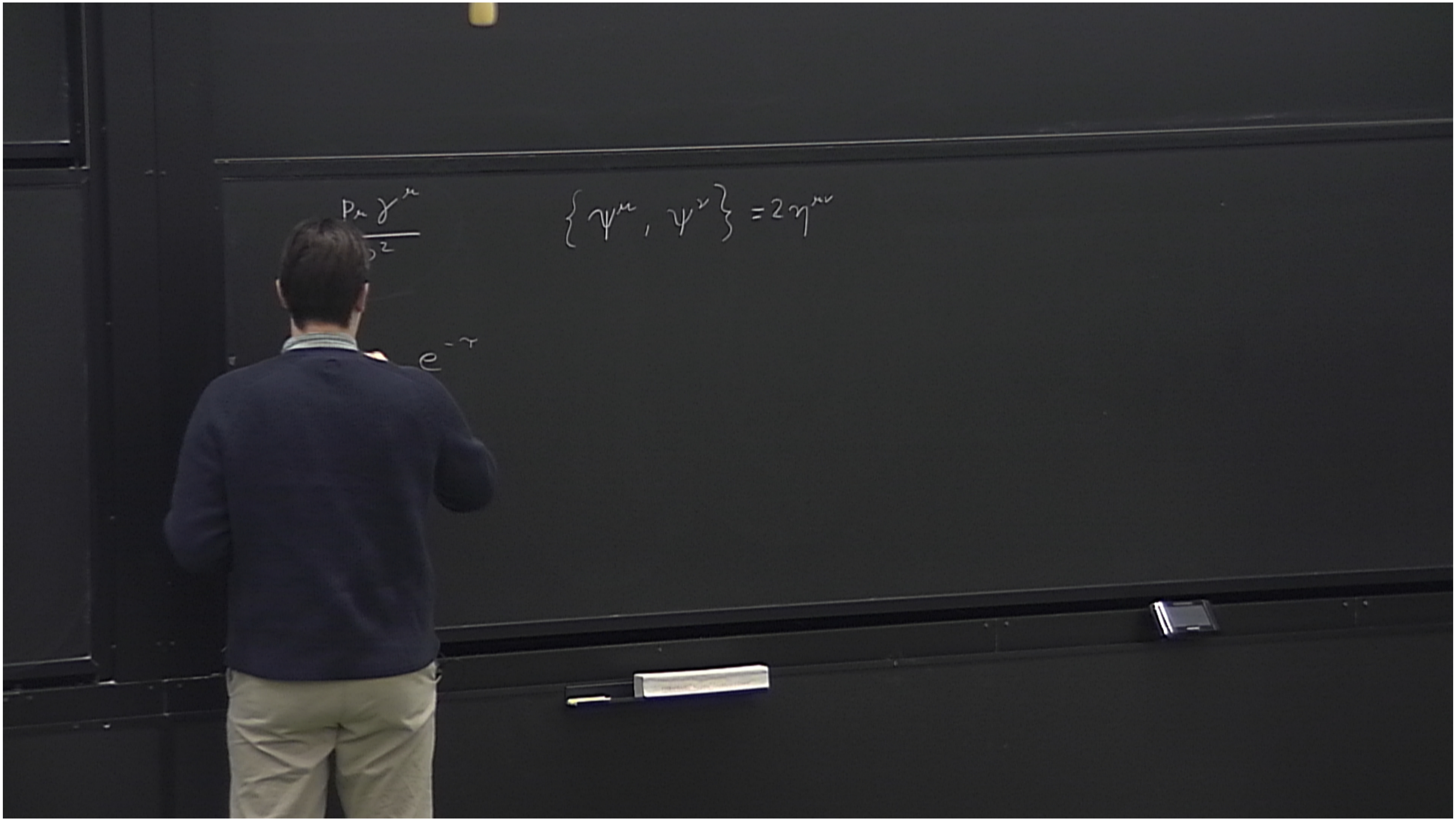
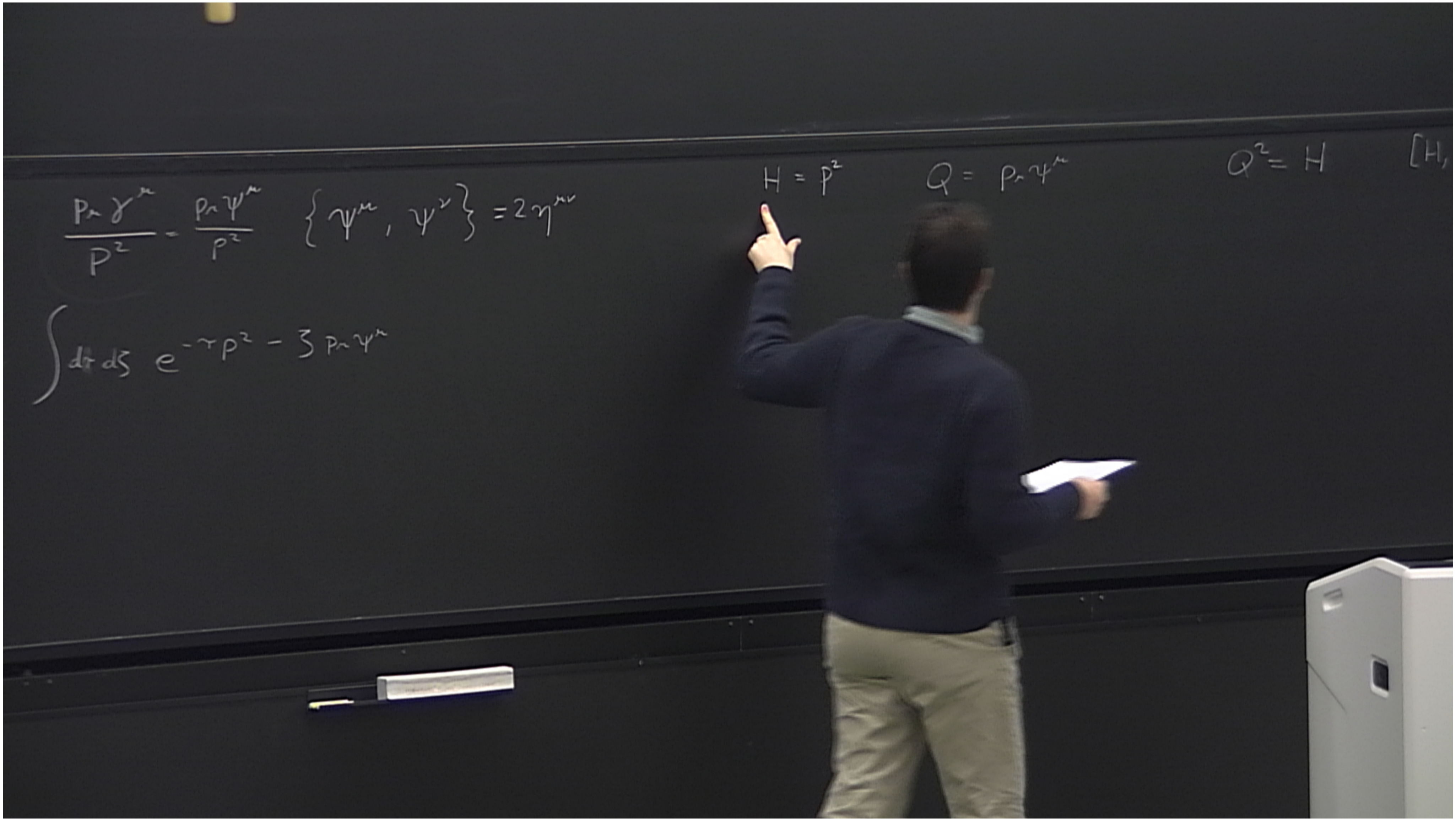
Title: 13/14 PSI - String Theory Review - Lecture 13

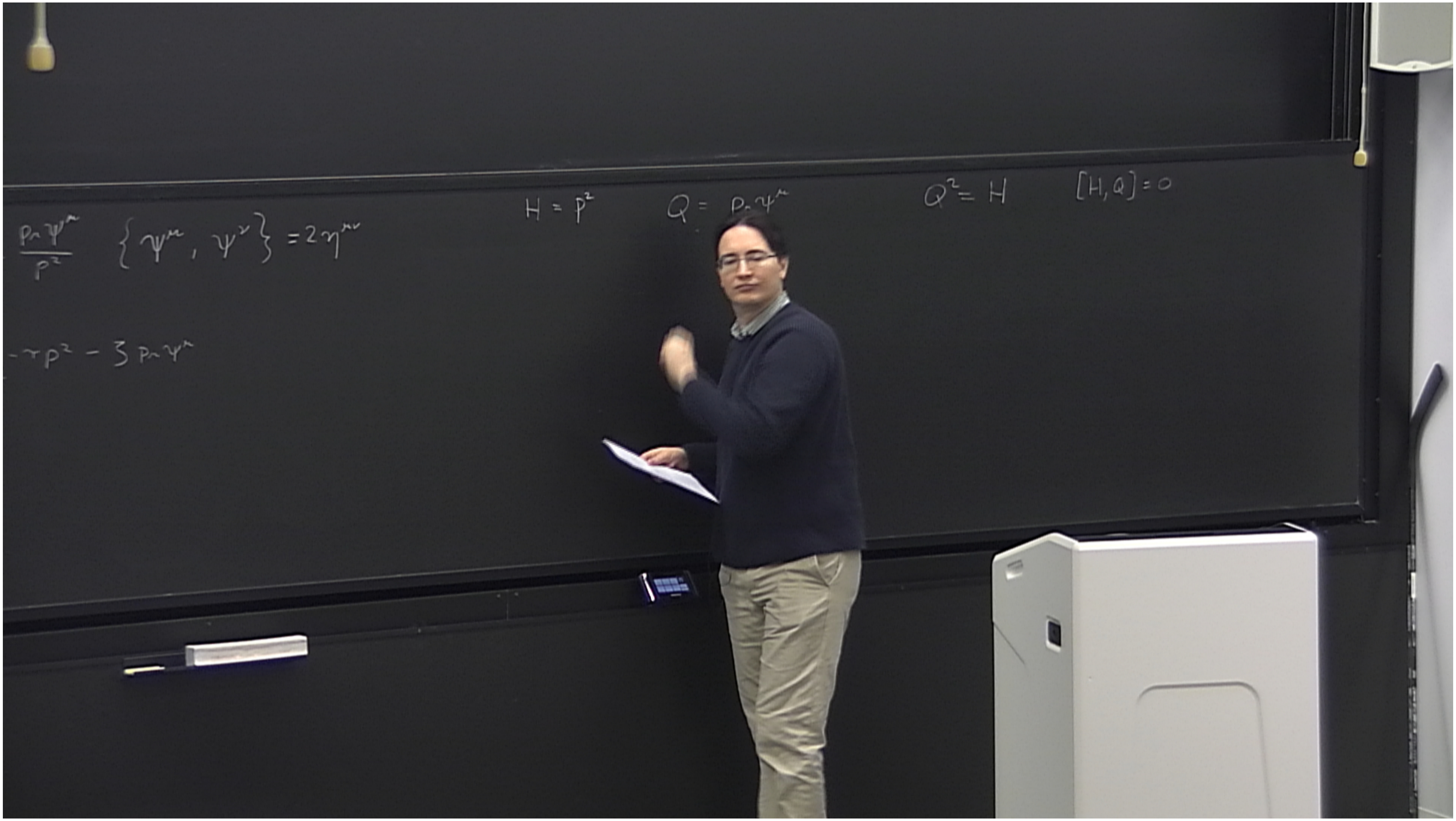
Date: Feb 13, 2014 10:15 AM

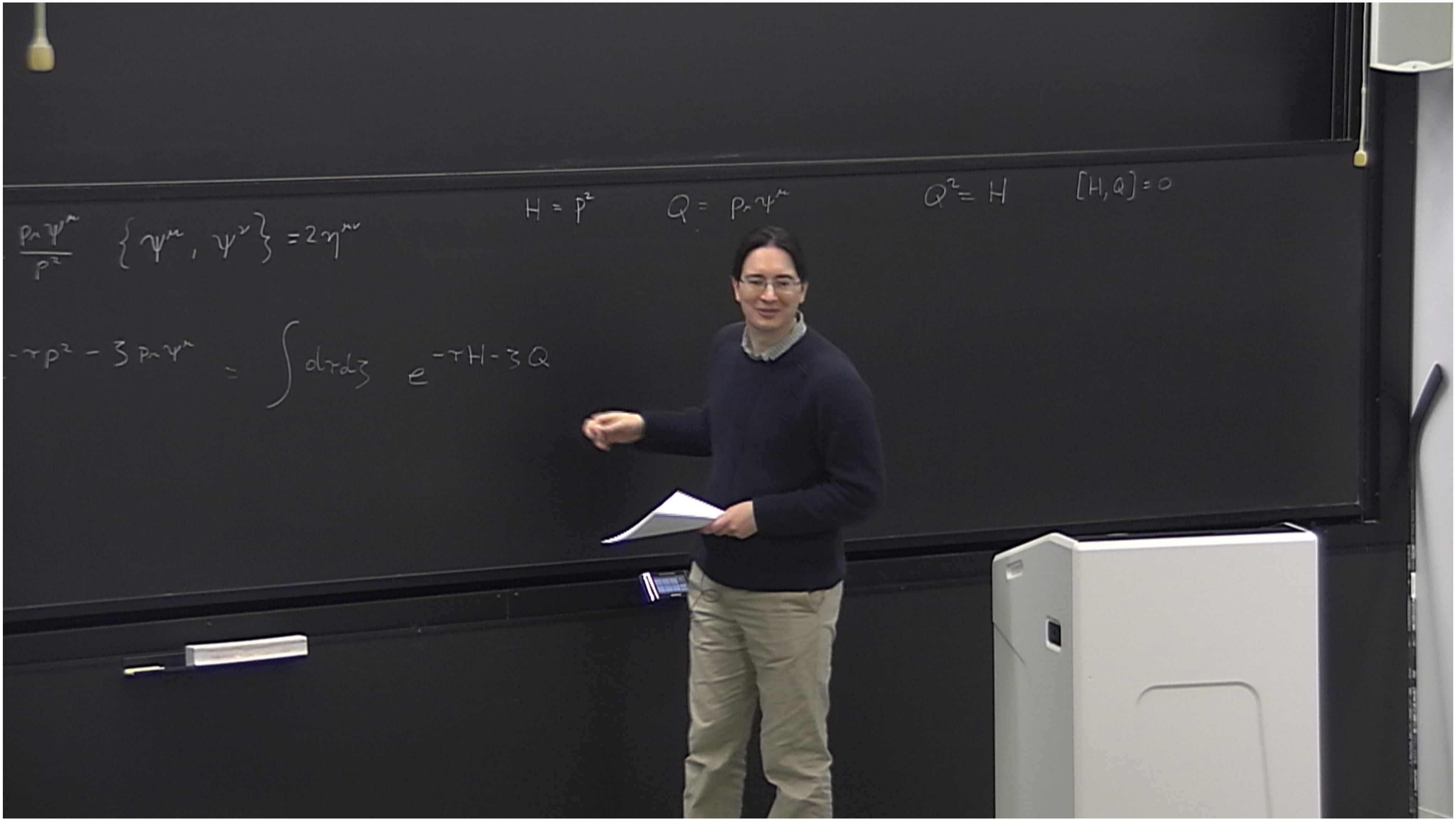
URL: http://pirsa.org/14020033

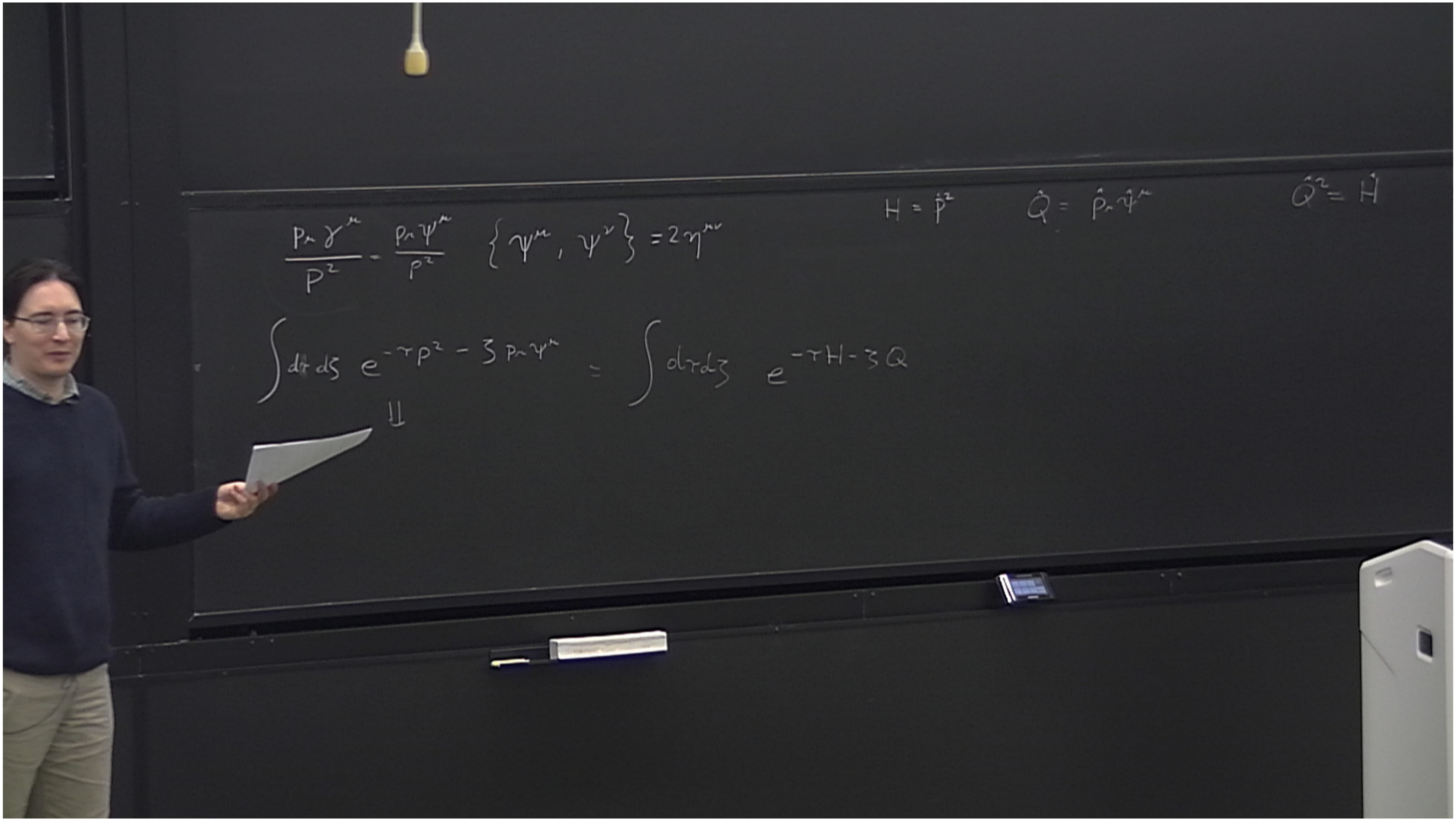
Abstract:

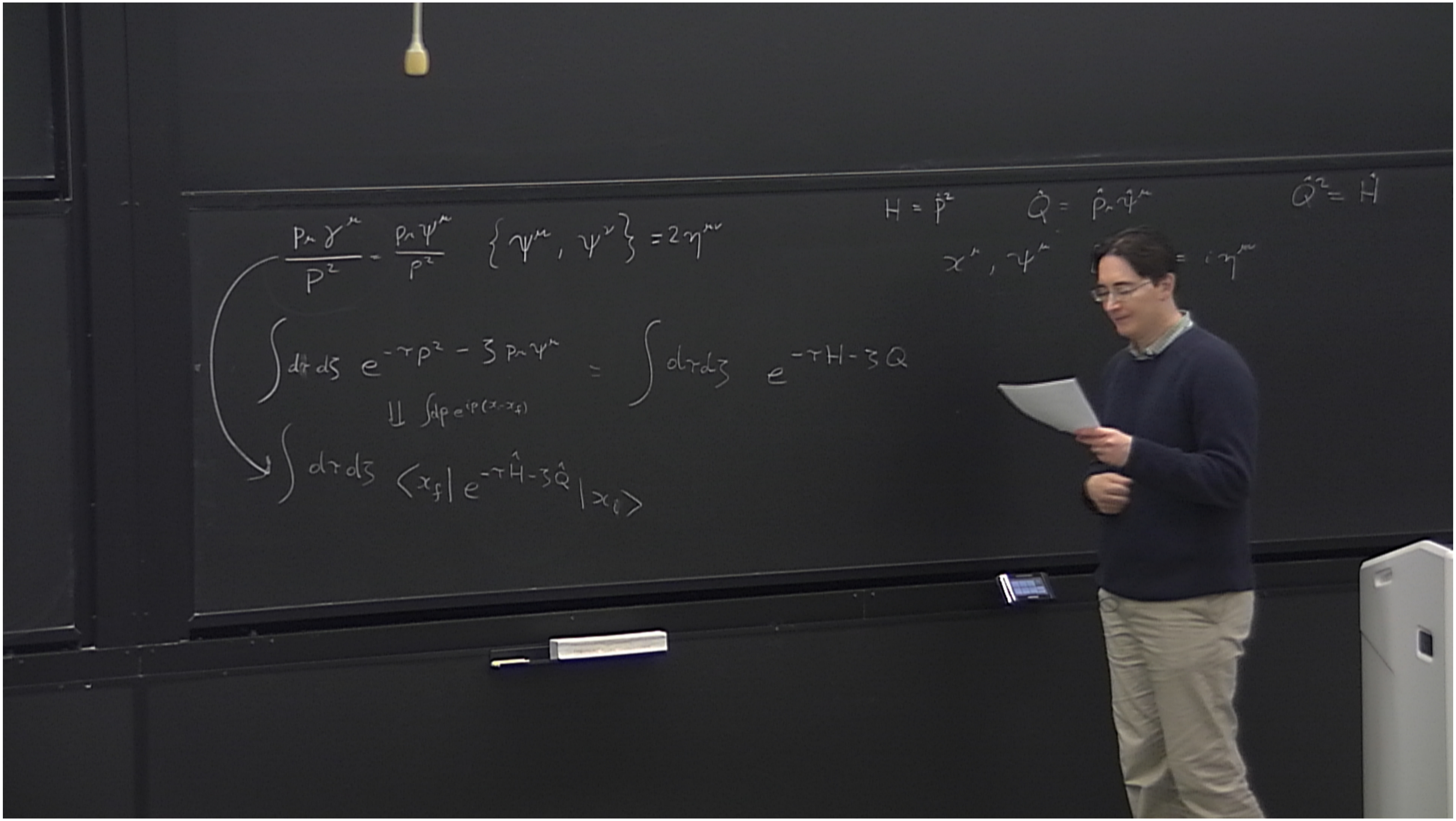

$p_\mu \gamma^\mu$
$\{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}$
$e^{-\tau}$

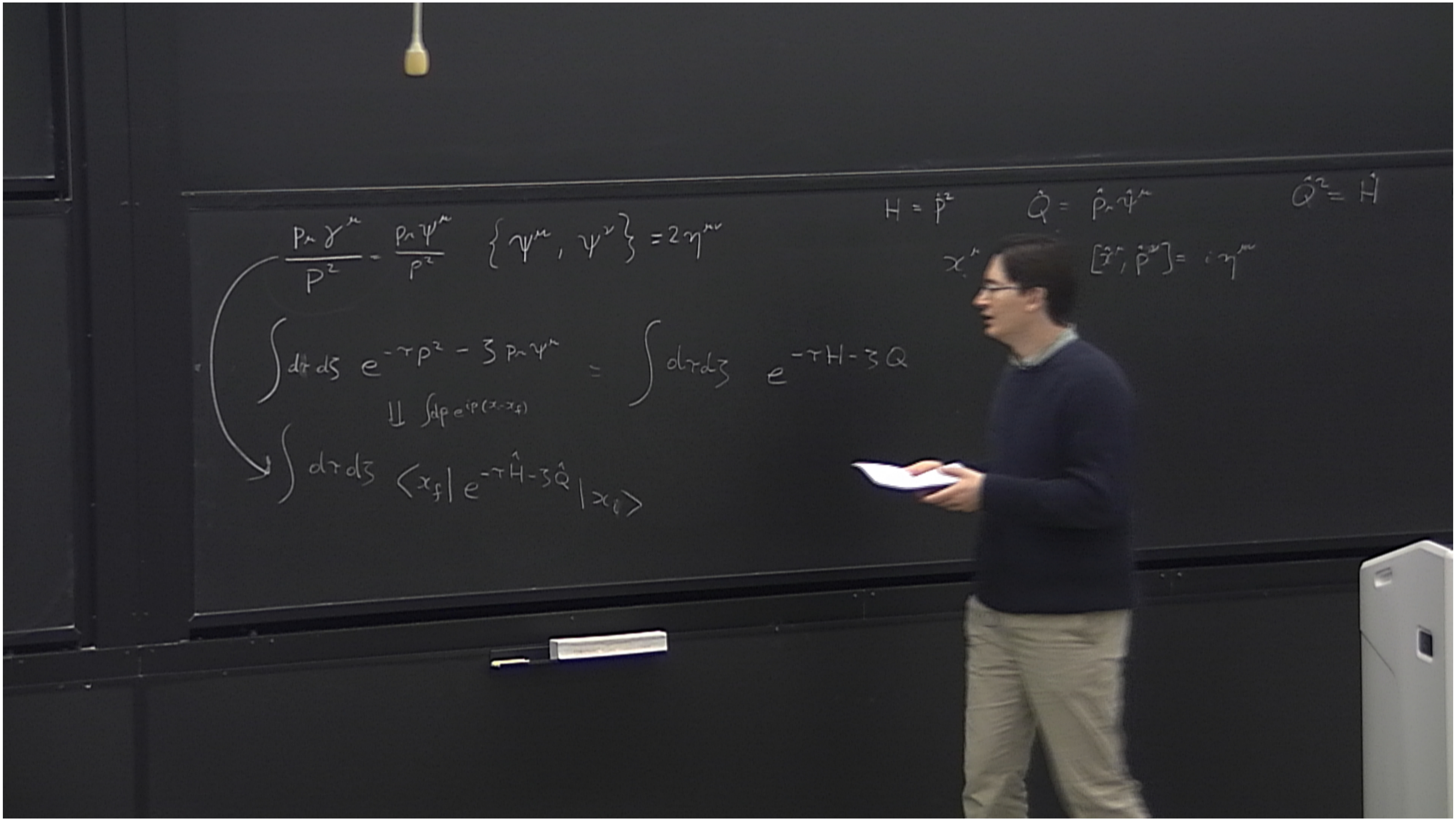

$\frac{p_\mu \gamma^\mu}{p^2} = \frac{p_\mu \psi^\mu}{p^2}$
$\{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}$
$\int d\tau \, d\zeta \; e^{-\tau p^2 - \zeta p_\mu \psi^\mu}$
$H = p^2$
$Q = p_\mu \psi^\mu$
$Q^2 = H$
$[H,$

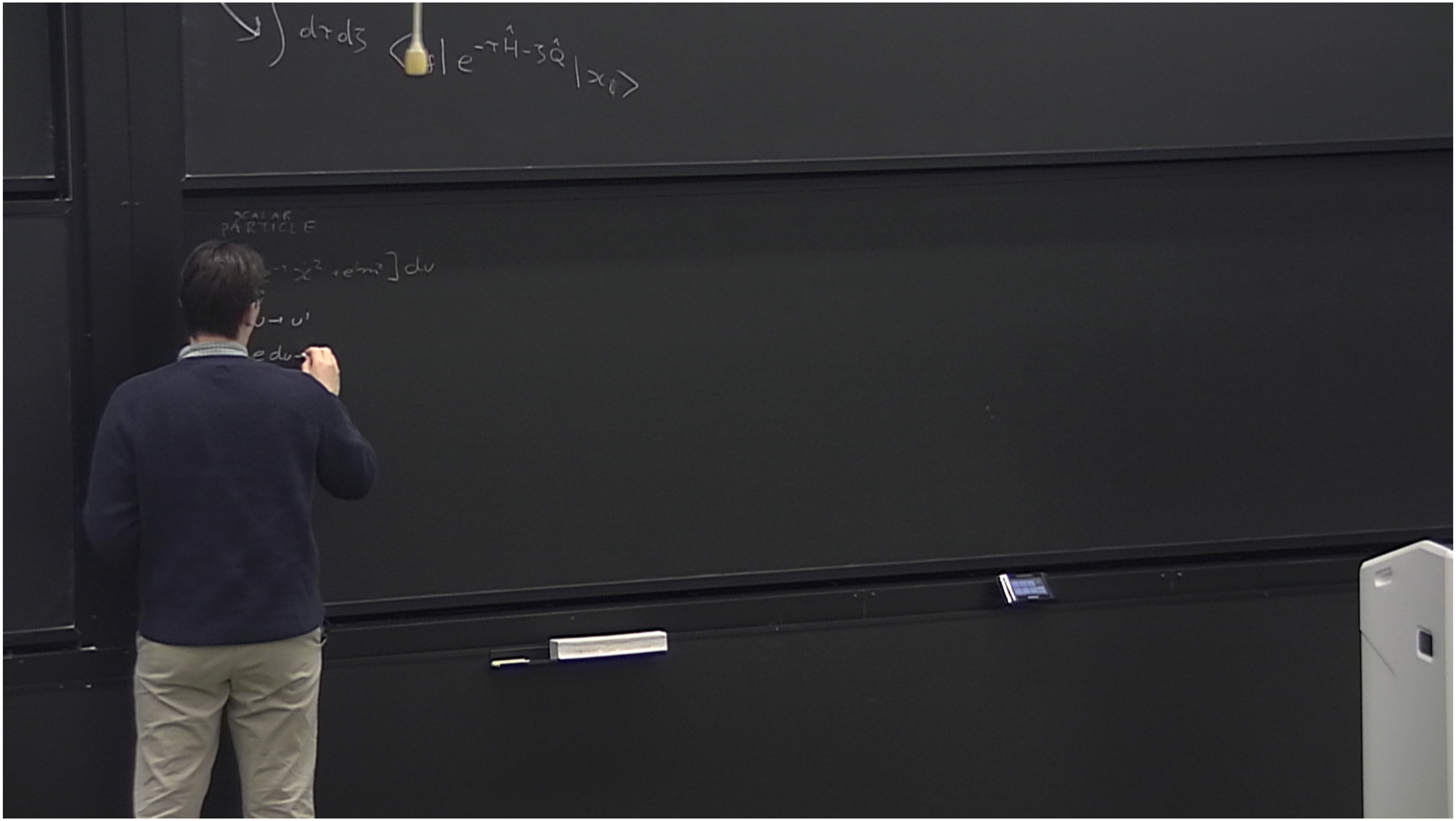

$H = p^2$
$Q = p_\mu \psi^\mu$
$Q^2 = H$
$[H, Q] = 0$
$\frac{p_\mu \psi^\mu}{p^2}$
$\{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}$
$\tau p^2 - \zeta p_\mu \psi^\mu$

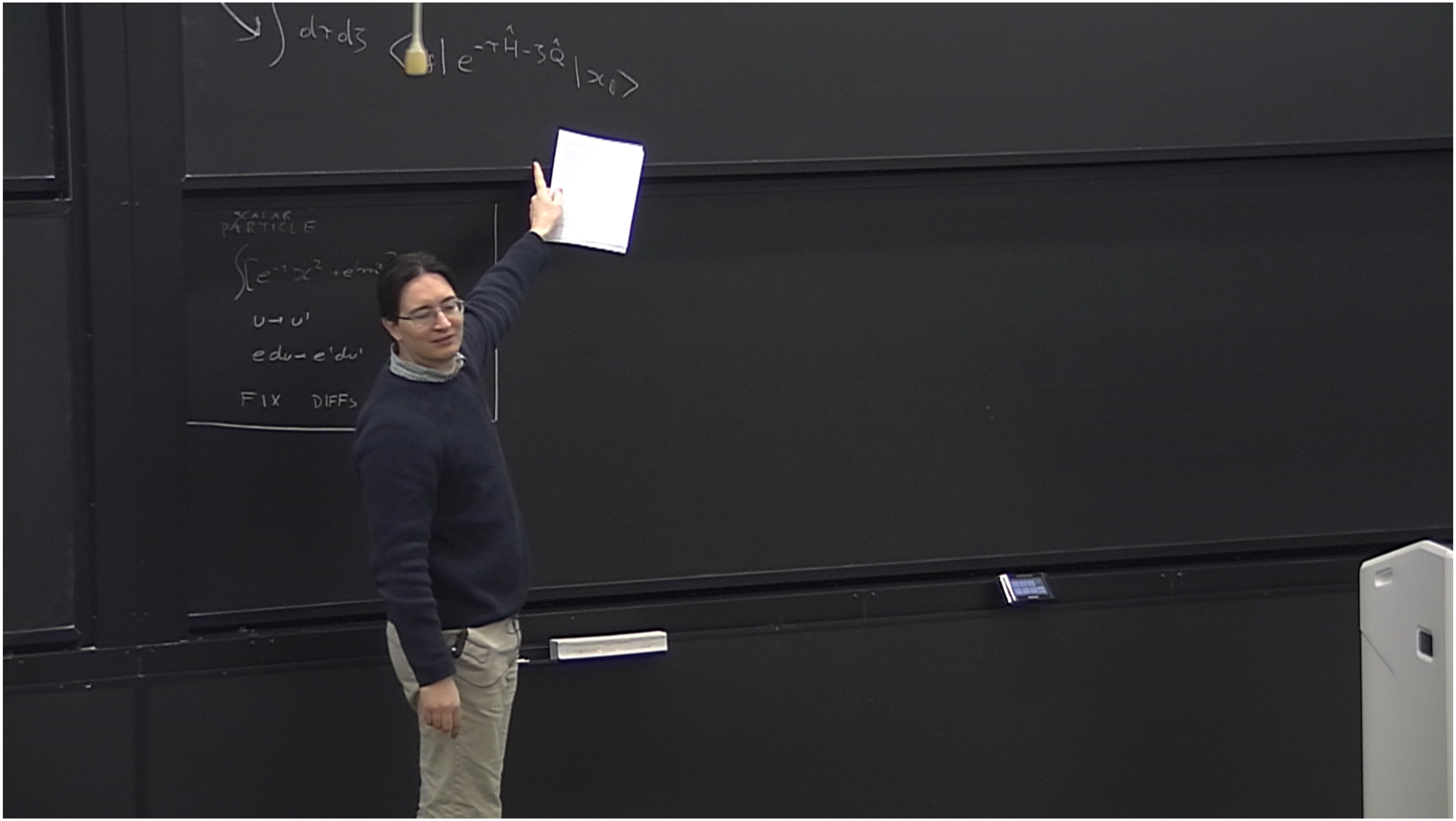

H = p²
Q = p_μ ψ^μ
Q² = H
[H, Q] = 0
{ψ^μ, ψ^ν} = 2η^μν
∫dτdζ e^{-τH-ζQ}

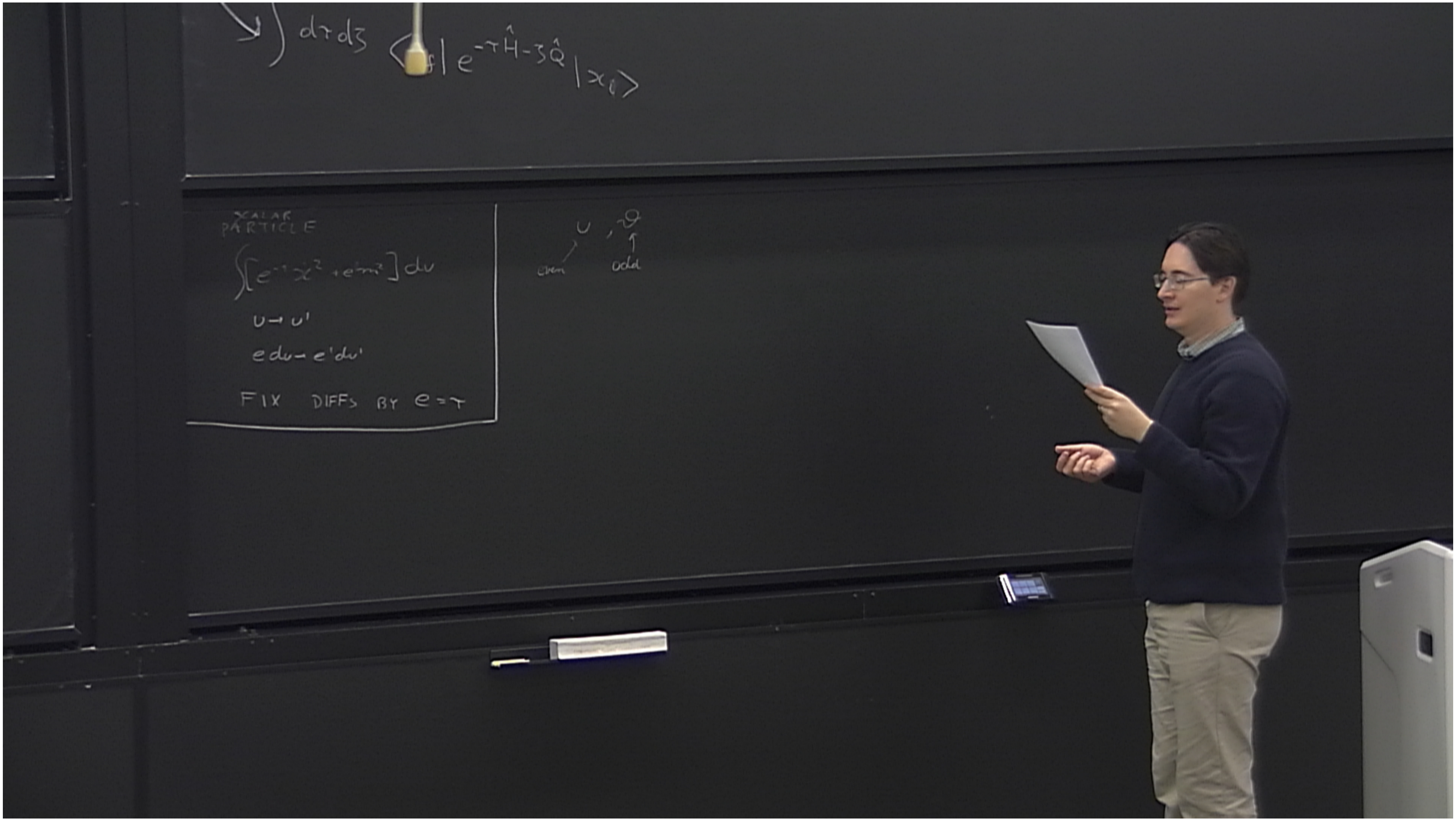

$\frac{p_\mu \gamma^\mu}{p^2} = \frac{p_\mu \psi^\mu}{p^2}$
$\{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}$
$H = \hat{p}^2$
$\hat{Q} = \hat{p}_\mu \hat{\psi}^\mu$
$\hat{Q}^2 = \hat{H}$
$\int d\tau\, d\zeta\; e^{-\tau p^2 - \zeta p_\mu \psi^\mu} = \int d\tau\, d\zeta\; e^{-\tau H - \zeta Q}$

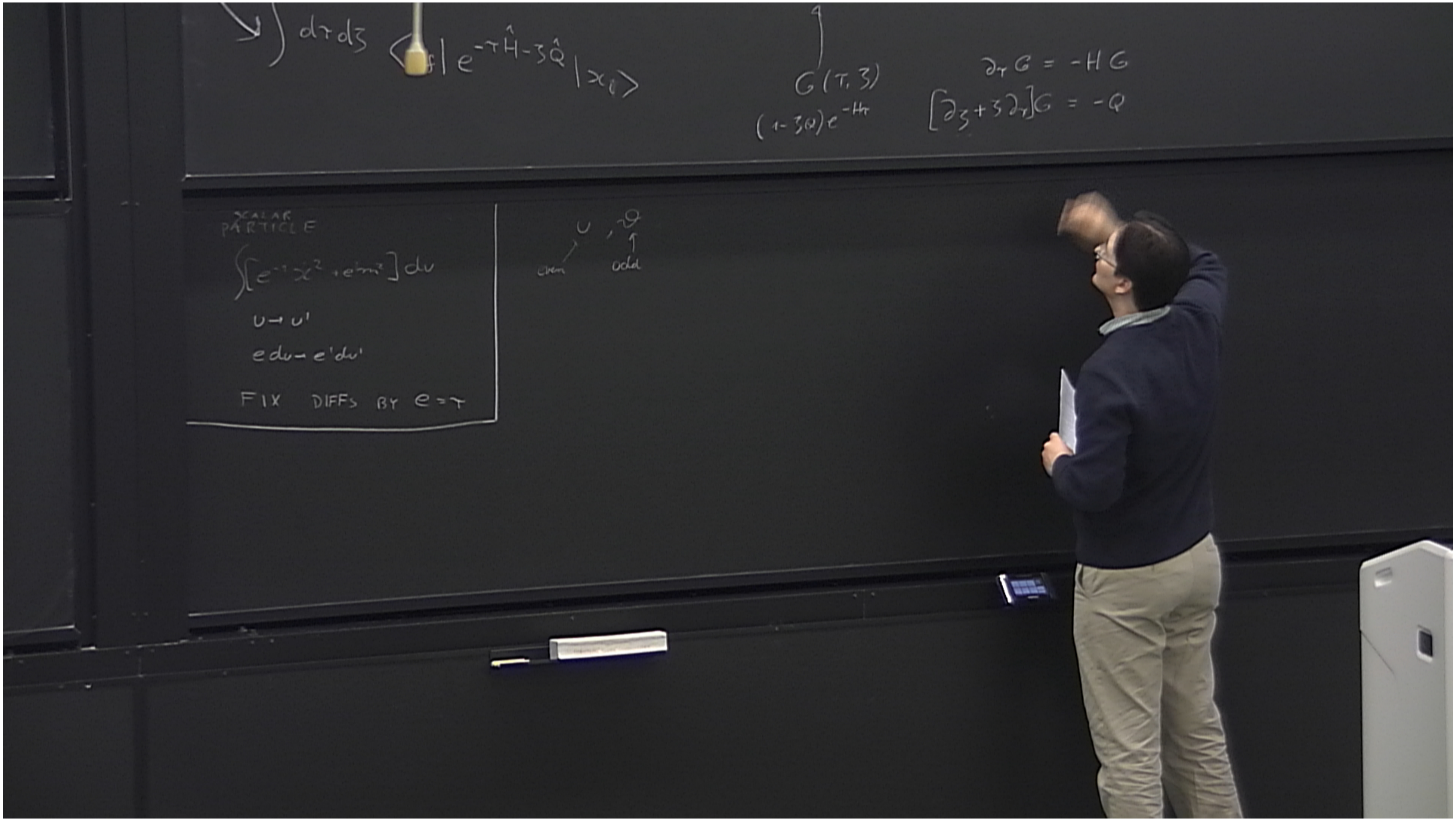

H = p̂²
Q̂ = p̂μ ψ̂μ
Q̂² = Ĥ
pμ γμ / p² = pμ ψμ / p²
{ψμ, ψν} = 2ημν
xμ, ψμ
= iημν
∫dτ dζ e^(−τp² − ζ pμ ψμ) = ∫dτ dζ e^(−τH − ζQ)
∫dp e^(ip(x−xf))
∫dτ dζ ⟨xf| e^(−τĤ − ζQ̂) |xi⟩

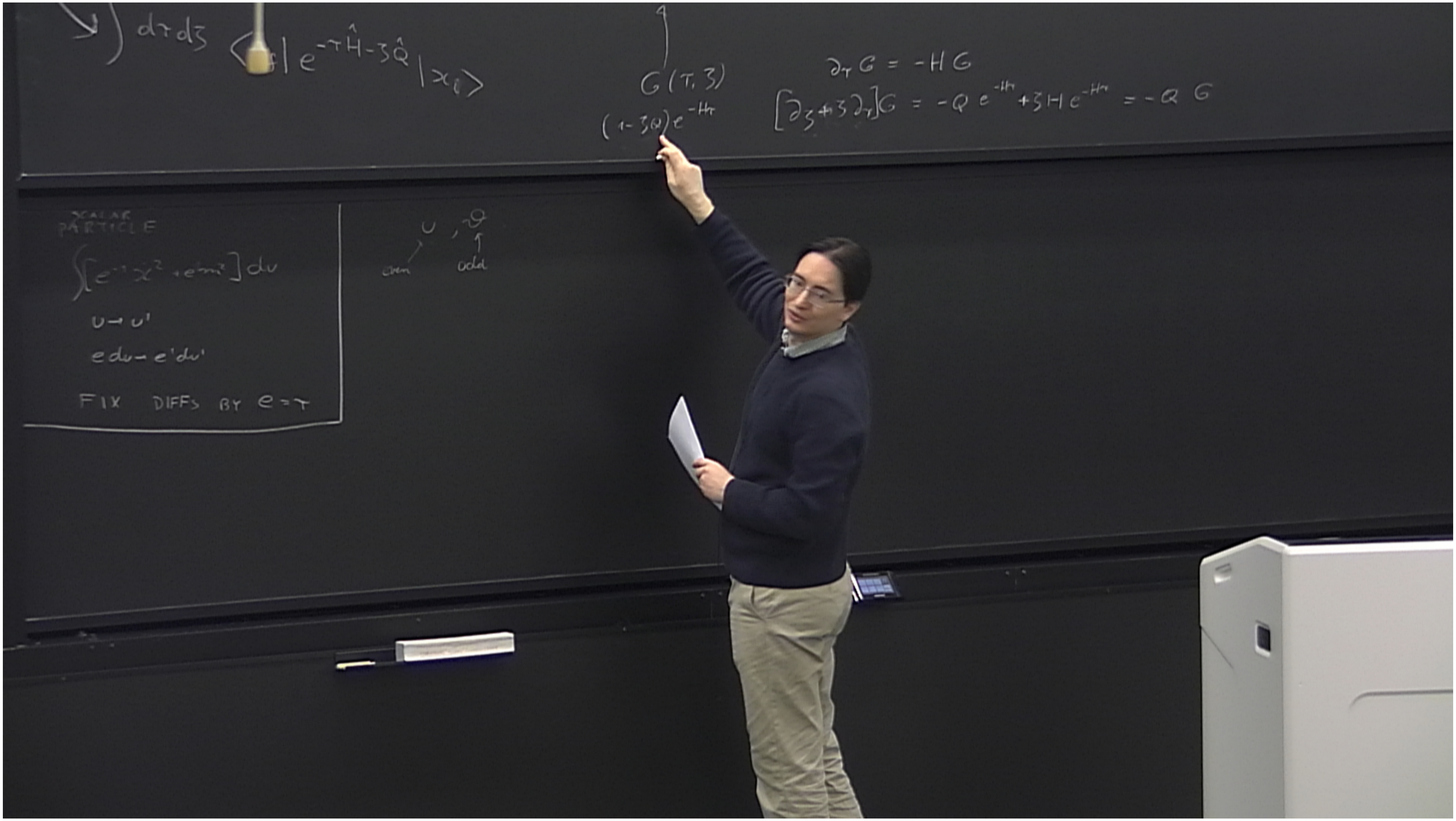


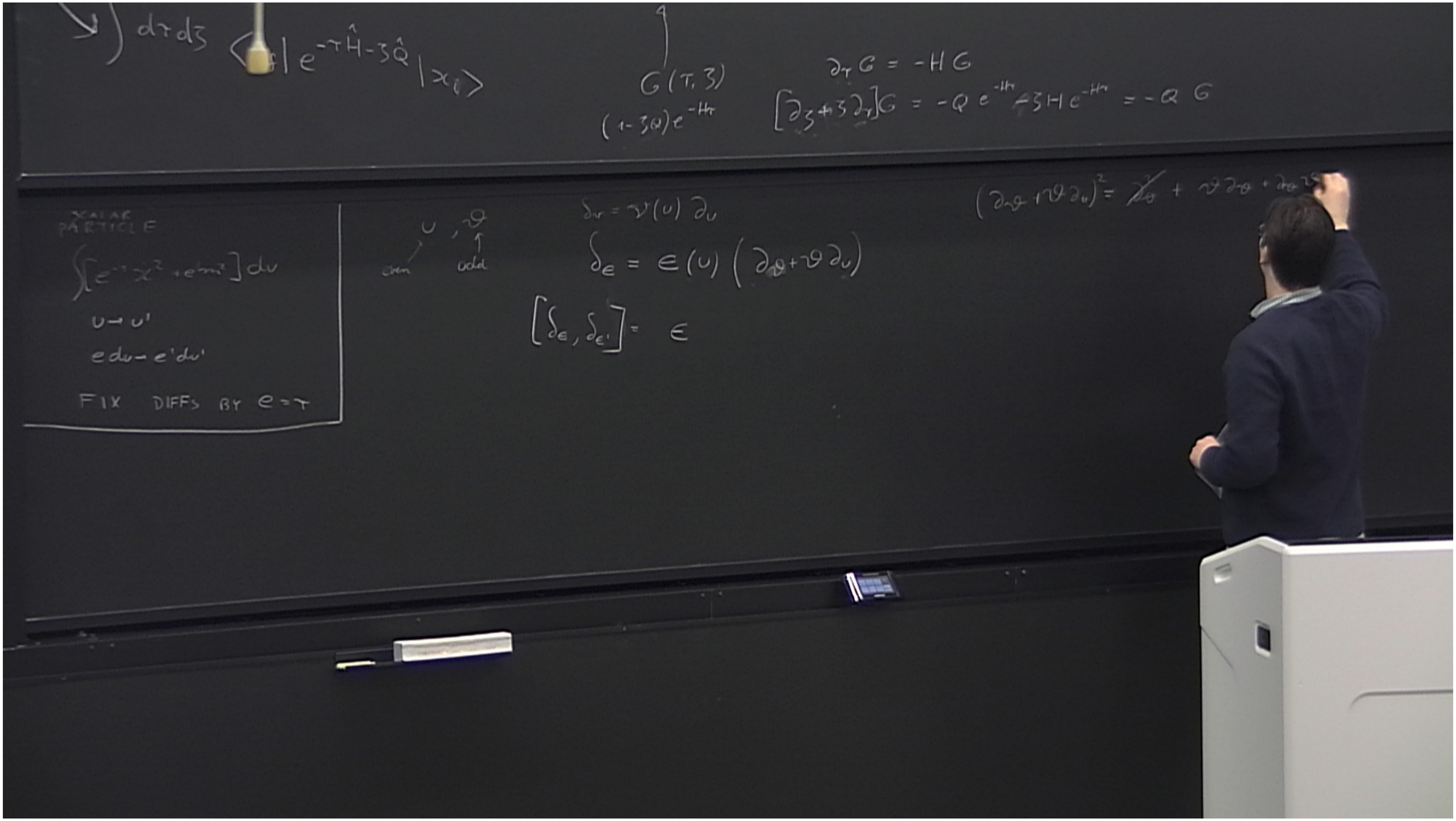

$\int d\tau\, dz \, \langle x_f | e^{-\tau \hat{H} - z\hat{Q}} | x_i \rangle$
SCALAR PARTICLE


∫ dτ dз ⟨ |e^{-τĤ-зQ̂}|x_i⟩
SCALAR PARTICLE
u → u'
e du → e'du'
FIX DIFFS

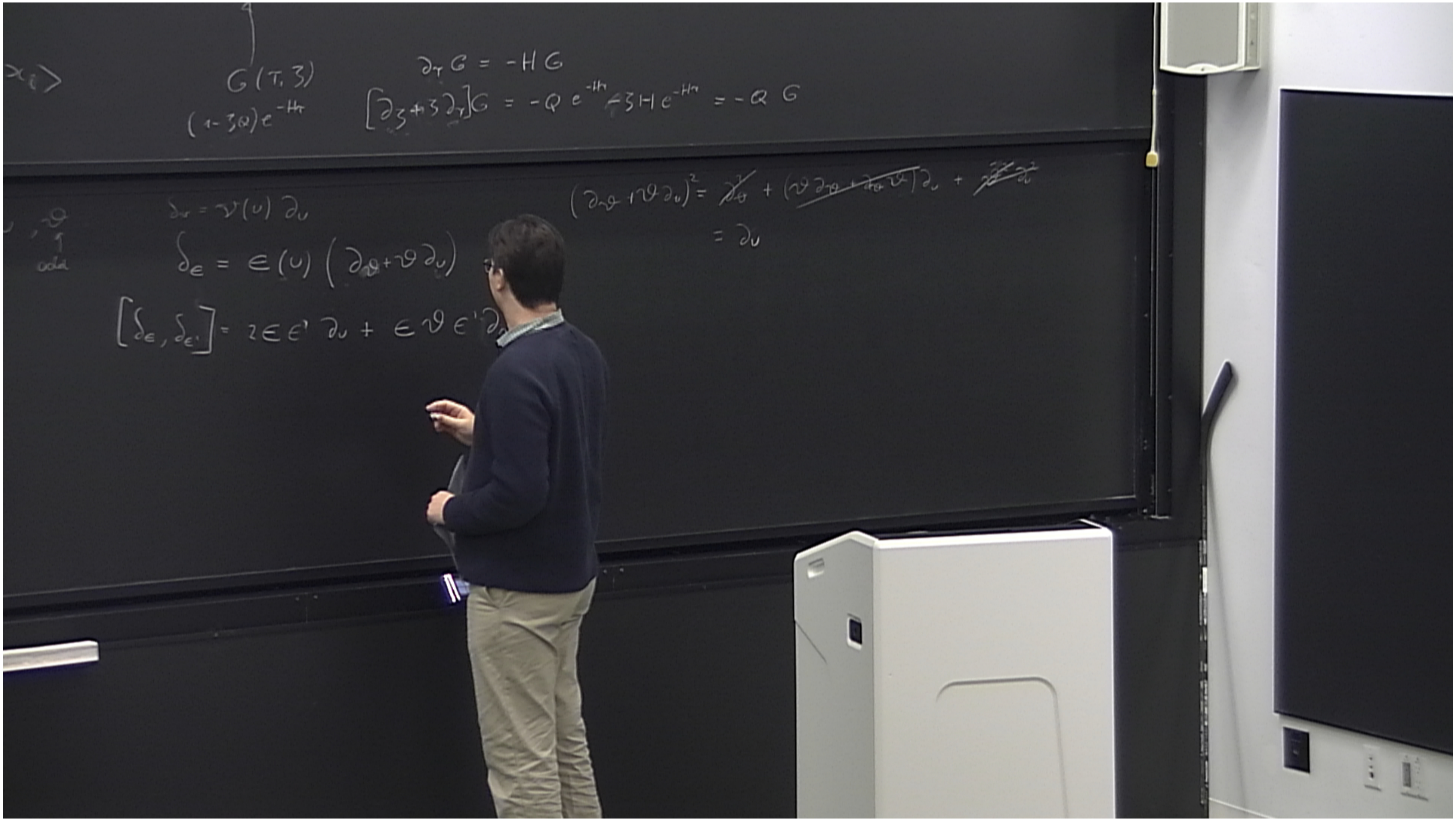

PARTICLE
even
odd
FIX DIFFS BY e = τ

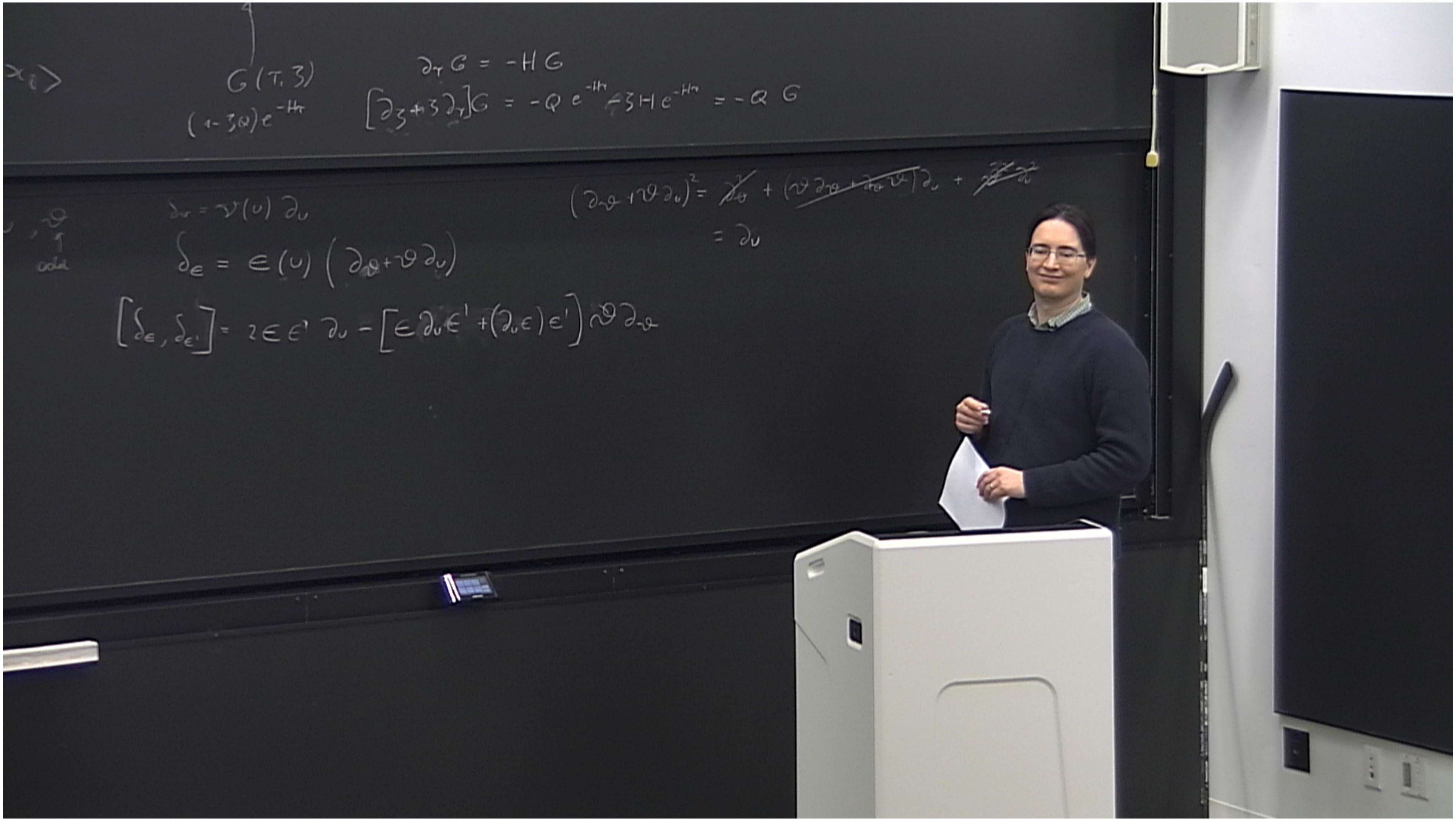

SCALAR PARTICLE
FIX DIFFS BY e = τ
even
odd

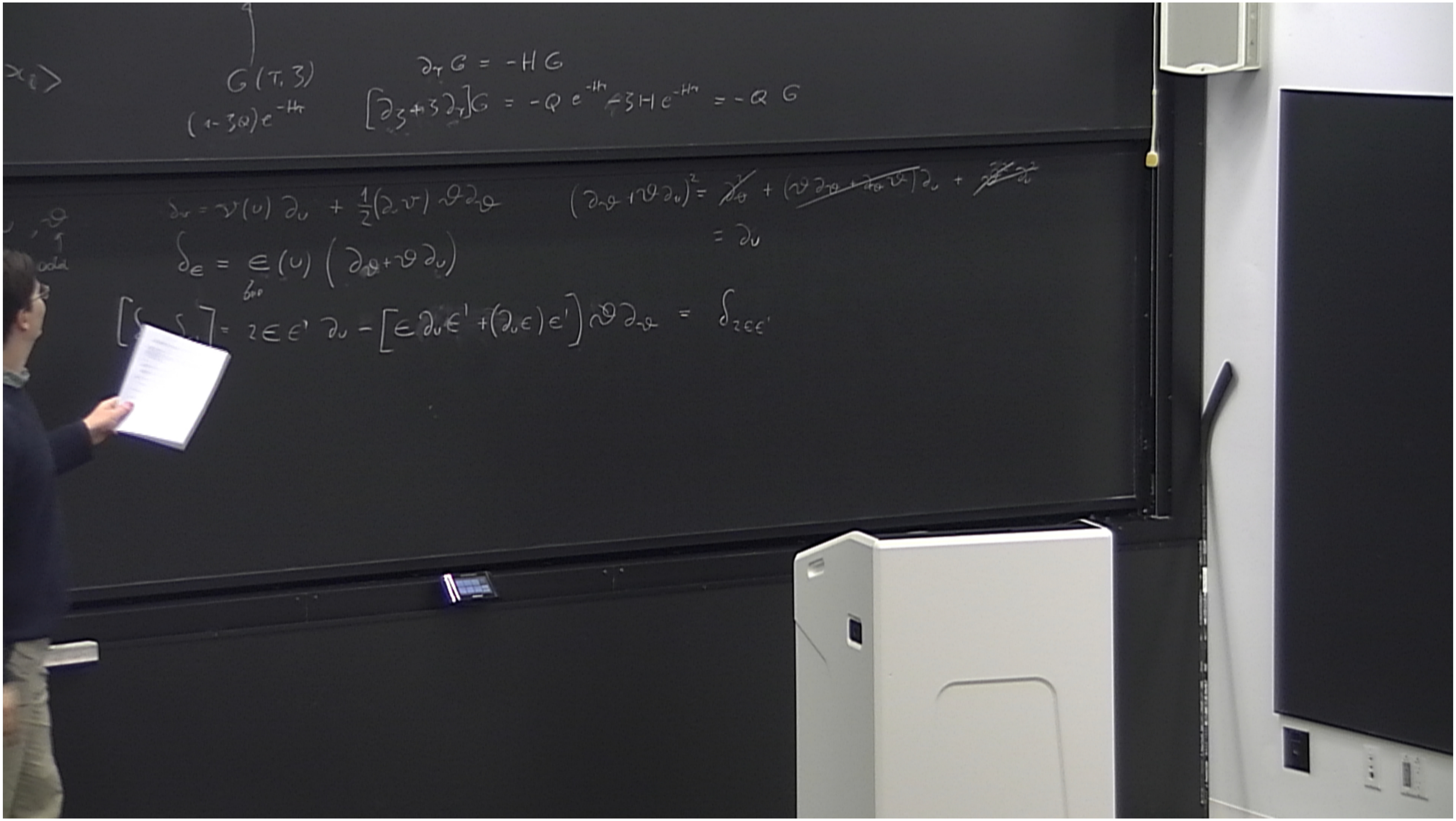

G(τ, ʒ)
∂τ G = -H G
SCALAR PARTICLE
u → u'
FIX DIFFS BY e = τ
even
odd

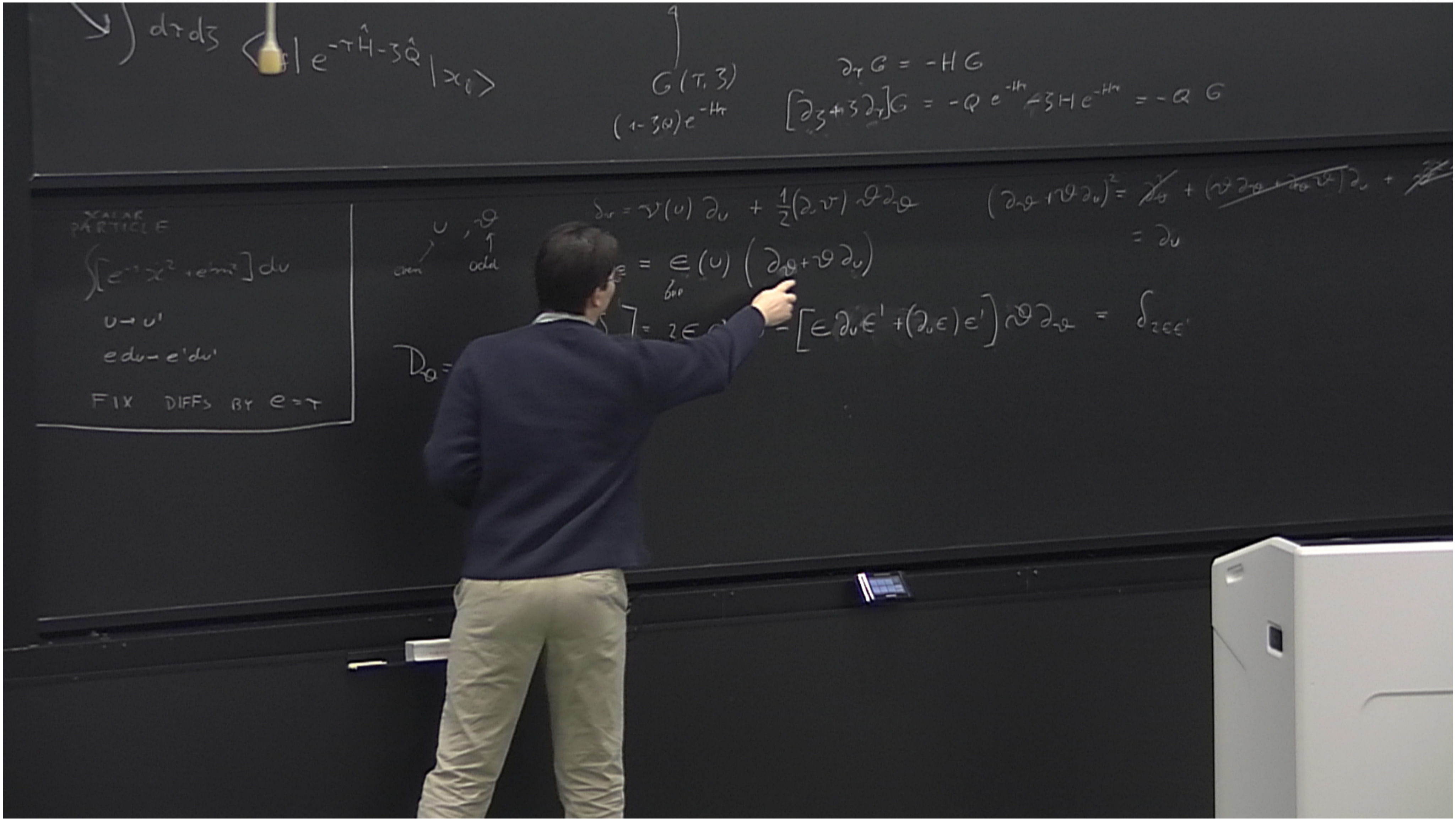


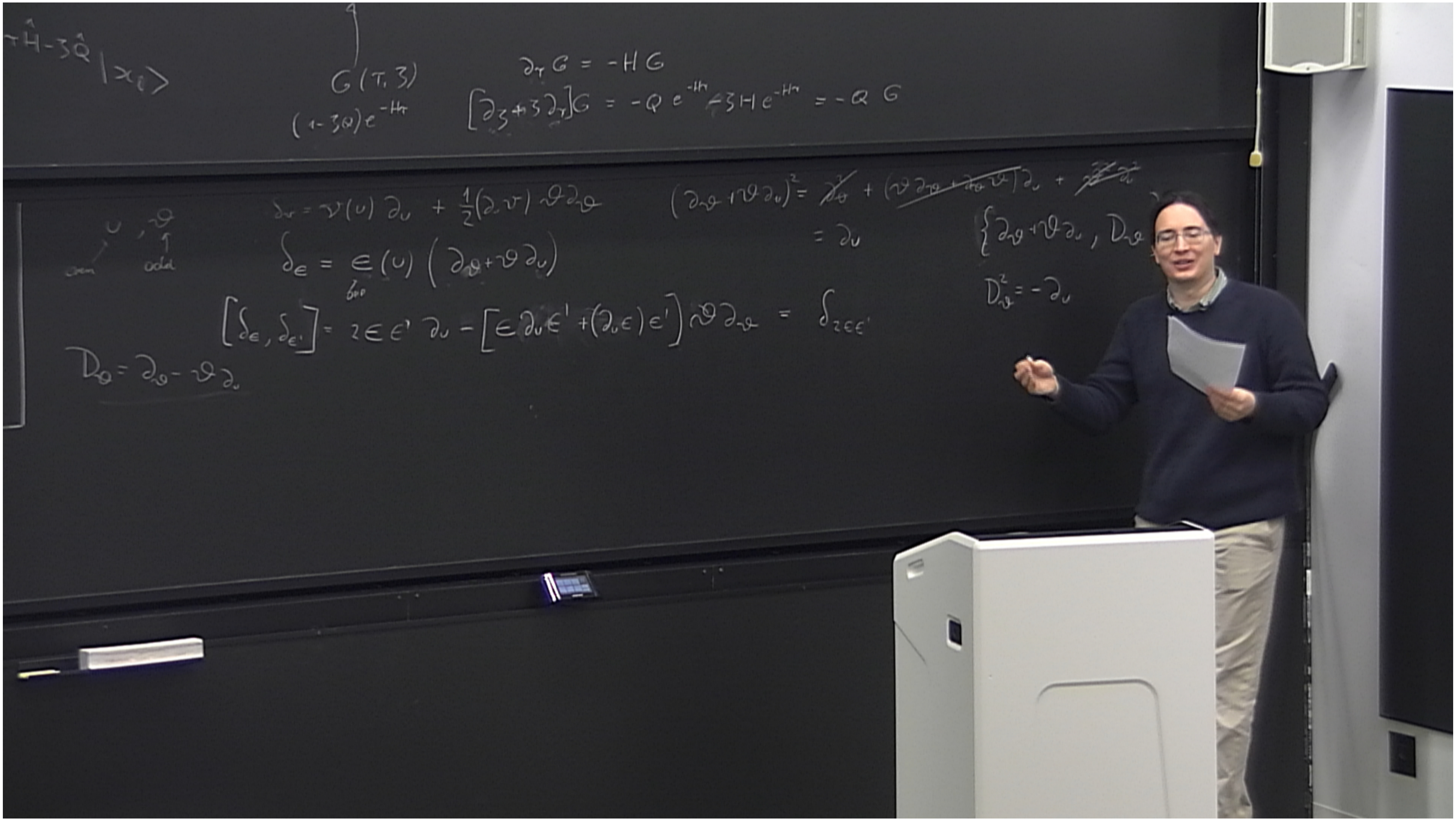
$G(\tau, 3)$
$\partial_\tau G = -HG$
SCALAR PARTICLE
$\int [e^{-1}\dot{x}^2 + e m^2] \, du$
$u \to u'$
$e\,du \to e'\,du'$
FIX DIFFS BY $e = \tau$
even
odd
$\delta_\epsilon = \epsilon(u)\,(\partial_\vartheta + \vartheta \partial_u)$
$[\delta_\epsilon, \delta_{\epsilon'}] = \epsilon$

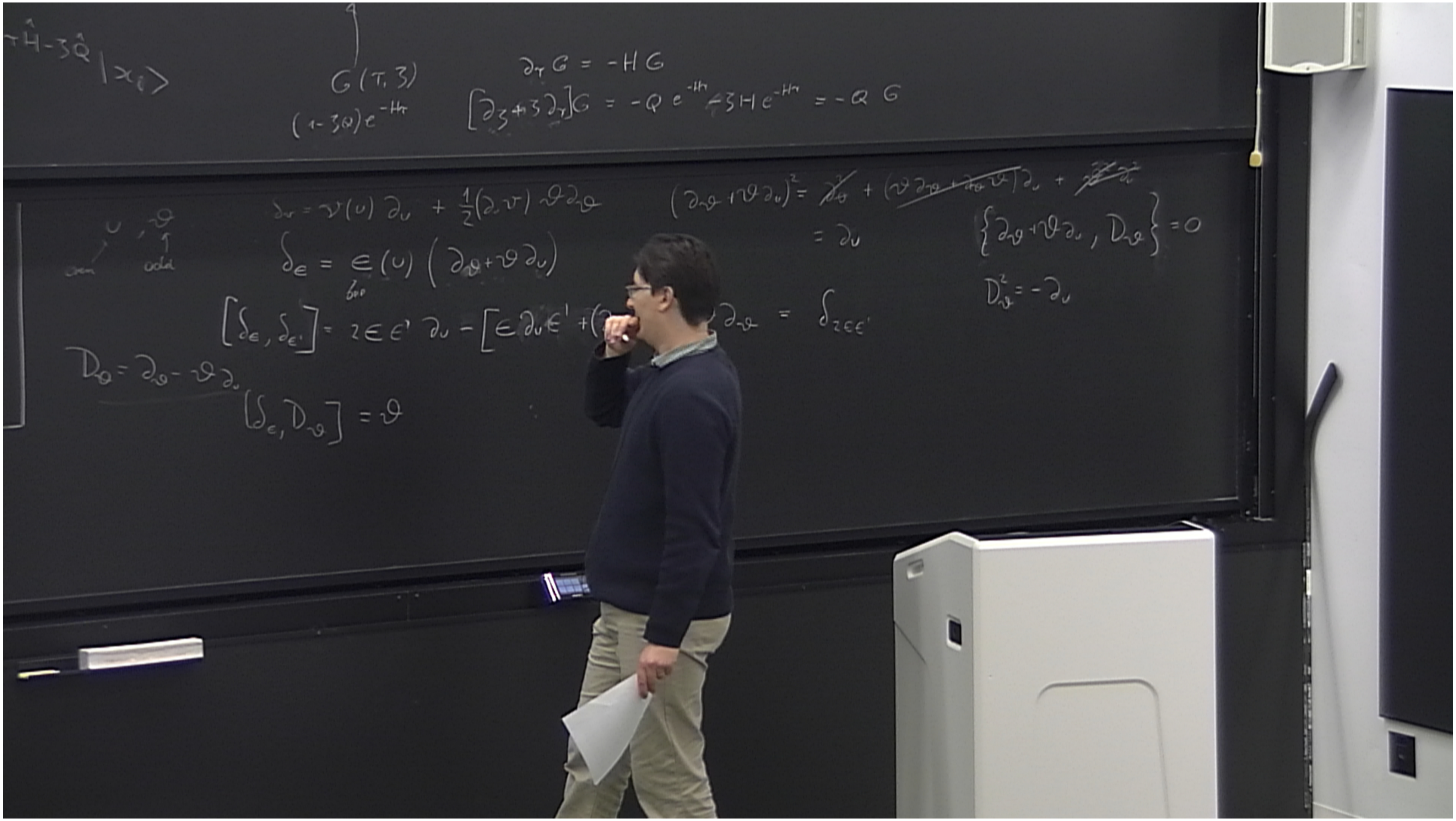

$G(\tau, z)$
$\partial_\tau G = -HG$
$\delta_v = v(u)\,\partial_u$
$\delta_\epsilon = \epsilon(u)\left(\partial_\vartheta + \vartheta\,\partial_u\right)$
$= \partial_u$

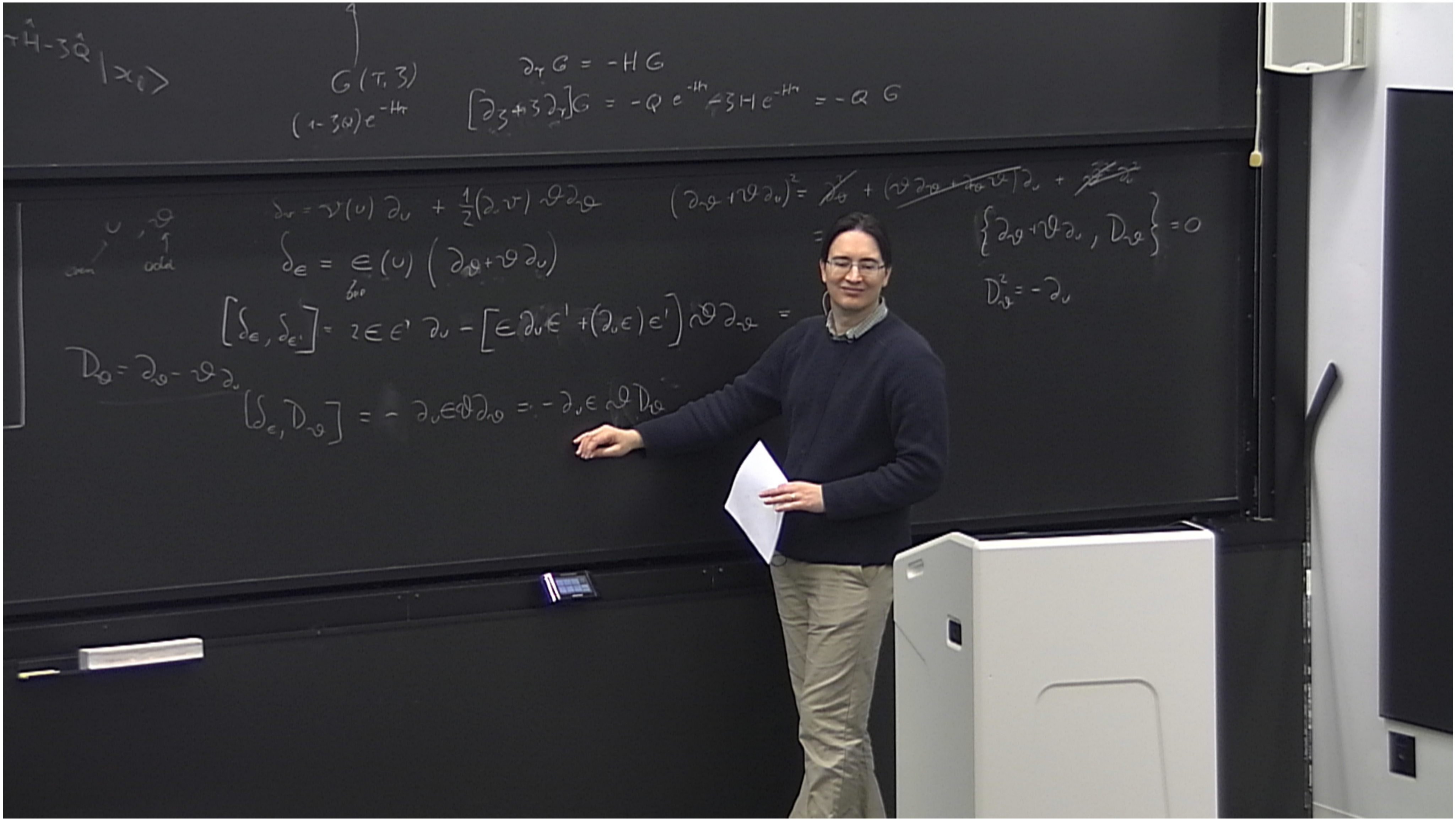

G(T, 3)
∂_T G = - H G
(1-3Q)e^{-Hτ}
[∂_3 + 3∂_T]G = - Q e^{-Hτ} - 3H e^{-Hτ} = - Q G
δ_ε = ε(u) (∂_θ + θ ∂_u)
= ∂_u
odd

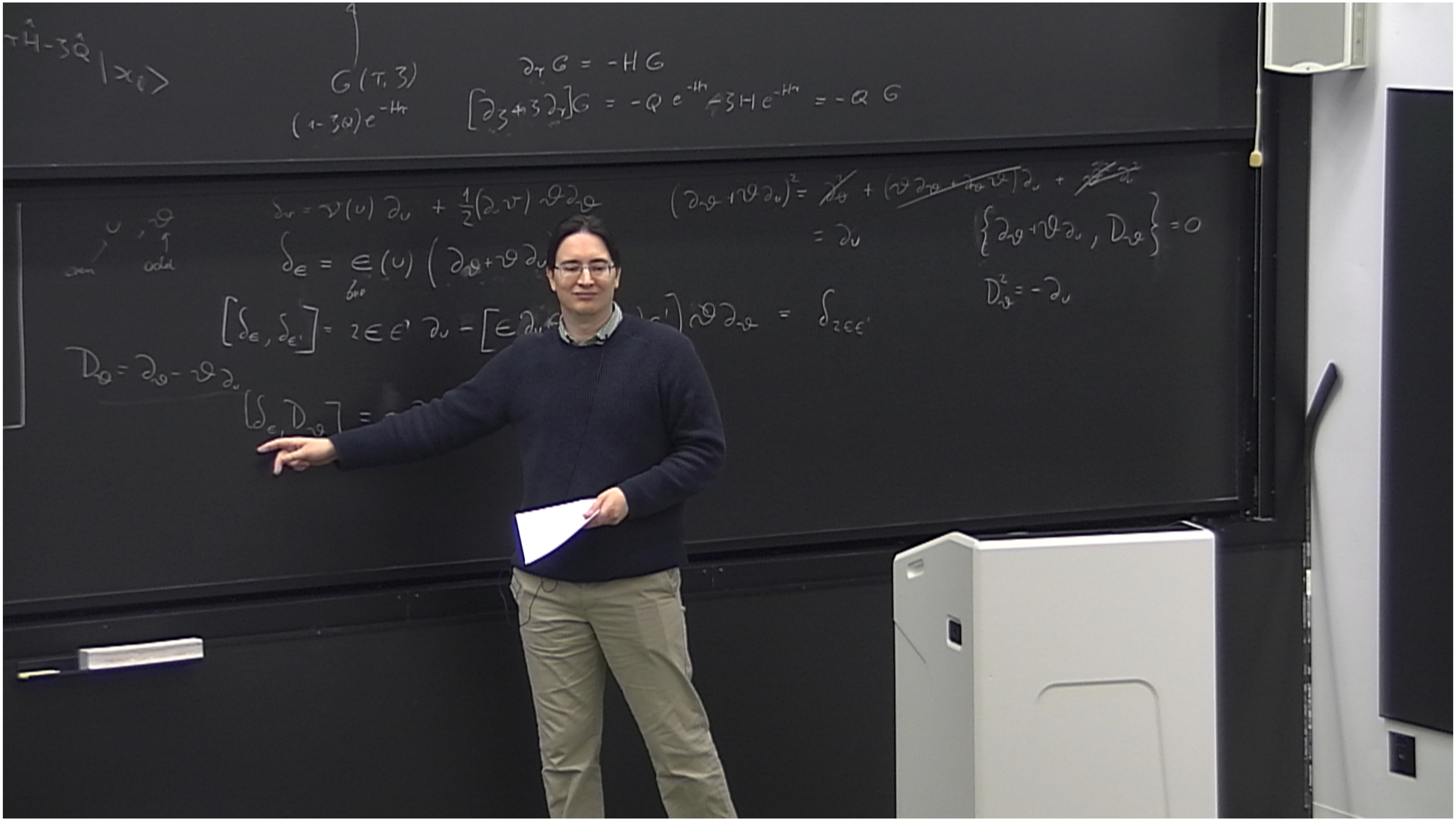

$G(\tau, \zeta)$
$\partial_\tau G = -HG$
$(1-3Q)e^{-H\tau}$
$[\partial_\zeta + 3\partial_\tau]G = -Qe^{-H\tau} - 3He^{-H\tau} = -QG$
$\delta_v = v(u)\partial_u + \frac{1}{2}(\partial_u v)\vartheta\partial_\vartheta$
$(\partial_\vartheta + \vartheta\partial_u)^2 = \partial_u$
odd
$\delta_\epsilon = \epsilon(u)(\partial_\vartheta + \vartheta\partial_u)$
$[\delta_\epsilon, \delta_{\epsilon'}] = 2\epsilon\epsilon'\partial_u - [\epsilon\partial_u\epsilon' + (\partial_u\epsilon)\epsilon']\vartheta\partial_\vartheta = \delta_{2\epsilon\epsilon'}$

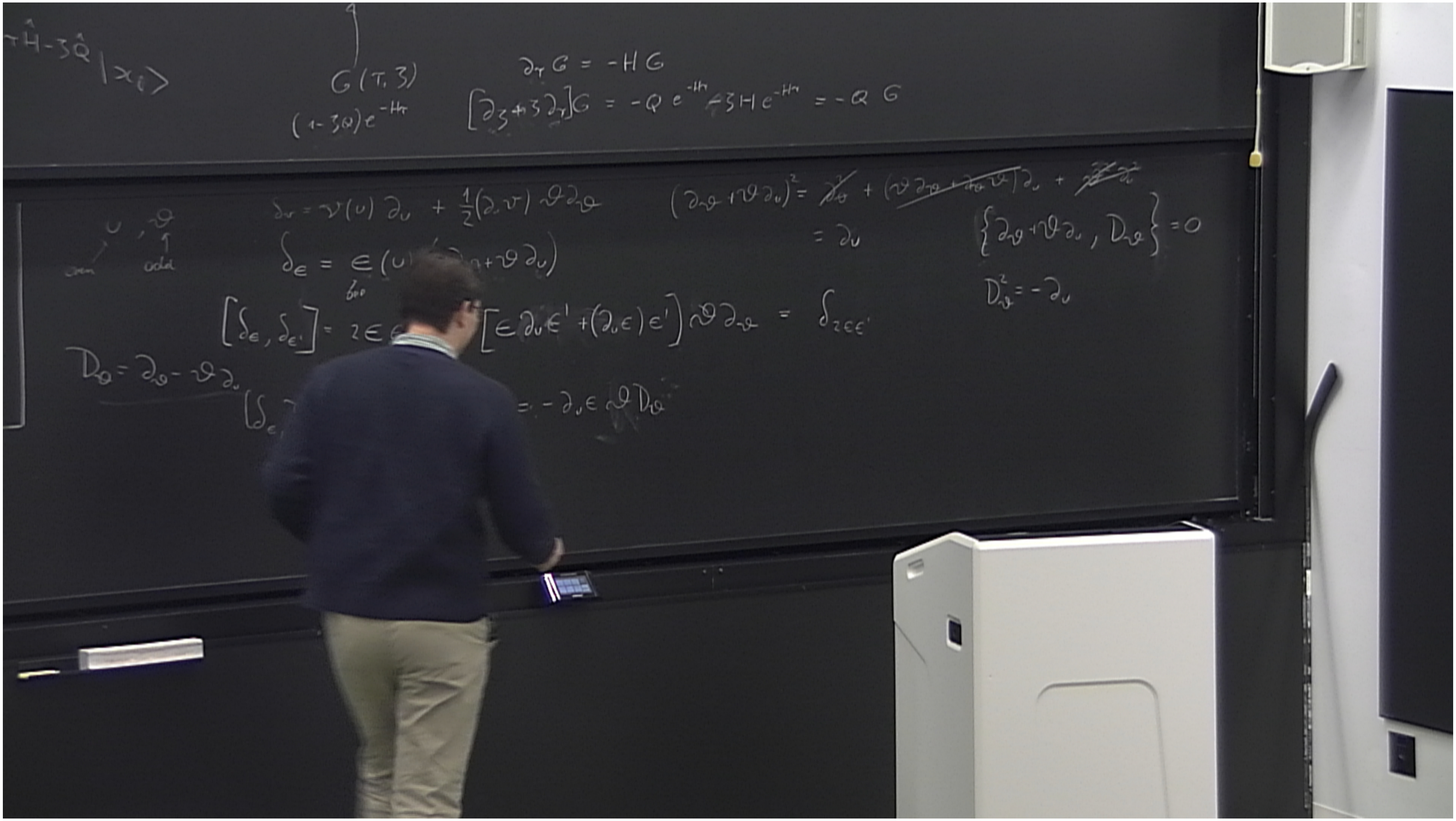

$\partial_\tau G = -HG$
FIX DIFFS BY $e = \tau$
even
odd

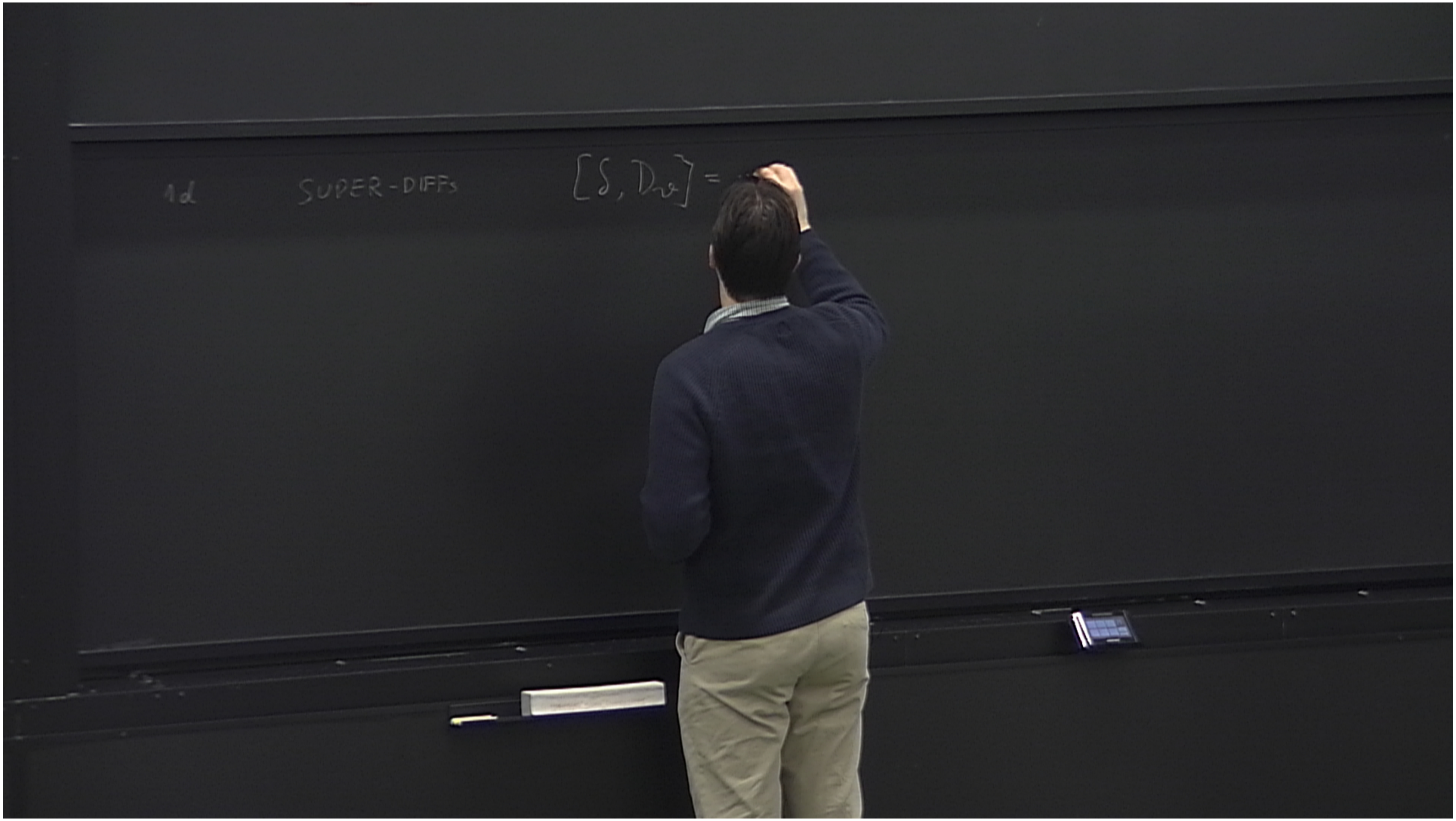

$\partial_\tau G = -HG$
$G(\tau, 3)$
$\delta_\epsilon = \epsilon(u)(\partial_\theta + \theta\partial_u)$
$D_\theta = \partial_\theta - \theta\partial_u$
$D_\theta^2 = -\partial_u$
even
odd

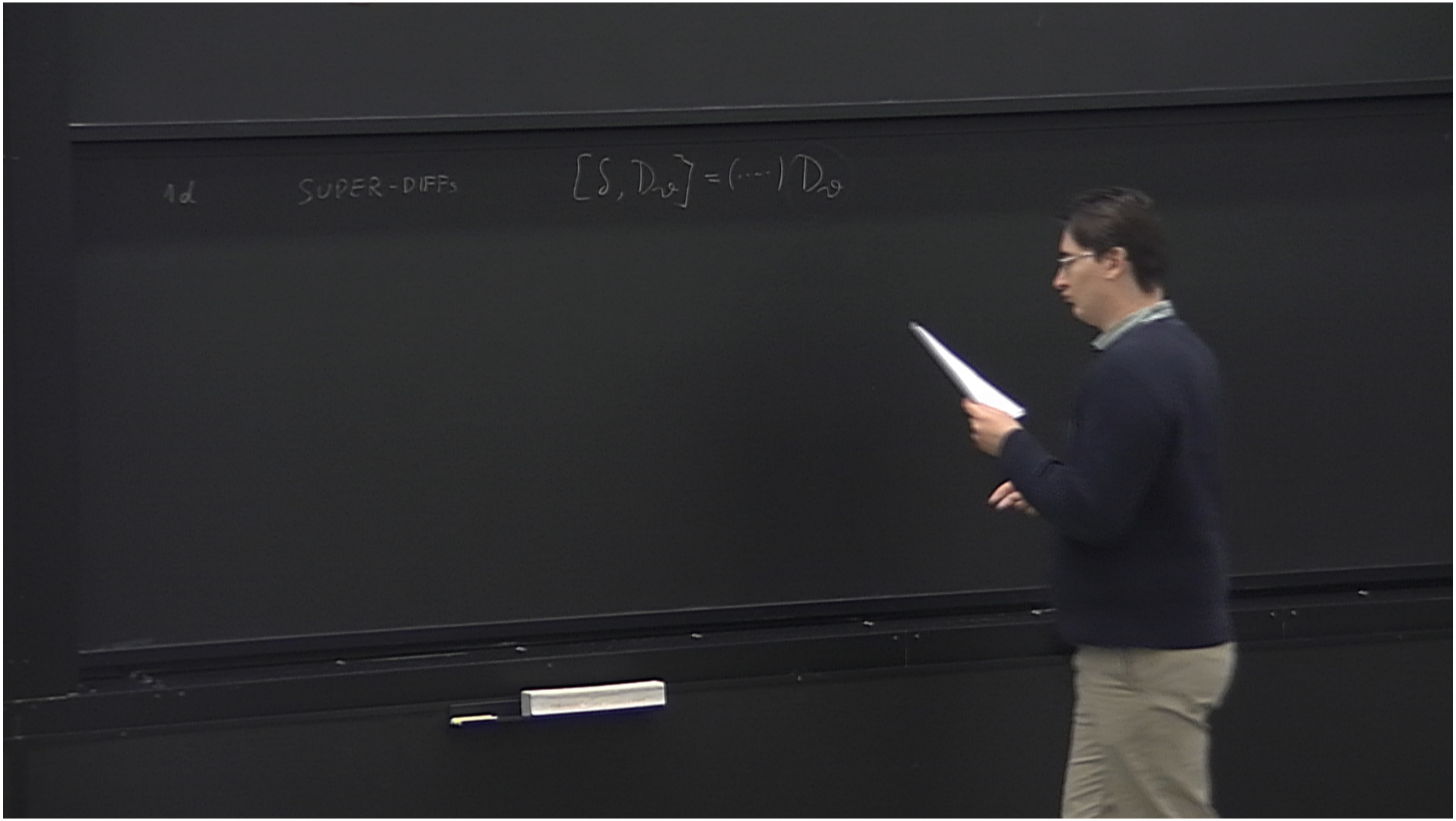

G(τ, 3)
∂τ G = −H G
(1−3Q)e^{−Hτ}
even
odd
D_ϑ² = −∂_u
[δ_ε, D_ϑ] = ϑ

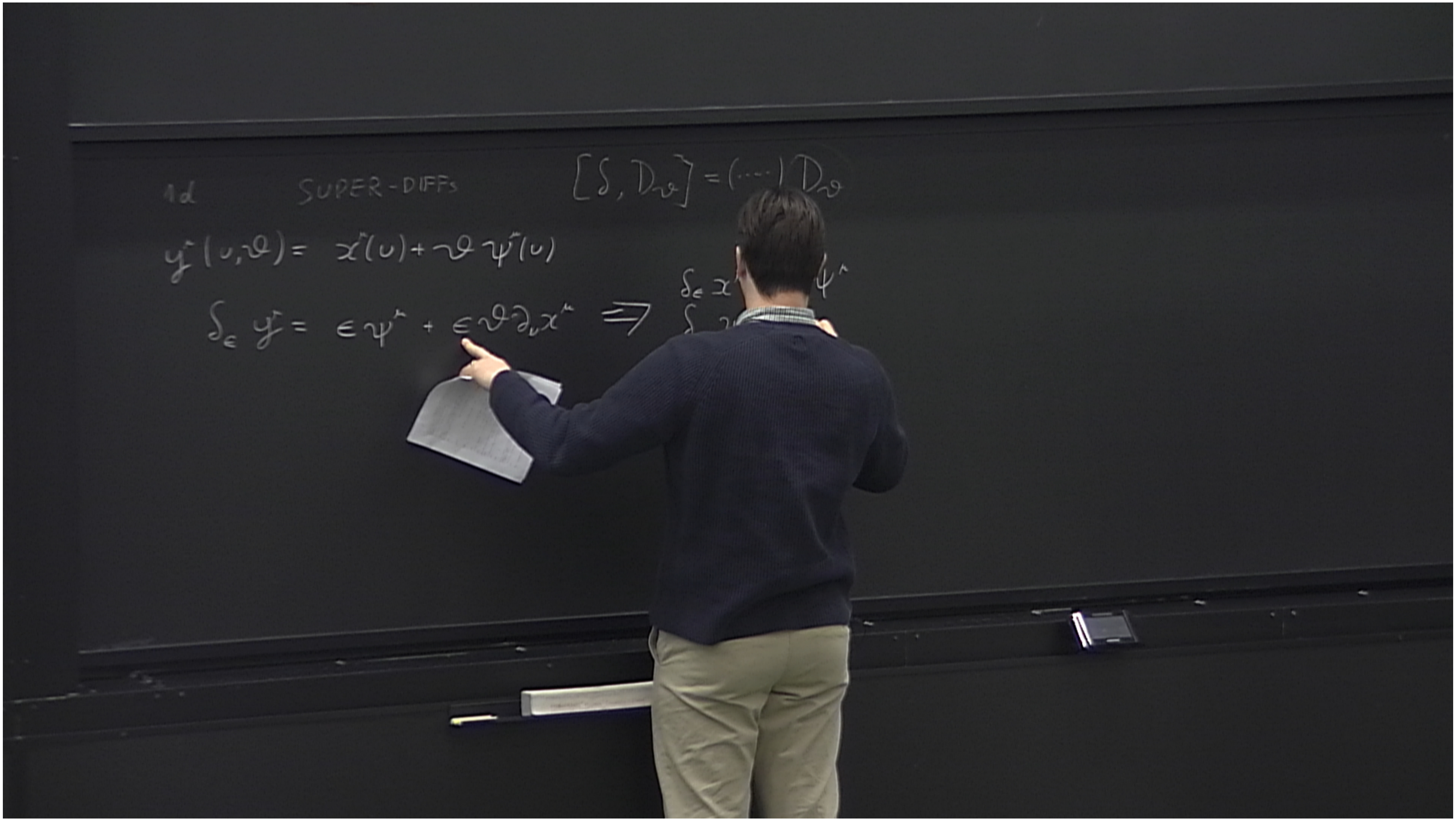


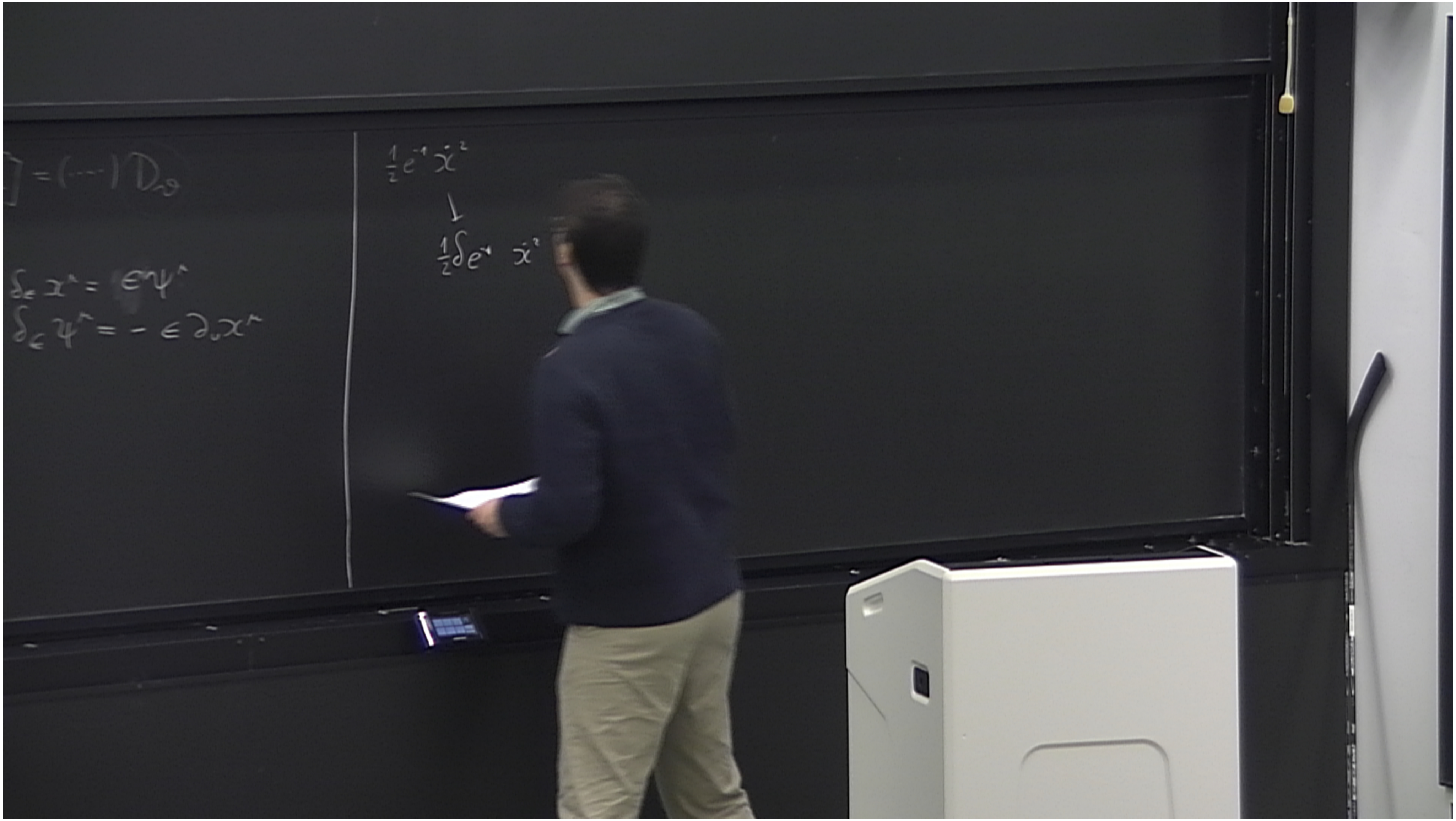


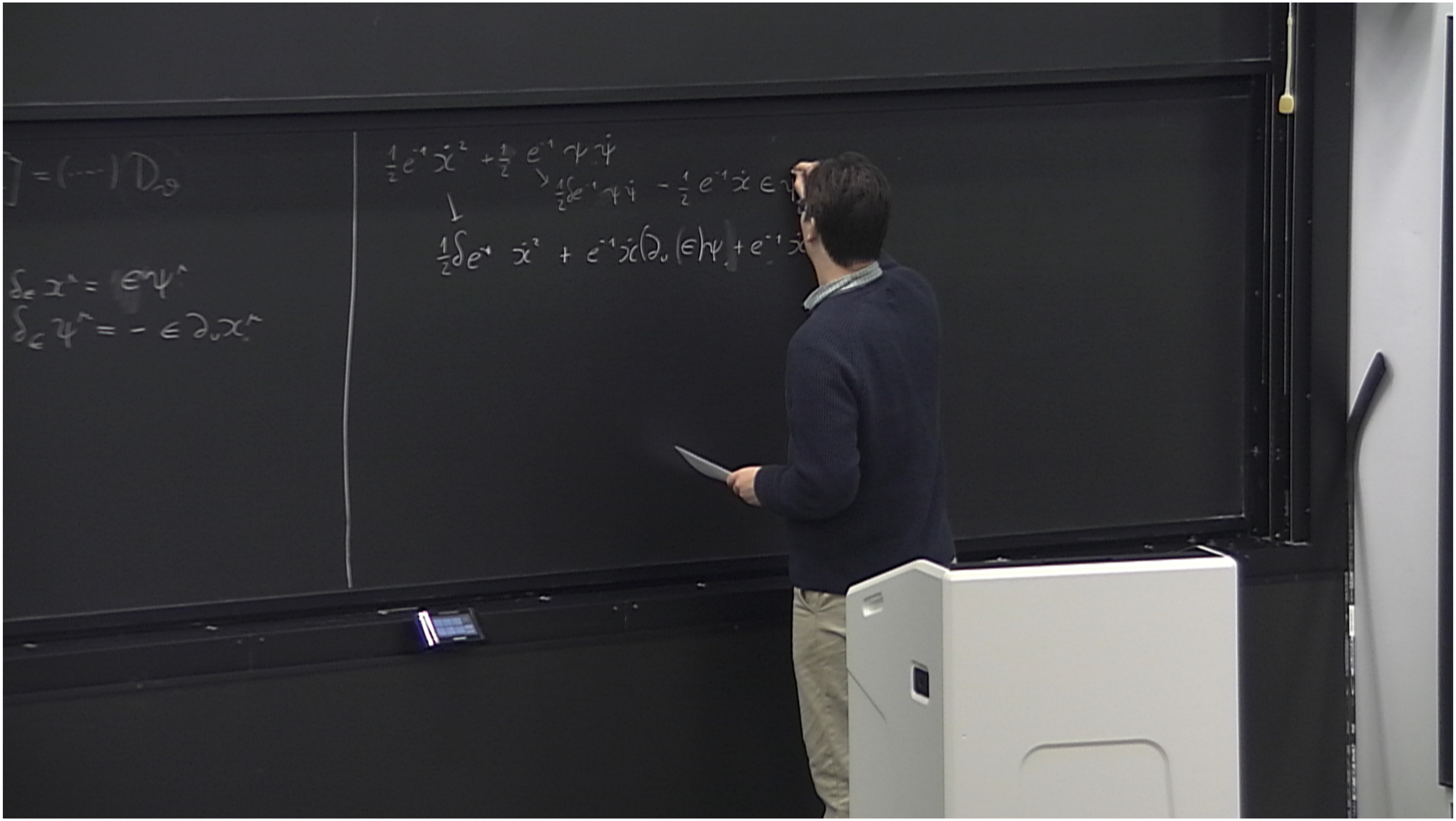

∂τ G = − H G
G(τ, ȝ)
{∂θ + θ∂u , Dθ} = 0
Dθ² = − ∂u
Dθ = ∂θ − θ∂u
odd


1d
SUPER-DIFFs
[S, D_θ] =

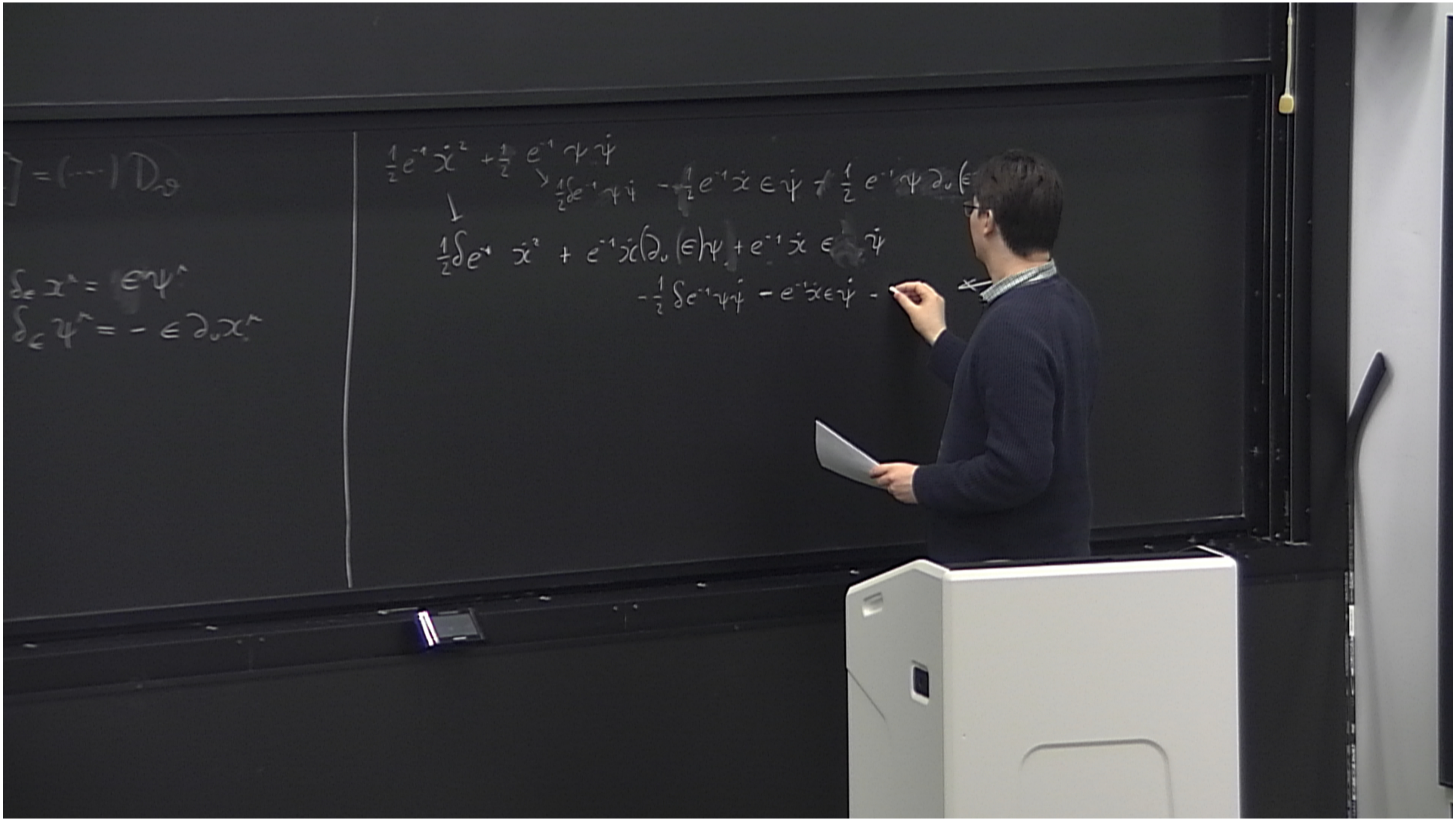

1d
SUPER-DIFFs
[δ, Dθ] = (· · · ·) Dθ

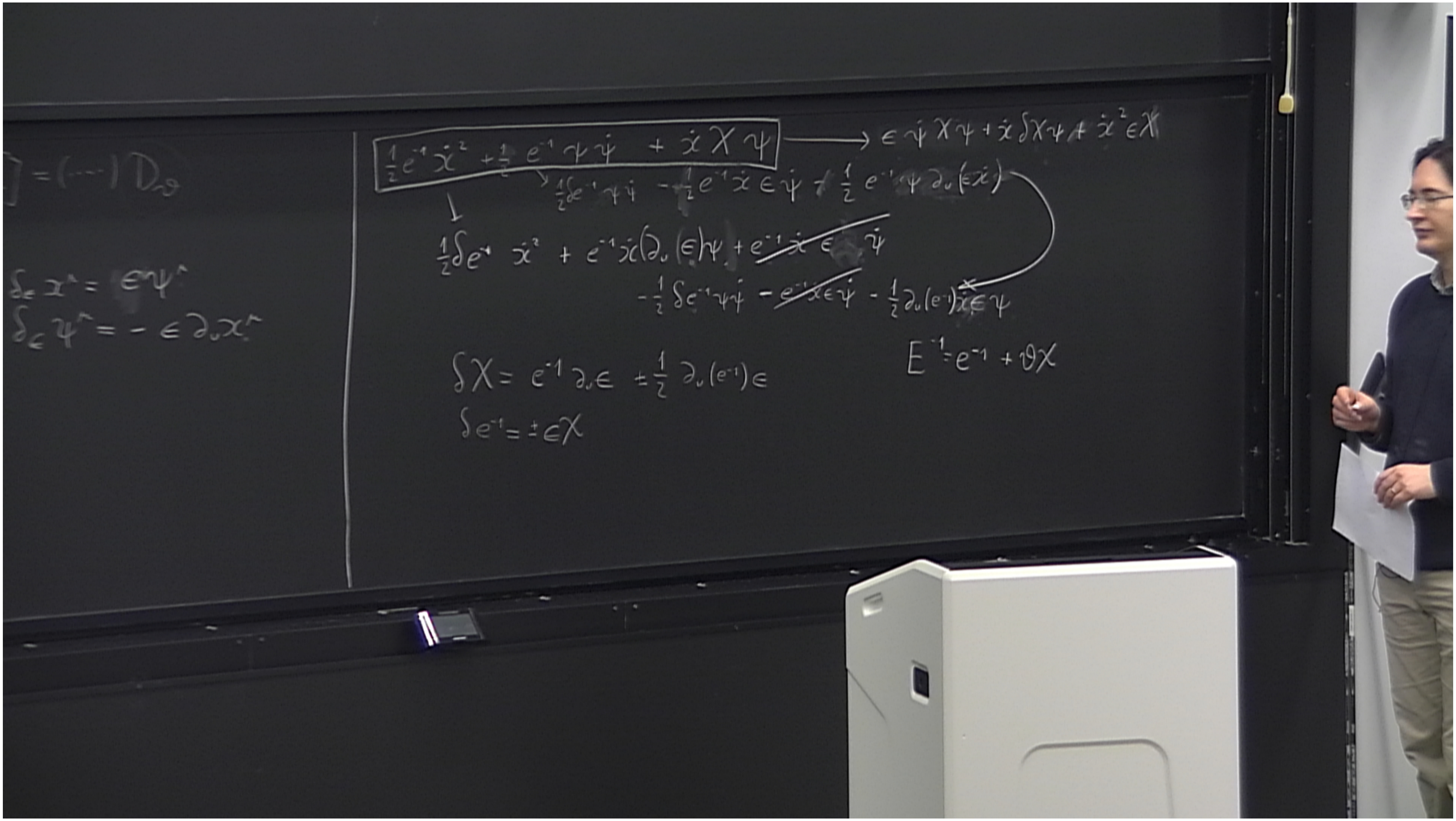

1d
SUPER-DIFFs
[δ, D_ϑ] = (....) D_ϑ
y^μ(υ,ϑ) = x^μ(υ) + ϑψ^μ(υ)
δ_ε y^μ = εψ^μ + εϑ∂_υ x^μ ⟹

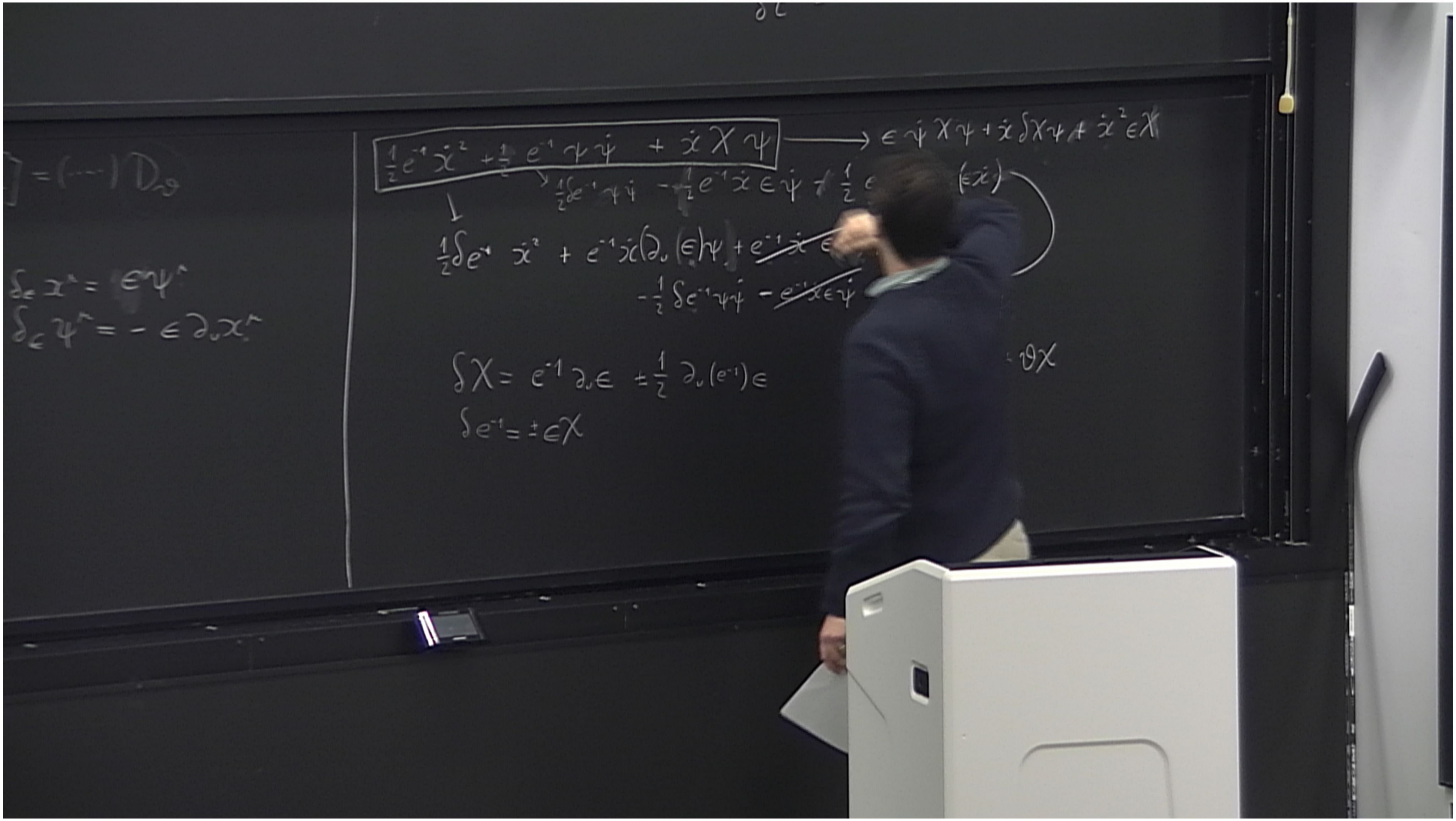

$\delta_\epsilon x^\mu = \epsilon\psi^\mu$
$\delta_\epsilon \psi^\mu = -\epsilon\partial_v x^\mu$
$\frac{1}{2}e^{-1}\dot{x}^2$
$\frac{1}{2}\delta e^{-1}\;\dot{x}^2$



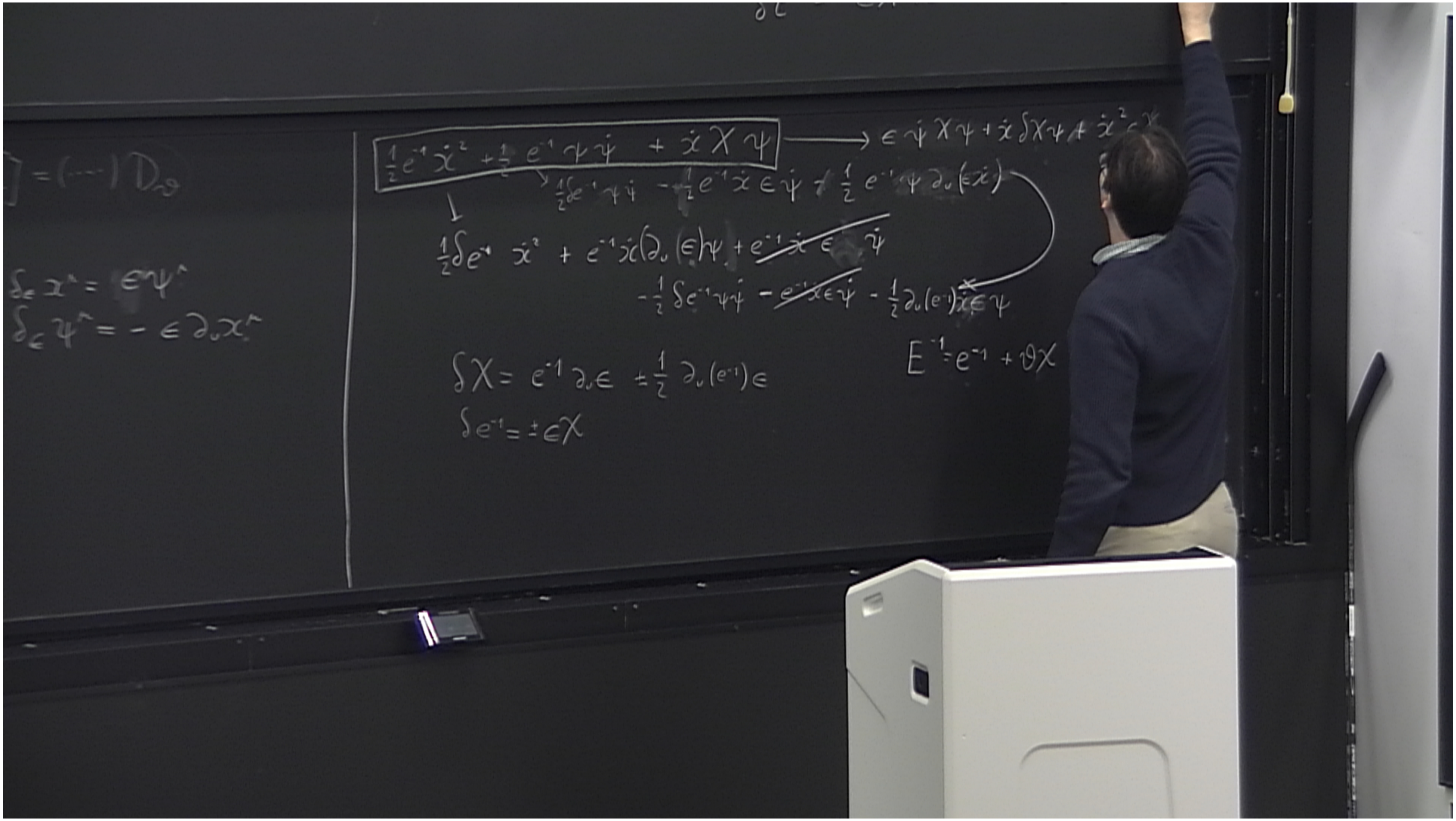
$\frac{1}{2}e^{-1}\dot{x}^2 + \frac{1}{2}e^{-1}\psi\dot{\psi}$
$\delta_\epsilon x^\mu = \epsilon\psi^\mu$
$\delta_\epsilon \psi^\mu = -\epsilon\partial_\tau x^\mu$


$\frac{1}{2} e^{-1} \dot{x}^2 + \frac{1}{2} e^{-1} \psi \dot{\psi}$
$\delta_\epsilon x^\mu = \epsilon \psi^\mu$
$\delta_\epsilon \psi^\mu = - \epsilon \partial_\tau x^\mu$


$\frac{1}{2} e^{-1} \dot{x}^2 + \frac{1}{2} e^{-1} \psi \dot{\psi} + \dot{x} \chi \psi$
$\delta_\epsilon x^\mu = \epsilon \psi^\mu$
$\delta_\epsilon \psi^\mu = - \epsilon \partial_\nu x^\mu$
$\delta \chi = e^{-1} \partial_\nu \epsilon \pm \frac{1}{2} \partial_\nu (e^{-1}) \epsilon$
$\delta e^{-1} = \pm \epsilon \chi$
$E^{-1} = e^{-1} + \theta \chi$



$\delta_\epsilon x^\mu = \epsilon\psi^\mu$
$\delta_\epsilon \psi^\mu = -\epsilon\partial_u x^\mu$
$\delta\chi = e^{-1}\partial_u\epsilon \pm \frac{1}{2}\partial_u(e^{-1})\epsilon$
$\delta e^{-1} = \pm\epsilon\chi$


δ_ε x^μ = ε ψ^μ
δ_ε ψ^μ = − ε ∂_υ x^μ
δχ = e^{-1} ∂_υ ε ± 1/2 ∂_υ(e^{-1}) ε
δe^{-1} = ± εχ
E^{-1} = e^{-1} + ϑχ

$$= \epsilon(\partial_\tau \ldots + \vartheta\partial_\tau)E^{-1} + (\cdots)E^{-1}$$

$$[\ \ ] = (\cdots)D_\vartheta$$

$$\delta_\epsilon x^\mu = \epsilon\psi^\mu$$

$$\delta_\epsilon \psi^\mu = -\epsilon\partial_\tau x^\mu$$

$$\boxed{\tfrac12 e^{-1}\dot x^2 + \tfrac12 e^{-1}\psi\dot\psi + \dot x\chi\psi} \longrightarrow \epsilon\dot\psi\chi\psi + \dot x\delta\chi\psi + \dot x^2\epsilon\chi$$

$$\tfrac12\delta e^{-1}\psi\dot\psi - \tfrac12 e^{-1}\dot x\epsilon\dot\psi + \tfrac12 e^{-1}\psi\partial_\tau(\epsilon\dot x)$$

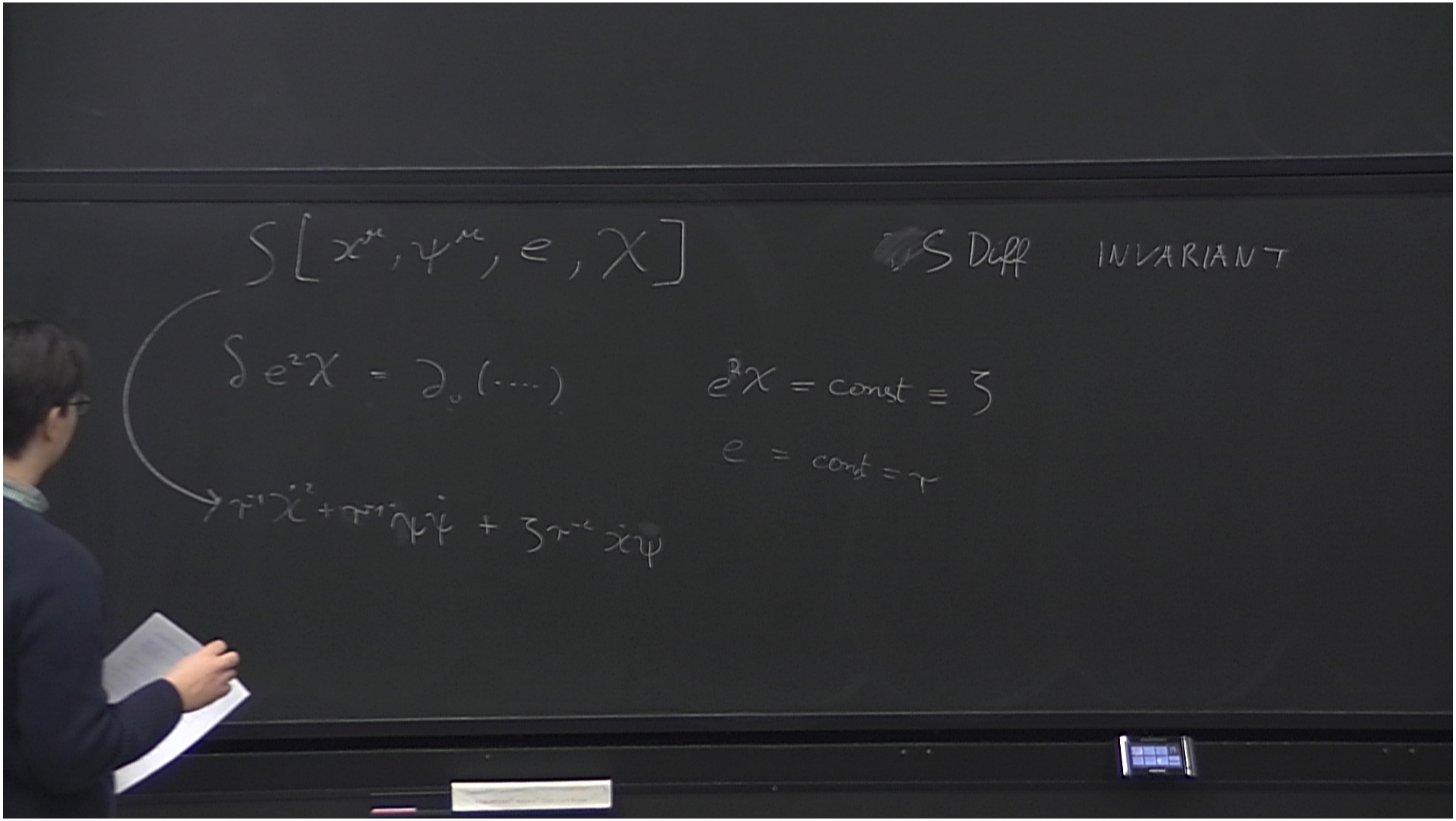
$$\tfrac12\delta e^{-1}\dot x^2 + e^{-1}\dot x(\partial_\tau(\epsilon)\psi) + \cancel{e^{-1}\dot x\epsilon\dot\psi}$$

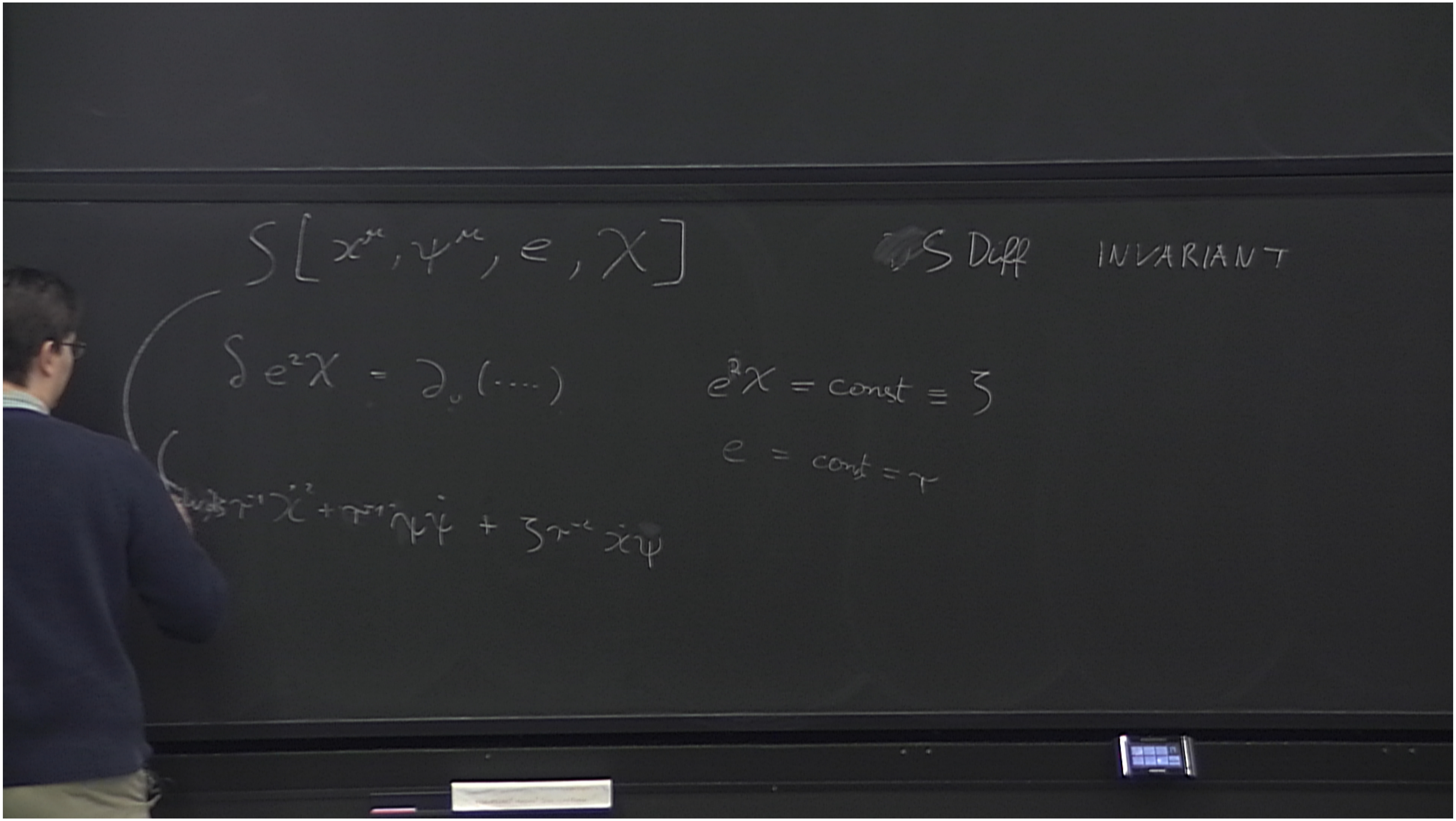
$$-\tfrac12\delta e^{-1}\psi\dot\psi - \cancel{e^{-1}\dot x\epsilon\dot\psi} - \tfrac12\partial_\tau(e^{-1})\dot x\epsilon\psi$$

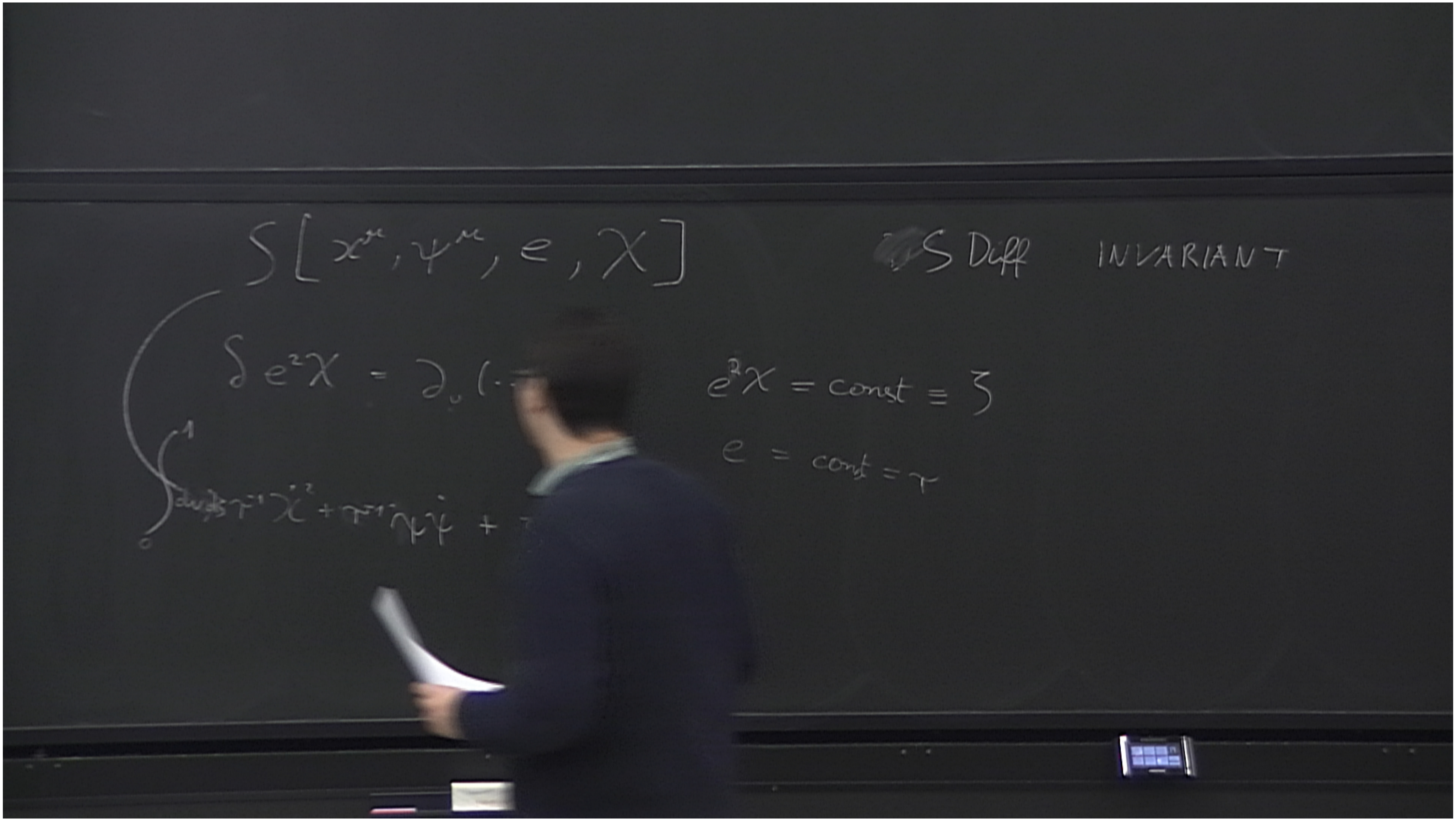
$$E^{-1} = e^{-1} + \vartheta\chi$$

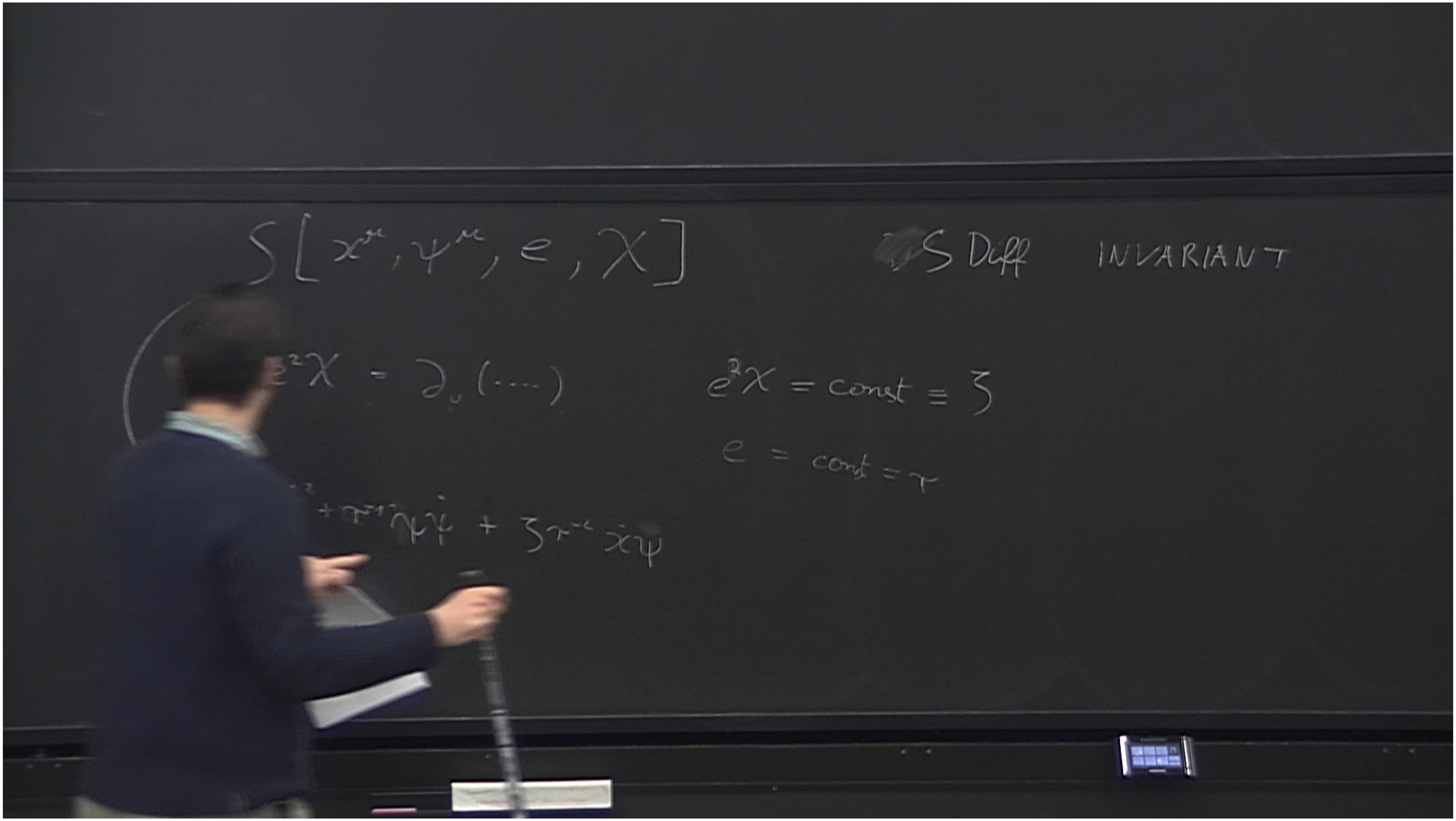
$$\delta\chi = e^{-1}\partial_\tau\epsilon \pm \tfrac12\partial_\tau(e^{-1})\epsilon$$

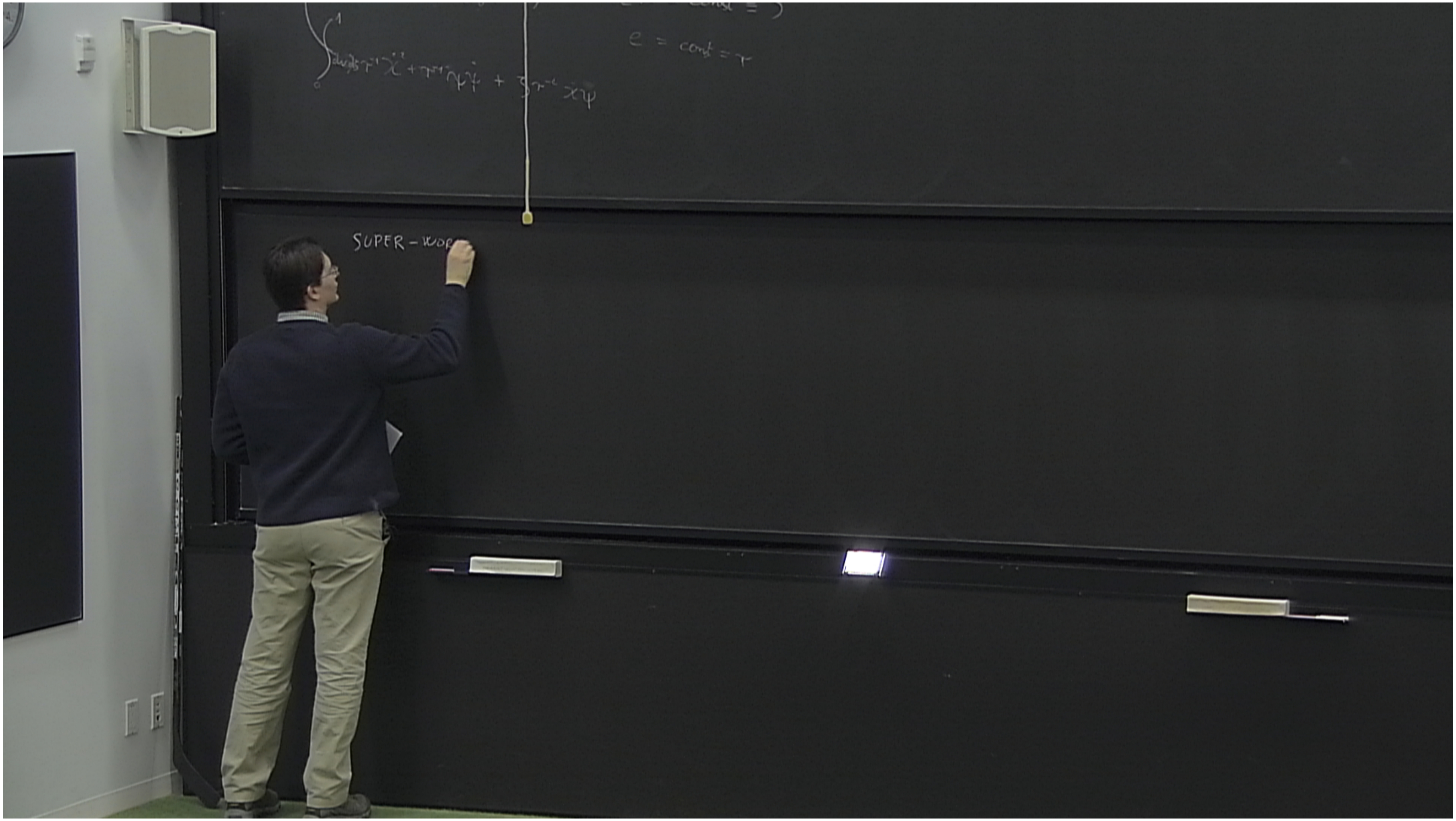
$$\delta e^{-1} = \pm\epsilon\chi$$

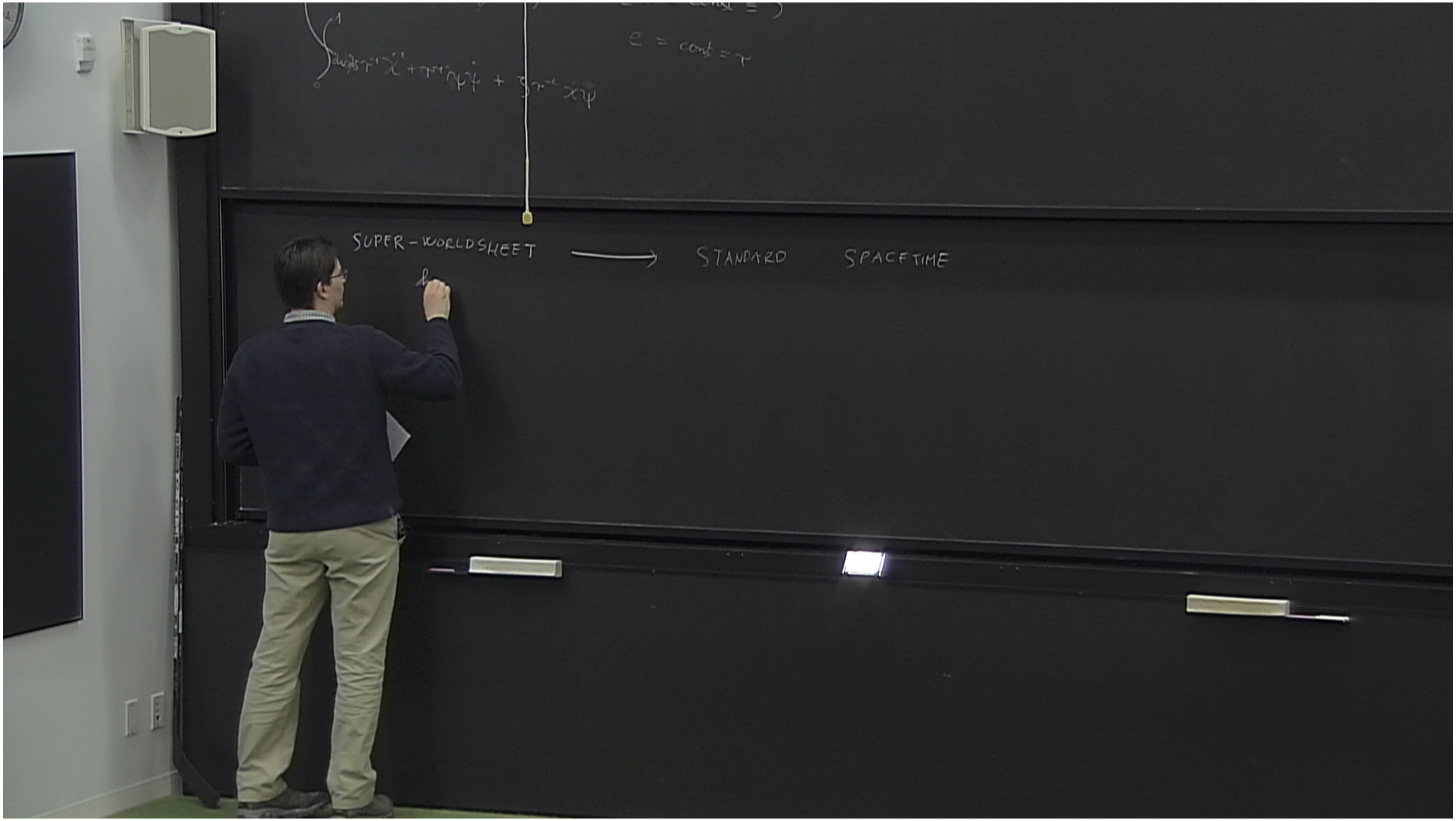

$S[x^\mu, \psi^\mu, e, \chi]$
S Diff INVARIANT
$\delta e^2\chi = \partial_0(\ldots.)$
$e^2\chi = const \equiv \zeta$
$e = const = \tau$
$\tau^{-1}\dot{x}^2 + \tau^{-1}\psi\dot{\psi} + \zeta\tau^{-2}\dot{x}\psi$


S[x^μ, ψ^μ, e, χ]
S Diff INVARIANT
δ e²χ = ∂₀(....)
e²χ = const ≡ ζ
e = const = τ
+ ζτ⁻¹ ẋψ


S[x^μ, ψ^μ, e, χ]
S Diff INVARIANT
δ e²χ = ∂ᵥ(
e²χ = const ≡ ζ
e = const = τ


$S[x^\mu, \psi^\mu, e, \chi]$
S Diff INVARIANT
$e^2\chi = \partial_v(\ldots)$
$e^2\chi = \text{const} \equiv \zeta$
$e = \text{const} = \tau$
$+ \tau^{-1}\psi\dot{\psi} + \zeta\tau^{-1}\dot{x}\psi$

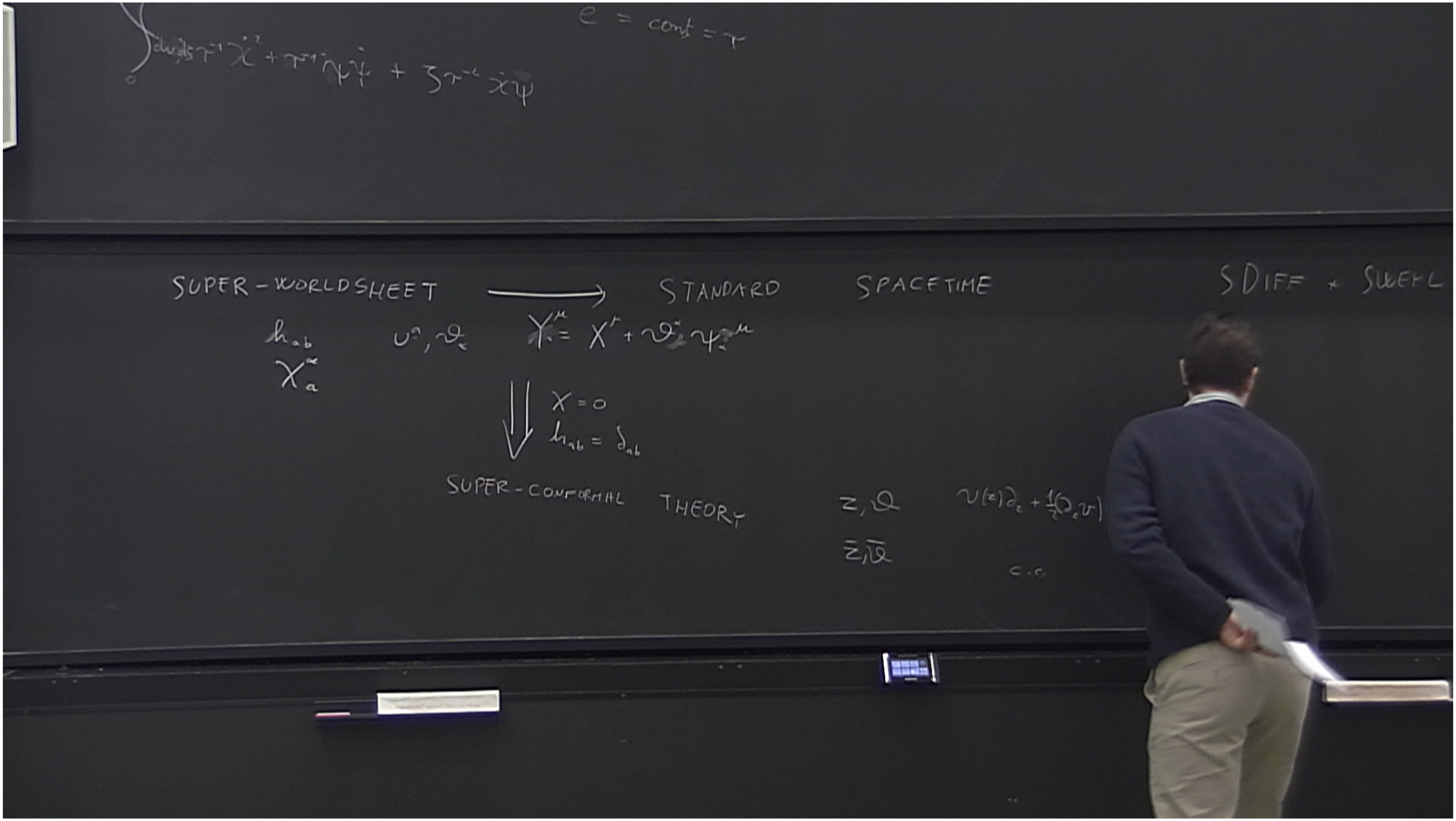


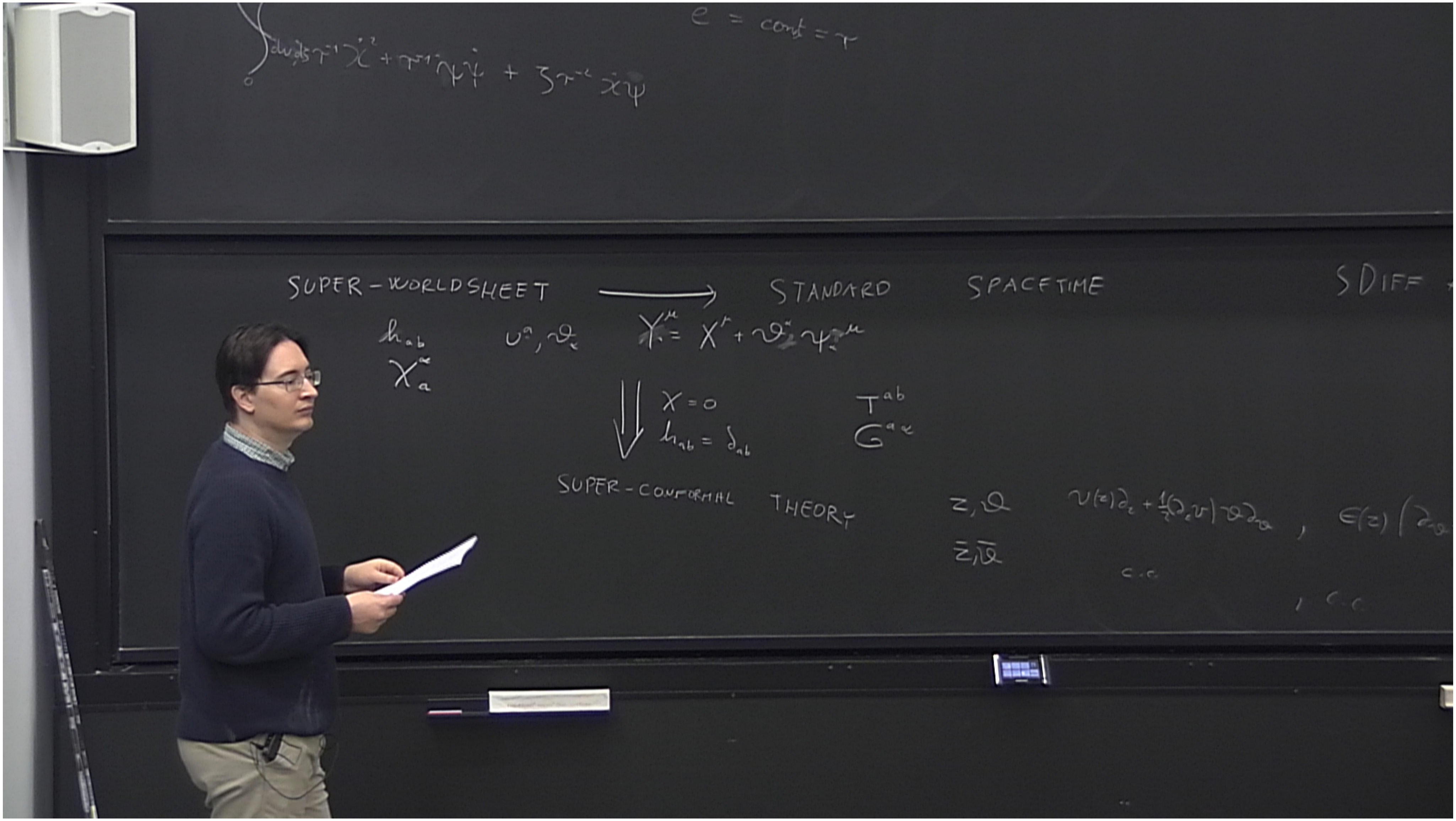

e = const = τ
SUPER-WORLDSHEET
STANDARD SPACETIME

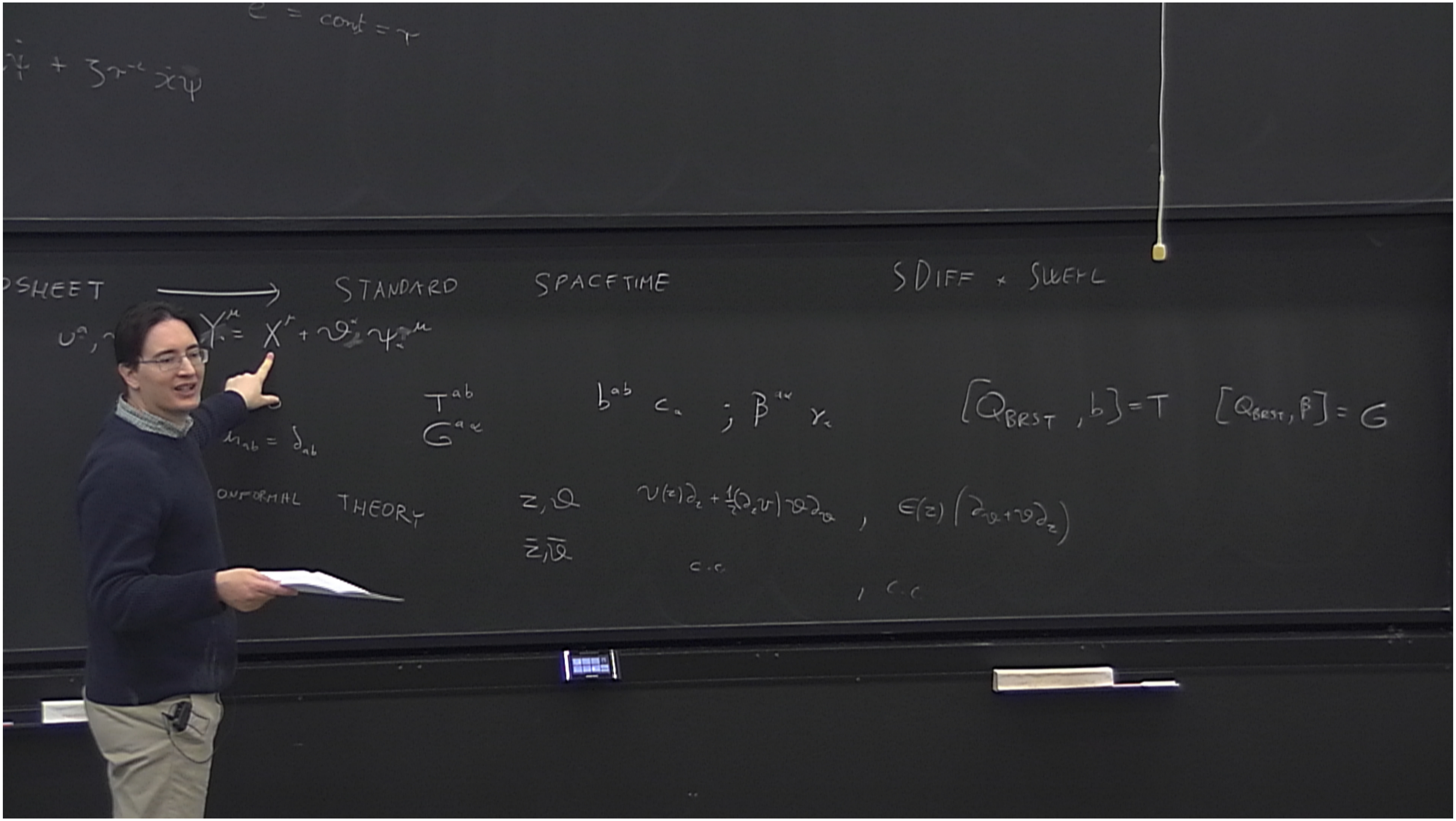
SUPER-WORLDSHEET $\longrightarrow$ STANDARD SPACETIME

h_{ab} $\quad u^{\alpha}, \vartheta_{\alpha}$ $\quad Y^{\mu} = X^{\mu} + \vartheta^{\alpha} \psi_{\alpha}^{\mu}$

χ_{a}^{α}


e = const = τ
SUPER-WORLDSHEET
STANDARD
SPACETIME
SDIFF × SWEYL
X = 0
SUPER-CONFORMAL
THEORY
c.c.


SUPER-WORLDSHEET
STANDARD
SPACETIME
SUPER-CONFORMAL
THEORY


STANDARD
SPACETIME
THEORY