Title: 13/14 PSI - Condensed Matter Review - Lecture 10

Date: Feb 10, 2014 09:00 AM

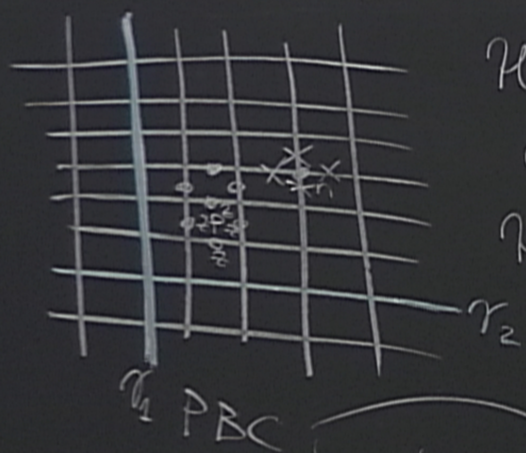
URL: http://pirsa.org/14020046

Abstract:

Quantum lattice $\mathbb{Z}_2$ Gauge Theory

Quantum lattice $\mathbb{Z}_2$ Gauge Theory

$$\hat{B}_p = \prod_{l\in p} \hat{\sigma}^z_l$$



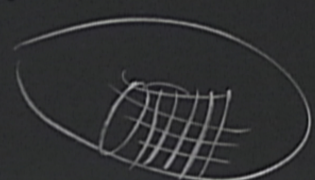
$$\mathcal{H}_{TOT} = \bigotimes_{i=1}^{2L^2} \mathbb{C}^2 \qquad \dim \mathcal{H}_{TOT} = 2^{2L^2}$$

$$\hat{G}(n) = \prod_{l\in n} \hat{\sigma}^x_l \qquad \prod_n \hat{G}(n) = \mathbb{1} = \prod_p \hat{B}_p$$

$$\boxed{\hat{W}^z_{1,2} = \prod_{l\in\gamma_{1,2}} \hat{\sigma}^z_l}$$

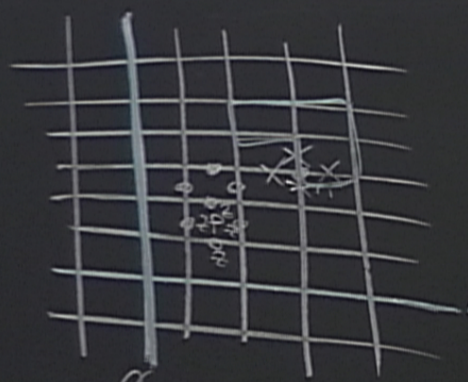
$$\mathcal{H}_{gauge} \equiv \{ \psi \in \mathcal{H}_{TOT} \mid \hat{G}(n)\psi = \psi \ \forall n \}$$

$$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$$



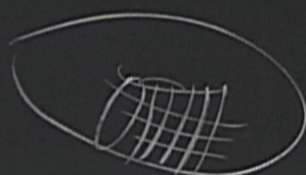
$$\mathcal{H}_{gauge} = \text{span}\{ |B_1 \cdots B_{L^2-1} \quad \rangle \}$$

Quantum lattice Z_2 Gauge Theory



γ_2

γ_1 PBC



$$\mathcal{H}_{TOT} = \bigotimes_{i=1}^{2L^2} \mathbb{C}^2 \qquad \dim \mathcal{H}_{TOT} = 2^{2L^2}$$

$$\hat{G}(n) = \prod_{\ell \in n} \hat{\sigma}^x_\ell \qquad \prod_n \hat{G}(n) = \mathbb{1} = \prod_p \hat{B}_p$$

$$\mathcal{H}_{gauge} \equiv \{ \psi \in \mathcal{H}_{TOT} \mid \hat{G}(n)\psi = \psi \ \forall n \}$$

$$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$$

$$\mathcal{H}_{gauge} = \text{span}\{ |B_1 \cdots B_{L^2-1}; W_1 W_2\rangle \}$$

$$\hat{B}_p = \prod_{\ell \in p} \hat{\sigma}^z_\ell$$

$$\boxed{\hat{W}^z_{1,2} = \prod_{\ell \in \gamma_{1,2}} \hat{\sigma}^z_\ell}$$

this $\mathcal{H}_{gauge}$ is <u>Not local</u>

$= \prod_{l \in p} \hat{\sigma}^z_l$

$[\hat{G}(n), \hat{B}_p] = 0$

$\hat{B}_p$

$$\boxed{\hat{W}^z_{1,2} = \prod_{l \in \gamma_{1,2}} \hat{\sigma}^z_l}$$

this $\mathcal{H}_{gauge}$ is <u>Not local</u>

$$H_{\mathbb{Z}_2} = -J\left(\sum_p \hat{B}_p - g \sum_l \hat{\sigma}^x_l\right) \qquad [\hat{G}(n), H_{\mathbb{Z}_2}] = 0$$

3D classical Ising gauge theory $\xrightarrow[\text{mapping}]{\text{C-Q}}$

$\boxed{g \gg 1}$ GS is unique, there is a gap
(LMS) $\langle \sigma^z_i \sigma^z_j \rangle \sim e^{-|i-j|/\xi}$

$\boxed{g \ll 1 \ (g=0)}$

Elitzur's Theorem

not G.I. quantities must have 0
exp. value

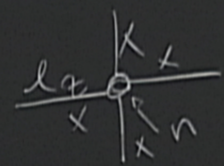
Elitzur's Theorem

not G.I. quantities must have 0 exp. value

$$\langle \Psi_0 | \hat{\sigma}^z_\ell | \Psi_0 \rangle = \langle \Psi_0 | G^\dagger(n)\, \hat{\sigma}^z_\ell\, G(n) | \Psi_0 \rangle$$

$$\langle\psi_0|\hat{\sigma}^z_\ell|\psi_0\rangle = \langle\psi_0|G^\dagger(n)\,\hat{\sigma}^z_\ell\,G(n)|\psi_0\rangle = -\langle\psi_0|\hat{\sigma}^z_\ell\,\underbrace{G^\dagger(n)G(n)}_{\mathbb{1}}|\psi_0\rangle = 0$$

$$(\hat{\sigma}^x)^2 = \mathbb{1}$$



$$\{G(n), \hat{\sigma}^z_\ell\} = 0$$

$= \prod_{\ell \in p} \hat{\sigma}^z_\ell$

$[\hat{G}(n), \hat{B}_p] = 0$

$\hat{W}^z_{1,2} = \prod_{\ell \in \gamma_{1,2}} \hat{\sigma}^z_\ell$

this $\mathcal{H}_{gauge}$ is Not local

$$H_{Z_2} = -J\left(\sum_p \hat{B}_p - g \sum_\ell \hat{\sigma}^x_\ell\right)$$

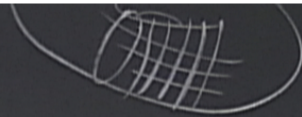
$[\hat{G}(n), H_{Z_2}] = 0$

3D classical Ising gauge theory $\xrightarrow[\text{mapping}]{\text{C-Q}}$

$\boxed{g >> 1}$ GS is unique, there is a gap (LMS) $\langle \sigma^x_i \sigma^x_j \rangle \sim e^{-|i-j|/\xi}$

$\langle \hat{\sigma}^z_\ell \rangle_{\Psi_0} = 0$ $\langle W_C \rangle \sim e^{-A(C)}$

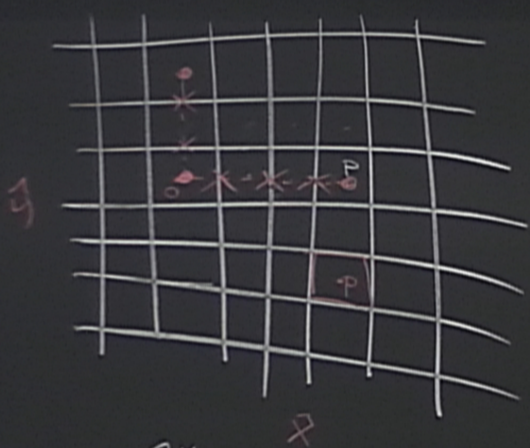
$\boxed{g << 1 \ (g=0)}$ $\langle \hat{\sigma}^z_\ell \rangle = 0$ $\langle W_C \rangle \sim e^{-C}$



$\mathcal{H}_{gauge} = \text{span}\{|B_1 \cdots B_{L^2-1}; W_1 W_2\rangle\}$

Not local

Duality



$$\begin{cases} \mu_1(p) = \hat{B}_p \\ \mu_3(p,\hat{\nu}) = \prod_{l=0}^{p} \hat{\sigma}^x(l,\nu) \end{cases}$$

$\hat{\nu} = \hat{x}, \hat{y}$

$$H_{\mathbb{Z}_2} \longrightarrow H_{Ising2D} = -J \sum_p \hat{\mu}_1(p) - J_g \sum_{l,\hat{\nu}} \mu_3(l,\hat{\nu})\mu_3(l+1,\hat{\nu})$$

$\hat{\nu} = \hat{x}, \hat{y}$

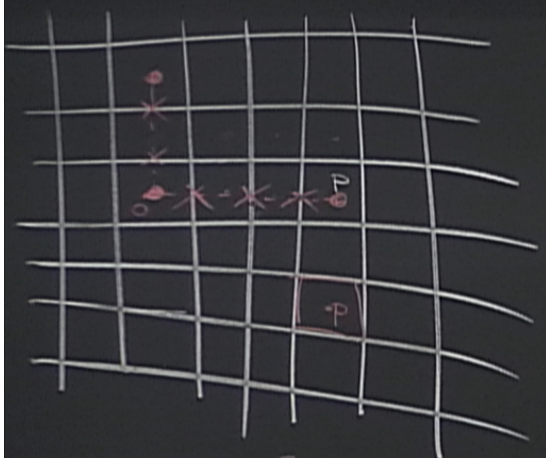
$$\hat{\sigma}^x_l = \mu_3(l,\hat{x})\mu_3(l+1,\hat{x})$$

$l \in \bar{V}$

$$
\begin{tikzcd}
\text{Classical} & \text{Ising Gauge 3D} \arrow[r, leftrightarrow, "\text{duality}"] \arrow[d, leftrightarrow, "\text{CQ}"'] & \text{Ising 3D} \arrow[d, leftrightarrow, "\text{CQ}"] & \boxed{3D} \\
\text{Quantum level} & \text{Quantum } Z_2 \arrow[r, "\text{duality}"] & \text{Quantum Ising} & \boxed{2D}
\end{tikzcd}
$$

$$J_g \sum_{l,\hat{v}} \mu_3(l,\hat{v})\,\mu_3(l+1,\hat{v})$$

$\hat{v} = \hat{x}, \hat{y}$



Duality

W_1, W_2 are fixed

$$\begin{cases}\mu_1(p) = \hat{B}_p \\ \mu_3(p,\hat{\nu}) = \prod_{\ell=0}^{p} \hat{\sigma}^x(\ell,\nu)\end{cases}$$

$\hat{\nu} = \hat{x}, \hat{y}$

$$H_{\mathbb{Z}_2} \longrightarrow H_{Ising\,2D} = -J\sum_p \hat{\mu}_1(p) - Jg\sum_{\ell,\hat{\nu}} \mu_3(\ell,\hat{\nu})\,\mu_3(\ell$$

$\hat{\nu} = \hat{x}, \hat{y}$

$\hat{x}$

$$\hat{\sigma}^x_\ell = \mu_3(\ell,\hat{x})\,\mu_3(\ell+1,\hat{x})$$

$\ell \in \bar{V}$

$$\boxed{g \gg 1 \quad \langle \mu_3 \rangle \neq 0 \rightarrow \langle \prod_{\ell}^{p} \hat{\sigma}^x_\ell \rangle \neq 0}$$

$x \in V$

$H_{\mathbb{Z}_2}$

- There is a QPT
- $g = 0$ GS : is st $B_p = 1$

$$E_0 = -JL^2$$

$$B_{\tilde{p}} = -1 \qquad E = -JL^2 + 2J$$

GS is 4-fold degenerate

$$\mathcal{H}_{GS} \equiv \mathrm{Span}\left\{ |B_{p_1} = +1 \dots B_p = +1,\ W_1 = \pm 1\ W_2 = \pm 1 \rangle \right\}$$

$\mu_3(\ell,\hat{x})\,\mu_3(\ell+1,\hat{x})$ $\Rightarrow \langle \prod_\ell \hat{\sigma}^x_\ell \rangle \neq 0$

$\ell \in \bar{V}$

$H_{\mathbb{Z}_2}$

- There is a QPT
- $g=0$ GS : is st $B_p = 1$

$$E_0 = -JL^2$$

$$B_{\tilde{p}} = -1 \qquad E = -JL^2 + 2J$$

GS is 4-fold degenerate

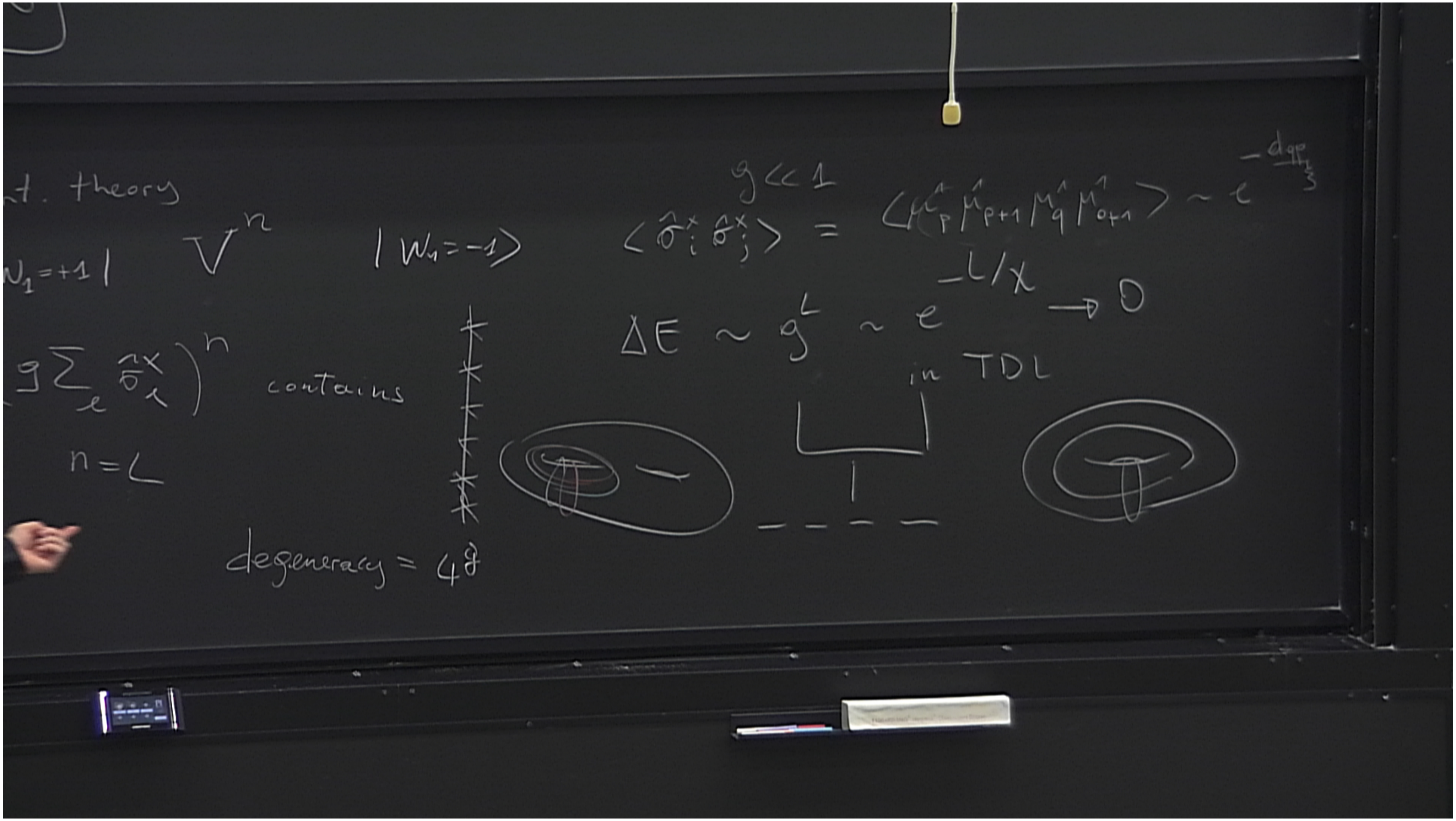
$$\mathcal{L}_{GS} \equiv \mathrm{Span}\left\{ |B_{p_1}=+1 \dots = B_p=+1,\ W_1=\pm 1\ W_2=\pm 1\rangle \right\}$$

Pert. theory

$$\langle W_1=+1|\ V^n\ |W_1=-1\rangle$$

$\left(g \sum_\ell \hat{\sigma}^x_\ell\right)^n$ contains

$$n=L$$



nt. theory
V^n
$W_1=+1|$
$|W_1=-1\rangle$
$g \ll 1$
$\langle \hat{\sigma}^x_i \hat{\sigma}^x_j \rangle = \langle \hat{\mu}_p \hat{\mu}_{p+1} \hat{\mu}_q \hat{\mu}_{q+1} \rangle \sim e^{-d_{qp}}$
$(g \sum_e \hat{\sigma}^x_e)^n$ contains
$n = L$
$\Delta E \sim g^L \sim e^{-L/x} \to 0$
in TDL
degeneracy = 4^g