

Title: 13/14 PSI - Condensed Matter Review - Lecture 7

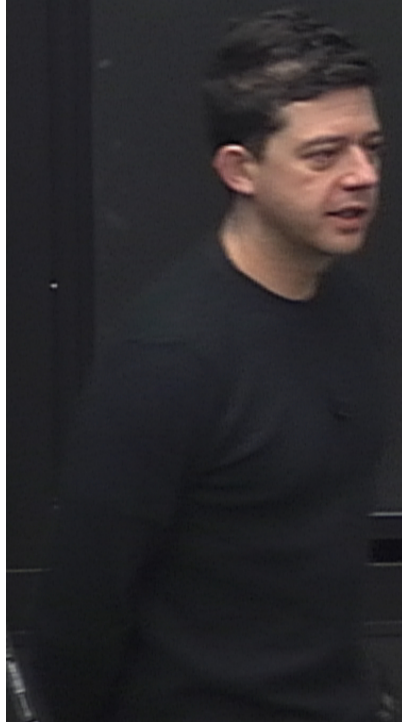
Date: Feb 05, 2014 09:00 AM

URL: <http://pirsa.org/14020041>

Abstract:



Lieb-Robinson Bounds



Lieb-Robinson Bounds

$$H = \sum_x \Phi_x$$

$$\text{diam}(X) \in \mathbb{R}$$

$$|X| \leq \sigma$$

$$\sup_x \|\Phi_x\| \leq 1$$

Lieb-Robinson Bounds

$$H = \sum_X \Phi_X \quad \text{diam}(X) \in \mathbb{R}$$

$$|X| \leq \sigma$$

$$\sup_X \|\Phi_X\| \leq K$$

$$\|[A(t), B]\| \leq C e^{-\frac{d-|t|}{\xi}}$$

$$\xi = a/\lambda$$

a is the lattice spacing

$$v = \frac{2K\sigma e^{\lambda R}}{\lambda}$$

Lieb-Robinson Bounds

$$H = \sum_X \Phi_X \quad \text{diam}(X) \in \mathbb{R}$$

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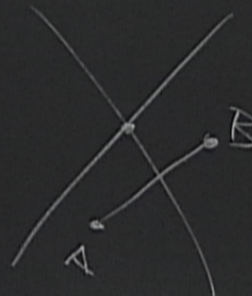
$$\sup_X \|\Phi_X\| \leq K$$

$$\sum = a/\lambda$$

a is the lattice spacing

$$\|[A(t), B]\|$$

$$v = \sum K$$



No Signaling

Lieb-Robinson Bounds

$$H = \sum \Phi_X \quad \text{diam}(X) \in \mathbb{R}$$

$$|X| \leq \sigma$$

$$\sup_X \|\Phi_X\| \leq K$$

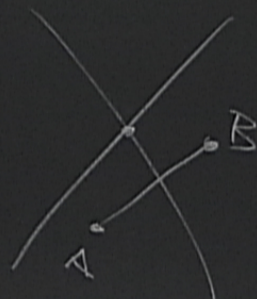
$$\|[A(t), B]\| \leq$$

$$\frac{d-vt}{\hbar}$$

$$\sum = a/\lambda$$

a is the lattice spacing

$$v = \dots$$



"No" Signaling

Lieb-Robinson Bounds

$$H = \sum_x \Phi_x$$

$$\text{diam}(X) \in \mathbb{R}$$

$$|X| \leq \sigma$$

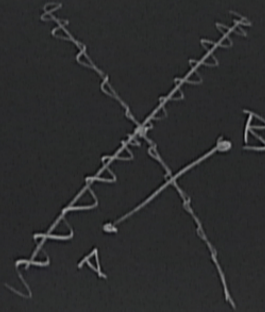
$$\sup_x \|\Phi_x\| \leq K$$

$$\sum = a_1$$

a is the lattice spacing

$$\|[A(t), B]\| \leq \frac{d \cdot v t}{\sum}$$

$$v = ?$$



"No" Signaling

Lieb-Robinson Bounds

$$H = \sum_x \Phi_x \quad \text{diam}(X) \in \mathbb{R}$$

$$\| [A(t), B] \| \leq \frac{d \cdot v t}{\delta}$$

$$v = ? \in \mathbb{R}$$

$$|X| \leq \delta$$

$$\sup_x \|\Phi_x\| \leq K$$

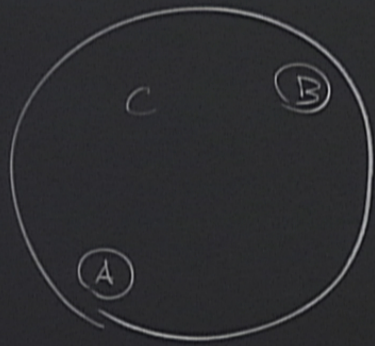
$$\delta = a/\lambda$$

a is the lattice spacing

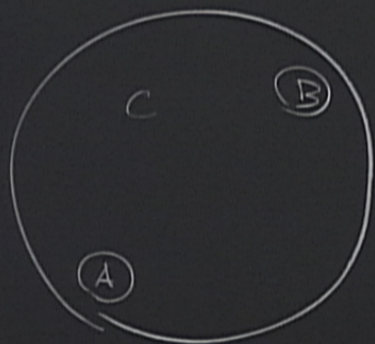


"No" Signaling



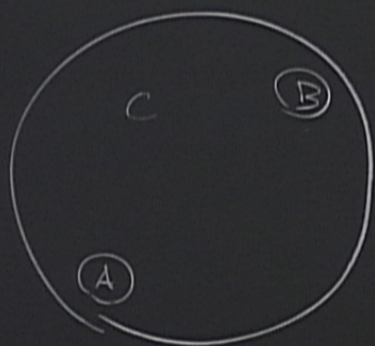


$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$



$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

ρ_0 initial state
for the whole system
(ABC)

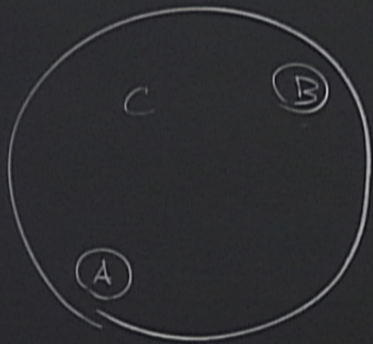


$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

ρ_0 initial state
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A can operate with
a unitary operation

$$U_A^k$$



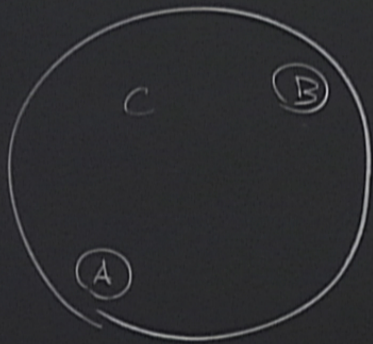
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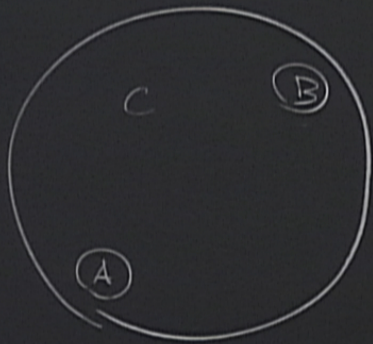
$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

ρ_0 initial state
for the whole system
(ABC)

A can operate with
a unitary operation

$$U_A^k \quad k \text{ labels the different operations}$$

$$U_A^0 = \mathbb{1}_A$$



$$\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

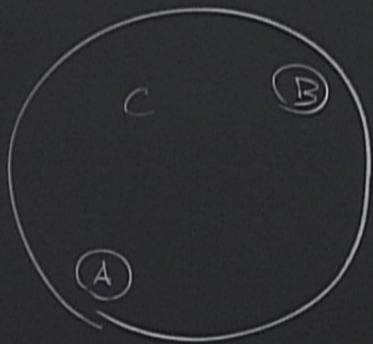
H on $\mathcal{H}_{ABC}$

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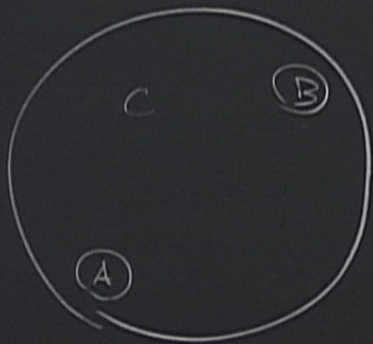
H on $\mathcal{H}_{ABC}$
which is local

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$$\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

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H on $\mathcal{H}_{ABC}$
which is local

$$\rho_{ABC}^k = U_A^k \rho_0 U_A^{k\dagger}$$

$$\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

H on $\mathcal{H}_{ABC}$
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$$\rho_{ABC}^K = U_A^K \rho_0 U_A^{K\dagger}$$

$$U_{ABC}(t) = e^{-iHt}$$

A can operate with
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U_A^K K labels
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$$U_A^0 = \mathbb{1}_A$$

(B)

$$\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

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$$\downarrow t$$
$$U_{ABC}(t) \rho_{ABC}^k U_{ABC}^\dagger(t)$$

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H on $\mathcal{H}_{ABC}$
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operate

$$U_A$$

$$U_A^0 = 1_A$$

$$\rho_{ABC}^K = U_A^K \rho_0 U_A^{K\dagger}$$

$$U_{ABC}(t) = e^{-iHt}$$

$$\text{Tr} [U_{ABC}(t) \rho_{ABC}^K U_{ABC}^\dagger(t)]$$

$$\mathcal{H}_{ABC} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

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$$\text{Tr}_{AC} [U_{ABC}(t) \rho_{ABC}^k U_{ABC}^\dagger(t)]$$

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$$\downarrow t$$

$$\text{Tr}_{AC} [U_{ABC}(t) \rho_{ABC}^k U_{ABC}^\dagger(t)] \equiv \sigma_B^k(t)$$

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$$\rho_{ABC}^k = U_A^k \rho_0 U_A^{k\dagger}$$

$$U_{ABC}(t) = e^{-iHt}$$

$$\begin{aligned} & \downarrow t \\ & \text{Tr}_{AC} [U_{ABC}(t) \rho_{ABC}^k U_{ABC}^\dagger(t)] \equiv \sigma_B^k(t) \\ & \sigma_B^0(t) \end{aligned}$$

B can measure $\hat{O}_B$

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$$O_B^0 = \text{Tr} [\hat{O}_B \sigma_B^0(t)]$$

O_B^k

B can measure $\hat{O}_B$

$$\begin{cases} \langle O_B^0 \rangle = \text{Tr} [\hat{O}_B \sigma_B^0(t)] \\ \langle O_B^K \rangle = \text{Tr} [\hat{O}_B \sigma_B^K(t)] \end{cases}$$

B can measure $\hat{O}_B$

$$\begin{cases} O_B^0 &= \text{Tr} [\hat{O}_B \sigma_B^0(t)] \\ O_B^K &= \text{Tr} [\hat{O}_B \sigma_B^K(t)] \end{cases}$$

$$|O_B^0(t) - O_B^K(t)|$$

B can measure $\hat{O}_B$

$$\begin{cases} O_B^0 = \text{Tr}[\hat{O}_B \sigma_B^0(t)] \\ O_B^K = \text{Tr}[\hat{O}_B \sigma_B^K(t)] \end{cases}$$

$$|O_B^0(t) - O_B^K(t)|$$

(trace distance)

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$$|O_B^0(t) - O_B^K(t)| = \text{Tr}[\hat{O}_B (\sigma_B^0(t) - \sigma_B^K(t))]$$

(trace distance)

B can measure $\hat{O}_B$

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distance)

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$$\begin{cases} O_B^0 = \text{Tr} [\hat{O}_B \sigma_B^0(t)] \\ O_B^K = \text{Tr} [\hat{O}_B \sigma_B^K(t)] \end{cases}$$

$$|O_B^0(t) - O_B^K(t)| = \left| \text{Tr} \left[\hat{O}_B (\sigma_B^0(t) - \sigma_B^K(t)) \right] \right| = \text{Tr} \left[\hat{O}_B U_t \Delta \sigma(0) U_t^\dagger \right]$$

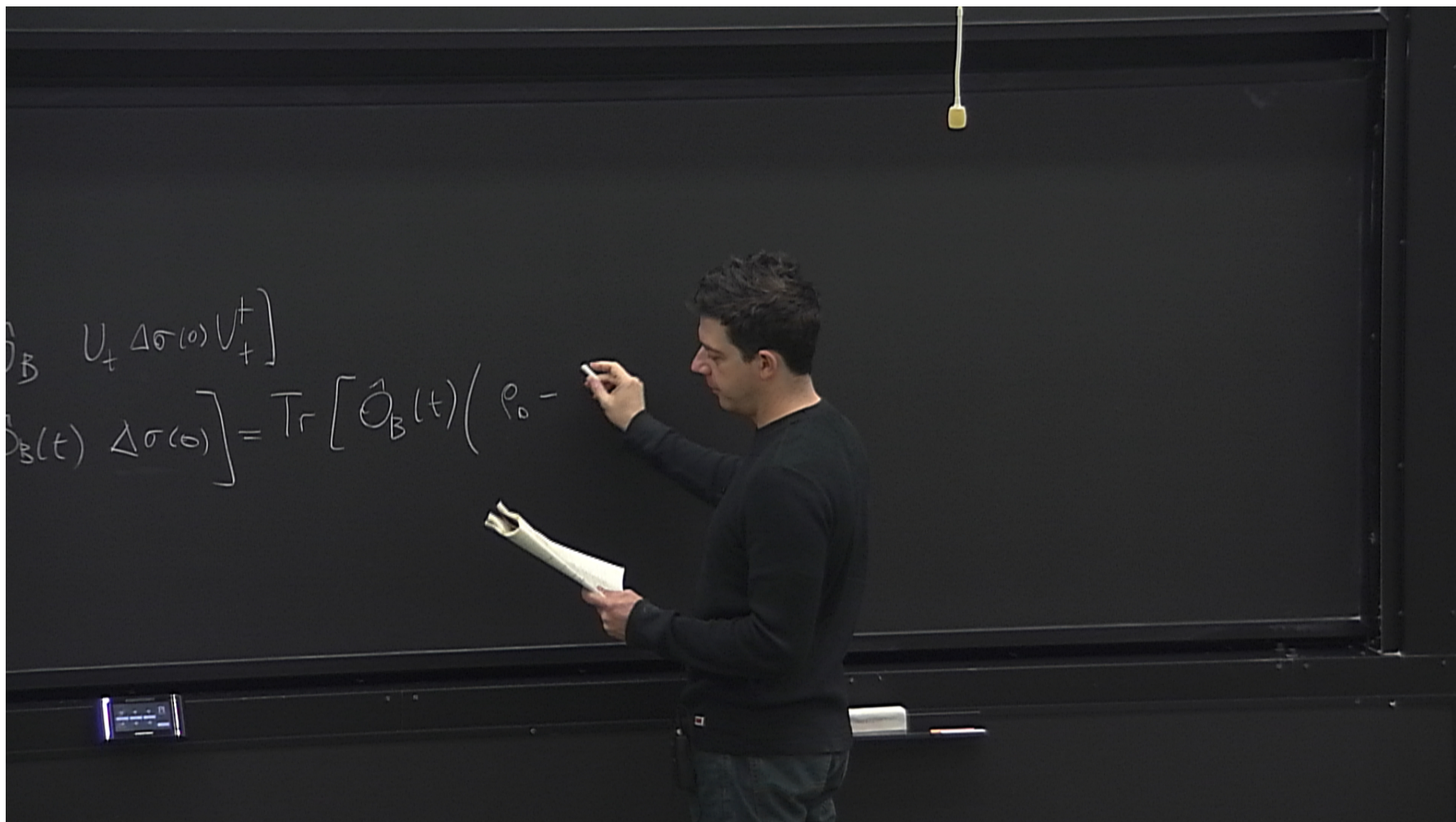
(trace distance)

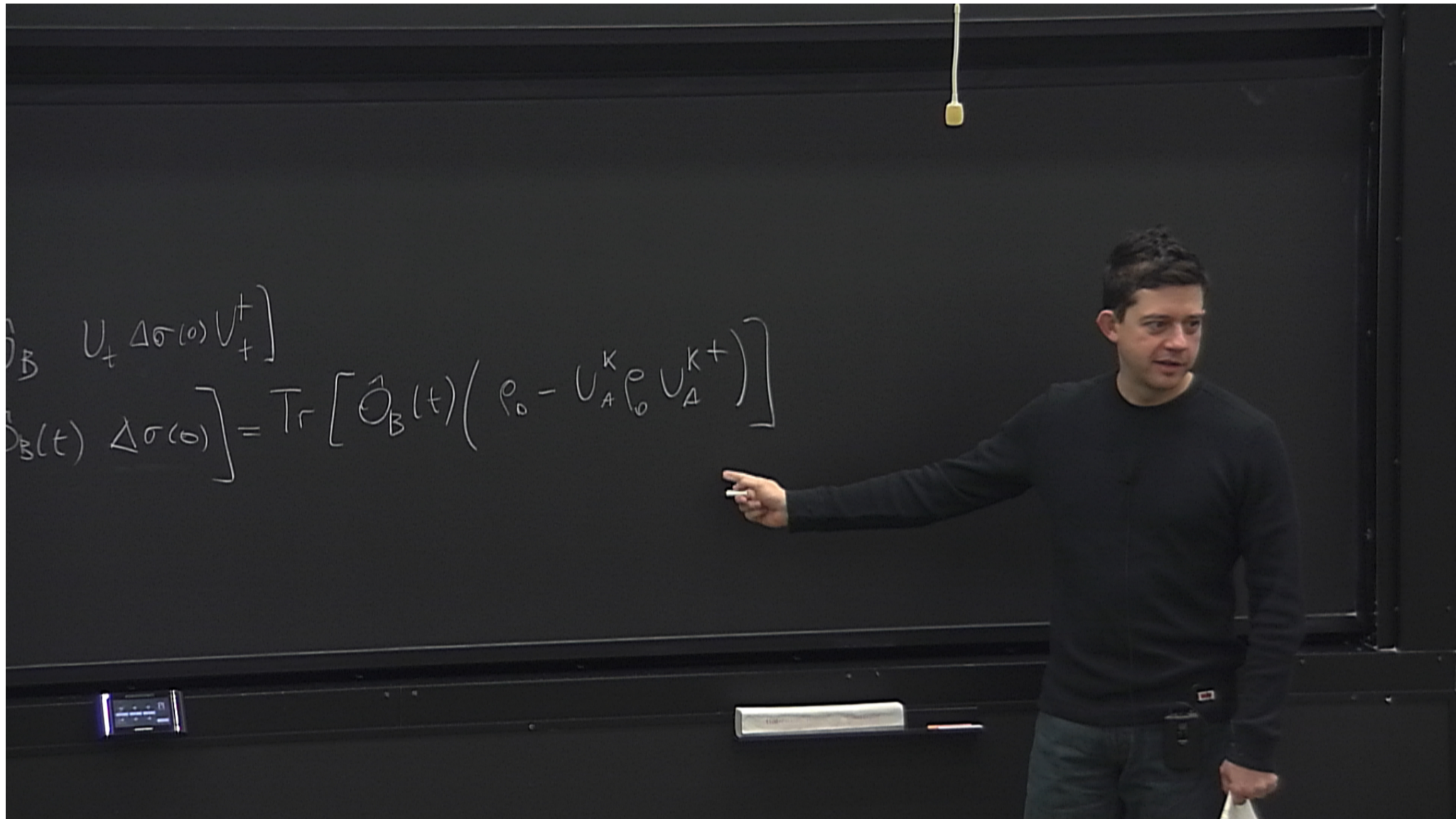
B can measure $\hat{O}_B$

$$\begin{cases} O_B^0 = \text{Tr} [\hat{O}_B \sigma_B^0(t)] \\ O_B^K = \text{Tr} [\hat{O}_B \sigma_B^K(t)] \end{cases}$$

$$|O_B^0(t) - O_B^K(t)| = \text{Tr} [\hat{O}_B (\sigma_B^0(t) - \sigma_B^K(t))] = \text{Tr} [\hat{O}_B U_t \Delta \sigma(0) U_t^\dagger] \\ = \text{Tr} [\hat{O}_B(t) \Delta \sigma(0)]$$

(trace distance)





$$\rho_B(t) \Delta \sigma_B(0) = \text{Tr} \left[\hat{\rho}_B(t) \left(\rho_0 - U_A^K \rho_0 U_A^{K\dagger} \right) \right]$$

$$\left[\hat{O}_B(t) \Delta \hat{\sigma}_B(0) \right] = \text{Tr} \left[\hat{O}_B(t) \left(\rho_0 - U_A^K \rho_0 U_A^{K\dagger} \right) \right] = \text{Tr} \left\{ \rho_0 U_A^{K\dagger} \left[U_A^K, \hat{O}_B(t) \right] \right\}$$

$$\left[\hat{O}_B(t) \Delta \hat{\sigma}_B(0) \right] = \text{Tr} \left[\hat{O}_B(t) \left(\rho_0 - U_A^K \rho_0 U_A^{K\dagger} \right) \right] = \text{Tr} \left\{ \rho_0 U_A^{K\dagger} \left[U_A^K, \hat{O}_B(t) \right] \right\}$$

$\mathbb{I} = U_A^{K\dagger} U_A^K$

$$H = \sum_x \psi_x^\dagger \psi_x$$

$$\| [A(t), B] \| \leq C e^{\frac{d \cdot t}{\xi}}$$

$$\sigma = \frac{2\pi\hbar c}{\lambda}$$

$$\sup_x \|\Phi_x\| \leq K$$

$$\xi = a$$

a is the lattice spacing

No signaling



(A)

ρ_0 initial state for the whole system (ABC)

A can operate with a unitary operation

U_A^k k labels the different operations

$U_A^0 = 1_A$

$$\rho_{ABC}^k = U_A^k \rho_0 U_A^{k\dagger}$$

$$U_{ABC}(t) = e^{-iH_{ABC}t}$$

$$\text{Tr}_{AC} [U_{ABC}(t) \rho_{ABC}^k U_{ABC}^\dagger(t)] = \sigma_B^k(t)$$

$$\sigma_B^0(t)$$

B can measure $\hat{O}_B$

$$\begin{cases} \sigma_B^0 = \text{Tr} [\hat{O}_B \sigma_B^0(t)] \\ \sigma_B^k = \text{Tr} [\hat{O}_B \sigma_B^k(t)] \end{cases}$$

$$|\sigma_B^0(t) - \sigma_B^k(t)| = |\text{Tr} [\hat{O}_B (\sigma_B^0(t) - \sigma_B^k(t))]| = |\text{Tr} [\hat{O}_B U_A^k \Delta \sigma_B^0 U_A^{k\dagger}]|$$

$$= |\text{Tr} [\hat{O}_B(t) \Delta \sigma_B^0]| = |\text{Tr} [\hat{O}_B(t) (\rho_0 - U_A^k \rho_0 U_A^{k\dagger})]| = |\text{Tr} [\rho_0 U_A^{k\dagger} [U_A^k, \hat{O}_B(t)]]|$$

$$\| = U_A^{k\dagger} U_A^k$$

(trace distance)

$$\begin{aligned}
 & \left| \sigma_B^0(t) - \sigma_B^K(t) \right| = \left| \text{Tr} \left[\hat{\sigma}_B U_t \Delta \sigma_B^{(0)} U_t^\dagger \right] \right| \\
 & = \left| \text{Tr} \left[\hat{\sigma}_B(t) \Delta \sigma_B^{(0)} \right] \right| = \left| \text{Tr} \left[\hat{\sigma}_B(t) \left(\rho_0 - U_A^K \rho_0 U_A^{K\dagger} \right) \right] \right| = \left| \text{Tr} \left[\rho_0 U_A^K \right] \right| \\
 & \leq 2 \|\hat{\sigma}_B\| e^{-\frac{\alpha - \nu t}{\xi}} \quad \quad \quad \mathbb{1} = U_A^{K\dagger} U_A^K
 \end{aligned}$$

① No signalling

① No signalling

$\|\Phi_x\|$ is whatever

$\|[\Phi_x, \Phi_y]\|$

① No signalling

$\|\Phi_x\|$ is whatever

$$\|[\Phi_x, \Phi_y]\| < \infty$$

then you have a bound

① No signalling

② What about correlations?

① No signalling

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$$\xi \rightarrow X > \xi$$

$$t = \frac{X - \xi}{\sigma} \quad ?$$

① No signalling

② What about correlations?

$$\xi \rightarrow X > \xi$$

$$t = \frac{X - \xi}{\sigma} \quad ?$$

$$|\langle O_A(t) O_B(t) \rangle|$$

① No signalling

② What about correlations?

$$\xi \rightarrow X \rightarrow \xi$$

$$t = \frac{X - \xi}{\sigma} ?$$

$$|\langle O_A(t) O_B(t) \rangle| \leq C \exp \left[-\frac{d - \sigma t}{\xi + \gamma X} \right]$$

① No signalling

② What about correlations?

$$\xi \rightarrow X \rightarrow \xi$$

$$|\langle O_A(t) O_B(t) \rangle| \leq C \exp \left[-\frac{d - \sigma t}{\xi + 2\chi} \right]$$

LMS Theorem

LMS Theorem
 $\langle A(\omega) B(\omega) \rangle_c$

LMS Theorem

$$\langle A(0) B(0) \rangle_c \leq C e^{-\frac{d-4t}{\xi}}$$

LMS Theorem

$$\langle A(0) B(0) \rangle_c \leq C e^{-\frac{L}{\xi}}$$

LMS Theorem

$$\langle A(0) B(0) \rangle_c \leq C e^{-\frac{L}{\xi}} \quad \xi^{-1} = \Delta$$

LMS Theorem

$$\underbrace{\langle A(0) B(0) \rangle_c}_L \leq C e^{-\frac{L}{\xi}}$$

$$\xi^{-1} = \Delta$$

$\{|E_n\rangle\}$ eigenstates of H

Be A_{mn}, B_{mn} the matrix elements

LMS Theorem

$$\underbrace{\langle A(0) B(0) \rangle}_L \leq C e^{-\frac{L}{\xi}}$$

$$\xi^{-1} = \Delta$$

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$$\underbrace{\langle A(0) | B(0) \rangle}_L \leq C e^{-\frac{L}{\xi}}$$

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$$f(t) = \langle [A(t), B] \rangle_{|E_0\rangle}$$

LMS Theorem

$$\underbrace{\langle A(0) B(0) \rangle}_L \leq C e^{-\frac{L}{\xi}} \quad \xi^{-1} = \Delta$$

$\{|E_n\rangle\}$ eigenstates of H

Be A_{mn}, B_{mn} the matrix elements

$$f(t) = \langle [A(t), B] \rangle_{|E_0\rangle} = \langle E_0 | e^{iHt} A e^{-iHt} B - B e^{iHt} A e^{-iHt} | E_0 \rangle$$

LMS Theorem

$$\underbrace{\langle A(0) | B(0) \rangle}_L \leq C e^{-\frac{L}{\xi}} \quad \xi^{-1} = \Delta$$

$\{|E_n\rangle\}$ eigenstates of H

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$$e^{-iHt} = \sum_n e^{-iE_n t} |E_n\rangle \langle E_n|$$

LMS Theorem

$$\underbrace{\langle A(0) B(0) \rangle}_L \leq C e^{-\frac{L}{\xi}} \quad \xi^{-1} = \Delta$$

$\{|E_n\rangle\}$ eigenstates of H

Be A_{mn}, B_{mn} the matrix elements

$$f(t) = \langle [A(t), B] \rangle_{|E_0\rangle} = \langle E_0 | e^{iHt} A e^{-iHt} B - B e^{iHt} A e^{-iHt} | E_0 \rangle$$

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S Theorem

$$\underbrace{B(0)}_L \leq C e^{-\frac{L}{\xi}} \quad \xi^{-1} = \Delta$$

$\{ \}$ eigenstates of H

A_{mn}, B_{mn} the matrix elements

$$\begin{aligned} &= \langle [A(t), B] \rangle_{|E_0\rangle} = \langle E_0 | e^{iHt} A e^{-iHt} B - B e^{iHt} A e^{-iHt} | E_0 \rangle \\ &= \sum_{nm} \left\{ e^{i(E_n - E_m)t} \langle E_0 | E_n \rangle A_{nm} B_{m0} - e^{i(E_n - E_m)t} B_{0n} A_{nm} \langle E_m | E_0 \rangle \right\} \end{aligned}$$

$$Ht = \sum_n e^{-iE_n t} |E_n\rangle \langle E_n|$$

$$\langle A(0) B(0) \rangle \leq C e^{-\frac{L}{\xi}} \quad \xi^{-1} = \Delta$$

$\{|E_n\rangle\}$ eigenstates of H

Be A_{mn}, B_{mn} the matrix elements

$$f(t) = \langle [A(t), B] \rangle_{|E_0\rangle} = \langle E_0 | e^{iHt} A e^{-iHt} B - B e^{iHt} A e^{-iHt} | E_0 \rangle$$

$$e^{-iHt} = \sum_n e^{-iE_n t} |E_n\rangle\langle E_n|$$

$$= \sum_{n,m} \left\{ e^{i(E_n - E_m)t} \langle E_0 | E_n \rangle A_{nm} B_{m0} - e^{i(E_m - E_n)t} B_{0n} A_{nm} \langle E_m | E_0 \rangle \right\}$$

$$f(t) = \sum_m \left\{ e^{-iE_m t} A_{0m} B_{m0} - e^{iE_m t} B_{0m} A_{m0} \right\}$$

$$f(t) = \sum_m \left\{ \underbrace{e^{-iE_m t} A_{0m} B_{m0}}_{f^+(t)} - \underbrace{e^{iE_m t} B_{0m} A_{m0}}_{f^-(t)} \right\}$$

$$f(t) = \sum_m \left\{ \underbrace{e^{-iE_m t} A_{0m} B_{m0}}_{f^+(t)} - \underbrace{e^{iE_m t} B_{0m} A_{m0}}_{f^-(t)} \right\}$$

$$\langle A \rangle_{\psi_0} = \langle B \rangle_{\psi_0} = 0$$



$$f(t) = \sum_m \left\{ \underbrace{e^{-iE_m t} A_{0m} B_{m0}}_{f^+(t)} - \underbrace{e^{iE_m t} B_{0m} A_{m0}}_{f^-(t)} \right\}$$

$$f^+(t) \xrightarrow{t \rightarrow 0} \langle A(0) B(0) \rangle$$

$$f(t) = \sum_m \left\{ \underbrace{e^{-iE_m t} A_{0m} B_{m0}}_{f^+(t)} - \underbrace{e^{iE_m t} B_{0m} A_{m0}}_{f^-(t)} \right\}$$

$$f^+(t) \xrightarrow{t \rightarrow 0} \langle A(0) B(0) \rangle$$

$$-O \frac{L}{R} + 2 \left(1 + \frac{1}{\sqrt{2\pi\sigma L \Delta E}} \right) \|A\| \|B\| e^{-\frac{\sigma L \Delta E}{2}}$$

$$|f^+(0)| \leq C \|A\| \|B\| e$$

$$f(t) = \sum_m \left\{ \underbrace{e^{-iE_m t} A_{0m} B_{m0}}_{f^+(t)} - \underbrace{e^{iE_m t} B_{0m} A_{m0}}_{f^-(t)} \right\}$$

$$f^+(t) \xrightarrow{t \rightarrow 0} \langle A(0) B(0) \rangle$$

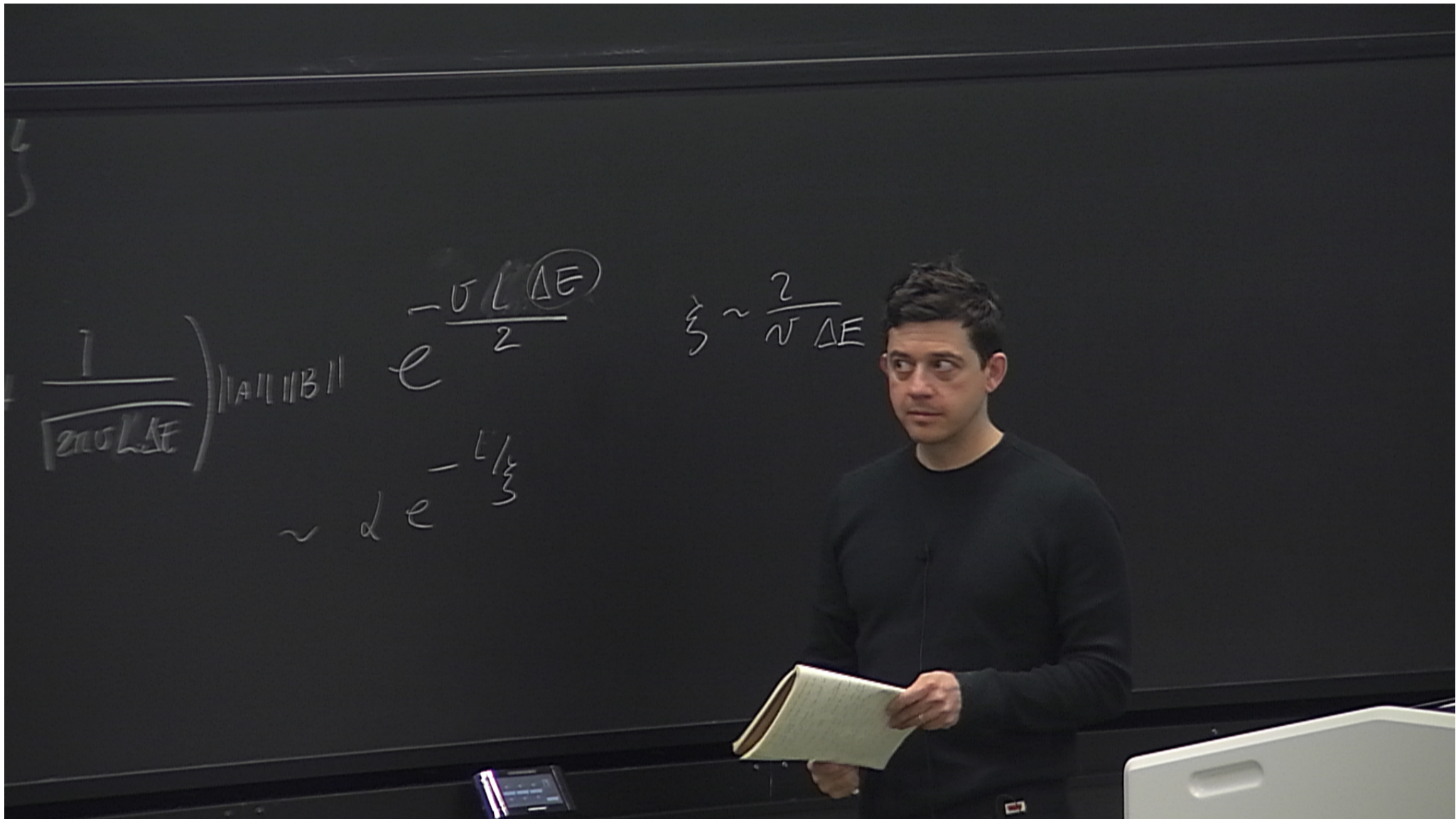
$$-O\left(\frac{L}{R}\right) + 2 \left(1 + \frac{1}{\sqrt{2\pi\sigma L \Delta E}} \right) \|A\| \|B\| e^{-\frac{\sigma L \Delta E}{2}}$$

$$|f^+(0)| \leq C \|A\| \|B\| e$$

$$f(t) = \sum_m \left\{ \underbrace{e^{-iE_m t} A_{0m} B_{m0}}_{f^+(t)} - \underbrace{e^{iE_m t} B_{0m} A_{m0}}_{f^-(t)} \right\}$$

$$f^+(t) \xrightarrow{t \rightarrow 0} \langle A(0) B(0) \rangle$$

$$|f^+(0)| \leq C \|A\| \|B\| e^{-O\left(\frac{L}{R}\right) + 2 \left(1 + \frac{1}{\sqrt{2\pi\sigma L \Delta E}}\right) \|A\| \|B\|} e^{-\frac{\sigma L \Delta E}{2}}$$



$$\frac{1}{\sqrt{2\pi\sigma L\Delta E}} \left(\|A\| \|B\| e^{-\frac{\sigma L \Delta E}{2}} \right) \quad \xi \sim \frac{2}{N \Delta E}$$

$$\sim d e^{-L/\xi}$$

$$\frac{1}{\sqrt{2\mu L \Delta E}} \left(\|A\| \|B\| e^{-\frac{\mu L \Delta E}{2}} \right) \quad \xi \sim \frac{2}{N \Delta E}$$

$$\sim d e^{-L/\xi} \quad \text{LMS theorem}$$

$$f(t) = \sum_m \left\{ \underbrace{e^{-iE_m t}}_{f^+(t)} A_{0m} B_{m0} - \underbrace{e^{iE_m t}}_{f^-(t)} B_{0m} A_{m0} \right\}$$

$$f^+(t) \xrightarrow{t \rightarrow 0} \langle A(0) B(0) \rangle$$

$$|f^+(0)| \leq c \|A\| \|B\| e^{-\theta\left(\frac{L}{R}\right)} + 2 \left(1 + \frac{1}{|2\pi\sigma L \Delta E|} \right) \|A\| \|B\| e^{-\frac{\sigma L \Delta E}{2}} \quad \xi \sim \frac{2}{\sigma L \Delta E}$$

$$\sim d e^{-\frac{L}{3}} \quad \text{LMS theorem}$$