Title: 13/14 PSI - Condensed Matter Review - Lecture 9

Date: Feb 07, 2014 09:00 AM

URL: http://pirsa.org/14020045

Abstract:

Elitzur's Theorem

It is impossible to spontaneously break a local "symmetry"

$$G(n) = \prod_{\ell \in n} \hat{\sigma}^x_\ell$$

$$H = -J \sum_{\Box} \sigma^z \sigma^z \sigma^z \sigma^z$$

$$\tilde{h}\sum_{\ell}\sigma^z_{\ell}$$

$$\langle \sigma^z(n,\nu)\rangle_h = \frac{1}{Z_h}\sum_{\{\sigma^z(n,\nu)\}} \sigma^z(n,\nu)\, e^{\left(\beta\sum \sigma^z\sigma^z\sigma^z\sigma^z + h\sum \sigma^z\right)}$$

Elitzur's Theorem

It is impossible to spontaneously break a local "symmetry"

$$G(n) = \prod_{\ell \in n} \hat{\sigma}^x_\ell$$

$$H = -J \sum_{\square} \sigma^z \sigma^z \sigma^z \sigma^z$$

$$\tilde{h} \sum_\ell \sigma^z_\ell$$

$$\langle \sigma^z(n,\nu) \rangle_h = \frac{1}{Z} \sum_{\{\sigma^z(n,\nu)\}} \quad \sigma^z(n,\nu$$

$$\sigma^z_\ell \rightarrow \sigma'^z_\ell = -\sigma^z_\ell$$

$$\ell = (n,\nu)$$

Elitzur's Theorem

It is impossible to spontaneously break a local "symmetry"

$$G(n) = \prod_{\ell \in n} \hat{\sigma}^x_\ell$$

$$H = -J \sum_{\Box} \sigma^z \sigma^z \sigma^z \sigma^z$$

$$\tilde{h} \sum_\ell \sigma^z_\ell$$

$$\langle \sigma^z(n,\nu) \rangle_h = \frac{1}{Z} \sum_{\{\sigma^z(n,\nu)\}} \sigma^z(n,\nu)$$

$$\sigma^z_\ell \to \sigma'^z_\ell = -\sigma^z_\ell$$

$$\ell = (n,\nu) \qquad \equiv \{\ell_n\}$$

$$h \sum \sigma^z = h \sum \sigma'^z - \sum \delta\sigma^z$$

$$\delta\sigma^z_{\ell_n} = \sigma'^z_{\ell_n} - \sigma^z_{\ell_n} = -2\sigma^z_{\ell_n}$$

0 when $\ell \notin \{\ell_n\}$

$$\sigma^z(n,\nu)\, e^{(\beta\sum \sigma^z\sigma^z\sigma^z\sigma^z + h\sum \sigma^z)}$$

$$= \frac{1}{Z_1}\sum_{\{\sigma\}} -\sigma'^z_\ell \exp\left[\beta\sum \sigma'\sigma'\sigma'\sigma' + h\sum \sigma'^z - \sum \delta\sigma^z\right]$$

$$= \left\langle -\sigma'^z_\ell\, e^{-\sum\delta\sigma^z} \right\rangle_h$$

$$\left| \langle \sigma^z_\ell \rangle_h - \langle -\sigma^z_\ell \rangle_h \right| = \left| \left\langle \sigma^z_\ell \left(e^{-\sum\delta\sigma^z} - 1 \right) \right\rangle_h \right|$$

$^z(n,\nu)\}$

σ^z_ℓ

$\delta\sigma^z$

$$\sigma^z(n,\nu)\, e^{(\beta\sum \sigma^z\sigma^z\sigma^z\sigma^z + h\sum \sigma^z)}$$

$$= \frac{1}{Z_1}\sum_{\{\sigma\}} -\sigma'^z_\ell \exp\Big[\underbrace{\beta\sum \sigma'\sigma'\sigma'\sigma' + h\sum \sigma'^z}\, - \sum \delta\sigma^z\Big]$$

$$= \left\langle -\sigma'^z_\ell\, e^{-\sum\delta\sigma^z} \right\rangle_h$$

$$\left| \langle \sigma^z_\ell \rangle_h - \langle -\sigma^z_\ell \rangle_h \right| = \left| \left\langle \sigma^z_\ell \left(e^{h\sum\delta\sigma^z} - 1 \right) \right\rangle_h \right| \leqslant \left| \langle \sigma^z_\ell \rangle_h \right| \left| e^{4dh} - 1 \right|$$

$$e^{-h\sum\delta\sigma^z} \leqslant e^{4dh} \qquad h \to 0$$

$\sigma^z(n,\nu)\}$

σ^z_ℓ

$\delta\sigma^z$

$$(\beta\sum \sigma^z\sigma^z\sigma^z\sigma^z + h\sum \sigma^z)$$

$$\sigma^z(n,\nu)\, e$$

$$= \frac{1}{Z_1} \sum_{\{\sigma\}} -\sigma'^z_\ell \exp\Big[\underbrace{\beta\sum \sigma'\sigma'\sigma'\sigma' + h\sum \sigma'^z}\; - \sum \delta\sigma^z\Big]$$

$$= \Big\langle -\sigma'^z_\ell e^{-\sum\delta\sigma^z} \Big\rangle_h$$

$$\left|\langle\sigma^z_\ell\rangle_h - \langle -\sigma^z_\ell\rangle_h\right| = \left|\Big\langle \sigma^z_\ell \big(e^{h\sum\delta\sigma^z} - 1\big)\Big\rangle_h\right| \leq \left|\langle\sigma^z_\ell\rangle_h\right| \left|e^{4dh} - 1\right|$$

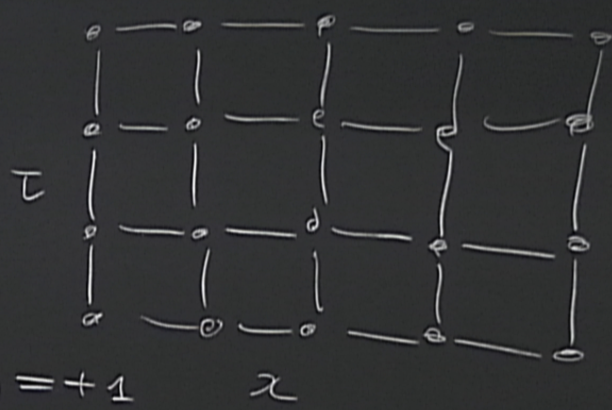
$$e^{-h\sum\delta\sigma^z} \leq e^{4dh} \qquad h \to 0 \quad \to 0$$

$$\Rightarrow \langle \sigma^z_\ell \rangle_{h\to 0} = 0$$

$$\sigma^z_\ell \qquad \delta\sigma^z \qquad \sigma^z(n,\nu)\}$$

0 when $\ell \notin \{\ell_n\}$

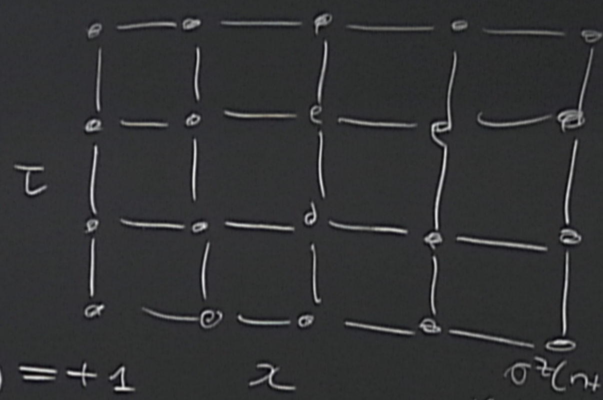
$d=2$



gauge fixing $\sigma^z(n,\tau) = +1$

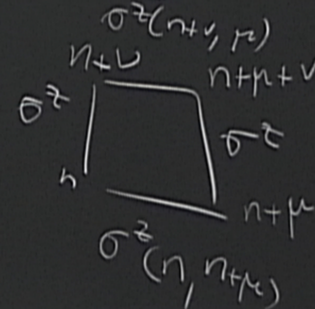
0 when $e \notin \{e_n\}$

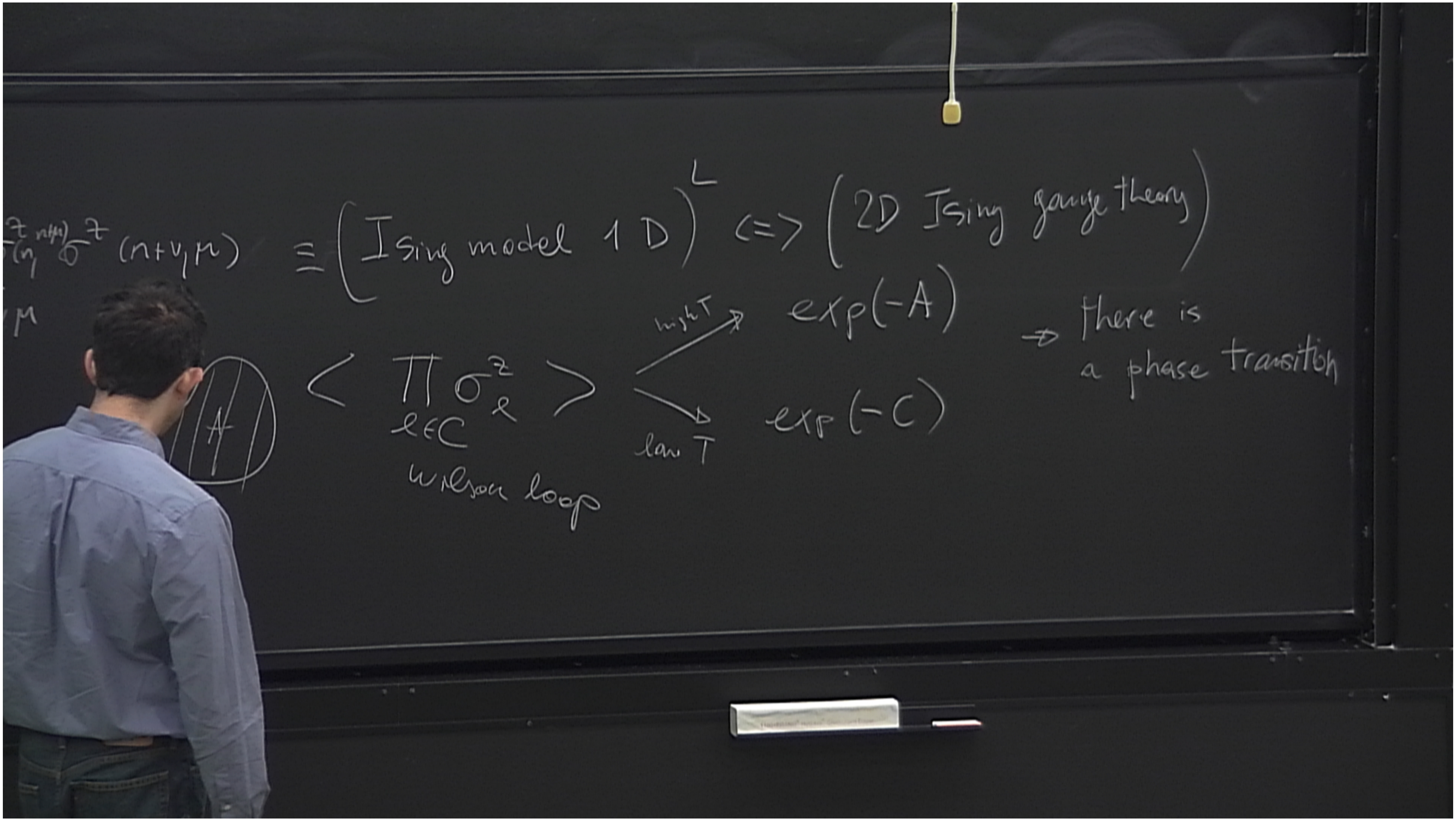
$d = 2$



$$H = -J \sum_{n,\nu\mu} \sigma^z(n, n+\mu)\, \sigma^z(n+\nu, \mu)$$

gauge fixing $\sigma^z(n, \tau) = +1$





(n+ν,μ) ≡ (Ising model 1D)^L <=> (2D Ising gauge theory)
high T
exp(-A)
there is
a phase transition
A
< Π σ^z_ℓ >
ℓ∈C
low T
exp(-C)
Wilson loop

$\sigma^z_{(n, n+\mu)}$

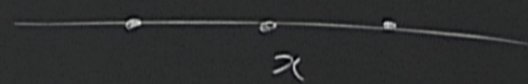
QPTs without symm. breaking

QM system $(H, \mathcal{H})$

free particle on a line
periodic potential

$$H = \frac{p^2}{2m} + V \cos \frac{2\pi x}{a}$$

$$\mathcal{H} = \{ |x\rangle, x \in \mathbb{R} \}$$



$$\hat{T}_y |x\rangle = |x+y\rangle$$

$\sigma^{z}_{(n,n+\mu)}$

QPTs without symm. breaking

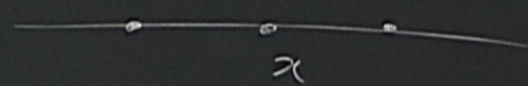
QM system $(H, \mathcal{H})$

free particle on a line

periodic potential

$$H = \frac{p^2}{2m} + V\cos\frac{2\pi x}{a}$$

$$\mathcal{H} = \{ |x\rangle, x \in \mathbb{R} \}$$



$$\hat{T}_y |x\rangle = |x+y\rangle$$

$$\hat{T}_a |x\rangle = |x+a\rangle$$

$$[\hat{T}_a, H] = 0$$

$|x\rangle = |x+y\rangle$

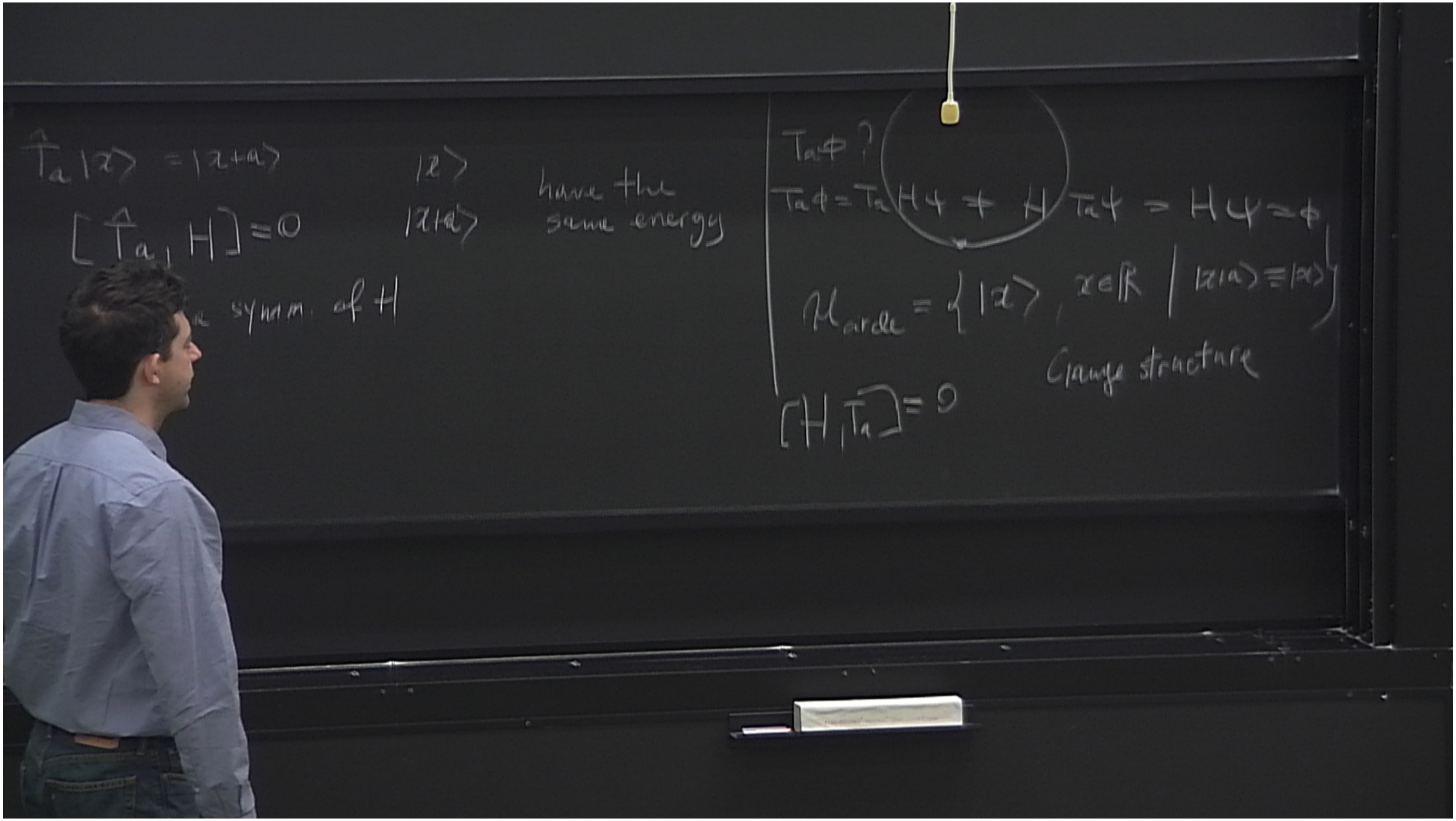
$\hat{T}_a |x\rangle = |x+a\rangle$

$[\hat{T}_a, H] = 0$

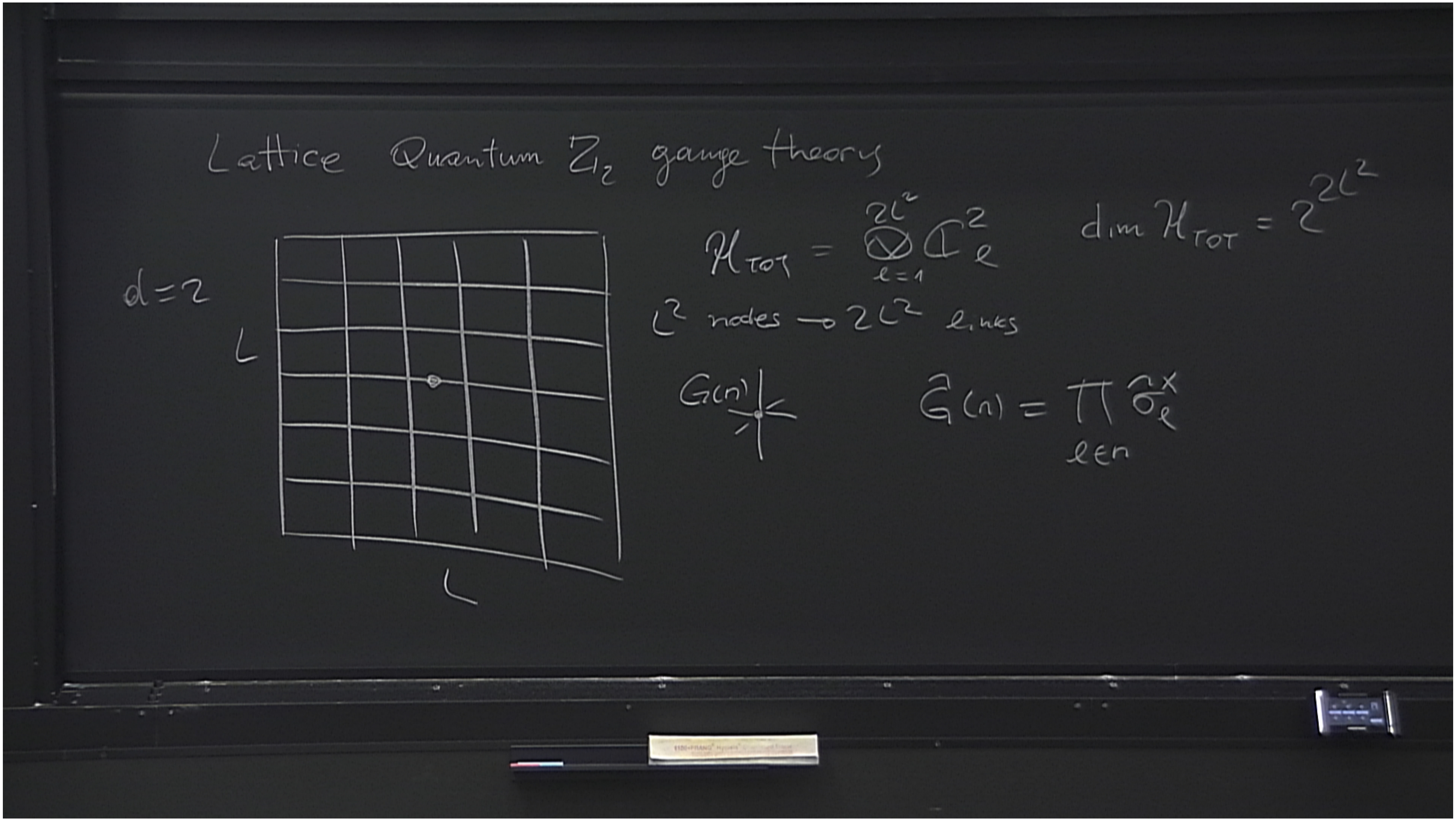
$\hat{T}_a$ a symm. of H

$|x\rangle$, $|x+a\rangle$ have the same energy

$\mathcal{H}_{circle} = \{ |x\rangle, \; x \in \mathbb{R} \mid |x+a\rangle \equiv |x\rangle \}$



T̂a|x> = |x+a>
|x>
|x+a>
have the same energy
[T̂a, H] = 0
symm. of H
Taφ ?
Taφ = Ta Hψ ≠ H Taψ = Hψ = φ
Hcircle = { |x>, x∈ℝ | |x+a> ≡ |x> }
Gauge structure
[H, Ta] = 0


Lattice Quantum $\mathbb{Z}_2$ gauge theory
d=2
L
L
$\mathcal{H}_{TOT} = \bigotimes_{\ell=1}^{2L^2} \mathbb{C}^2_\ell$
$\dim \mathcal{H}_{TOT} = 2^{2L^2}$
L^2 nodes $\to$ $2L^2$ links
G(n)
$\hat{G}(n) = \prod_{\ell \in n} \hat{\sigma}^x_\ell$

$$= 2^{2L^2}$$

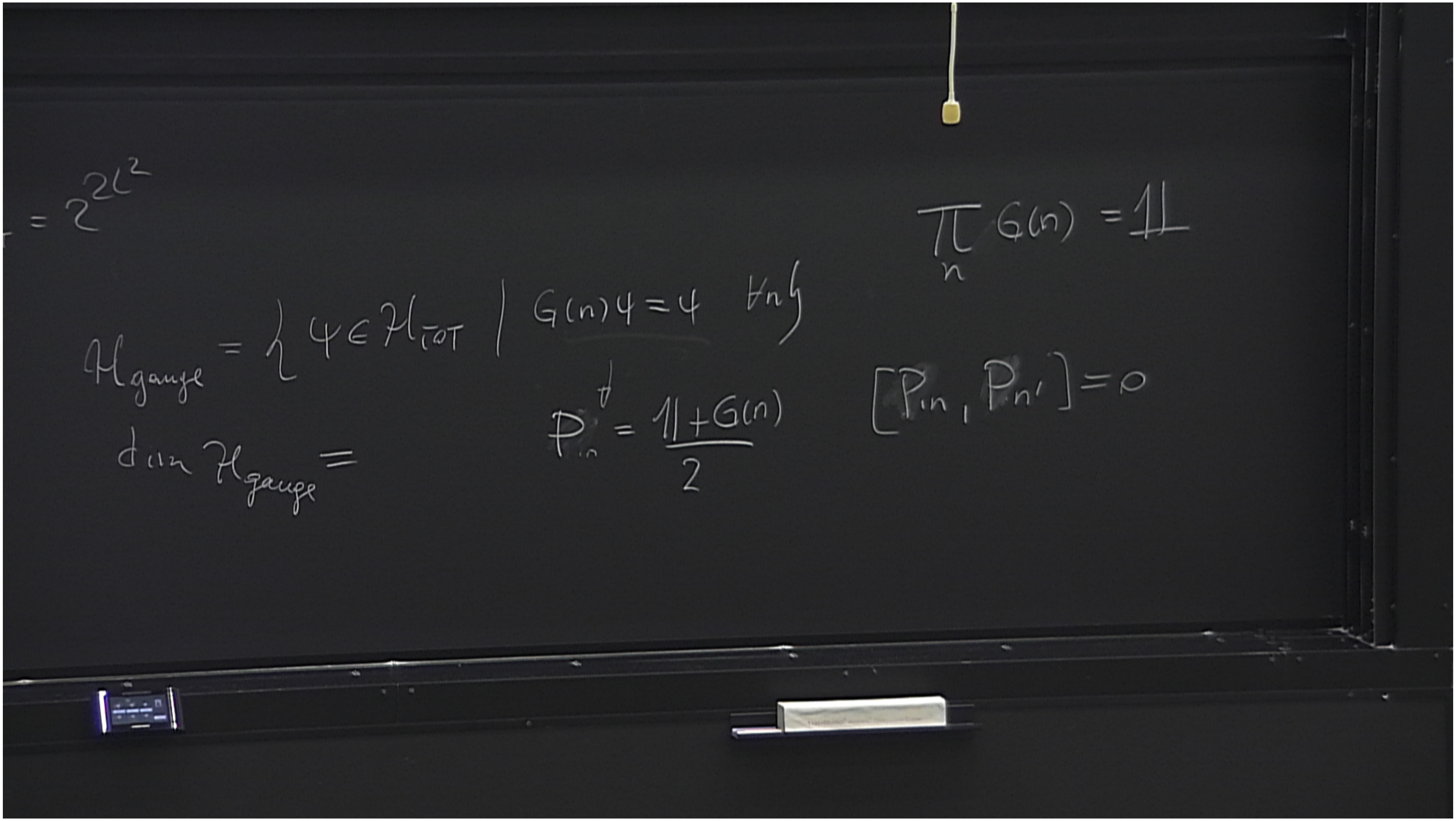
$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$$

$= 2^{2L^2}$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$$

$$\dim \mathcal{H}_{gauge} =$$

$$\Pi_n = \frac{\mathbb{1} + G(n)}{2} \qquad [\Pi_n , \Pi_{n'}] = 0$$



$= 2^{2L^2}$
$\prod_n G(n) = \mathbb{1}$
$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \ \forall n \}$
$P_n = \frac{\mathbb{1} + G(n)}{2}$
$[P_n, P_{n'}] = 0$
$\dim \mathcal{H}_{gauge} =$

$$\dim \mathcal{H}_{TOT} = 2^{2L^2}$$

$$\prod_n G(n) = \mathbb{1}$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$$

$$\hat{\sigma}^x_\ell$$

$$\dim \mathcal{H}_{gauge} = 2^{2L^2-(L^2-1)} = 2^{L^2+1}$$

$$P_n^\dagger = \frac{\mathbb{1}+G(n)}{2}$$

$$[P_n, P_{n'}] = 0$$

$$\dim \mathcal{H}_{TOT} = 2^{2L^2}$$

$$\hat{B}_p = \prod_{\ell \in p} \hat{\sigma}^z_\ell$$

$$[\hat{B}_p , \hat{G}(n)] = 0$$

$$\prod_n G(n) = \mathbb{1}$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \ \forall n \}$$

$$\hat{\sigma}^x_\ell$$

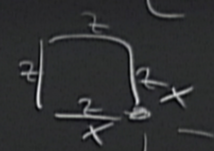
$$\dim \mathcal{H}_{gauge} = 2^{2L^2-(L^2-1)} = 2^{L^2+1}$$

$$P_n^\dagger = \frac{\mathbb{1}+G(n)}{2}$$

$$[P_n , P_{n'}] = 0$$

$$\dim \mathcal{H}_{TOT} = 2^{2L^2}$$

$$\hat{B}_p = \prod_{\ell \in p} \hat{\sigma}^z_\ell$$



$$[\hat{B}_p, \hat{G}(n)] = 0$$

$$\prod_n G(n) = \mathbb{1}$$

$$\hat{\sigma}^x_\ell$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$$

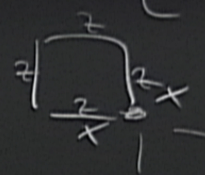
$$\dim \mathcal{H}_{gauge} = 2^{2L^2 - (L^2 - 1)} = 2^{L^2+1}$$

$$P_n^\dagger = \frac{\mathbb{1} + G(n)}{2}$$

$$[P_n, P_{n'}] = 0$$

$\dim \mathcal{H}_{TOT} = 2^{2L^2}$

$$\hat{B}_p = \prod_{l \in p} \hat{\sigma}_l^z$$



$$[\hat{B}_p, \hat{G}(n)] = 0$$

$$\prod_n G(n) = \mathbb{1}$$

$$\prod_p B_p = \mathbb{1}$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$$

$\hat{\sigma}_l^x$

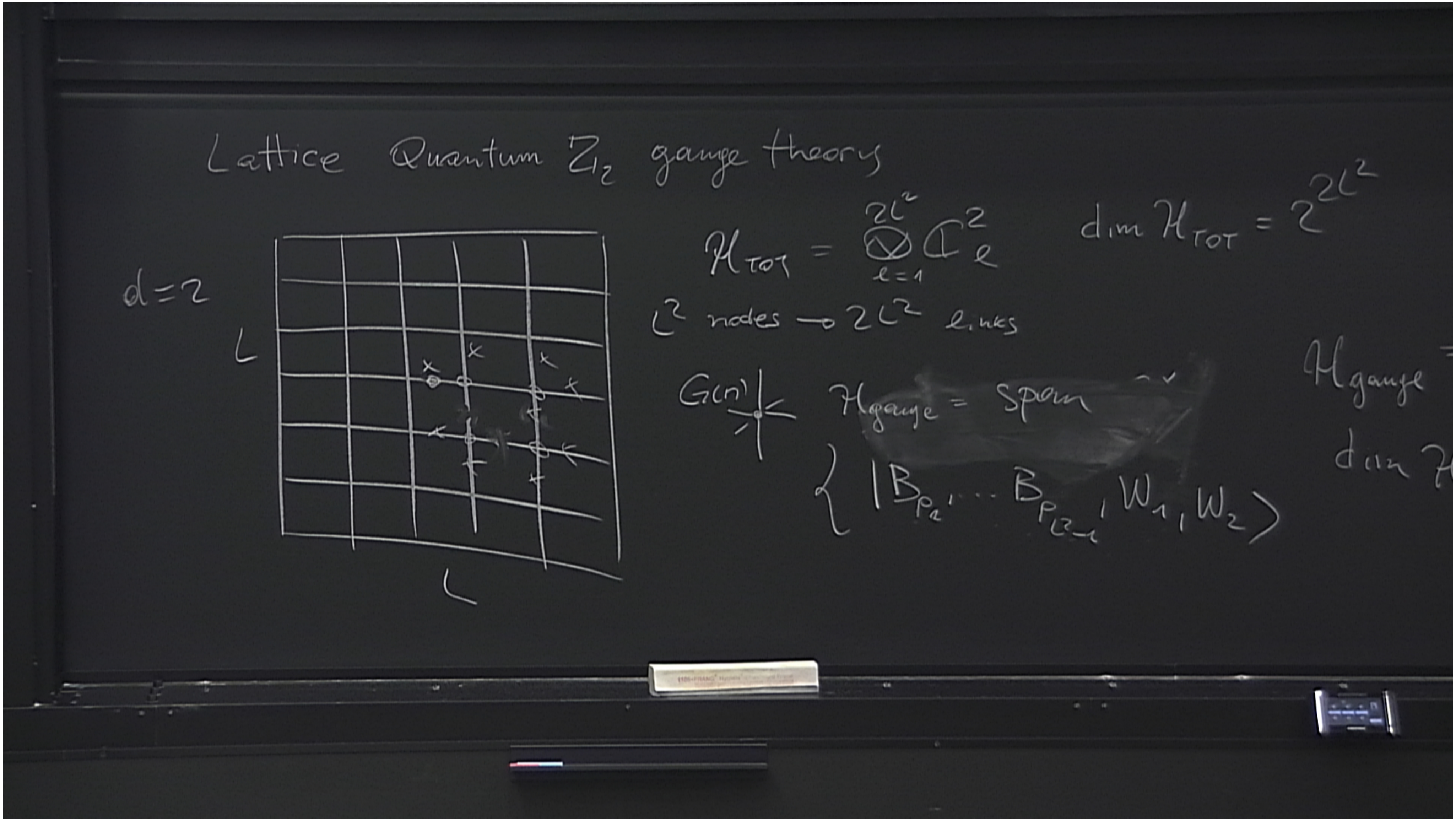
$$2L^2 - (L^2 - 1)$$

$$\dim \mathcal{H}_{gauge} = 2^{2L^2-(L^2-1)} = 2^{L^2+1}$$

$$P_n = \frac{\mathbb{1} + G(n)}{2}$$

$$[P_n, P_{n'}] = 0$$

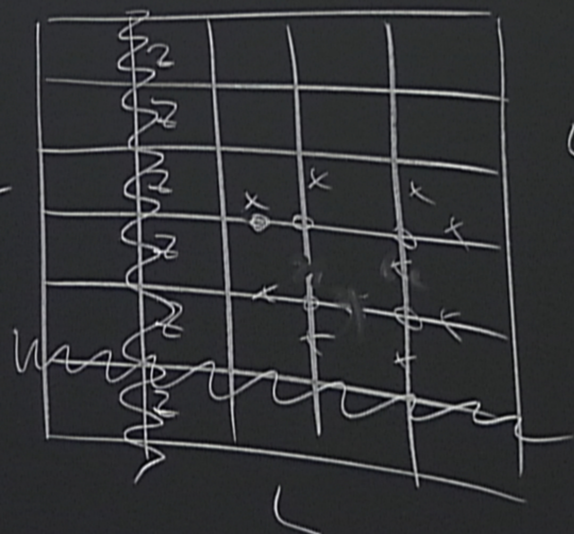
$L^2 - 1$


Lattice Quantum Z2 gauge theory
d=2
L
L
H_TOT = ⊗_{l=1}^{2L^2} C^2_l
dim H_TOT = 2^{2L^2}
L^2 nodes → 2L^2 links
G(n)
H_gauge = span
{ |B_{P_1}, ... B_{P_{L^2-1}}, W_1, W_2 >
H_gauge
dim

Lattice Quantum $\mathbb{Z}_2$ gauge theory

d=2

L



L

$$\mathcal{H}_{TOT} = \bigotimes_{\ell=1}^{2L^2} \mathbb{C}^2_\ell \qquad \dim \mathcal{H}_{TOT} = 2^{2L^2}$$

L^2 nodes $\to$ $2L^2$ links

$G(n)$

$$\mathcal{H}_{gauge} = \mathrm{span}\{|B_{p_2},\dots,B_{p_{L^2-1}}, W_1, W_2\rangle$$

$\mathcal{H}_{gauge} =$

$\dim \mathcal{H}$