

Title: 13/14 PSI - String Theory Review - Lecture 14

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Abstract:

SUPER WORLD SHEET $\longrightarrow$ SPACE TIME
 $\Downarrow$ GAUGE FIXING
 SUPER-CONFORMAL THEORY

$$T, \bar{T}, \quad G, \bar{G}$$

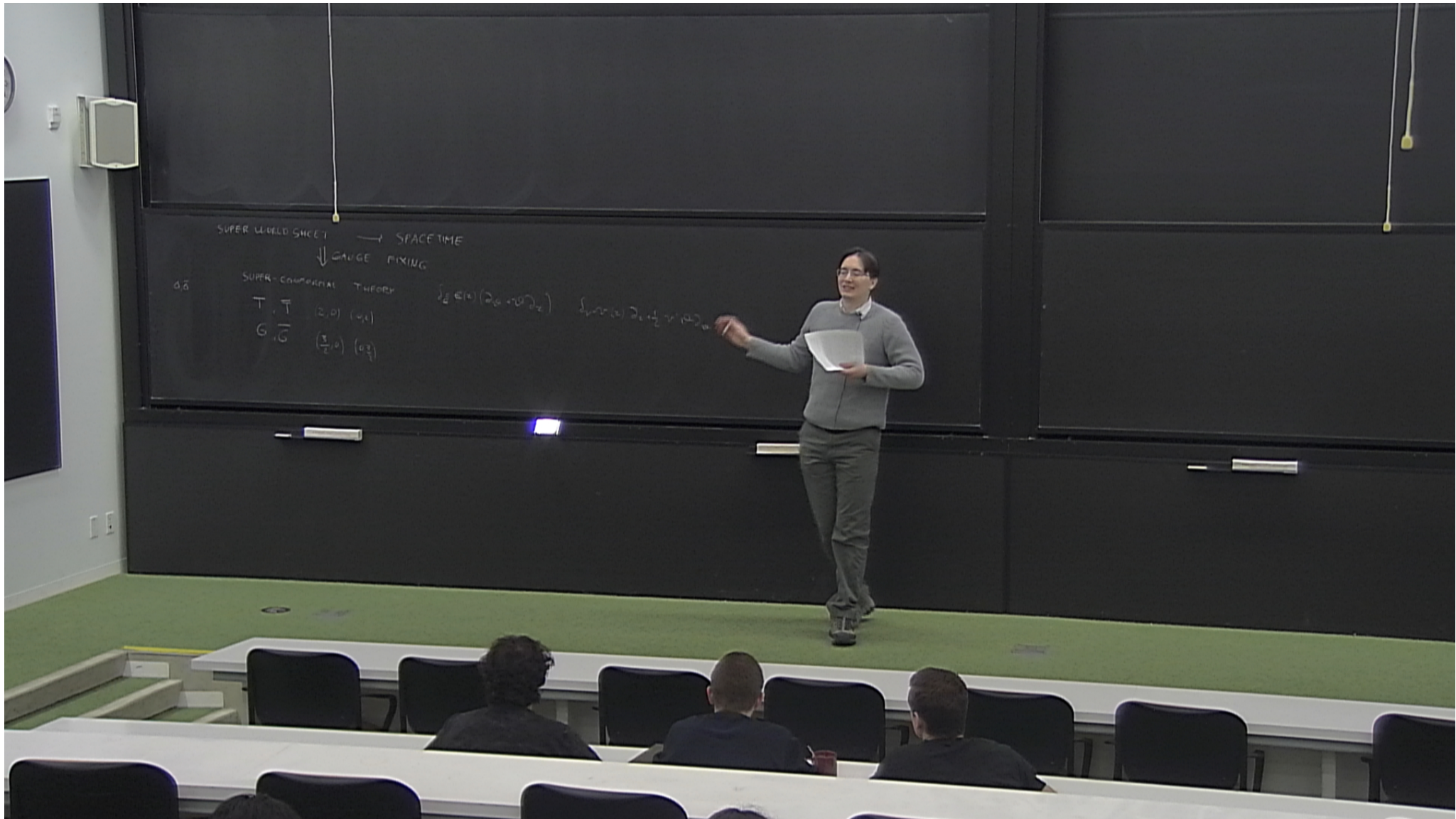
$$\delta_{\xi} \epsilon(z) (\partial_z + v \partial_{\bar{z}}) \quad \delta_v v(z) \partial_z + \frac{1}{2} v' \partial_{\bar{z}}$$

SUPER WORLD SHEET $\longrightarrow$ SPACE TIME
 $\Downarrow$ GAUGE FIXING
 SUPER-CONFORMAL THEORY

$$\delta_{\xi} \epsilon(z) (\partial_x + \psi \partial_z)$$

$$\delta_{\psi} \psi(z) \partial_z + \frac{1}{2} \psi' \psi \partial_z$$

$T, \bar{T}$
 $G, \bar{G}$



SUPER WORLD SHEET $\longrightarrow$ SPACE TIME

$\Downarrow$ GAUGE FIXING

$\delta, \bar{\delta}$

SUPER-CONFORMAL THEORY

$T, \bar{T} \quad (2, 0) \quad (0, 2)$

$G, \bar{G} \quad (\frac{3}{2}, 0) \quad (0, \frac{3}{2})$

$$\int \frac{1}{2\pi} \int d^2z \, E(z) (\partial_{\bar{z}} + v \partial_z)$$

$$\int v = v(z) \partial_z + \frac{1}{2} v'(z) \partial_{\bar{z}}$$

FREE SCALAR + FREE FERMION , BOTH MASSLESS

$$=ss \quad \gamma(z, \bar{z}, \theta, \bar{\theta}) = X(z, \bar{z}) + \gamma \psi + \bar{\gamma} \bar{\psi} + \gamma \bar{\gamma} F$$

BOTH MASSLESS

$$Y(z, \bar{z}, \psi, \bar{\psi}) = X(z, z) + \psi \psi + \bar{\psi} \bar{\psi} + \psi \bar{\psi} F$$

$$\uparrow$$

$$\psi(z) = \left(\frac{dz'}{dz}\right)^{\frac{1}{2}} \psi(z')$$

$$\partial X(z) = \frac{dz'}{dz} \partial X(z')$$

$$\partial \bar{\partial} X = 0$$

$$\bar{\partial} \psi = 0$$

$$\partial \bar{\psi} = 0$$

$$F = 0$$

BOTH MASSLESS

$$Y(z, \bar{z}, \psi, \bar{\psi}) = X(z, \bar{z}) + \psi \psi + \bar{\psi} \bar{\psi} + \psi \bar{\psi} F$$

$$S = \int \frac{1}{2} \partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} + F^2$$

$$\psi(z) = \left(\frac{dz'}{dz} \right)^{\frac{1}{2}} \psi(z')$$

$$\partial X(z) = \frac{dz'}{dz} \partial X(z')$$

$$\bar{\partial} \bar{X} = 0 \quad \bar{\partial} \psi = 0 \quad \partial \bar{\psi} = 0 \quad F = 0$$

FREE SCALAR, + FREE FERMION, BOTH MASSLESS

$$\Psi(z, \bar{z}, \theta, \bar{\theta}) = X(z, \bar{z}) + \theta \psi(z, \bar{z}) + \bar{\theta} \bar{\psi}(z, \bar{z}) + \theta \bar{\theta} F(z, \bar{z})$$

$$X = x$$

$$[a_m, a_n] = m \delta_{m+n, 0}$$

$$\psi =$$

$$\{b_m, b_n\} = \delta_{m+n, 0}$$

$$S = \int d^2z \left[\frac{1}{2} \partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} \right] + F^2$$

$$T = \frac{1}{2} \partial X \partial X + \psi \partial \psi \quad \partial \bar{\partial} X = 0$$

$$G = \psi \partial X$$

FREE SCALAR + FREE FERMION, BOTH MASSLESS

$$Y(z, \bar{z}, \theta, \bar{\psi}) = X(z, \bar{z})$$

$$X = x + \dots$$

$$[a_m, a_n] = m \delta_{m+n, 0}$$

$$\psi = \dots \psi_n$$

$$\{\psi_m, \psi_n\} = \delta_{m+n, 0}$$

$$\begin{matrix} b & c \\ \bar{b} & \bar{c} \end{matrix}$$

$$Q_{\text{BRST}} = \int c T + \gamma G + \dots dz + \text{cc}$$

$$S = \int \frac{1}{2} \partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \psi + F^2$$

$$T = \frac{1}{2} \partial X \partial X + \psi \partial \psi \quad \partial \bar{\partial} X = 0$$

$$G = \psi \partial X$$

FREE SCALAR + FREE FERMION, BOTH MASSLESS

$$\Psi(z, \bar{z}, \theta, \bar{\theta}) = X(z, \bar{z})$$

$$X = x + \dots$$

$$[a_m, a_n] = m \delta_{m+n, 0}$$

$$\psi = \dots \psi_n$$

$$\{\psi_m, \psi_n\} = \delta_{m+n, 0}$$

$$\begin{matrix} b & c \\ \beta & \gamma \end{matrix}$$

$$Q_{\text{BRST}} = \int c T + \gamma G + \dots dz + \text{cc}$$

$$S = \int \frac{1}{2} \partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \psi + F^2 \quad \left(\frac{1}{2}, 0 \right)$$

$$T = \frac{1}{2} \partial X \partial X + \psi \partial \psi \quad \partial \bar{\partial} X = 0$$

$$G = \frac{1}{2} \psi \partial X$$

H MASSLESS $\mathcal{V}(z, \bar{z}, \psi, \bar{\psi}) = X(z, \bar{z}) + \psi \bar{\psi} + \bar{\psi} \psi + \psi \bar{\psi} F$

$$S = \int \frac{1}{2} \partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} + F^2$$

$\uparrow$
 $(\frac{1}{2}, 0) \rightarrow \psi(z) = \left(\frac{dz'}{dz}\right)^{\frac{1}{2}} \psi(z')$
 $\partial X(z) = \frac{dz'}{dz} \partial X(z')$

$T = \frac{1}{2} \partial X \partial X + \frac{1}{2} \psi \partial \psi$ $\partial \bar{\partial} X = 0$ $\bar{\partial} \psi = 0$ $\partial \bar{\psi} = 0$ $F = 0$

$G = \frac{1}{2} \psi \partial X$

$dz + cc$

$$z = e^s \quad \psi(s) = e^{\frac{s}{2}} \psi(z) \quad (NS) \quad \begin{array}{l} \text{STATES} \\ \text{FOR ANTIPERIODIC} \\ \text{FERMIONS} \end{array} \quad \longleftrightarrow \quad 1, \psi, \dots$$

$$\psi(\sigma=2\pi) = -\psi(\sigma=0)$$

$$(R) \quad \text{PERIODIC} \quad \longleftrightarrow$$

$$\psi(\sigma=2\pi) = \psi(\sigma=0)$$

$$\psi(e^{2\pi i} z) = -\psi(z)$$

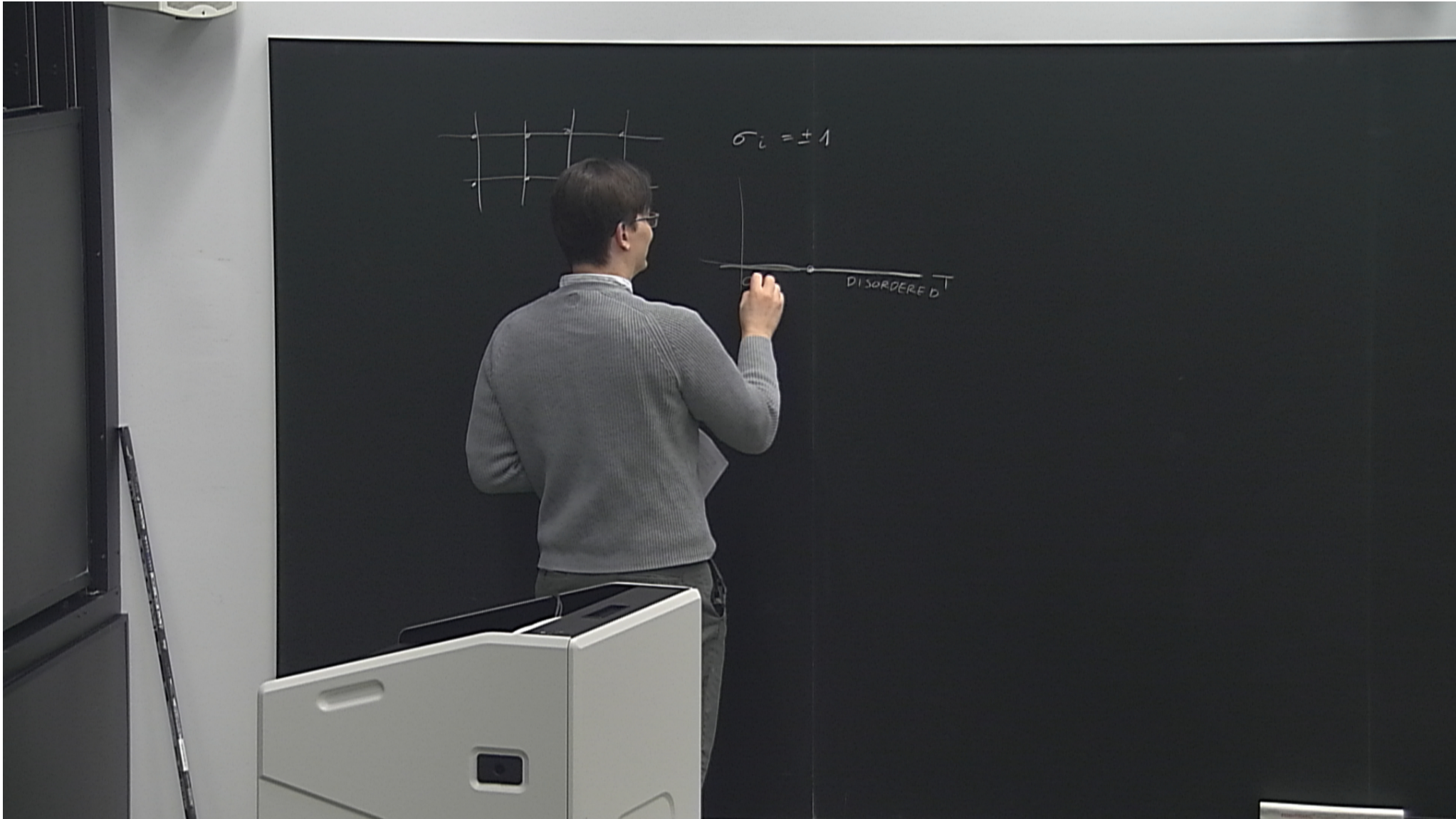

$$z = e^s \quad \psi(s) = e^{\frac{s}{2}} \psi(z) \quad (NS) \quad \begin{array}{l} \text{STATES} \\ \text{FOR ANTIPERIODIC} \\ \text{FERMIONS} \end{array} \quad \longleftrightarrow \quad 1, \psi, \dots$$

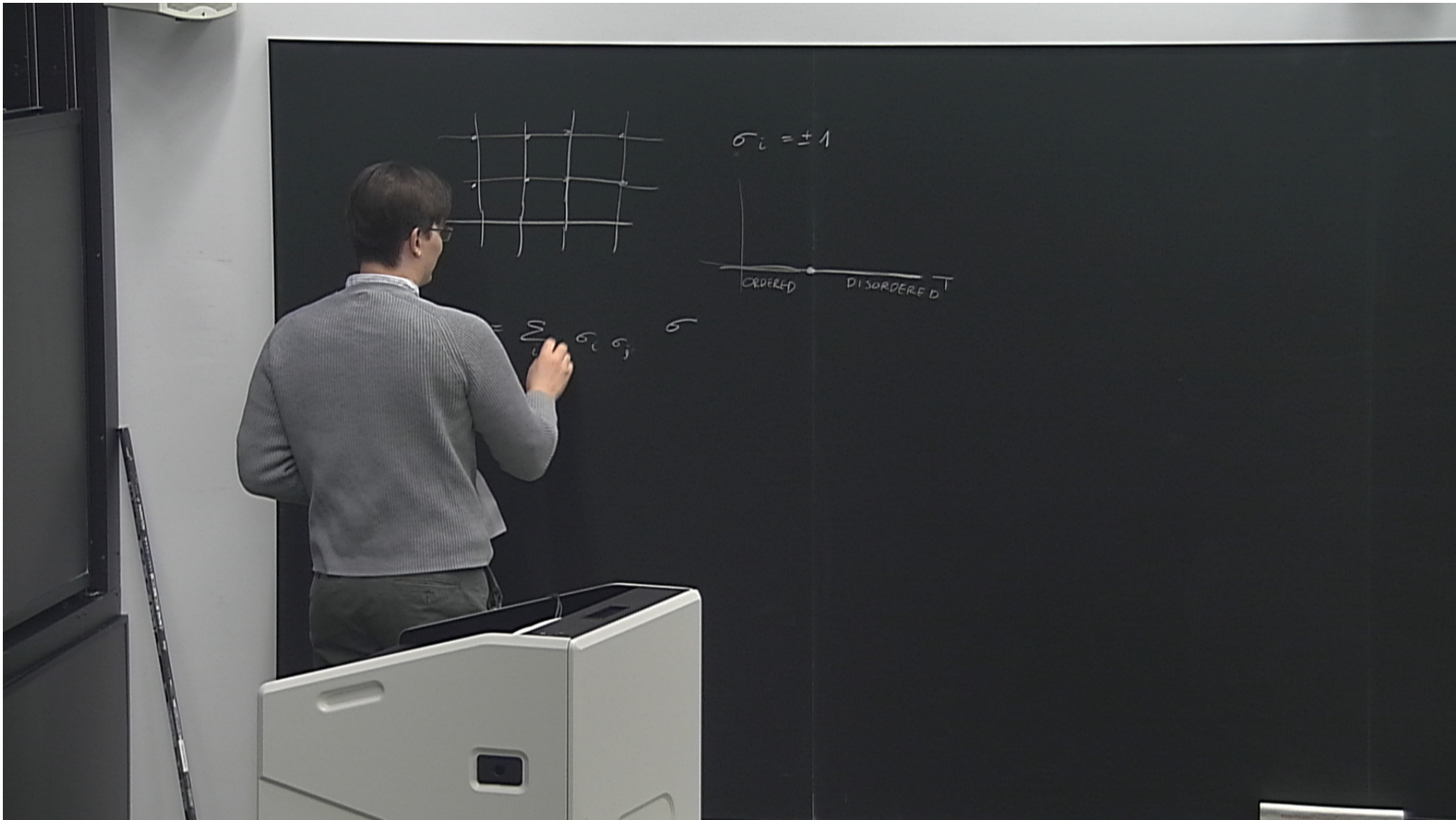
$$\psi(\sigma=2\pi) = -\psi(\sigma=0)$$

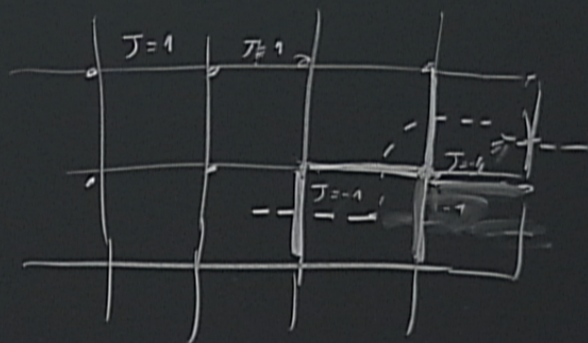
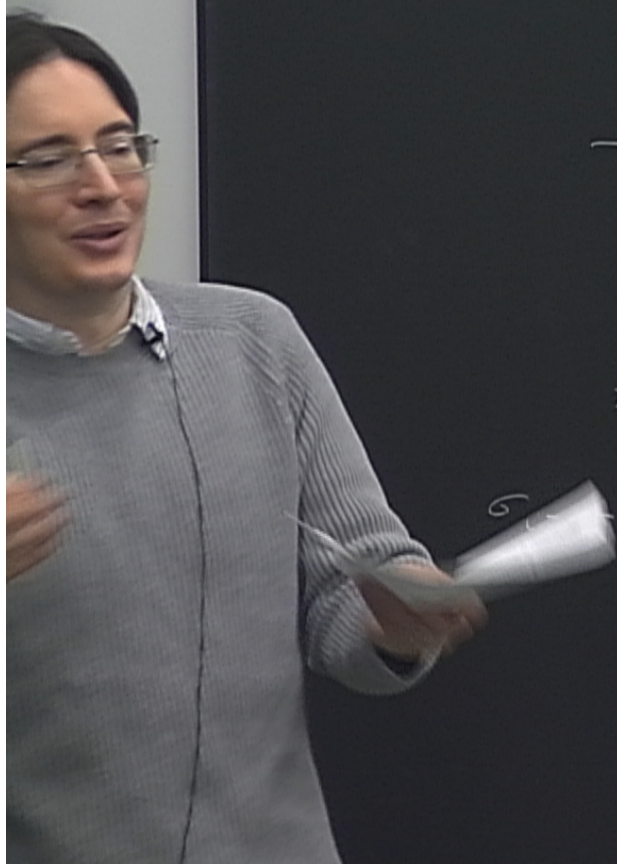
$$(R) \quad \begin{array}{l} \text{PERIODIC} \\ \psi(\sigma=2\pi) = \psi(\sigma=0) \end{array} \quad \longleftrightarrow$$

$$\psi(e^{2\pi i} z) = -\psi(z)$$

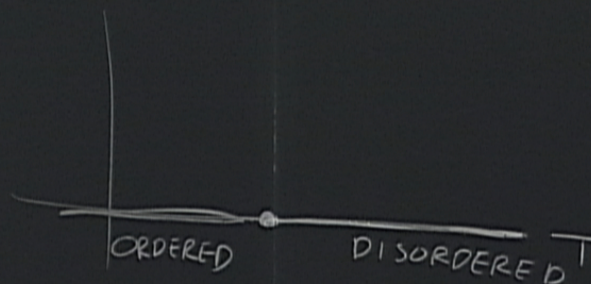

TWIST FIELDS



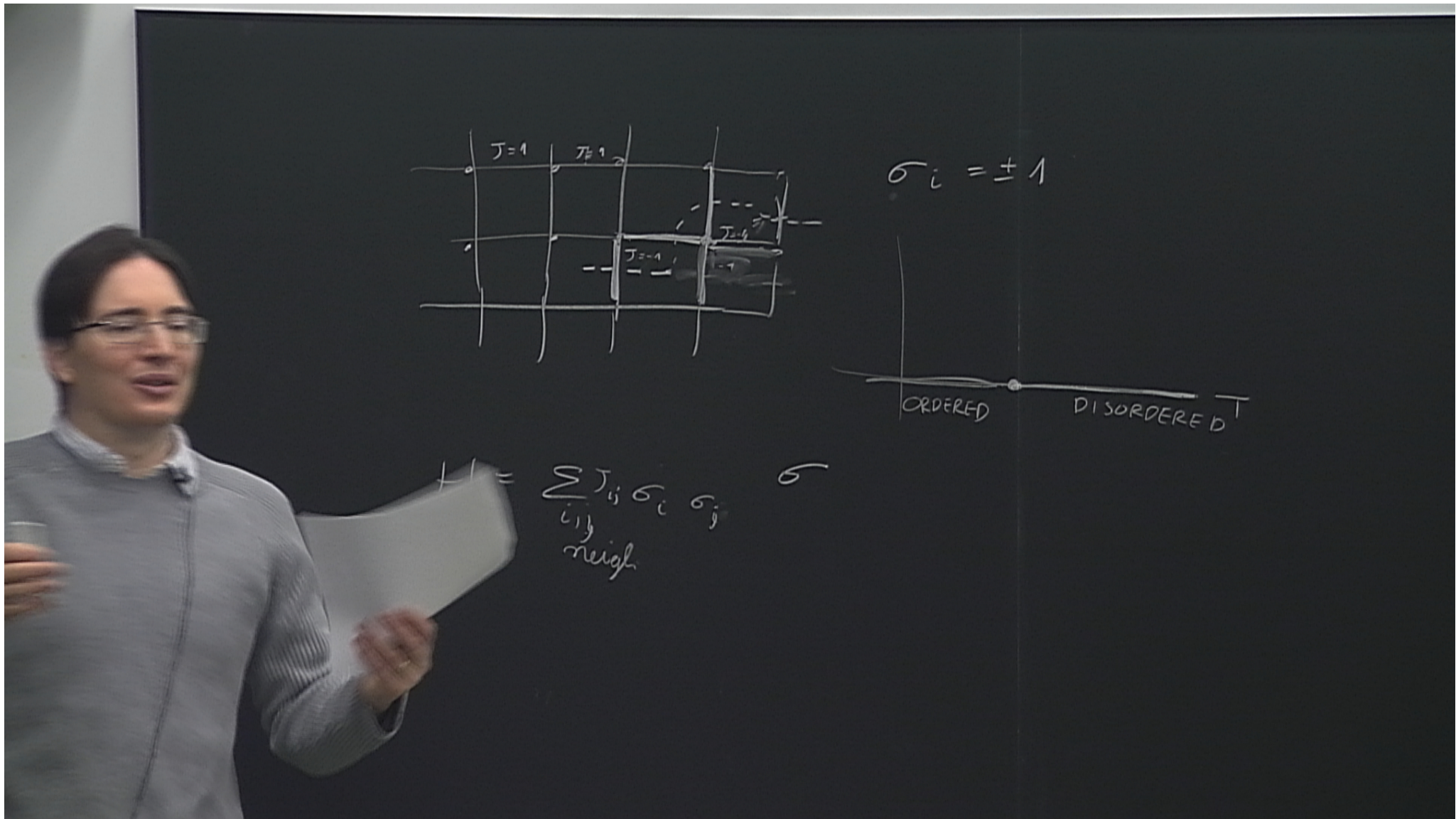


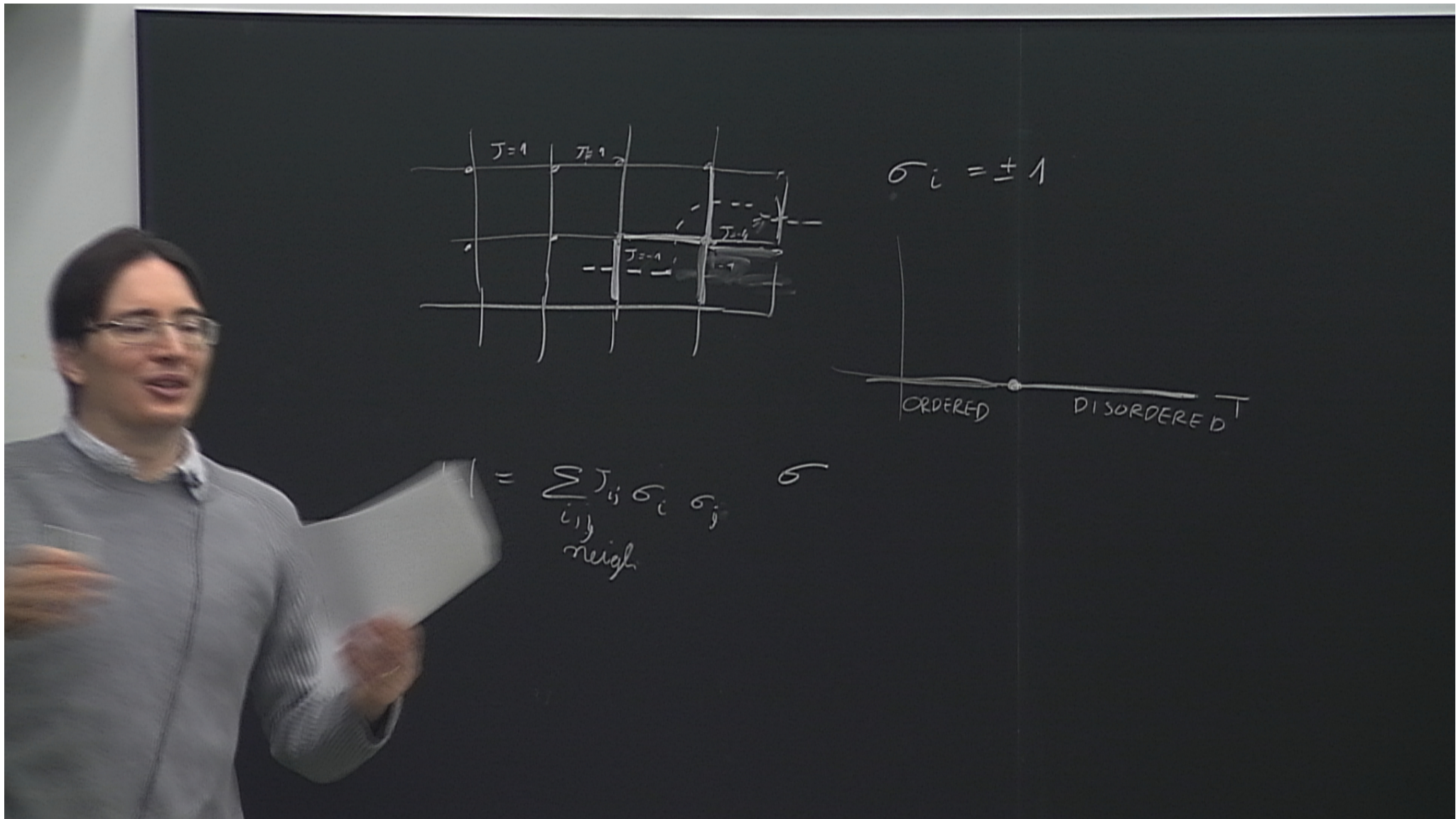


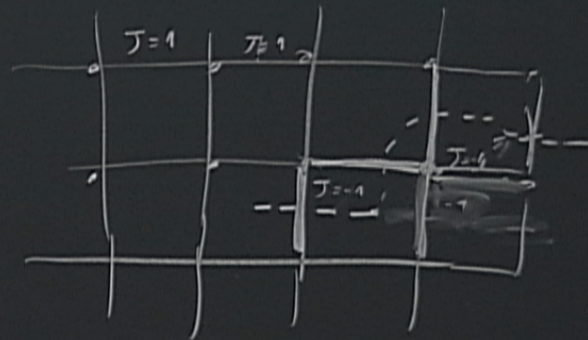
$$\sigma_i = \pm 1$$



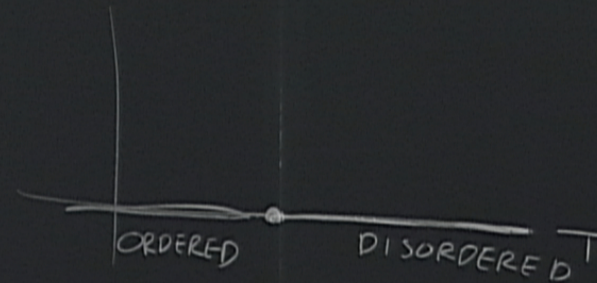
$$H = \sum_{\substack{i,j \\ \text{neigh}}} J_{ij} \sigma_i \sigma_j$$





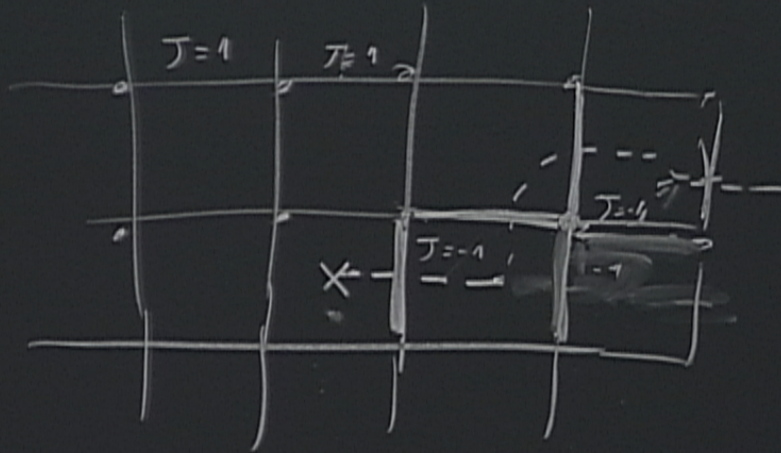


$$\sigma_i = \pm 1$$

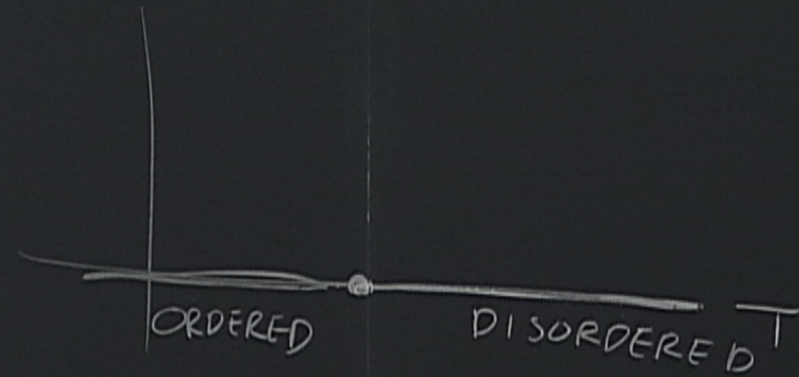


$$H = \sum_{i,j \text{ neigh}} J_{ij} \sigma_i \sigma_j$$

$$\sigma_i \rightarrow -\sigma_i$$

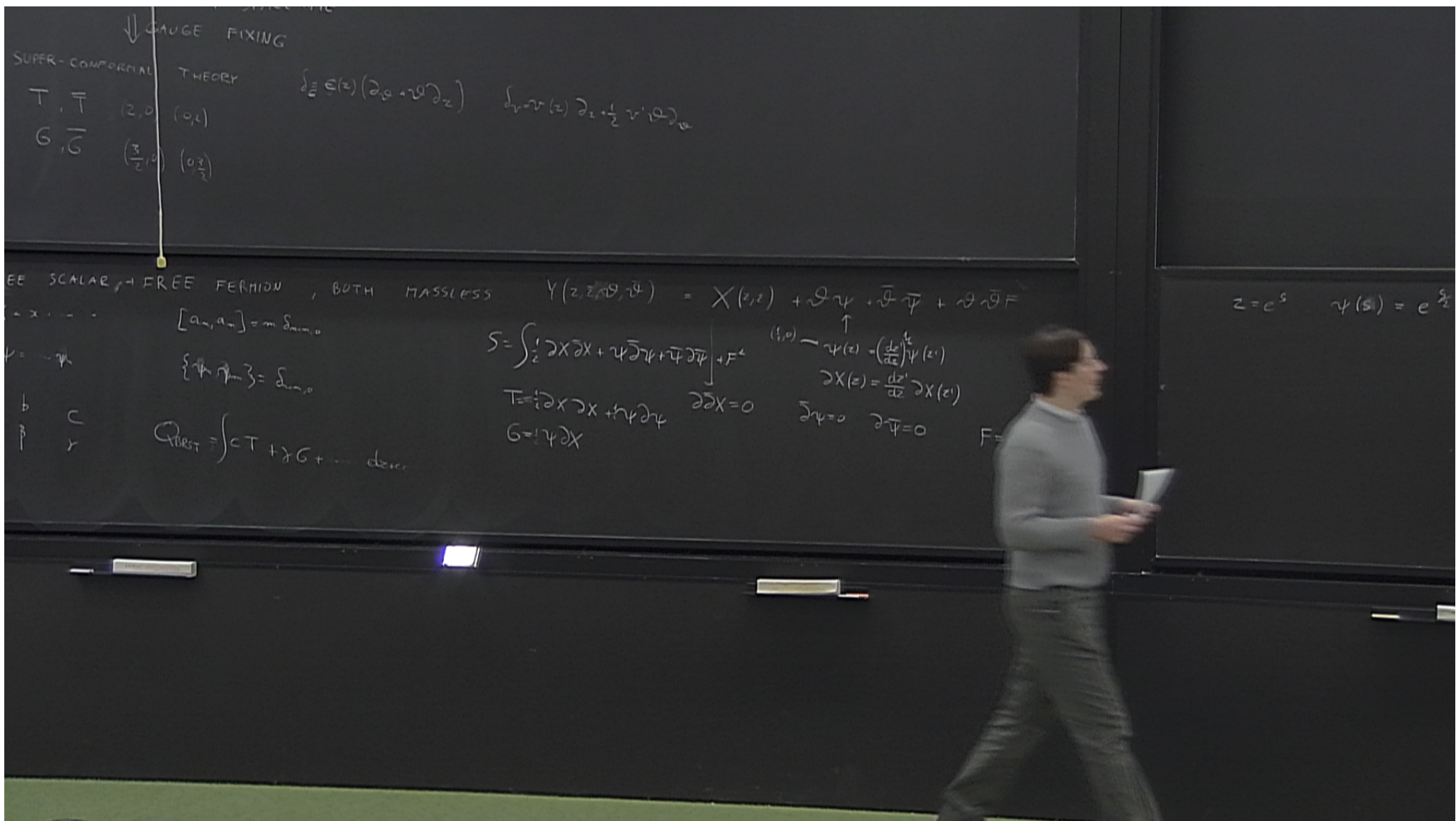


$$\sigma_i = \pm 1$$



$$H = \sum_{i,j \text{ neigh}} J_{ij} \sigma_i \sigma_j$$

$$\sigma_i = \pm 1$$



↓ GAUGE FIXING

SUPER-CONFORMAL THEORY

$$\delta_\epsilon \equiv \epsilon(z) (\partial_\theta + \gamma \partial_z) \quad \delta_\gamma = \gamma(z) \partial_z + \frac{1}{2} \gamma'(z) \partial_\theta$$

$$\begin{array}{cc} T, \bar{T} & (2,0) \quad (0,2) \\ G, \bar{G} & (\frac{3}{2},0) \quad (0,\frac{3}{2}) \end{array}$$

FREE SCALAR, + FREE FERMION, BOTH MASSLESS

$$\mathcal{Y}(z, z', \theta, \bar{\theta}) = X(z, z') + \theta \psi + \bar{\theta} \bar{\psi} + \theta \bar{\theta} F$$

$$[a_m, a_n] = m \delta_{m+n,0}$$

$$\{\psi_m, \bar{\psi}_n\} = \delta_{m+n,0}$$

$$S = \int_z \left[\partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} \right] + F^2$$

$$\begin{aligned} \uparrow \\ \psi(z) &= \left(\frac{dz'}{dz} \right)^{\frac{1}{2}} \psi(z') \\ \partial X(z) &= \frac{dz'}{dz} \partial X(z') \end{aligned}$$

$$\begin{aligned} T &= i \partial X \bar{\partial} X + \psi \bar{\partial} \psi \\ G &= i \psi \partial X \end{aligned} \quad \begin{aligned} \partial \bar{\partial} X &= 0 \\ \bar{\partial} \psi &= 0 \quad \partial \bar{\psi} = 0 \end{aligned}$$

$$F =$$

$$Q_{BRST} = \int c T + \gamma G + \dots dz$$

$$z = e^s \quad \psi(s) = e^{\frac{s}{2}}$$

$$NS) \quad \psi = \sum_n c_n e^{-(n+\frac{1}{2})S}$$

$$\psi_m \quad m \in \mathbb{Z} + \frac{1}{2}$$

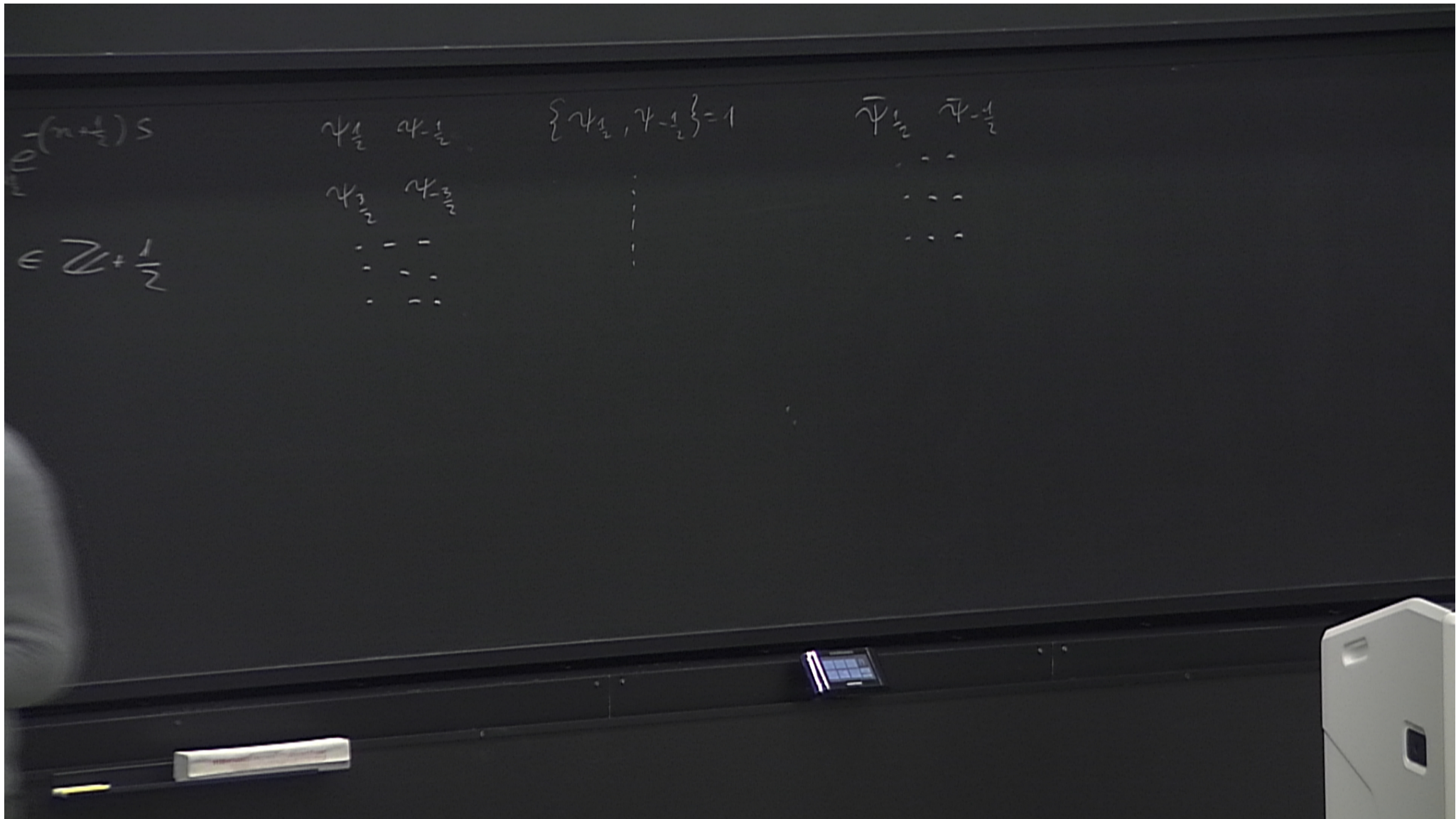
$$\psi_{\frac{1}{2}} \quad \psi_{-\frac{1}{2}}$$

$$\psi_{\frac{3}{2}} \quad \psi_{-\frac{3}{2}}$$

$$\begin{matrix} - & - \\ - & - \\ - & - \end{matrix}$$

$$\{\psi_{\frac{1}{2}}, \psi_{-\frac{1}{2}}\} = 1$$

⋮



$$\psi_{\frac{1}{2}} \quad \psi_{-\frac{1}{2}}$$

$$\{\psi_{\frac{1}{2}}, \psi_{-\frac{1}{2}}\} = 1$$

$$\bar{\psi}_{\frac{1}{2}} \quad \bar{\psi}_{-\frac{1}{2}}$$

$$\psi_{\frac{3}{2}} \quad \psi_{-\frac{3}{2}}$$

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$$R) \quad \psi = \sum \psi_m e^{-ms}$$

$$\psi_m \quad m \in \mathbb{Z} \quad \psi_0$$

$$\{\psi_1, \psi_{-1}\} = 1$$

$\vdots$
 $\vdots$
 $\vdots$

$$R) \quad \psi = \sum \psi_n^\mu e^{-ns}$$

$$\psi_n^\mu \quad n \in \mathbb{Z}$$

$$\{\psi_1^\mu, \psi_{-1}^\nu\} = \eta^{\mu\nu}$$

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$$



$$R) \quad \psi^\mu = \sum \psi_m^\mu e^{-m\sigma}$$

$$\psi_m^\mu \quad m \in \mathbb{Z}$$

$$\{\psi_1^\mu, \psi_{-1}^\nu\} = \eta^{\mu\nu}$$

- - -
- - -
- - -

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$$

$$|A, \bar{A}\rangle$$

$$\{\bar{\psi}_0^\mu, \bar{\psi}_0^\nu\} = \eta^{\mu\nu}$$

A SPINOR INDEX
FOR ψ_0^μ

$$\bar{A} \quad || \quad \bar{\psi}_0^\mu$$

$$R) \quad \psi = \sum \psi_n^\mu e^{-n\sigma}$$

$$\psi_n^\mu \quad n \in \mathbb{Z}$$

$$\{\psi_1^\mu, \psi_{-1}^\nu\} = \eta^{\mu\nu}$$

$$\vdots$$

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$$

$$|A, \bar{A}\rangle$$

$$\{\bar{\psi}_0^\mu, \bar{\psi}_0^\nu\} = \eta^{\mu\nu}$$

A SPINOR INDEX
FOR ψ_0^μ

$$\bar{A} \quad || \quad \bar{\psi}_0^\mu$$

$$R) \quad \psi^\mu = \sum \psi_m^\mu e^{-m\sigma}$$

$$\psi_m^\mu \quad m \in \mathbb{Z}$$

$$\{\psi_1^\mu, \psi_{-1}^\nu\} = \eta^{\mu\nu}$$

$$\vdots$$

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$$

$$|A, \bar{A}\rangle$$

$$\{\bar{\psi}_0^\mu, \bar{\psi}_0^\nu\} = \eta^{\mu\nu}$$

A SPINOR INDEX
FOR ψ_0^μ

$$\bar{A} \quad || \quad \bar{\psi}_0^\mu$$

NS
NS

$$\psi^\mu = \sum_n \psi_{n+\frac{1}{2}}^\mu e^{-\left(n+\frac{1}{2}\right)S}$$

$$\psi^\mu \quad n \in \mathbb{Z} + \frac{1}{2}$$

$$\begin{matrix} \psi_{\frac{1}{2}}^\mu & \psi_{-\frac{1}{2}}^\mu \\ \psi_{\frac{3}{2}}^\mu & \psi_{-\frac{3}{2}}^\mu \\ \vdots & \vdots \end{matrix}$$

$$\{\psi_{\frac{1}{2}}^\mu, \psi_{-\frac{1}{2}}^\nu\} = \eta^{\mu\nu}$$

$$\begin{matrix} \bar{\psi}_{\frac{1}{2}}^\mu & \bar{\psi}_{-\frac{1}{2}}^\mu \\ \vdots & \vdots \end{matrix}$$

NS R

$$\psi_{\frac{1}{2}}, \dots$$

$$\bar{\psi}_0, \dots$$

(RR)

$$\psi^\mu = \sum \psi_m^\mu e^{-ms}$$

$$\psi_m^\mu \quad m \in \mathbb{Z}$$

$$\{\psi_1^\mu, \psi_{-1}^\nu\} = \eta^{\mu\nu}$$

$$\{\psi_0^\mu, \psi_0^\nu\} = \eta^{\mu\nu}$$

$$|A, \bar{A}\rangle$$

$$\{\bar{\psi}_0^\mu, \bar{\psi}_0^\nu\} = \eta^{\mu\nu}$$

A · SPINOR INDEX
FOR ψ_0^μ

$$\bar{A} \quad || \quad \bar{\psi}_0^\mu$$

$$(\partial_z + v \partial_z) \quad \partial_r = v(z) \partial_z + \frac{1}{2} v' \partial_\theta$$

$$P^\mu \tilde{a}_\mu |P\rangle$$

$$\sum_{n=-\infty}^{\infty} \alpha_n \alpha_{-n} = \frac{1}{2} \alpha_n \alpha_{-n}$$

H MASSLESS

$$\mathcal{Y}(z, \bar{z}, \theta, \bar{\theta}) = X(z, \bar{z}) + \theta \psi + \bar{\theta} \bar{\psi} + \theta \bar{\theta} F$$

$$S = \int \frac{1}{2} \partial X \bar{\partial} X + \psi \bar{\partial} \psi + \bar{\psi} \partial \bar{\psi} + F^2$$

$$\left(\frac{1}{2}, 0\right) \rightarrow \psi(z) = \left(\frac{dz'}{dz}\right)^{\frac{1}{2}} \psi(z')$$

$$\partial X(z) = \frac{dz'}{dz} \partial X(z')$$

$$T = \frac{1}{2} \partial X \partial X + \frac{1}{2} \psi \partial \psi$$

$$\partial \bar{\partial} X = 0$$

$$\bar{\partial} \psi = 0$$

$$\partial \bar{\psi} = 0$$

$$F = 0$$

$$G = \frac{1}{2} \psi \partial X$$

$$dz + c.c.$$

FREE SCALAR + FREE FERMION, BOTH MASSLESS

$$X = x + \dots$$

$$[a_m, a_n] = m \delta_{m+n, 0}$$

$$\psi = \dots \psi_m$$

$$\{\psi_m, \psi_n\} = \delta_{m+n, 0}$$

$$\begin{array}{c} b \\ \beta \end{array} \quad \begin{array}{c} c \\ \gamma \end{array}$$

$$Q_{\text{BRST}} = \int c T + \gamma G + \dots dz + \text{c.c.}$$

NS NS

$$L_n |P_{HY}\rangle = 0 \quad n \geq 0$$

$$G_n | \rangle = 0 \quad n \geq 0$$

$$|P\rangle$$

$$P^2 = 1$$

TACHYON !

$$\psi_{-\frac{1}{2}}^\mu \bar{\psi}_{-\frac{1}{2}}^\nu |P\rangle$$

$$P^2 = 0$$

GRAVITON + B + DILATON

$$\psi = \dots \psi_n$$

$$\{\psi_m, \psi_n\} = \delta_{m+n,0}$$

$$\begin{matrix} b & c \\ \bar{b} & \bar{c} \end{matrix}$$

$$Q_{BRST} = \int cT + \gamma G + \dots dz + cc$$

$$T = \frac{i}{2} \partial X \partial X + i \psi \partial \psi$$

$$G = \frac{i}{2} \psi \partial X$$

$$\partial \bar{\partial} X = 0$$

$$\bar{\partial} \psi = 0$$

$$\partial \bar{\psi} = 0$$

$$F = 0$$

$$\partial X(z) = \frac{dz'}{dz} \partial X(z')$$

NS NS

$$L_n |P\rangle = 0 \quad n \geq 0$$

$$G_n |P\rangle = 0 \quad n \geq 0$$

$$L_0 |P\rangle = (p^2 - \frac{1}{2}) |P\rangle$$

$$|P\rangle \quad p^2 = 1 \quad \text{TACHYON}$$

$$\psi_{-1/2}^\mu \bar{\psi}_{-1/2}^\nu |P\rangle \quad p^2 = 0 \quad \text{GRAVITON} + B + \text{DILATON}$$

$$(-1)^F \psi = -\psi (-1)^F$$

$$(-1)^F \bar{\psi} = -\bar{\psi} (-1)^F$$

$$(-1)^F \beta = -\beta (-1)^F$$

$$[(-1)^F, Q] = 0$$

$$(-1)^F \bar{\psi} = -\bar{\psi} (-1)^F$$

$$\psi = \dots \psi_n$$

$$\begin{matrix} b & c \\ \beta & \gamma \end{matrix}$$

$$\{\psi_m, \psi_n\} = \delta_{m+n,0}$$

$$Q_{BRST} = \int (cT + \gamma G + \dots) dz + cc$$

$$T = -\frac{1}{2} \partial X \partial X + \dots \psi \partial$$

$$G = \frac{1}{2} \psi \partial X$$

NS NS

$$L_n |P\rangle = 0 \quad n \geq 0$$

$$G_m |P\rangle = 0 \quad m \geq 0$$

$$(-1)^F \quad (-1)^{\bar{F}}$$

$$L_0 |P\rangle = (P^2 - \frac{1}{2}) |P\rangle$$

$$L_0 |P\rangle = (P^2 - \frac{1}{2}) |P\rangle$$

$$\begin{array}{c} |P\rangle \xrightarrow{\quad} \overline{P}^2 = 1 \text{ --- TACHYON --- } \end{array}$$

$$\psi_{-\frac{1}{2}}^\mu \bar{\psi}_{-\frac{1}{2}}^\nu |P\rangle \quad P^2 = 0 \quad \text{GRAVITON} + B + \text{DILATON} + \dots$$

β γ

$$Q_{\text{BRST}} = \int (cT + \gamma G + \dots) dz + \text{cc}$$

$$G = \frac{1}{2} \psi \partial X$$

NS NS

$$L_n |P\rangle = 0 \quad n \geq 0$$

$$G_m |P\rangle = 0 \quad m \geq 0$$

$$(-1)^F \quad (-1)^{\bar{F}} \quad L_0 |P\rangle = (P^2 - \frac{1}{2}) |P\rangle$$

GSO

$$|P\rangle \quad P^2 = -1 \quad \text{TACHYON!}$$

$$\psi_{-\frac{1}{2}}^\mu \bar{\psi}_{-\frac{1}{2}}^\nu |P\rangle$$

$$P^2 = 0$$

GRAVITON + B + DILATON + +

RNS

$$\bar{\psi}_{\frac{1}{2}}^\mu |P\rangle$$

$$P^2 = 0$$

$\bar{p}$ $\begin{matrix} c \\ r \end{matrix}$

$$Q_{BRST} = \int (cT + \gamma G + \dots) dz + cc$$

$$G = \frac{1}{2} \psi \partial X$$

NS NS

$$L_n |P_{NS}\rangle = 0 \quad n \geq 0$$

$$G_m |P_{NS}\rangle = 0 \quad m \geq 0$$

$$(-1)^F \quad (-1)^{\bar{F}} \quad L_0 |P\rangle =$$

GSO

$$\text{---} |P\rangle \text{---} \quad p^2 = -1 \quad \text{---} \text{TACHYON!} \text{---}$$

$$\psi_{-\frac{1}{2}}^\mu \bar{\psi}_{-\frac{1}{2}}^\nu |P\rangle$$

$$p^2 = 0$$

GRAVITON + B + DILATON + +

$$RNS \quad |e_\mu^i \bar{\psi}_{-\frac{1}{2}}^\mu |q, p\rangle$$

$$p^2 = 0$$

$$e_\mu^i = p_\mu v^i$$

