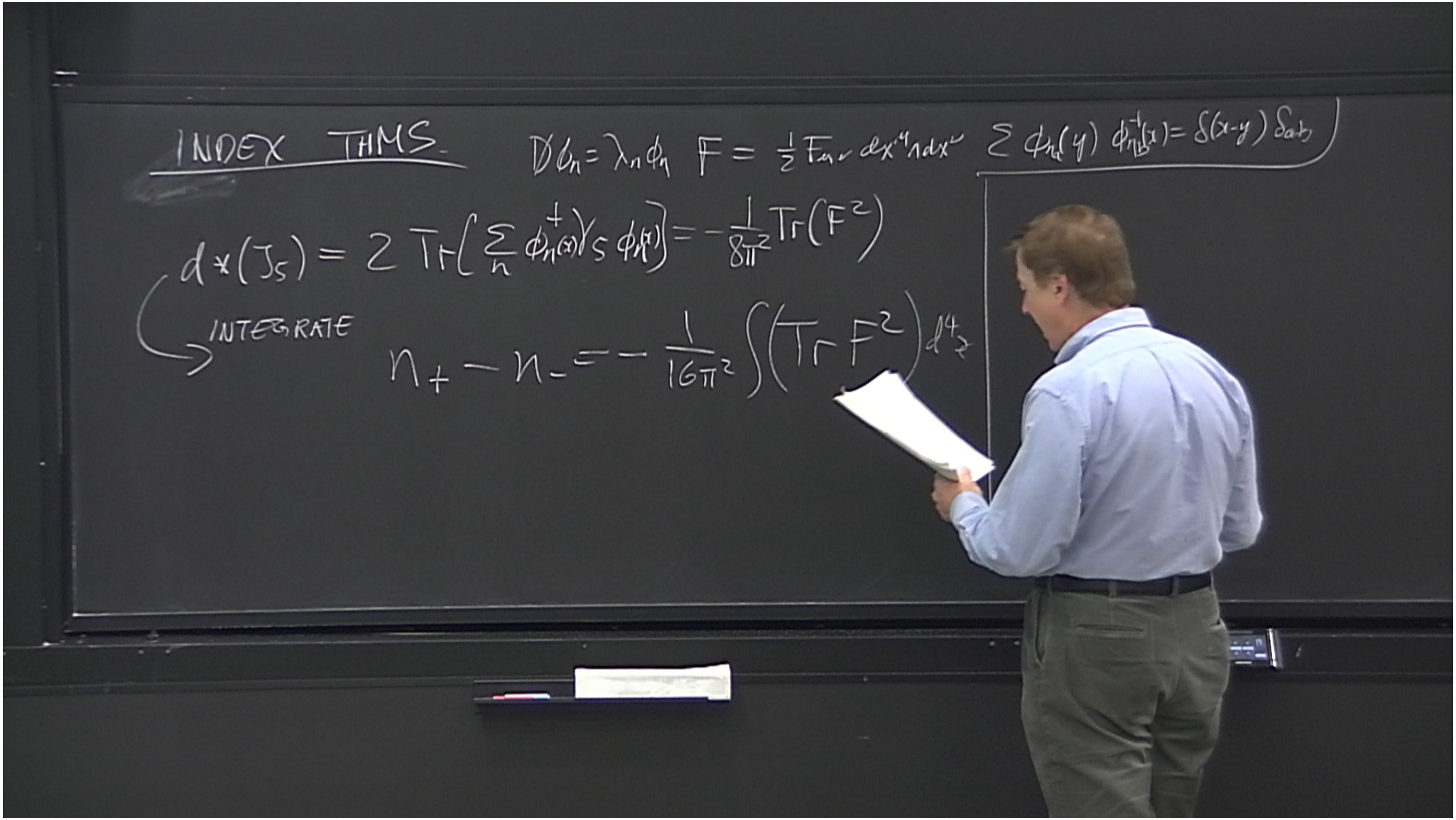

INDEX THMS.
$F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$
$d*(J_5) = 2\,\mathrm{Tr}\left(\sum_n \phi_n^\dagger(x)\gamma_5\phi_n(x)\right) = -\frac{1}{8\pi^2}\mathrm{Tr}(F^2)$

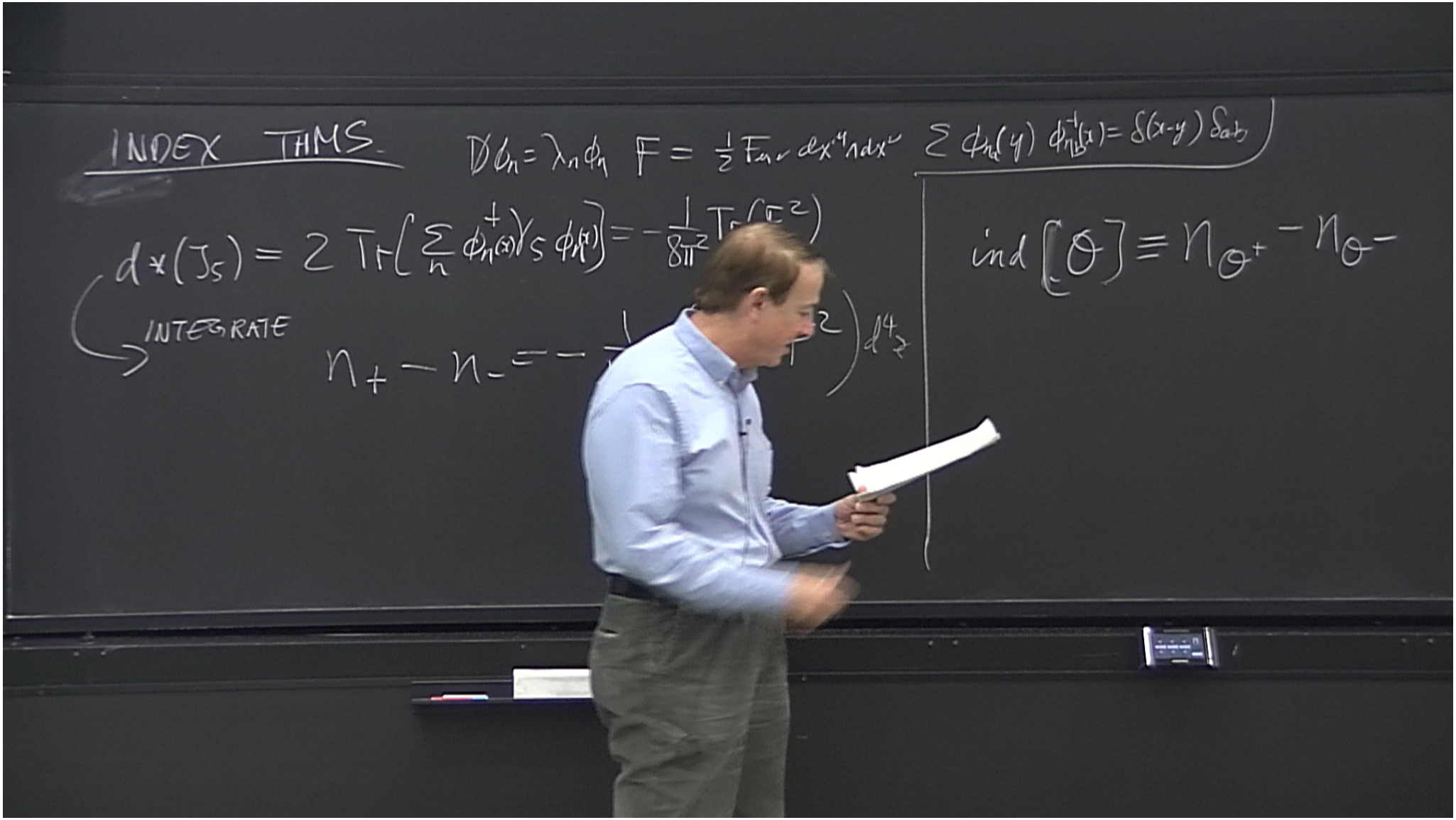

INDEX THMS.
$F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$
$= 2\,\mathrm{Tr}\left(\sum_n \phi_n^\dagger(x)\gamma_5\phi_n(x)\right) = -\frac{1}{8\pi^2}\mathrm{Tr}(F^2)$
$n_+ - n_- = -\frac{1}{16\pi^2}\int \left(\mathrm{Tr}\, F^2\right) d^4x$

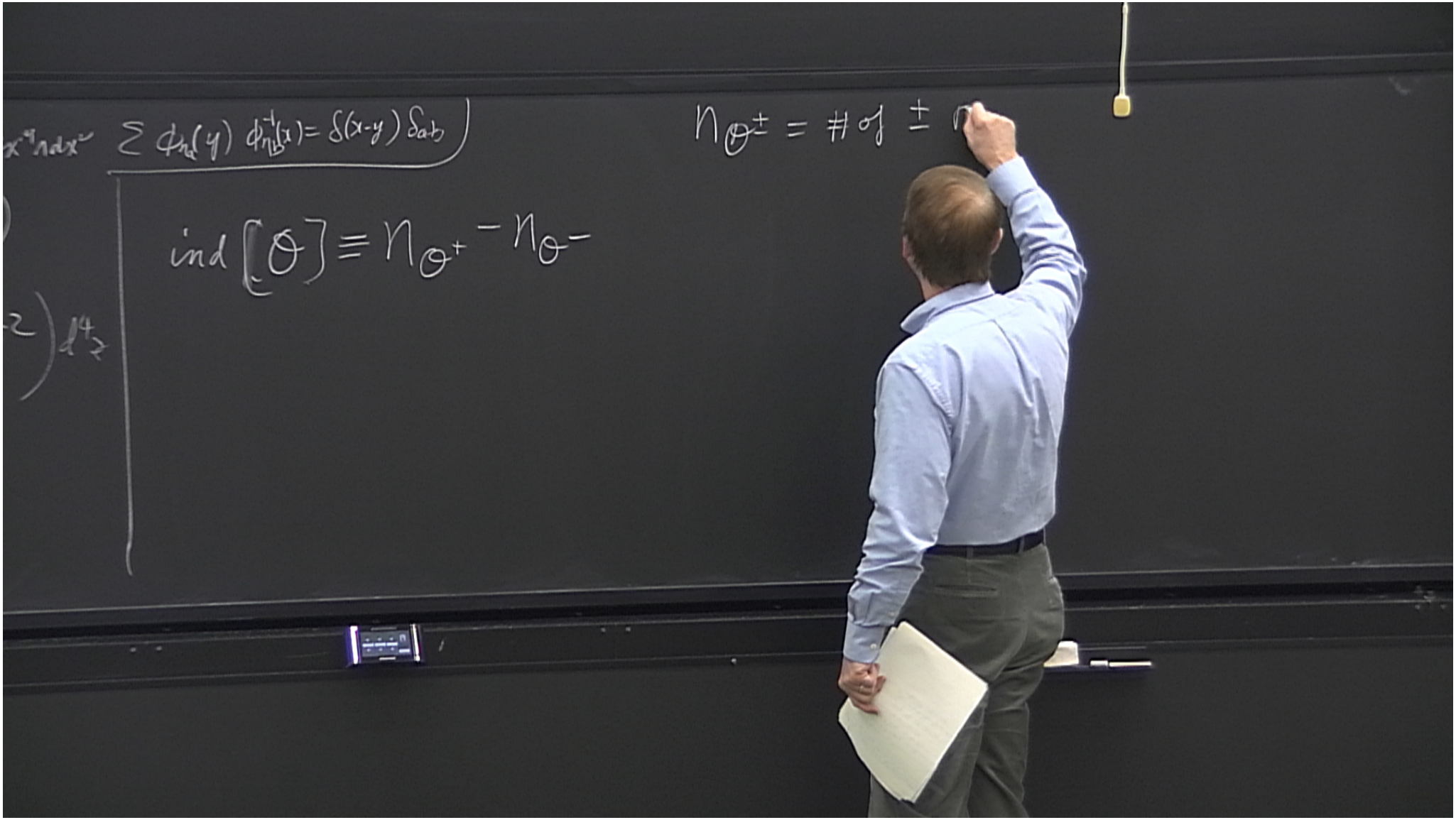

INDEX THMS.
$F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$
$d*(J_5) = 2\,\mathrm{Tr}\left[\sum_n \phi_n^\dagger(x)\gamma_5\phi_n(x)\right] = -\frac{1}{8\pi^2}\mathrm{Tr}(F^2)$
INTEGRATE
$n_+ - n_- = -\frac{1}{16\pi^2}\int\left(\mathrm{Tr}\,F^2\right)d^4x$


INDEX THMS.
INTEGRATE


INDEX THMS.
$\not{D}\phi_n = \lambda_n \phi_n$
$F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$
$\sum \phi_n(y)\, \phi_n^\dagger(x) = \delta(x-y)$
$d*(J_5) = 2\,\mathrm{Tr}\left[\sum_n \phi_n^\dagger(x)\gamma_5 \phi_n(x)\right] = -\frac{1}{8\pi^2}\mathrm{Tr}(F^2)$
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$F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$
$\sum \phi_{n\alpha}(y)\, \phi^{-1}_{n\beta}(x) = \delta(x-y)\,\delta_{\alpha\beta}$
$d*(J_5) = 2\,\mathrm{Tr}\left[\sum_n \phi^\dagger_n(x)\gamma_5\phi_n(x)\right] = -\frac{1}{8\pi^2}\mathrm{Tr}(F^2)$
INTEGRATE
$n_+ - n_- = -\frac{1}{8\pi^2}\int \mathrm{Tr}(F^2)\, d^4x$
$\mathrm{ind}[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$


$\sum \phi_{n_\alpha}(y)\, \phi^{-1}_{n_\beta}(x) = \delta(x-y)\,\delta_{\alpha\beta}$
$n_{\mathcal{O}^\pm}$ = # of ±
$\mathrm{ind}[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$

$\sum \phi_{n_\alpha}(y)\, \phi^{-1}_{n_B}(x) = \delta(x-y)\,\delta_{ab}$

$n_{\mathcal{O}^\pm}$ = # of $\pm$ DUALITY SCANS TO $\mathcal{O}$

$$\text{ind}\,[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$$



$\sum \phi_{n_A}(y)\, \phi^{-1}_{n_B}(x) = \delta(x-y)\,\delta_{ab}$

$n_{\mathcal{O}^\pm}$ = # of $\pm$ DUALITY SCANS TO $\mathcal{O}\mathcal{I} = 0$

$$\text{ind}\,[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$$

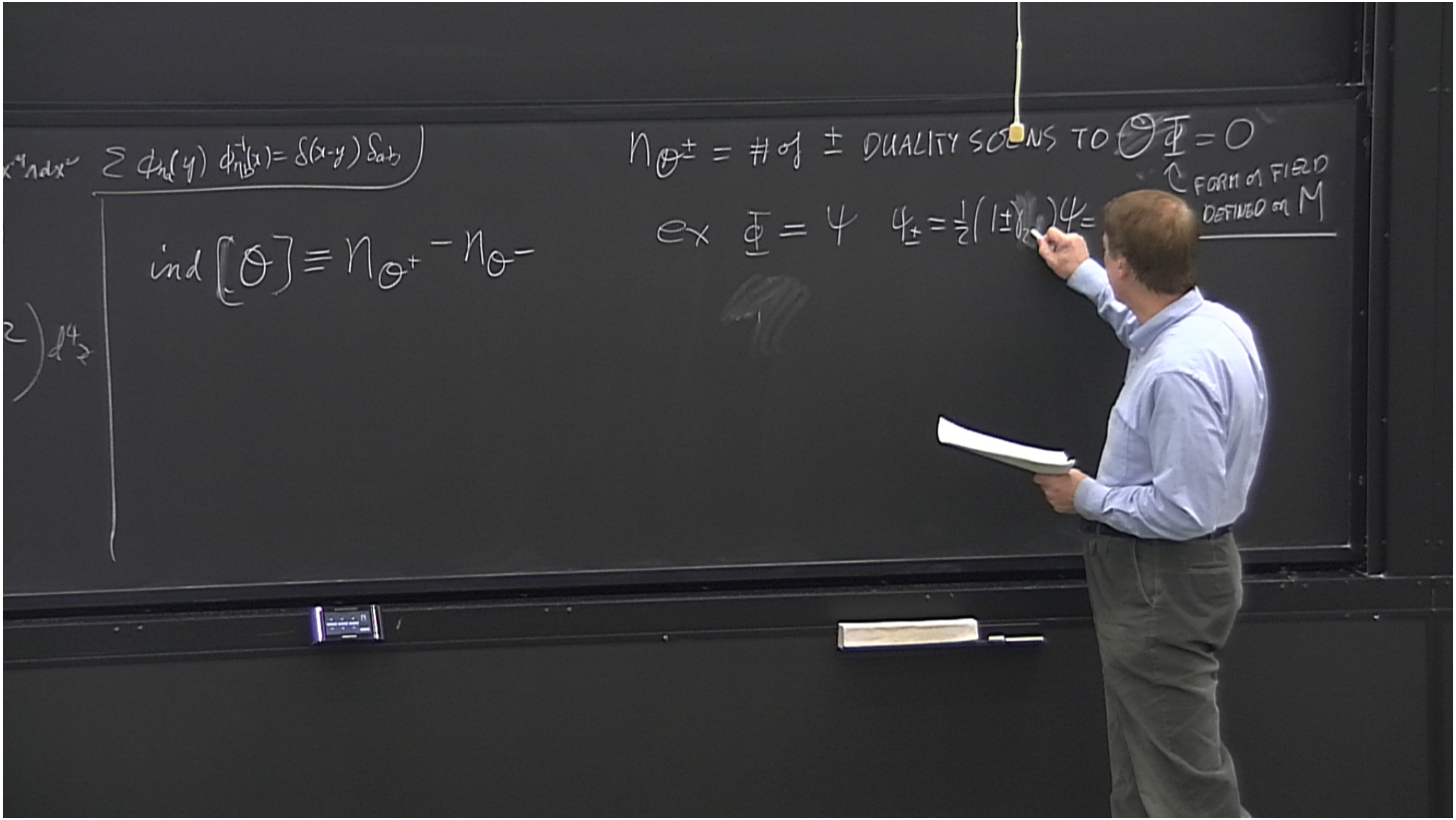
$\int d^4x \, n dx^\nu$ $\sum \phi_{n_a}(y)\, \phi^{-1}_{n_b}(x) = \delta(x-y)\, \delta_{ab}$

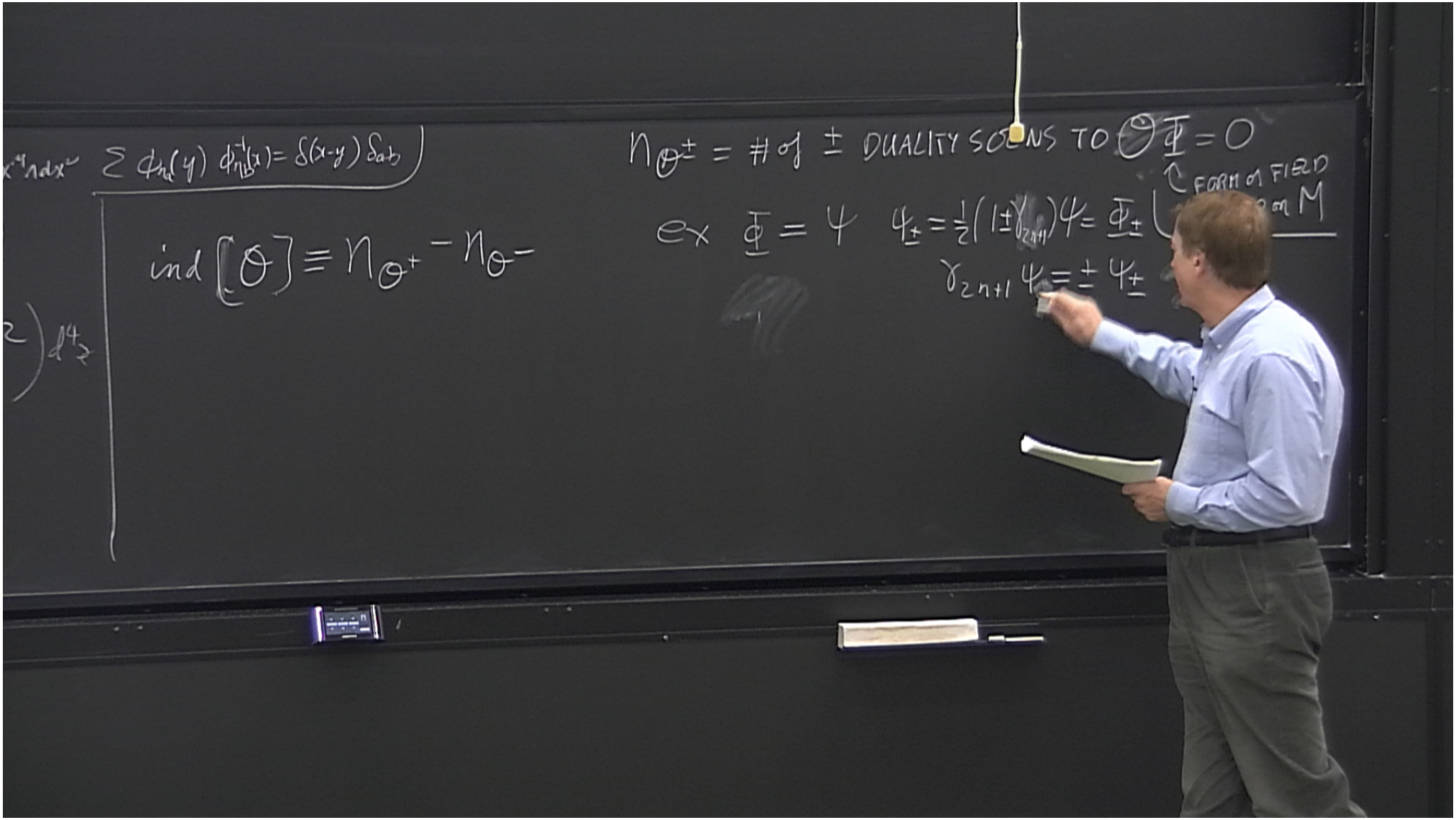
$n_{\mathcal{O}^\pm}$ = # of $\pm$ DUALITY SCONS TO $\mathcal{O}\Phi = 0$

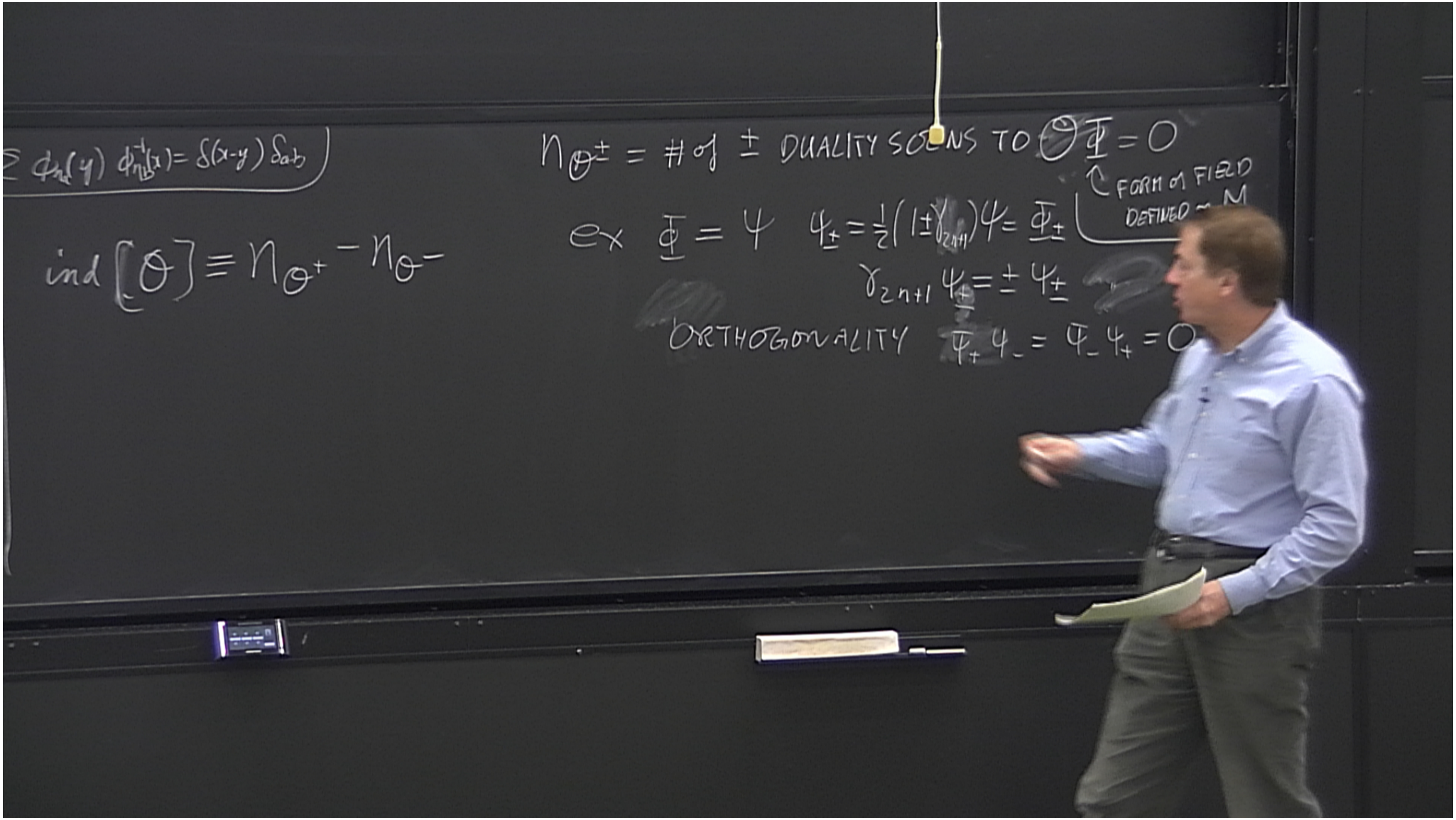
↑ FORM of FIELD DEFINED on M

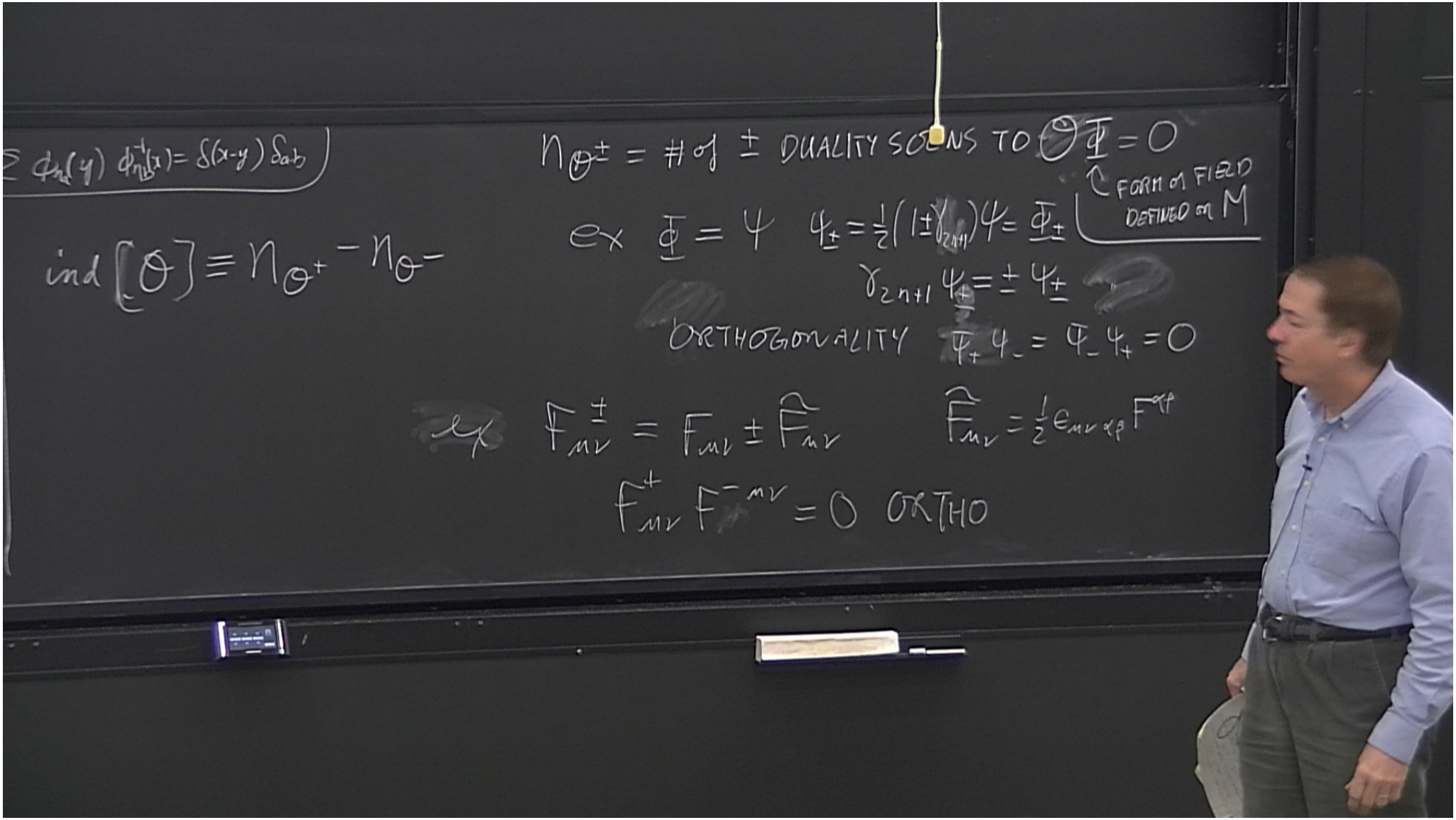
$$\text{ind}[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$$

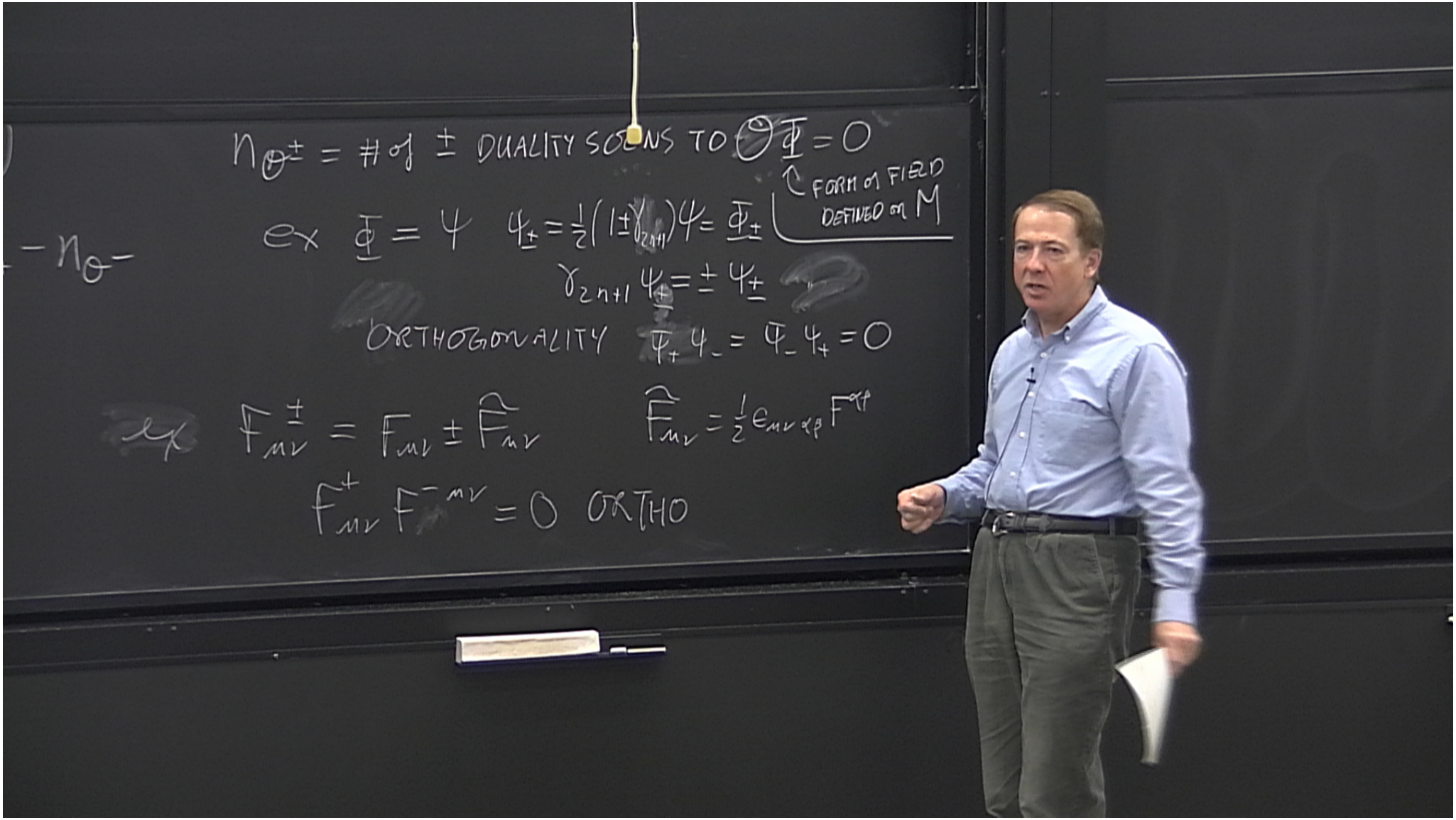
$\left.\right) d^4z$


$\sum \phi_n(y)\,\phi_n^{-1}(x) = \delta(x-y)\,\delta_{ab}$
$n_{\mathcal{O}^\pm}$ = # of ± DUALITY SOLNS TO $\mathcal{O}\Phi = 0$
FORM of FIELD DEFINED on M
ex $\Phi = \psi$ $\psi_\pm = \frac{1}{2}(1 \pm \gamma_{\cdot})\psi =$
$\mathrm{ind}[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$


$\sum \phi_n(y)\,\phi_n^{-1}(x) = \delta(x-y)\,\delta_{ab}$
$n_{\Theta^\pm}$ = # of ± DUALITY SOLNS TO $\Theta\Phi = 0$
FORM of FIELD
on M
ex $\Phi = \Psi$ $\Psi_\pm = \frac{1}{2}(1 \pm \gamma_{2n+1})\Psi = \Phi_\pm$
$\gamma_{2n+1}\Psi_\pm = \pm\Psi_\pm$
$\mathrm{ind}[\Theta] \equiv n_{\Theta^+} - n_{\Theta^-}$


$\sum \phi_n(y)\, \phi^{-1}_n(x) = \delta(x-y)\,\delta_{ab}$
$\mathrm{ind}\,[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$
$n_{\mathcal{O}^\pm}$ = # of $\pm$ DUALITY SOLNS TO $\mathcal{O}\Phi = 0$
FORM OF FIELD DEFINED ~ M
ex $\Phi = \psi$ $\psi_\pm = \frac{1}{2}(1 \pm \gamma_{2n+1})\psi = \Phi_\pm$
$\gamma_{2n+1}\psi_\pm = \pm\psi_\pm$
ORTHOGONALITY $\Phi_+ \psi_- = \Phi_- \psi_+ = 0$


$\sum \phi_n(y)\, \phi^{\dagger}_n(x) = \delta(x-y)\,\delta_{ab}$
$n_{\mathcal{O}^\pm}$ = # of $\pm$ DUALITY SOLNS TO $\mathcal{O}\Phi = 0$
FORM OF FIELD DEFINED ON M
$\text{ind}[\mathcal{O}] \equiv n_{\mathcal{O}^+} - n_{\mathcal{O}^-}$
ex $\mathcal{O} = \gamma$ $\psi_\pm = \frac{1}{2}(1 \pm \gamma_{2n+1})\psi = \Phi_\pm$
$\gamma_{2n+1}\psi_\pm = \pm\psi_\pm$
ORTHOGONALITY $\Phi_+\psi_- = \Phi_-\psi_+ = 0$
ex $F^\pm_{\mu\nu} = F_{\mu\nu} \pm \hat{F}_{\mu\nu}$ $\hat{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$
$F^+_{\mu\nu}F^{-\mu\nu} = 0$ ORTHO


$n_{\Theta^\pm}$ = # of ± DUALITY SOLNS TO $\Theta\Phi = 0$
FORM of FIELD DEFINED on M
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$\mathcal{D}\psi_n = \lambda_n \psi_n$ $F = \frac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu$ $\sum \phi_n(y)\, \phi_n^{-1}(x) = \delta(x-y)\, \delta_{ab}$

$\left[\bar{\psi}_n(x) \gamma_5 \psi_n(x)\right] = -\frac{1}{8\pi^2} \mathrm{Tr}(F^2)$

$= -\frac{1}{16\pi^2} \int \left(\mathrm{Tr}\, F^2\right) d^4x$

$\mathrm{ind}\left[\mathcal{D}\right] \equiv n_{\mathcal{D}^+} - n_{\mathcal{D}^-}$

$\mathcal{D} \rightarrow$ ELLIPTIC

$M \rightarrow$ MUST ADMIT A DEFINITION of DUALITY/CHIRALITY on Φ

$n_{\mathcal{D}^\pm}$ = # of $\pm$ DUALITY SOLNS TO

ex $\Phi = \psi$ $\psi_\pm = \frac{1}{2}(1 \pm \gamma_{2n+1})\psi =$

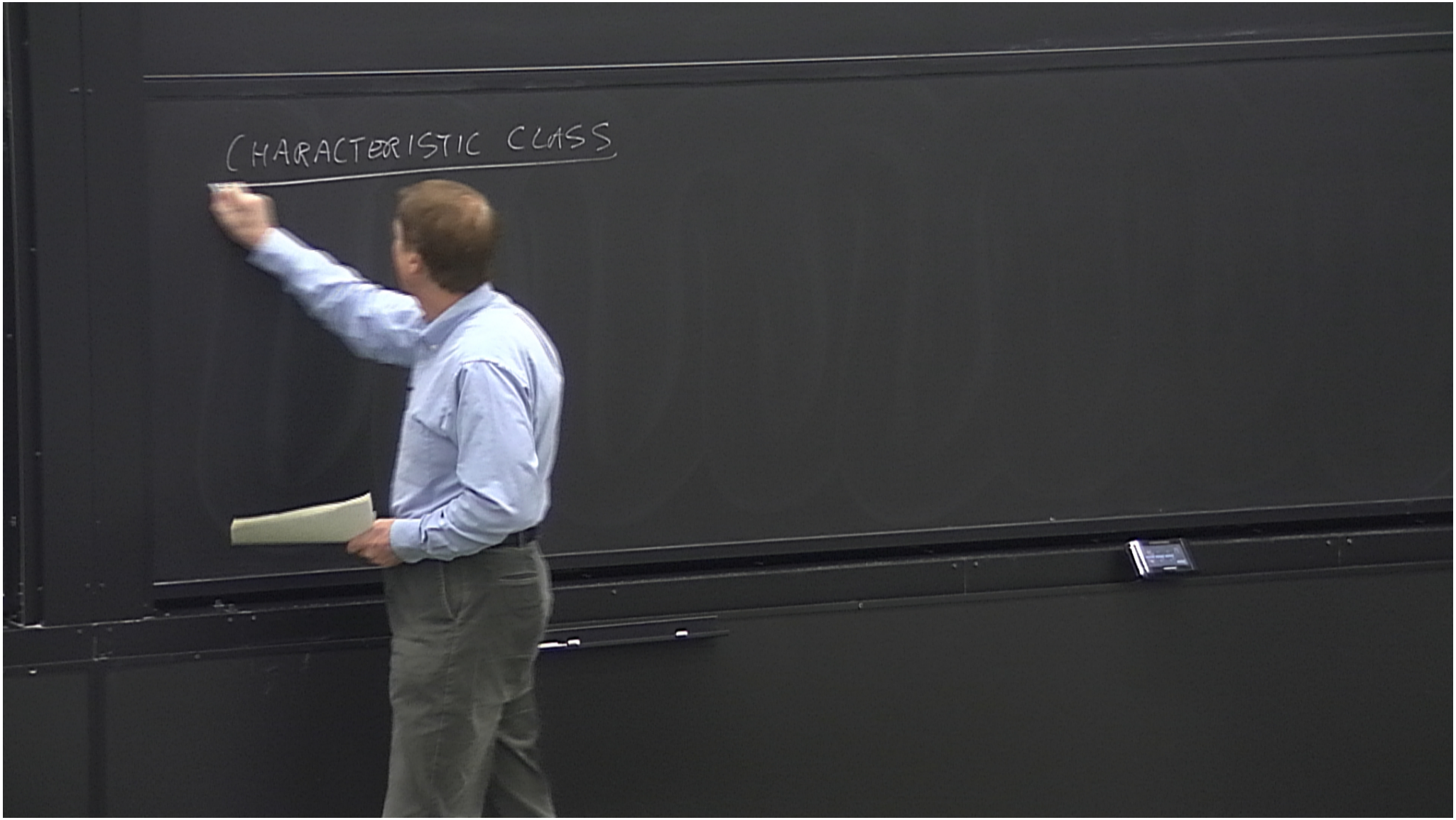
$\gamma_{2n+1} \psi_\pm = \pm$

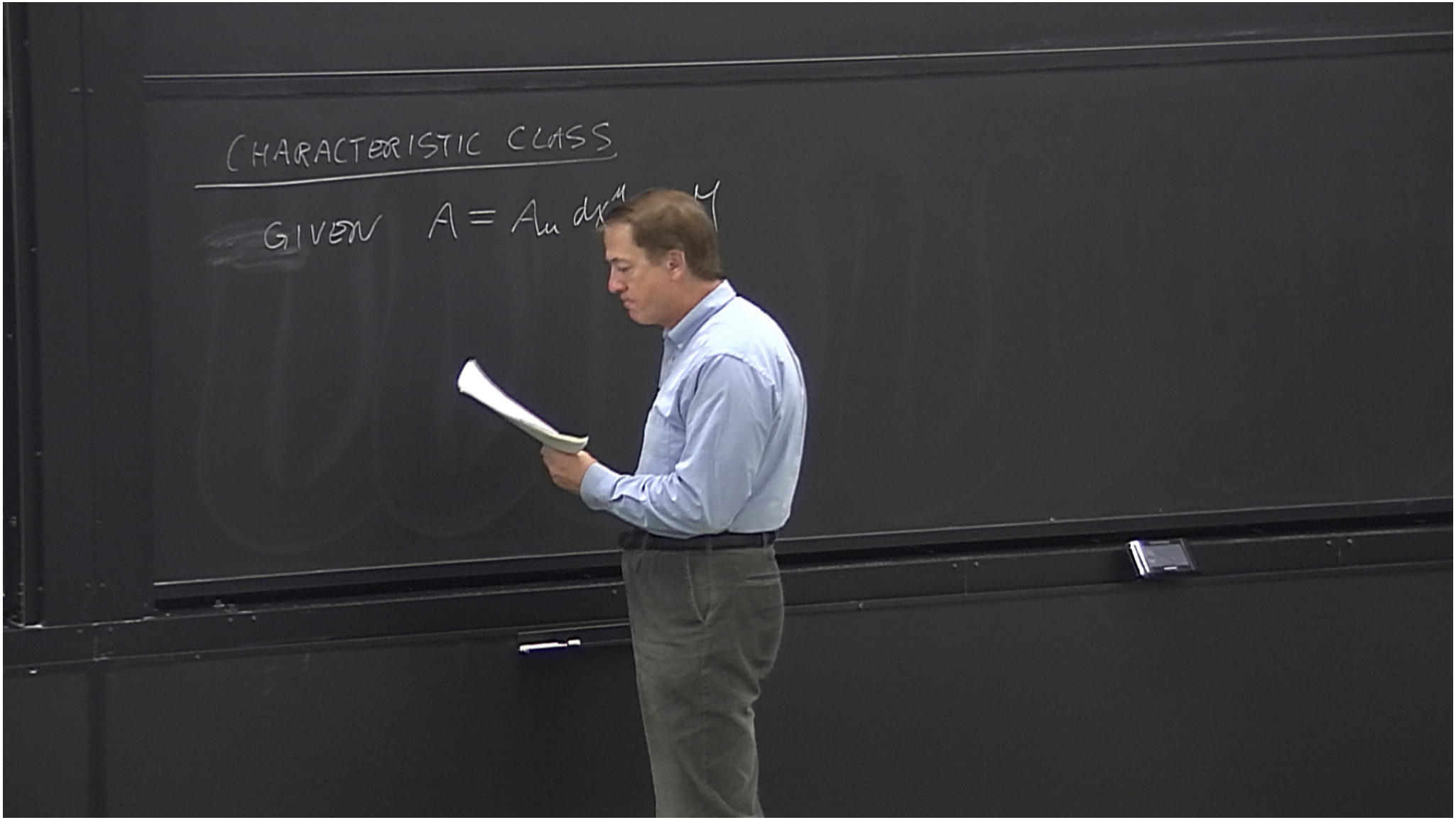
ORTHOGONALITY $\psi_+ \psi_-$

ex

$F^\pm_{\mu\nu} = F_{\mu\nu} \pm \tilde{F}_{\mu\nu}$ $\tilde{F}_{\mu\nu} =$

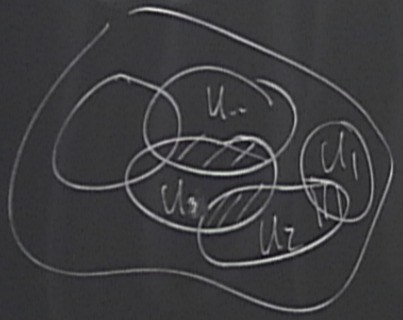
$F^+_{\mu\nu} F^{-\mu\nu} = 0$ ORTHO


CHARACTERISTIC CLASS


CHARACTERISTIC CLASS
GIVEN

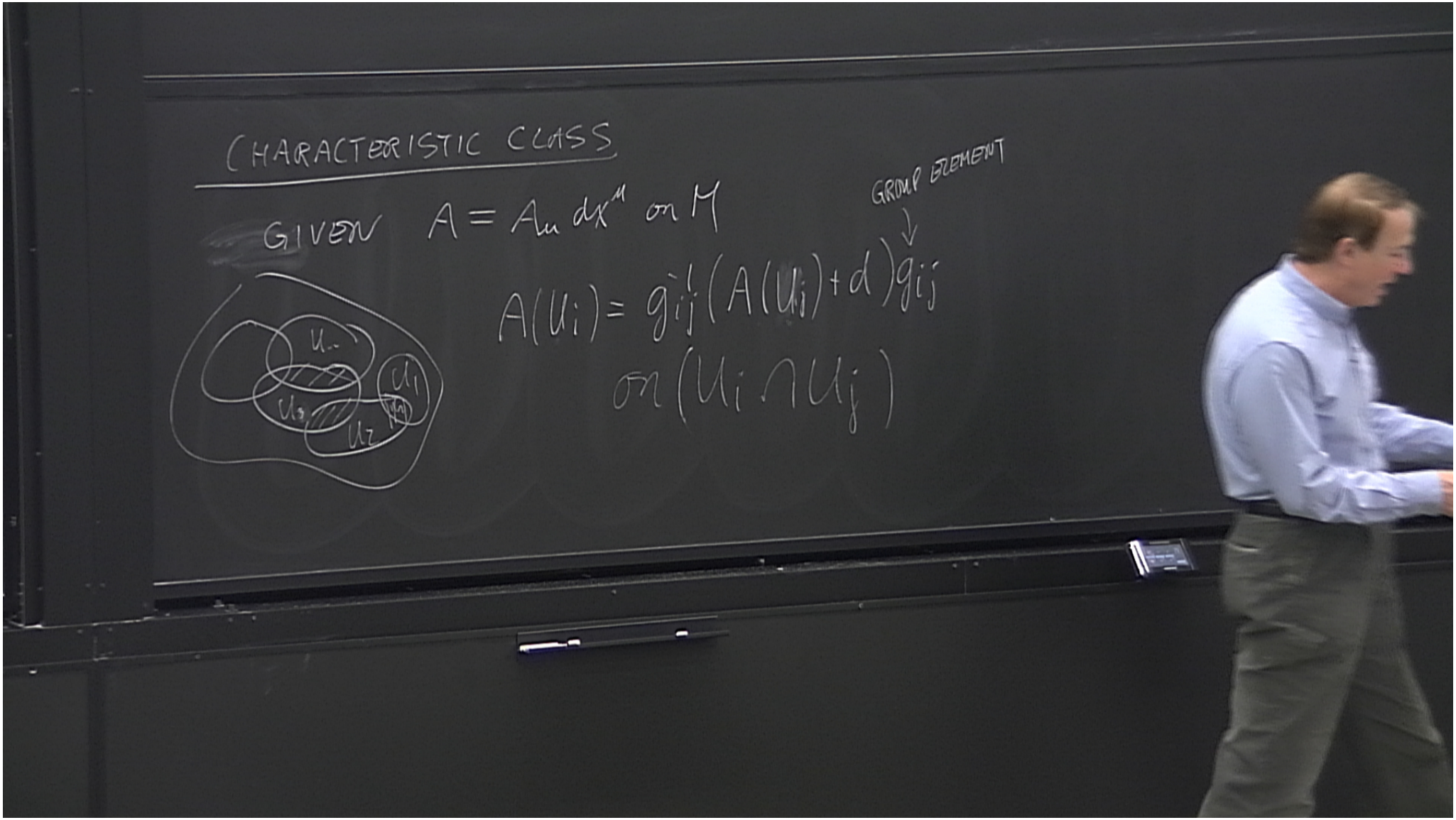
CHARACTERISTIC CLASS

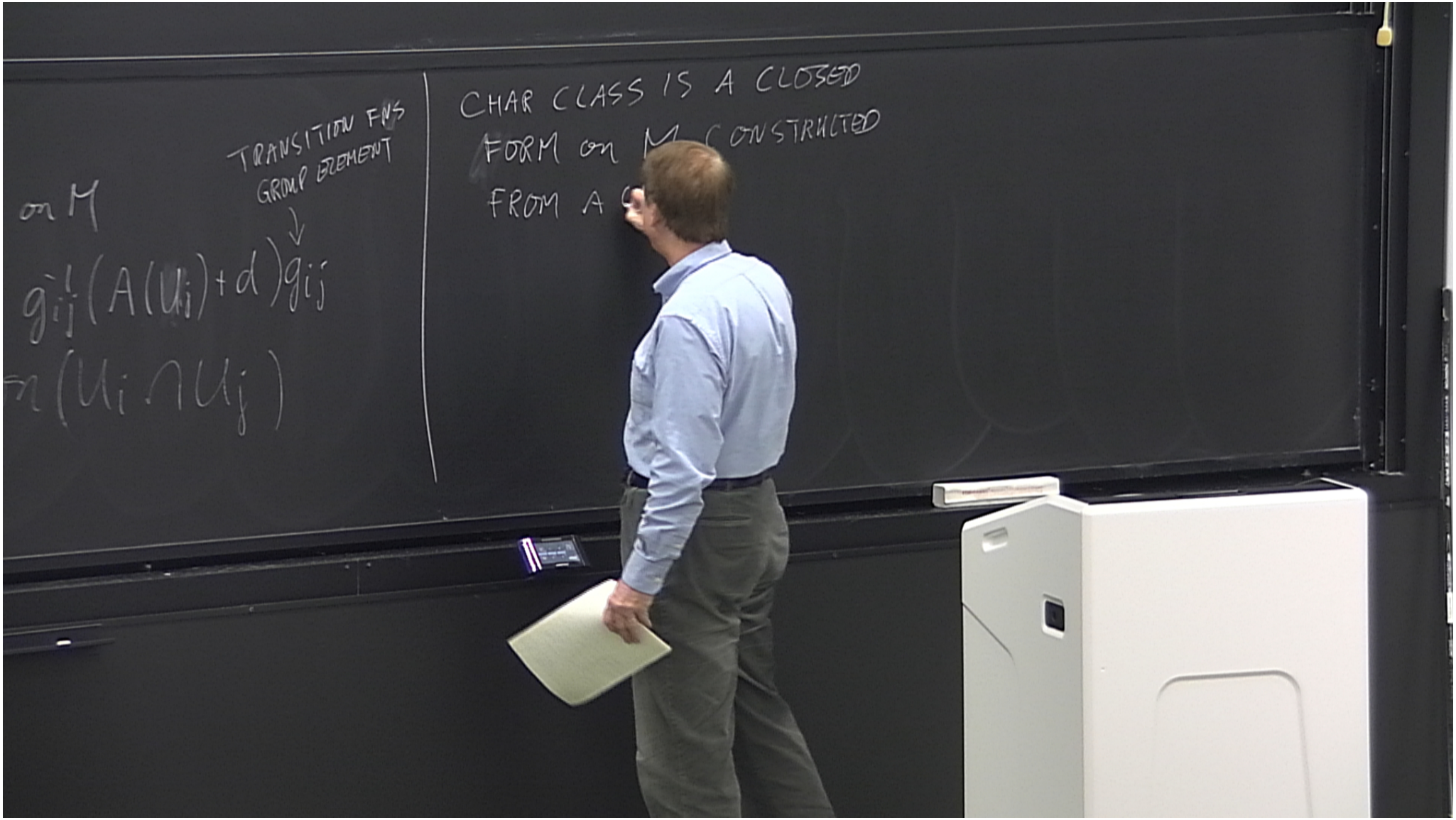
GIVEN $A = A_\mu dx^\mu$ on M

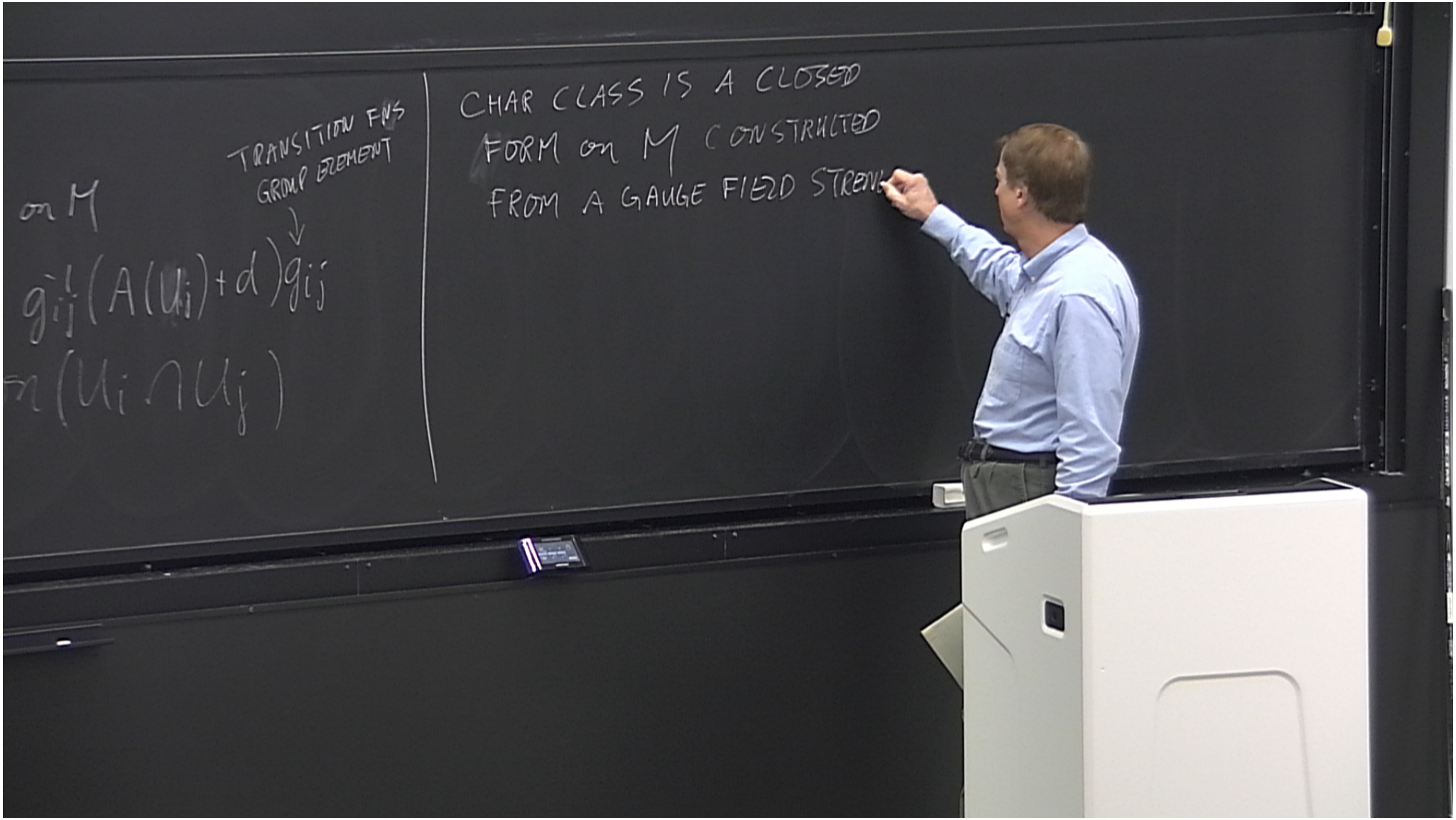


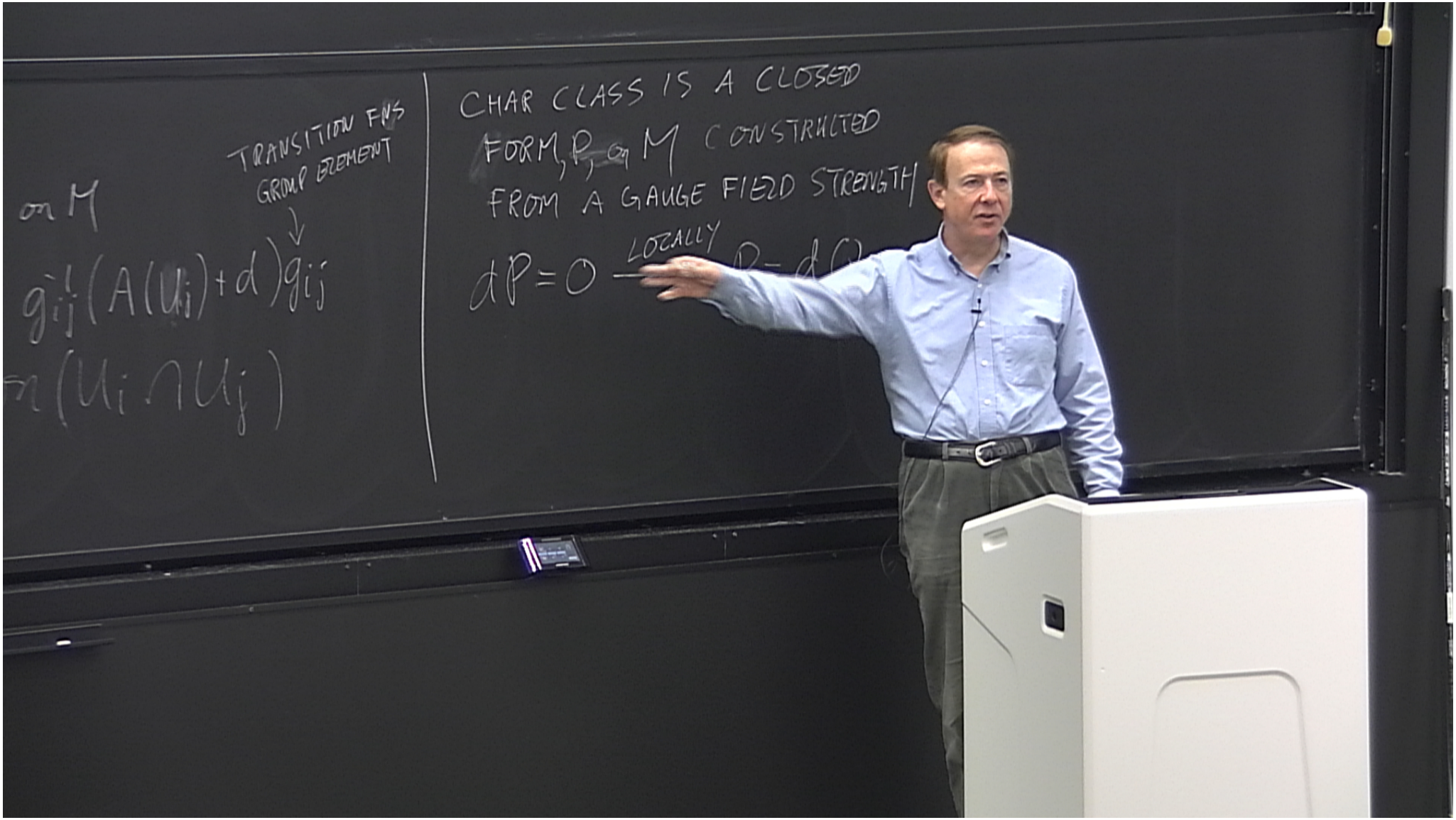
$$A(U_i) = g_{ij}^{-1}(A(U_j) + d)g_{ij}$$

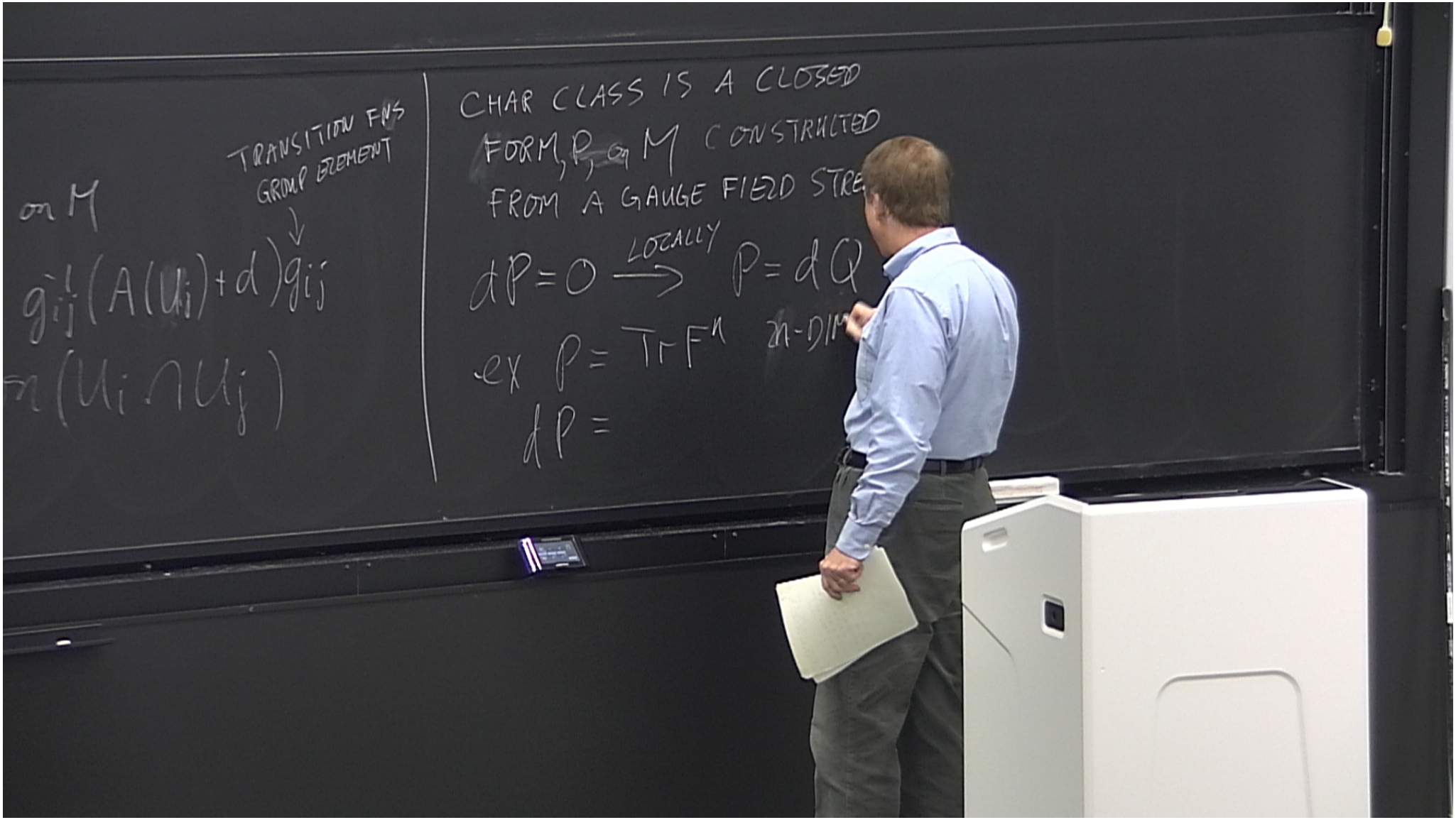
on $(U_i \cap U_j)$

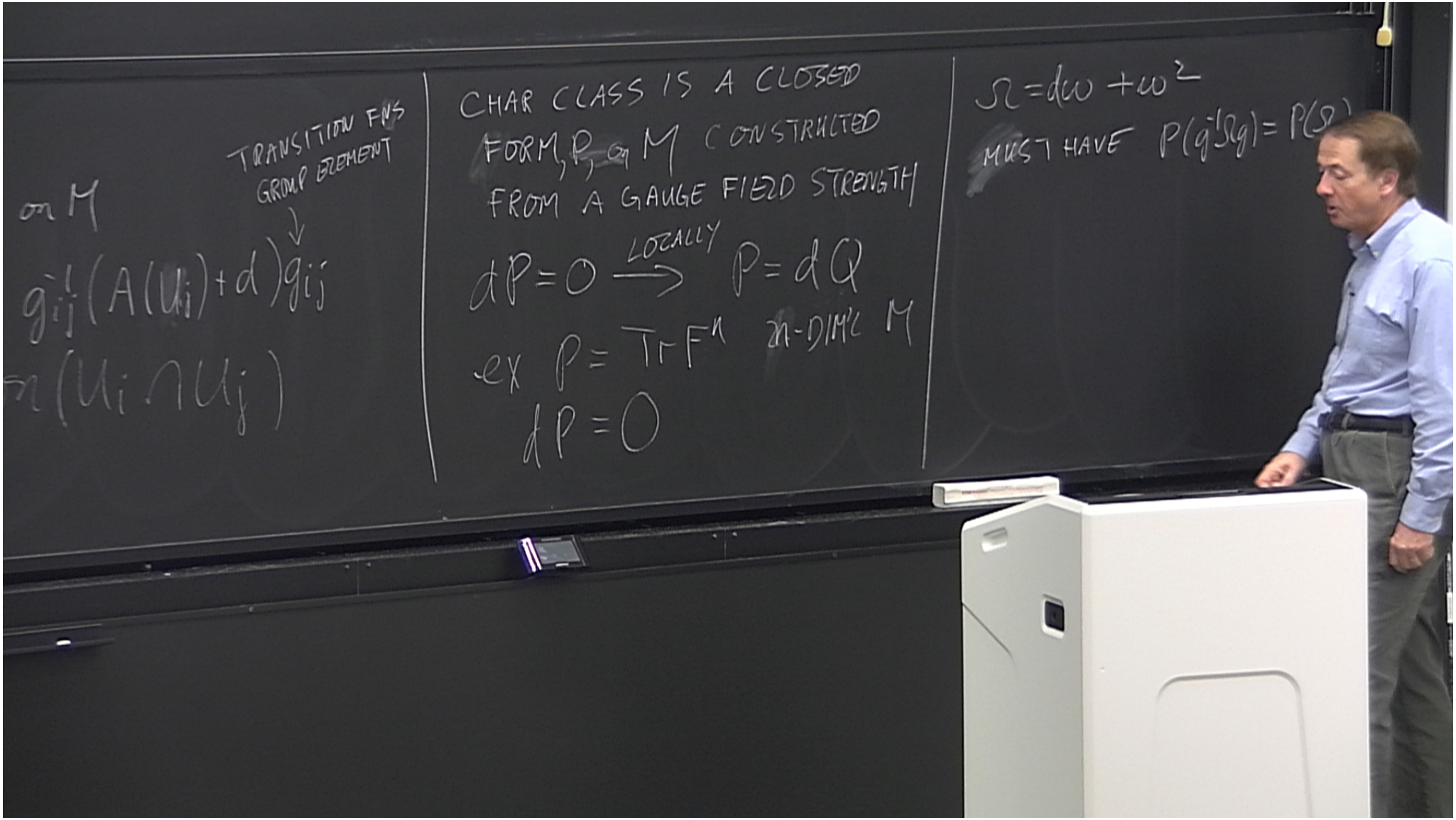

CHARACTERISTIC CLASS
GIVEN $A = A_\mu dx^\mu$ on M
GROUP ELEMENT
$A(U_i) = g_{ij}^{-1}(A(U_j) + d)g_{ij}$
on $(U_i \cap U_j)$

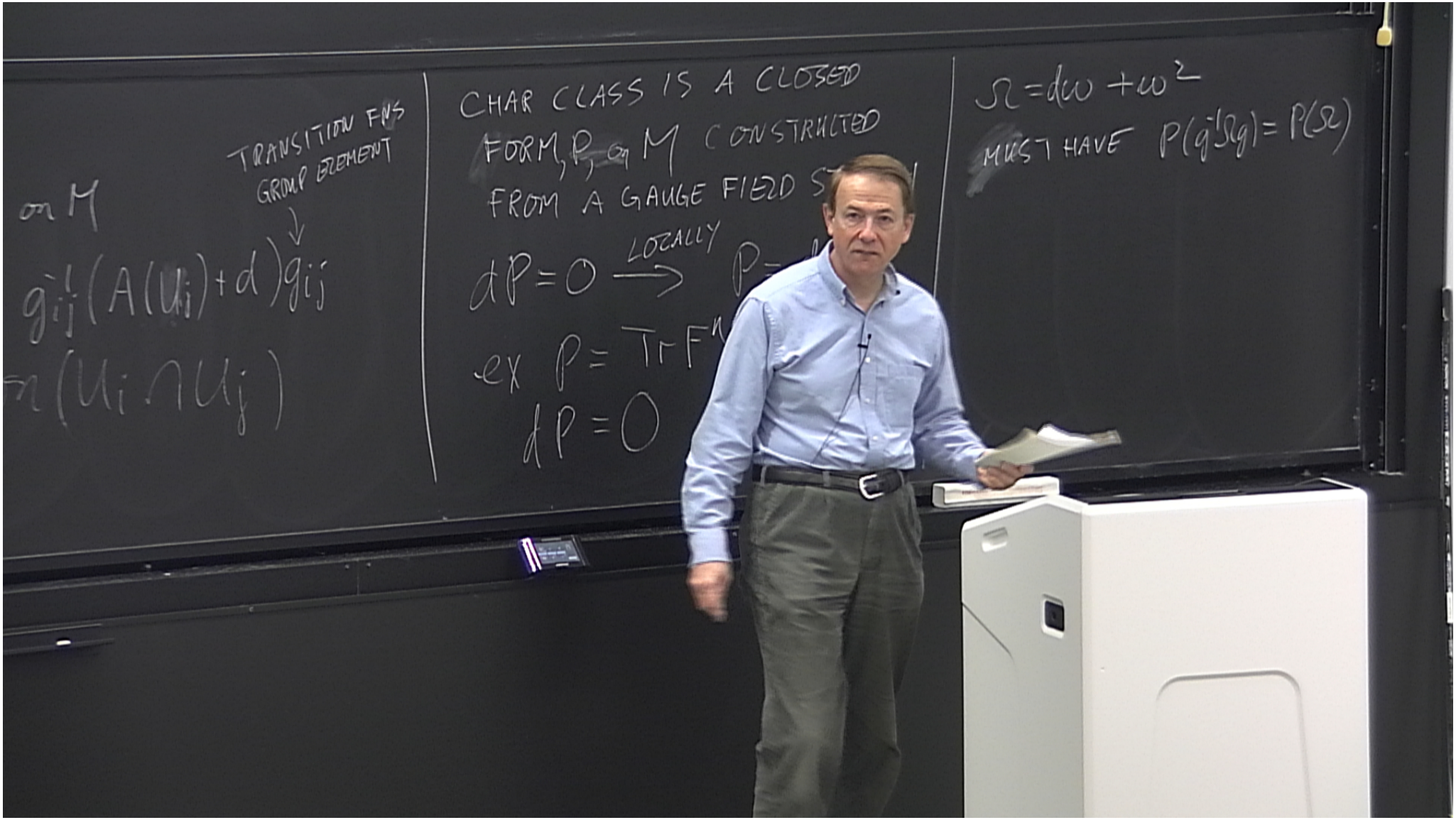

on M
TRANSITION FNS
GROUP ELEMENT
CHAR CLASS IS A CLOSED
FORM ON M CONSTRUCTED
FROM A

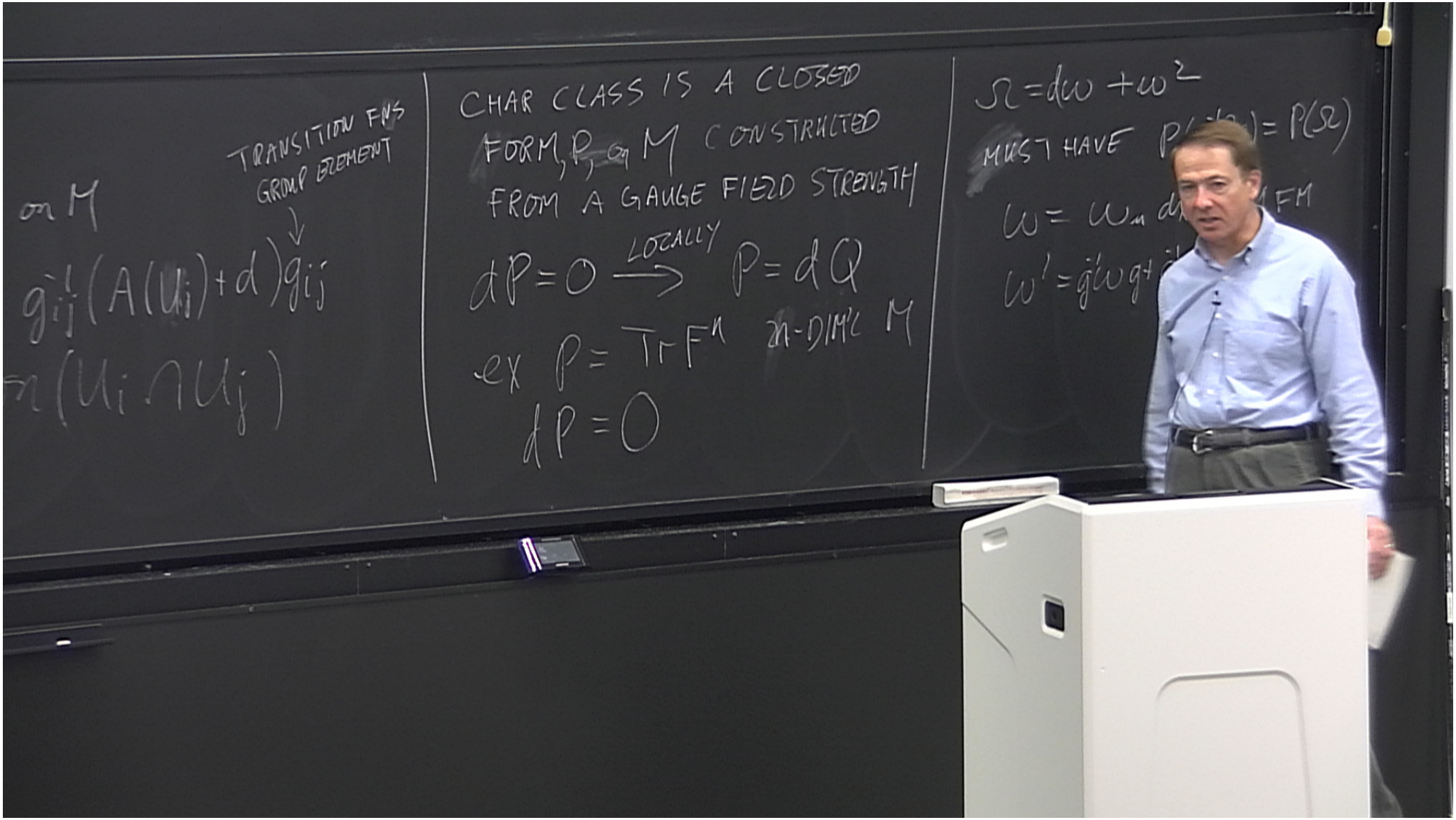

on M
TRANSITION FNS
GROUP ELEMENT
$g_{ij}^{-1}(A(U_i)+d)g_{ij}$
$\Omega(U_i \cap U_j)$
CHAR CLASS IS A CLOSED
FORM on M CONSTRUCTED
FROM A GAUGE FIELD

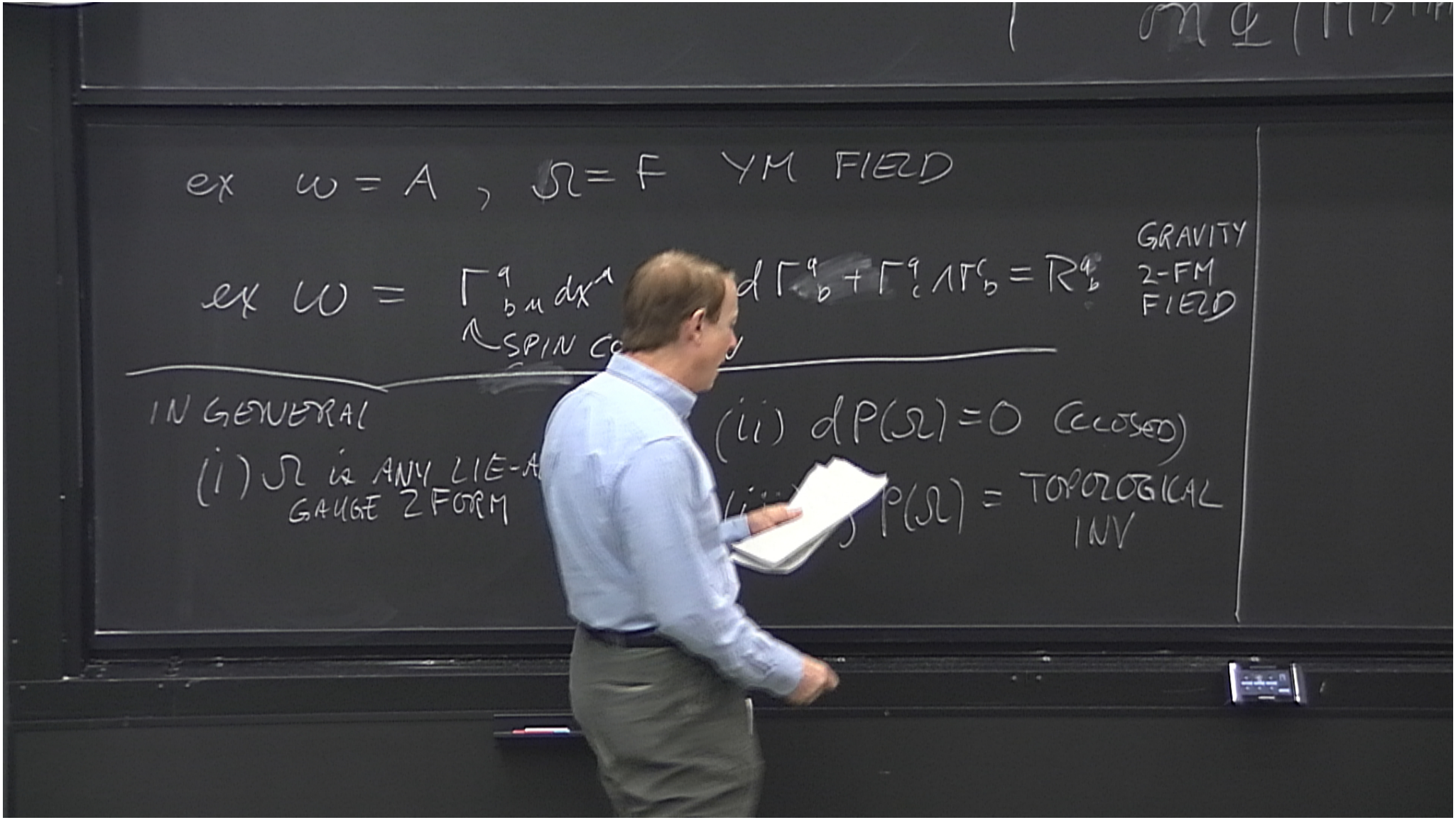

TRANSITION FNS
GROUP ELEMENT
CHAR CLASS IS A CLOSED
FORM, P, ON M CONSTRUCTED
FROM A GAUGE FIELD STRENGTH
LOCALLY


on M
TRANSITION FNS
GROUP ELEMENT
$g_{ij}^{-1}(A(U_i)+d)g_{ij}$
$\pi(U_i \cap U_j)$
CHAR CLASS IS A CLOSED
FORM, P, on M CONSTRUCTED
FROM A GAUGE FIELD STRE
$dP=0 \xrightarrow{\text{LOCALLY}} P=dQ$
ex $P=\mathrm{Tr}F^n$ 2n-DIM
$dP=$


on M
TRANSITION FNS
GROUP ELEMENT
$g_{ij}^{-1}(A(U_i)+d)g_{ij}$
$U_i \cap U_j$
CHAR CLASS IS A CLOSED
FORM, P, OF M CONSTRUCTED
FROM A GAUGE FIELD STRENGTH
$dP=0 \xrightarrow{\text{LOCALLY}} P=dQ$
ex $P=\mathrm{Tr}F^n$ $2n$-DIM'L M
$dP=0$
$\Omega=d\omega+\omega^2$
MUST HAVE $P(g^{-1}\Omega g)=P(\Omega)$


on M
TRANSITION FNS
GROUP ELEMENT
$g_{ij}^{-1}(A(U_i)+d)g_{ij}$
$U_i \cap U_j$
CHAR CLASS IS A CLOSED
FORM, P, ON M CONSTRUCTED
FROM A GAUGE FIELD
$dP=0$
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on M
TRANSITION FNS
GROUP ELEMENT
$g_{ij}^{-1}(A(U_i)+d)g_{ij}$
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CHAR CLASS IS A CLOSED
FORM, P, ON M CONSTRUCTED
FROM A GAUGE FIELD STRENGTH
$dP=0 \xrightarrow{\text{LOCALLY}} P=dQ$
ex $P=\mathrm{Tr}F^n$ 2n-DIM'L M
$dP=0$
$\Omega=d\omega+\omega^2$
MUST HAVE $P(\Omega)=P(\Omega)$
$\omega=\omega_m$
FM
$\omega'=g\omega g+$


ex ω = A , Ω = F YM FIELD
GRAVITY
2-FM
FIELD
IN GENERAL
(i) Ω is ANY LIE-A
GAUGE 2FORM
(ii) dP(Ω) = 0 (CLOSED)
P(Ω) = TOPOLOGICAL
INV

ex $\omega = A$, $\Omega = F$ YM FIELD

ex $\omega = \Gamma^{q}_{b\,\mu}\,dx^{\mu}$ $\Omega = d\Gamma^{q}_{b} + \Gamma^{q}_{c} \wedge \Gamma^{c}_{b} = R^{q}_{b}$ GRAVITY 2-FM FIELD

↖ SPIN CONNECTION

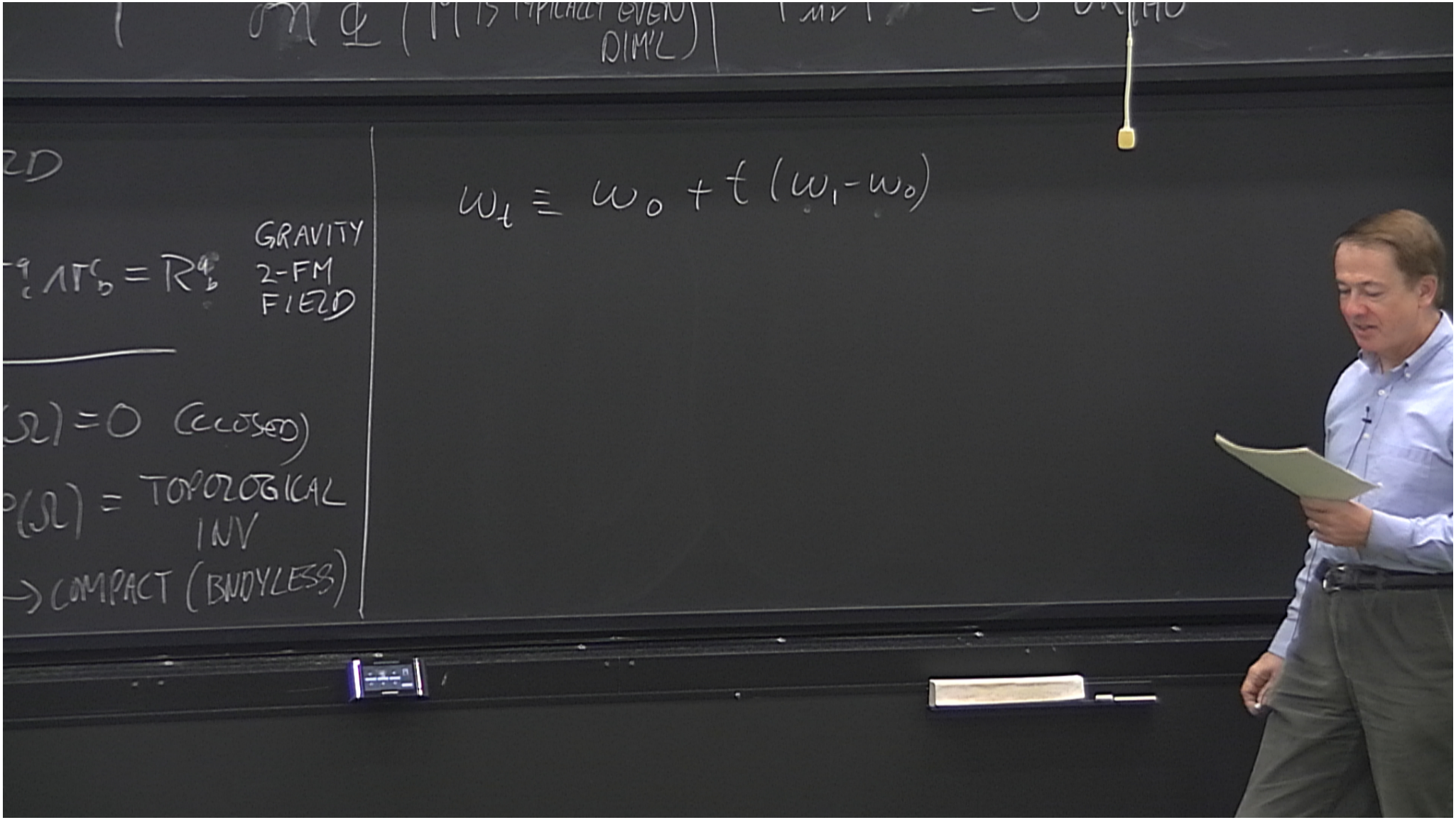
IN GENERAL

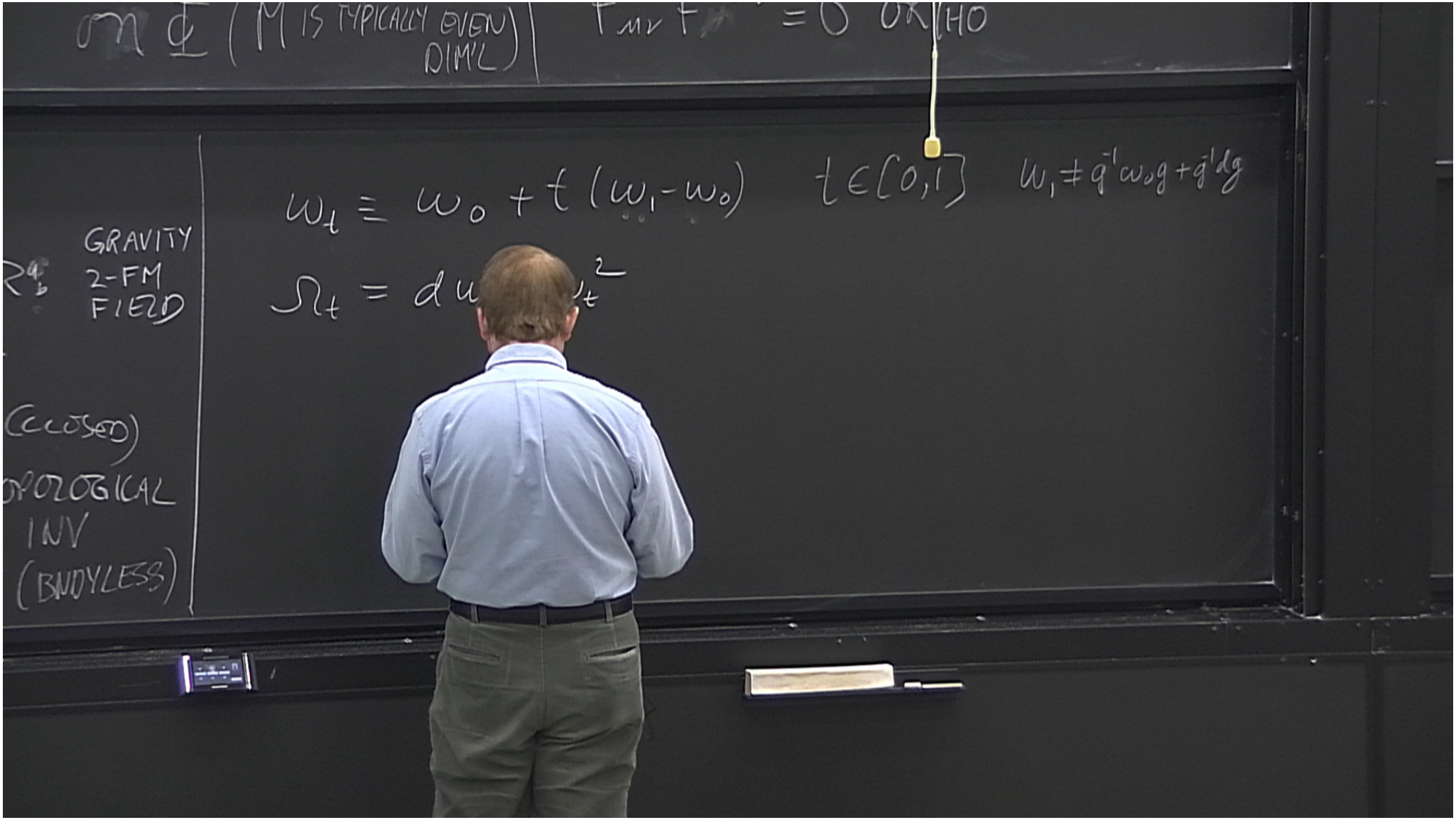
(i) Ω is ANY LIE-ALG VALUED GAUGE 2 FORM

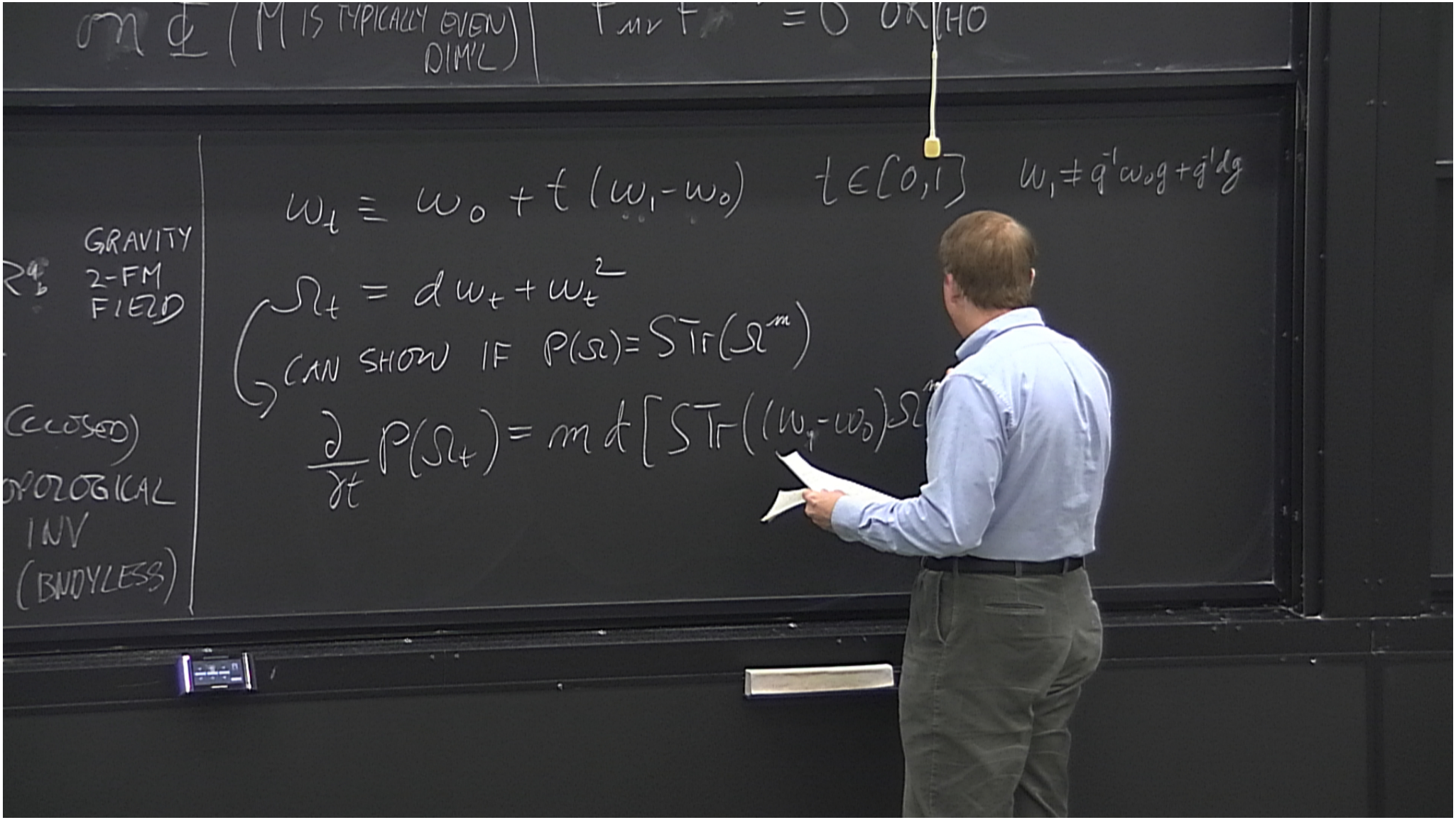
(ii) $dP(\Omega) = 0$ (CLOSED)

(iii) $\int_M P(\Omega)$ = TOPOLOGICAL INV

→ COMPACT (BNDYLESS)


$\omega_t \equiv \omega_0 + t(\omega_1 - \omega_0)$
GRAVITY
2-FM
FIELD
R^a_b
=0 (CLOSED)
TOPOLOGICAL
INV
COMPACT (BNDYLESS)


(M IS TYPICALLY EVEN DIM'L)
$\equiv 0$ OR (HO
$\omega_t \equiv \omega_0 + t(\omega_1 - \omega_0)$
$t \in [0,1]$
$\omega_1 \neq g^{-1}\omega_0 g + g^{-1}dg$
GRAVITY
2-FM
FIELD
(CLOSED)
TOPOLOGICAL
INV
(KNOTLESS)


M IS TYPICALLY EVEN DIM'L
$\omega_t \equiv \omega_0 + t(\omega_1 - \omega_0)$ $t \in [0,1]$ $\omega_1 \equiv g^{-1}\omega_0 g + g^{-1}dg$
GRAVITY
2-FM
FIELD
(CLOSED)
TOPOLOGICAL
INV
(BUDYLESS)
$\Omega_t = d\omega_t + \omega_t^2$
CAN SHOW IF $P(\Omega) = STr(\Omega^m)$
$\frac{\partial}{\partial t} P(\Omega_t) = m\, d[STr(($

$\mathcal{M}$ (M IS TYPICALLY EVEN DIM'L) | $F_{\mu\nu} + \ldots = 0$ OR (HO

GRAVITY
2-FM
FIELD

(CLOSED)
TOPOLOGICAL
INV
(BNDYLESS)

$\omega_t \equiv \omega_0 + t(\omega_1 - \omega_0) \quad t \in [0,1] \quad \omega_1 \neq g^{-1}\omega_0 g + g^{-1}dg$

$\Omega_t = d\omega_t + \omega_t^2$

↳ CAN SHOW IF $P(\Omega) = \mathrm{STr}(\Omega^m)$

$$\frac{\partial}{\partial t} P(\Omega_t) = m\, d\left[\mathrm{STr}\left((\omega_1 - \omega_0)\Omega_t^{m-1}\right)\right]$$

$$\Rightarrow \int_M P(\Omega_1) = \int_M P(\Omega_0)$$

$\mathcal{M}$ (M IS TYPICALLY EVEN DIM'L) | $F_{\mu\nu} + \ldots = 0$ OR (HO

GRAVITY
2-FM
FIELD

(CLOSED)
TOPOLOGICAL
INV
(BNDYLESS)

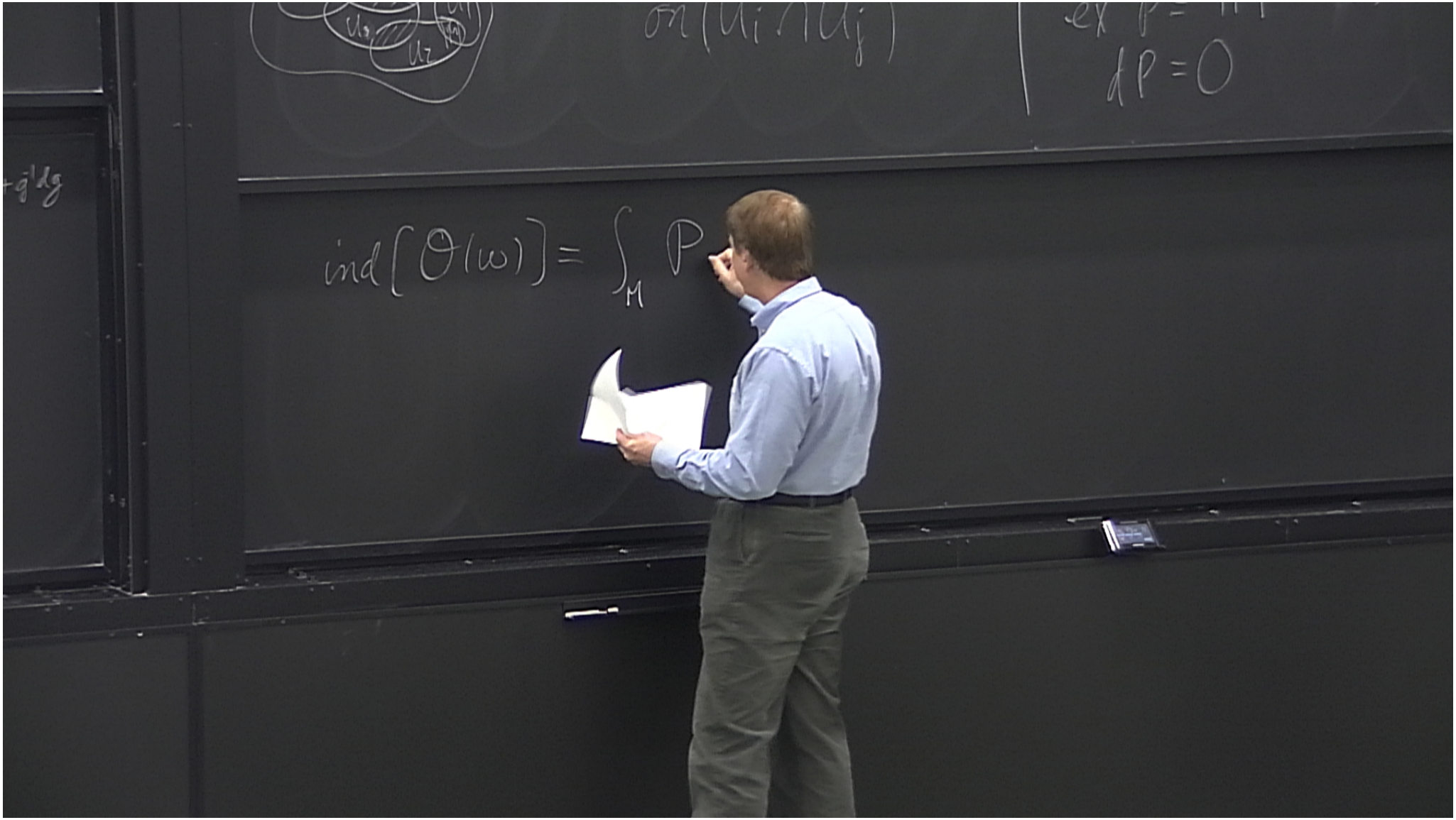
$\omega_t \equiv \omega_0 + t(\omega_1 - \omega_0) \quad t \in [0,1] \quad \omega_1 \neq g^{-1}\omega_0 g + g^{-1}dg$

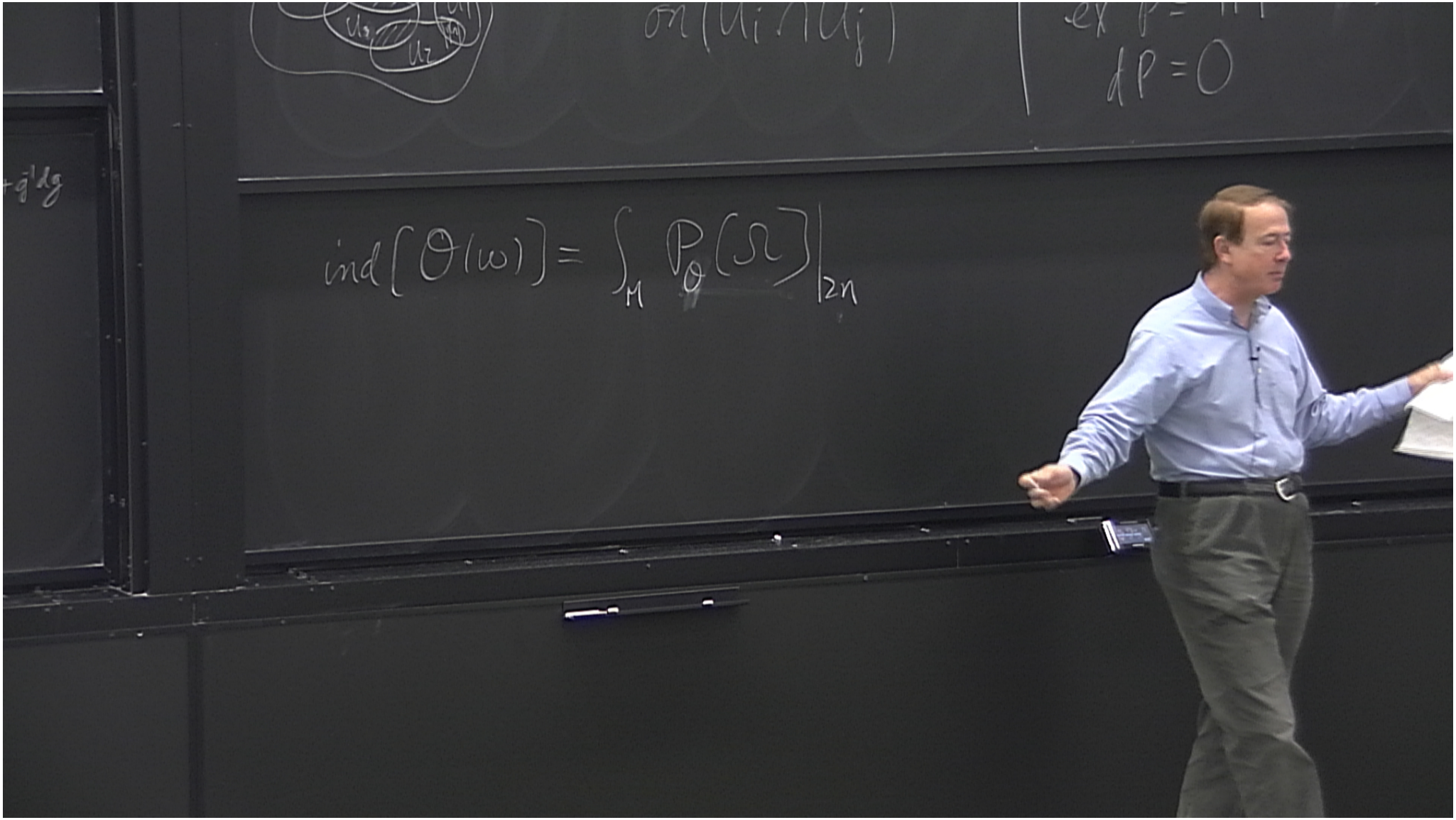
$\Omega_t = d\omega_t + \omega_t^2$

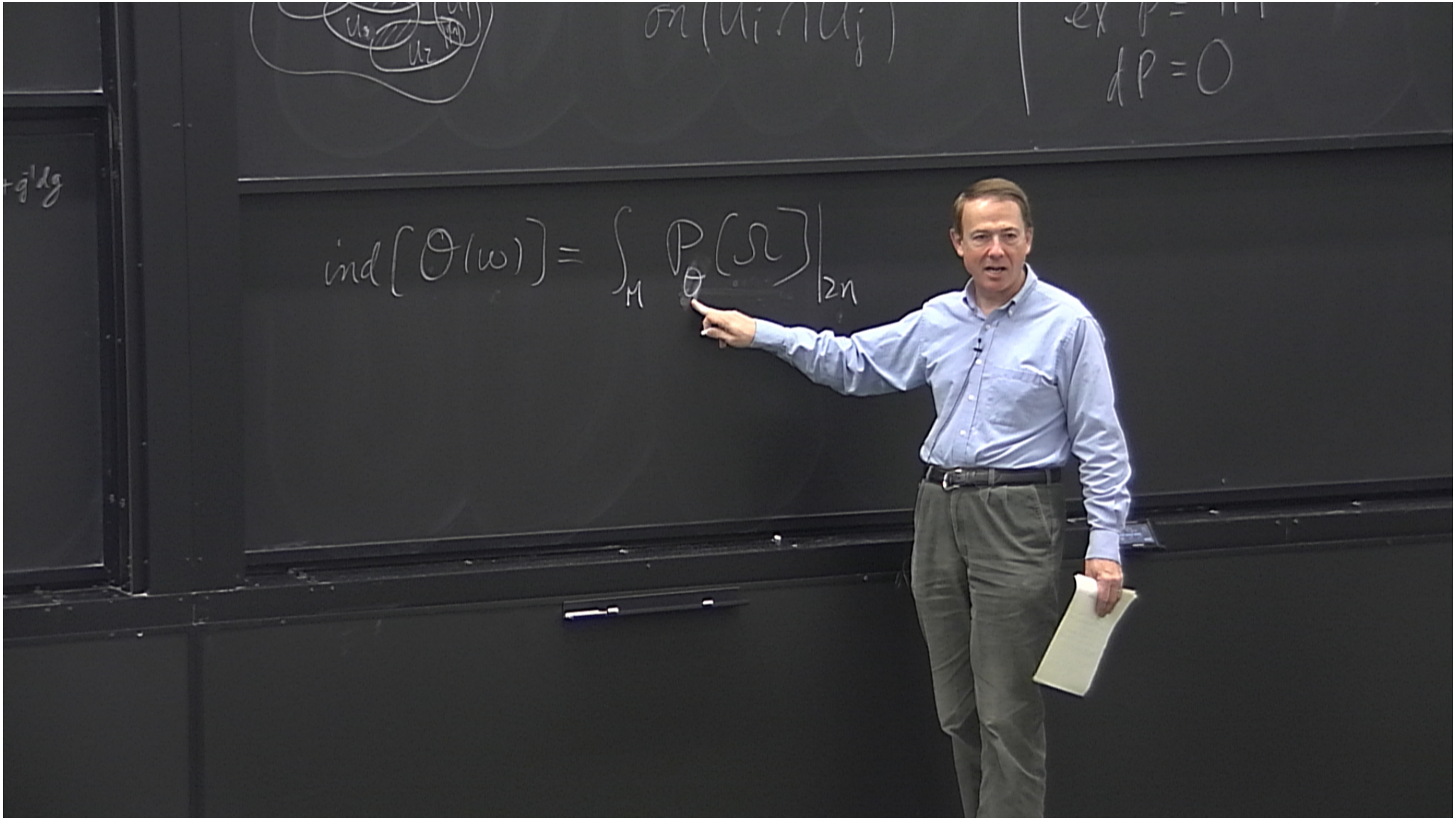
CAN SHOW IF $P(\Omega) = \mathrm{STr}(\Omega^m)$

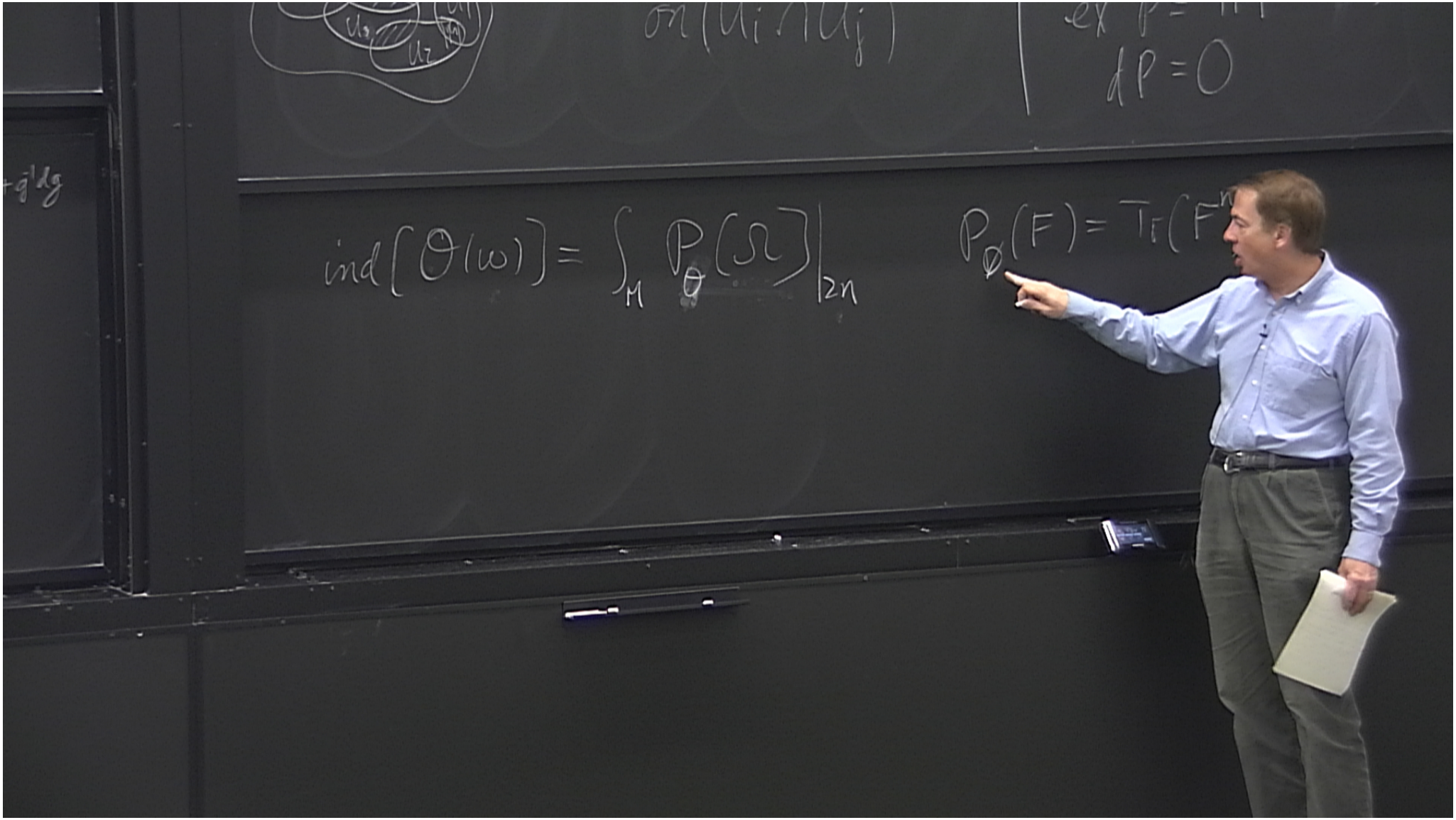
$\frac{\partial}{\partial t} P(\Omega_t) = m\, d\left[\mathrm{STr}\left((\omega_1 - \omega_0)\Omega_t^{m-1}\right)\right]$

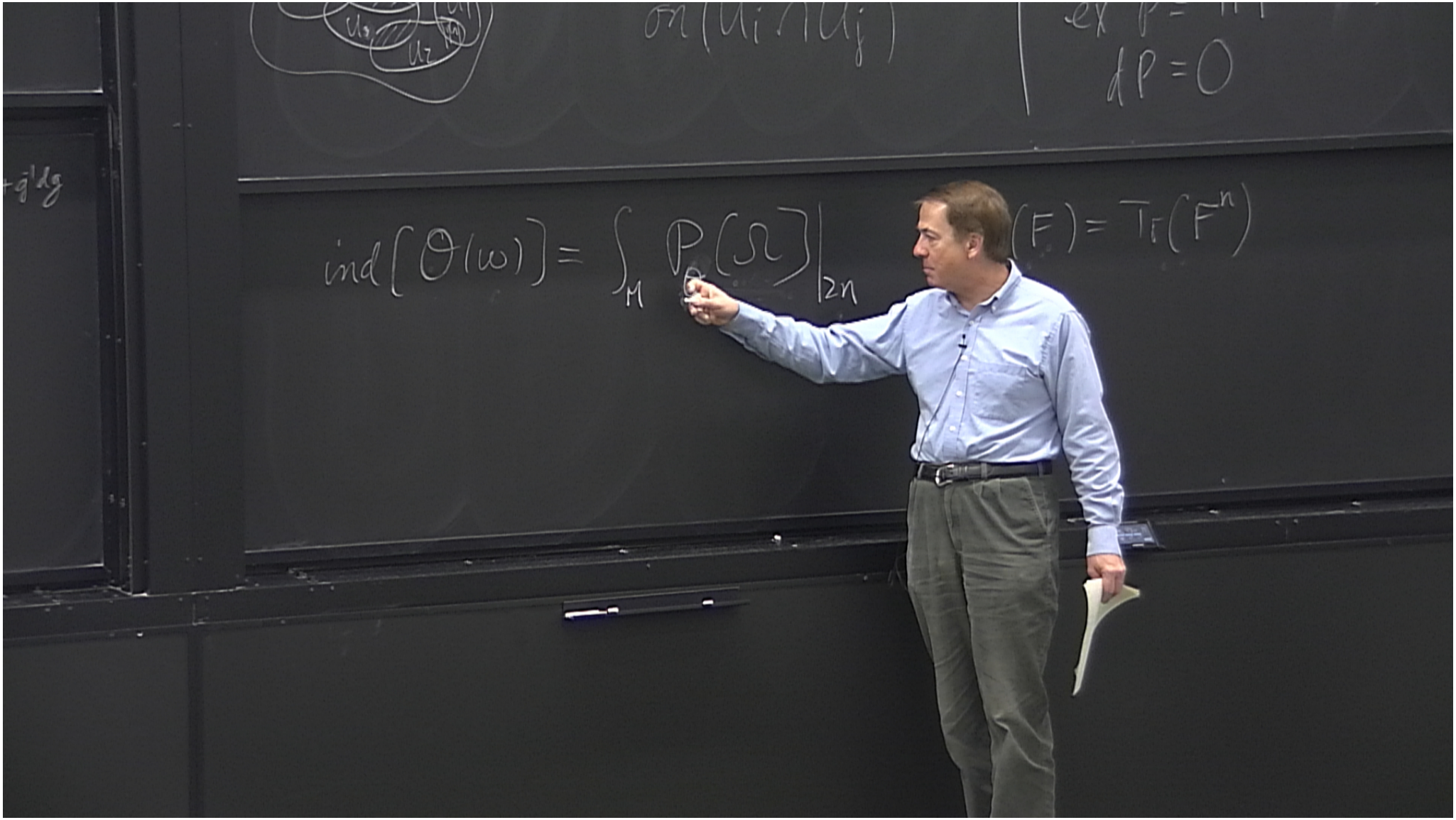
$\Rightarrow \int_M P(\Omega_1) = \int_M P(\Omega_0)$

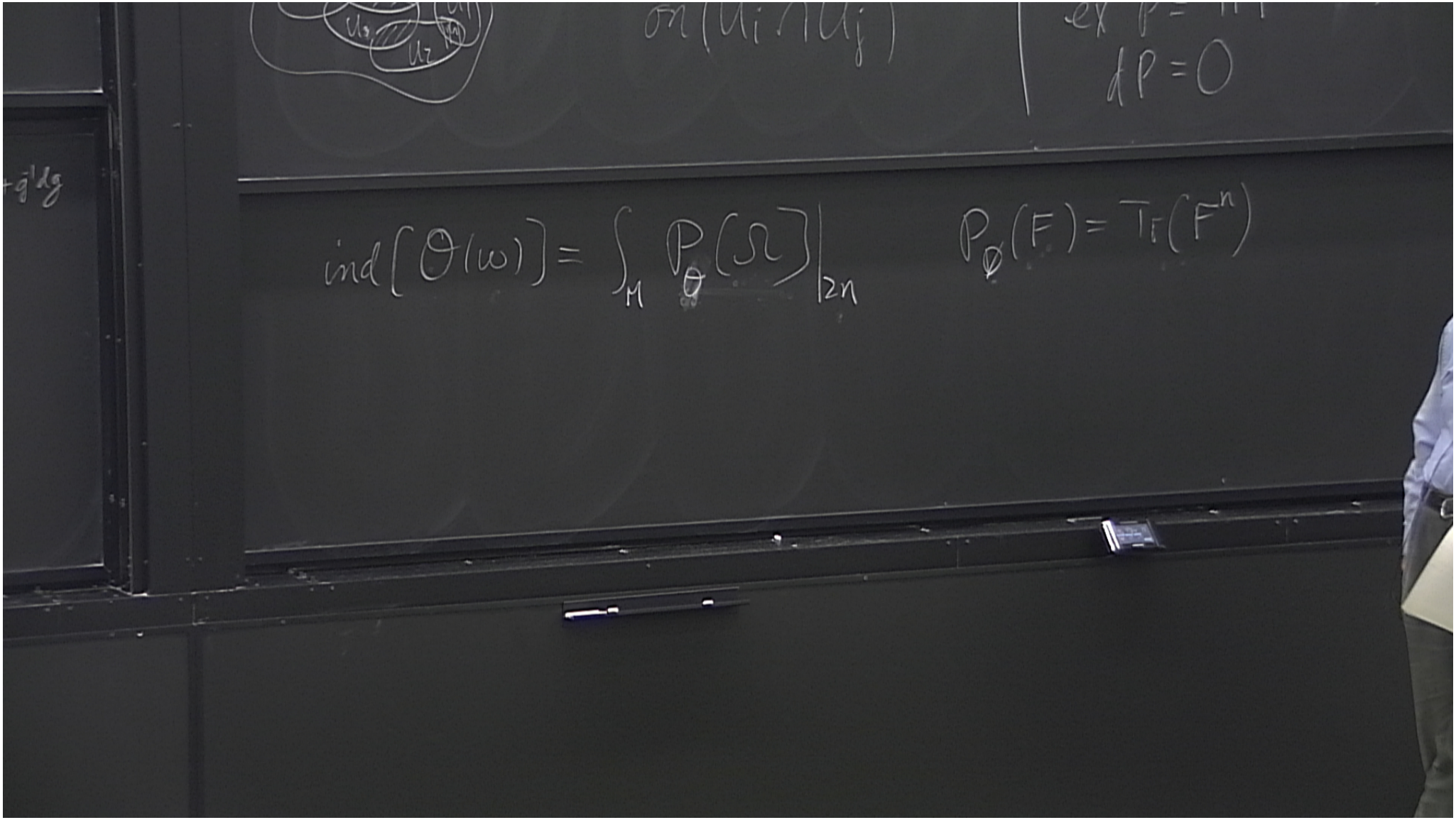

ind[Θ(ω)] = ∫M P
dP = 0

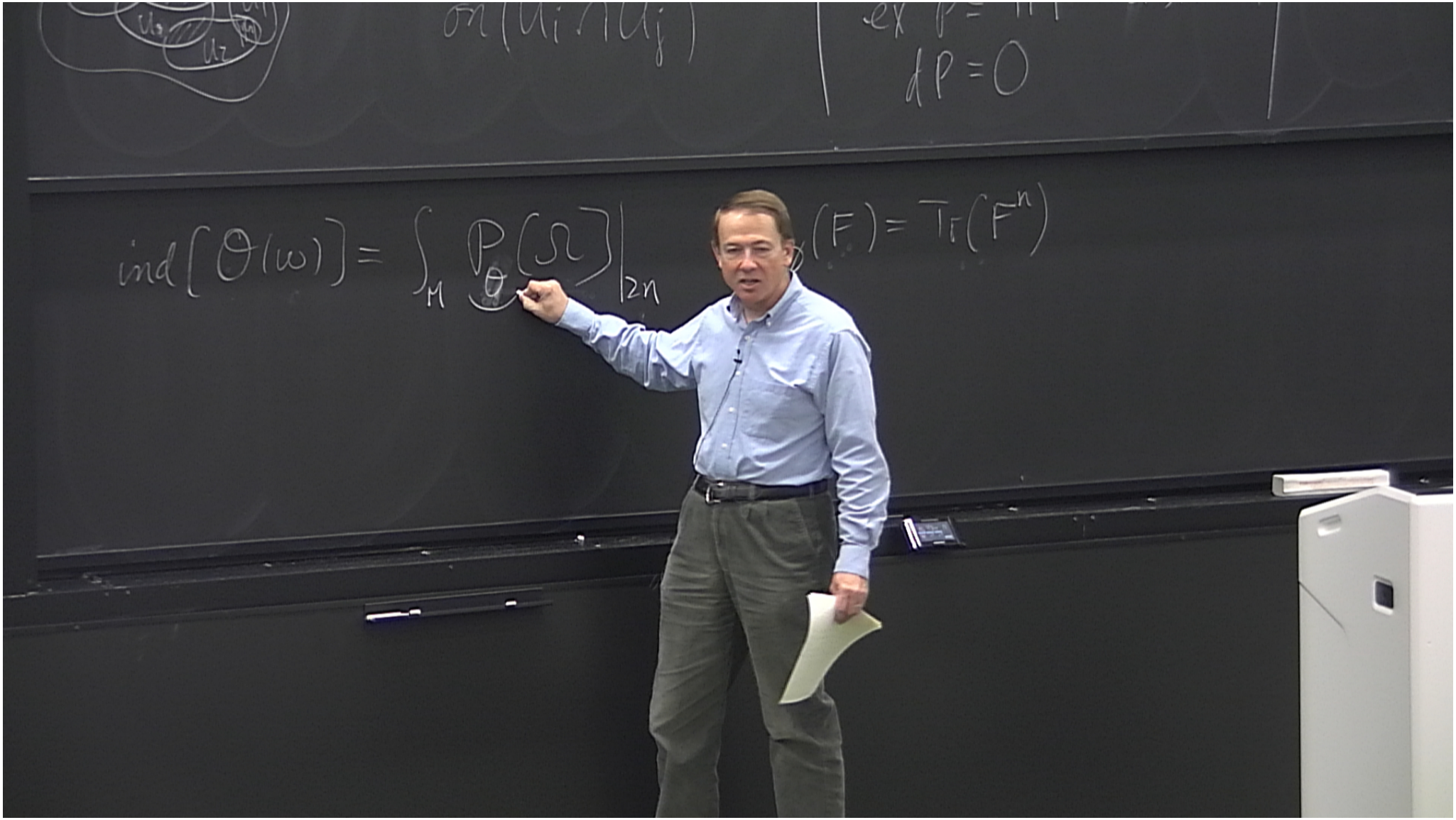

$\mathrm{ind}[\Theta(\omega)] = \int_M P_\Theta(\Omega)\Big|_{2n}$
$dP = 0$

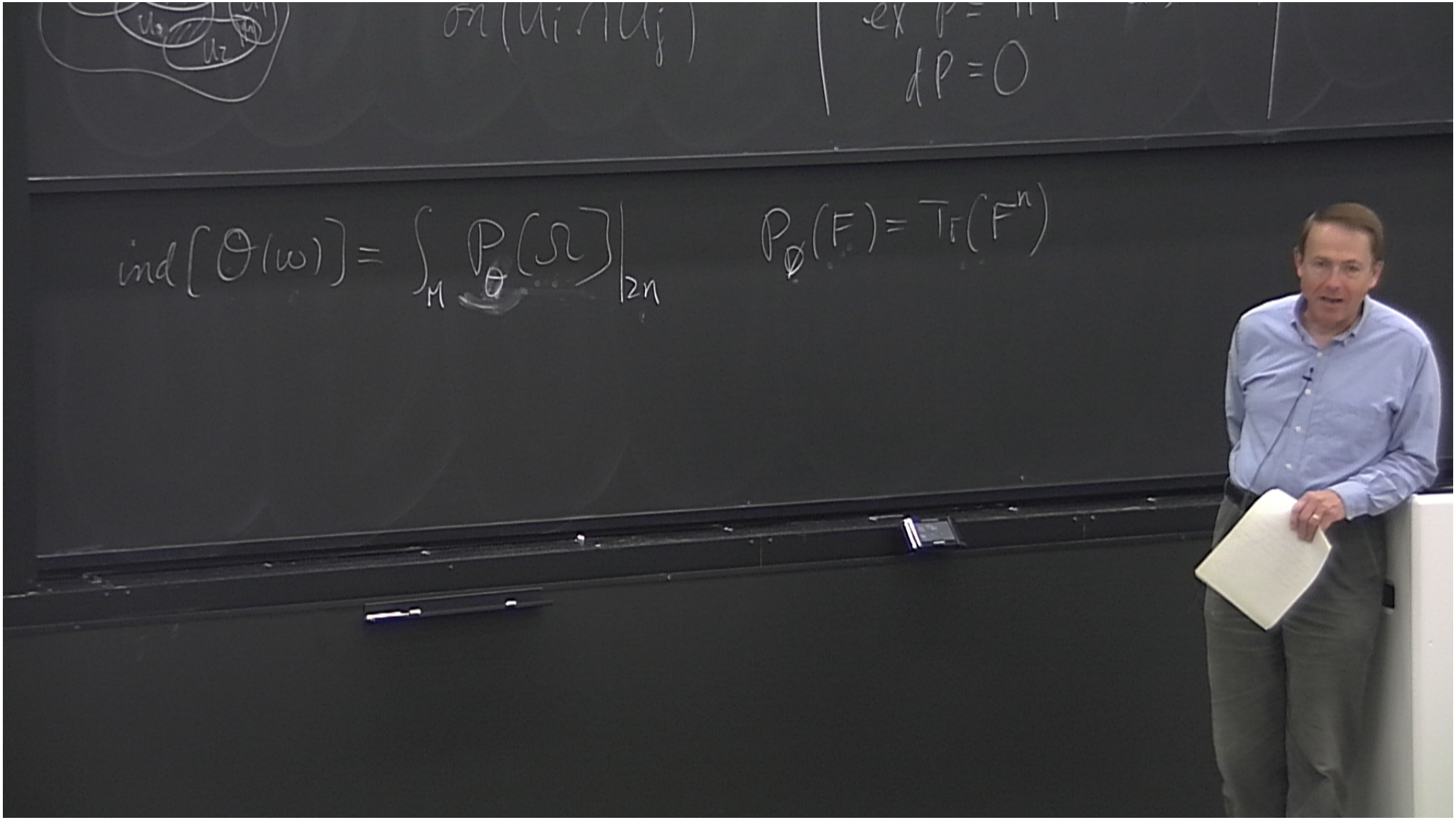

$\mathrm{ind}[\theta(\omega)] = \int_M P_\theta(\Omega)\Big|_{2n}$
$dP = 0$

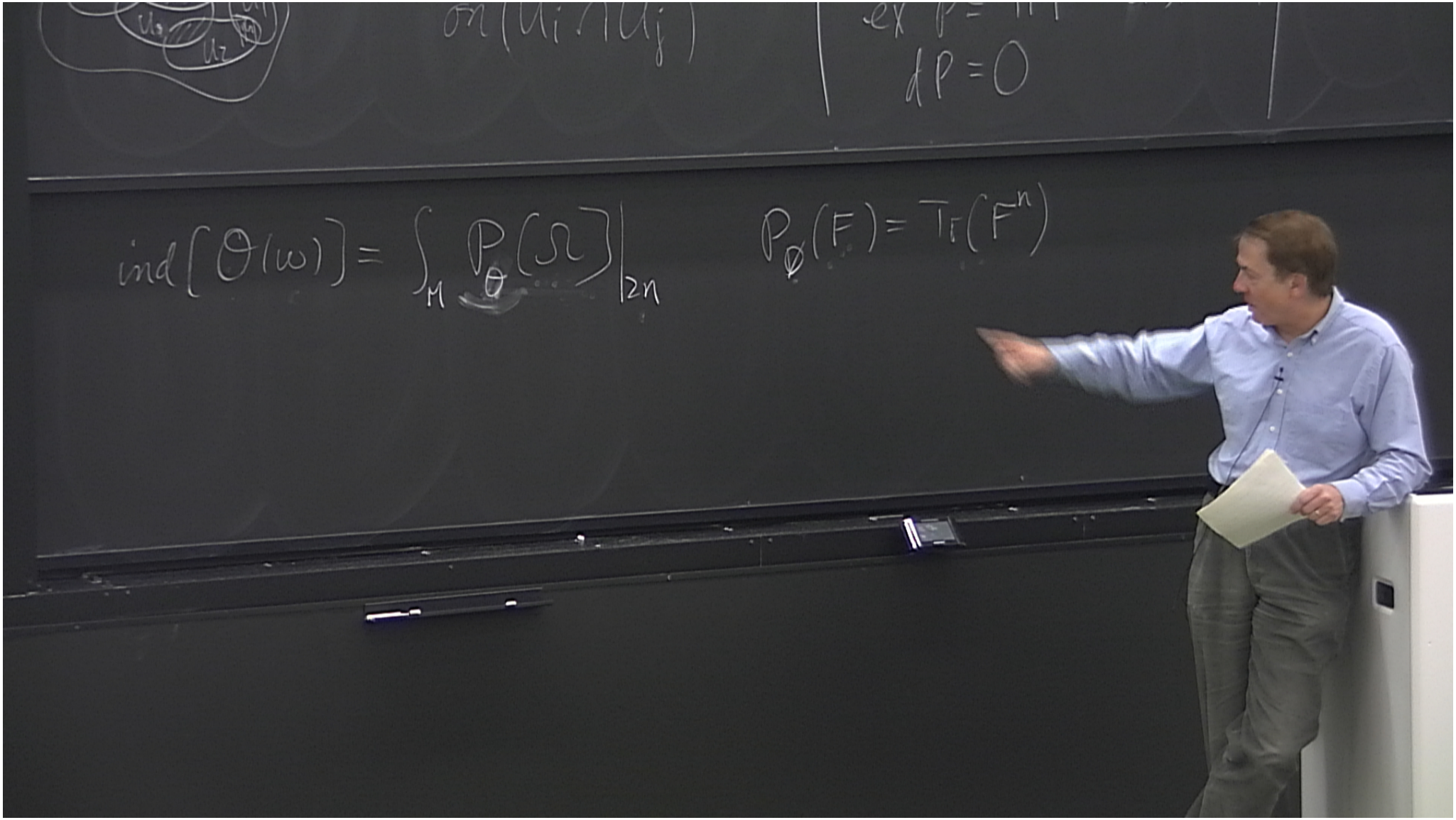

$\mathrm{ind}[\theta(\omega)] = \int_M P_\theta(\Omega)\Big|_{2n}$
$P_\emptyset(F) = \mathrm{Tr}(F^$
$dP = 0$


$\mathrm{ind}[\theta(\omega)] = \int_M P(\Omega)\Big|_{2n}$
$(F) = T_i(F^n)$
$dP = 0$


$\mathrm{ind}[\theta(\omega)] = \int_M P_\theta(\Omega)\Big|_{2n}$
$P_\emptyset(F) = T_i(F^n)$
$dP = 0$


$dP = 0$
$\mathrm{ind}[\Theta(\omega)] = \int_M \Theta(\Omega)|_{2n}$
$(F) = T_i(F^n)$


$ind[\Theta(\omega)] = \int_M P_\theta(\Omega)\Big|_{2n}$
$P_\emptyset(F) = T_i(F^n)$
$dP = 0$


$ind[\theta(\omega)] = \int_M P_\theta(\Omega)|_{2n}$
$P_\emptyset(F) = T_i(F^n)$
$dP = 0$