Title: 13/14 PSI - Beyond the Standard Model - Lecture 4

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Abstract:

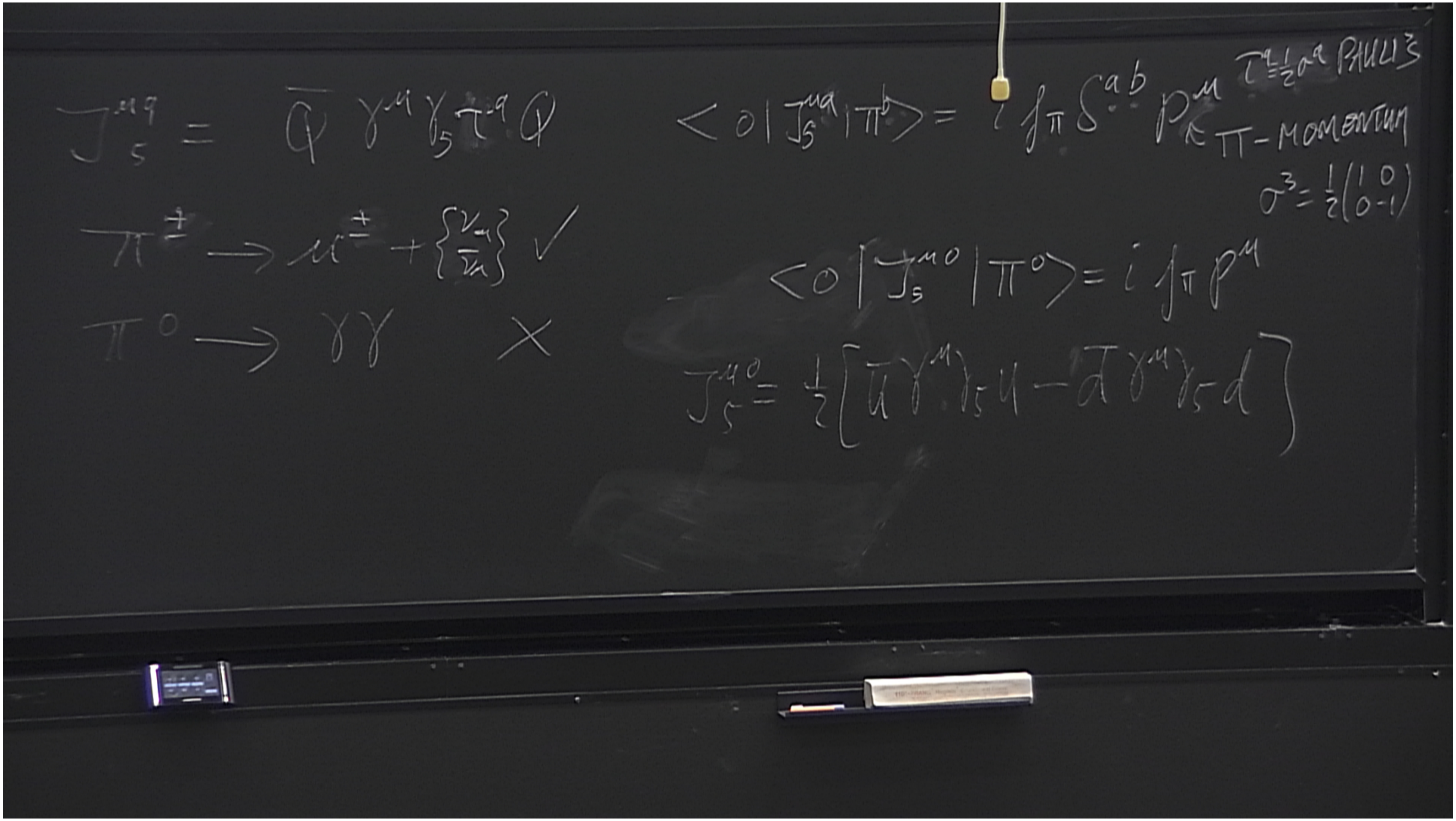
IMPLICATIONS of ANOMALIES

(A) PION DECAY $\pi^0 \to 2\gamma$

CURRENT ALG &
MASSLESS QCD $\to SU_L(2) \otimes SU_R(2)$

$\pi^a \to$ GOLDSTONE BOSONS
ASSOCIATED WITH $SU_L(2) \otimes SU_R(2)$
$m_\pi \simeq 0$ $\longrightarrow SU(2)$

$J_5^{\mu a} =$


$J_5^{\mu a} = \bar{Q} \gamma^\mu \gamma_5 \tau^a Q$
$\pi^\pm \to \mu^\pm + \{\nu_\mu, \bar{\nu}_\mu\}$ ✓
$\pi^0 \to \gamma\gamma$ ×
$\tau^a = \frac{1}{2}\sigma^a$ PAULI'S
$\langle 0 | J_5^{\mu a} | \pi^b \rangle = i f_\pi \delta^{ab} p^\mu$
π-MOMENTUM
$\sigma^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\langle 0 | J_5^{\mu 0} | \pi^0 \rangle = i f_\pi p^\mu$
$J_5^{\mu 0} = \frac{1}{2}\left[\bar{u}\gamma^\mu\gamma_5 u - \bar{d}\gamma^\mu\gamma_5 d\right]$

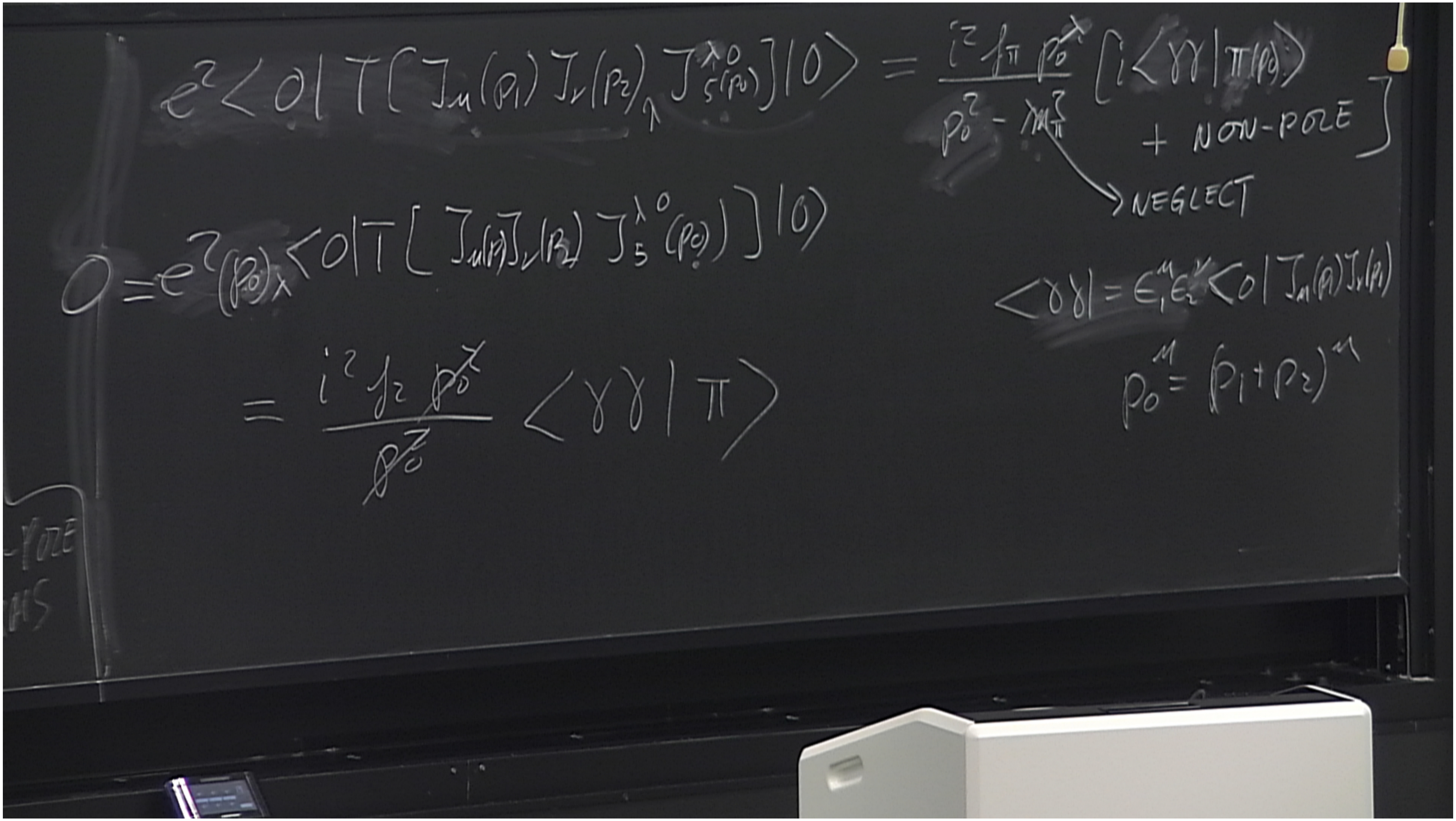
$$\langle \gamma\gamma | \pi \rangle = ?$$

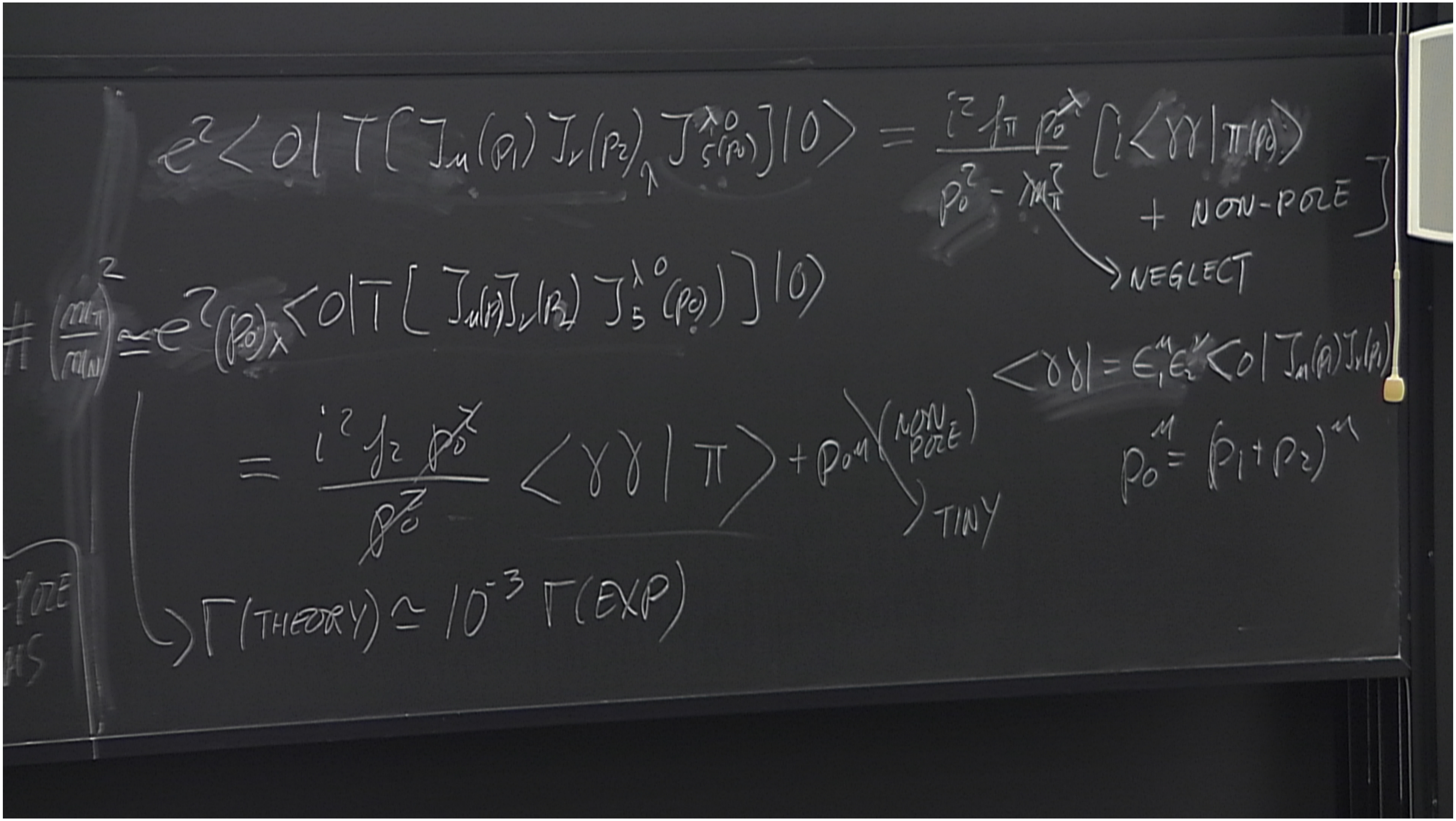
LSZ THM

$$\langle 0 | T[\varphi(k_1) \cdots \varphi(k_n)] | 0 \rangle$$

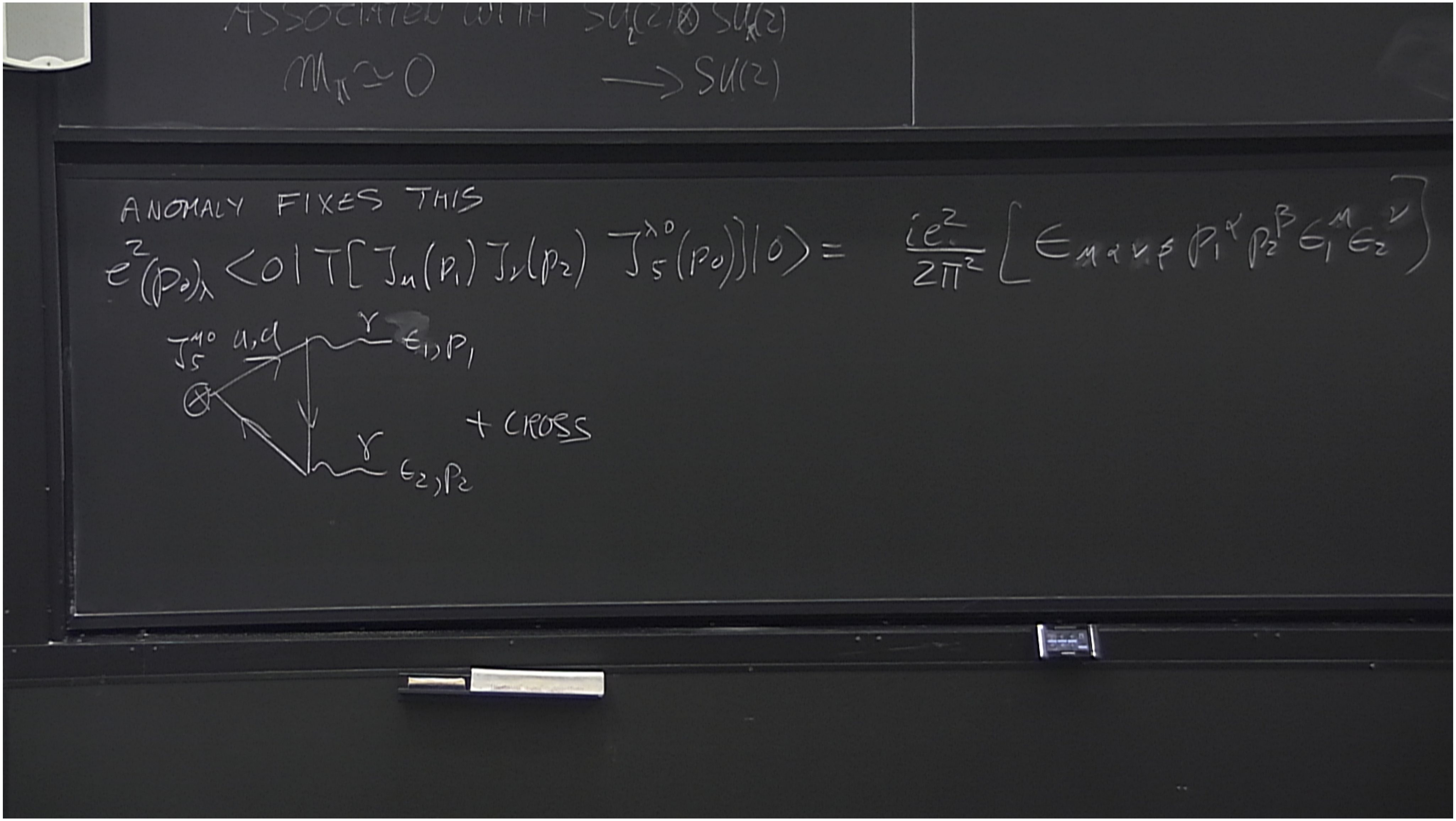
$$= \sum_{I,p} \langle 0 | \varphi(k_1) | \psi^I(p) \rangle \langle \psi^I(p) | \varphi(k_2) \cdots \varphi(k_n) | 0 \rangle$$

$$= i \frac{\langle 0 | \varphi(0) | \psi(k_1) \rangle}{k_1^2 - m_\psi^2} \langle \psi^I(k_1) | \varphi(k_2) \cdots \varphi(k_n) | 0 \rangle + \text{NON-POLE TERMS}$$


e^2 <0| T[J_μ(p_1) J_ν(p_2) J_5^{λ0}(p_0)] |0> = i^2 f_π p_0^λ / (p_0^2 − m_π^2) [i <γγ|π(p_0)>
+ NON-POLE]
NEGLECT
0 = e^2 (p_0)_λ <0|T[J_μ(p_1) J_ν(p_2) J_5^{λ0}(p_0)] |0>
= i^2 f_π p_0^λ / p_0^2 <γγ|π>
<γγ| = ε_1^μ ε_2^ν <0| J_μ(p_1) J_ν(p_1)
p_0^μ = (p_1 + p_2)^μ

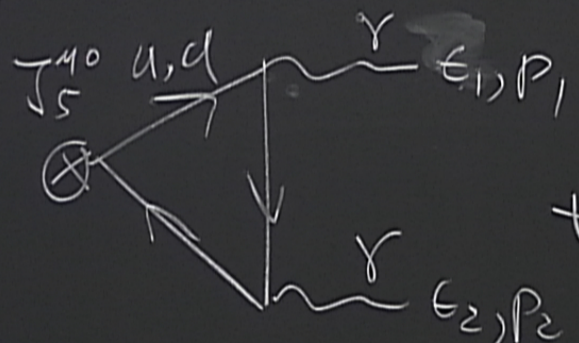


$e^2 \langle 0 | T[J_\mu(p_1) J_\nu(p_2) J_5^{\lambda 0}(p_0)] | 0 \rangle = \frac{i^2 f_\pi p_0^\lambda}{p_0^2 - m_\pi^2} \left[i \langle \gamma\gamma | \pi(p_0) \rangle + \text{NON-POLE} \right]$
NEGLECT
$\# \left(\frac{m_\pi}{m_N}\right)^2 \simeq e^2 (p_0)_\lambda \langle 0|T[J_\mu(p_1) J_\nu(p_2) J_5^{\lambda 0}(p_0)]|0\rangle$
$= \frac{i^2 f_\pi p_0^\lambda}{p_0^2} \langle \gamma\gamma | \pi \rangle + p_{0\mu} (\text{NON-POLE})$
TINY
$\Gamma(\text{THEORY}) \simeq 10^{-3}\, \Gamma(\text{EXP})$
$\langle \gamma\gamma | = \epsilon_1^\mu \epsilon_2^\nu \langle 0 | J_\mu(p_1) J_\nu(p_2)$
$p_0^\mu = (p_1 + p_2)^\mu$


ASSOCIATED WITH $SU_L(2) \otimes SU_R(2)$
$m_\pi \simeq 0 \longrightarrow SU(2)$
ANOMALY FIXES THIS
$e^2 (p_0)_\lambda \langle 0 | T[J_\mu(p_1) J_\nu(p_2) J_5^{\lambda 0}(p_0)] | 0 \rangle = \frac{ie^2}{2\pi^2} [\epsilon_{\mu\alpha\nu\beta} p_1^\alpha p_2^\beta \epsilon_1^\mu \epsilon_2^\nu]$
$J_5^{\mu 0}$
u, d
γ
ϵ_1, p_1
ϵ_2, p_2
+ CROSS

ANOMALY FIXES THIS

$$e^2 (p_0)_\lambda \langle 0 | T[J_\mu(p_1) J_\nu(p_2) J_5^{\lambda 0}(p_0)] | 0 \rangle = \frac{ie^2}{2\pi^2}\left[\epsilon_{\mu\alpha \ldots} \right.$$



+ CROSS

$$\Gamma = \frac{1}{2!}\left(\frac{1}{2 m_\pi}\right)\int \ldots$$

SU(2)

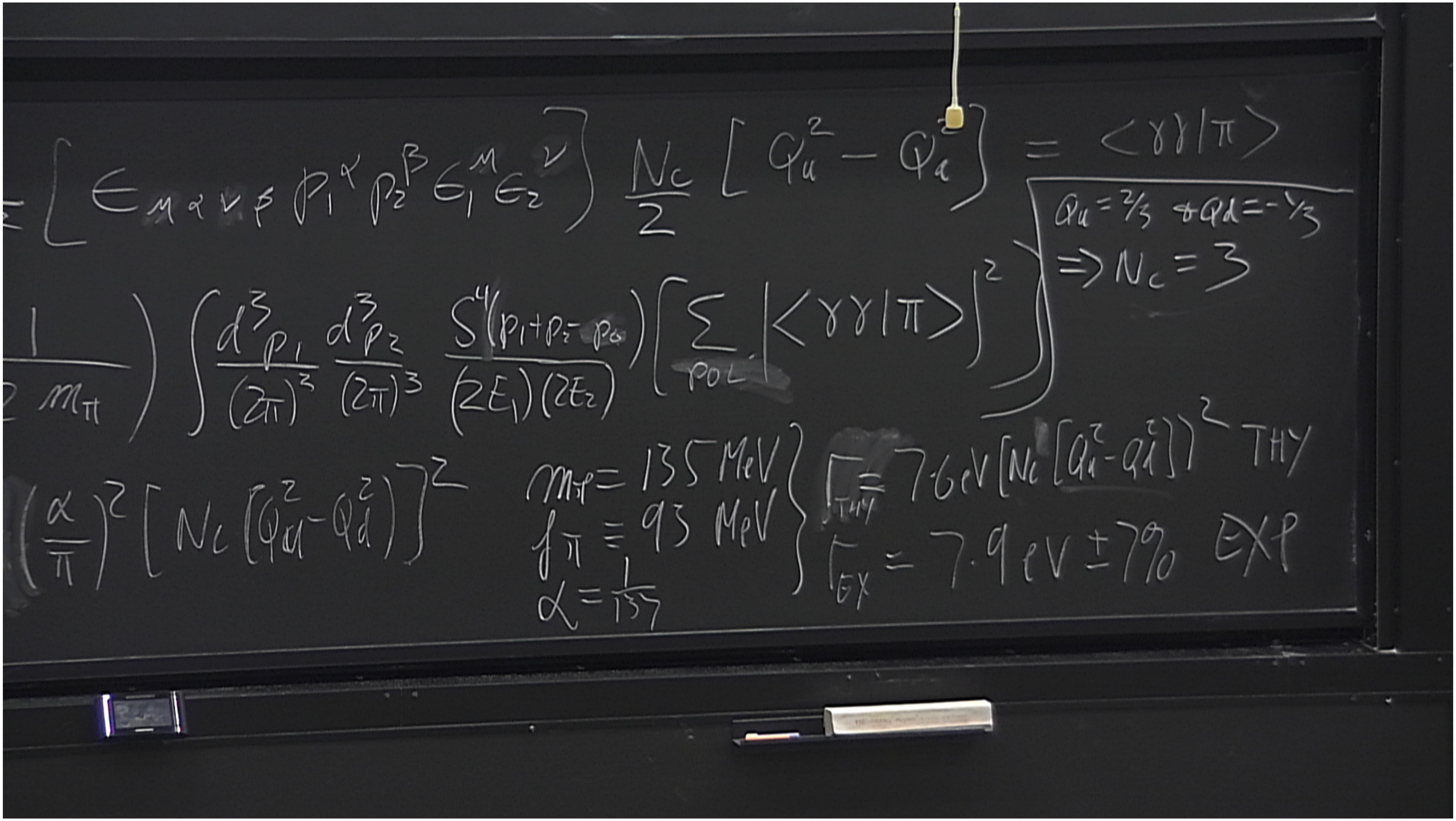
$$(p_0)]|0\rangle = \frac{ie^2}{2\pi^2}\left[\epsilon_{\mu\alpha\nu\beta}\, p_1^{\alpha} p_2^{\beta} \epsilon_1^{\mu} \epsilon_2^{\nu}\right] \frac{N_c}{2}\left[Q_u^2 - Q_d^2\right] = \langle\gamma\gamma|\pi\rangle$$

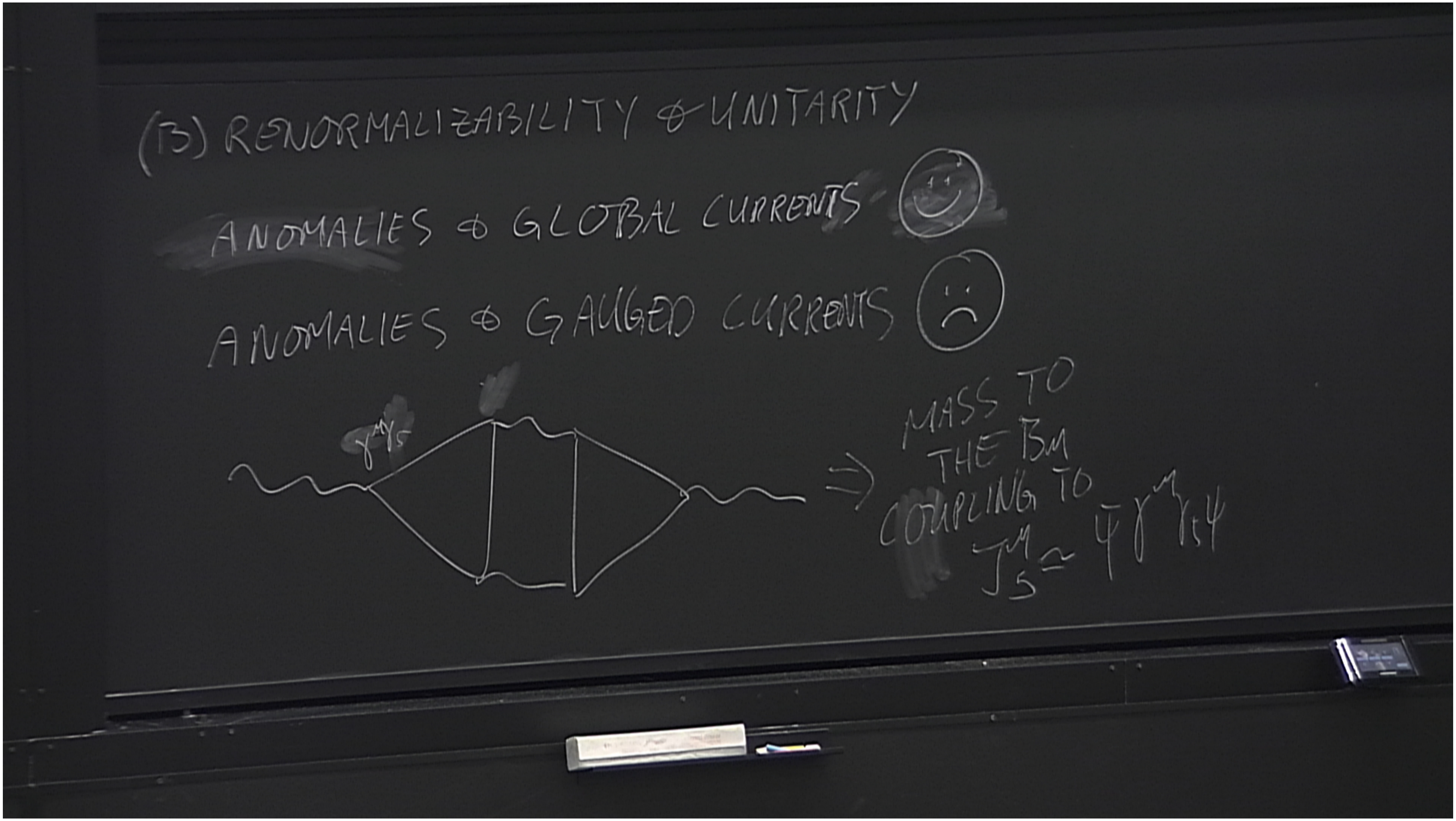
$$\Gamma = \frac{1}{2!}\left(\frac{1}{2 m_\pi}\right)\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{\delta^4(p_1+p_2-p_0)}{(2E_1)(2E_2)}\left[\sum_{pol}\left|\langle\gamma\gamma|\pi\rangle\right|^2\right]$$

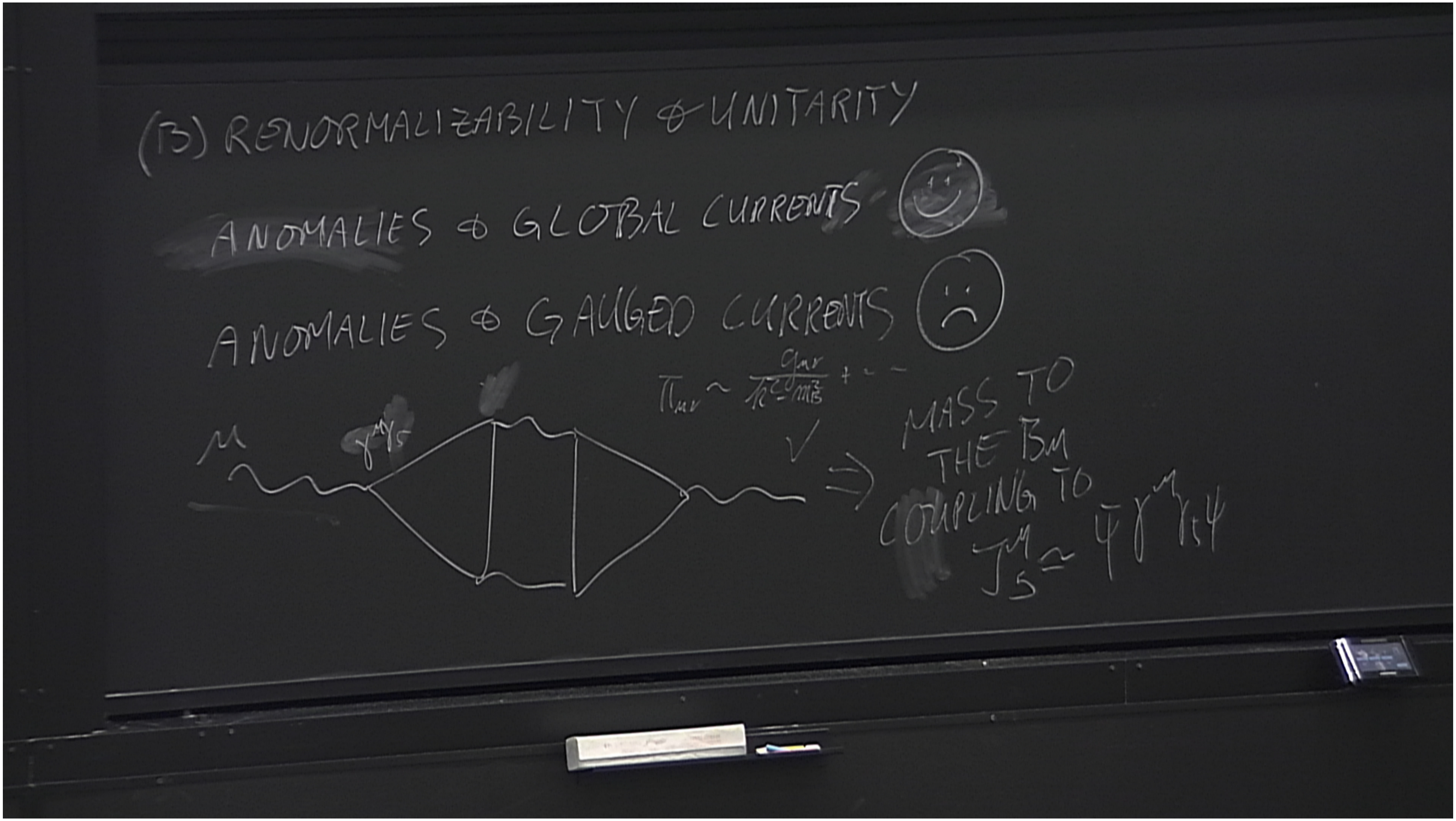
$$= \frac{m_\pi^3}{64\pi f_\pi^2}\left(\frac{\alpha}{\pi}\right)^2\left[N_c\left(Q_u^2 - Q_d^2\right)\right]^2$$

$m_\pi = 135$ MeV
$f_\pi = 93$ MeV
$\alpha = \frac{1}{137}$

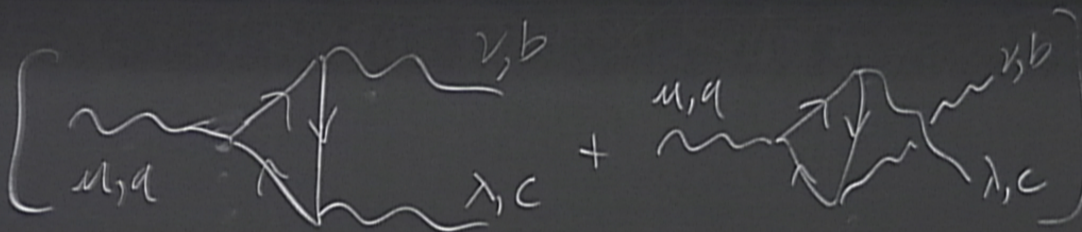
$\Gamma_{THY} = 7.6\,\text{eV}\left[N_c^2\left[Q_u^2 - Q_d^2\right]\right]^2$ THY
$\Gamma_{EX} = 7.9\,\text{eV} \pm 7\%$ EXP


[∈ μανβ p1^α p2^β ∈1^μ ∈2^ν] Nc/2 [Qu² − Qd²] = <γγ|π>
Qu = 2/3 + Qd = −1/3
⇒ Nc = 3
1/(2 mπ) ∫ d³p1/(2π)³ d³p2/(2π)³ δ⁴(p1+p2−pπ)/((2E1)(2E2)) [Σ_POL |<γγ|π>|²]
(α/π)² [Nc (Qu² − Qd²)]²
mπ = 135 MeV
fπ = 93 MeV
α = 1/137
Γ_THY = 7.6 eV [Nc [Qu² − Qd²]]² THY
Γ_EX = 7.9 eV ± 7% EXP


(B) RENORMALIZABILITY & UNITARITY
ANOMALIES & GLOBAL CURRENTS
ANOMALIES & GAUGED CURRENTS
$\gamma^\mu \gamma_5$
⇒ MASS TO
THE BM
COUPLING TO
$J_5^\mu \simeq \bar{\psi} \gamma^\mu \gamma_5 \psi$


(B) RENORMALIZABILITY & UNITARITY
ANOMALIES & GLOBAL CURRENTS
ANOMALIES & GAUGED CURRENTS
MASS TO
THE Bμ
COUPLING TO

$-ip_0^\mu$ [



]

$$\sim \mathrm{Tr}[T^a T^b T^c] + \mathrm{Tr}[T^a T^c T^b]\ \epsilon_{\nu\alpha\lambda\beta}\, p_1^\alpha p_2^\beta$$

$$\sim \mathrm{Tr}[T^a \{T^b, T^c\}]$$

RENORM OK $\Rightarrow A^{abc} = \mathrm{Tr}\left[T^a\{T^b, T^c\}\right] = 0$

IN TERMS of LH & RH FERMIONS

$$A^{abc} = \mathrm{Tr}\left[T_L^a\{T_L^b, T_L^c\}\right] - \mathrm{Tr}\left[T_R^a\{T_R^b, T_R^c\}\right]$$

HOW TO GET $A^{abc} = 0$?

$f_\pi = 93\ \mathrm{MeV}$

$\alpha = \frac{1}{137}$

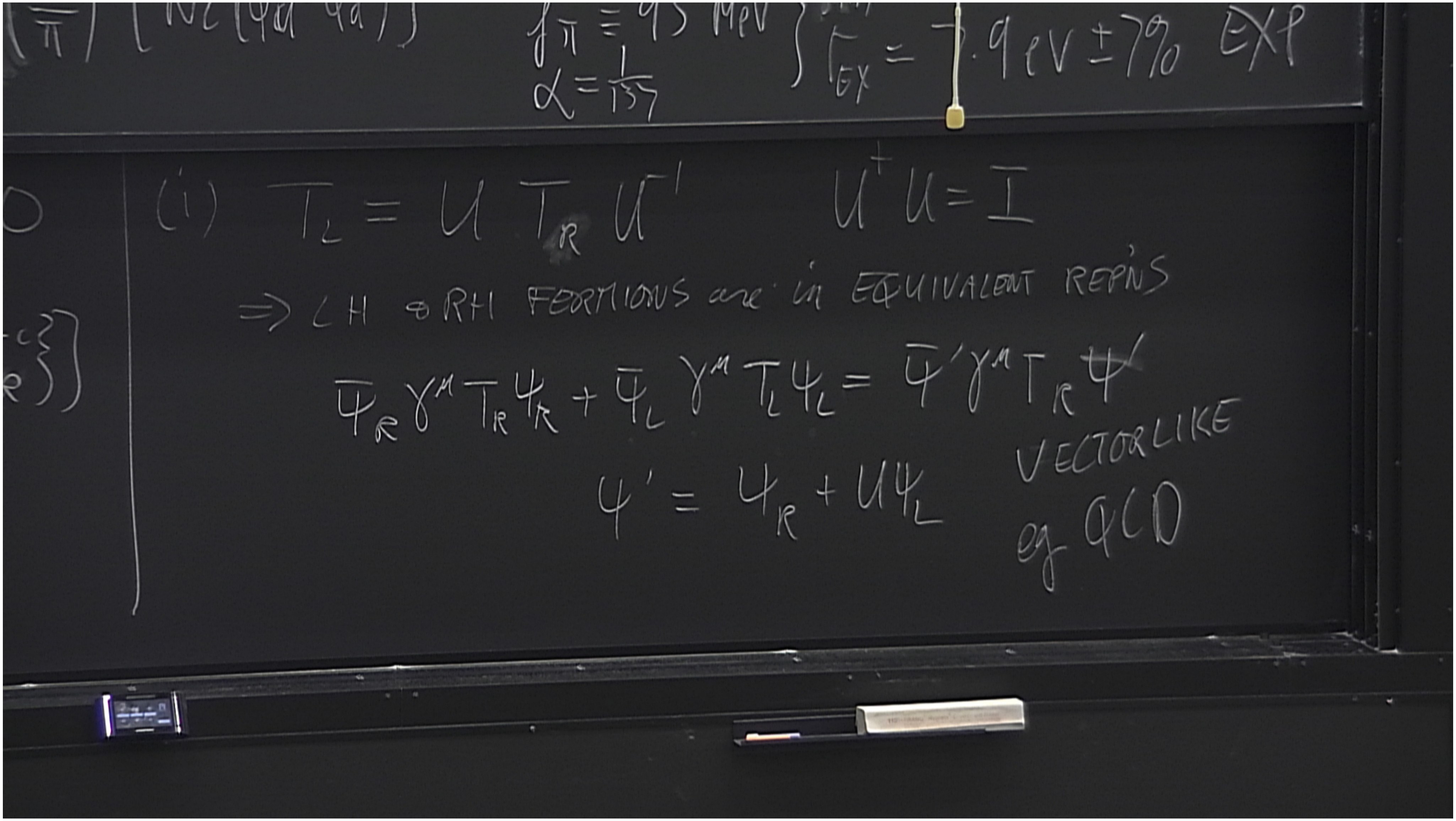
$\Gamma_{EX} = 7.9\ \mathrm{eV} \pm 7\%$ EXP

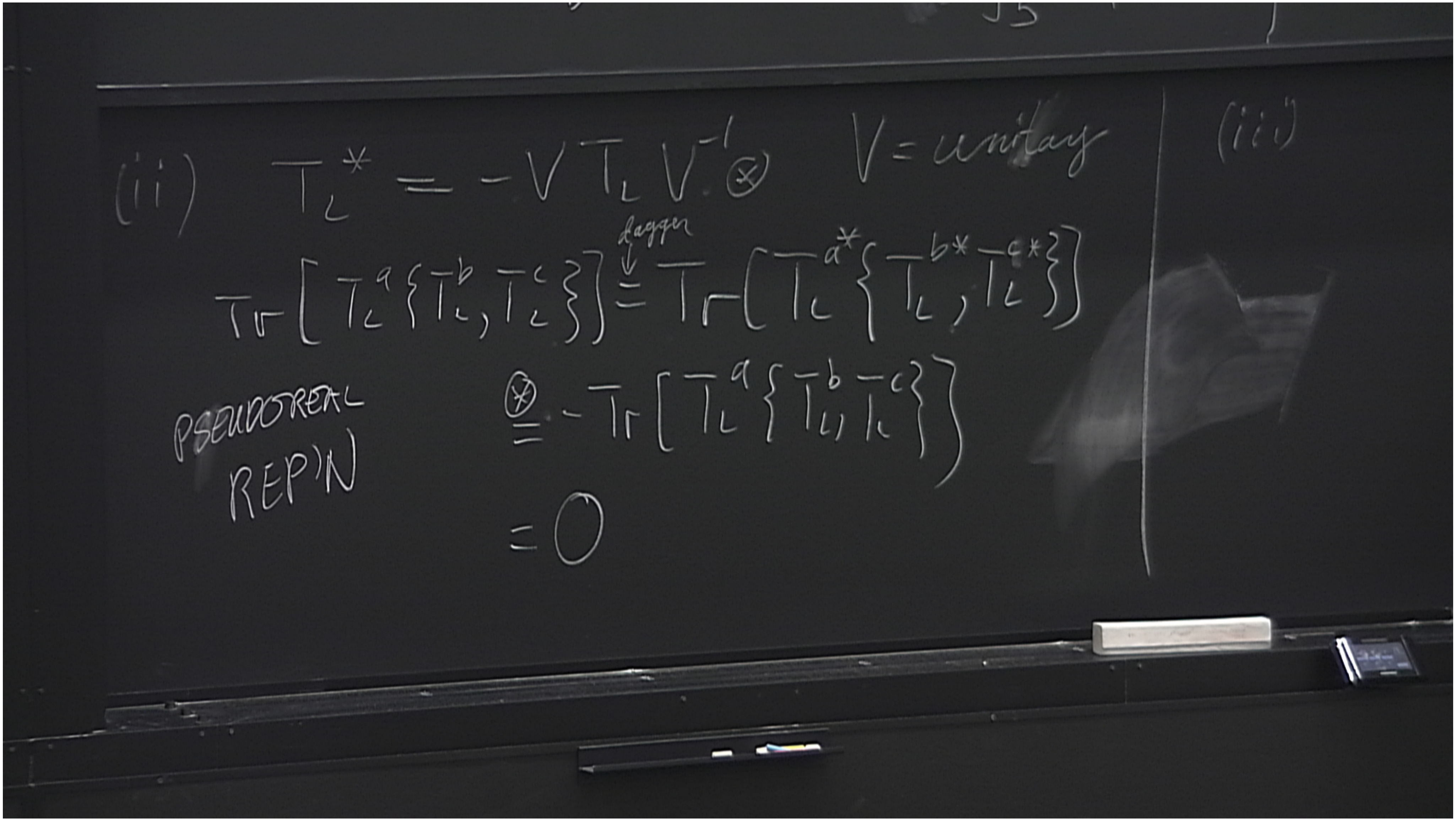
(i) $T_L = U\, T_R\, U^{-1}$ $\qquad U^\dagger U = I$

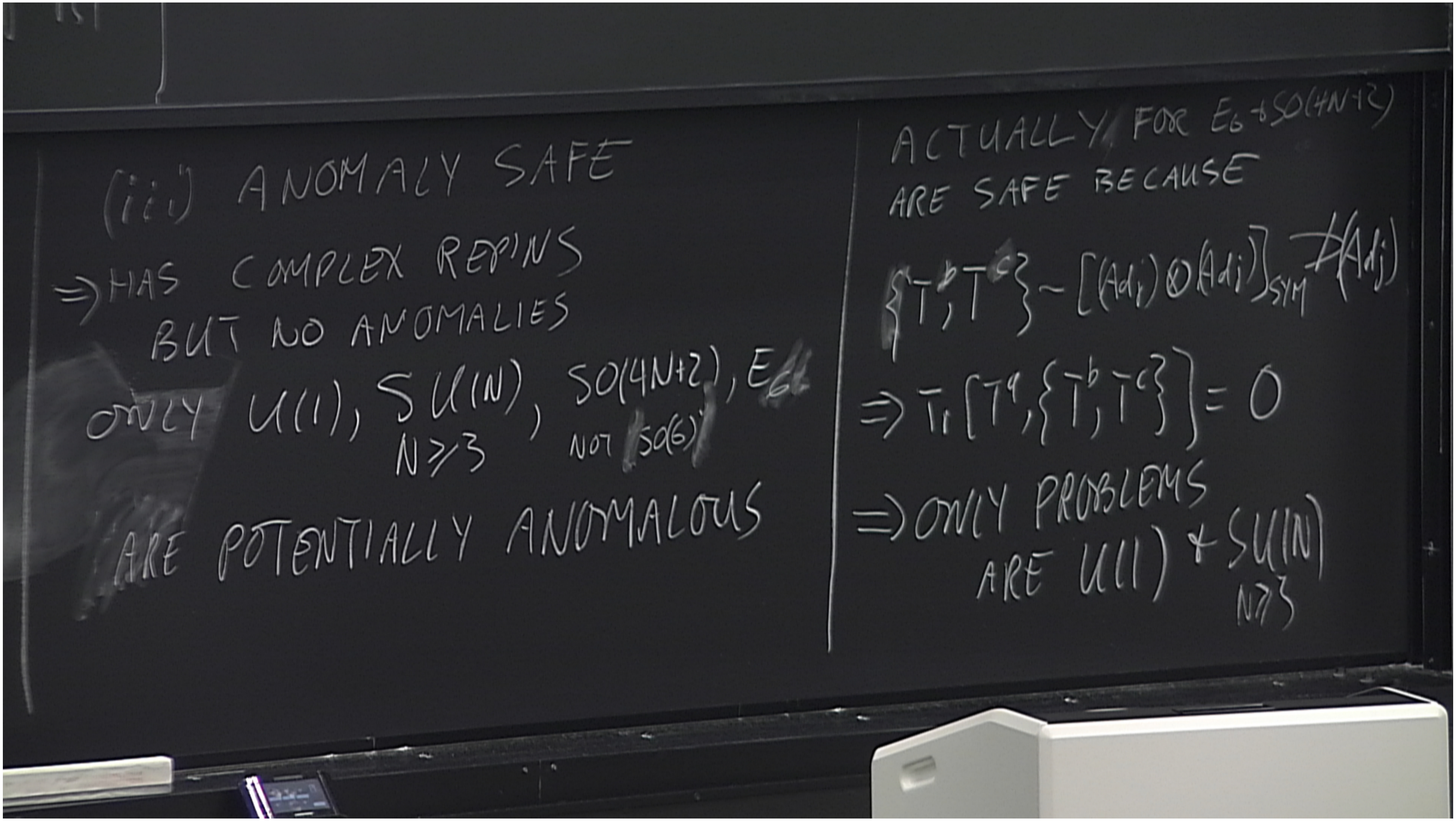
$\Rightarrow$ LH & RH FERMIONS are in EQUIVALENT REP'NS

$$\bar{\psi}_R \gamma^\mu T_R \psi_R + \bar{\psi}_L \gamma^\mu T_L \psi_L = \bar{\psi}' \gamma^\mu T_R \psi'$$

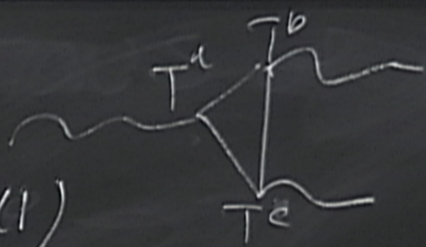
$$\psi' = \psi_R + U \psi_L$$


(i) $T_L = U T_R U^{-1}$
$U^{\dagger} U = I$
⇒ LH & RH FERMIONS are in EQUIVALENT REP'NS
$\bar{\psi}_R \gamma^\mu T_R \psi_R + \bar{\psi}_L \gamma^\mu T_L \psi_L = \bar{\psi}' \gamma^\mu T_R \psi'$
$\psi' = \psi_R + U \psi_L$
VECTORLIKE
eg QCD


(ii) $T_L^* = -V T_L V^{-1}$ ⊗ V = unitary
dagger
$\mathrm{Tr}[T_L^a\{T_L^b, T_L^c\}] = \mathrm{Tr}(T_L^{a*}\{T_L^{b*}, T_L^{c*}\})$
PSEUDOREAL REP'N
⊗ $= -\mathrm{Tr}[T_L^a\{T_L^b, T_L^c\}]$
$= 0$
(iii)


(iii) ANOMALY SAFE
⇒ HAS COMPLEX REP'NS
BUT NO ANOMALIES
ONLY U(1), SU(N), SO(4N+2), E6
N≥3
NOT SO(6)
ARE POTENTIALLY ANOMALOUS
ACTUALLY, FOR E6 & SO(4N+2)
ARE SAFE BECAUSE
{T^a, T^b} ~ [(Adj) ⊗ (Adj)]_SYM ∌ (Adj)
⇒ Tr [T^a, {T^b, T^c}] = 0
⇒ ONLY PROBLEMS
ARE U(1) & SU(N)
N≥3

STD MODEL



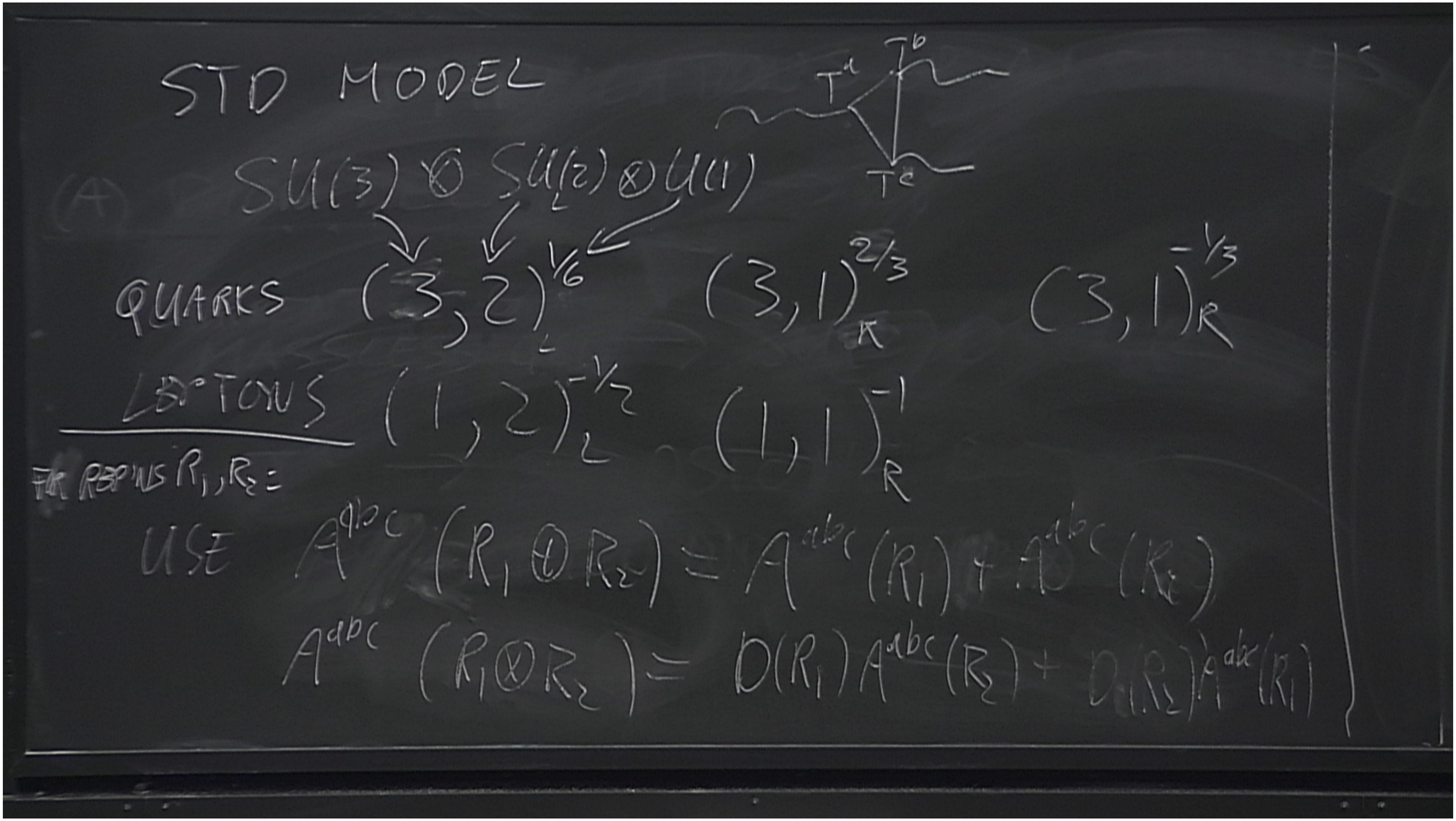
$SU(3) \otimes SU(2)_L \otimes U(1)$

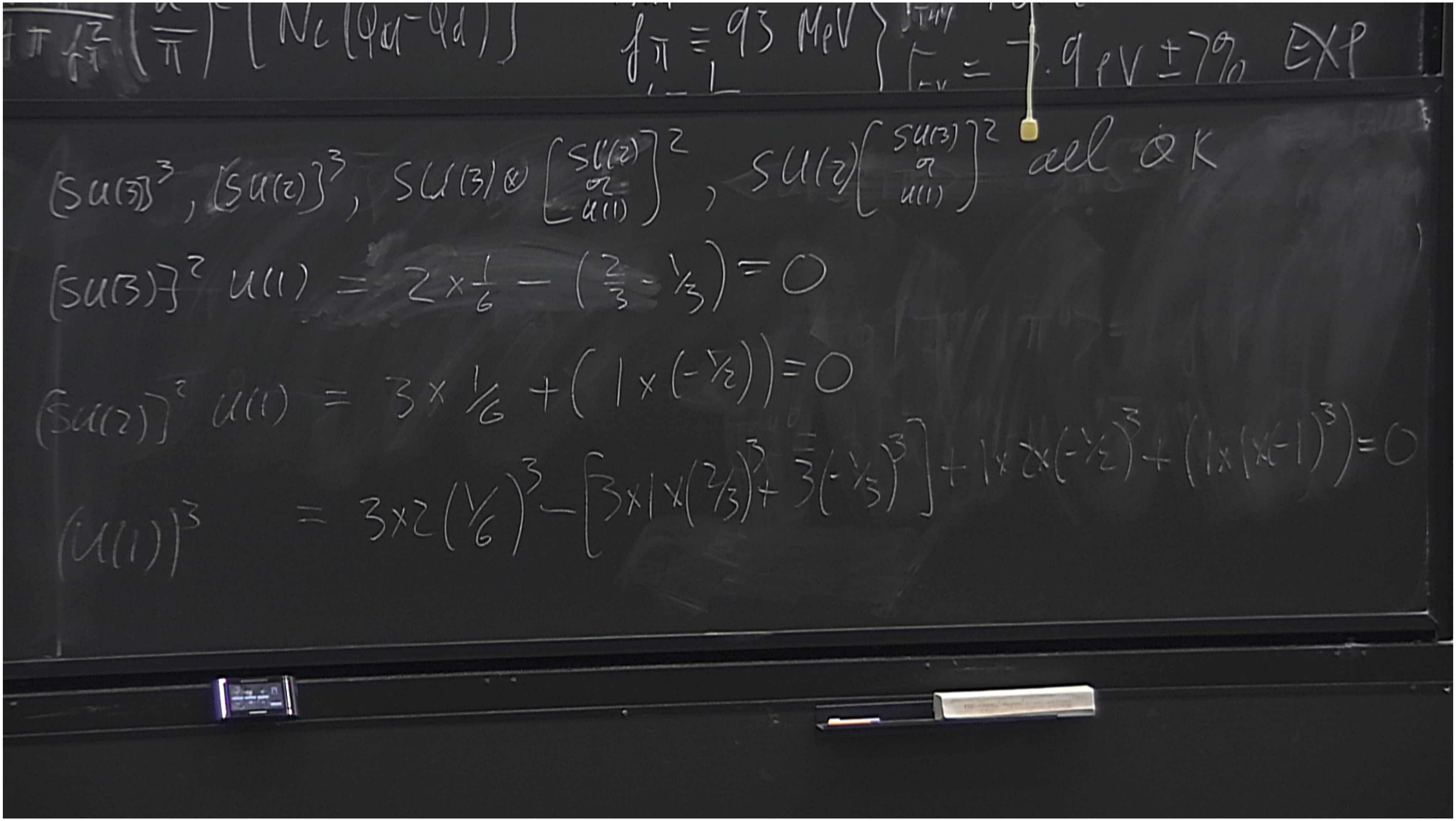
QUARKS $(3,2)_L^{1/6}$ $(3,1)_R^{2/3}$ $(3,1)_R^{-1/3}$

LEPTONS $(1,2)_L^{-1/2}$ $(1,1)_R^{-1}$

USE $A^{abc}(R_1 \oplus R_2) = A^{abc}(R_1) + A^{abc}(R_2)$

$A^{abc}(R_1 \otimes R_2) = D(R_1)A^{abc}(R_2) + D(R_2)A^{abc}(R_1)$


STD MODEL
T^a T^b T^c
SU(3) ⊗ SU(2)_L ⊗ U(1)
QUARKS (3,2)_L^{1/6} (3,1)_R^{2/3} (3,1)_R^{-1/3}
LEPTONS (1,2)_L^{-1/2} (1,1)_R^{-1}
FOR REP'NS R_1, R_2 =
USE A^{abc}(R_1 ⊕ R_2) = A^{abc}(R_1) + A^{abc}(R_2)
A^{abc}(R_1 ⊗ R_2) = D(R_1) A^{abc}(R_2) + D(R_2) A^{abc}(R_1)


f_π ≅ 93 MeV
.9 eV ± 7% EXP
[SU(3)]^3, [SU(2)]^3, SU(3) ⊗ [SU(2) or U(1)]^2, SU(2) [SU(3) or U(1)]^2 all OK
[SU(3)]^2 U(1) = 2 × 1/6 − (2/3 − 1/3) = 0
[SU(2)]^2 U(1) = 3 × 1/6 + (1 × (−1/2)) = 0
(U(1))^3 = 3×2 (1/6)^3 − [3×1×(2/3)^3 + 3(−1/3)^3] + 1×2×(−1/2)^3 + (1×1×(−1)^3) = 0