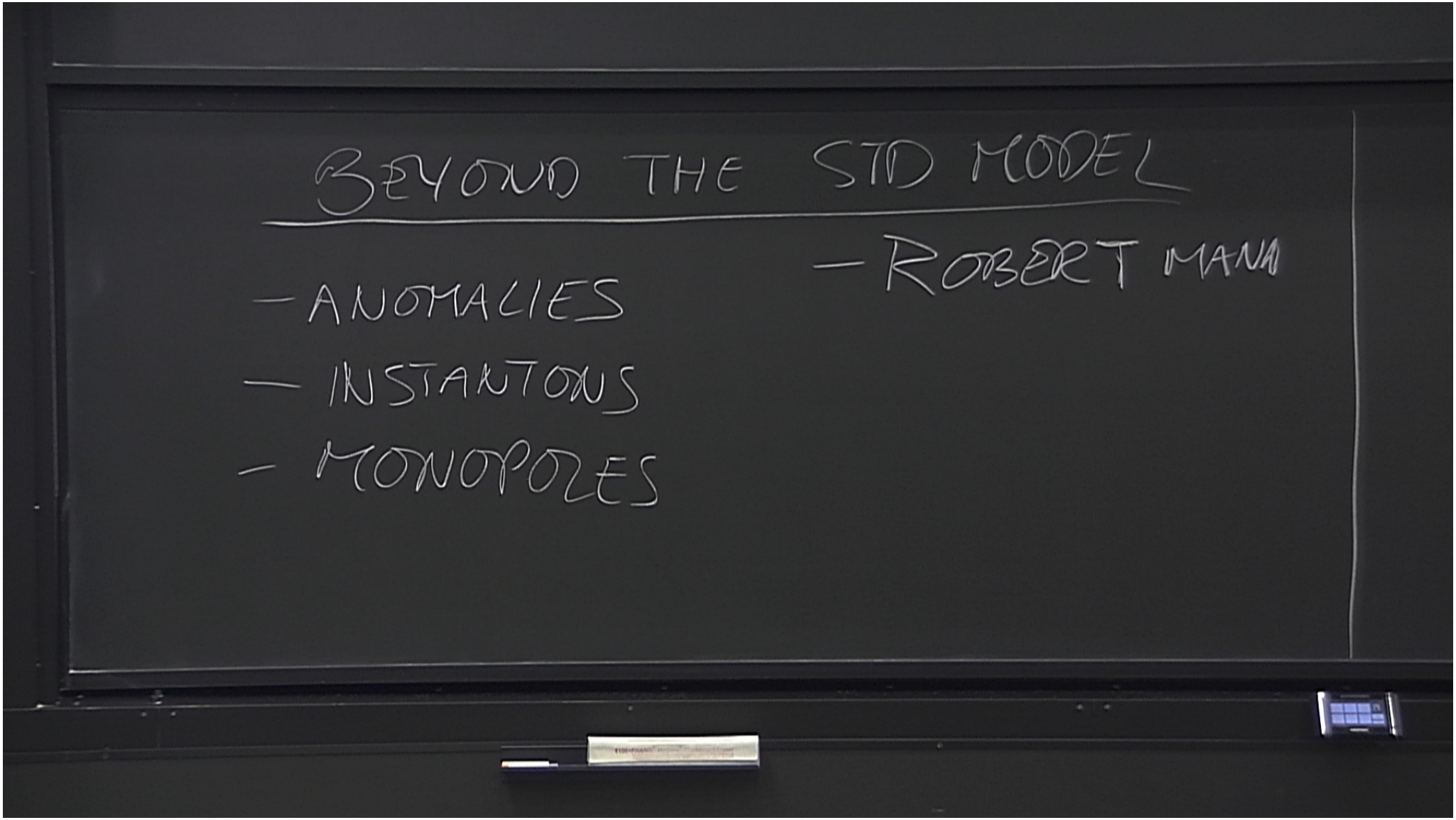
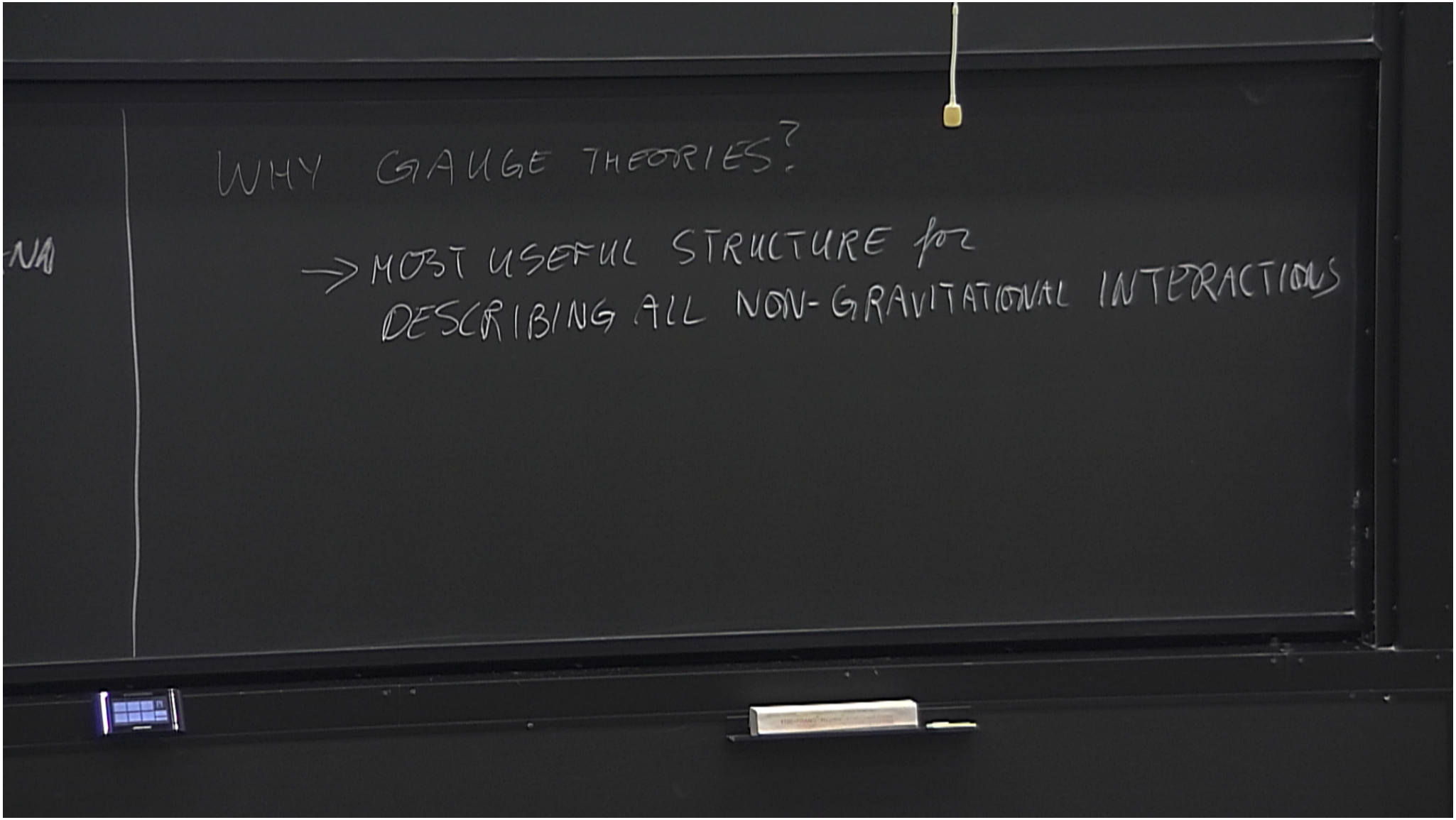
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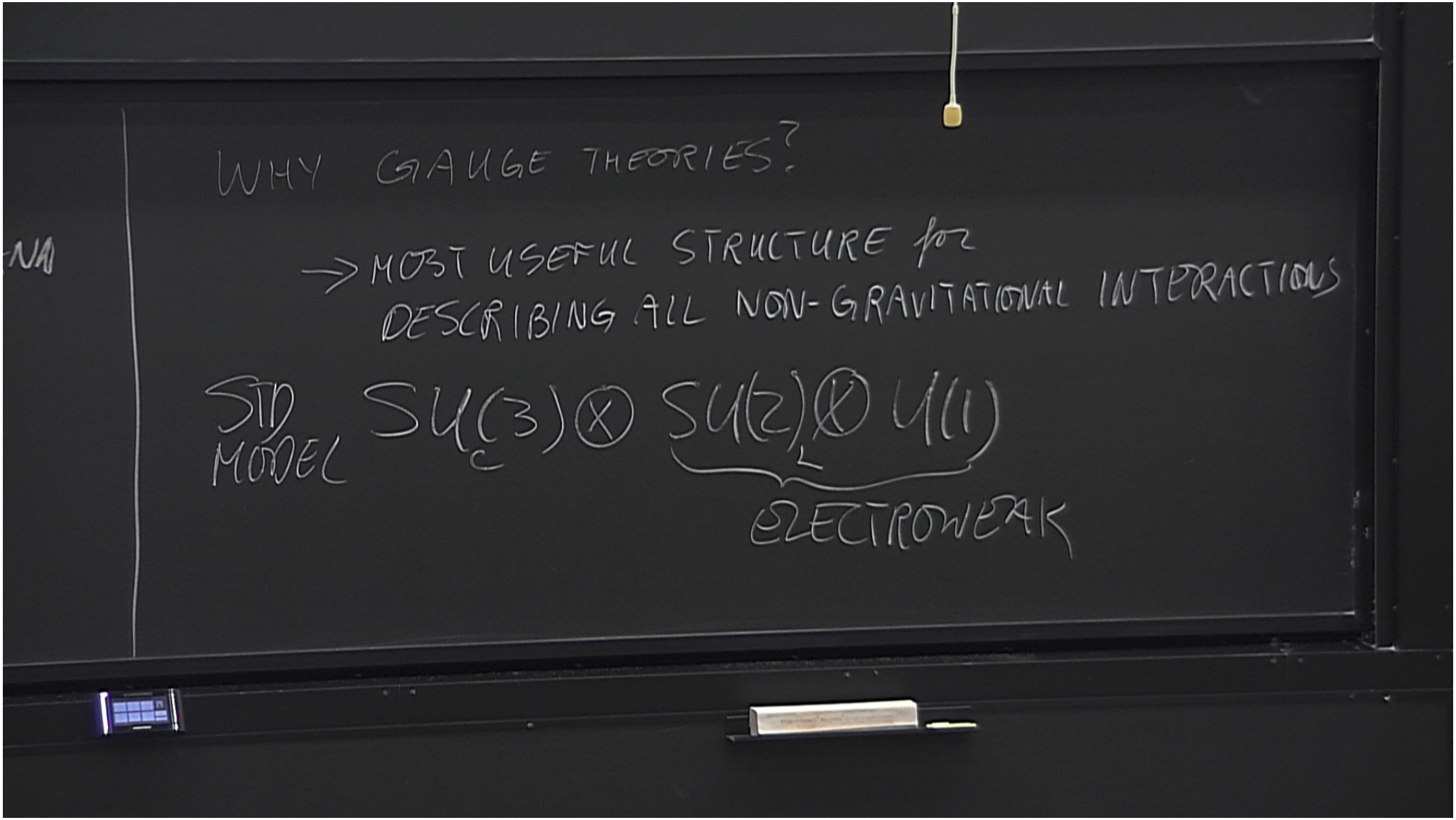
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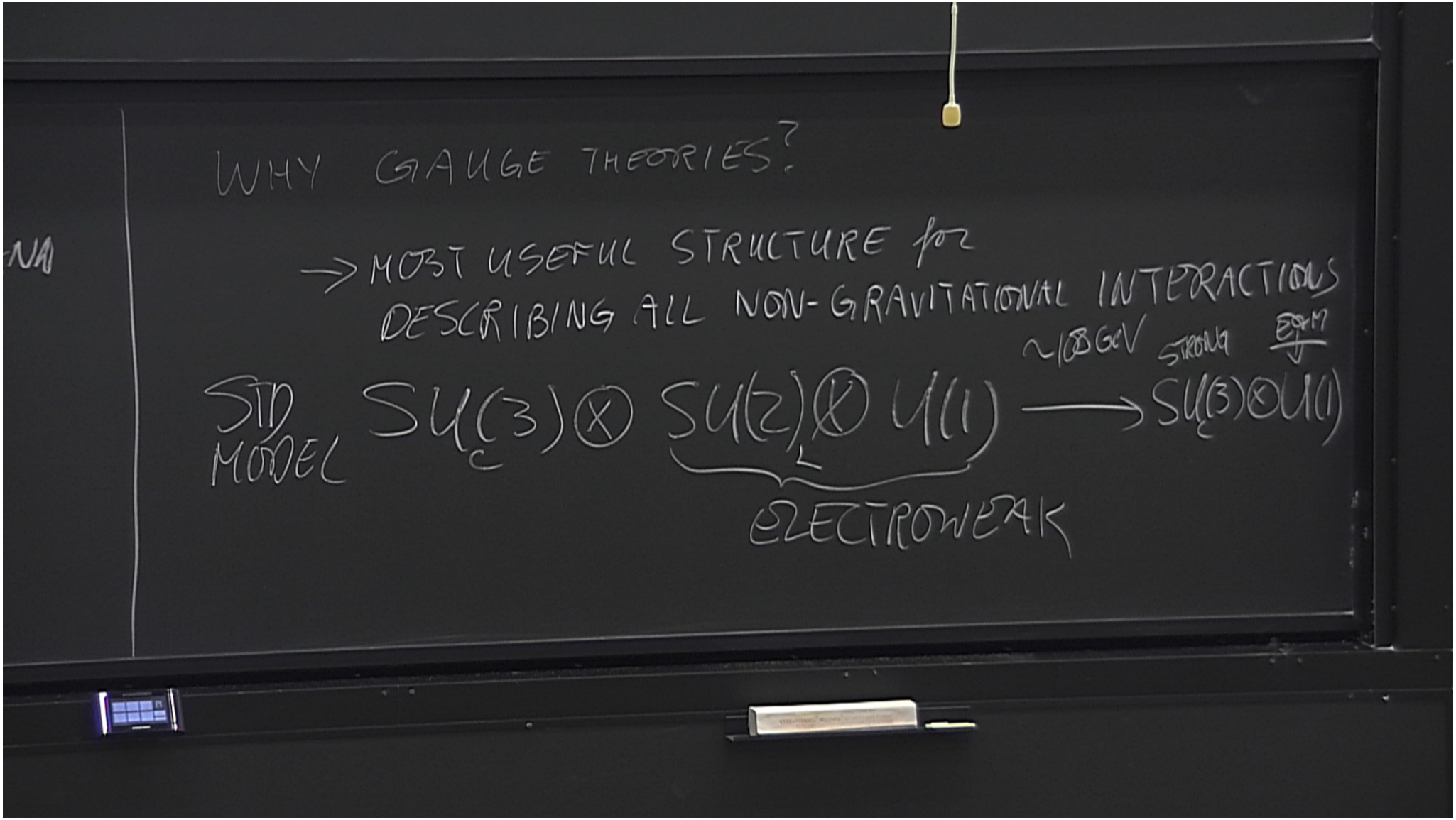
URL: http://pirsa.org/14020051

Abstract:


BEYOND THE STD MODEL
- ROBERT MANN
- ANOMALIES
- INSTANTONS
- MONOPOLES


WHY GAUGE THEORIES?
⇒ MOST USEFUL STRUCTURE for
DESCRIBING ALL NON-GRAVITATIONAL INTERACTIONS

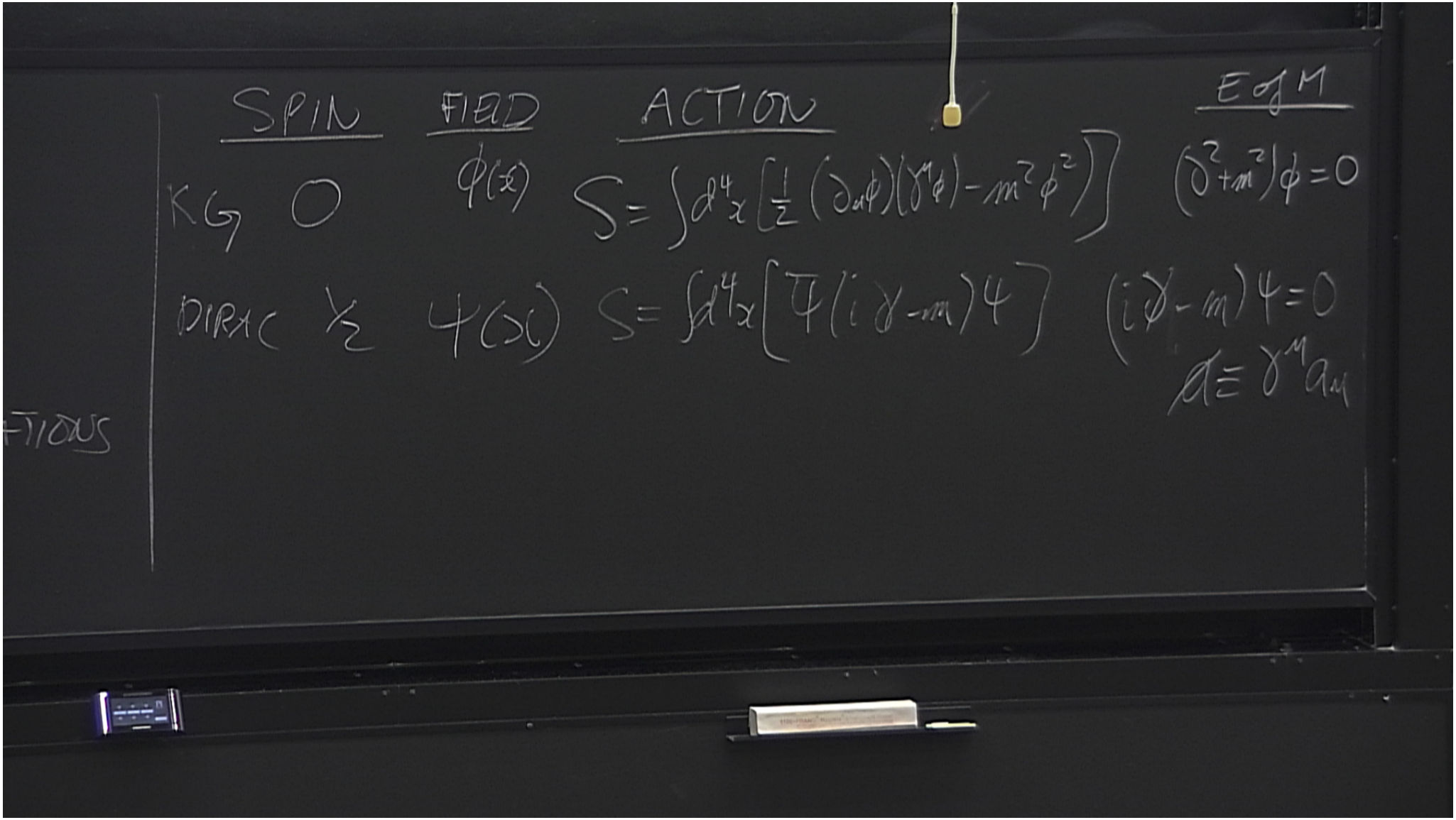

WHY GAUGE THEORIES?
⇒ MOST USEFUL STRUCTURE for
DESCRIBING ALL NON-GRAVITATIONAL INTERACTIONS
STD MODEL
$SU(3)_C \otimes SU(2)_L \otimes U(1)$
ELECTROWEAK

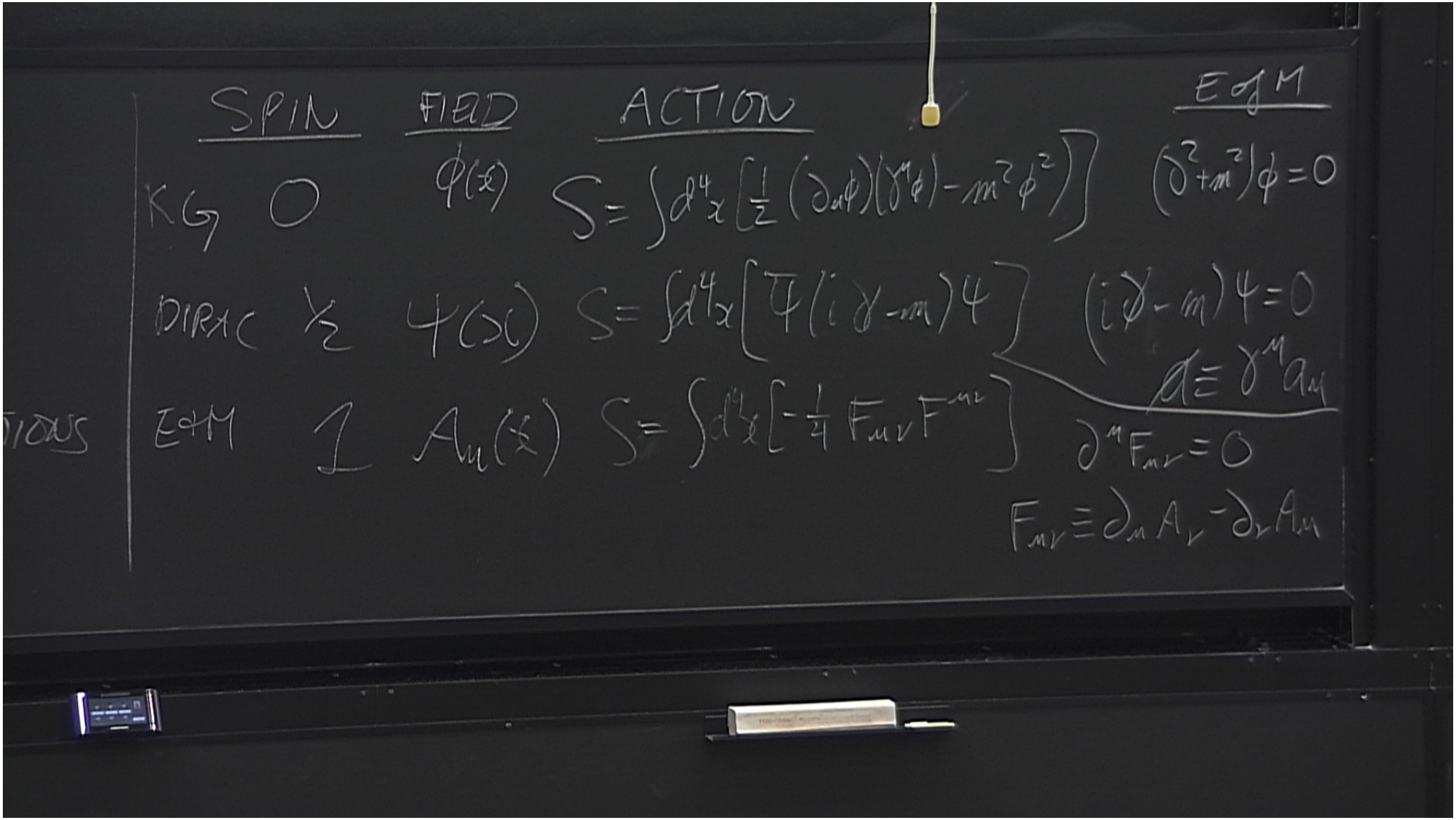

WHY GAUGE THEORIES?
→ MOST USEFUL STRUCTURE for
DESCRIBING ALL NON-GRAVITATIONAL INTERACTIONS
STD MODEL
$SU(3)_C \otimes SU(2)_L \otimes U(1)$
ELECTROWEAK
~100 GeV
STRONG
EM
$\longrightarrow SU(3) \otimes U(1)$

HOW TO BUILD A GAUGE THEORY

1) LAWS of PHYSICS are INVARIANT
UNDER $SO(3,1) \otimes T_4$ (POINCARÉ GRP)

2) FUNDAMENTAL (or ELEMENTARY) PARTICLES
CORRESPOND to IRREDUCIBLE REPRESENTATIONS
of the POINCARÉ GRP (WIGNER 1939)
$\Rightarrow$ BUILD Rel WAVE EQN'S


SPIN
FIELD
ACTION
EofM
KG 0
$\phi(x)$
$S=\int d^4x\left[\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)-m^2\phi^2\right]$
$(\partial^2+m^2)\phi=0$
DIRAC ½
$\psi(x)$
$S=\int d^4x\left[\bar{\psi}(i\partial\!\!\!/-m)\psi\right]$
$(i\partial\!\!\!/-m)\psi=0$
$a\!\!\!/\equiv\gamma^\mu a_\mu$
FTIONS


SPIN
FIELD
ACTION
EofM
KG 0
$\phi(x)$
$S=\int d^4x\left[\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)-m^2\phi^2\right]$
$(\partial^2+m^2)\phi=0$
DIRAC ½
$\psi(x)$
$S=\int d^4x\left[\bar{\psi}(i\not{\partial}-m)\psi\right]$
$(i\not{\partial}-m)\psi=0$
$\not{a}\equiv\gamma^\mu a_\mu$
EoM
1
$A_\mu(x)$
$S=\int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$
$\partial^\mu F_{\mu\nu}=0$
$F_{\mu\nu}\equiv\partial_\mu A_\nu-\partial_\nu A_\mu$

1 the POINCARÉ GRP (WIGNER 1939)

⇒ BUILD Rel WAVE EQN'S

DIRAC EQN

$(i\gamma - m)\psi = 0$

PHASE SYM $\psi' = e^{i\alpha}\psi$

$(i\gamma - m)\psi' = 0$

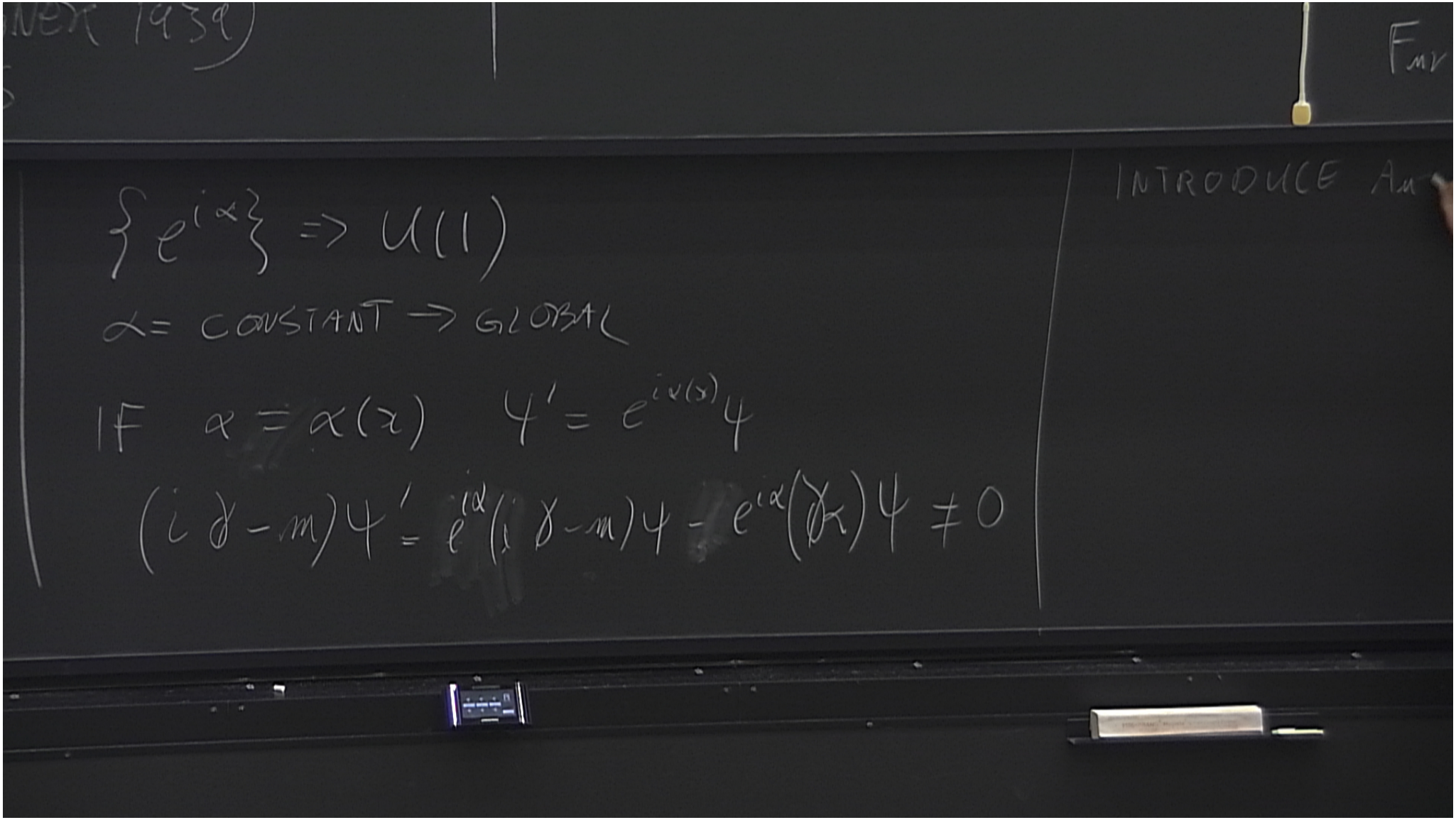
ONLY SYM when $m \neq 0$

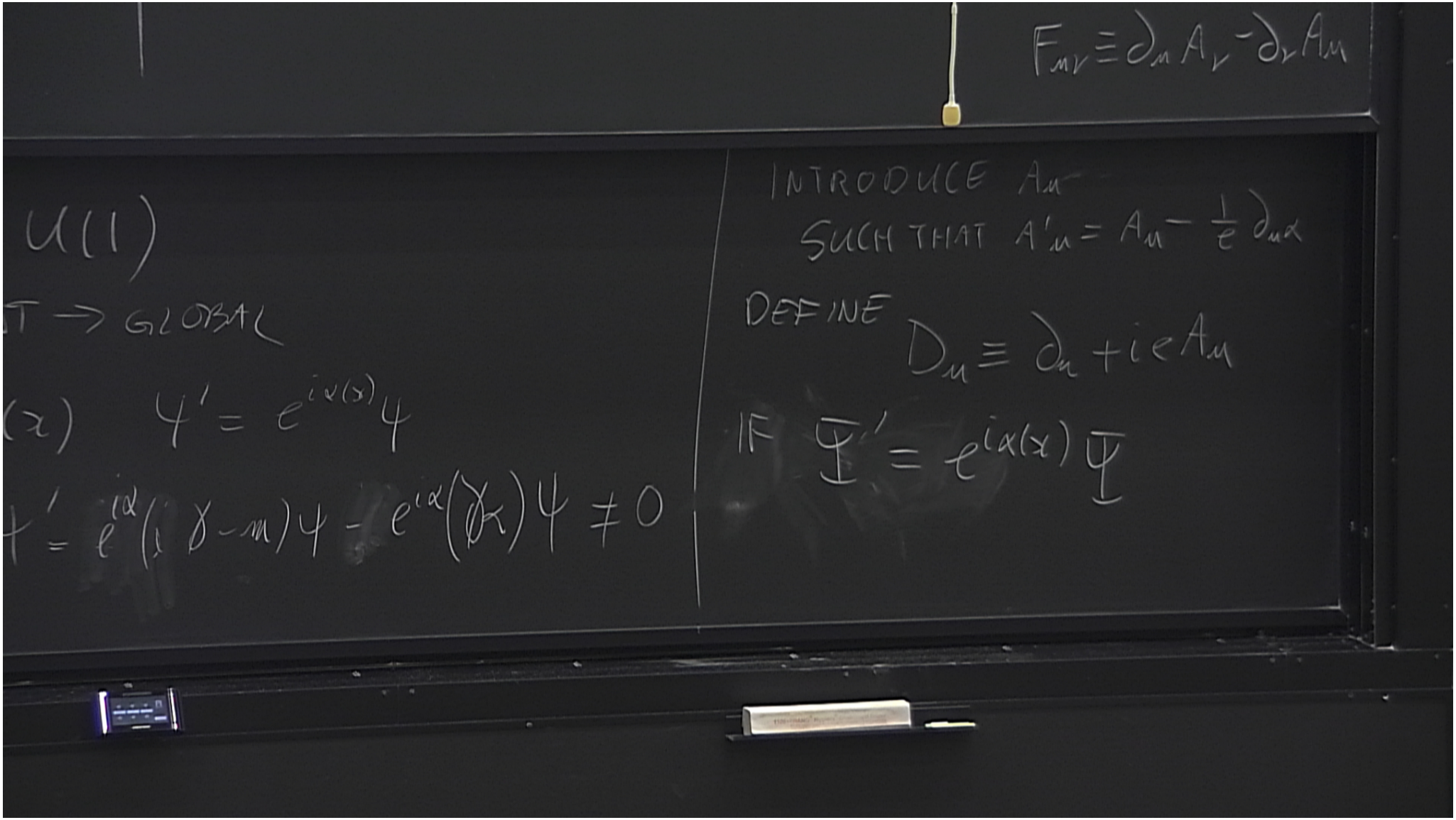
$\{e^{i\alpha}\} \Rightarrow U(1)$

α = CONSTANT → GLOBAL

IF $\alpha = \alpha(x)$ $\psi' = e^{i\alpha(x)}\psi$

$(i\gamma - m)\psi' \neq 0$


$\{e^{i\alpha}\} \Rightarrow U(1)$
α = CONSTANT → GLOBAL
IF $\alpha = \alpha(x)$ $\psi' = e^{i\alpha(x)}\psi$
$(i\partial - m)\psi' = e^{i\alpha}(i\partial - m)\psi - e^{i\alpha}(\partial_\mu)\psi \neq 0$
INTRODUCE A_μ


$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$
U(1)
T → GLOBAL
(x) $\psi' = e^{i\alpha(x)}\psi$
$\not{\partial}' = e^{i\alpha}(i\not{\partial} - m)\psi - e^{i\alpha}(\not{\partial}\alpha)\psi \neq 0$
INTRODUCE A_μ
SUCH THAT $A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\alpha$
DEFINE $D_\mu \equiv \partial_\mu + ieA_\mu$
IF $\Psi' = e^{i\alpha(x)}\Psi$

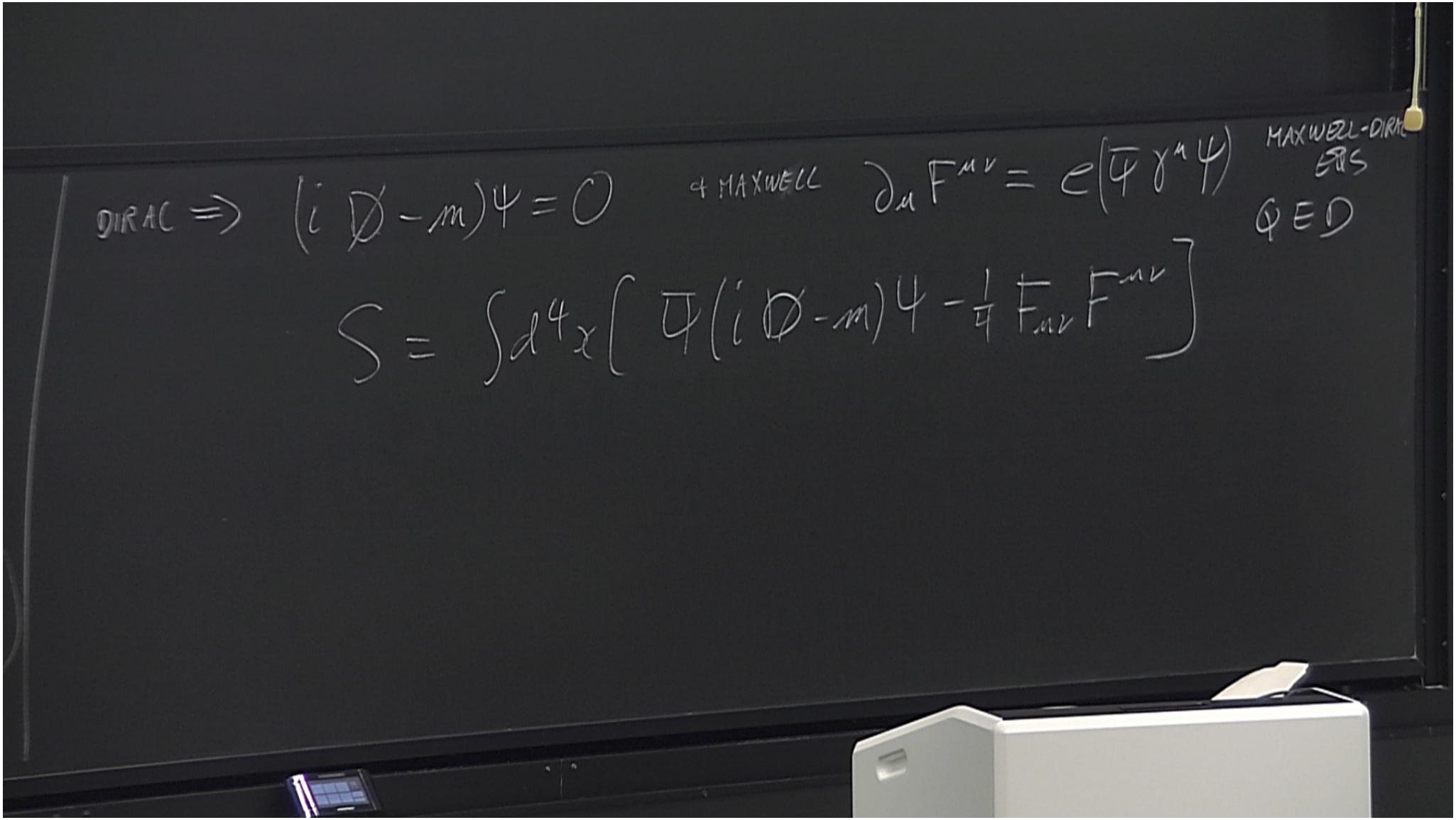
$$(D_\mu \Psi)' = e^{i\alpha(x)} (D_\mu \Psi)$$

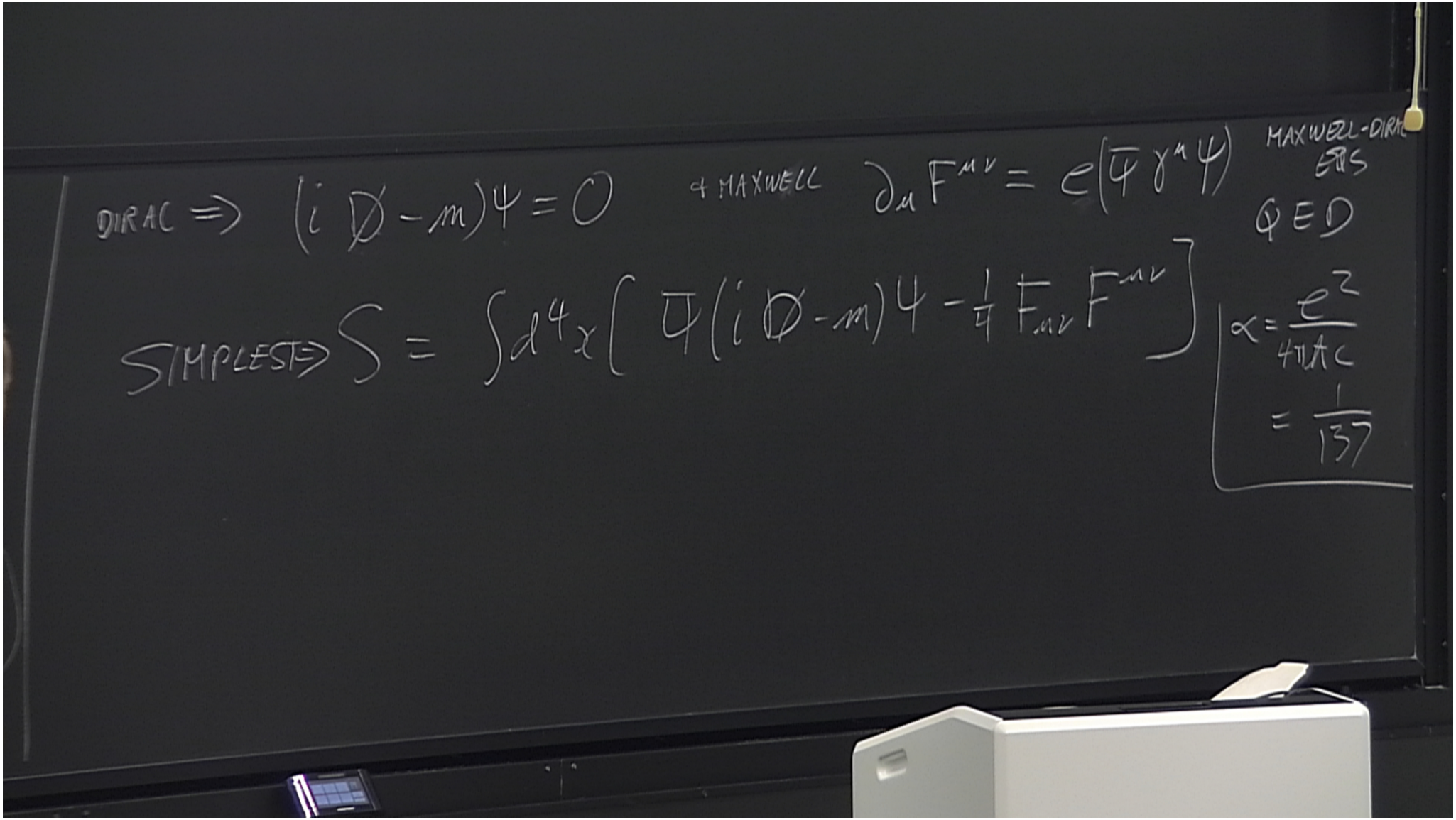
$$(D_\mu \Psi)' = (\partial_\mu \Psi' + ieA'_\mu)$$

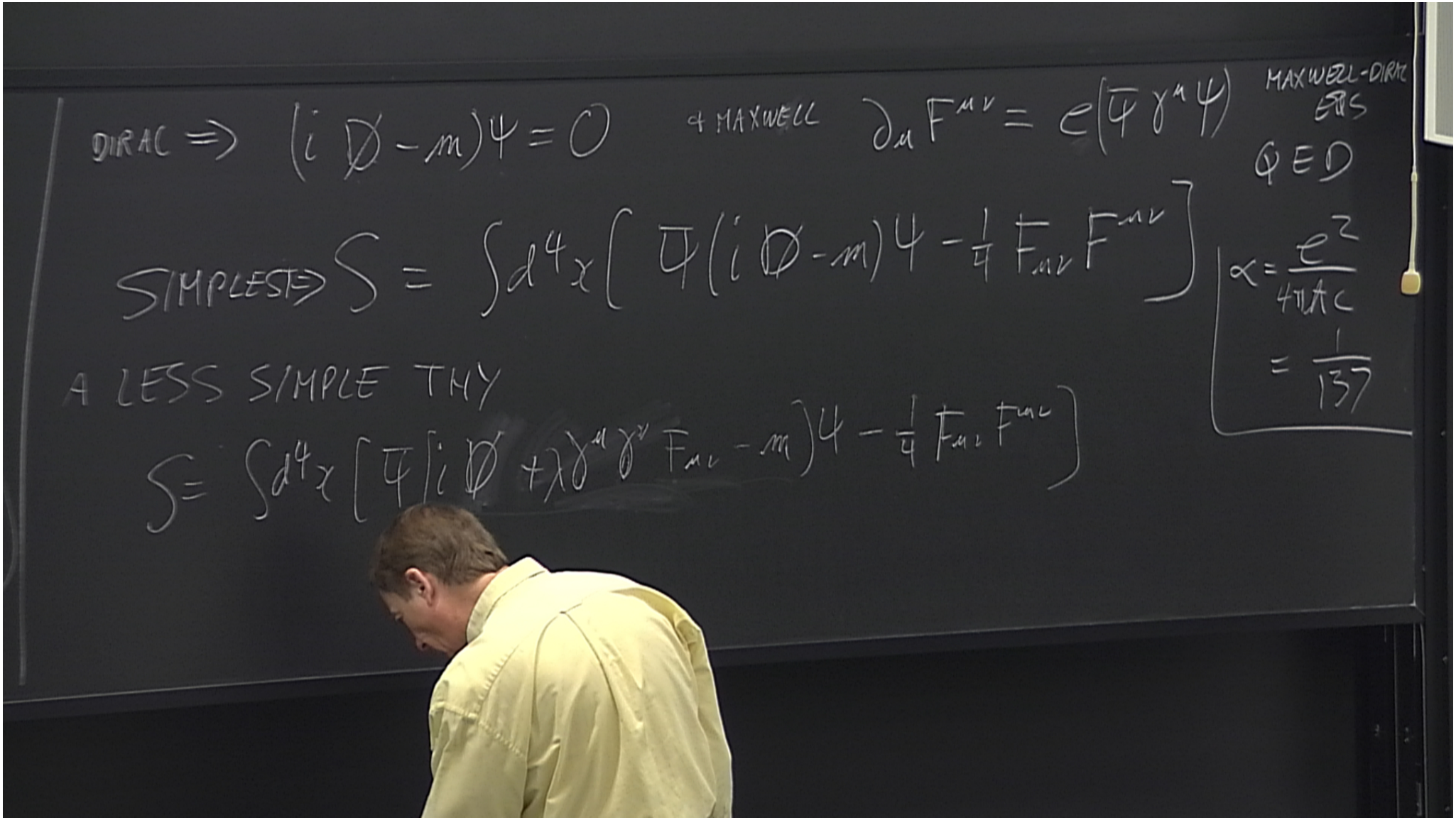
$$= \partial_\mu (e^{i\alpha}\Psi) + ie(A_\mu - \tfrac{1}{e}\partial_\mu \alpha)(e^{i\alpha}\Psi)$$

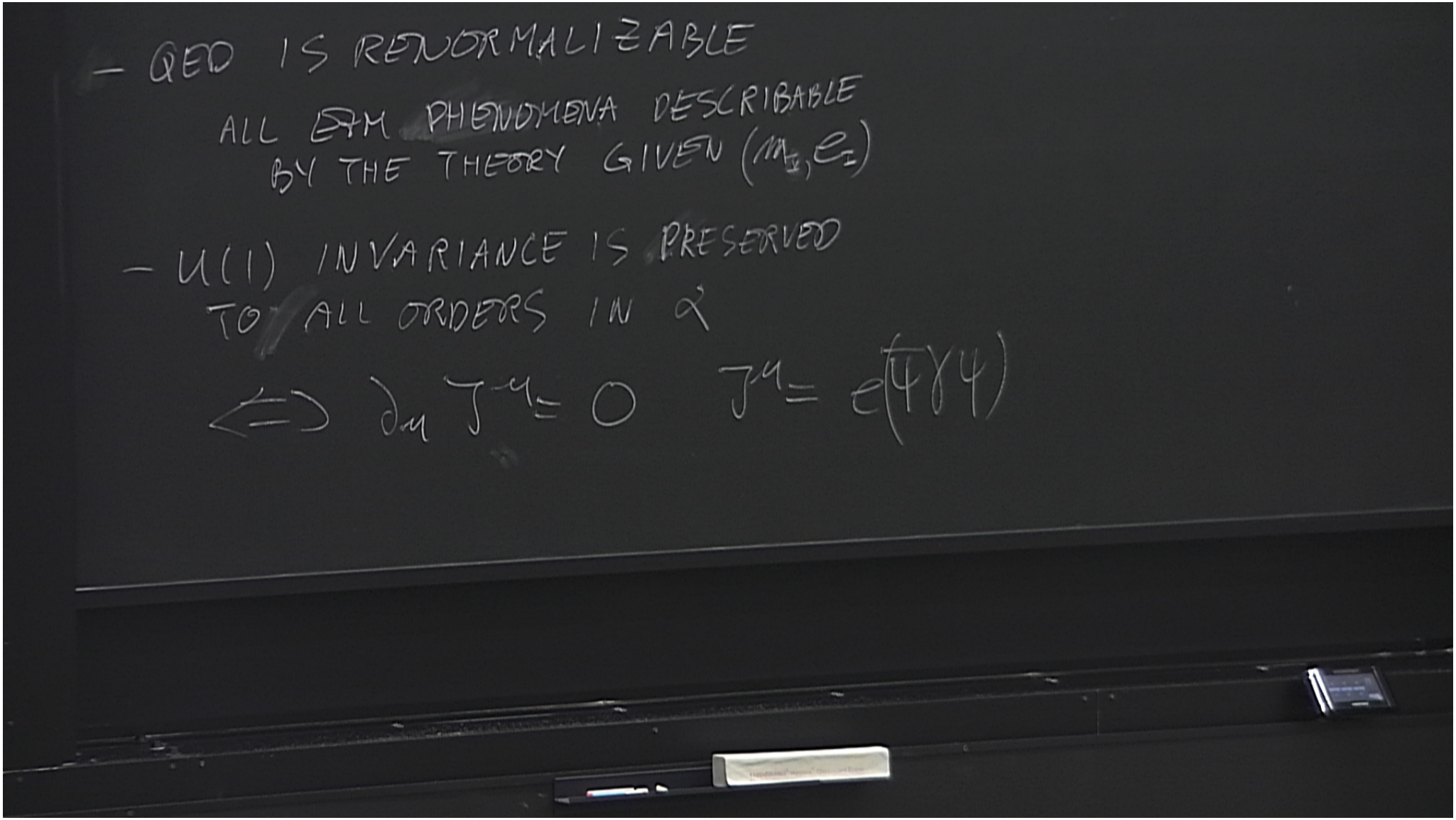
$$= e^{i\alpha}\left[\partial_\mu \Psi + i(\partial_\mu \alpha)\Psi + ieA_\mu \Psi - i(\partial_\mu \alpha)\Psi\right]$$

$$= e^{i\alpha}(D_\mu \Psi)$$


DIRAC $\Rightarrow$ $(i\not{D} - m)\psi = 0$
+ MAXWELL $\partial_\mu F^{\mu\nu} = e(\bar{\psi}\gamma^\nu\psi)$
MAXWELL-DIRAC EQNS
QED
$S = \int d^4x \left[\bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$


DIRAC ⇒ $(i\not{D} - m)\psi = 0$
+ MAXWELL $\partial_\mu F^{\mu\nu} = e(\bar{\psi}\gamma^\nu\psi)$
MAXWELL-DIRAC EQNS
QED
SIMPLEST ⇒ $S = \int d^4x \left[\bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$
$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$


DIRAC ⇒ $(i\not{D} - m)\psi = 0$
+ MAXWELL $\partial_\mu F^{\mu\nu} = e(\bar{\psi}\gamma^\nu\psi)$
MAXWELL-DIRAC EQNS
QED
SIMPLEST ⇒ $S = \int d^4x \left[\bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$
$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$
A LESS SIMPLE THY
$S = \int d^4x \left[\bar{\psi}(i\not{D} + \lambda\gamma^\mu\gamma^\nu F_{\mu\nu} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$


- QED IS RENORMALIZABLE
ALL EM PHENOMENA DESCRIBABLE
BY THE THEORY GIVEN (m_e, e)
- U(1) INVARIANCE IS PRESERVED
TO ALL ORDERS IN e^2
$\Leftrightarrow \partial_\mu J^\mu = 0$ $J^\mu = e(\bar{\psi}\gamma\psi)$

$\alpha)$

$$\alpha \cdot T = \alpha^a T_a$$

$$[T_a, T_b] = i C^c_{ab} T_c$$

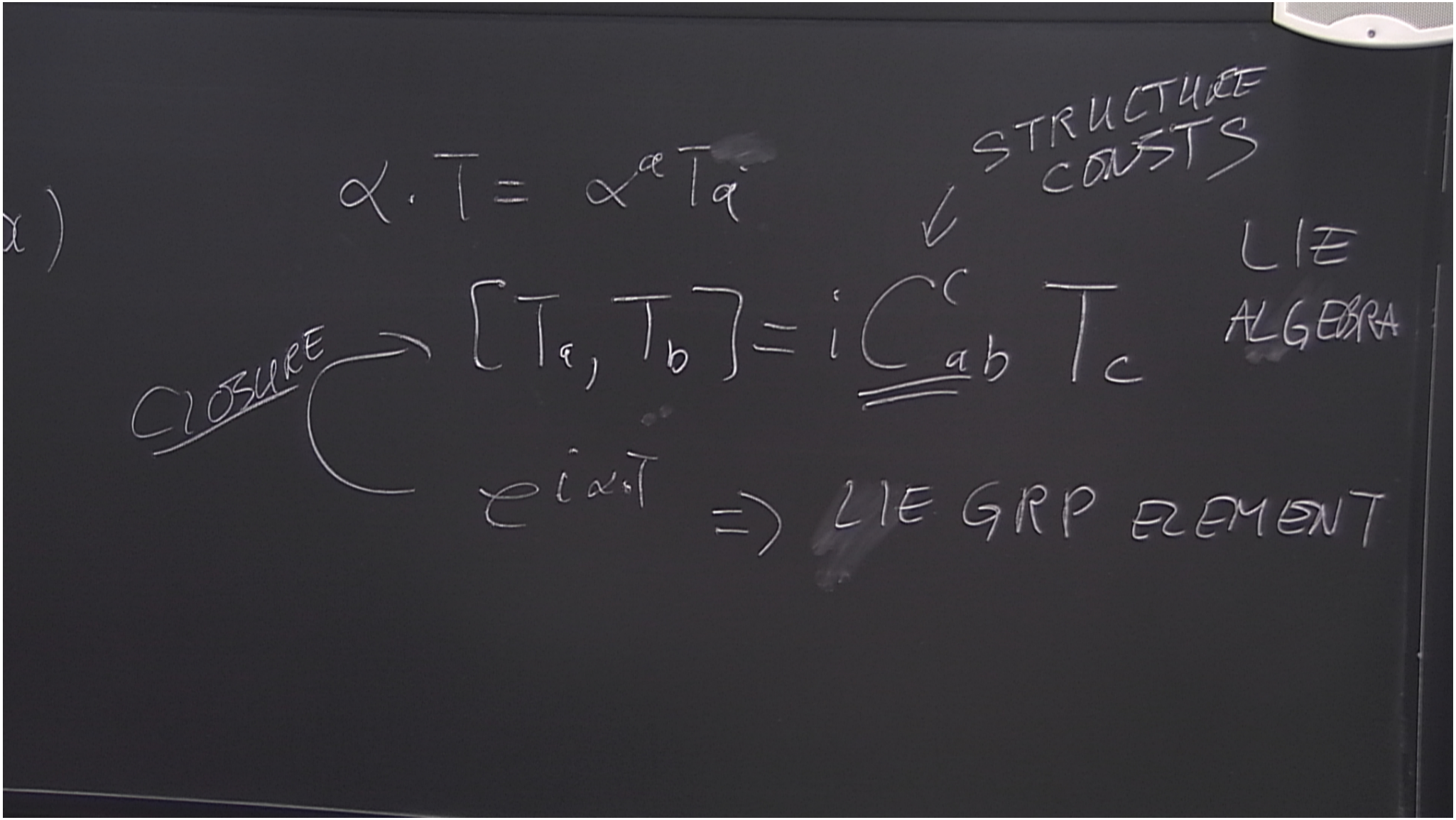
LIE ALGEBRA

$e^{i\alpha \cdot T} \Rightarrow$ LIE GRP ELEMENT

$\alpha)$ $\quad \alpha \cdot T = \alpha^a T_a$

CLOSURE $\rightarrow [T_a, T_b] = i C^c_{ab} T_c$ LIE ALGEBRA

$e^{i\alpha \cdot T} \Rightarrow$ LIE GRP ELEMENT


α)
$\alpha \cdot T = \alpha^a T_a$
STRUCTURE CONSTS
LIE ALGEBRA
CLOSURE
$[T_a, T_b] = i C^c_{ab} T_c$
$e^{i\alpha \cdot T}$
$\Rightarrow$ LIE GRP ELEMENT

YANG-MILLS

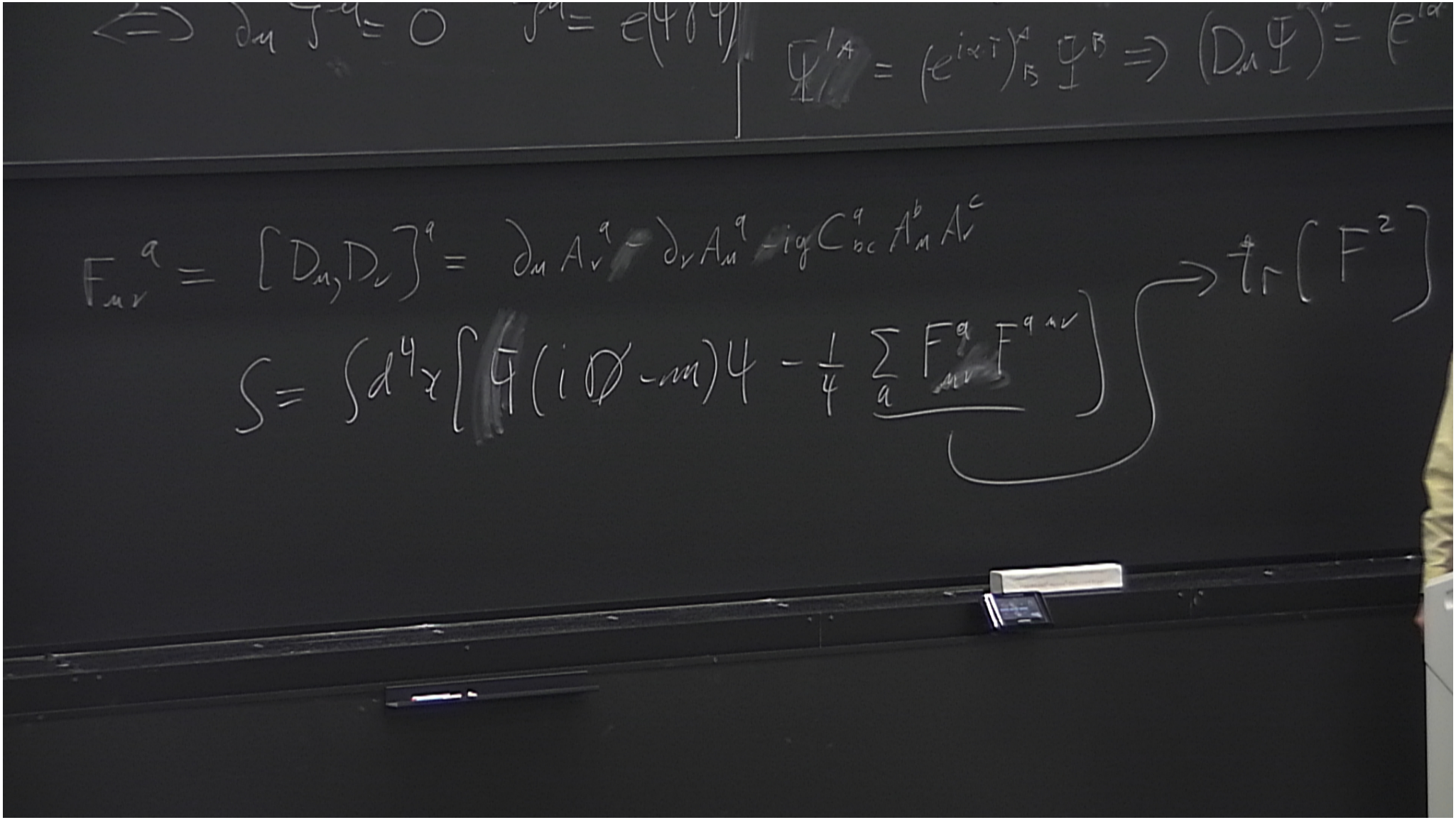
$$\psi'^{A}(x) = \left(e^{i\alpha(x)\cdot T}\right)^{A}_{B}\,\psi^{B}(x)$$

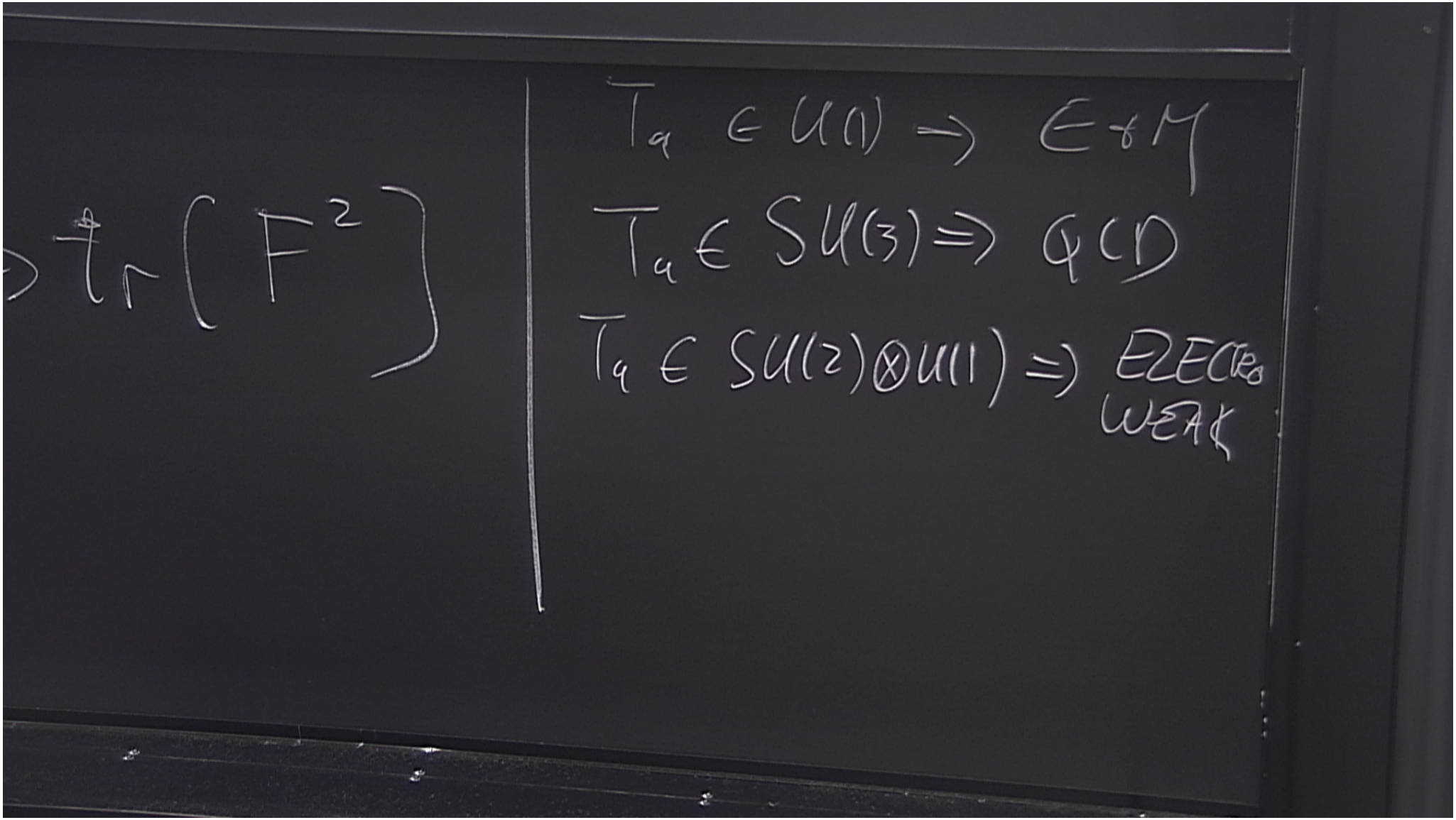
$$\alpha \cdot T = \alpha^{a} T_a$$

$$(A'^{a}_{\mu} T_a) = \left(e^{i\alpha\cdot T}\right)(A^{a}_{\mu} T_a)\left(e^{-i\alpha\cdot T}\right) + \frac{i}{g}\left[e^{-i\alpha\cdot T}\,\partial_\mu\, e^{i\alpha\cdot T}\right] \qquad [T_a, T_b] = i$$

$$D_\mu \equiv \partial_\mu + ig A^{a}_{\mu} T_a \qquad e^{i\alpha\cdot T} \Rightarrow$$

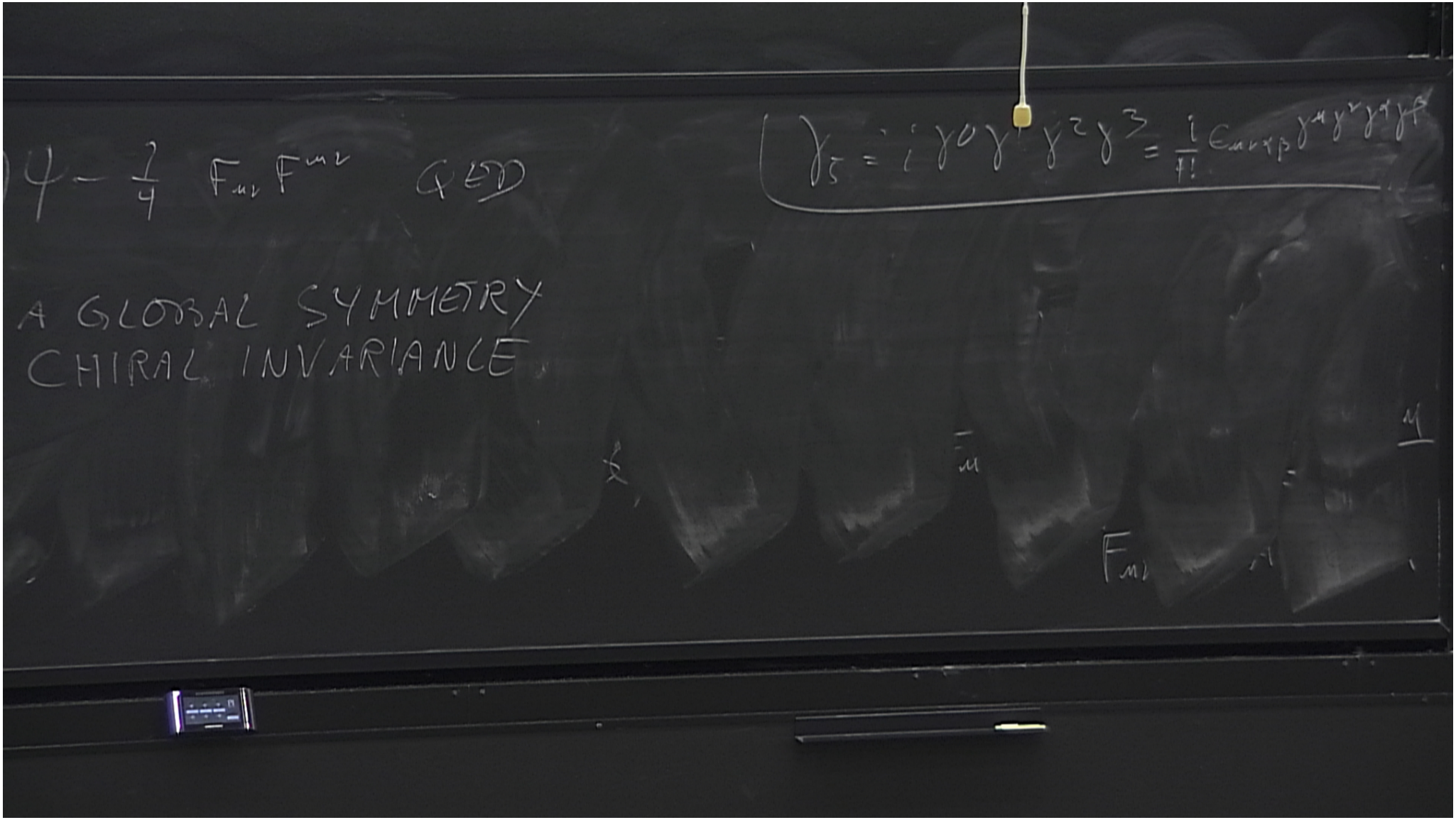
$$\Psi'^{A} = \left(e^{i\alpha\cdot T}\right)^{A}_{B}\,\Psi^{B} \Rightarrow \left(D_\mu \Psi\right)'^{A} = \left(e^{i\alpha\cdot T}\right)^{A}_{B}\left(D_\mu \Psi\right)^{B}$$


$\Leftrightarrow \partial_\mu j^\mu = 0 \quad j^\mu = e(\bar{\psi}\gamma^\mu\psi)$
$\Psi'^A = (e^{i\alpha T})^A_B \Psi^B \Rightarrow (D_\mu \Psi)' = (e^{i\alpha}$
$F^a_{\mu\nu} = [D_\mu, D_\nu]^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - igC^a_{bc} A^b_\mu A^c_\nu$
$S = \int d^4x \left[\bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} \sum_a F^a_{\mu\nu} F^{a\,\mu\nu} \right] \rightarrow \mathrm{tr}(F^2)$


$\Rightarrow \mathrm{tr}(F^2)$
$T_q \in U(1) \Rightarrow$ EM
$T_q \in SU(3) \Rightarrow$ QCD
$T_q \in SU(2) \otimes U(1) \Rightarrow$ ELECTRO WEAK

$S = \int d^4x\, \mathcal{L} \Rightarrow \mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ QED

IF $m = 0$ $\psi' = e^{i\theta\gamma_5}\psi$ IS A GLOBAL SYMMETRY

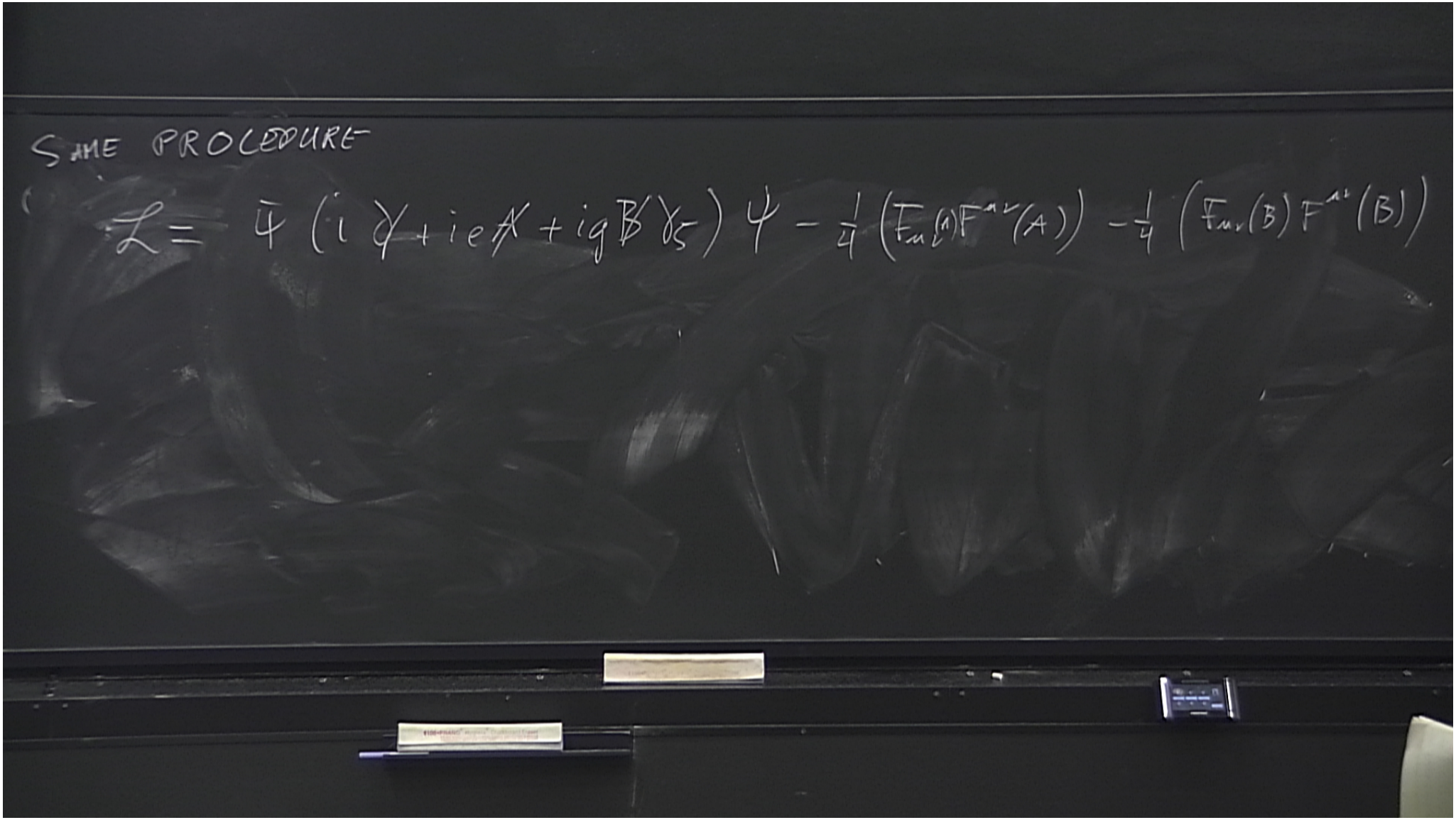

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ QED
$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$
A GLOBAL SYMMETRY
CHIRAL INVARIANCE

$S = \int d^4x\, \mathcal{L} \Rightarrow \mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F$

IF $m = 0$ $\psi' = e^{i\theta\gamma_5}\psi$ IS A GLOBAL
CHIRAL INV

WHAT IF $\theta = \theta(x)$?

INTRODUCE $B'_\mu = B_\mu - \frac{1}{g}\partial_\mu\theta$

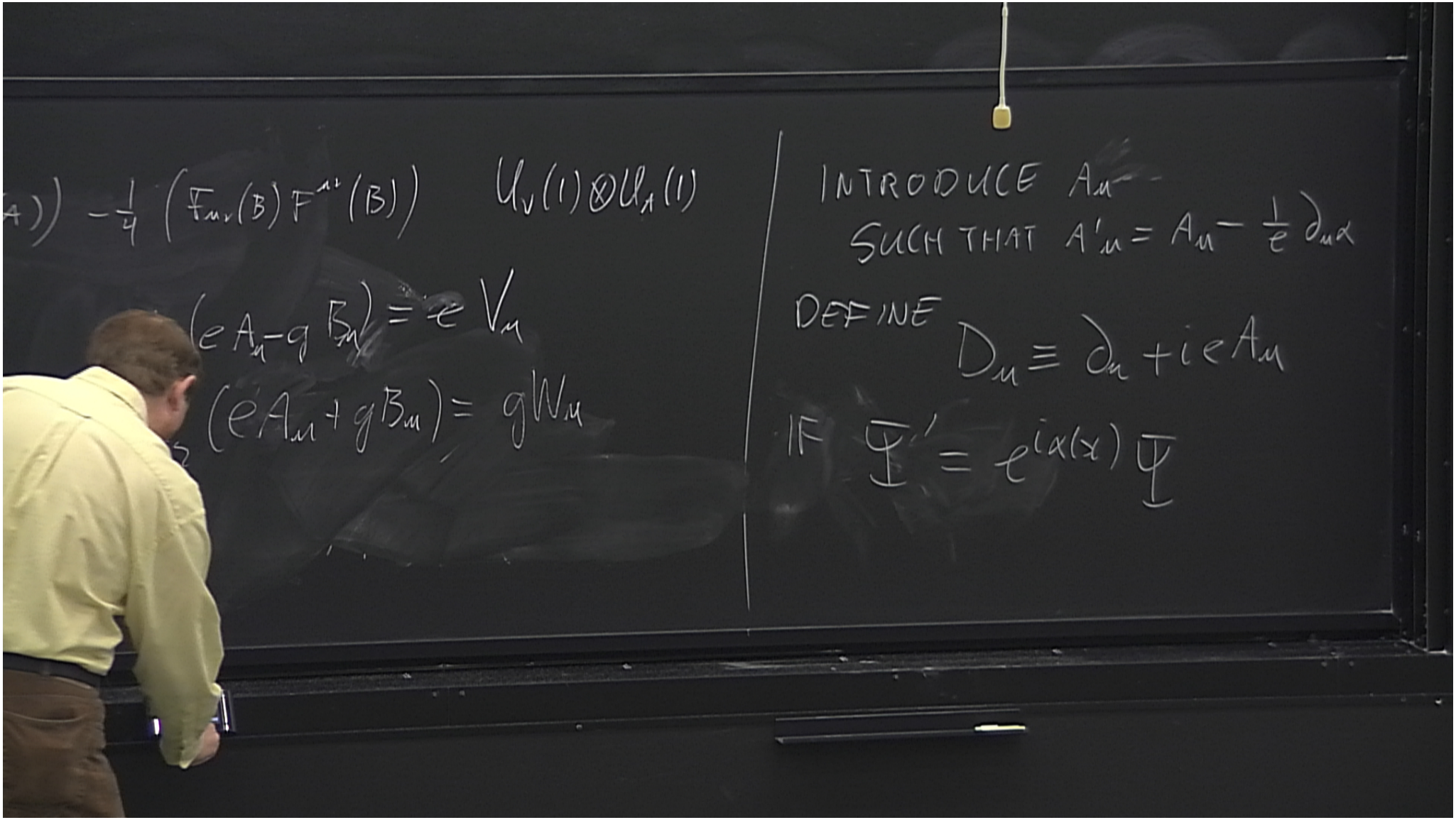

SAME PROCEDURE
$\mathcal{L} = \bar{\psi}\left(i\gamma + ie\not{A} + igB\gamma_5\right)\psi - \frac{1}{4}\left(F_{\mu\nu}(A)F^{\mu\nu}(A)\right) - \frac{1}{4}\left(F_{\mu\nu}(B)F^{\mu\nu}(B)\right)$

SAME PROCEDURE

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu + ie\not{A} + ig\not{B}\gamma_5\right)\psi - \frac{1}{4}\left(F_{\mu\nu}(A)F^{\mu\nu}(A)\right) - \frac{1}{4}\left(F_{\mu\nu}(B)\right.$$

$$\psi = \psi_L + \psi_R \qquad \psi_L = \frac{1}{2}(1-\gamma_5)\psi$$

$$\bar{\psi} = \bar{\psi}_L + \psi_R \qquad \psi_R = \frac{1}{2}(1+\gamma_5)\psi$$


$-\frac{1}{4}\left(F_{\mu\nu}(B)F^{\mu\nu}(B)\right)$
$U_V(1)\otimes U_A(1)$
$(eA_\mu - gB_\mu) = eV_\mu$
$(eA_\mu + gB_\mu) = gW_\mu$
INTRODUCE A_μ
SUCH THAT $A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\alpha$
DEFINE $D_\mu \equiv \partial_\mu + ieA_\mu$
IF $\Psi' = e^{i\alpha(x)}\Psi$

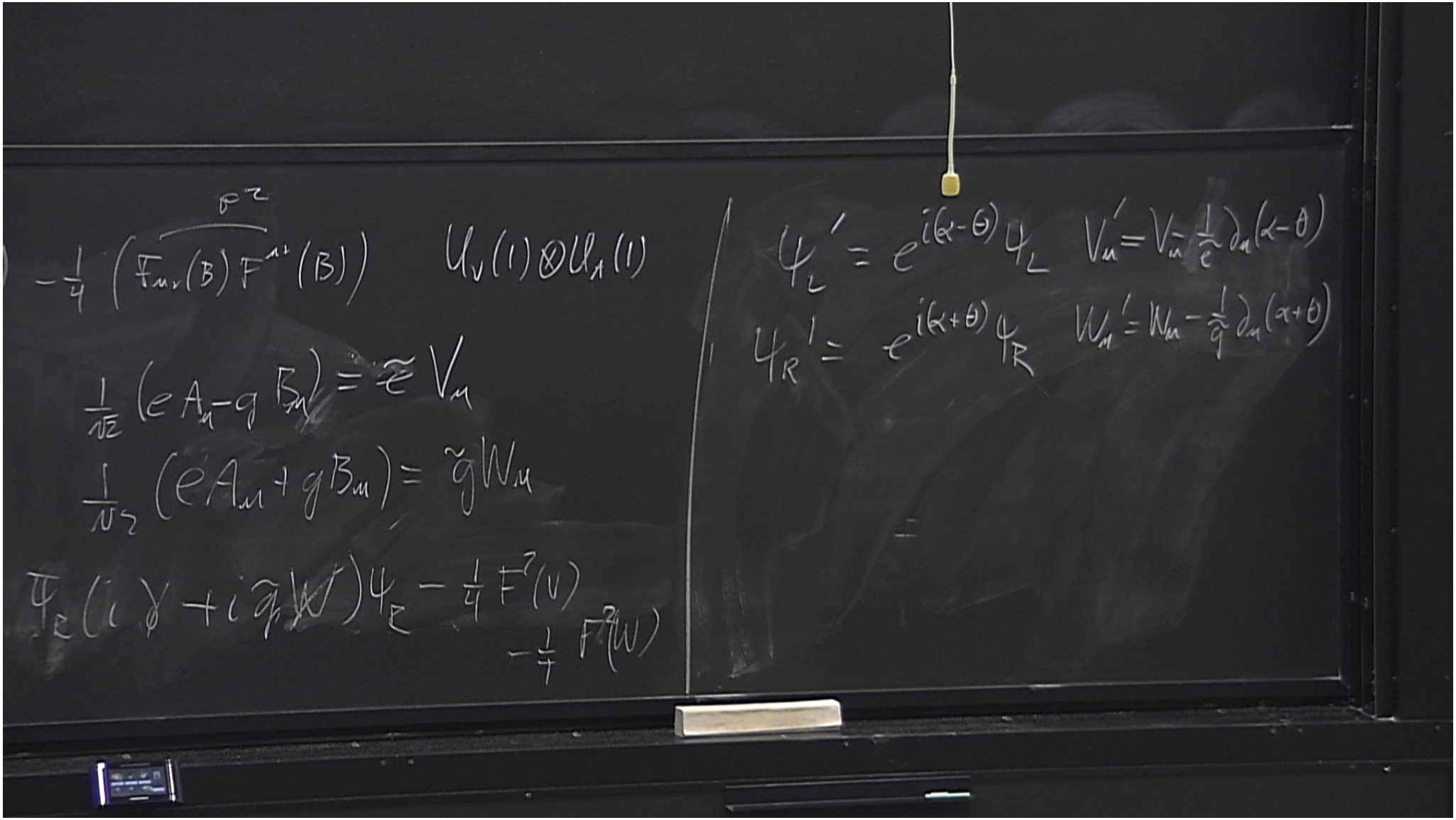
$+ ie\not{A} + ig\not{B}\gamma_5)\psi - \frac{1}{4}(F_{\mu\nu}(A)F^{\mu\nu}(A)) - \frac{1}{4}(F_{\mu\nu}(B)F^{\mu\nu}(B))$ $\quad U_V(1)\otimes U_A(1)$

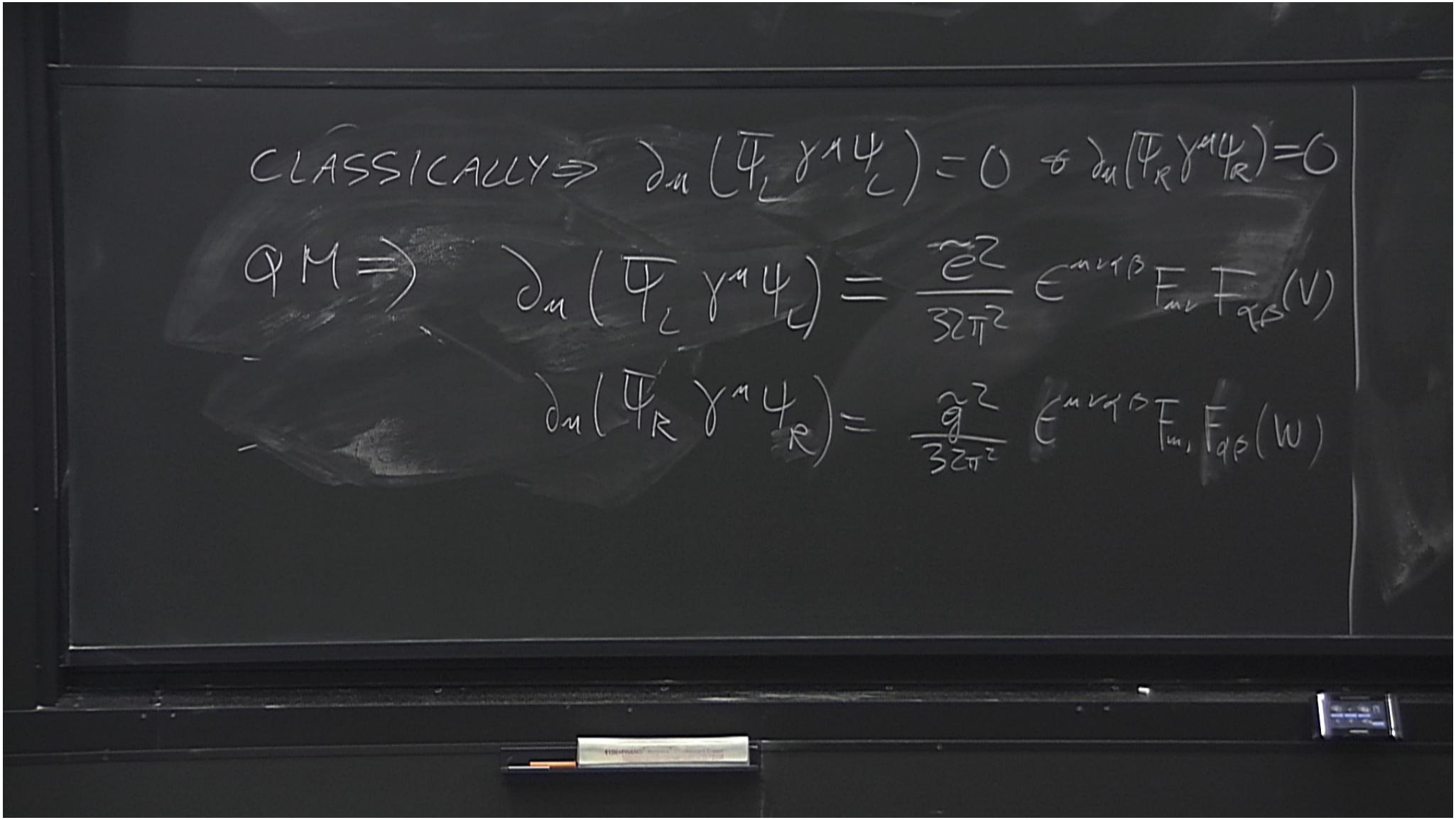
$+\psi_R$ $\quad \psi_L = \frac{1}{2}(1-\gamma_5)\psi$ $\quad \frac{1}{\sqrt{2}}(eA_\mu - gB_\mu) = eV_\mu$

$+\psi_R$ $\quad \psi_R = \frac{1}{2}(1+\gamma_5)\psi$ $\quad \frac{1}{\sqrt{2}}(eA_\mu + gB_\mu) = gW_\mu$

$\mathcal{L} = \bar{\psi}_L(i\partial\!\!\!/ + ie\not{V})\psi_L + \bar{\psi}_R(i\partial\!\!\!/ + ig\not{W})\psi_R - \frac{1}{4}F^2(V)$

$-\frac{1}{4}F^2(W)$


$-\frac{1}{4}\left(\overline{F_{\mu\nu}(B)F^{\mu\nu}(B)}\right)$
$U_V(1)\otimes U_A(1)$
$\frac{1}{\sqrt{2}}(eA_\mu - gB_\mu) = \tilde{e}V_\mu$
$\frac{1}{\sqrt{2}}(eA_\mu + gB_\mu) = \tilde{g}W_\mu$
$\Psi_E(i\partial\!\!\!/ + i\tilde{g}W\!\!\!/)\Psi_E - \frac{1}{4}F^2(V)$
$-\frac{1}{4}F^2(W)$
$\Psi_L' = e^{i(\alpha-\theta)}\Psi_L \quad V_\mu' = V_\mu - \frac{1}{\tilde{e}}\partial_\mu(\alpha-\theta)$
$\Psi_R' = e^{i(\alpha+\theta)}\Psi_R \quad W_\mu' = W_\mu - \frac{1}{\tilde{g}}\partial_\mu(\alpha+\theta)$


CLASSICALLY $\Rightarrow \partial_\mu(\bar{\psi}_L \gamma^\mu \psi_L) = 0$ & $\partial_\mu(\bar{\psi}_R \gamma^\mu \psi_R) = 0$
QM $\Rightarrow \partial_\mu(\bar{\psi}_L \gamma^\mu \psi_L) = \frac{\tilde{e}^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}(V)$
$\partial_\mu(\bar{\psi}_R \gamma^\mu \psi_R) = \frac{\tilde{g}^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}(W)$