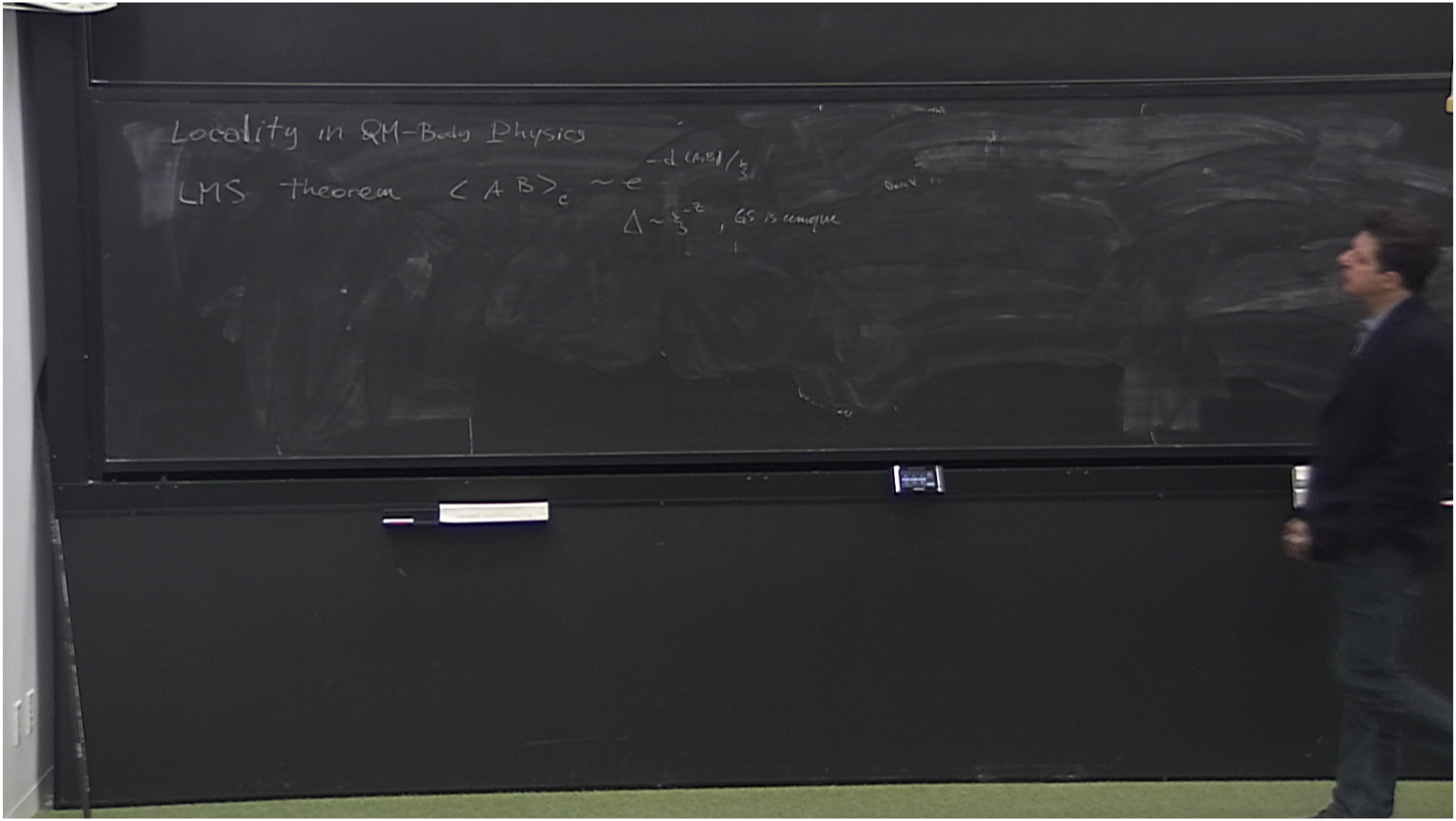
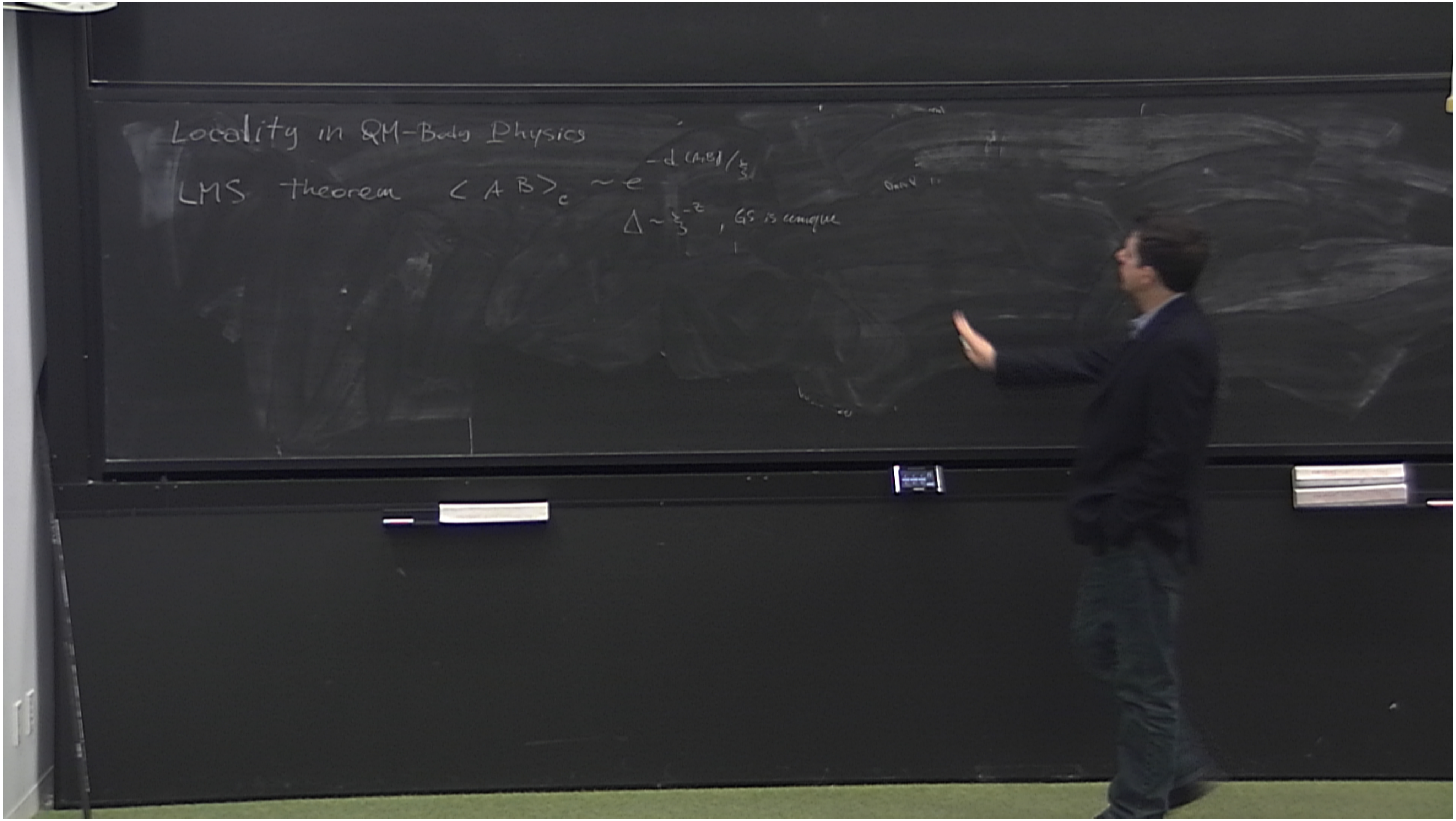
Title: 13/14 PSI - Condensed Matter Review - Lecture 6

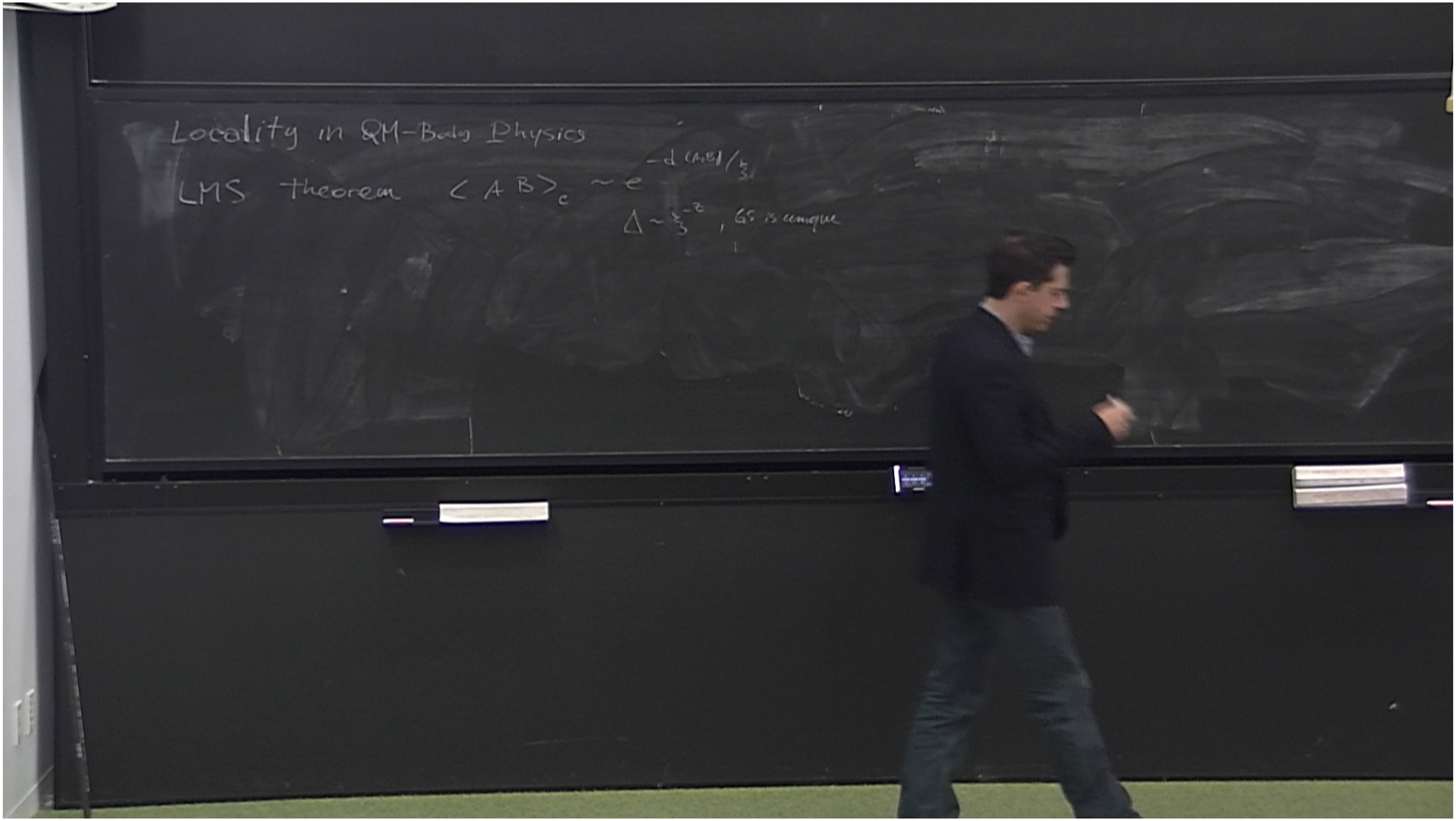
Date: Feb 04, 2014 09:00 AM

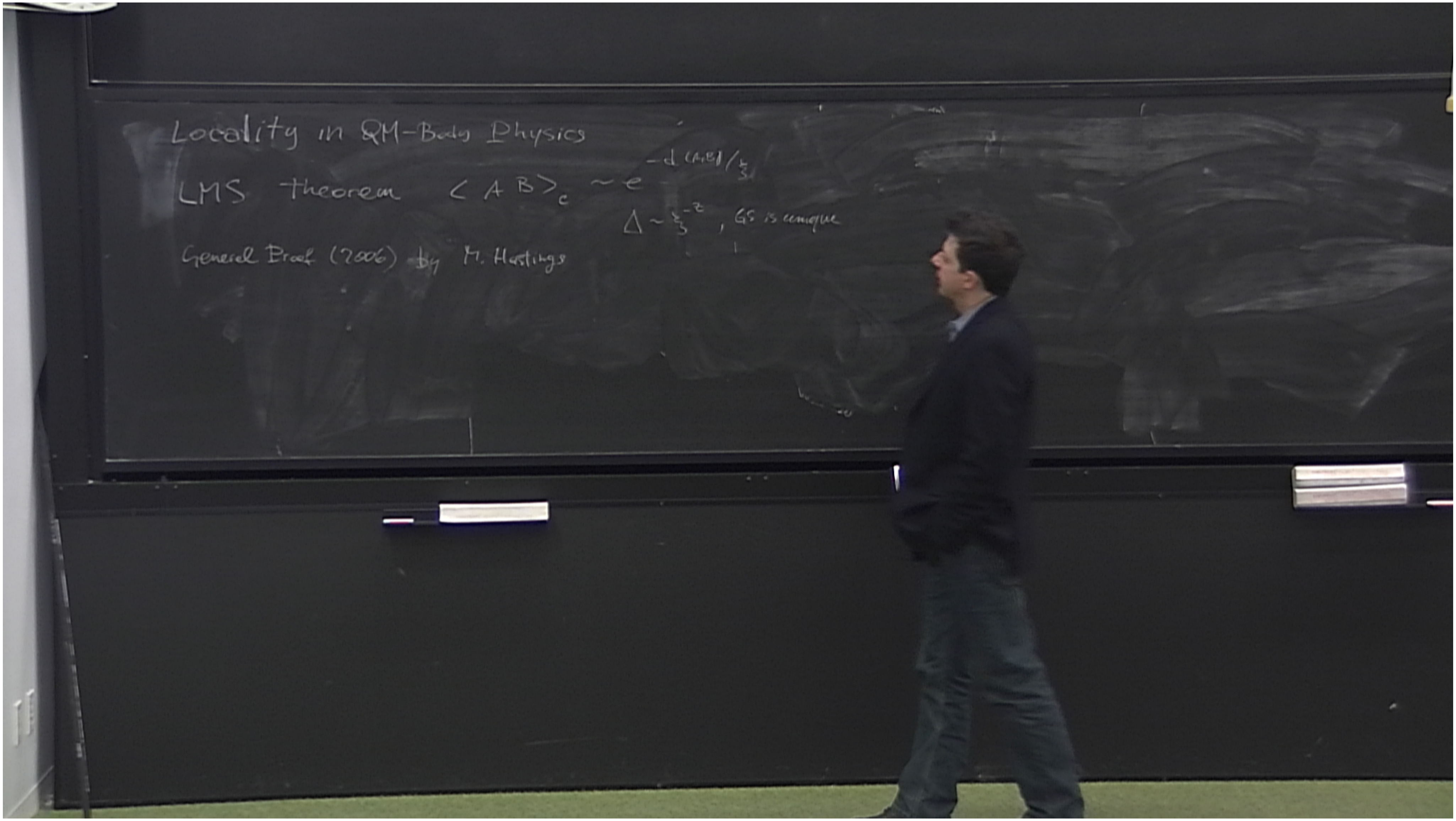
URL: http://pirsa.org/14020040

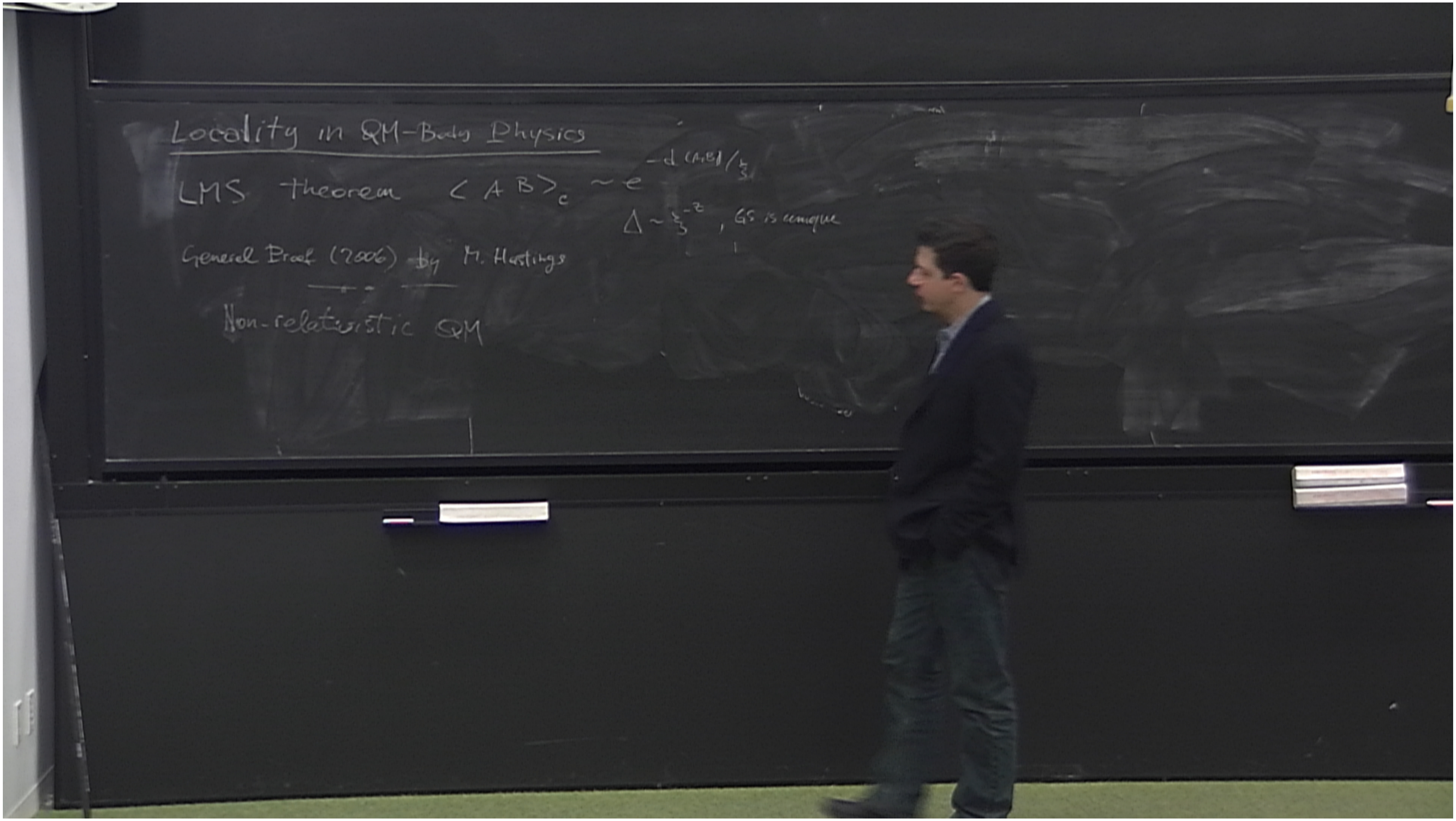
Abstract:

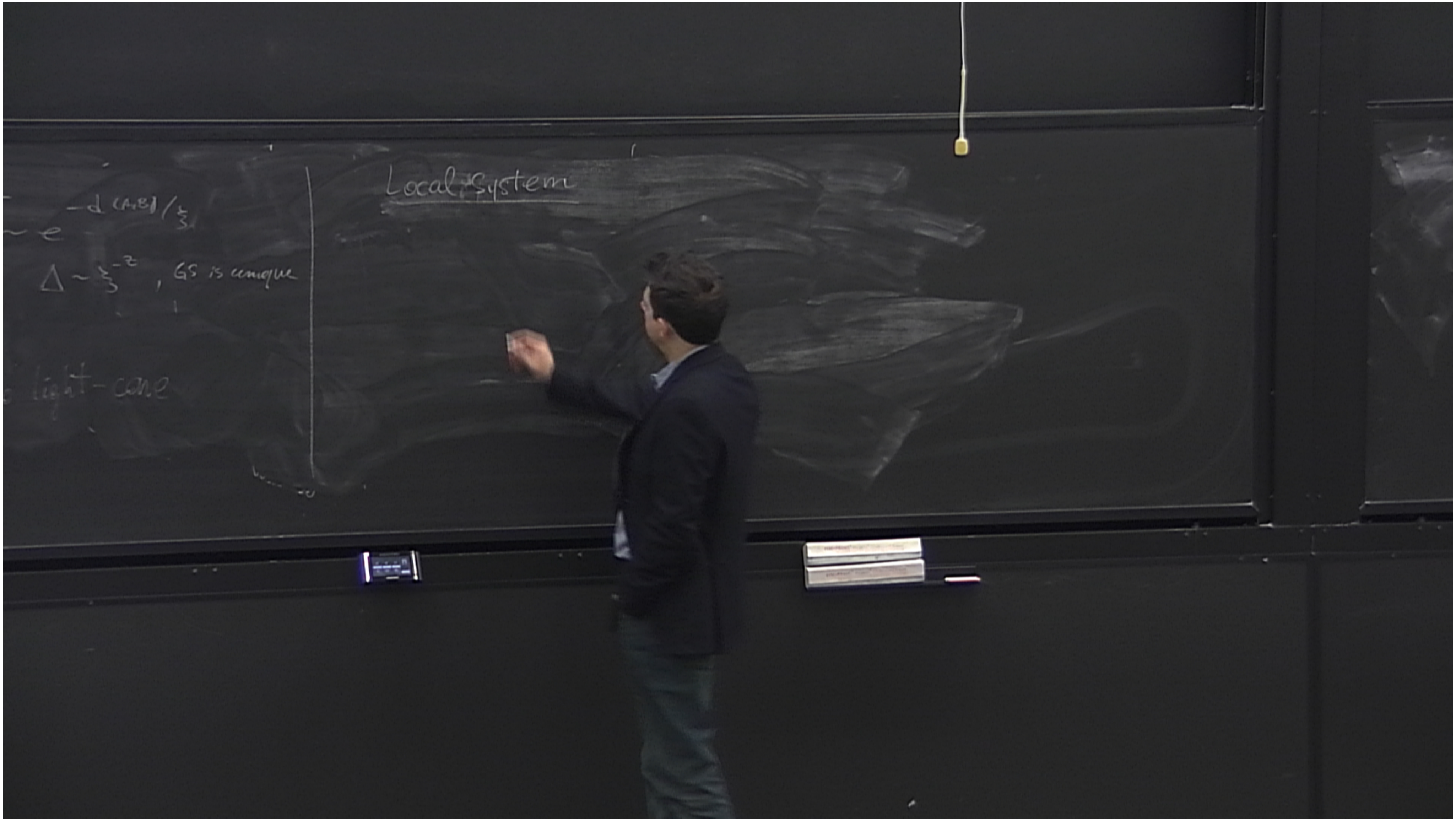

Locality in QM-Body Physics
LMS theorem ⟨A B⟩c ~ e^(-d(A,B)/ξ)
Δ ~ ξ^(-z), GS is unique

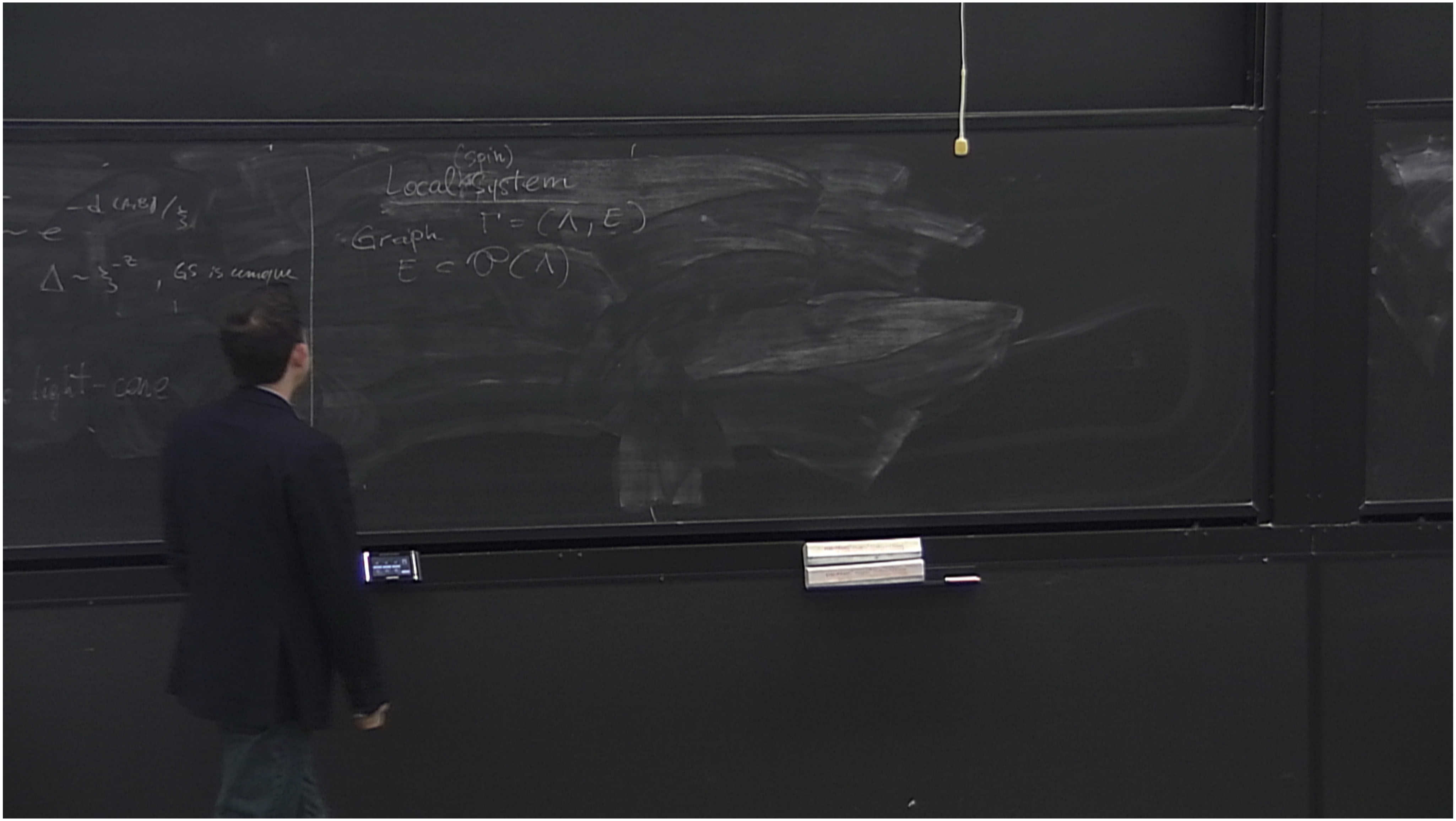

Locality in QM-Body Physics
LMS theorem $\langle A B \rangle_c \sim e^{-d(A,B)/\xi}$
$\Delta \sim \xi^{-z}$, GS is unique

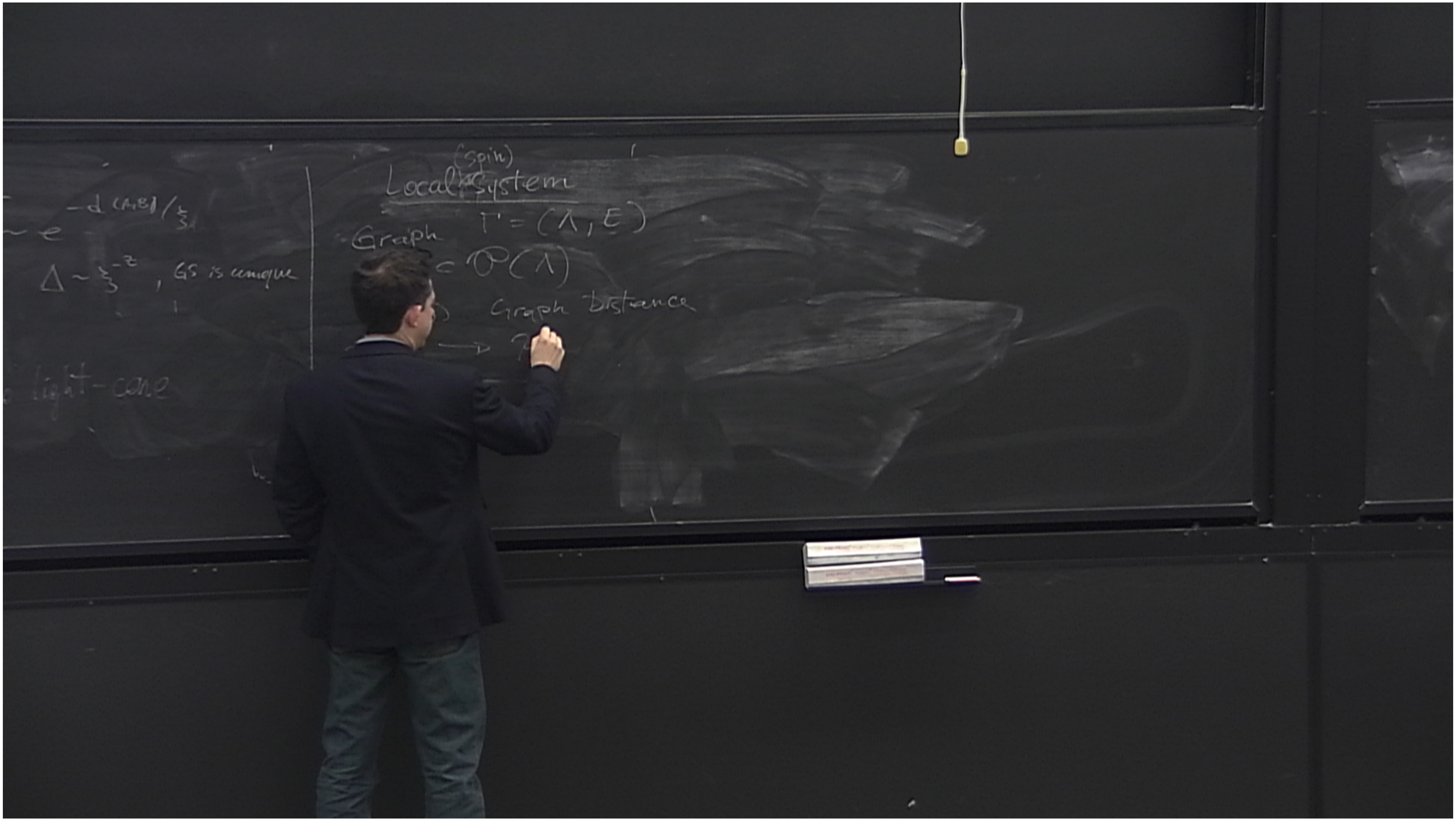

Locality in QM-Body Physics
LMS theorem ⟨A B⟩_c ~ e^{-d(A,B)/ξ}
Δ ~ ξ^{-z}, GS is unique

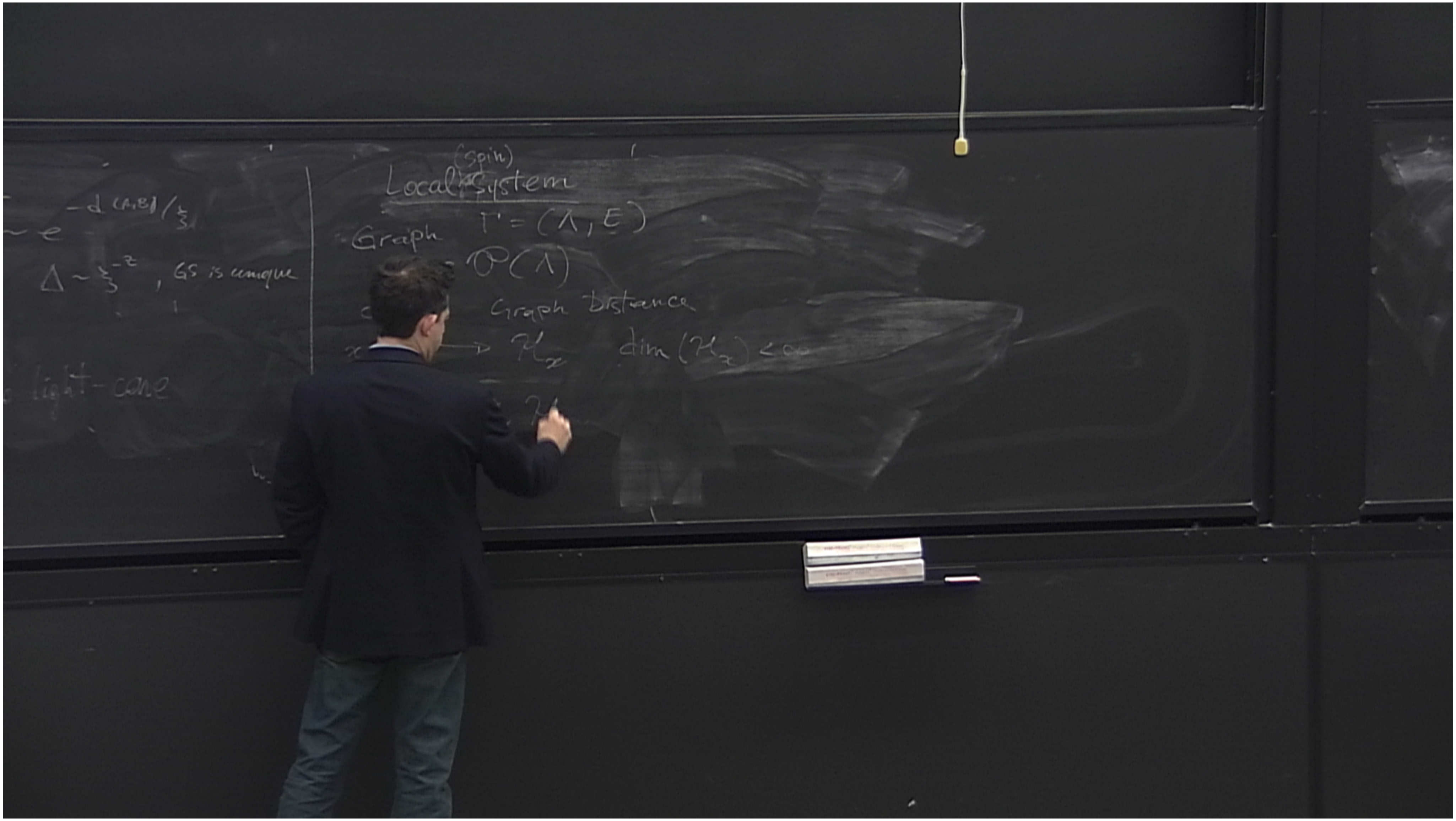

Locality in QM-Body Physics
LMS theorem $\langle A B \rangle_c \sim e^{-d(A,B)/\xi}$
$\Delta \sim \xi^{-z}$, GS is unique
General Proof (2006) by M. Hastings

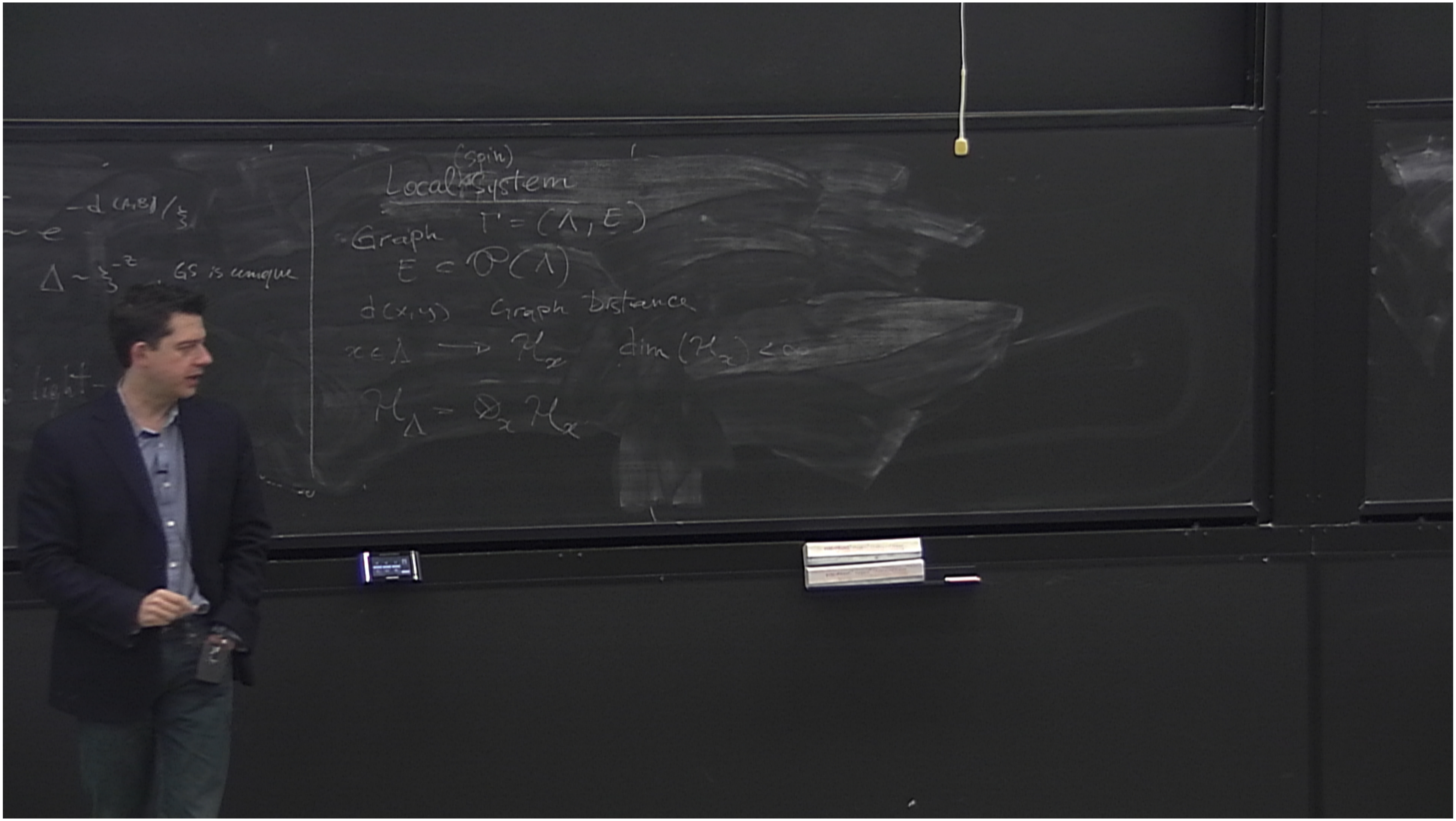

Locality in QM-Body Physics
LMS theorem $\langle A B \rangle_c \sim e^{-d(A,B)/\xi}$
$\Delta \sim \xi^{-z}$, GS is unique
General Proof (2006) by M. Hastings
Non-relativistic QM

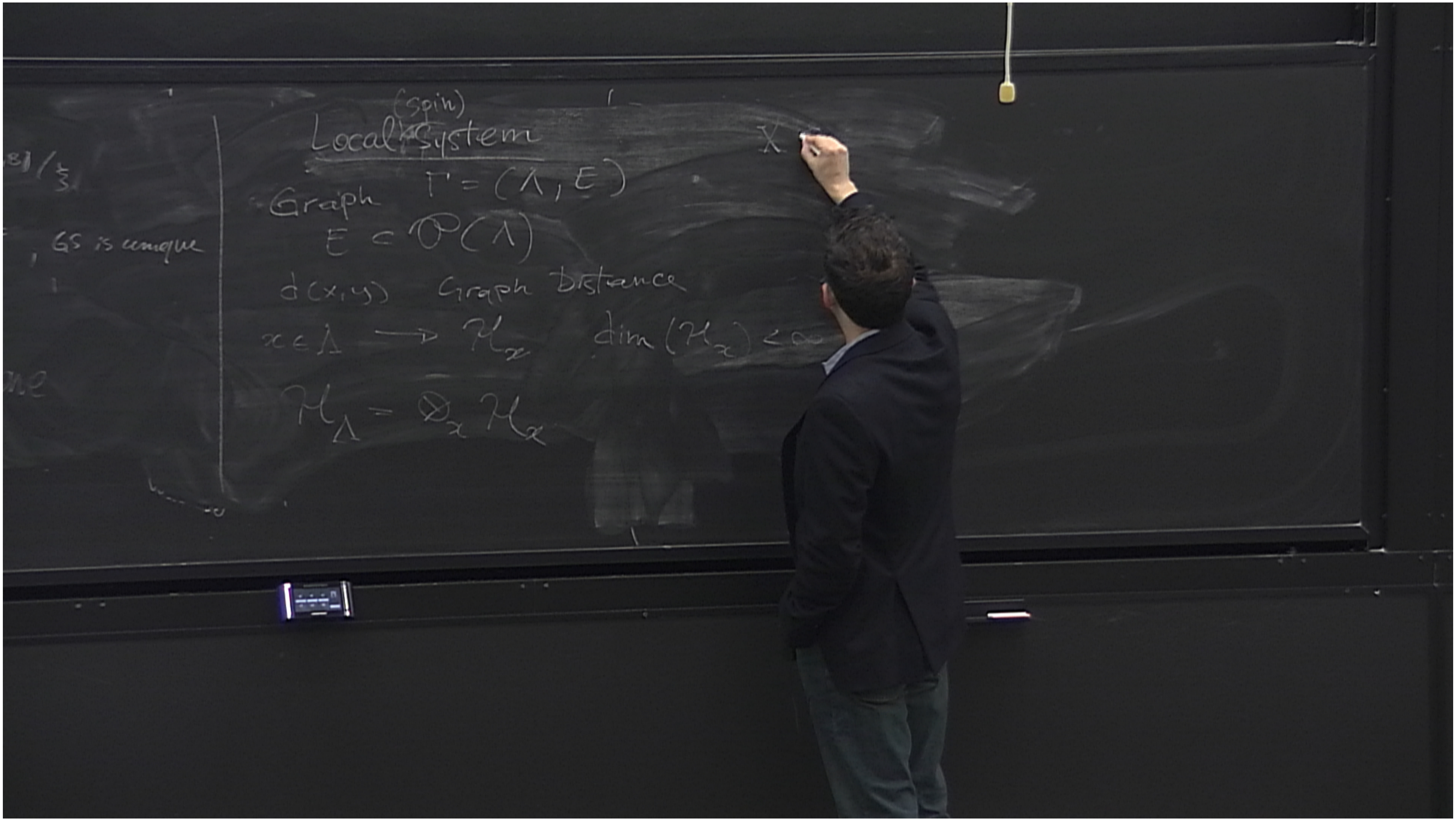

Local System
GS is unique
light-cone

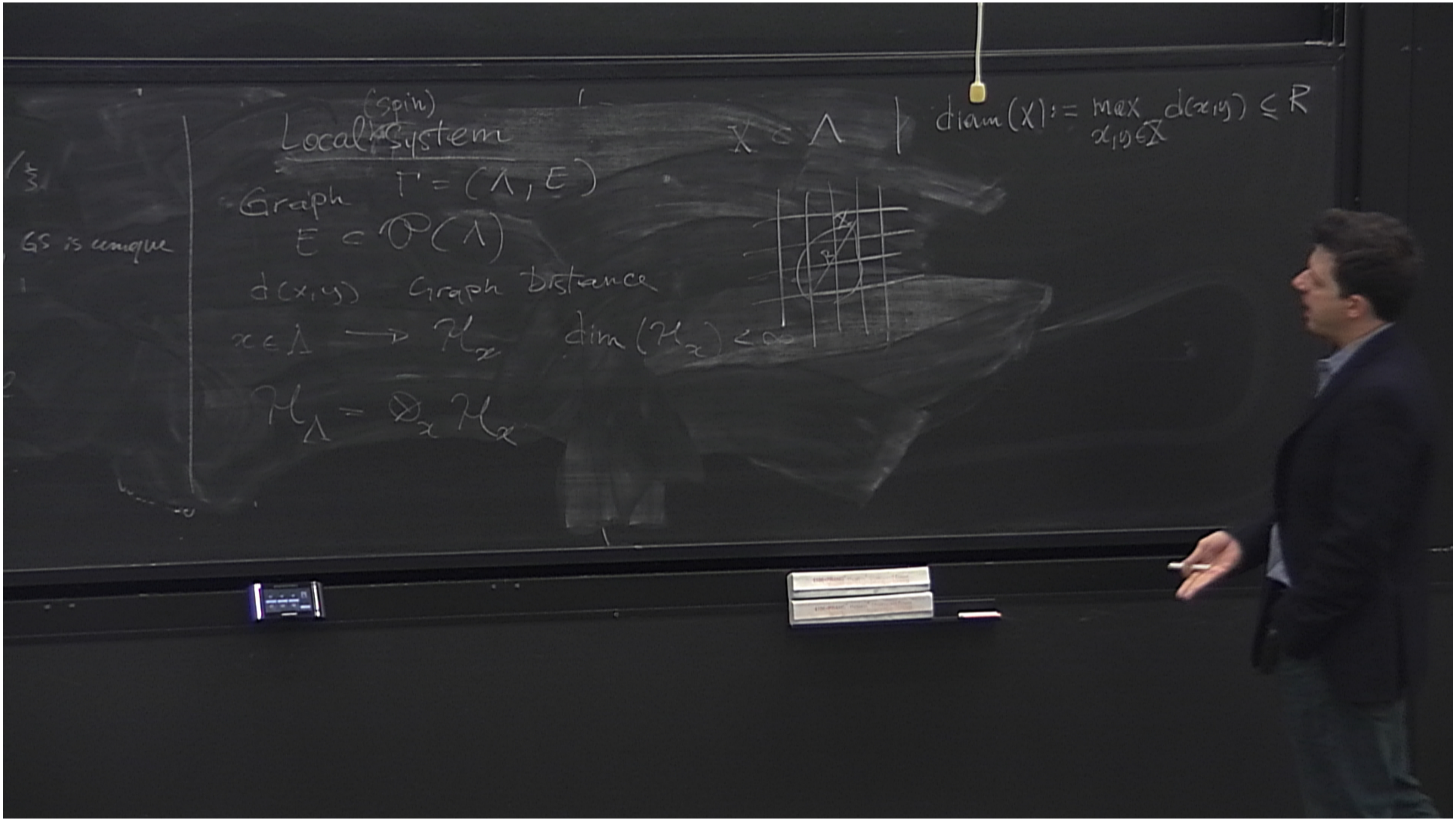

(spin)
Local System
Graph Γ = (Λ, E)
E ⊂ P(Λ)
GS is unique
light-cone

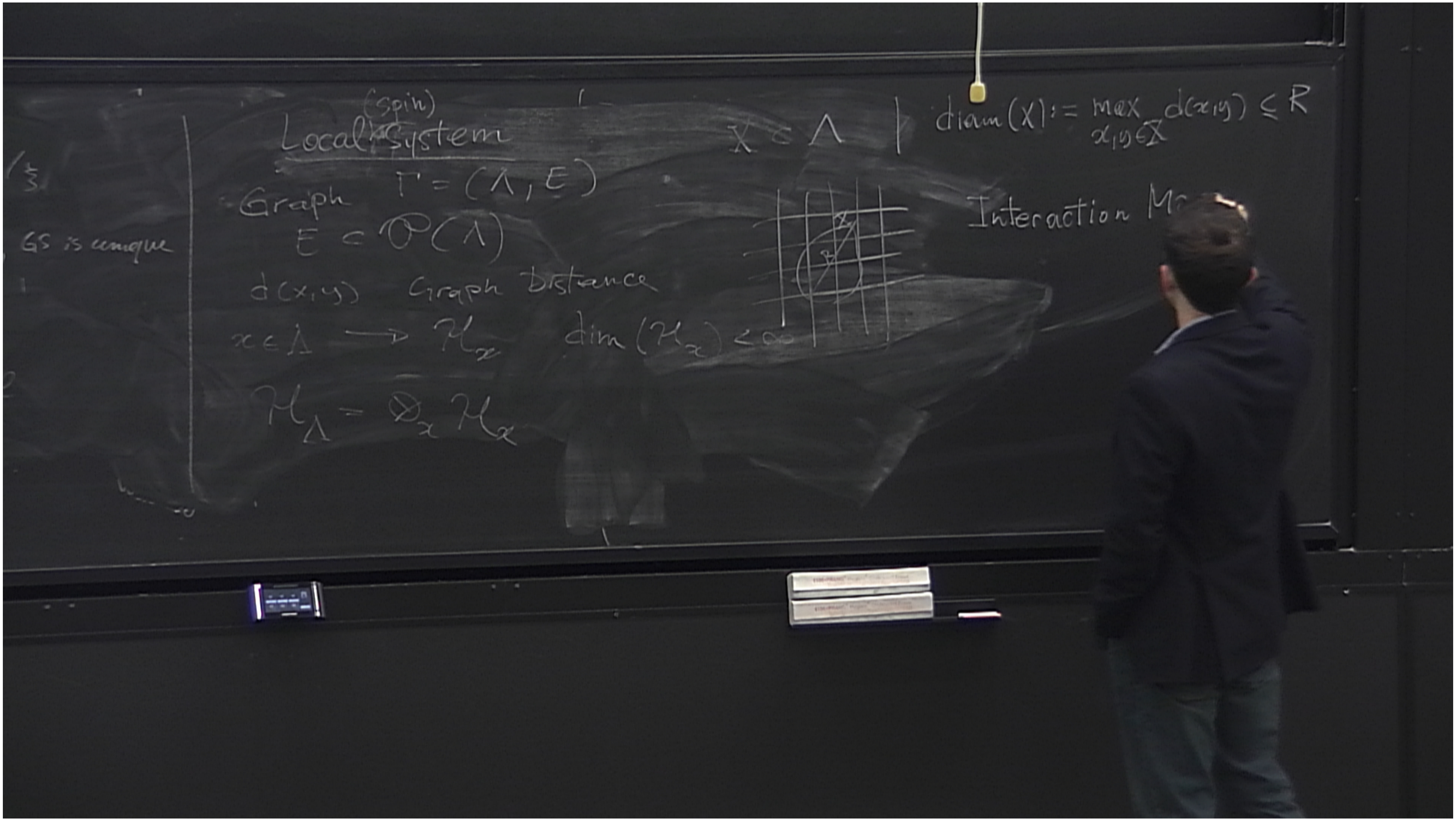

(spin)
Local System
Graph
Graph Distance
GS is unique
light-cone


(spin)
Local System
Graph Γ = (Λ, E)
Graph Distance
dim(H_x) < ∞
Δ ~ ξ^-z , GS is unique
light-cone

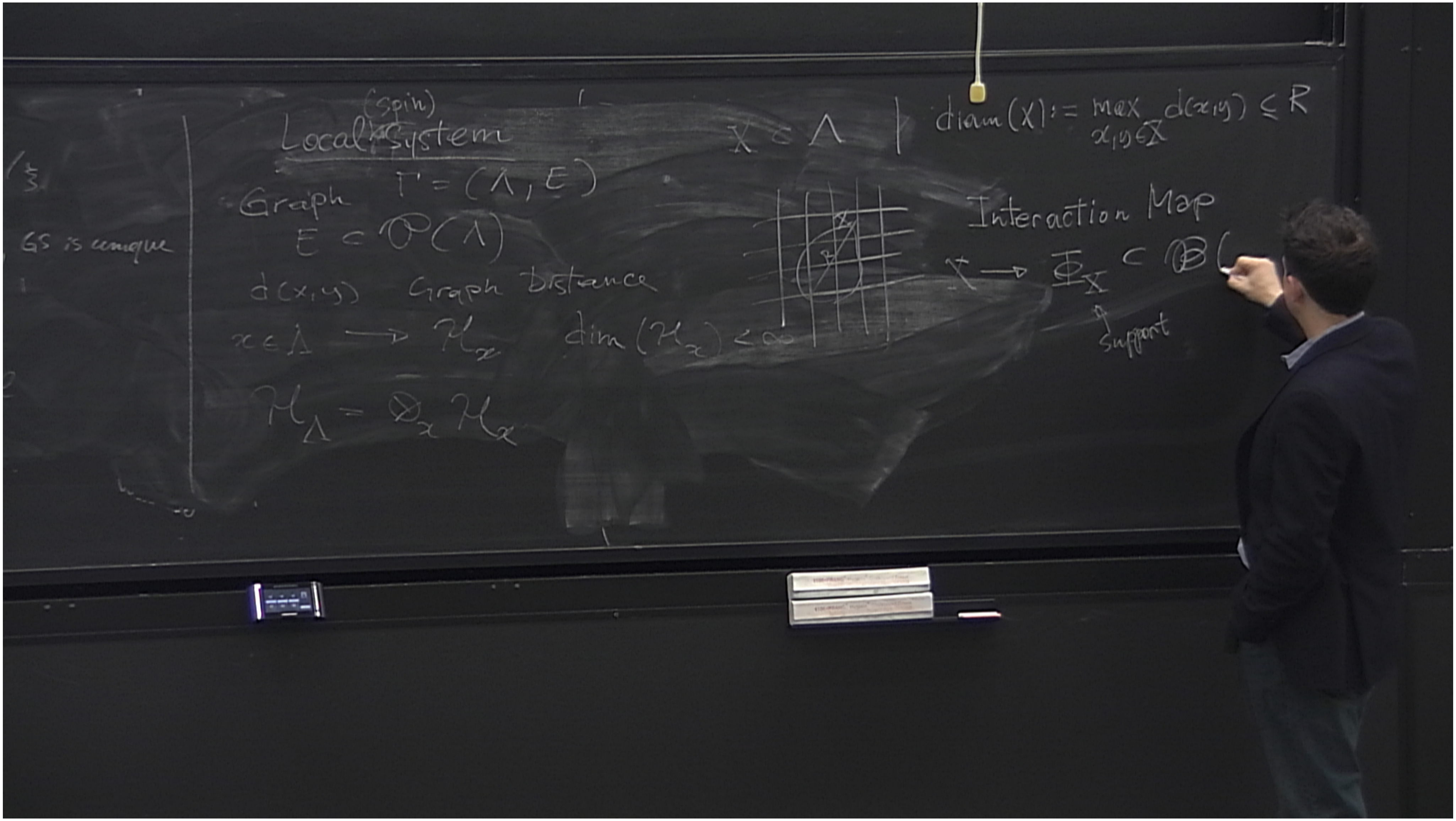

$\sim e^{-d(A,B)/\xi}$
$\Delta \sim \xi^{-z}$
GS is unique
(spin)
Local System
Graph $\Gamma = (\Lambda, E)$
$E \subset \mathcal{P}(\Lambda)$
$d(x,y)$ Graph Distance
$x \in \Lambda \longrightarrow \mathcal{H}_x$ $\dim(\mathcal{H}_x) < \infty$
$\mathcal{H}_\Lambda = \otimes_x \mathcal{H}_x$

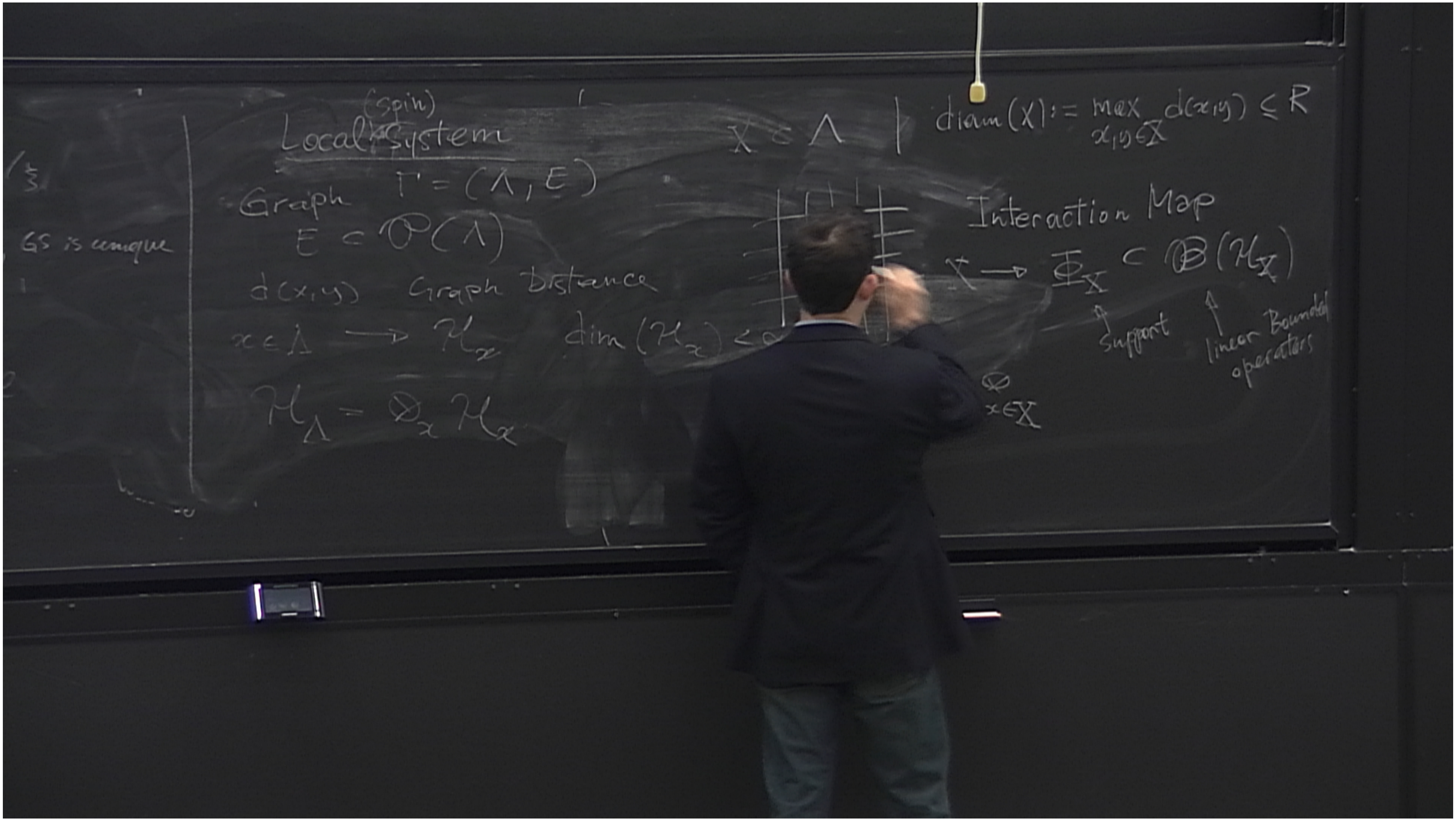

(spin)
Local System
Graph $\Gamma = (\Lambda, E)$
$E \subset \mathcal{P}(\Lambda)$
$d(x,y)$ Graph Distance
$x \in \Lambda \longrightarrow \mathcal{H}_x$ $\dim(\mathcal{H}_x) < \infty$
$\mathcal{H}_\Lambda = \otimes_x \mathcal{H}_x$
GS is unique

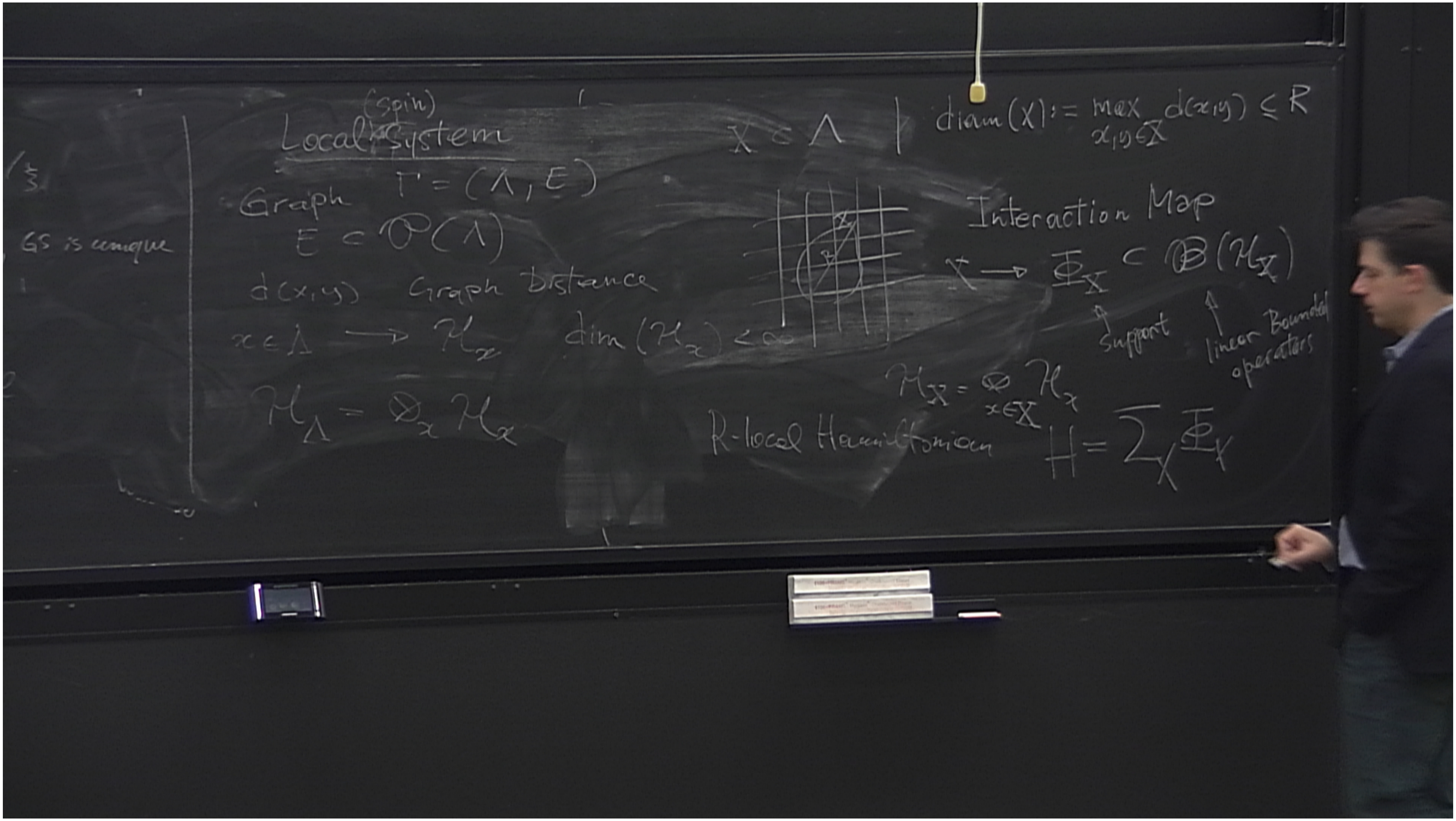

Local (spin) System
Graph Γ = (Λ, E)
E ⊂ P(Λ)
d(x,y) Graph Distance
x ∈ Λ → H_x dim(H_x) < ∞
H_Λ = ⊗_x H_x
X ⊂ Λ
diam(X) := max_{x,y∈X} d(x,y) ≤ R
GS is unique



(spin)
Local System
Graph Γ = (Λ, E)
E ⊂ 𝒫(Λ)
d(x,y) Graph Distance
x ∈ Λ → ℋ_x dim(ℋ_x) < ∞
ℋ_Λ = ⊗_x ℋ_x
X ⊂ Λ
diam(X) := max_{x,y∈X} d(x,y) ≤ R
Interaction M
GS is unique


GS is unique
Local (spin) System
Graph Γ = (Λ, E)
E ⊂ 𝒫(Λ)
d(x,y) Graph Distance
x ∈ Λ ⟶ ℋ_x dim(ℋ_x) < ∞
ℋ_Λ = ⊗_x ℋ_x
X ⊂ Λ
diam(X) := max_{x,y∈X} d(x,y) ≤ R
Interaction Map
X ⟶ Φ_X ⊂ 𝔅(
support


GS is unique
Local (spin) System
Graph Γ = (Λ, E)
E ⊂ ℘(Λ)
d(x,y) Graph Distance
x ∈ Λ ⟶ H_x dim(H_x) < ∞
H_Λ = ⊗_x H_x
X ⊂ Λ
diam(X) := max_{x,y∈X} d(x,y) ≤ R
Interaction Map
X ⟶ Φ_X ⊂ B(H_X)
Support
linear Bounded operators
⊗_{x∈X}


GS is unique
Local (spin) System
Graph $\Gamma = (\Lambda, E)$
$E \subset \mathcal{P}(\Lambda)$
$d(x,y)$ Graph Distance
$x \in \Lambda \longrightarrow \mathcal{H}_x$
$\dim(\mathcal{H}_x) < \infty$
$\mathcal{H}_\Lambda = \otimes_x \mathcal{H}_x$
$X \subset \Lambda$
$\operatorname{diam}(X) := \max_{x,y \in X} d(x,y) \leq R$
Interaction Map
$X \longrightarrow \Phi_X \subset \mathcal{B}(\mathcal{H}_X)$
Support
linear Bounded operators
$\mathcal{H}_X = \otimes_{x \in X} \mathcal{H}_x$
R-local Hamiltonian
$H = \sum_X \Phi_X$

(spin)

Local System

$X \subset \Lambda$

$\mathrm{diam}(X) := \max_{x,y \in X} d(x,y) \leq R$

Graph $\Gamma = (\Lambda, E)$

$E \subset \mathcal{P}(\Lambda)$

$d(x,y)$ Graph Distance

$x \in \Lambda \longrightarrow \mathcal{H}_x \quad \dim(\mathcal{H}_x) < \infty$

$\mathcal{H}_\Lambda = \otimes_x \mathcal{H}_x$

Interaction Map

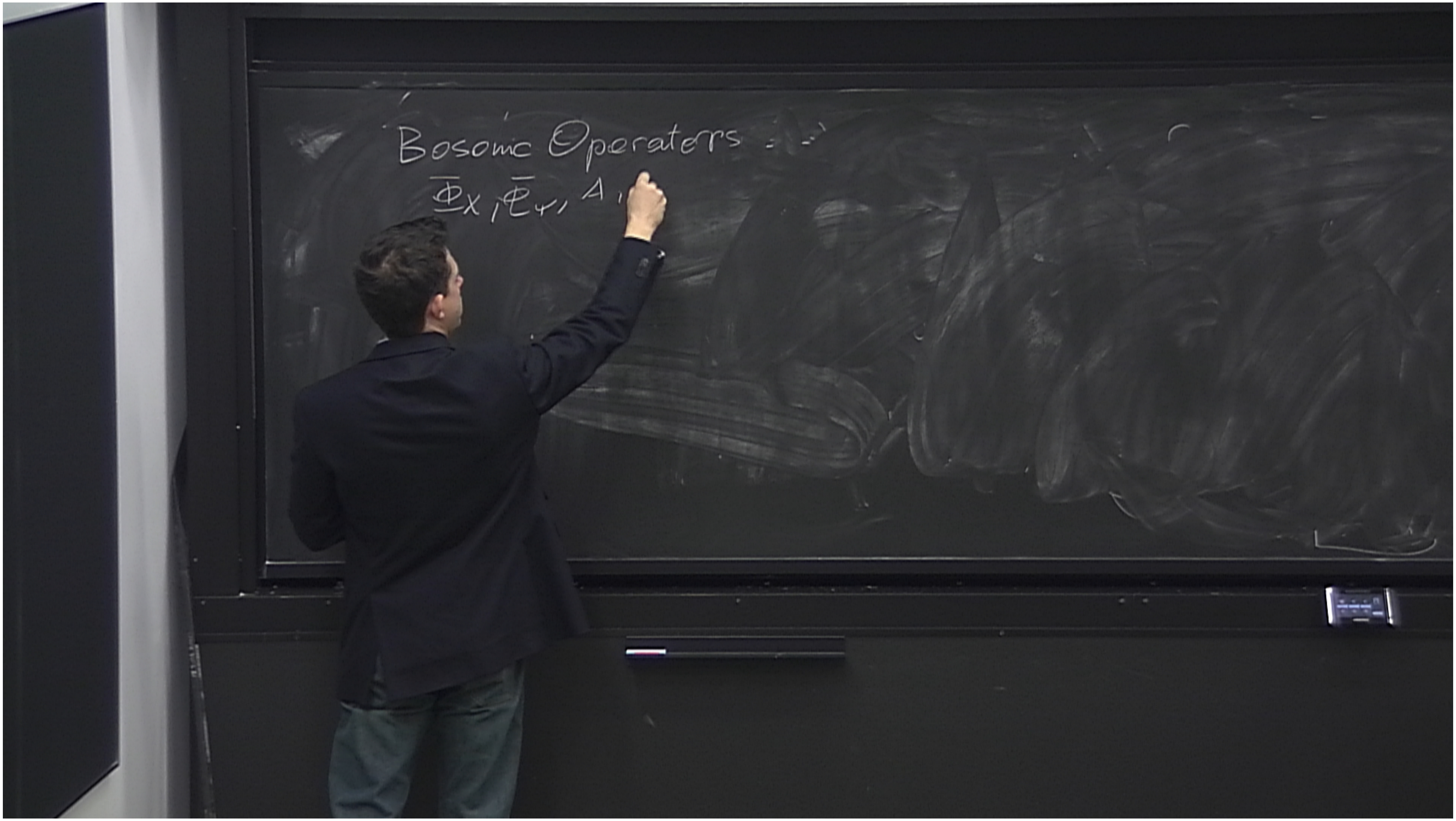
$X \longrightarrow \Phi_X \subset \mathcal{B}(\mathcal{H}_X)$

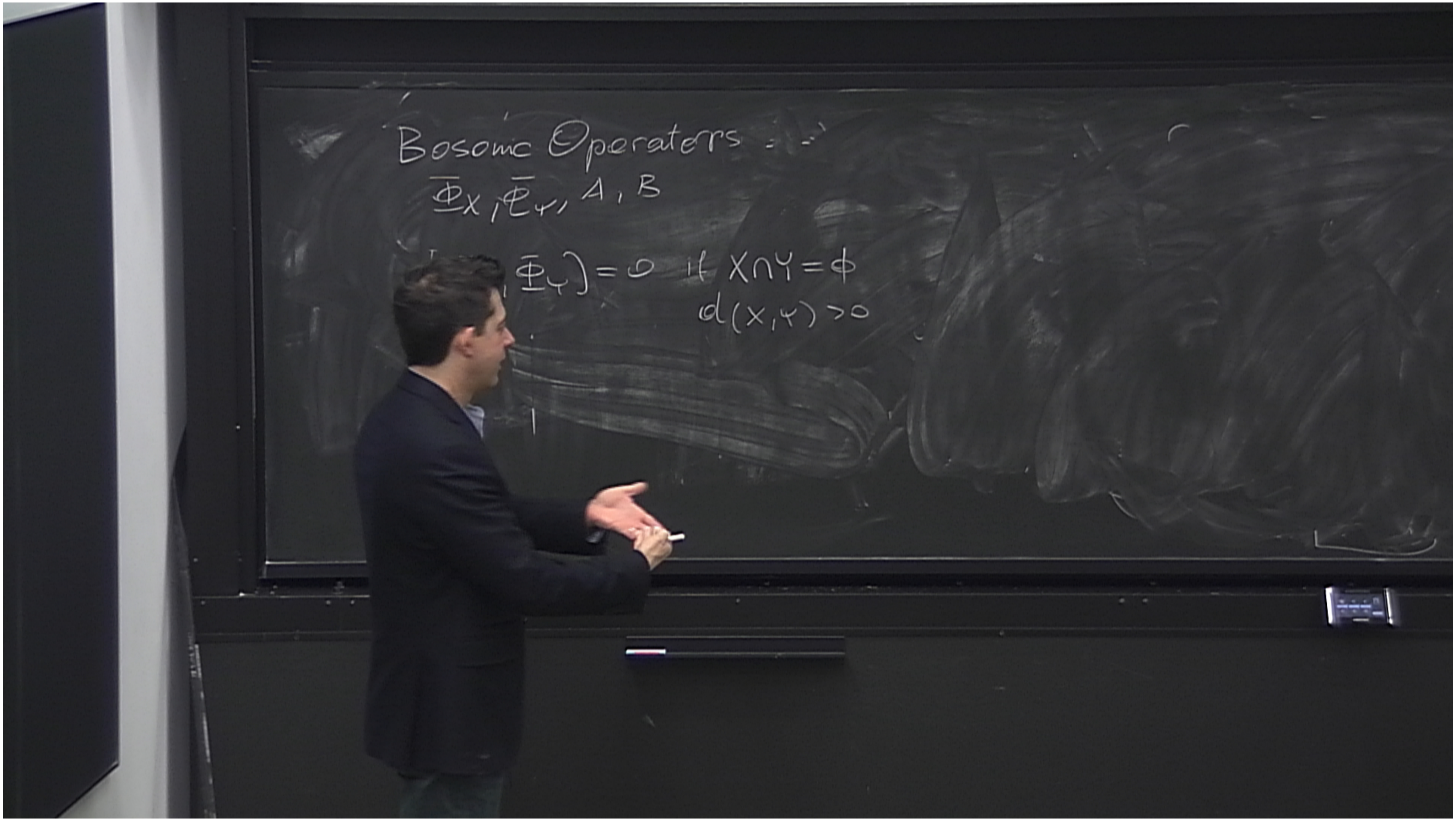
Support

linear Bounded operators

$\mathcal{H}_X = \bigotimes_{x \in X} \mathcal{H}_x$

R-local Hamiltonian $H = \sum_X \Phi_X$


Bosonic Operators
$\Phi_X, \Phi_Y, A,$

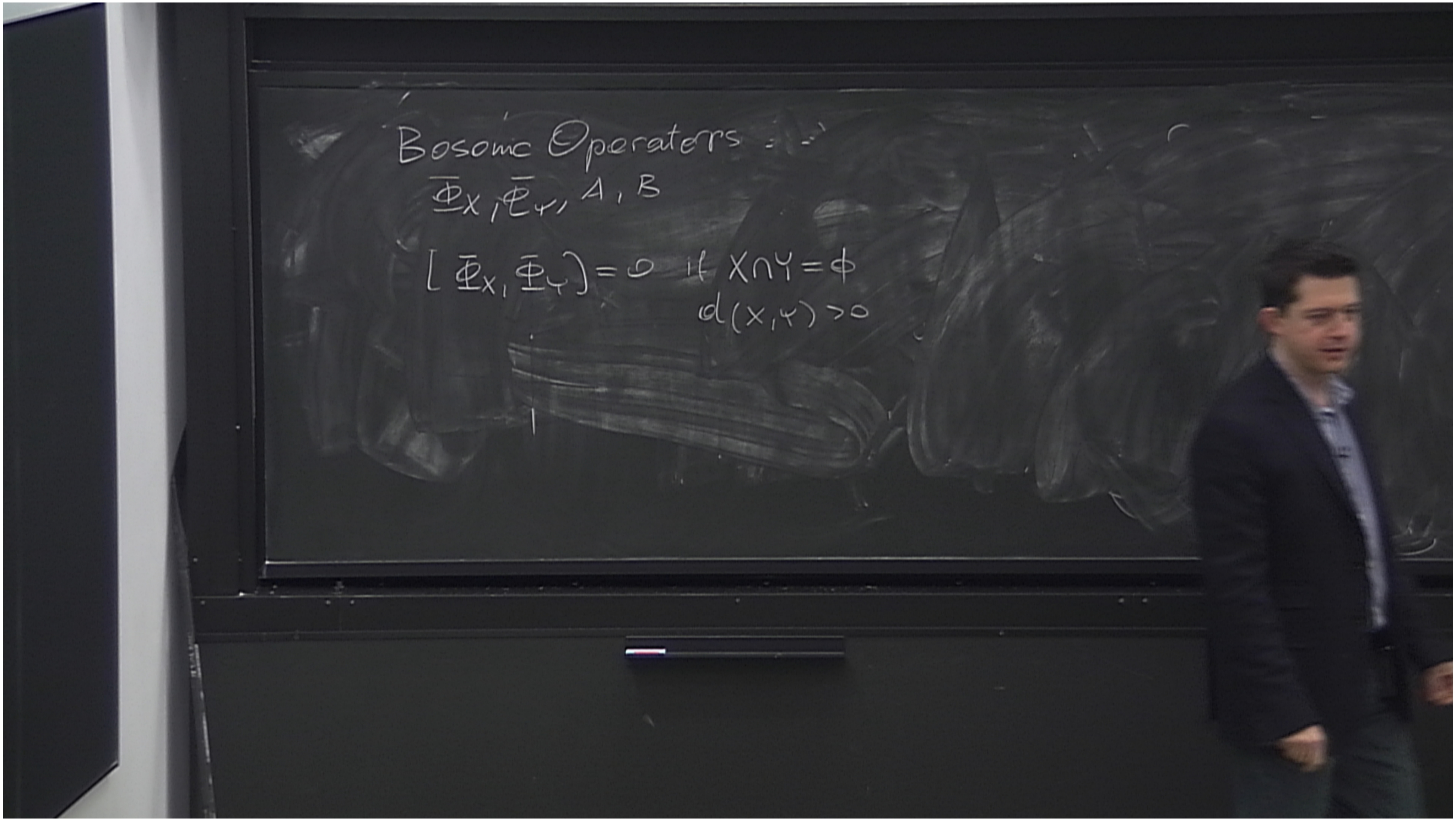

Bosonic Operators
Φ_X, Φ_Y, A, B
Φ_Y] = 0 if X∩Y = φ
d(X,Y) > 0

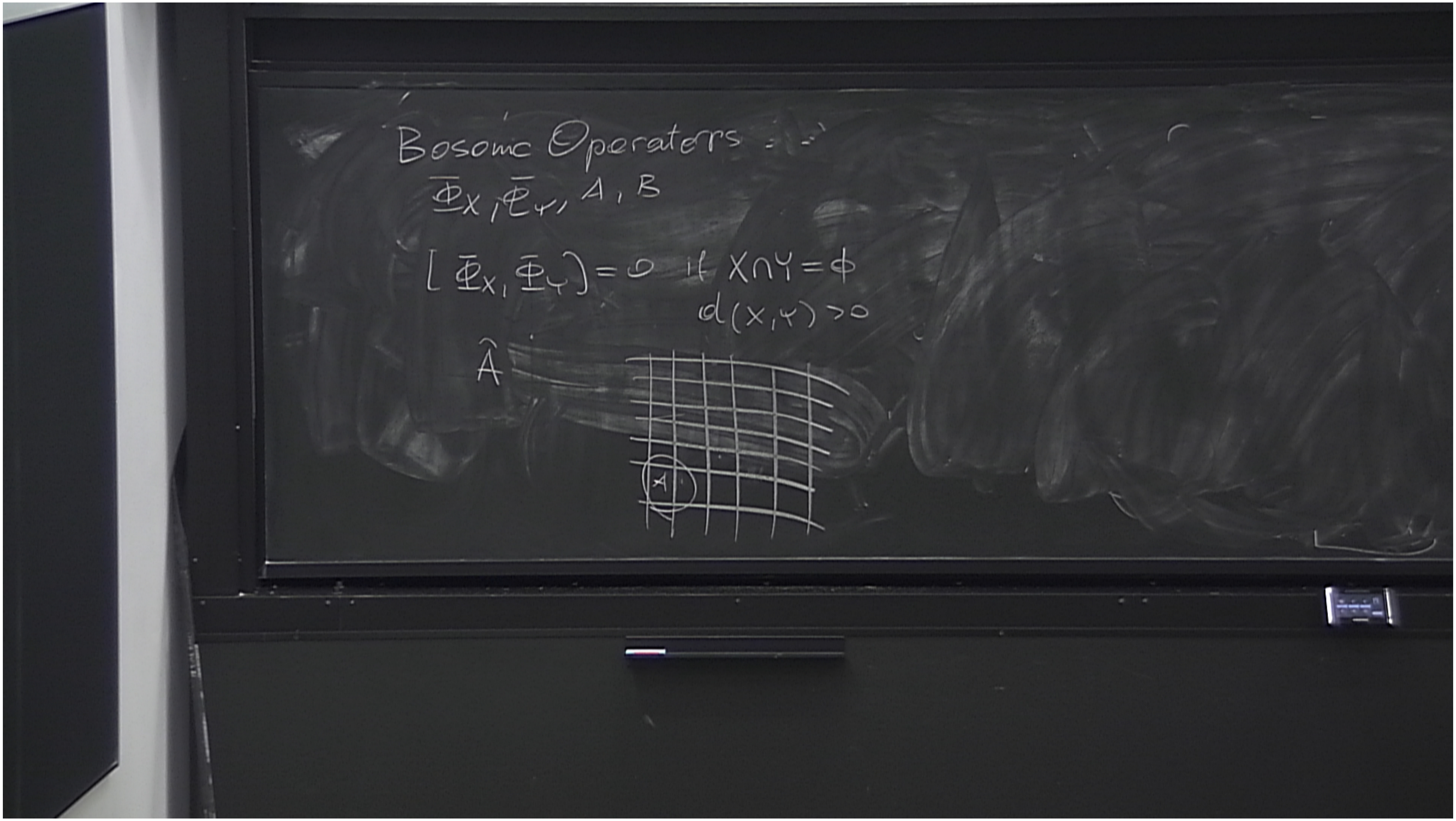
Bosonic Operators

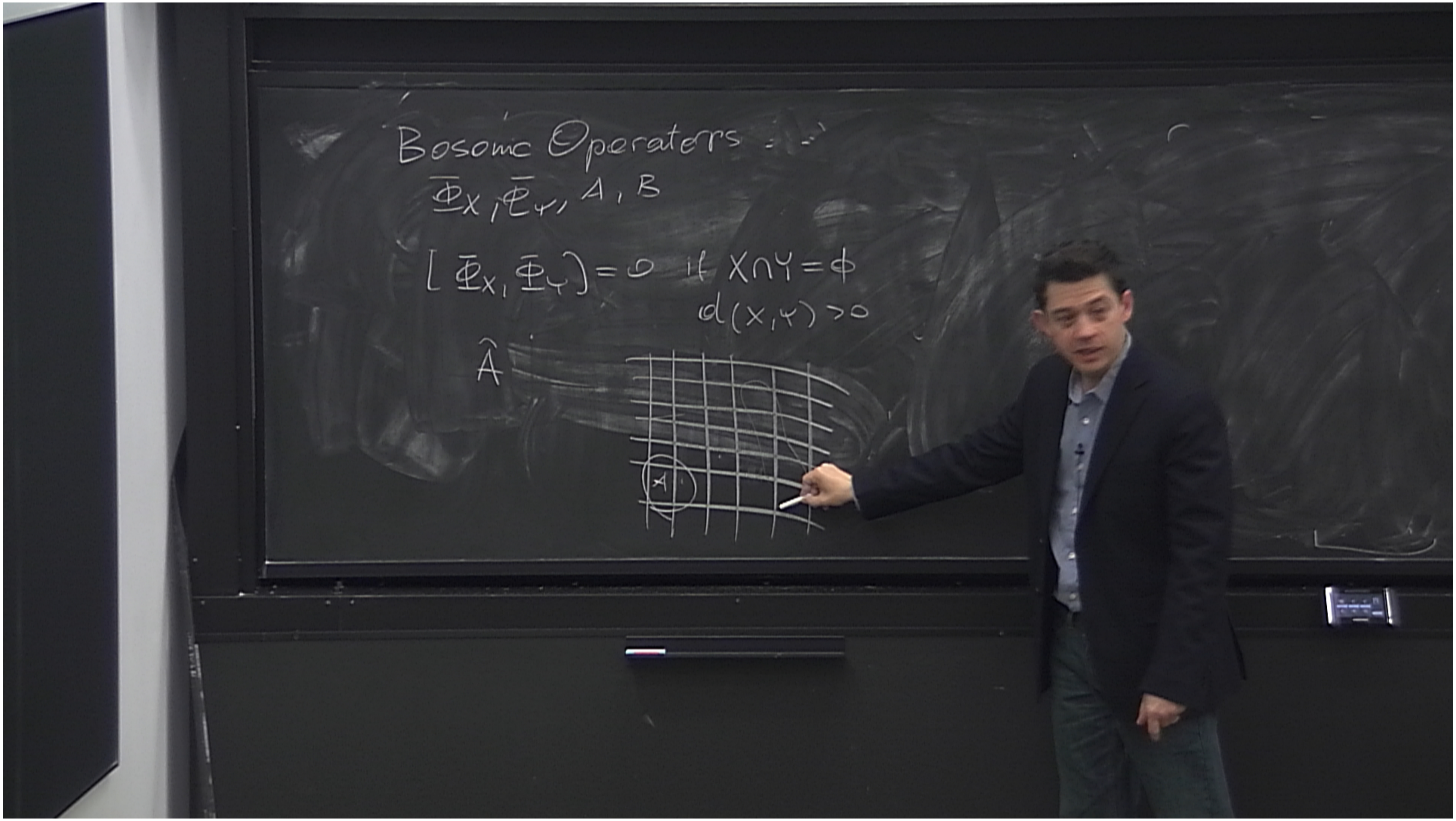
Φ_X, Φ_Y, A, B

$[\Phi_X, \Phi_Y] = 0$ if $X \cap Y = \phi$

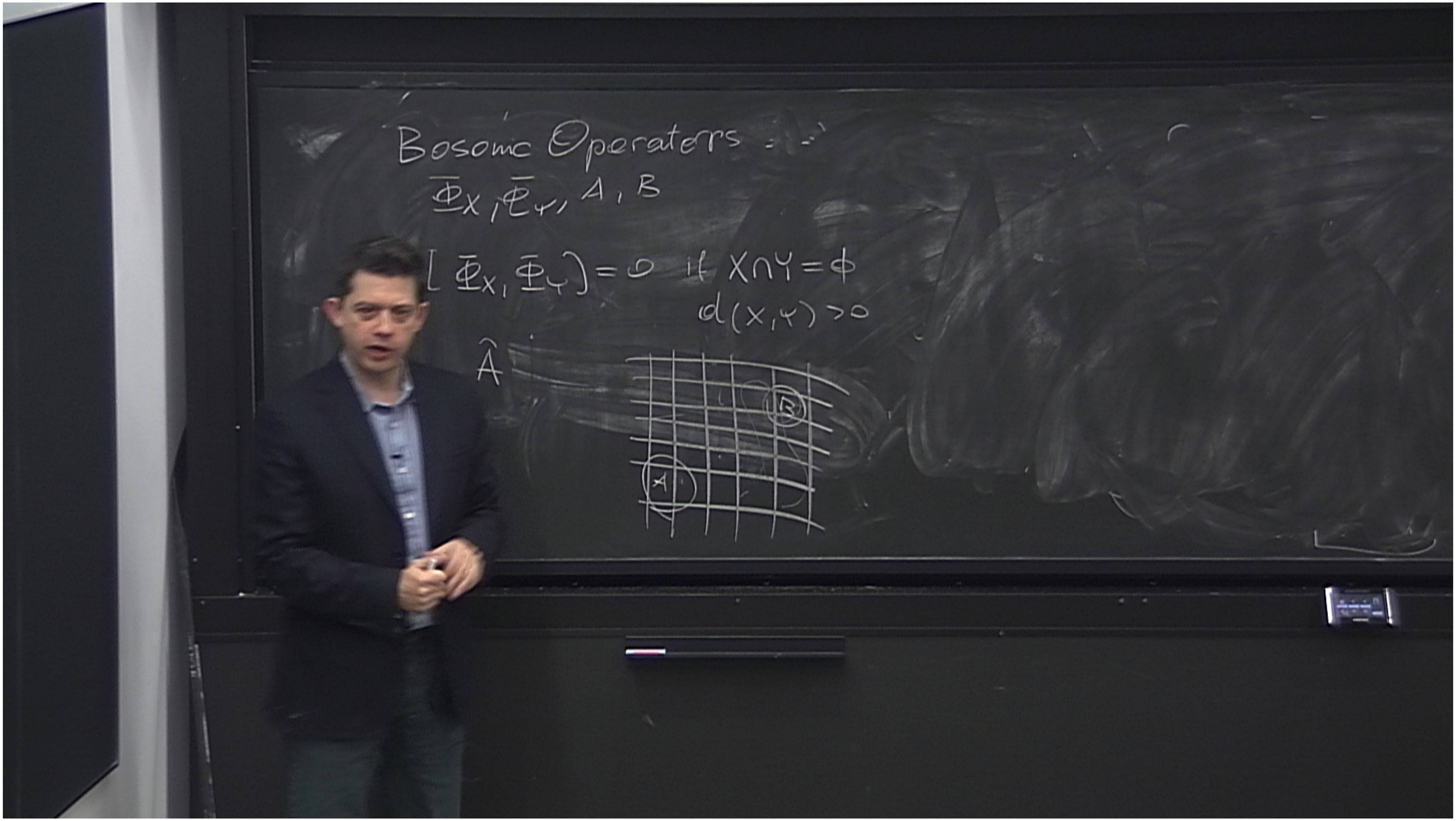
$d(X, Y) > 0$


Bosonic Operators :
$\bar{\Phi}_X, \bar{\Phi}_Y, A, B$
$[\bar{\Phi}_X, \bar{\Phi}_Y] = 0$ if $X \cap Y = \phi$
$d(X,Y) > 0$

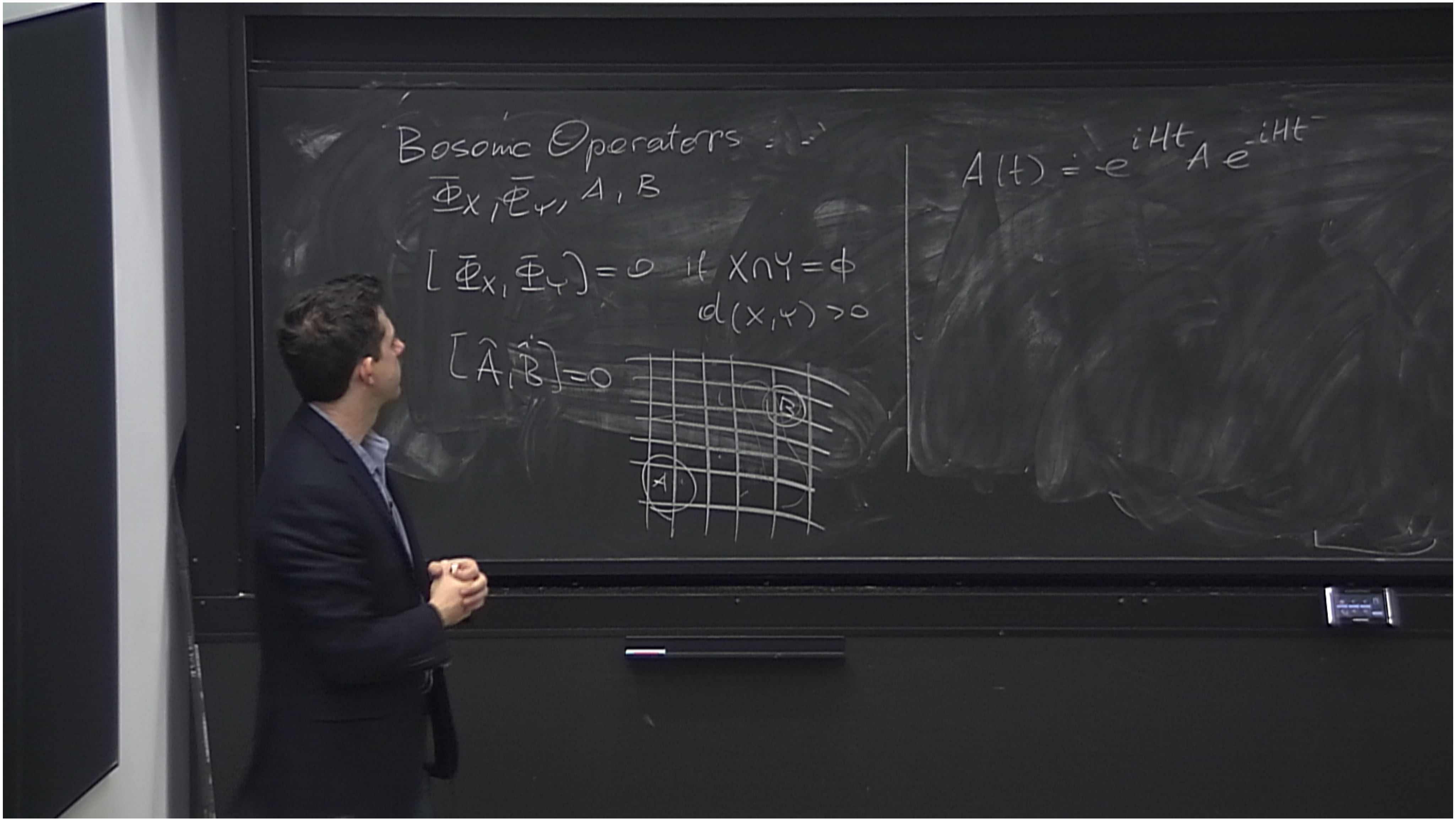

Bosonic Operators
Φ_X, Φ_Y, A, B
$[\Phi_X, \Phi_Y] = 0$ if $X \cap Y = \phi$
$d(X,Y) > 0$
$\hat{A}$
A



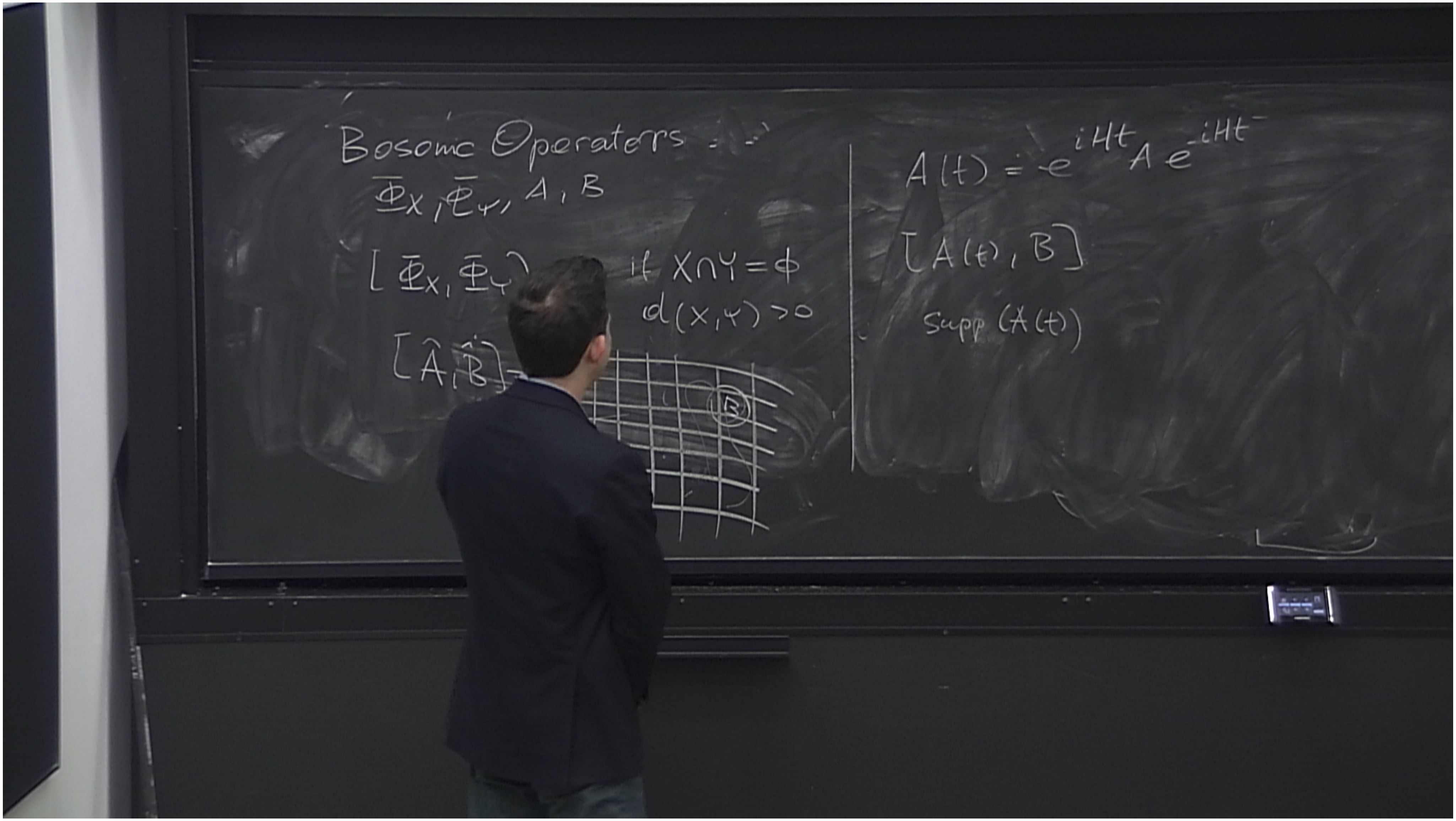
Bosonic Operators
Φ_X, Φ_Y, A, B
$[\Phi_X, \Phi_Y] = 0$ if $X \cap Y = \phi$
$d(X,Y) > 0$
$\hat{A}$
A


Bosonic Operators
Φ_X, Φ_Y, A, B
$[\Phi_X, \Phi_Y] = 0$ if $X \cap Y = \phi$
$d(X,Y) > 0$
$\hat{A}$
B
A


Bosonic Operators
Φ_X, Φ_Y, A, B
$[\Phi_X, \Phi_Y]$
if $X \cap Y = \phi$
$d(X,Y) > 0$
$[\hat{A}, \hat{B}] =$
B



Bosonic Operaters
Φ_X, Φ_Y, A, B
$[\Phi_X, \Phi_Y] = 0$ if $X \cap Y = \phi$
$d(X,Y) > 0$
$[\hat{A}, \hat{B}] = 0$
B
A
$A(t) = e^{iHt} A e^{-iHt}$


Bosonic Operators
Φ_X, Φ_Y, A, B
$[\Phi_X, \Phi_Y]$
if $X \cap Y = \phi$
$d(X,Y) > 0$
$[\hat{A}, \hat{B}]$
$A(t) = e^{iHt} A e^{-iHt}$
$[A(t), B]$
Supp $(A(t))$

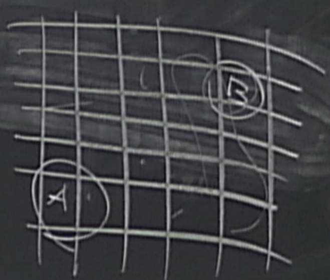
Bosonic Operators:

Φ_X, Φ_Y, A, B

$[\Phi_X, \Phi_Y] = 0$ if $X \cap Y = \phi$

$d(X,Y) > 0$

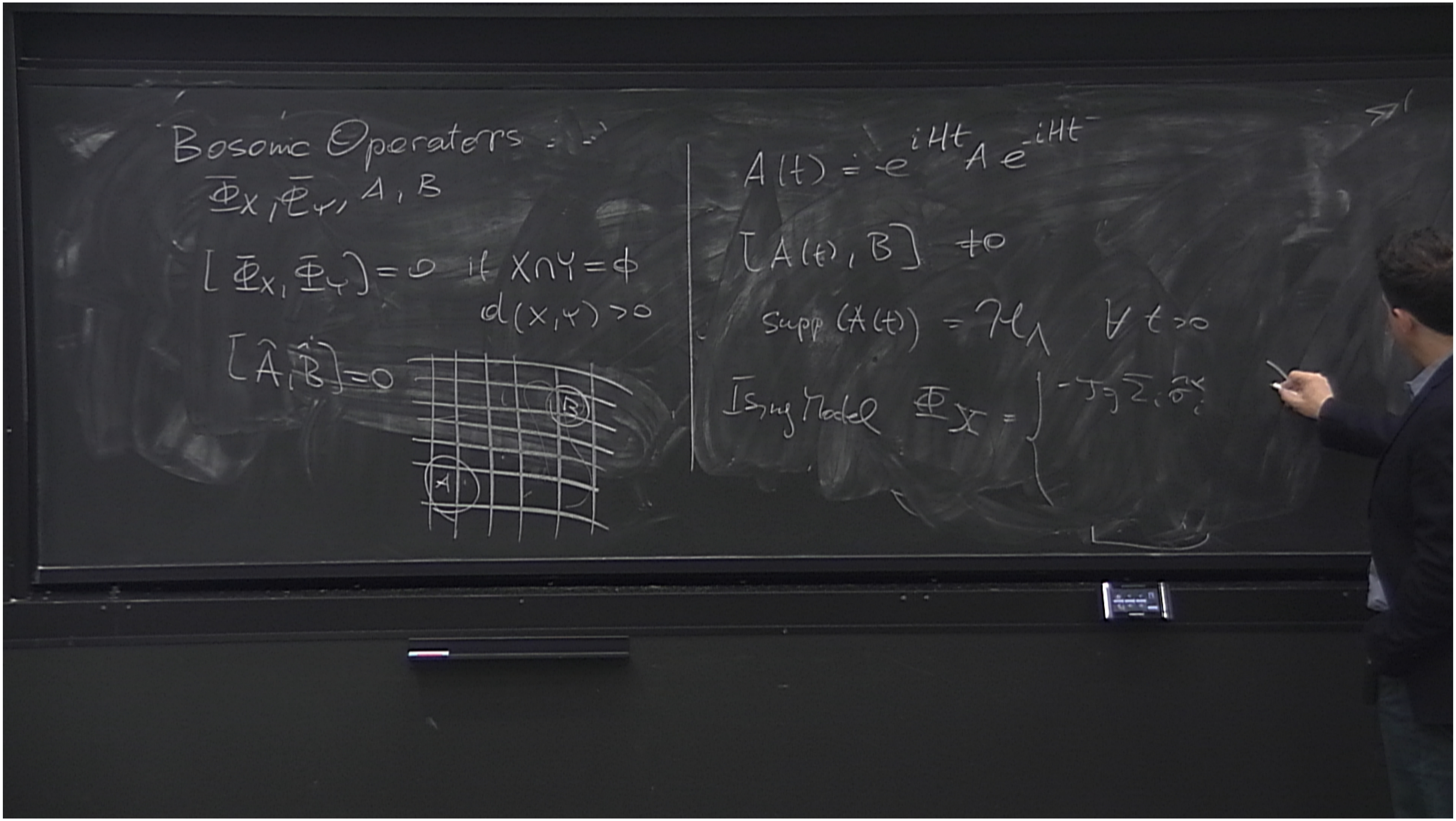
$[\hat{A}, \hat{B}] = 0$

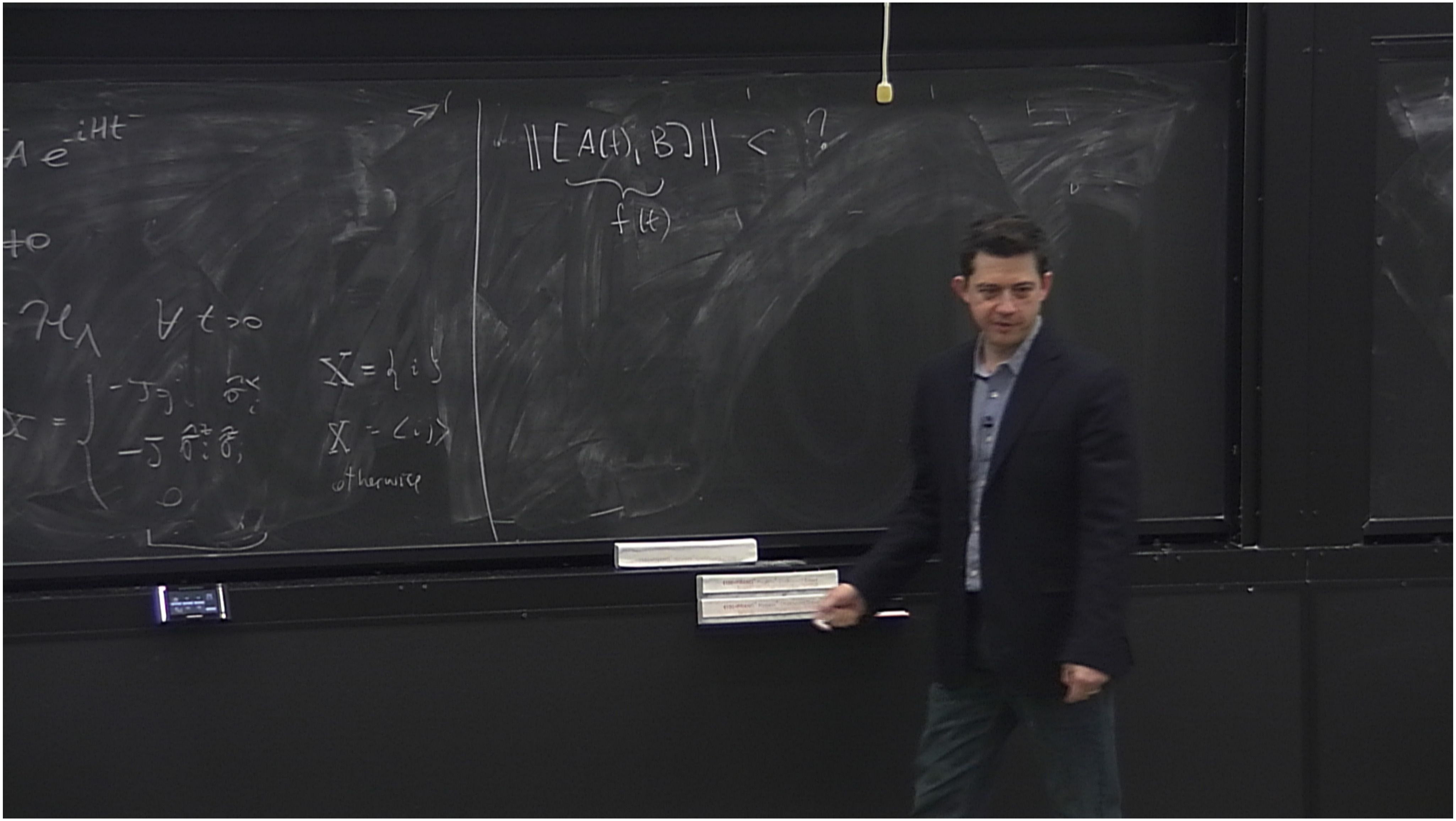


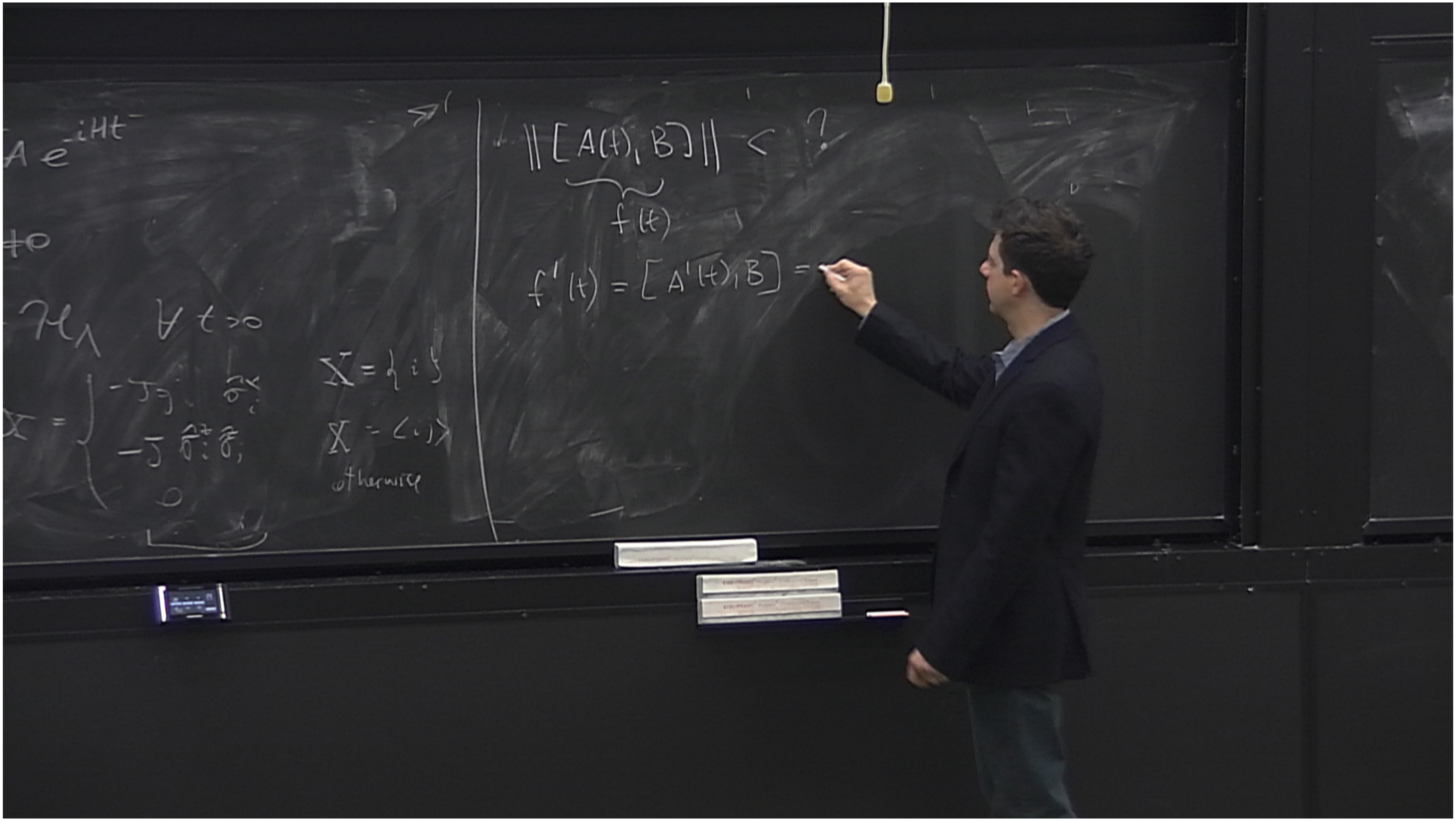
$A(t) := e^{iHt} A e^{-iHt}$

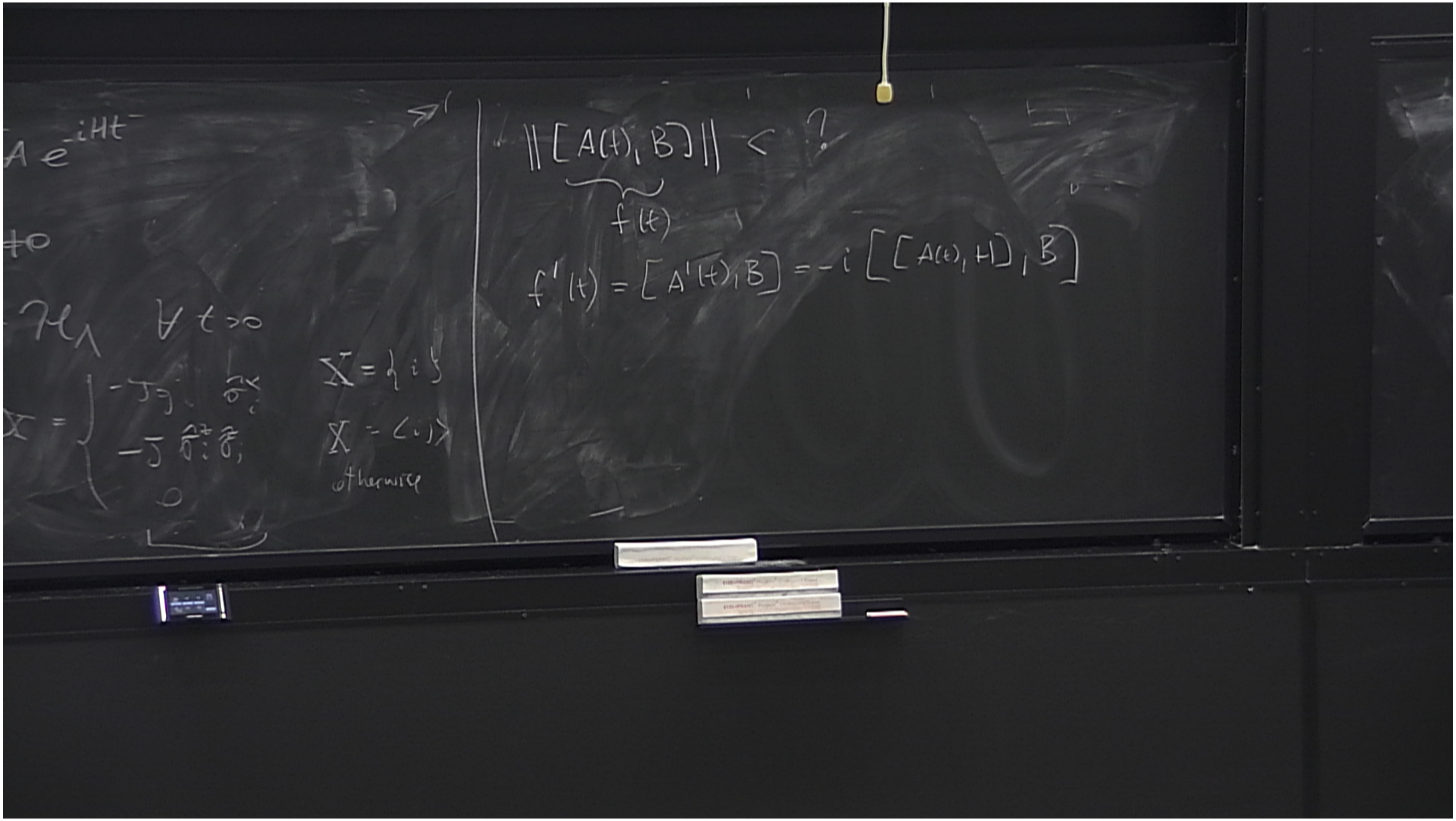
$[A(t), B] \neq 0$

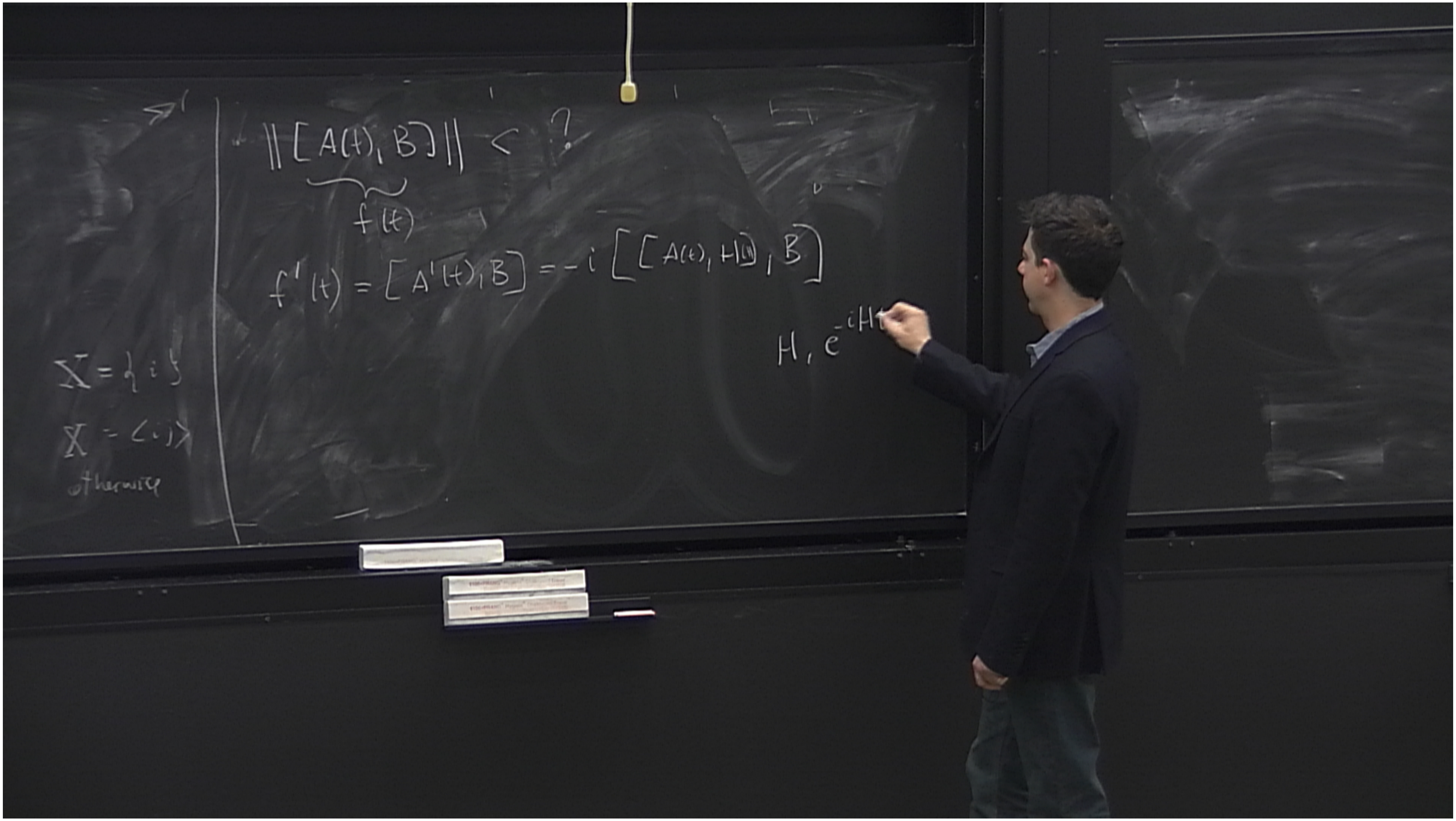
$\mathrm{Supp}(A(t)) = \mathcal{H}_\Lambda \quad \forall t > 0$

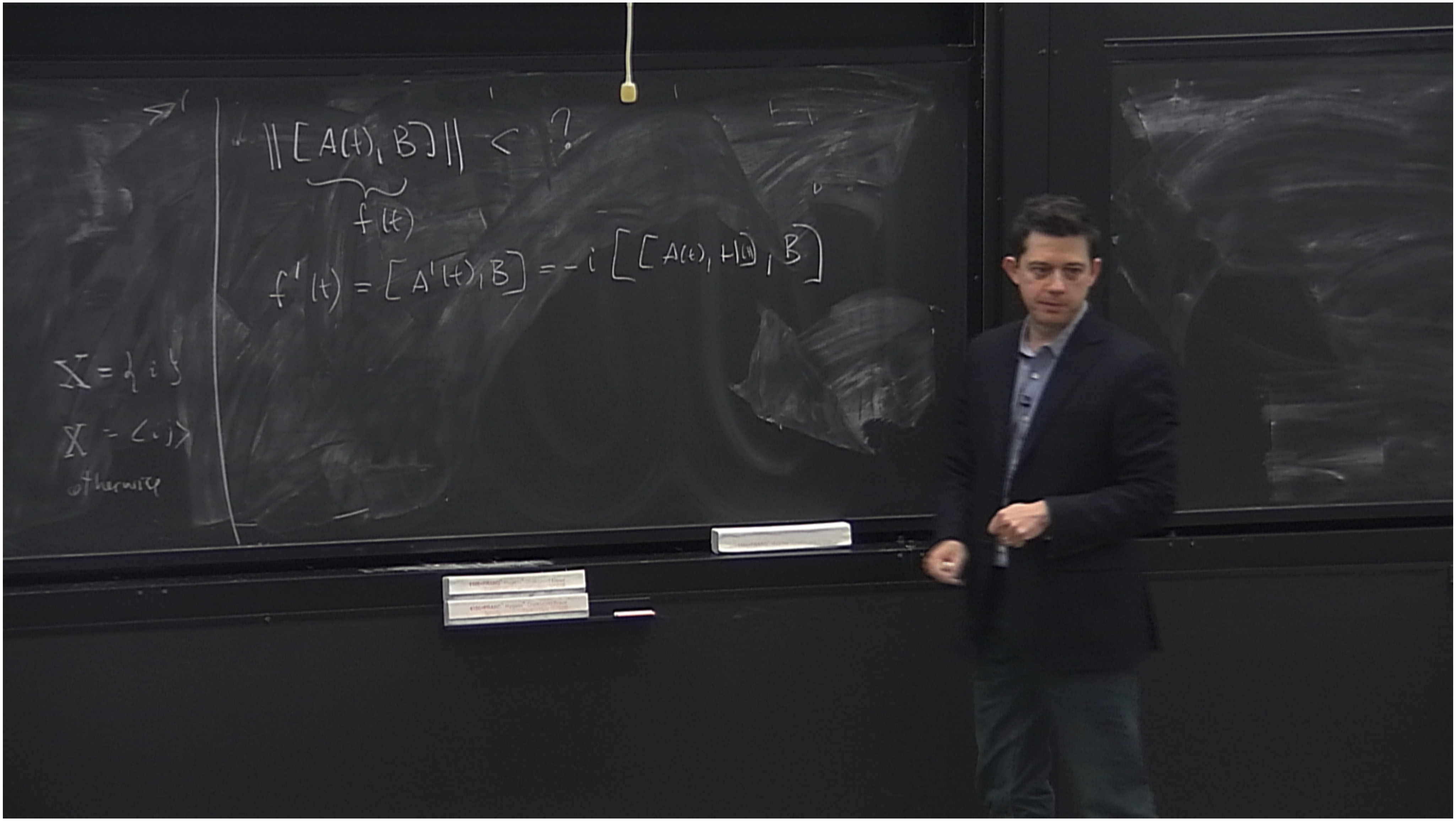

Bosonic Operators :
Φ_X, Φ_Y, A, B
[Φ_X, Φ_Y] = 0 if X∩Y = φ
d(X,Y) > 0
[Â, B̂] = 0
A
B
A(t) = e^{iHt} A e^{-iHt}
[A(t), B] ≠ 0
Supp (A(t)) = H_Λ ∀ t > 0
Ising Model Φ_X =

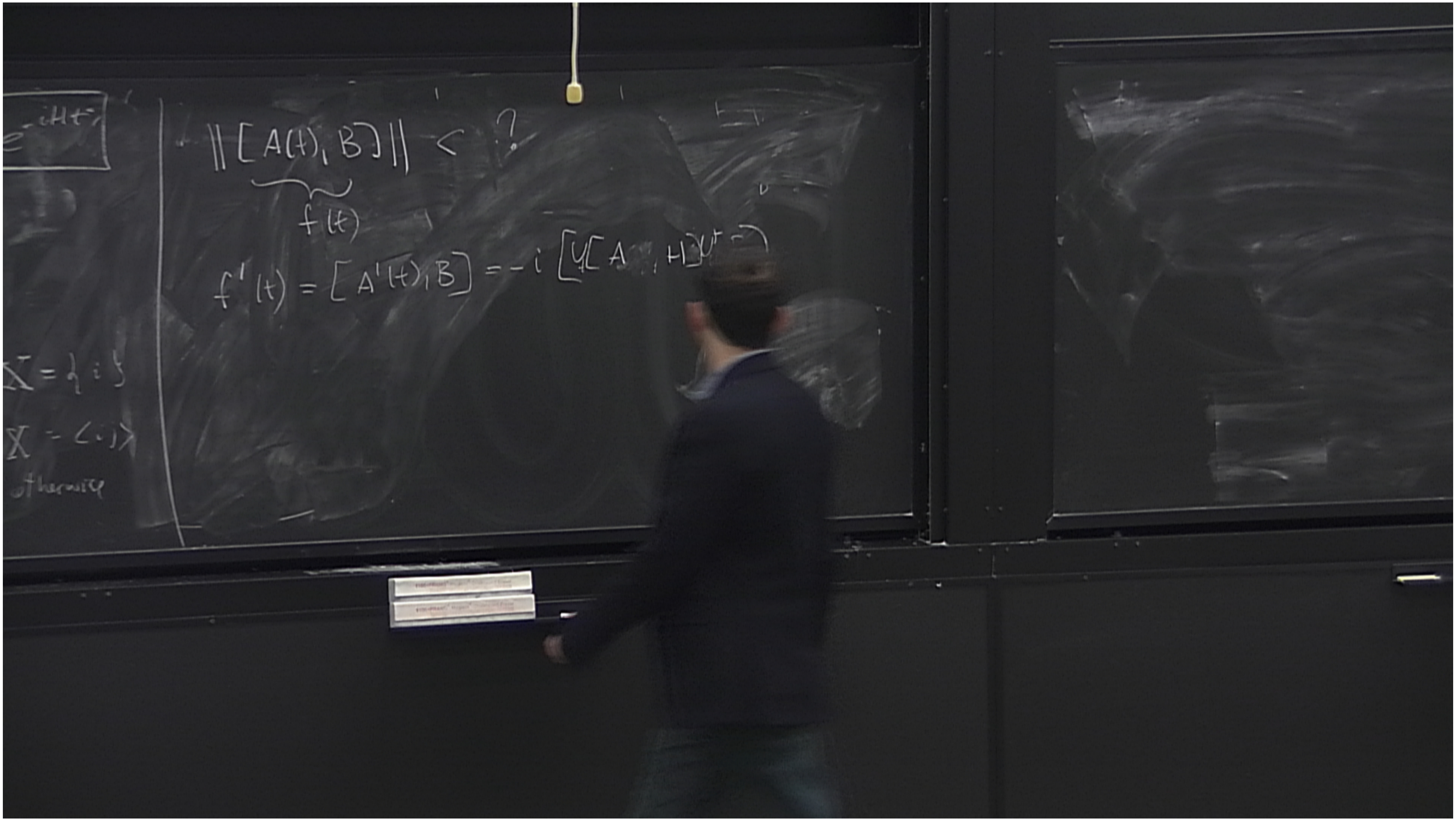

||[A(t), B]|| < ?
f(t)
∀ t > 0
X = {i}
otherwise

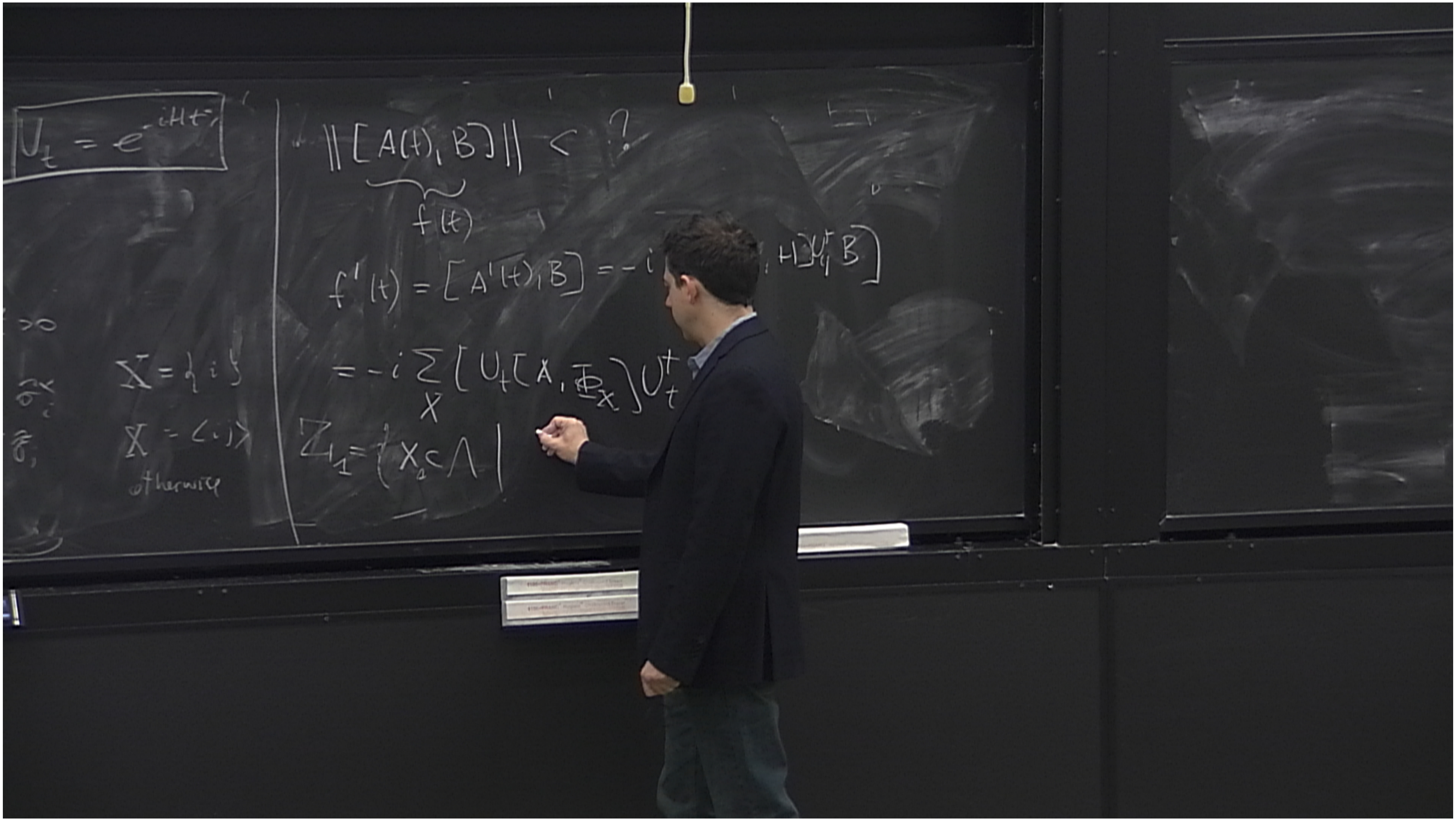

||[A(t), B]|| < ?
f(t)
f'(t) = [A'(t), B] =
∀ t > 0
X = {i}
otherwise

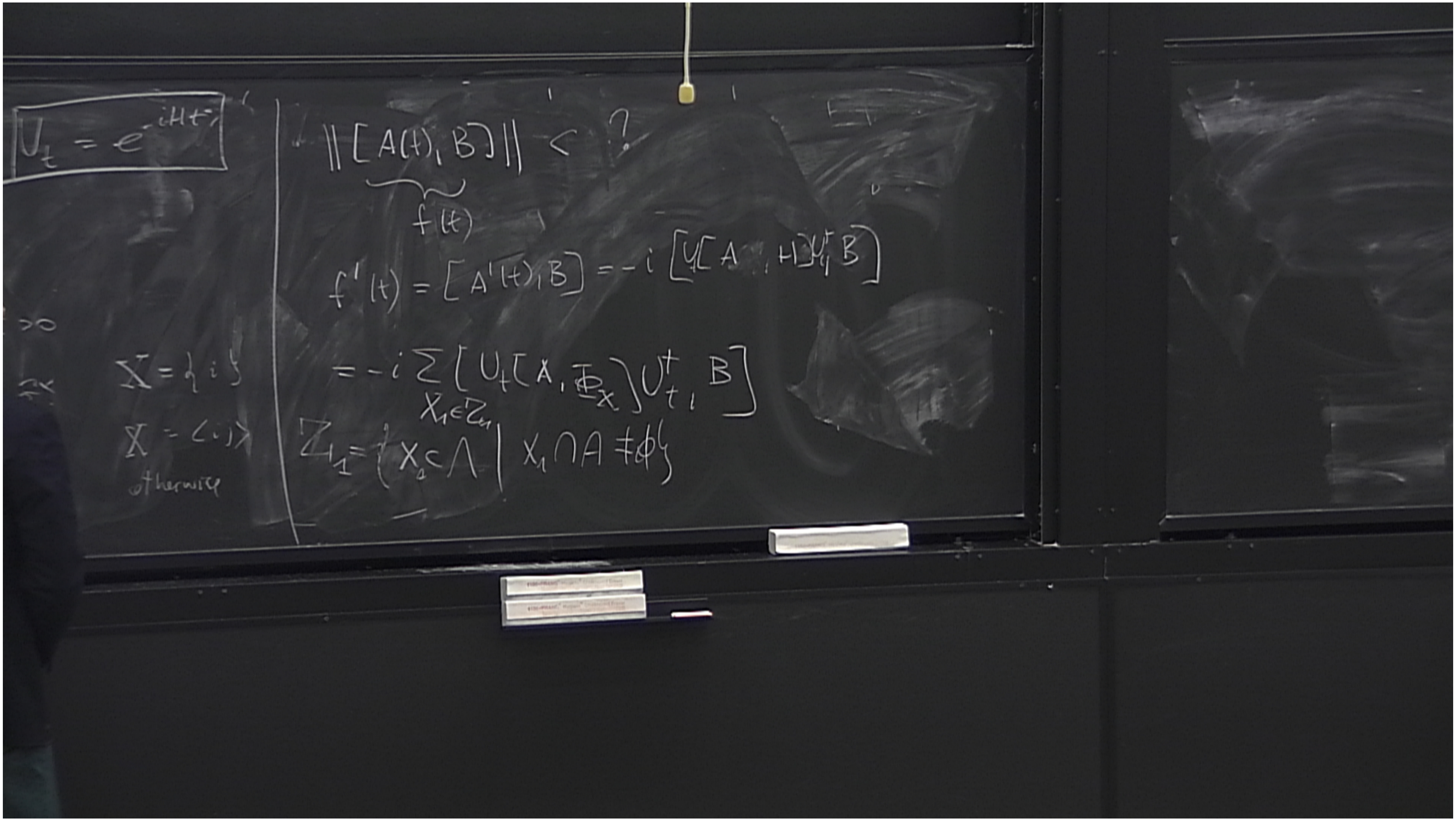

$A e^{-iHt}$
$\|[A(t), B]\| < ?$
$f(t)$
$f'(t) = [A'(t), B] = -i\,[[A(t), H], B]$
$\forall\, t > 0$
$X = \{i\}$
$X = \langle i,j \rangle$
otherwise

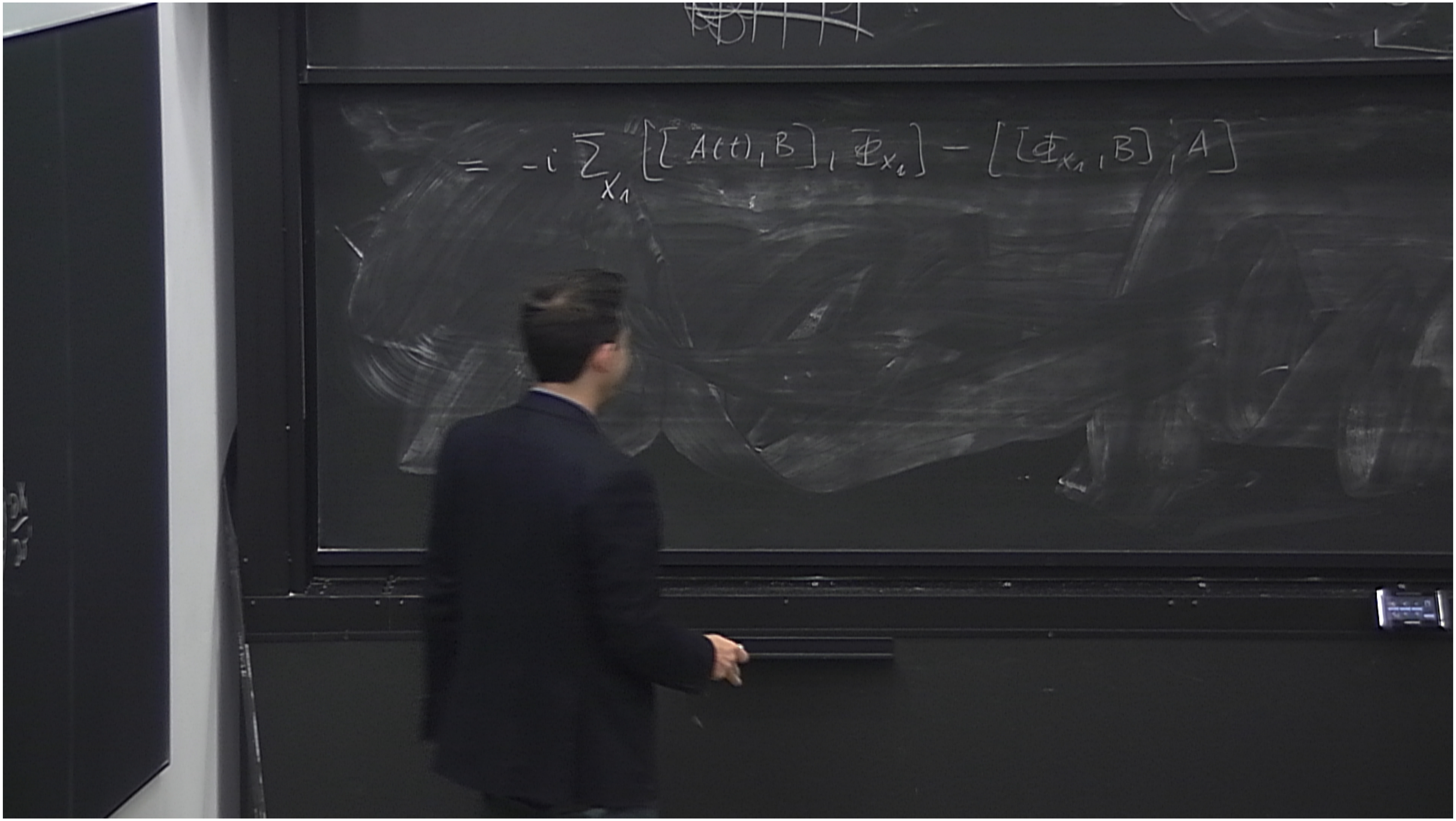

||[A(t), B]|| < ?
f(t)
f'(t) = [A'(t), B] = -i [[A(t), H(t)], B]
H, e-iH
X = {i}
X = ⟨i⟩
otherwise

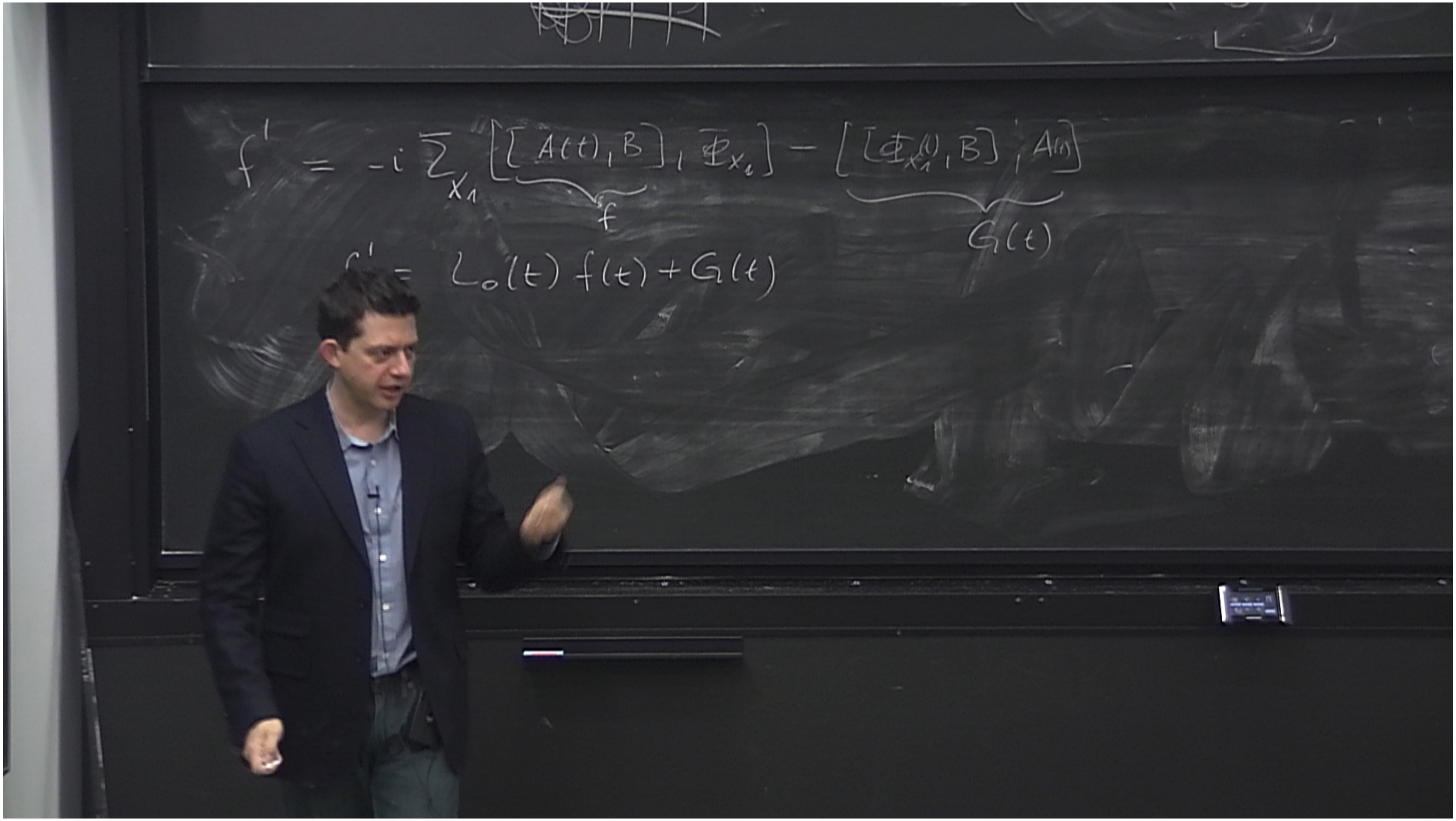

||[A(t), B]|| < ?
f(t)
f'(t) = [A'(t), B] = -i [[A(t), H(t)], B]
X = {i}
X = <i, j>
otherwise

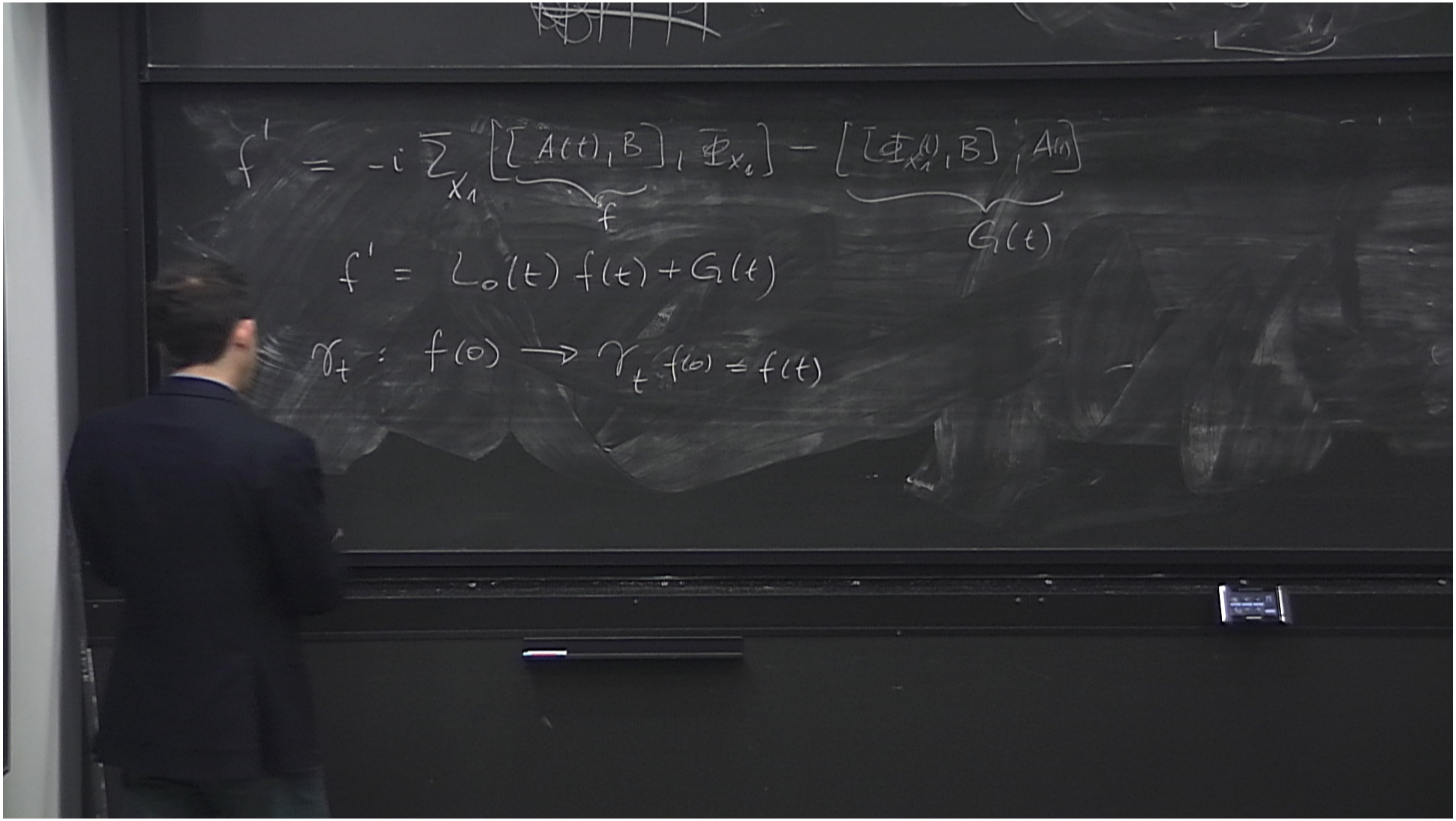

||[A(t), B]|| < ?
f(t)
f'(t) = [A'(t), B] = -i [U[A, H]U†


$U_t = e^{-iHt}$
$\|[A(t), B]\| < ?$
$f(t)$
$f'(t) = [A'(t), B]$
otherwise


$U_t = e^{-iHt}$
$\| [A(t), B] \| < ?$
$f(t)$
$f'(t) = [A'(t), B] = -i [U_t [A, H] U_t^\dagger, B]$
$= -i \sum_{X_1 \in Z_1} [U_t [A, \Phi_X] U_t^\dagger, B]$
$Z_1 = \{ X_1 \subset \Lambda \mid X_1 \cap A \neq \phi \}$
$X = \{i\}$
$X = \langle i, j \rangle$
otherwise


$= -i \sum_{X_1} \left[[A(t), B], \Phi_{X_1} \right] - \left[[\Phi_{X_1}, B], A \right]$


f' = -i Σ_{x1} [[A(t), B], Φ_{x1}] - [[Φ_{x1}(t), B], A(t)]
f
G(t)
f' = L_0(t) f(t) + G(t)



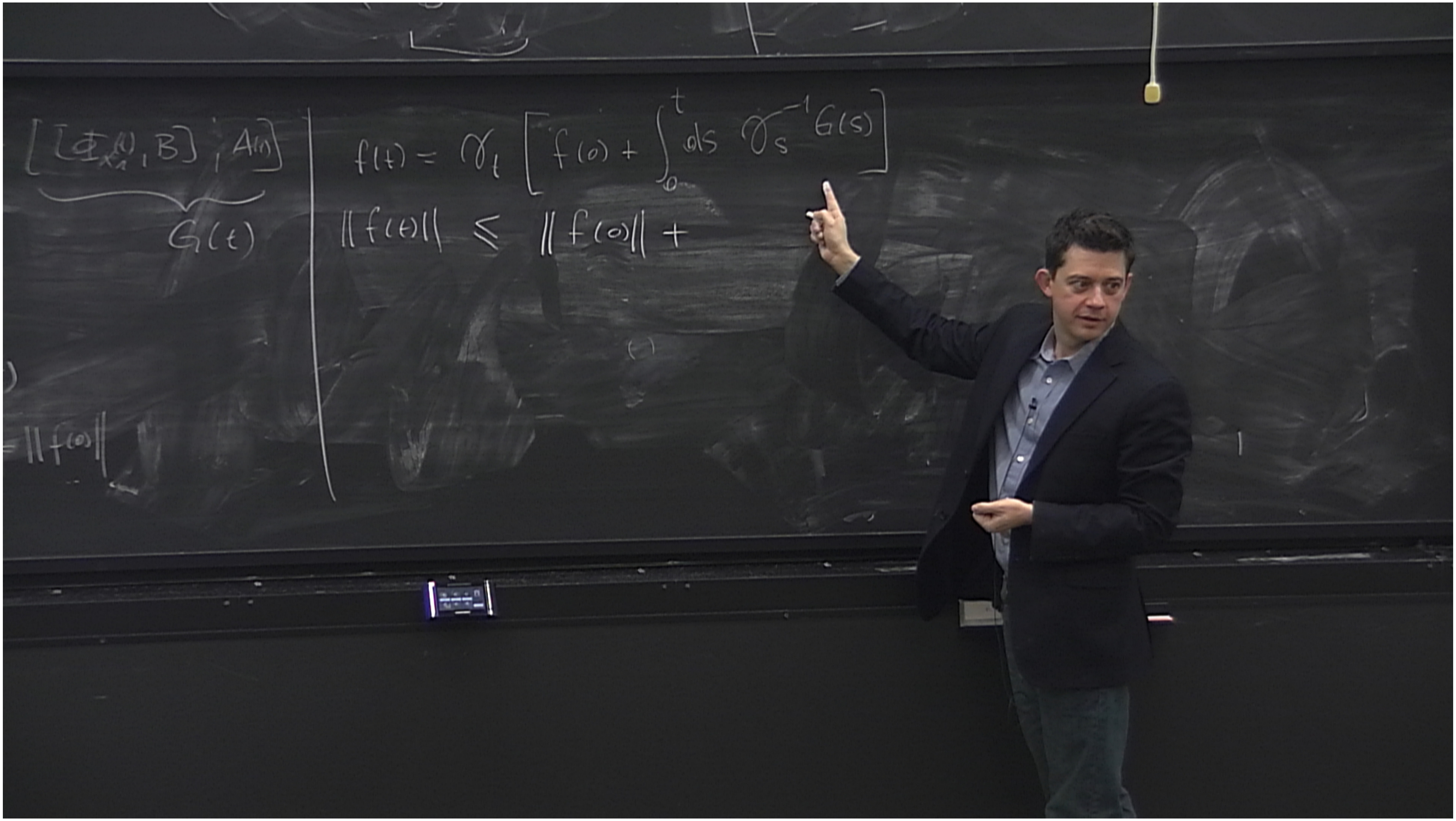
$f' = -i \sum_{x_1} \big[\underbrace{[A(t), B]}_{f}, \Phi_{x_1}\big] - \underbrace{\big[[\Phi_{x_1}(t), B], A(t)\big]}_{G(t)}$
$f' = L_0(t) f(t) + G(t)$
$\gamma_t : f(0) \longrightarrow \gamma_t f(0) = f(t)$

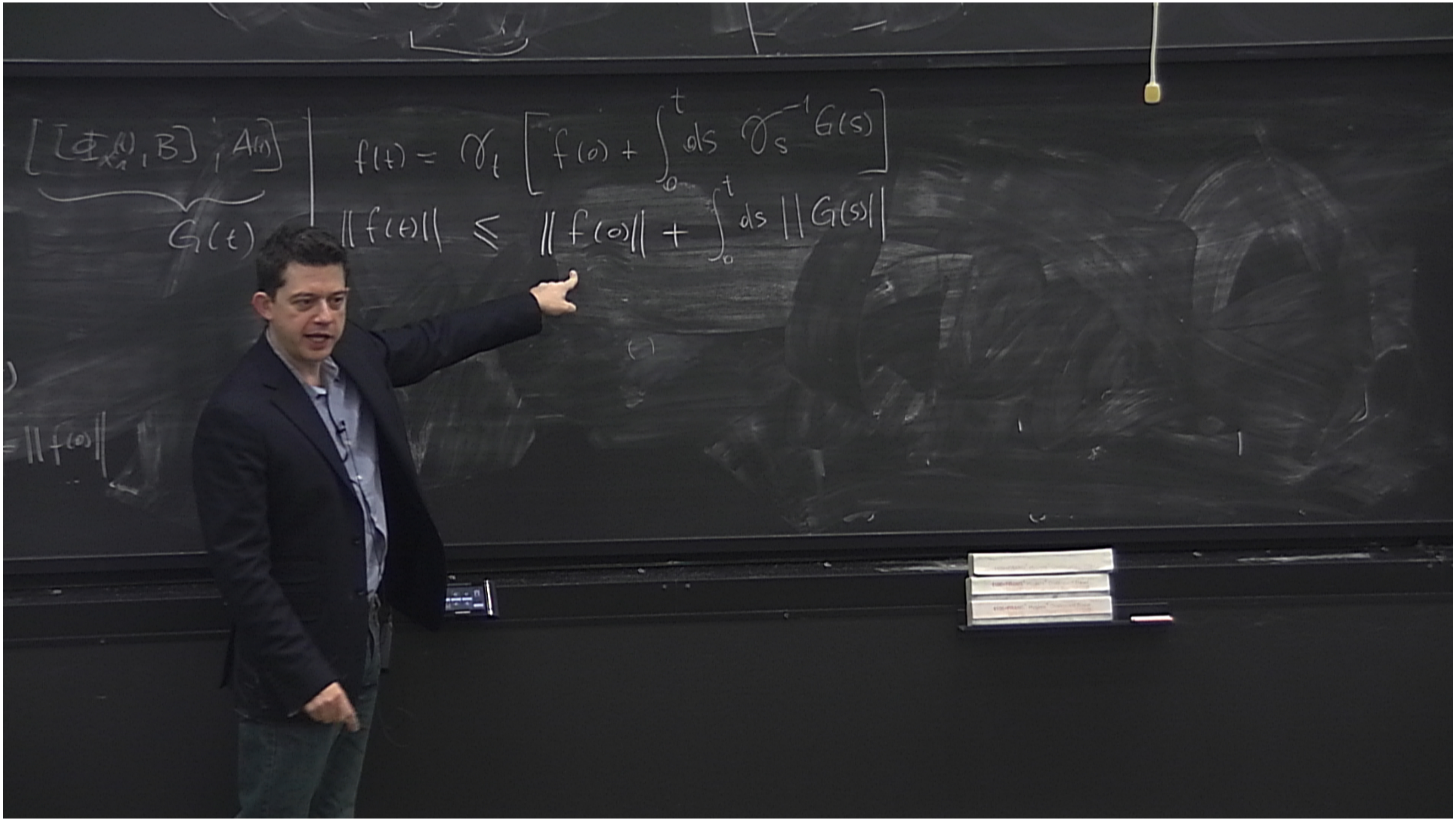
$$f' = -i \sum_{x_1} \Big[\underbrace{[A(t), B]}_{f}, \Phi_{x_1} \Big] - \underbrace{\Big[[\Phi_{x_1}(1), B], A(1) \Big]}_{G(t)}$$

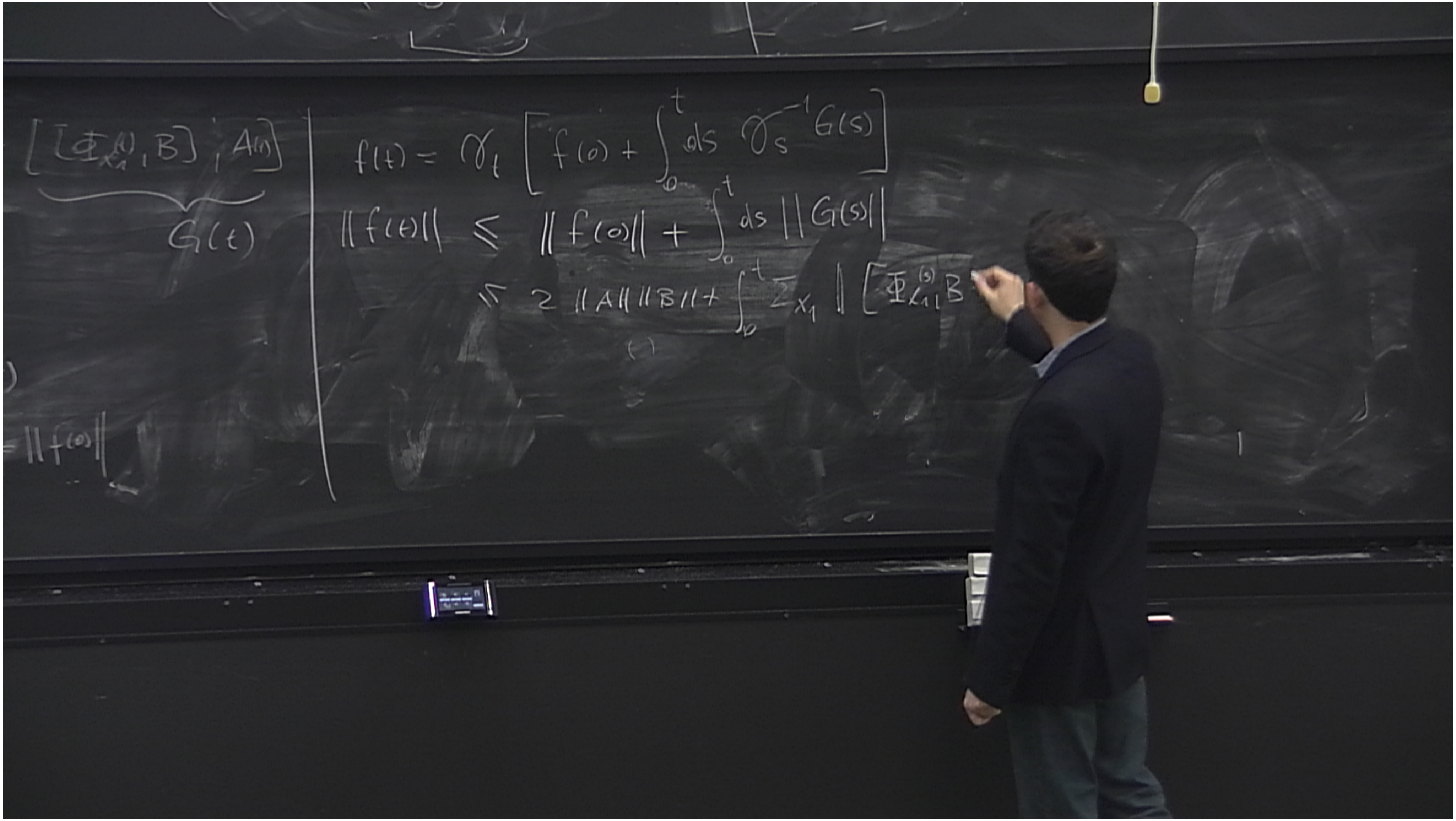
$$f' = L_0(t) f(t) + G(t)$$

$$\gamma_t : f(0) \longrightarrow \gamma_t f(0) = f(t)$$

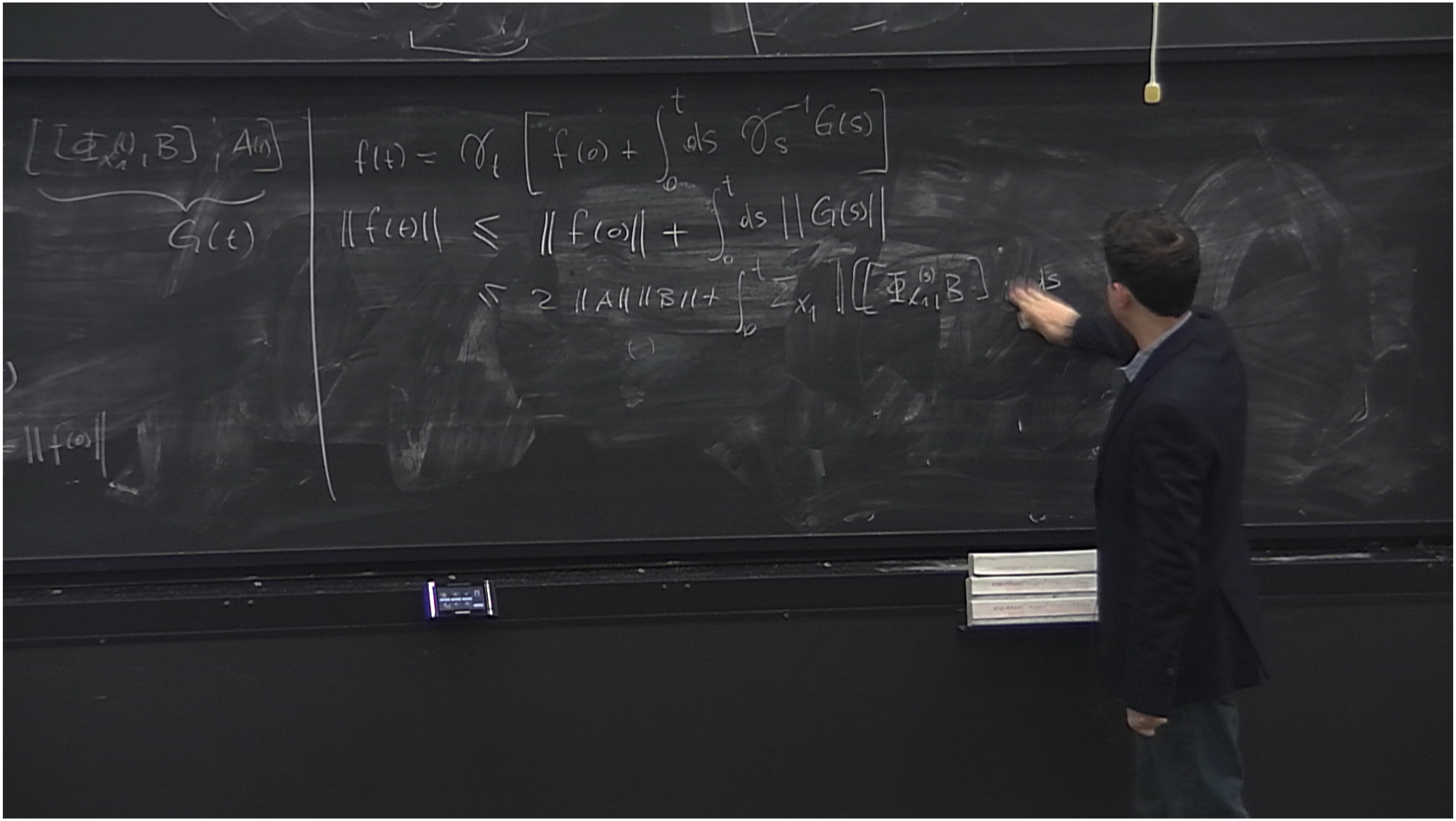
norm-preserving $\quad \| f(t) \| = \| \gamma_t f(0) \| = \| f(0) \|$


$[[\Phi_{X_1}(t), B], A(t)] = G(t)$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| +$
$\|f(0)\|$

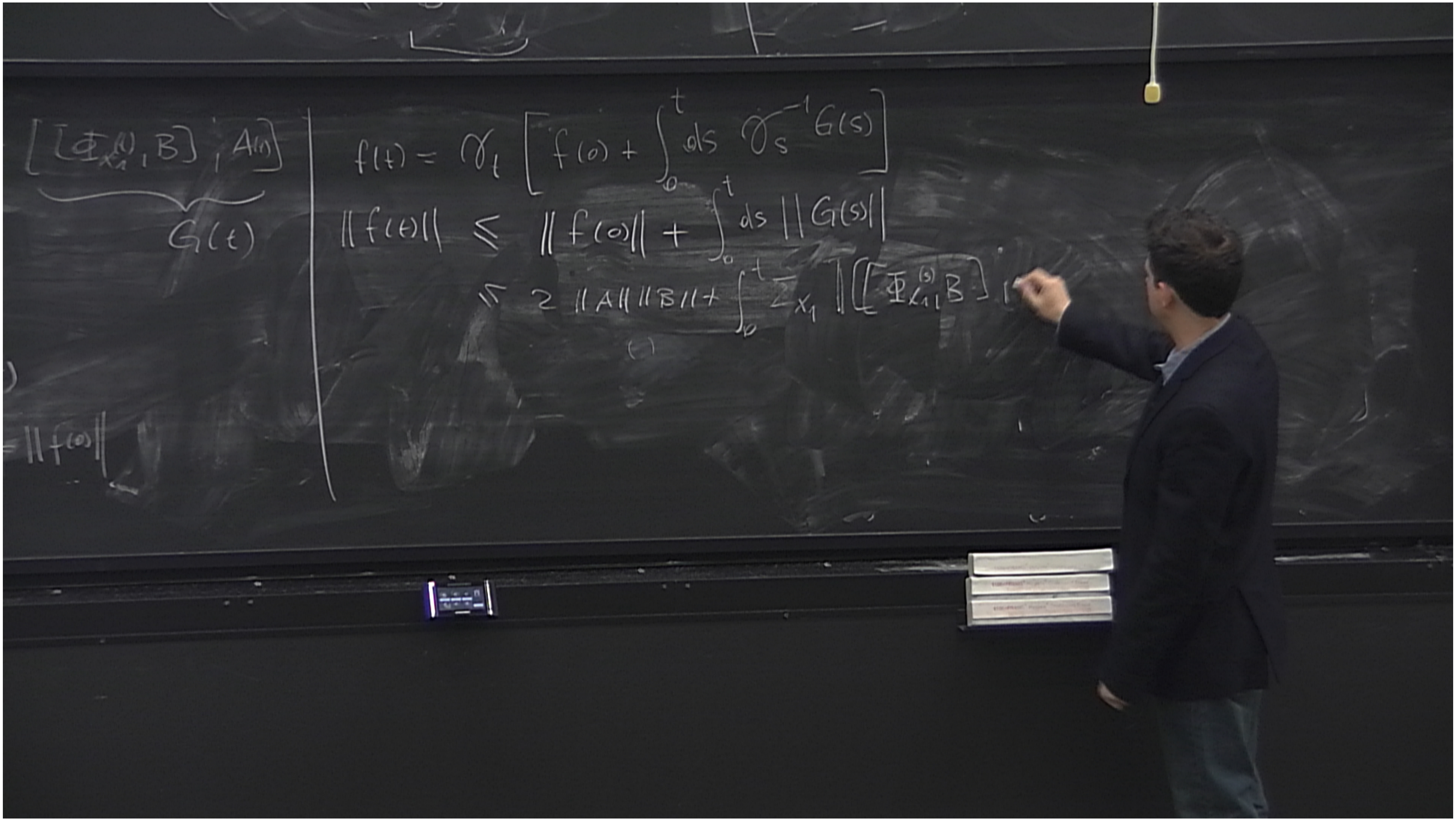

$[[\Phi_{X_1}(t), B], A(t)]$
$G(t)$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| + \int_0^t ds\, \|G(s)\|$
$= \|f(0)\|$

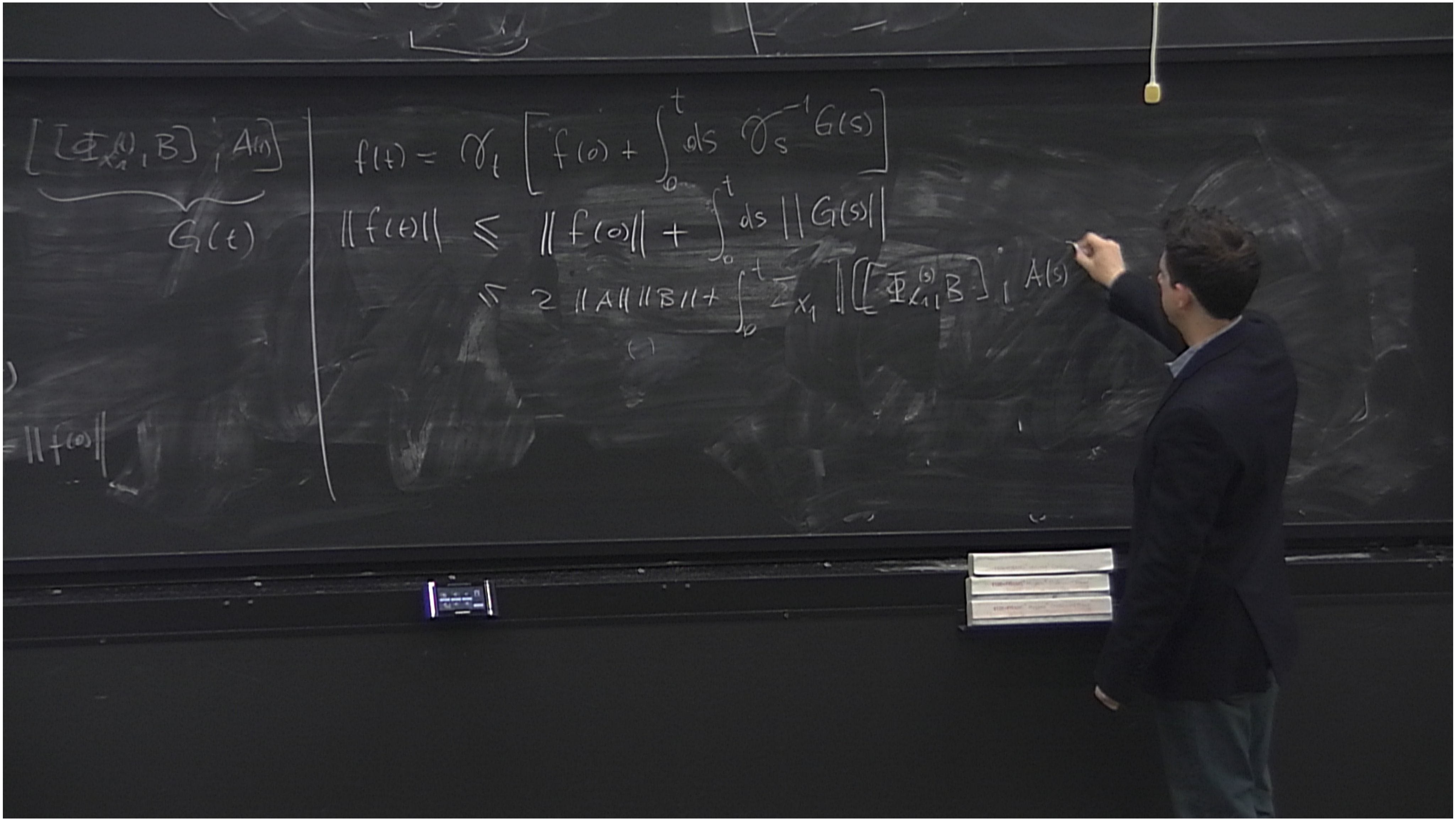


$[[\Phi_{X_1}^{(t)}, B], A(t)]$
$G(t)$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| + \int_0^t ds\, \|G(s)\|$
$\leq 2\|A\|\|B\| + \int_0^t \sum_{X_1} \| [\Phi_{X_1}^{(s)}, B$
$= \|f(0)\|$

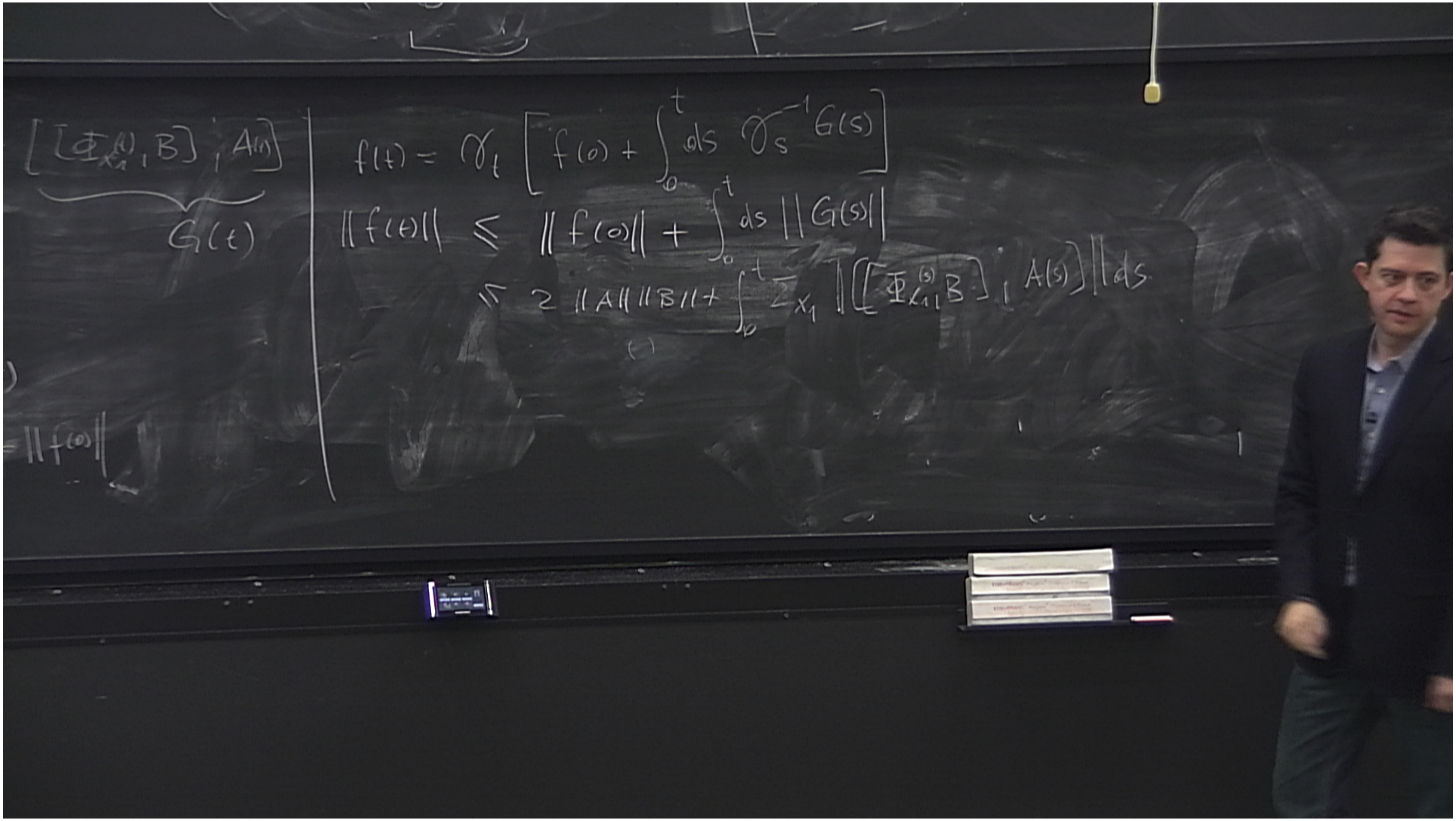


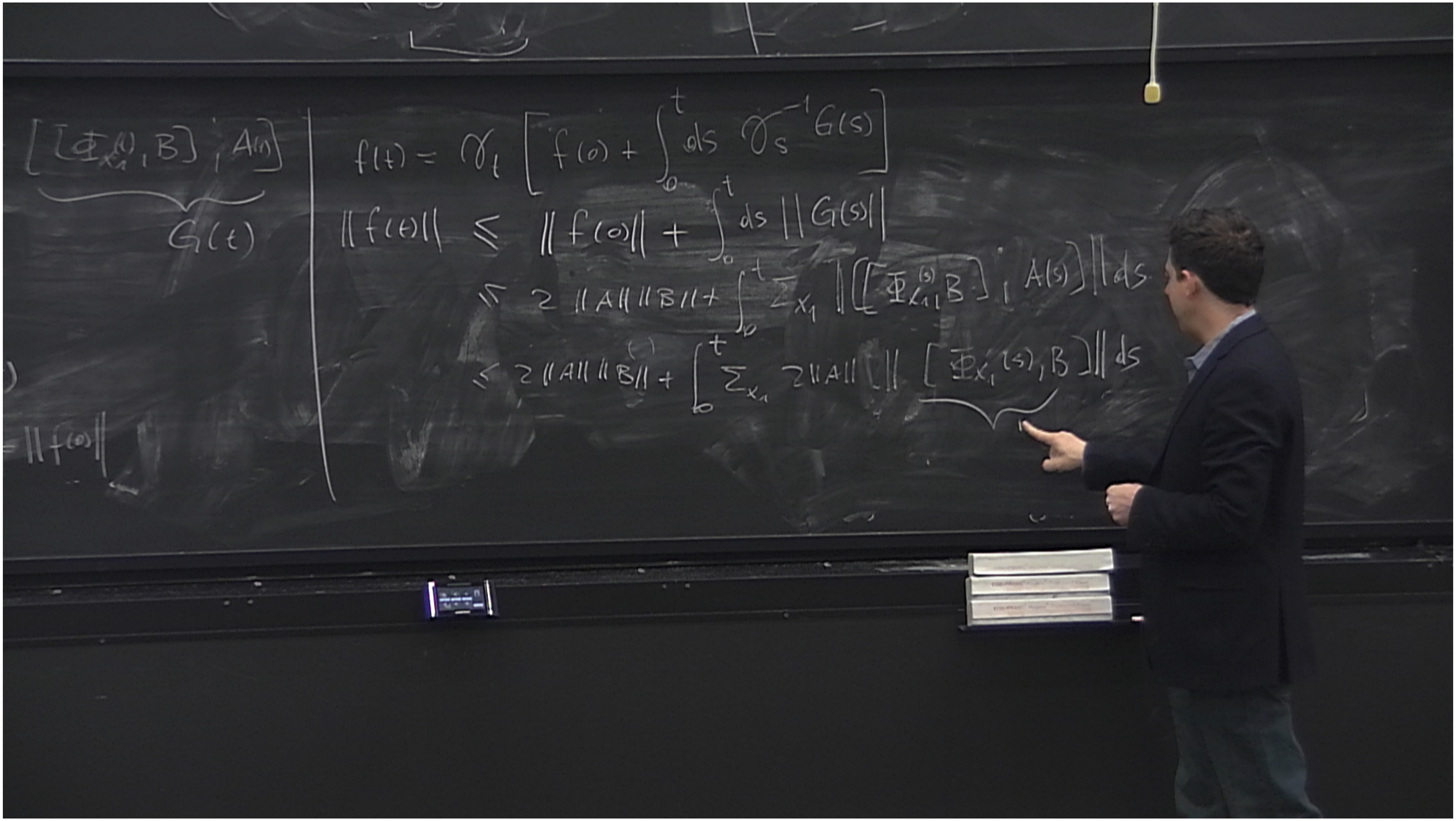
$[[\Phi^{(1)}_{X_1}, B], A(1)]$
$G(t)$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| + \int_0^t ds\, \|G(s)\|$
$\leq 2\|A\|\|B\| + \int_0^t \sum_{X_1} \|[\Phi^{(s)}_{X_1}, B]\| ds$
$= \|f(0)\|$

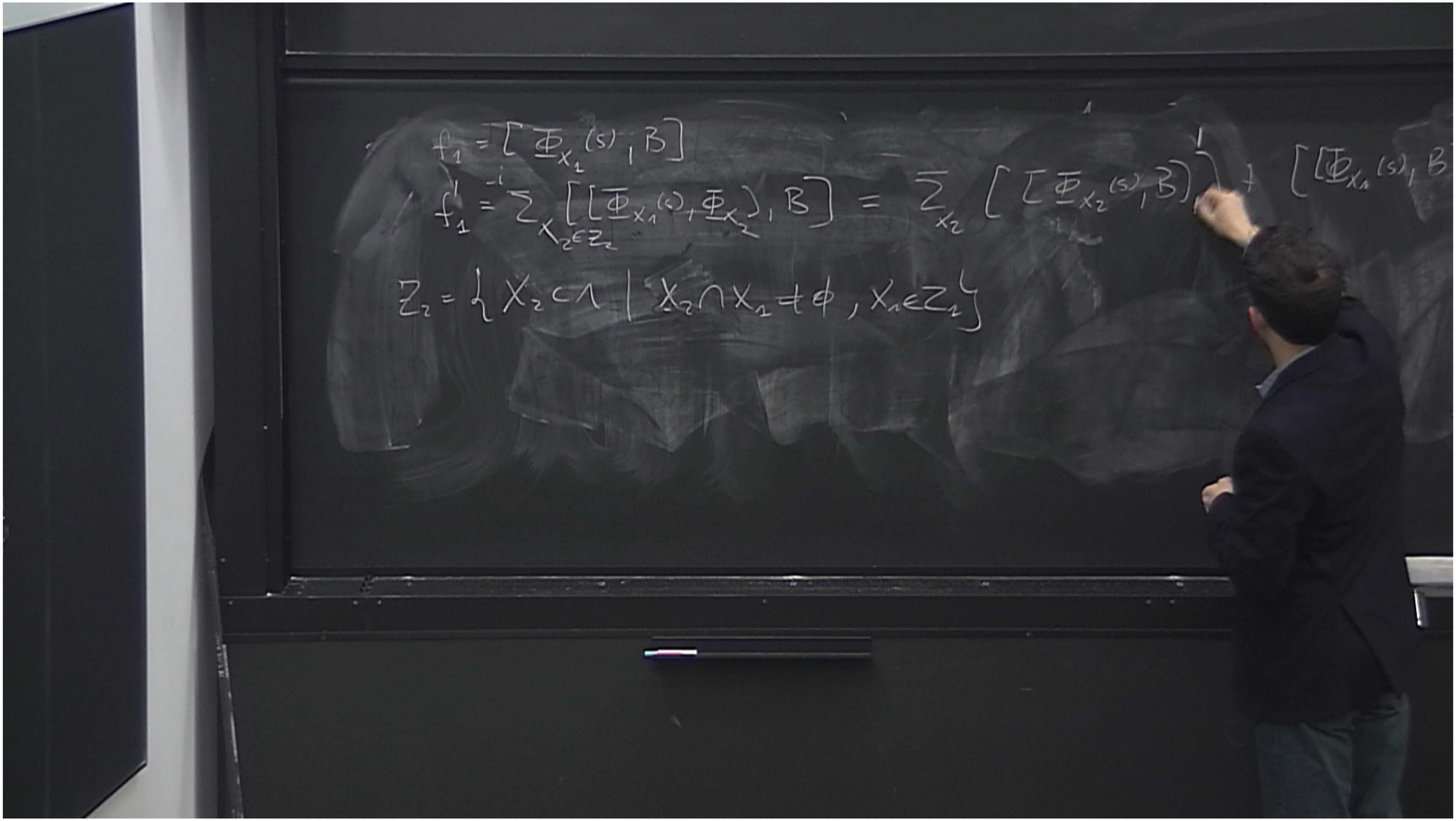

$\underbrace{[[\Phi^{(1)}_{X_1}, B], A(1)]}_{G(t)}$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| + \int_0^t ds\, \|G(s)\|$
$\leq 2\|A\|\|B\| + \int_0^t \sum_{X_1} \|[[\Phi^{(s)}_{X_1}, B]$
$= \|f(0)\|$

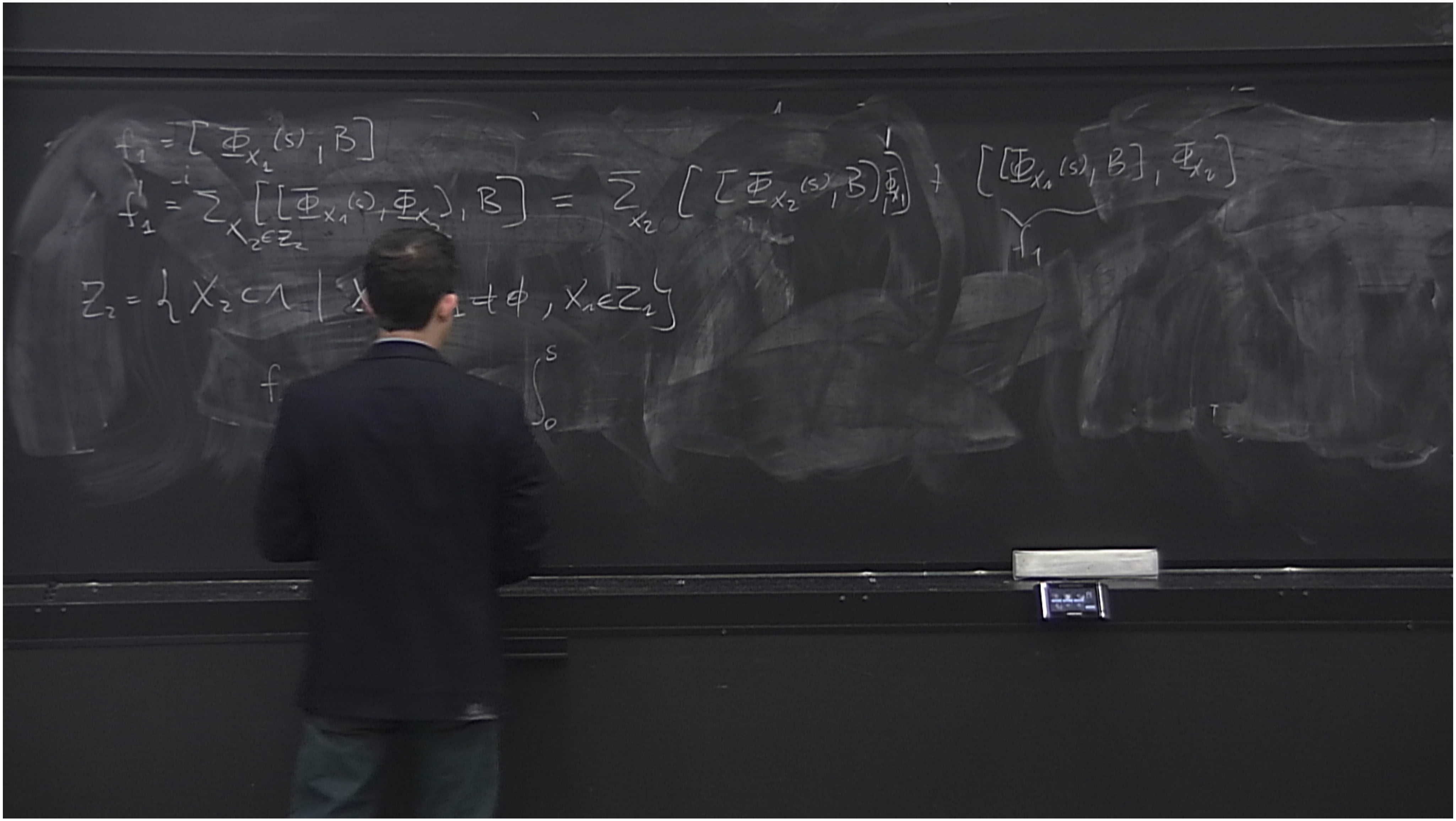


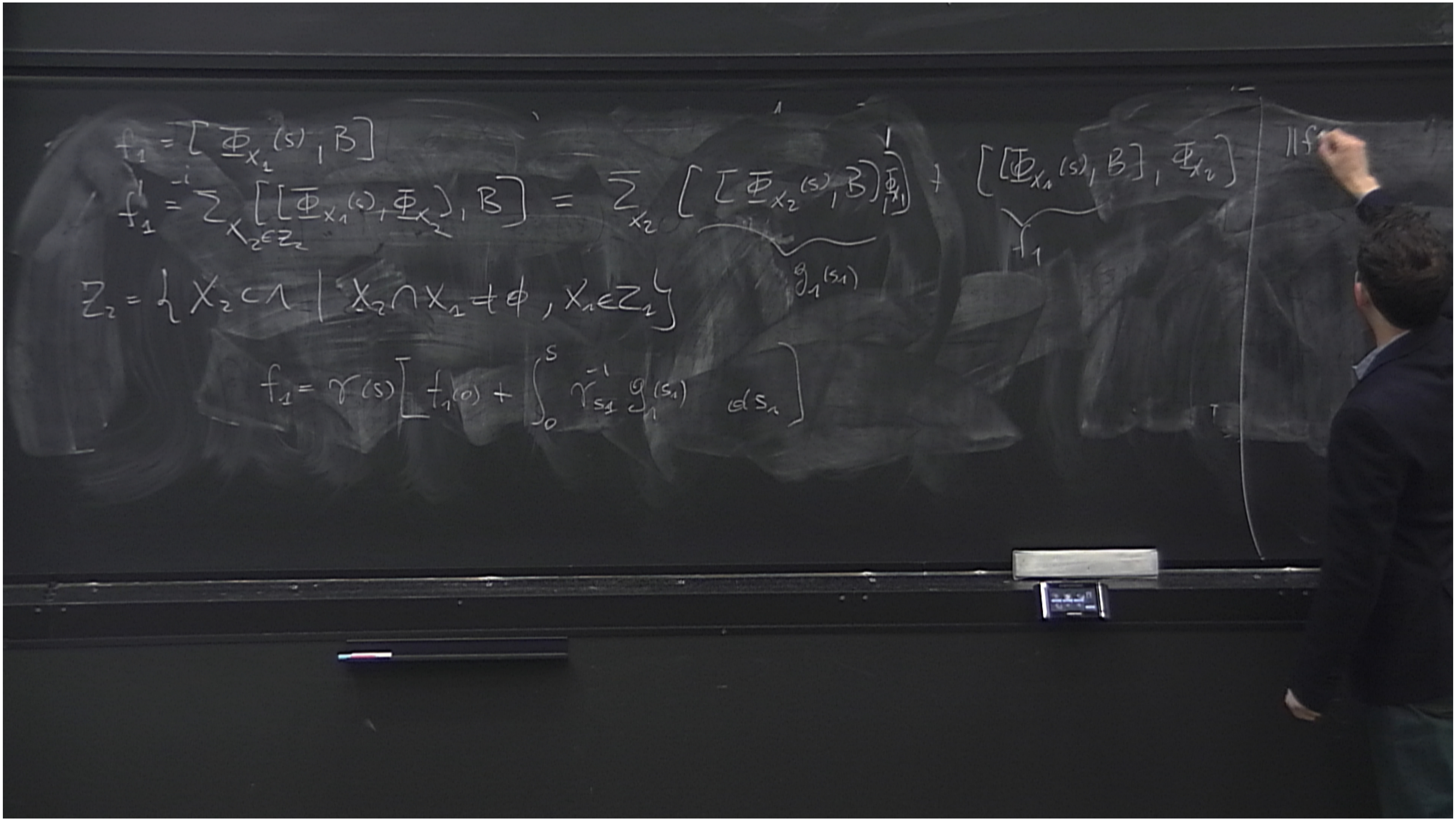
$\underbrace{\left[[\Phi^{(t)}_{X_1}, B], A(t) \right]}_{G(t)}$
$= \|f(0)\|$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| + \int_0^t ds\, \|G(s)\|$
$\leq 2\|A\|\|B\| + \int_0^t \sum_{X_1} \|[[\Phi^{(s)}_{X_1}, B], A(s)$

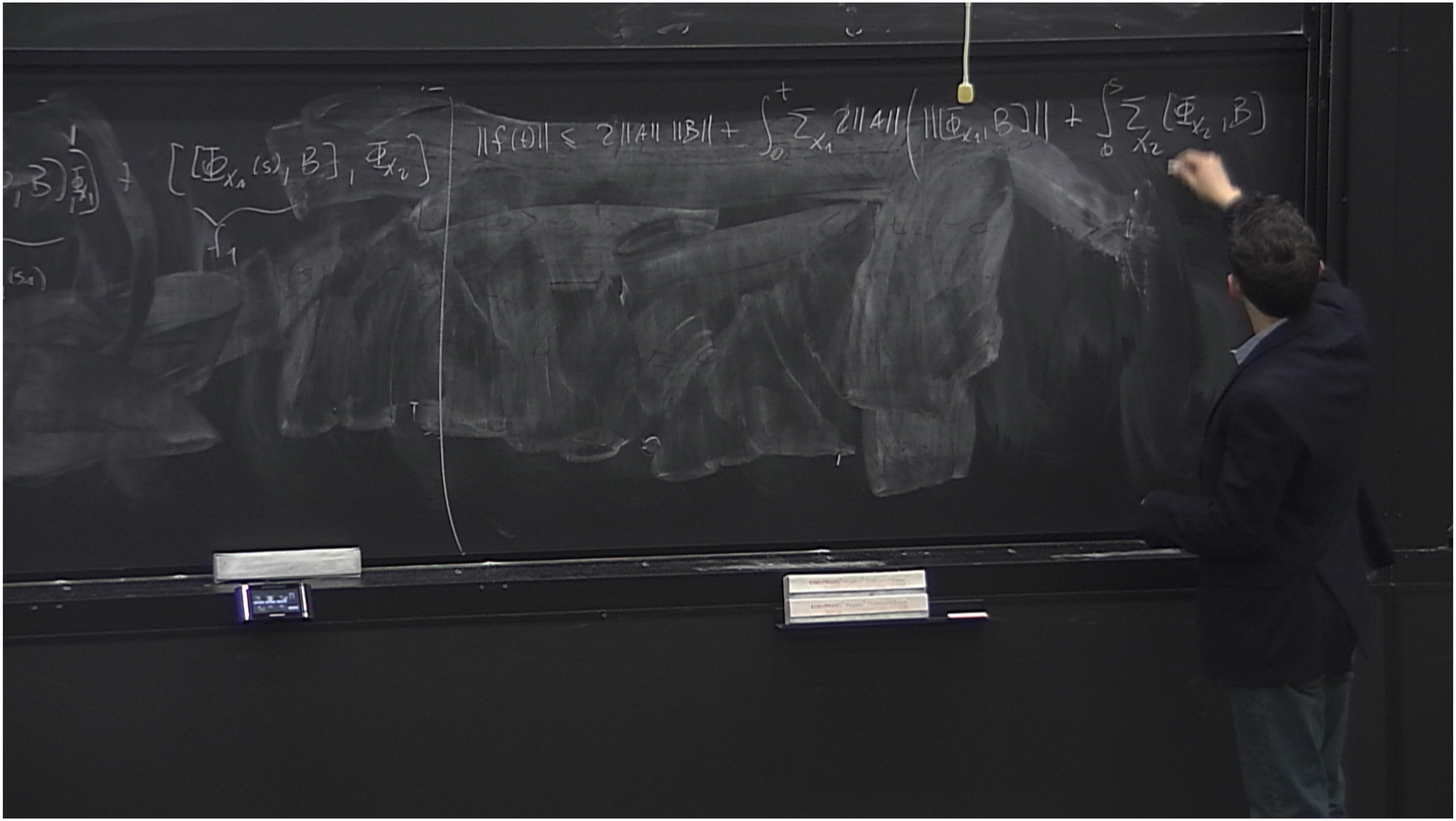

$\underbrace{[[\Phi^{(1)}_{X_1}, B], A(t)]}_{G(t)}$
$= \|f(0)\|$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| + \int_0^t ds\, \|G(s)\|$
$\leq 2\|A\|\|B\| + \int_0^t \sum_{X_1} \|[[\Phi^{(s)}_{X_1}, B], A(s)]\| ds$

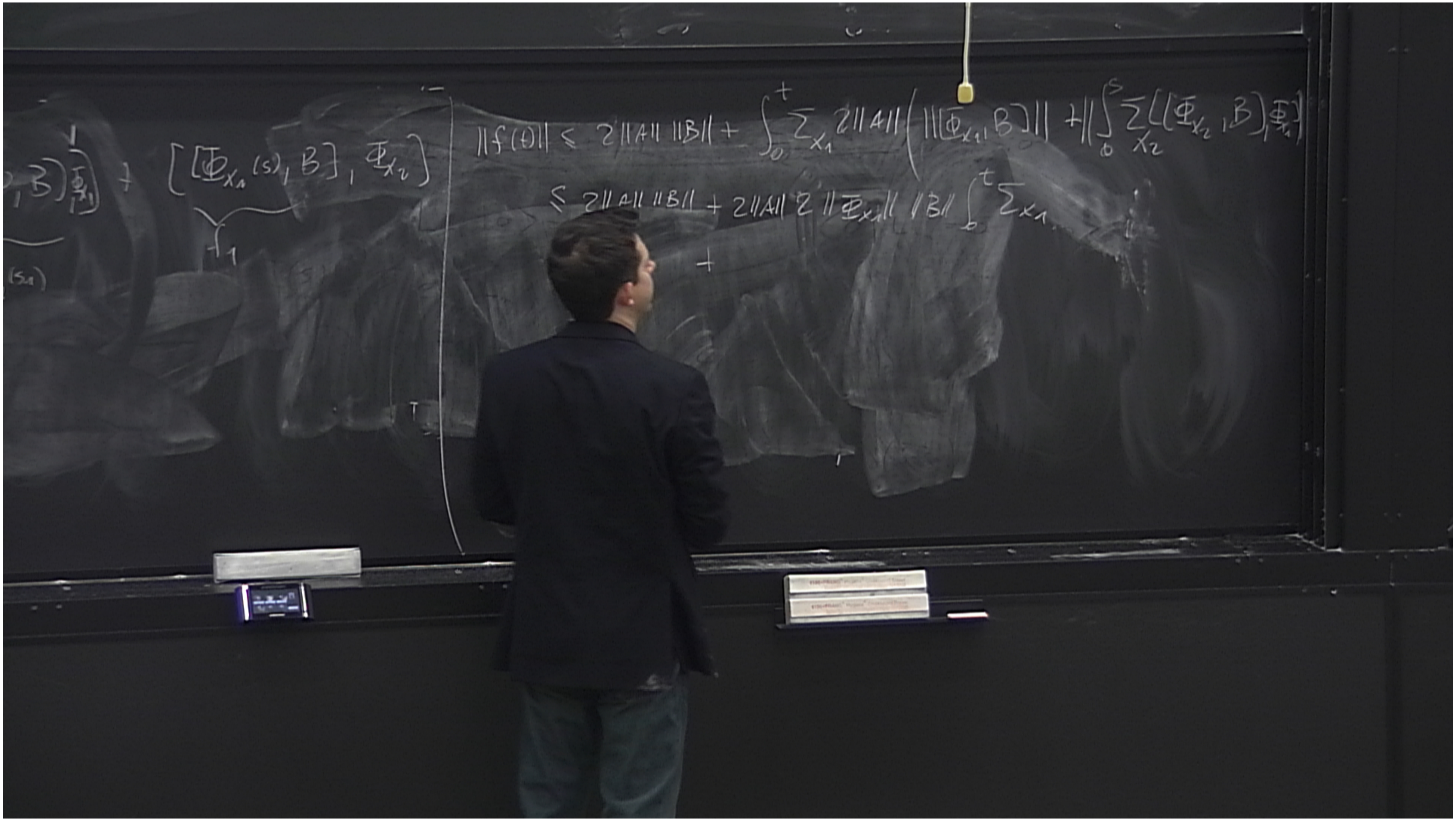

$[[\Phi_{X_1}^{(t)}, B], A(t)]$
$G(t)$
$f(t) = \gamma_t \left[f(0) + \int_0^t ds\, \gamma_s^{-1} G(s) \right]$
$\|f(t)\| \leq \|f(0)\| + \int_0^t ds\, \|G(s)\|$
$\leq 2\|A\|\|B\| + \int_0^t \sum_{X_1} \|[[\Phi_{X_1}^{(s)}, B], A(s)]\| ds$
$\leq 2\|A\|\|B\| + \int_0^t \sum_{X_1} 2\|A\| \|[\Phi_{X_1}(s), B]\| ds$
$= \|f(0)\|$


f₁ = [Φ_{X₁}(s), B]
f₁' = -i Σ_{X₂∈Z₂} [[Φ_{X₁}(s), Φ_{X₂}], B] = Σ_{X₂} [[Φ_{X₂}(s), B] + [Φ_{X₁}(s), B
Z₂ = { X₂ ⊂ Λ | X₂ ∩ X₁ ≠ φ, X₁ ∈ Z₁ }

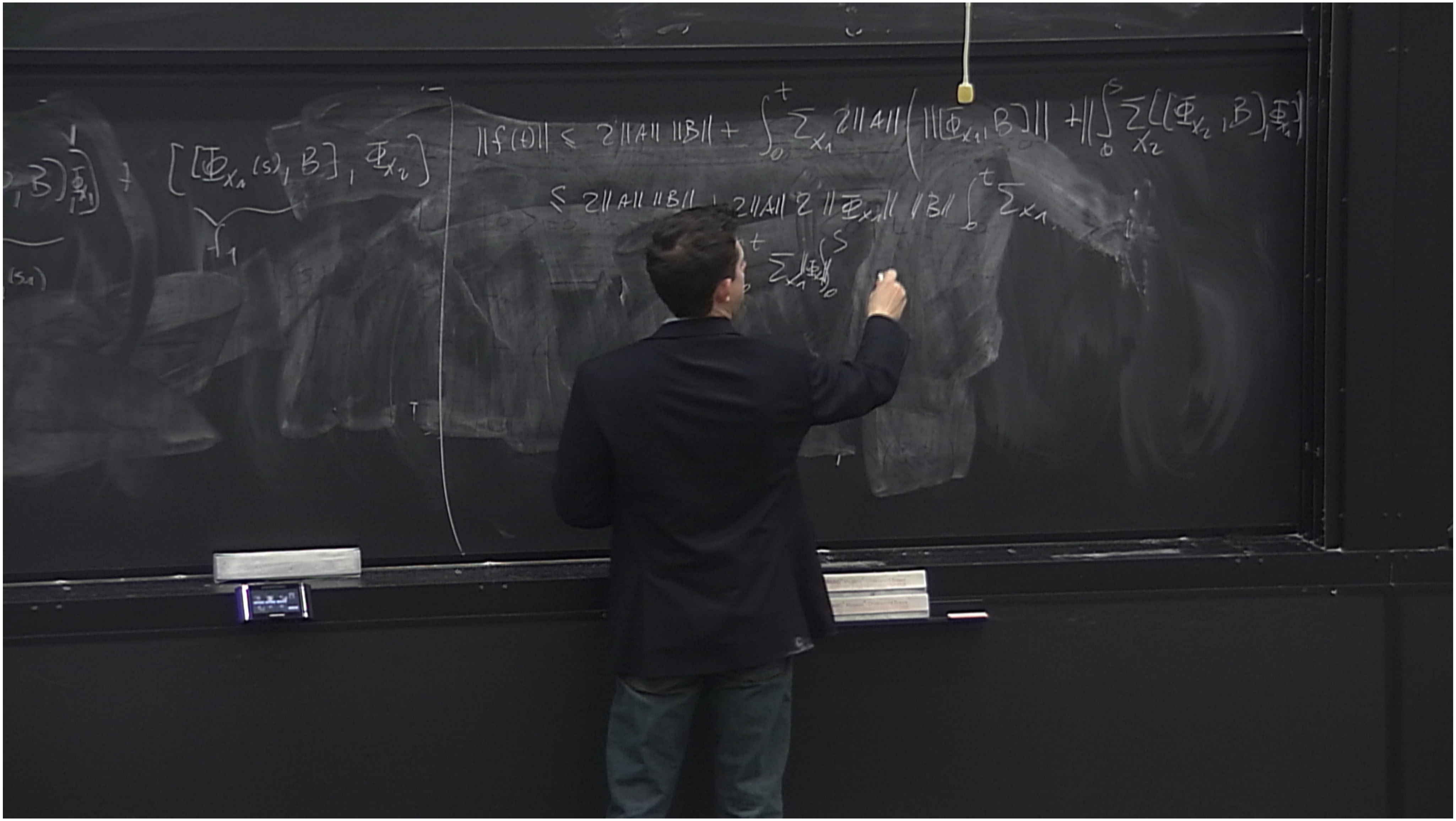

$f_1 = [\Phi_{X_1}(s), B]$
$f_1 = \sum_{X_2 \in Z_2} [[\Phi_{X_1}(s), \Phi_{X_2}], B] = \sum_{X_2} [[\Phi_{X_2}(s), B] \Phi_{X_1}] + [[\Phi_{X_1}(s), B], \Phi_{X_2}]$
$Z_2 = \{X_2 \subset \Lambda \mid X_2 \cap X_1 \neq \phi, X_1 \in Z_1\}$

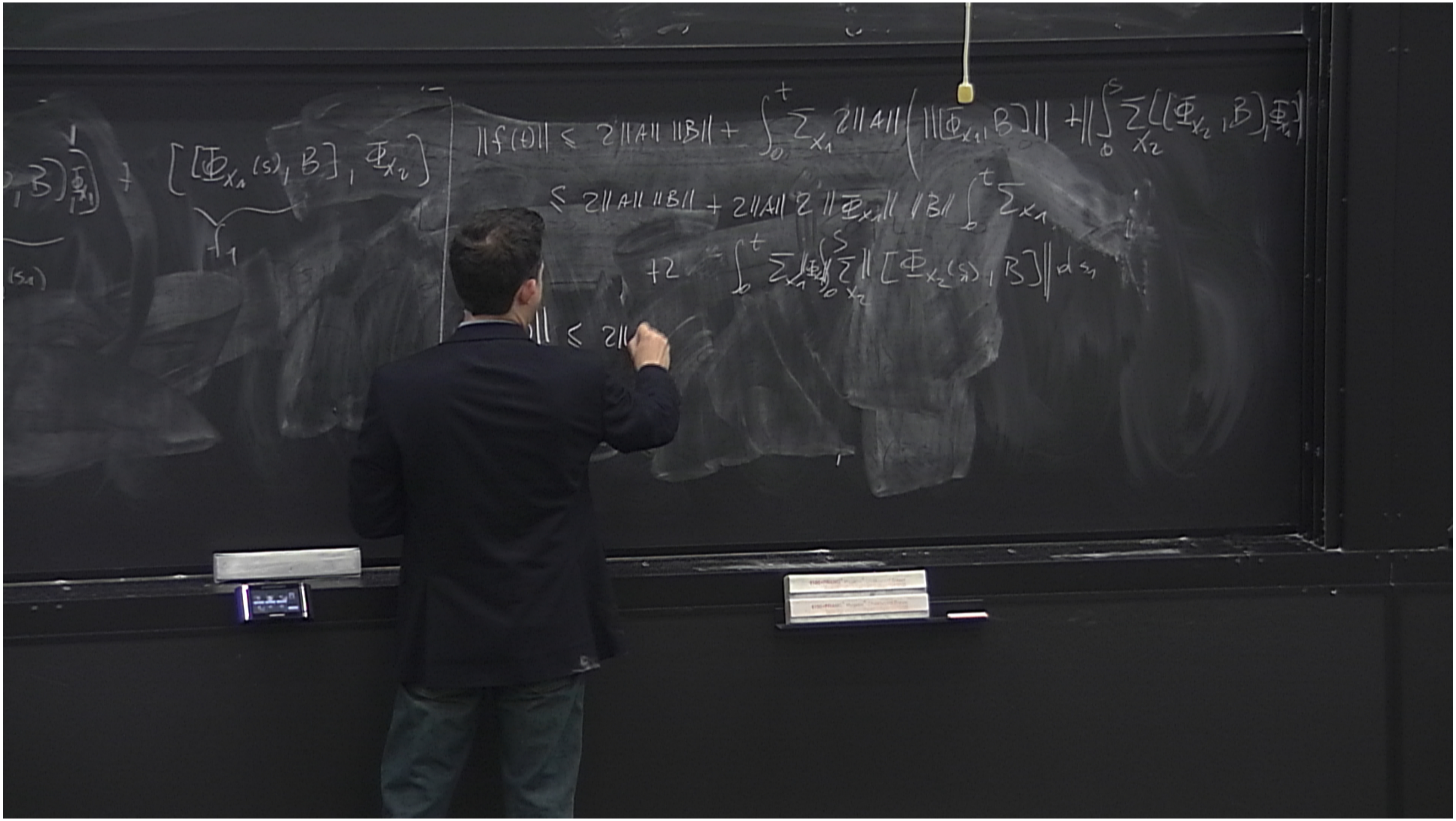

$f_1 = [\Phi_{X_1}(s), B]$
$f_1' = -i \sum_{X_2 \in Z_2} [[\Phi_{X_1}(s), \Phi_{X_2}], B] = \sum_{X_2} \left(\underbrace{[[\Phi_{X_2}(s), B], \Phi_{X_1}]}_{g_1(s_1)} + \underbrace{[[\Phi_{X_1}(s), B], \Phi_{X_2}]}_{f_1} \right)$
$Z_2 = \{ X_2 \subset \Lambda \mid X_2 \cap X_1 \neq \phi, X_1 \in Z_1 \}$
$f_1 = \gamma(s)\left[f_1(0) + \int_0^s \gamma_{s_1}^{-1} g_1(s_1) \, ds_1 \right]$
$\|f$

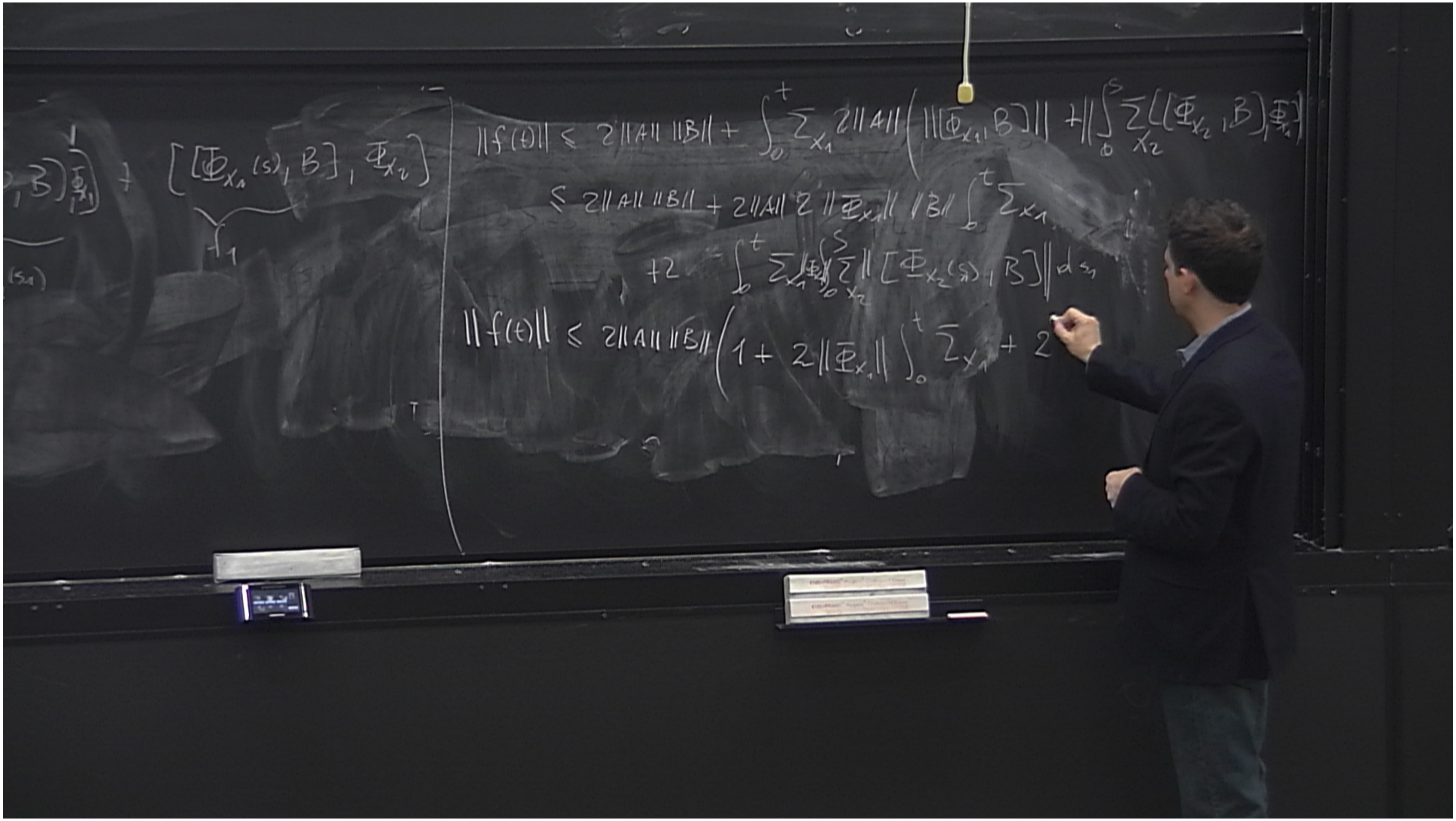

$[[\Phi_{X_1}(s), B], \Phi_{X_2}]$
$\|f(t)\| \leq 2\|A\|\,\|B\| + \int_0^t \sum_{X_1} 2\|A\| \left(\|[\Phi_{X_1}, B]\| + \int_0^s \sum_{X_2} [\Phi_{X_2}, B] \right.$



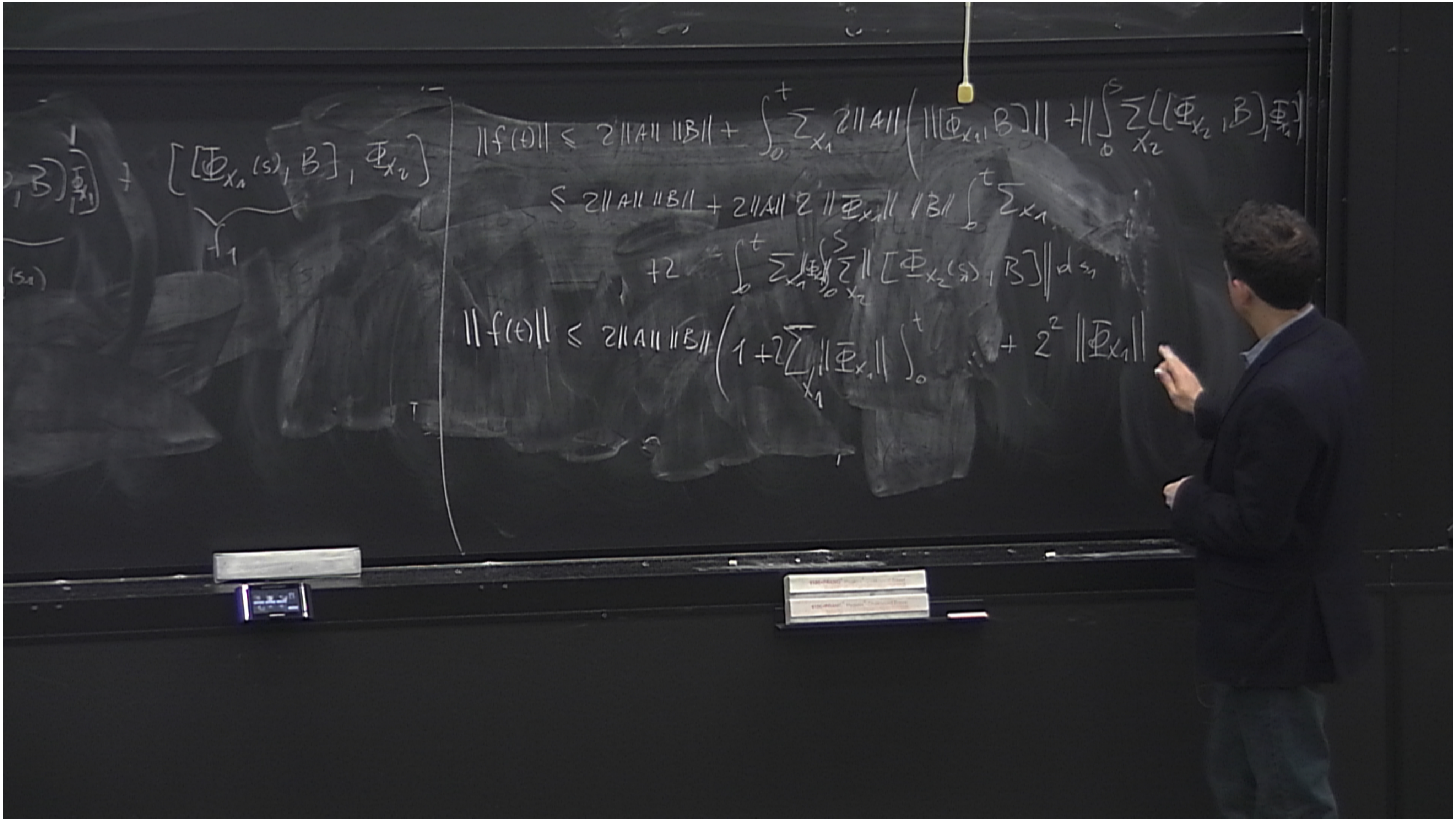
[[Φx1(s), B], Φx2]
||f(t)|| ≤ 2||A|| ||B|| + ∫0t Σx1 2||A|| (||[Φx1, B]|| + ||∫0s Σx2 [[Φx2, B], Φx1]
≤ 2||A|| ||B|| + 2||A|| Σ ||Φx1|| ||B|| ∫0t Σx1


||f(t)|| ≤ 2||A|| ||B|| +


$[[\Phi_{X_1}(s), B], \Phi_{X_2}]$
$\|f(t)\| \leq 2\|A\|\,\|B\| + \int_0^t \sum_{X_1} 2\|A\| \left(\|[\Phi_{X_1}, B]\| + \|\int_0^s \sum_{X_2} [[\Phi_{X_2}, B], \Phi_{X_1}]\right.$
$\leq 2\|A\|\,\|B\| + 2\|A\| \sum \|\Phi_{X_1}\|\,\|B\| \int_0^t \sum_{X_1}$
$+2 \int_0^t \sum_{X_1} \|\Phi_{X_1}\| \int_0^s \sum_{X_2} \|[\Phi_{X_2}(s), B]\| \, ds$



||f(t)|| ≤ 2||A|| ||B|| + ∫_0^t Σ_{X_1} 2||A|| (||[Φ_{X_1}, B]|| + ||∫_0^s Σ_{X_2} [[Φ_{X_2}, B], Φ_{X_1}]
≤ 2||A|| ||B|| + 2||A|| 2 ||Φ_{X_1}|| ||B|| ∫_0^t Σ_{X_1}
+2 ∫_0^t Σ_{X_1} ||Φ|| ∫_0^s Σ_{X_2} ||[Φ_{X_2}(s), B]|| ds
||f(t)|| ≤ 2||A|| ||B|| (1 + 2||Φ_{X_1}|| ∫_0^t Σ_{X_1} + 2
[[Φ_{X_1}(s), B], Φ_{X_2}]


||f(t)|| ≤ 2||A|| ||B|| + ∫_0^t Σ_{X_1} 2||A|| (||[Φ_{X_1}, B]|| + ||∫_0^s Σ_{X_2} [[Φ_{X_2}, B], Φ_{X_1}]||
≤ 2||A|| ||B|| + 2||A|| Σ ||Φ_{X_1}|| ||B|| ∫_0^t Σ_{X_1}
+2 ∫_0^t Σ_{X_1} ∫_0^s Σ_{X_2} ||[Φ_{X_2}(s_1), B]|| ds_1
||f(t)|| ≤ 2||A|| ||B|| (1 + 2 Σ_{X_1} ||Φ_{X_1}|| ∫_0^t + 2^2 ||Φ_{X_1}||
[[Φ_{X_1}(s), B], Φ_{X_2}]

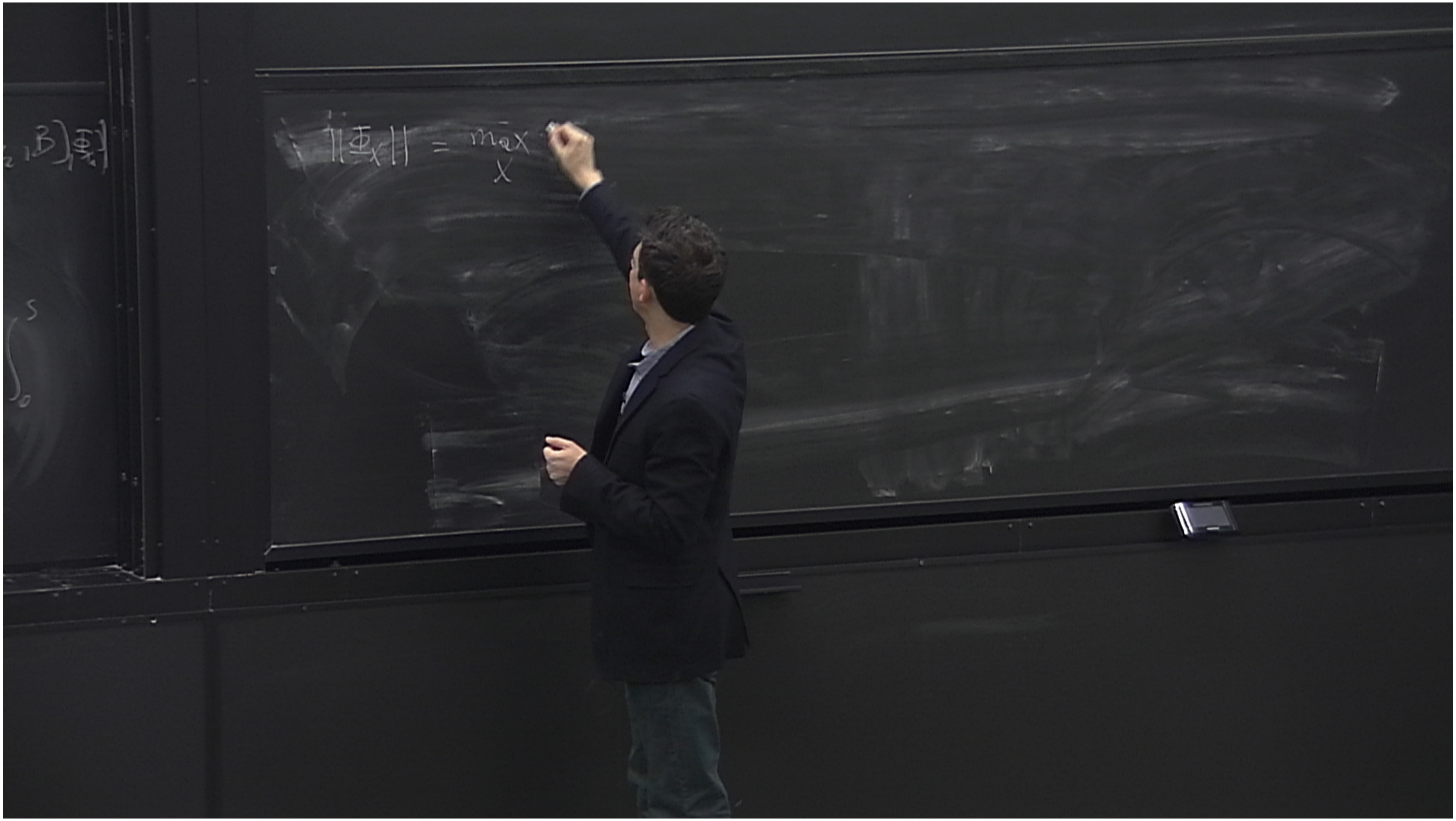
$$[[\Phi_{X_1}(s), B], \Phi_{X_2}]$$

$$\|f(t)\| \leq 2\|A\|\,\|B\| + \int_0^t \sum_{X_1} 2\|A\| \left(\|[\Phi_{X_1}, B]\| + \|\int_0^s \sum_{X_2} [[\Phi_{X_2}, B], \Phi_{X_1}]\right.$$

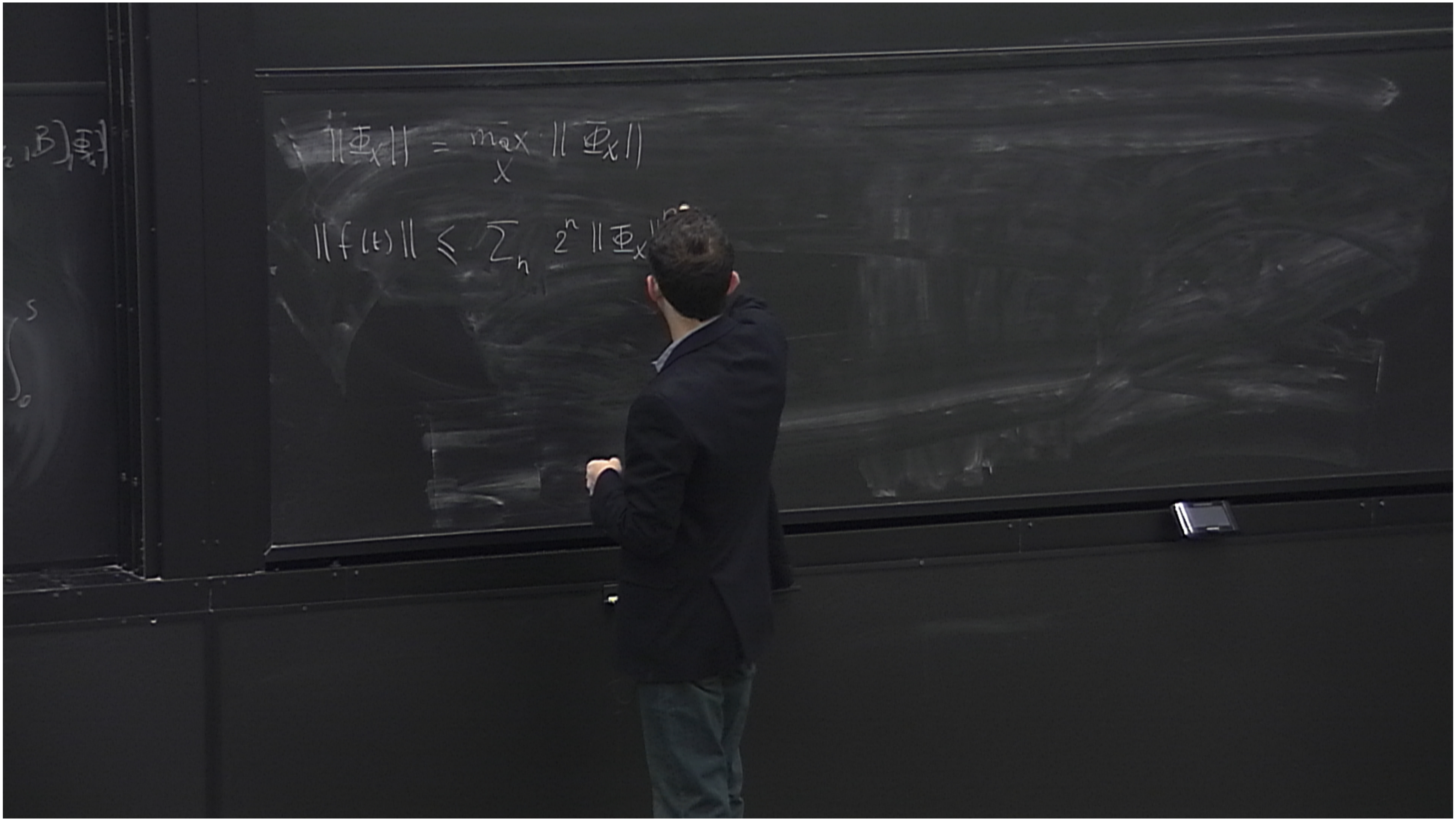
$$\leq 2\|A\|\,\|B\| + 2\|A\|\, 2\, \|\Phi_{X_1}\|\, \|B\| \int_0^t \sum_{X_1}$$

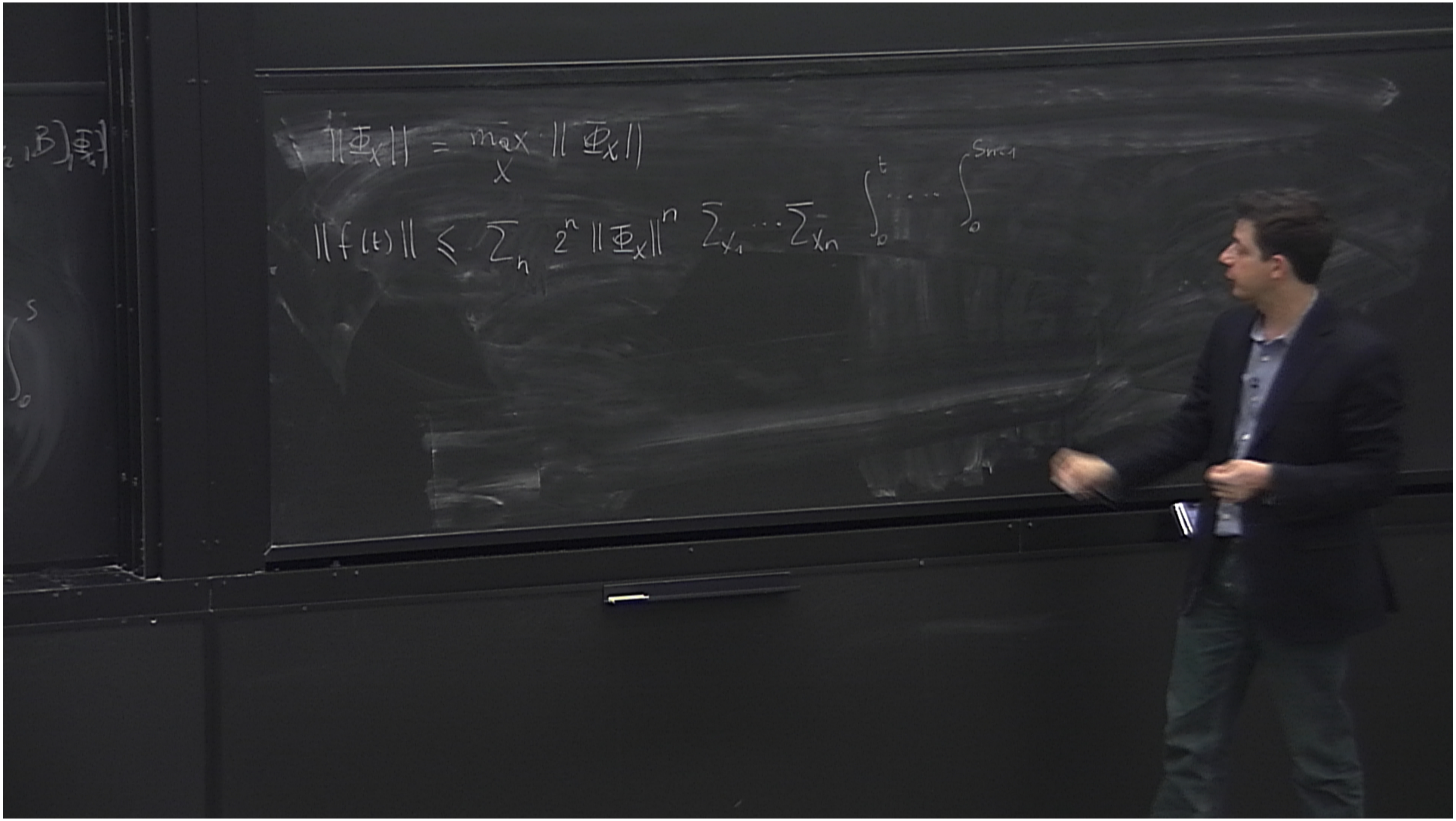
$$+2 \int_0^t \sum_{X_1} \|\Phi_{X_1}\| \int_0^s \sum_{X_2} \|[\Phi_{X_2}(s), B]\|\, ds$$

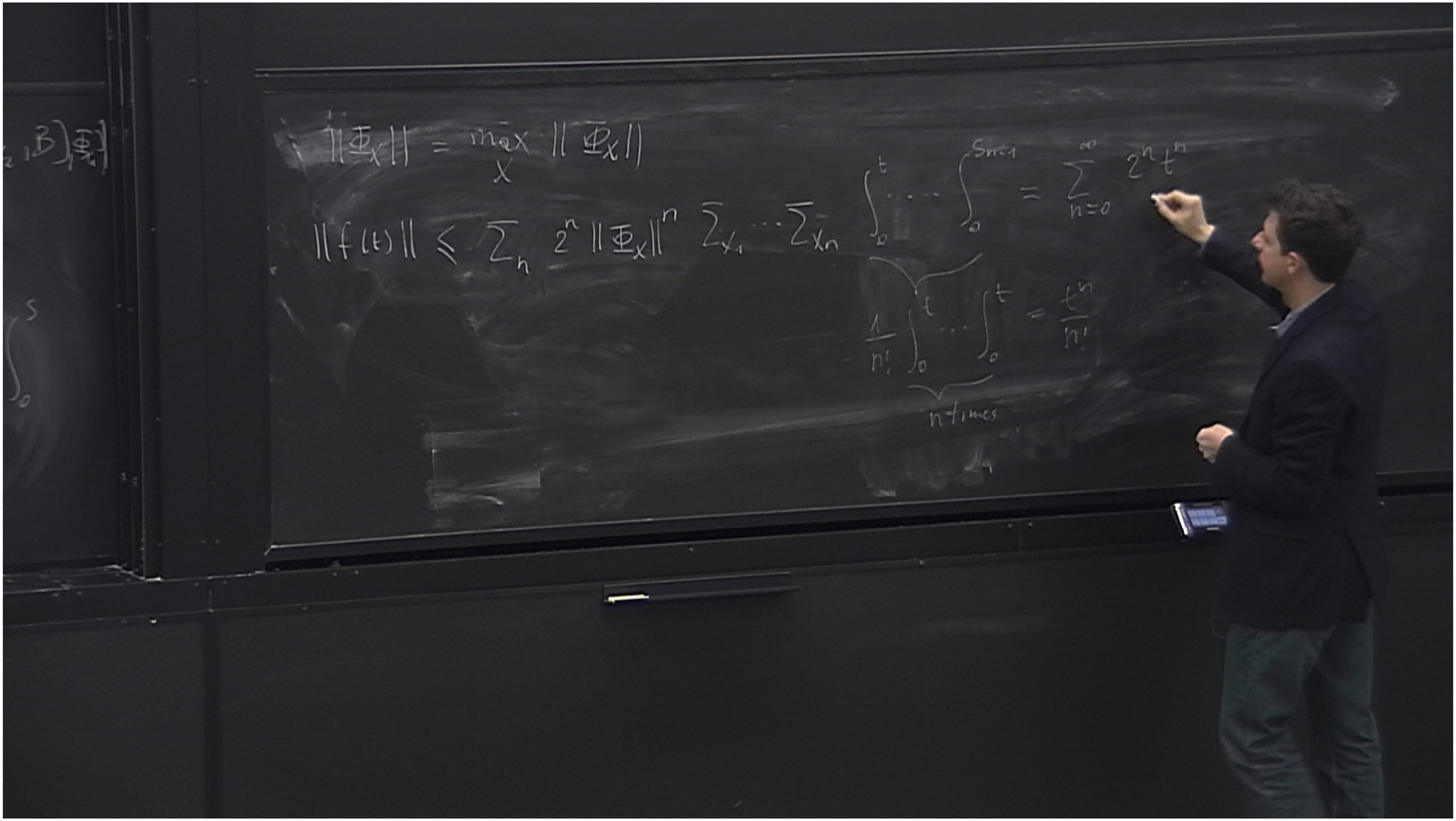
$$\|f(t)\| \leq 2\|A\|\,\|B\| \left(1 + 2\sum_{X_1} \|\Phi_{X_1}\| \int_0^t + \sum_{X_1}\sum_{X_2} 2^2\, \|\Phi_{X_1}\|\, \|\Phi_{X_2}\| \int_0^t \int_0^s\right.$$

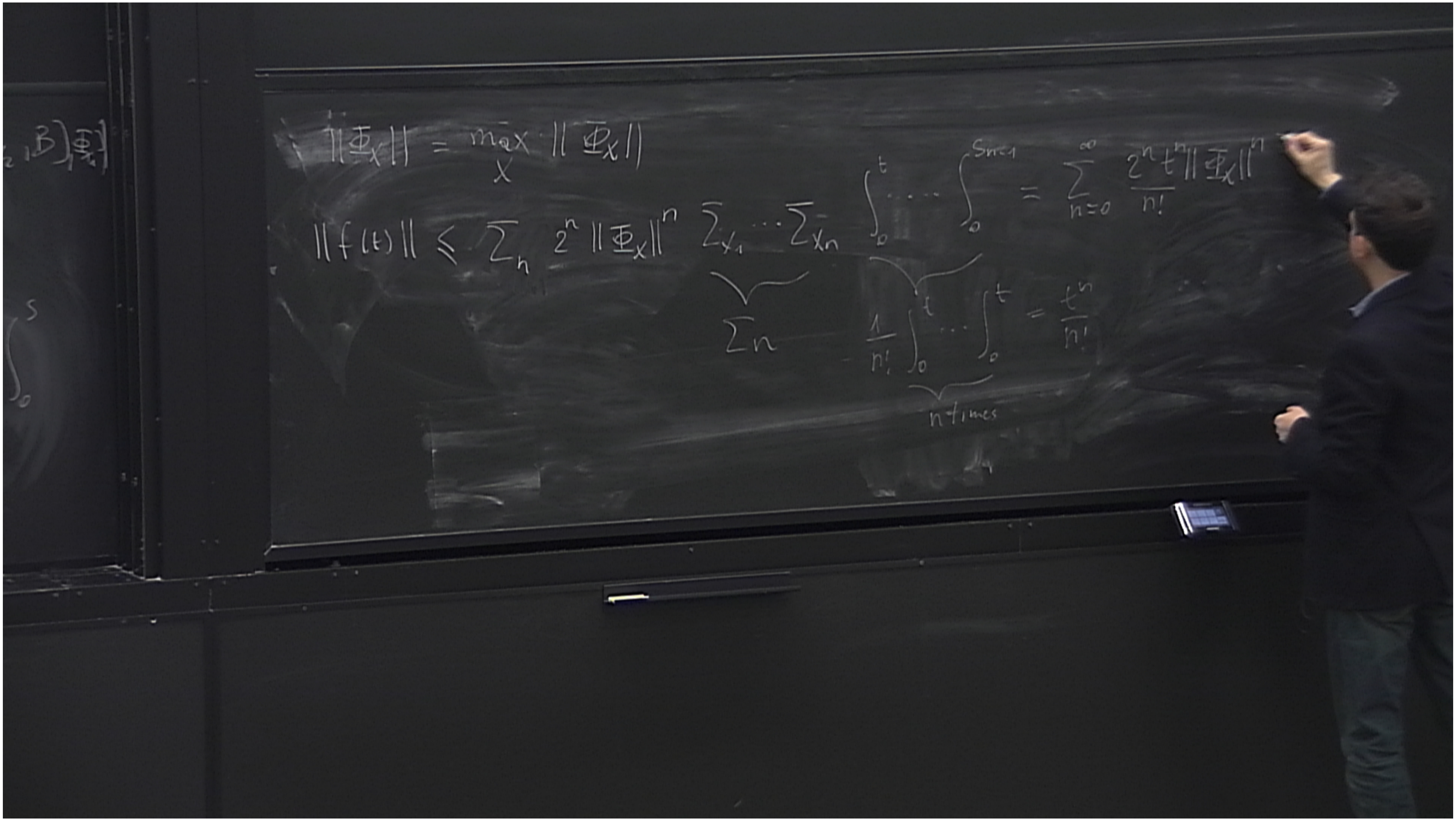


$\|\Phi_X\| = \max_X$


$\|\Phi_x\| = \max_X \|\Phi_X\|$
$\|f(t)\| \leq \sum_n 2^n \|\Phi_x\|$

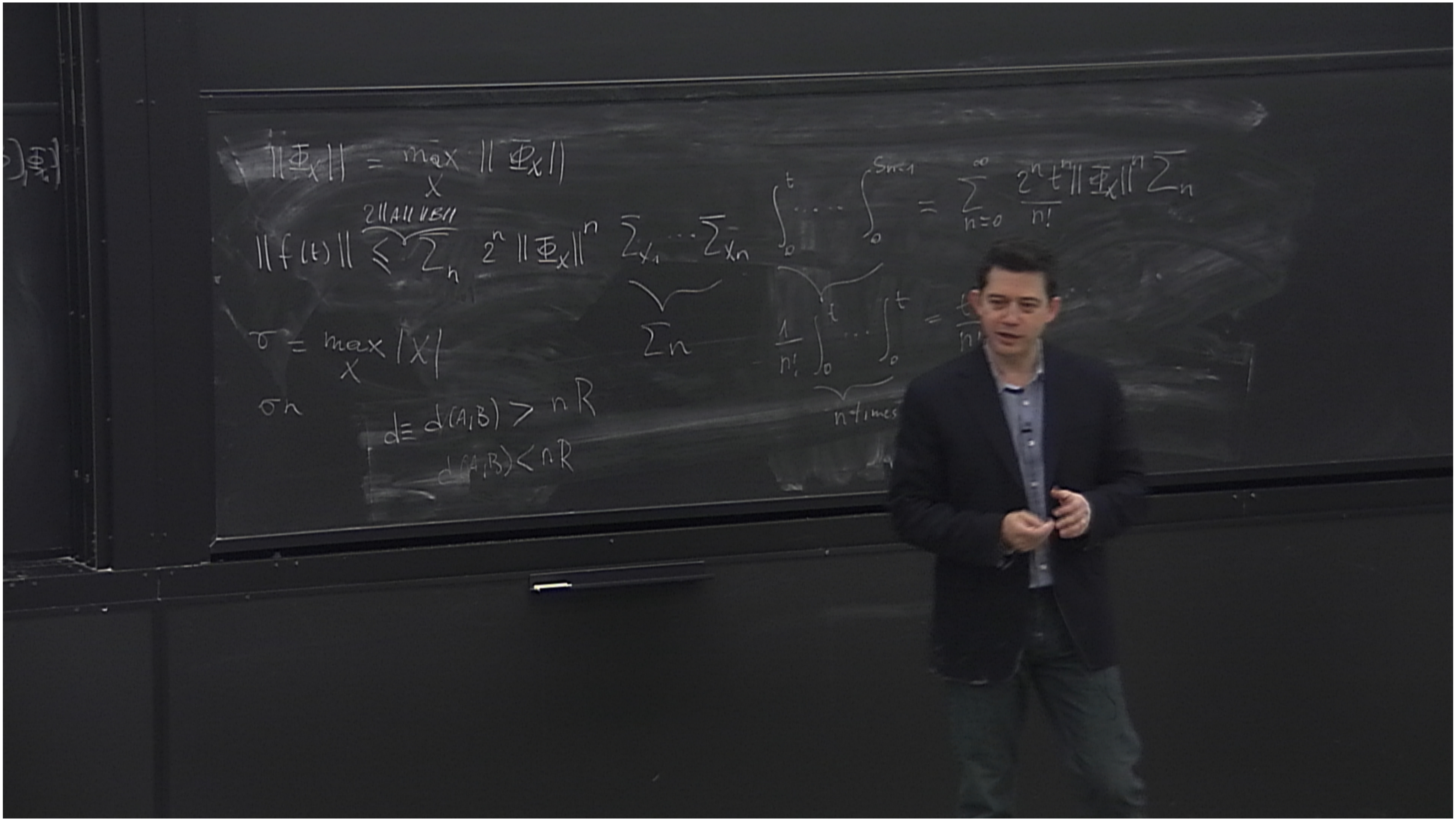

$\|\Phi_X\| = \max_X \|\Phi_X\|$
$\|f(t)\| \leq \sum_n 2^n \|\Phi_X\|^n \sum_{X_1} \cdots \sum_{X_n} \int_0^t \cdots \int_0^{s_{n-1}}$

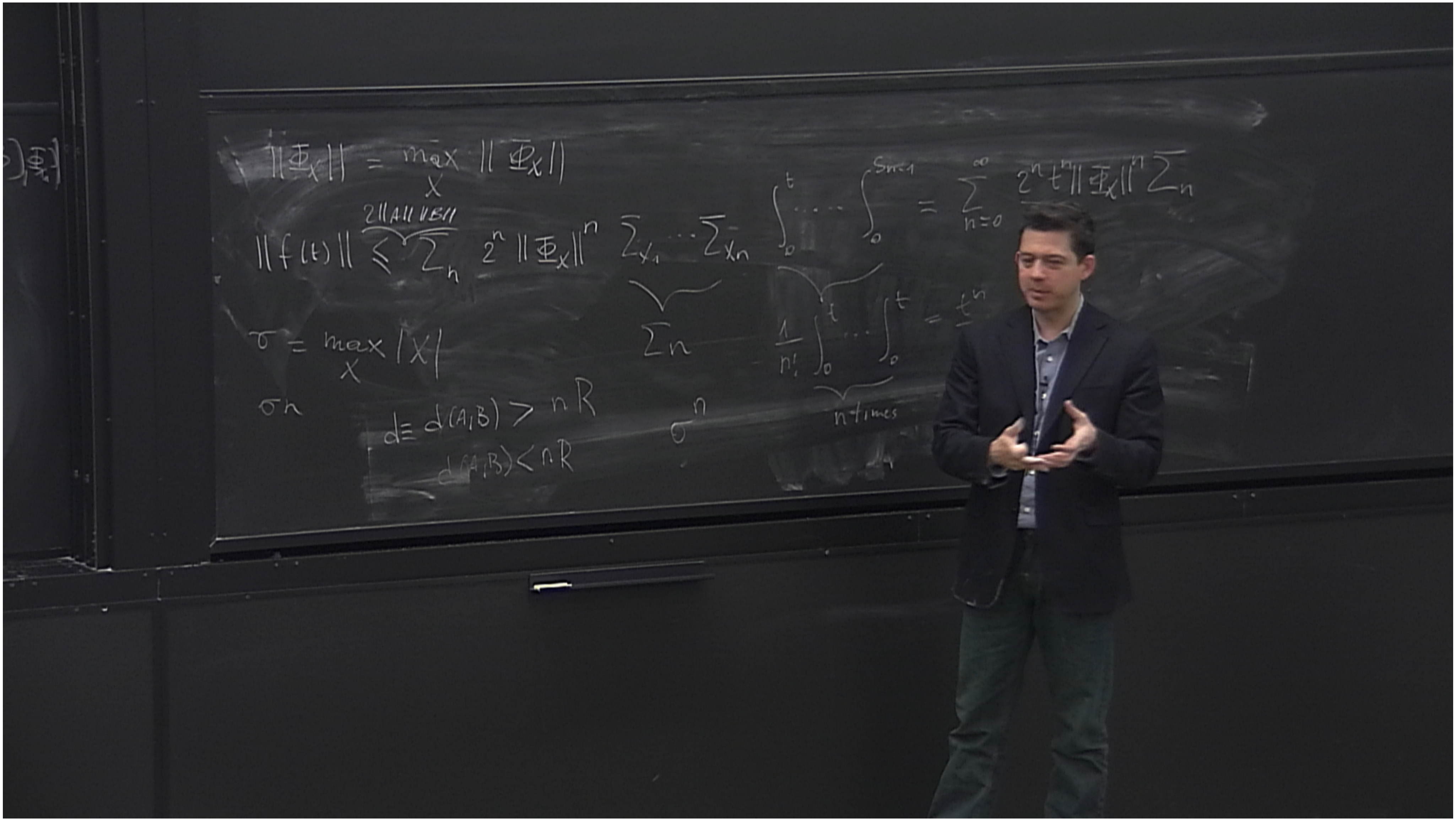

$\|\Phi_X\| = \max_X \|\Phi_X\|$
$\|f(t)\| \leq \sum_n 2^n \|\Phi_X\|^n \sum_{X_1} \cdots \sum_{X_n} \int_0^t \cdots \int_0^{s_{n-1}} = \sum_{n=0}^{\infty} 2^n t^n$
$\frac{1}{n!} \underbrace{\int_0^t \cdots \int_0^t}_{n \text{ times}} = \frac{t^n}{n!}$

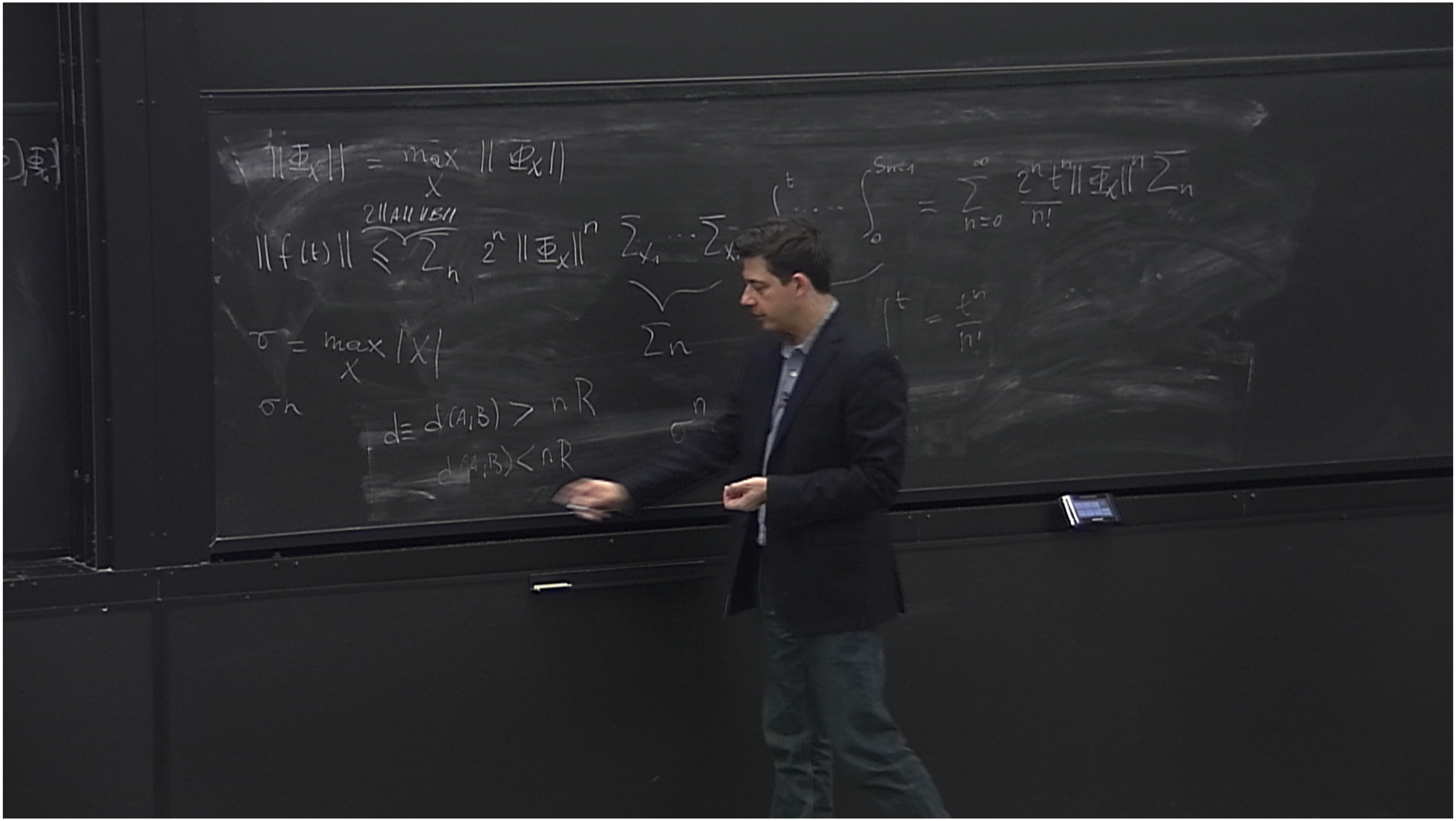


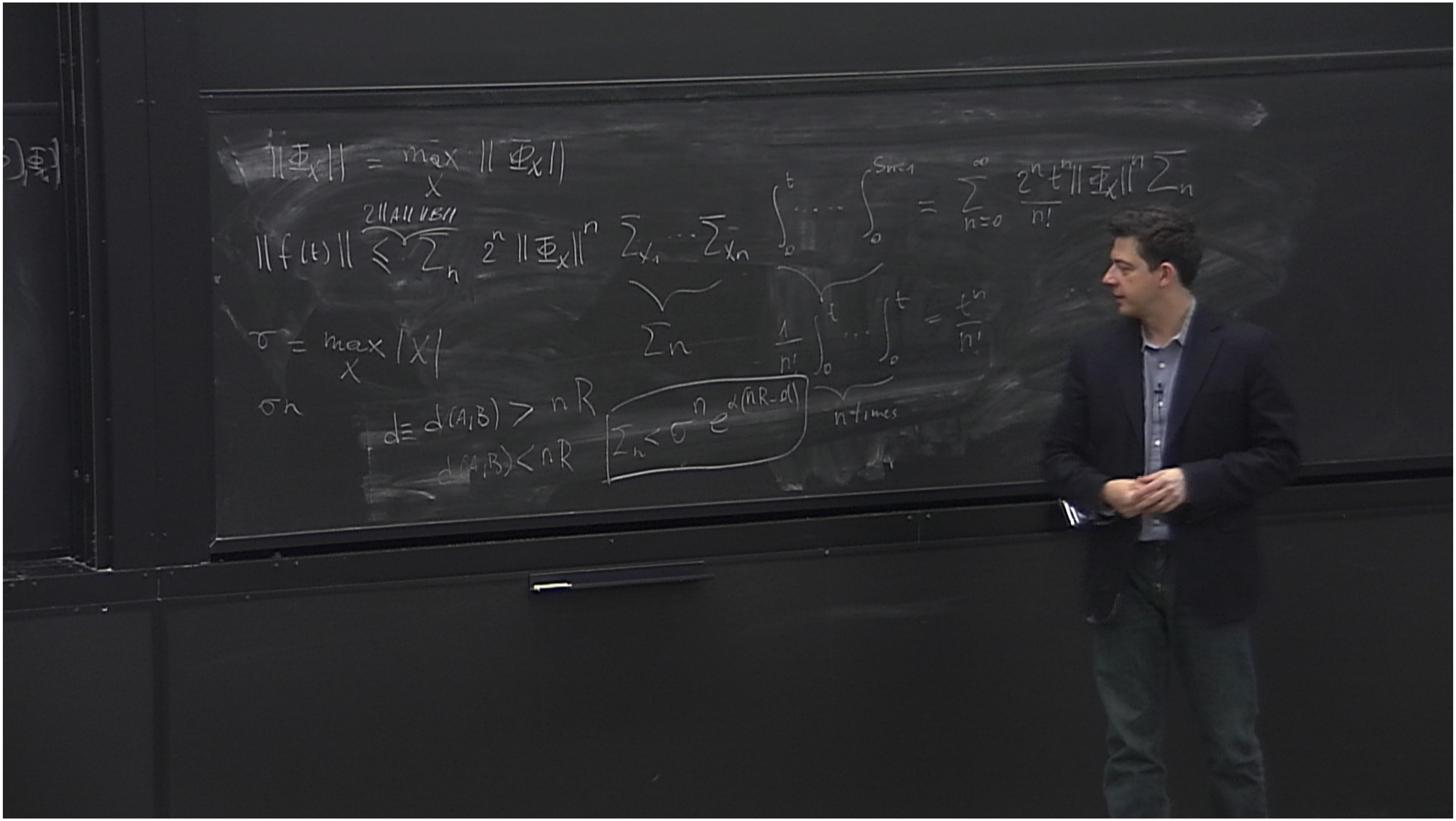
$\|\Phi_x\| = \max_X \|\Phi_X\|$
$\|f(t)\| \leq \sum_n 2^n \|\Phi_x\|^n \underbrace{\sum_{X_1} \cdots \sum_{X_n}}_{\sum_n} \int_0^t \cdots \int_0^{s_{n-1}} = \sum_{n=0}^{\infty} \frac{2^n t^n}{n!} \|\Phi_x\|^n$
$\frac{1}{n!} \underbrace{\int_0^t \cdots \int_0^t}_{n \text{ times}} = \frac{t^n}{n!}$

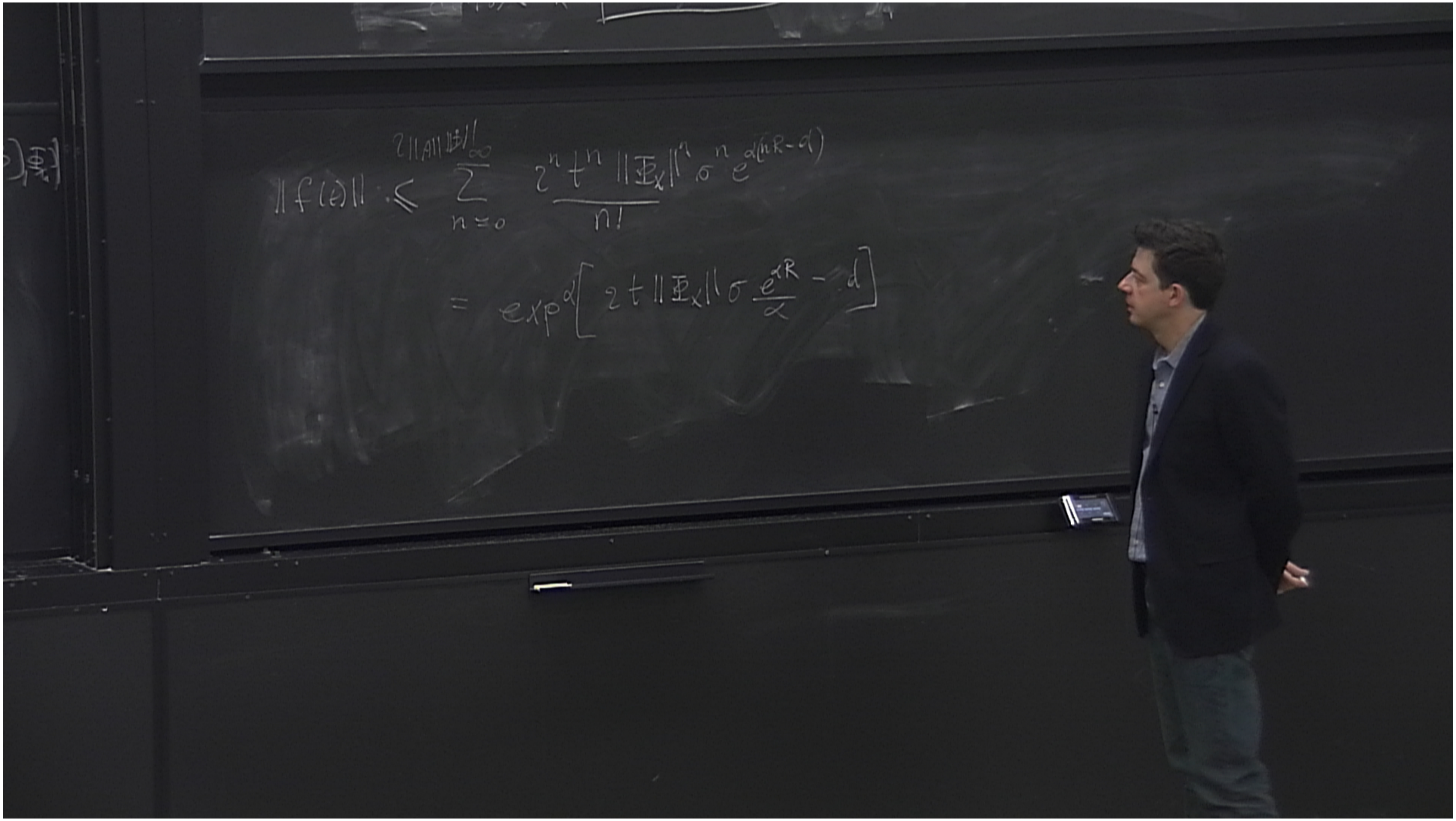

n times

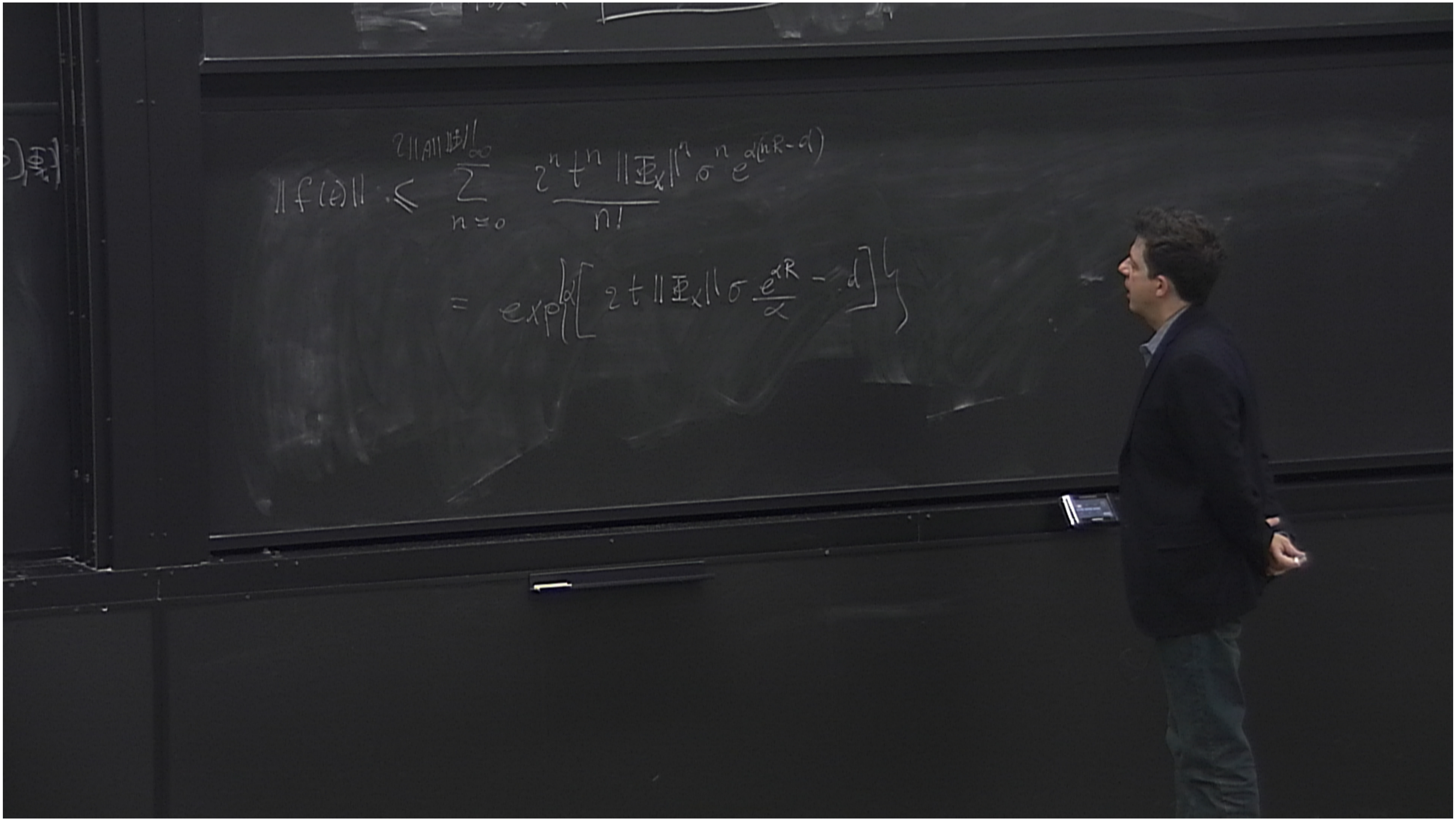

d(A,B) > nR
d(A,B) < nR
n-times


||Φ_X|| = max_X ||Φ_X||
||f(t)|| ≤ Σ_n 2^n ||Φ_X||^n Σ_{X_1} … Σ_{X_n}
2||A|| ||B||
Σn
σ = max_X |X|
σn
d ≡ d(A,B) > nR
d(A,B) < nR
σ^n
1/n!
n times
= t^n
= Σ_{n=0}^∞ 2^n t^n ||Φ_X||^n Σ_n

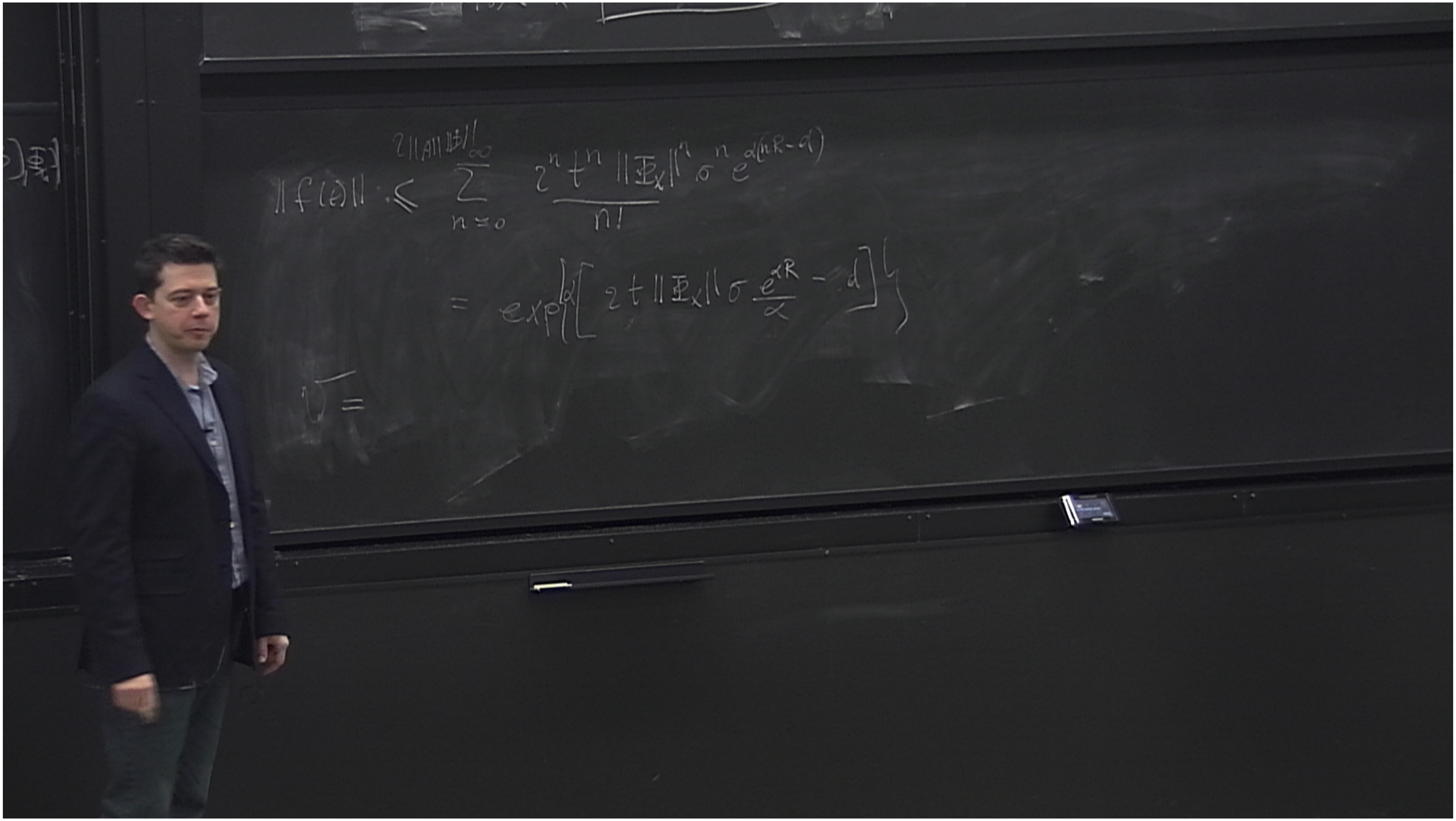

||Φ_X|| = max_X ||Φ_X||
||f(t)|| ≤ Σ_n 2^n ||Φ_X||^n Σ_{X_1} ... Σ_{X_n}
2||A|| ||B||
Σ_n
σ = max_X |X|
σ^n
d ≡ d(A,B) > nR
d(A,B) < nR
= Σ_{n=0}^∞ 2^n t^n/n! ||Φ_X||^n Σ_n
= t^n/n!

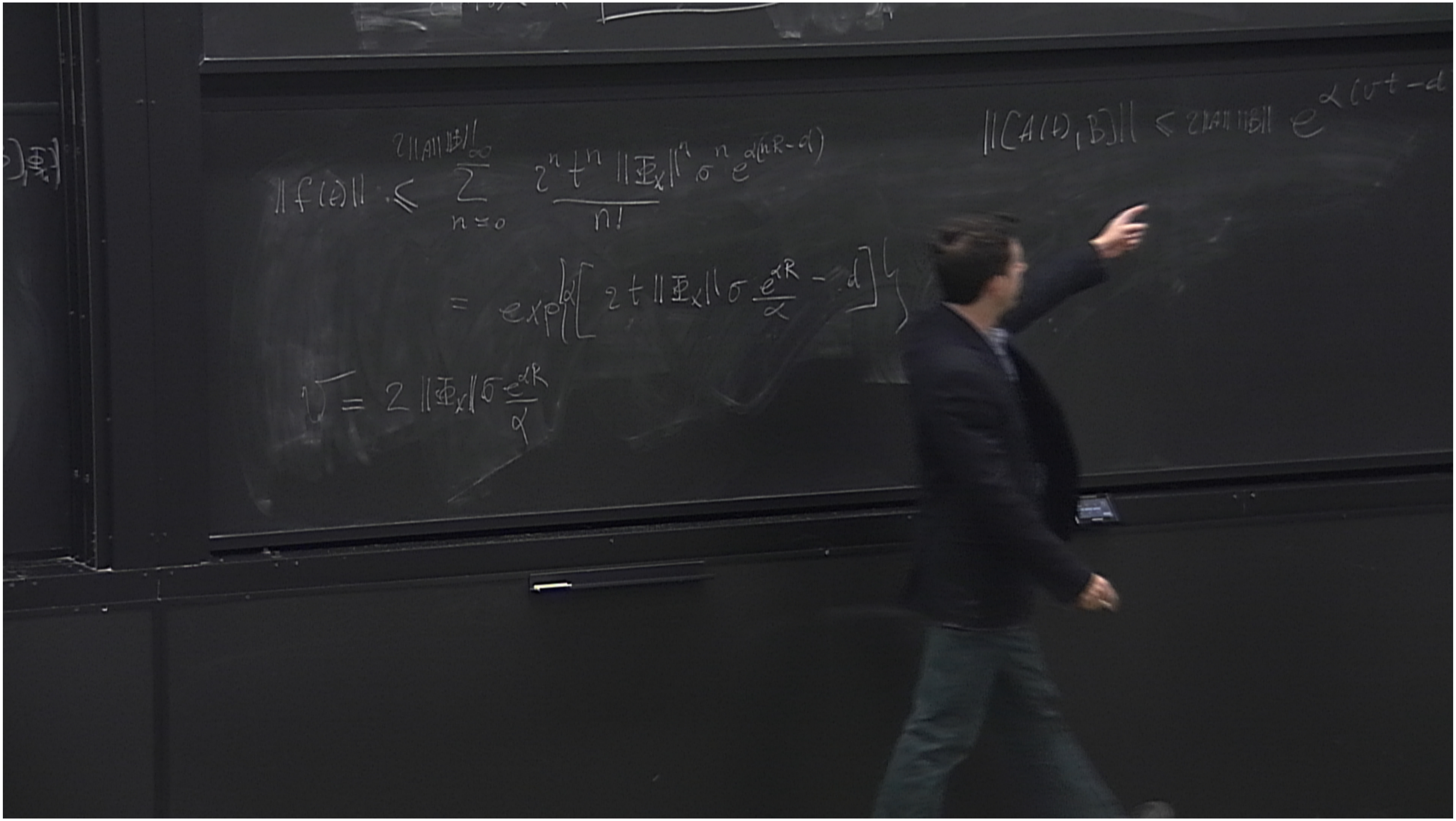

n times


$\|f(t)\| \leqslant \sum_{n=0}^{\infty} \frac{2^n t^n \|\Phi_x\|^n \sigma^n e^{\alpha(nR-d)}}{n!}$
$= \exp \alpha\left[2t\|\Phi_x\|\sigma \frac{e^{\alpha R}}{\alpha} - d\right]$



$\|f(t)\| \leqslant \sum_{n=0}^{\infty} \frac{2^n t^n \|\Phi_x\|^n \sigma^n e^{\alpha(nR-d)}}{n!}$
$2\|A\|\|\Phi\|$
$= \exp\{\alpha[2t\|\Phi_x\|\sigma \frac{e^{\alpha R}}{\alpha} - d]\}$


$2\|A\|\,\|b\|$
$\|f(t)\| \leq \sum_{n=0}^{\infty} \frac{2^n t^n \|\Phi_x\|^n \sigma^n e^{\alpha(nR-d)}}{n!}$
$= \exp\left\{\alpha\left[2t\|\Phi_x\|\sigma \frac{e^{\alpha R}}{\alpha} - d\right]\right\}$
$v =$


$\|[A(t),B]\| \leq 2\|A\|\|B\| e^{\alpha(vt-d)}$
$\|f(t)\| \leq 2\|A\|\|B\| \sum_{n=0}^{\infty} \frac{2^n t^n \|\Phi_x\|^n \sigma^n e^{\alpha(nR-d)}}{n!}$
$= \exp\left\{\alpha\left[2t\|\Phi_x\|\sigma \frac{e^{\alpha R}}{\alpha} - d\right]\right\}$
$v = 2\|\Phi_x\|\sigma \frac{e^{\alpha R}}{\alpha}$