

Title: 13/14 PSI - String Theory Review - Lecture 10

Date: Feb 10, 2014 10:15 AM

URL: <http://pirsa.org/14020028>

Abstract:

$$\int \sqrt{\det \left[G_{\mu\nu}(x) \frac{\partial X^\mu}{\partial u^a} \frac{\partial X^\nu}{\partial u^b} \right]}$$

$$\frac{1}{2} \int \sqrt{-h} h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X^\nu}{\partial u^b} G_{\mu\nu}(x)$$

$S_{\mu\nu}(X)$

$$T_a^a = \beta_{\sigma\tau}^G \partial_\mu X^\sigma \partial^\mu X^\tau$$

$$e^i(\mu) \\ \mu \partial_\mu e^2 = \beta(e^2)$$

$$\mu \partial_\mu G_{\sigma\tau}(X, \mu) = \beta_{\sigma\tau}^G[G]$$

$S_{\mu\nu}(X)$

$$T_a^a = \beta_{\sigma\tau}^G[X] \partial_\mu X^\sigma \partial^\mu X^\tau + \dots$$

$$e(\mu) \\ \mu \partial_\mu e^2 = \beta(e^2)$$

$$\mu \partial_\mu G_{\sigma\tau}(X, \mu) = \beta_{\sigma\tau}^G[G]$$

$G_{\mu\nu}(X)$

$$T_a^a = \beta_{\sigma\tau}^G[X] \partial_\mu X^\sigma \partial^\mu X^\tau + \dots$$

$$p^2 = 2$$

$$e^2(\mu) \\ \mu \partial_\mu e^2 = \beta(e^2)$$

$$\mu \partial_\mu G_{\sigma\tau}^*(X, \mu) = \beta_{\sigma\tau}^G[G]$$

$$\sqrt{\det \left[G_{\mu\nu}(x) \frac{\partial X^\mu}{\partial u^a} \frac{\partial X^\nu}{\partial u^b} \right]}$$

$$\frac{\alpha'}{2} \int \sqrt{-h} h^{ab} \frac{\partial X^\mu}{\partial u^a} \frac{\partial X^\nu}{\partial u^b} G_{\mu\nu}(x)$$

$$X^\mu = x^\mu + \sqrt{\alpha'} y^\mu$$

$$G_{\mu\nu} = \eta_{\mu\nu} + \dots$$

$G_{\mu\nu}(X)$

$$T_{\alpha\beta}^a = \beta_{\sigma\tau}^G[X] \partial_\mu X^\sigma \partial^\nu X^\tau + \dots$$

$$\mu \partial_\mu e^2 = \beta(e^2)$$

$$\beta_{\sigma\tau}^G = \alpha' R_{\sigma\tau}[G]$$

$$\mu \partial_\mu G_{\sigma\tau}(X, \mu) = \beta_{\sigma\tau}^G[G]$$

$G_{\mu\nu}(X)$

$$T_a^a = \beta_{\sigma\tau}^G[X] \partial_\mu X^\sigma \partial^\mu X^\tau + \dots$$

$$e^2(\mu) \\ \mu \partial_\mu e^2 = \beta(e^2)$$

$$\beta_{\sigma\tau}^G = \alpha^1 R_{\sigma\tau}[G] \\ + \alpha^2 \dots$$

$$\mu \partial_\mu G_{\sigma\tau}^G(X, \mu) = \beta_{\sigma\tau}^G[G]$$

$$\beta^G = 0 \quad \equiv \quad R_{\sigma\tau}[G] = 0$$

$S_{\mu\nu}(X)$

$$T_{\alpha\beta}^a = \beta_{\sigma\tau}^G [X] \partial_\mu X^\sigma \partial^\mu X^\tau + \dots$$

$$e^2(\mu) \\ \mu \partial_\mu e^2 = \beta(e^2)$$

$$\beta_{\sigma\tau}^G = \alpha^1 R_{\sigma\tau}[G] \\ + \alpha^2 \dots$$

$$\mu \partial_\mu G_{\sigma\tau}^2(X, \mu) = \beta_{\sigma\tau}^G [G]$$

$$\beta_{\sigma\tau}^G = \frac{\delta S[G]}{\delta G_{\sigma\tau}} \stackrel{\beta=0}{=} R_{\sigma\tau}[G] = 0$$

$$G_{\mu\nu} = \eta_{\mu\nu} + \delta G_{\mu\nu}$$

$$A_n = \int DX D_b D_c e^{S^X + S^{bc} + \int d^4p \int d^4x \delta G_{\mu\nu}(p) e^{i p X} \partial_\mu X^\nu \bar{\partial} X^\mu}$$

ccV ccV ccV

$$= A_n(0) + A_n + \delta G_{\mu\nu}(p) e^{i p X} \partial_\mu X^\nu \bar{\partial} X^\mu$$

$$\delta G_{\mu\nu}(x) \alpha_{-1}^{\mu} \bar{a}_{-1}^{\nu} |P\rangle$$

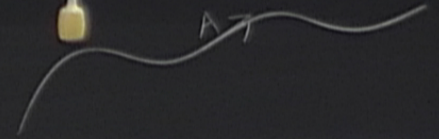


$$i \int d^2u \delta G_{\mu\nu}(p) e^{i p X} \partial X^{\mu} \bar{\partial} X^{\nu}$$

ccv cc

PARTICLE IN QED

$$\int A_n \frac{dx^n}{dt} dt$$



$$p^2 = 2$$

$$-\int \epsilon^{ab} B_{mn} [X] \partial_a X^m \partial_b X^n d^2x^c$$

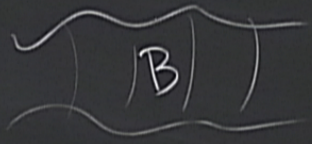
$$E^{12} = 1$$

$$E^{21} = -1$$

$$E^{11} = 0$$

$$E^{22} = 0$$

PARTICLE



$$-\int \epsilon^{ab} B_{mn} [X] \partial_a X^m \partial_b X^n \, d^2x$$

$$\begin{aligned} E^{12} &= 1 \\ E^{21} &= -1 \\ E^{11} &= 0 \\ E^{22} &= 0 \end{aligned}$$

PARTICLE

TACHYON



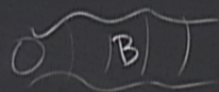
$$-\int \epsilon^{ab} B_{ab}[X] \partial_a X^\mu \partial_b X^\nu d\sigma^0 d\sigma^1$$

$$E^{12} = 1$$

$$E^{21} = -1$$

$$E^{11} = 0$$

$$E^{22} = 0$$



PARTICLE IN QED

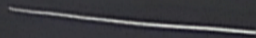
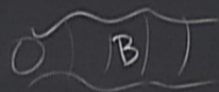
TACHYON

$$\int T(X)$$

$$\beta^G[G, B]$$

$$\int A_\mu \frac{dx^\mu}{dt} dt$$

$$-\int \epsilon^{ab} B_{ab}[X] \partial_a X^\mu \partial_b X^\nu d\sigma d\tau \quad \begin{matrix} E^{12} = 1 \\ E^{21} = -1 \\ E^{11} = 0 \\ E^{22} = 0 \end{matrix}$$



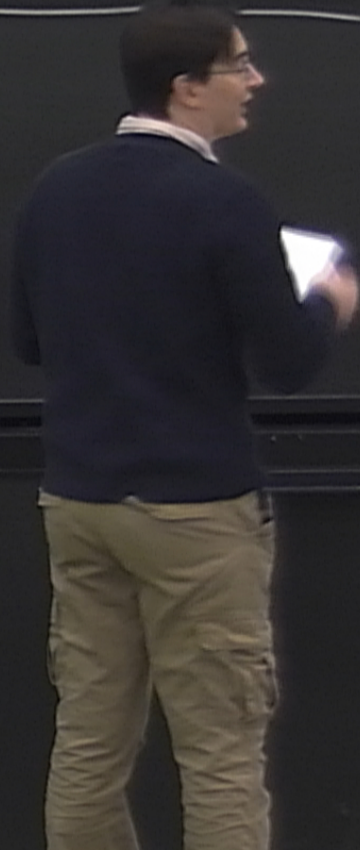
$$\begin{matrix} \beta^G [G, B, T, \Phi] = 0 \\ \beta^B [\dots] = 0 \\ \beta^T [\dots] = 0 \end{matrix} \quad \beta^\Phi [\dots] = 0$$

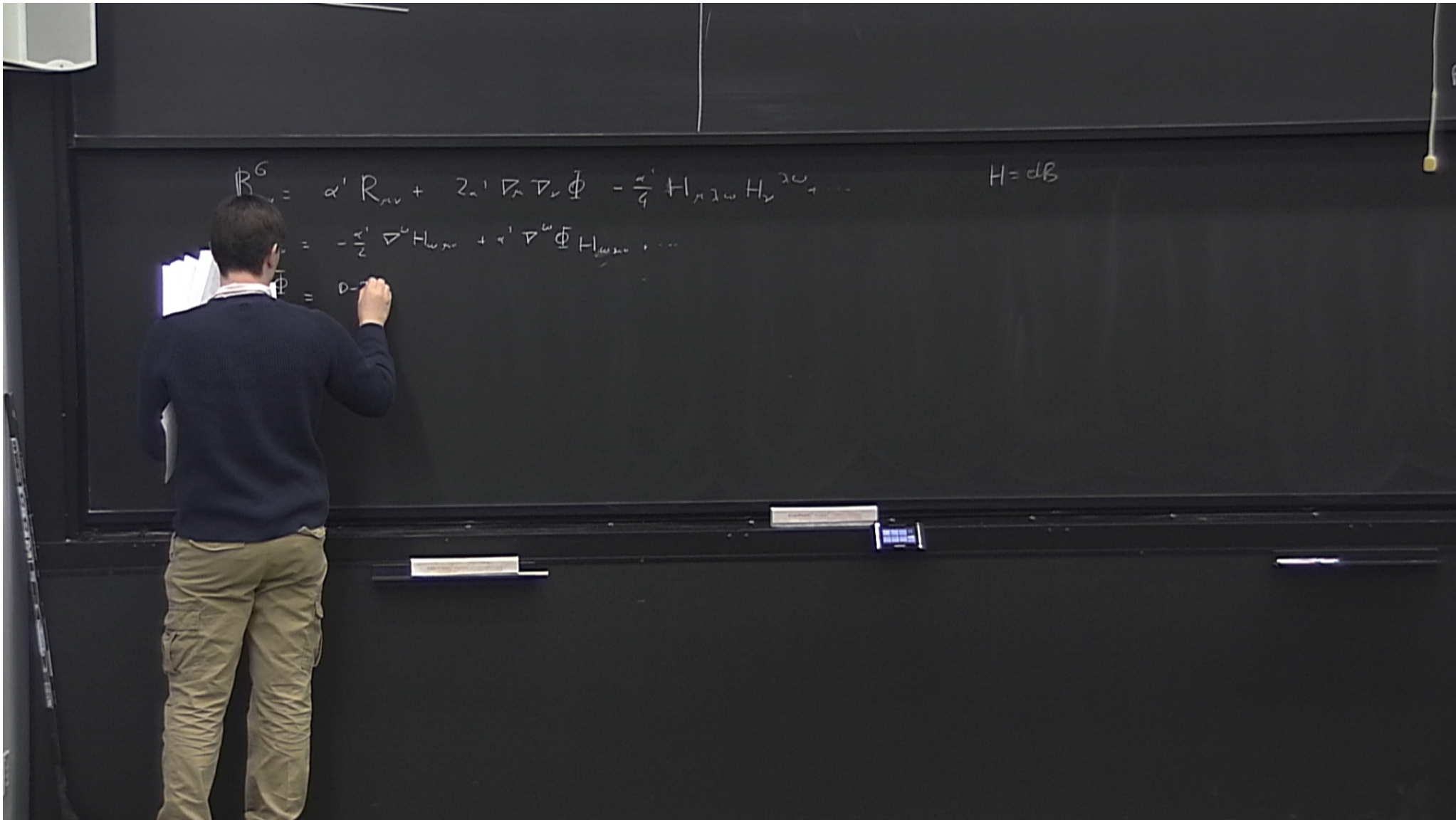
PARTICLE IN QED

$$\int A_\mu \frac{dx^\mu}{dt} dt$$

TACHYON

$$\int T(X) d\sigma d\tau$$





$$p^z = 2$$

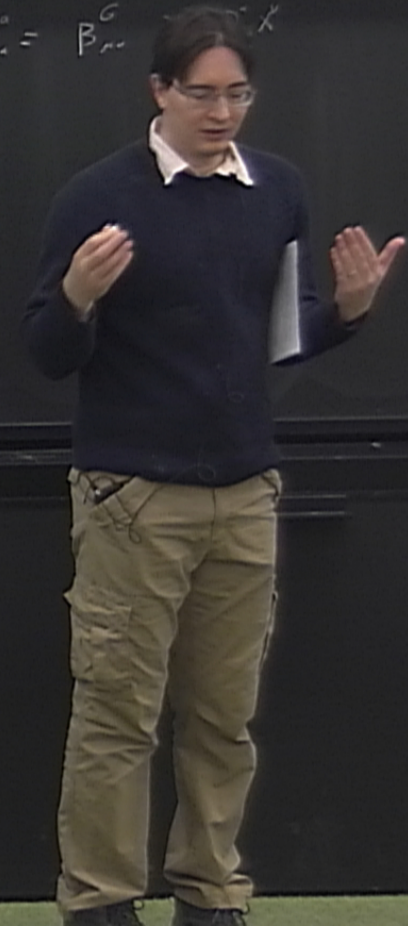
$$\beta_{\mu\nu}^G = \alpha' R_{\mu\nu} + 2\alpha' \nabla_\mu \nabla_\nu \Phi - \frac{\alpha'}{4} H_{\mu\lambda\omega} H_{\nu}{}^{\lambda\omega} + \dots$$

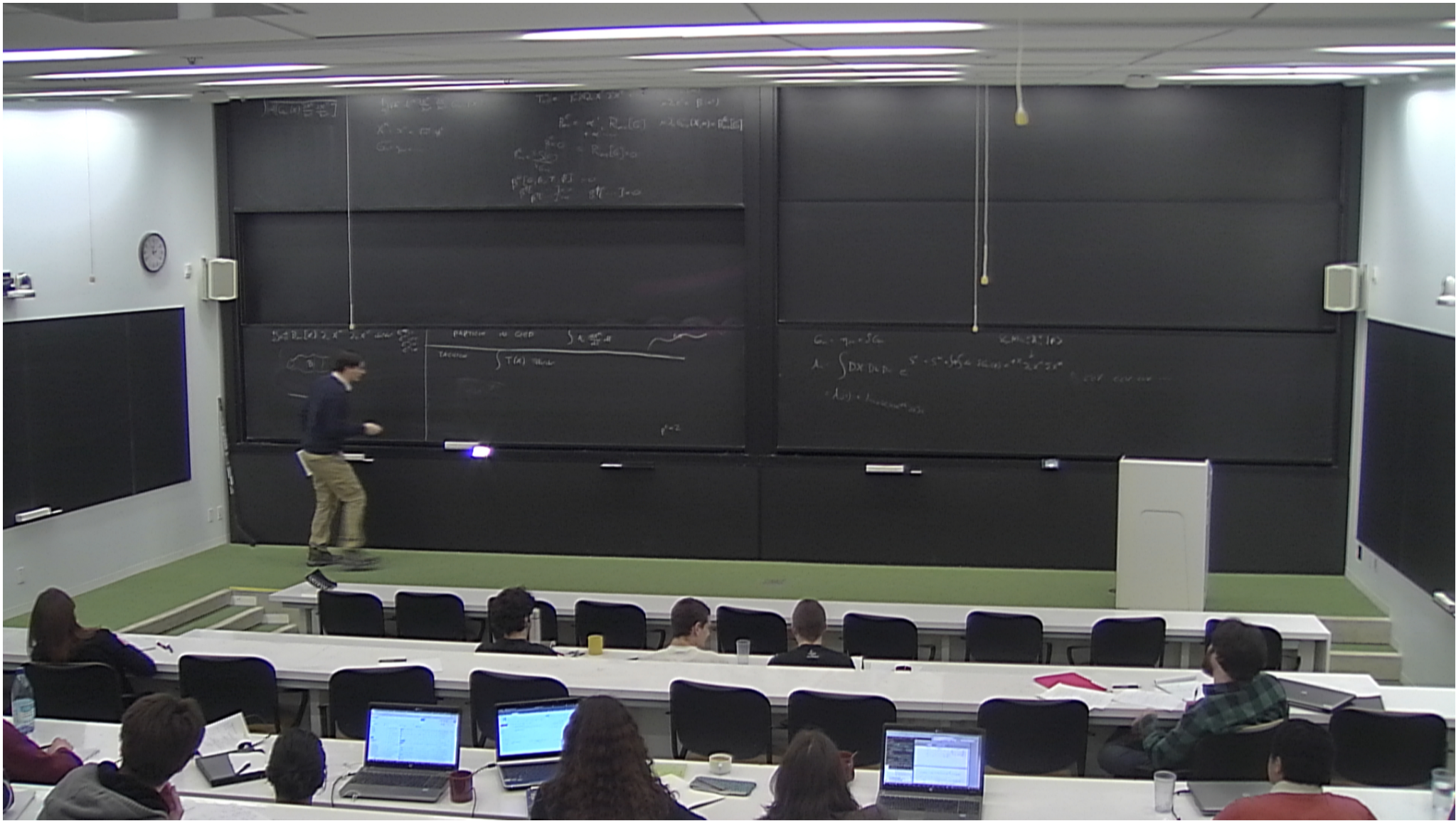
$$H = dB$$

$$\beta_{\mu\nu}^B = -\frac{\alpha'}{2} \nabla^\lambda H_{\lambda\mu\nu} + \alpha' \nabla^\omega \Phi H_{\omega\mu\nu} + \dots$$

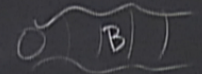
$$T_{\mu\nu} = \beta_{\mu\nu}^G + \dots$$

$$\beta^\Phi = \frac{D-26}{2} - \frac{\alpha'}{2} \nabla^2 \Phi + \alpha' \nabla_\mu \Phi \nabla^\mu \Phi - \frac{\alpha'}{24} H_{\mu\nu\lambda} H^{\mu\nu\lambda}$$





$$\frac{1}{2} \int d^4x \epsilon^{ab} B_{ab} [X] \partial_a X^\mu \partial_b X^\nu d\nu d\nu' \quad \begin{matrix} \epsilon^{0123} = 1 \\ \epsilon^{0132} = -1 \\ \epsilon^{0120} = 0 \\ \epsilon^{0102} = 0 \end{matrix}$$



PARTICLE IN QED

$$\int A_\mu \frac{dx^\mu}{dt} dt$$

CHTON

$$\int T(X) d\nu d\nu'$$

ATON

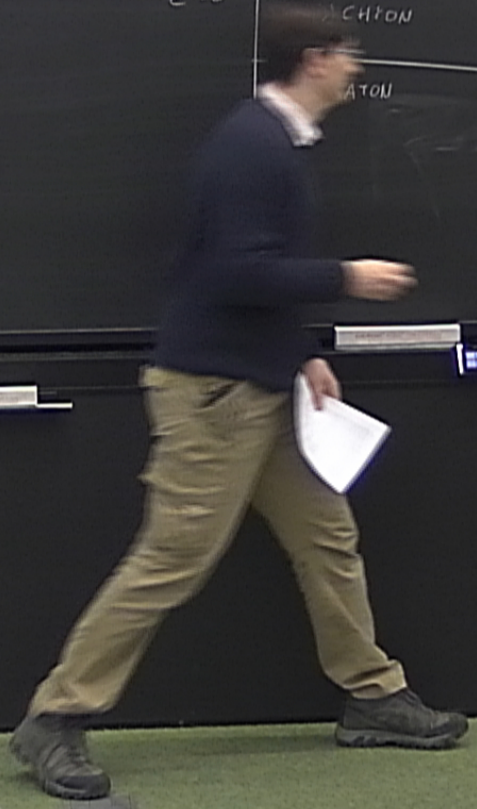
$$\int R^{(2)} \Phi(X) d\nu d\nu'$$

$$\partial(b\epsilon) = R^{(1)}$$

$$\int b\epsilon \partial X^\mu e^{i p X}$$

$$\int \partial b\epsilon e^{i p X}$$

$$p^2 = 2$$



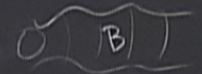
$$\frac{1}{2} \int d^4x \epsilon^{abcd} B_{ab} [X] \partial_c X^\mu \partial_d X^\nu d\omega d\omega'$$

$$E^{0123} = 1$$

$$E^{0132} = -1$$

$$E^{0213} = 0$$

$$E^{0231} = 0$$



PARTICLE IN QED

$$\int A_\mu \frac{dx^\mu}{dt} dt$$

TACHYON

$$\int d\omega d\omega'$$

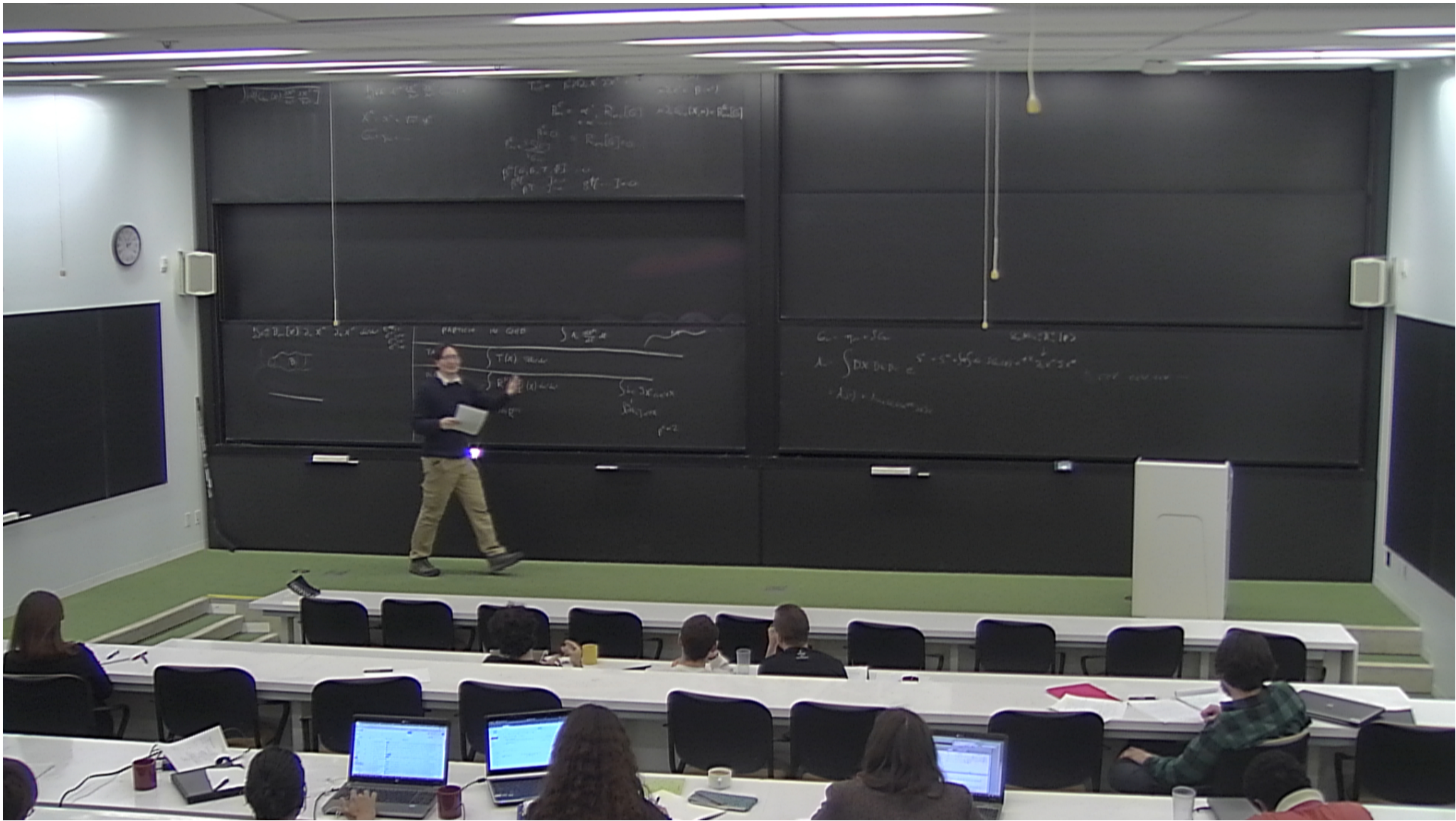
DILATON

$$\Phi(x) d\omega d\omega'$$

$$\int b_c \bar{\psi} X^\mu e^{ipX}$$

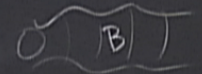
$$\int \bar{\psi} b_c e^{ipX}$$

$$p^2 = -2$$



$$\frac{1}{2\pi\alpha'} \int \epsilon^{ab} B_{ab} [X] \partial_a X^\mu \partial_b X^\nu d\tau d\sigma$$

$$\begin{aligned} E^{01} &= 1 \\ E^{12} &= F_1 \\ E^{23} &= 0 \\ E^{31} &= 0 \end{aligned}$$



PARTICLE IN QED

$$\int A_\mu \frac{dx^\mu}{dt} dt$$

TACHYON $\int T(X) d\tau d\sigma$

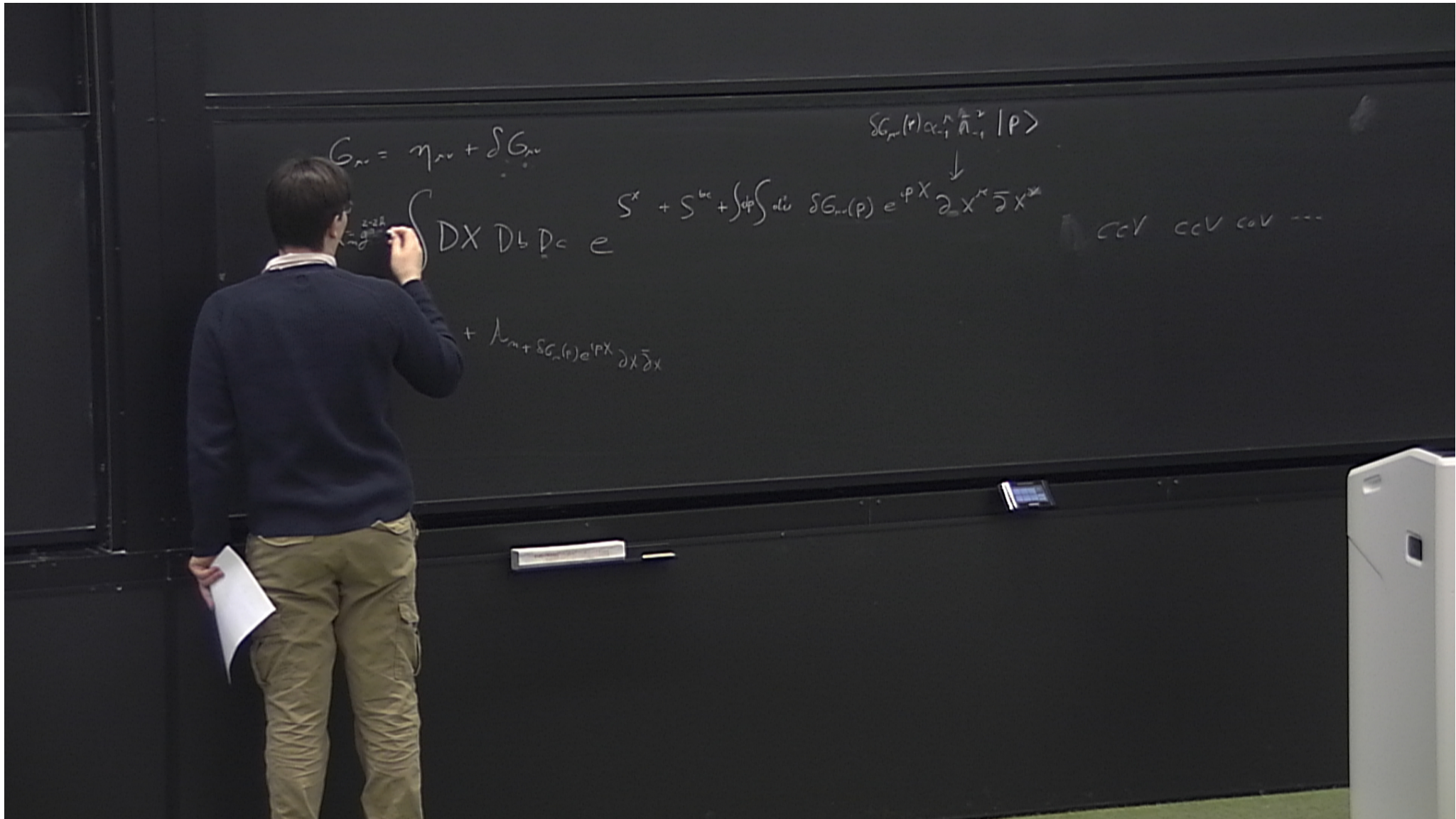
DILATON $\int R^{(2)} \Phi(X) d\tau d\sigma$

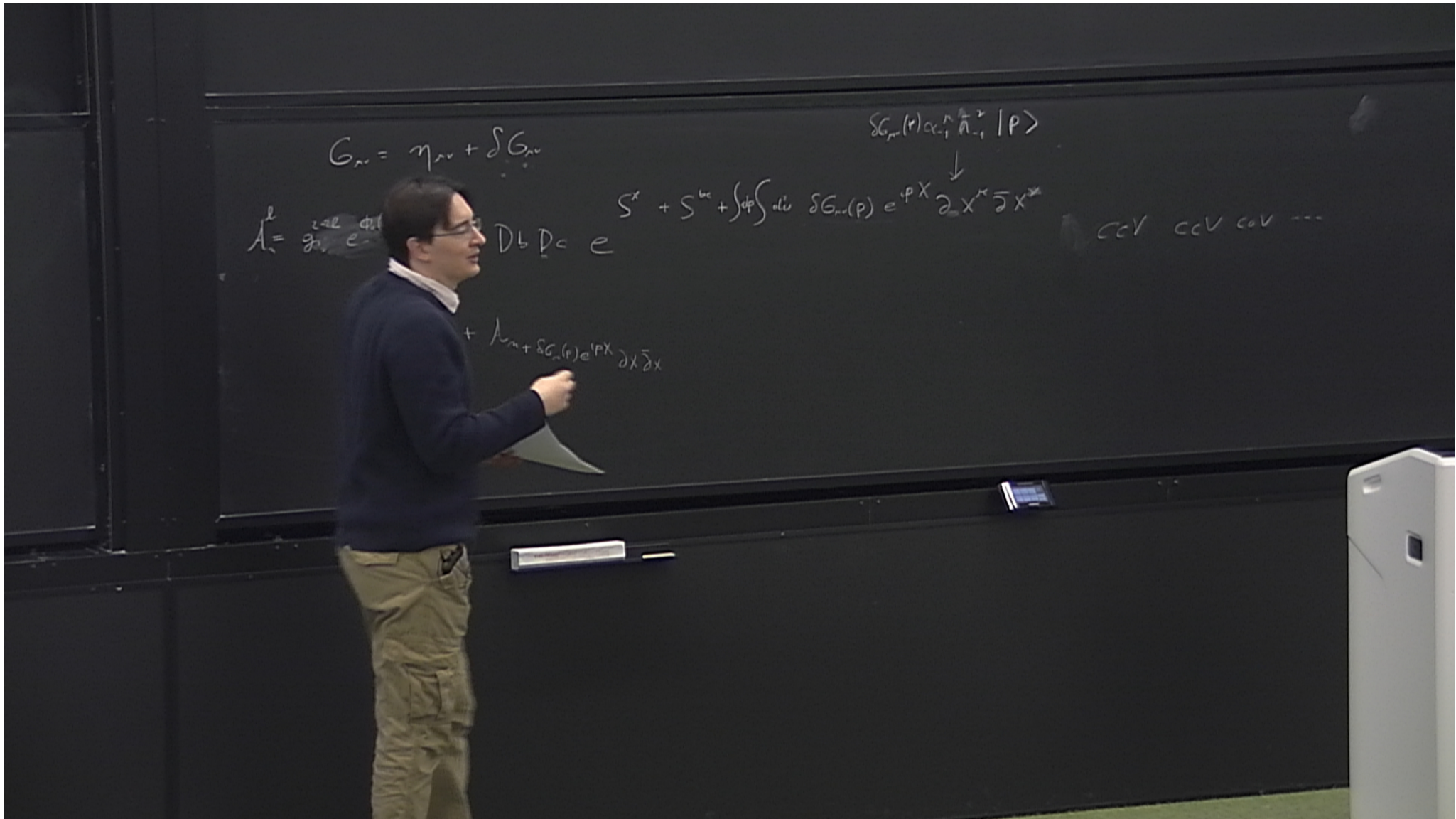
$$\partial_\mu (b c) \partial_\nu \rho^{(\mu)}$$

$$b c \partial X^\mu e^{i p \cdot X}$$

$$\langle \partial X^\mu \partial X^\nu \rangle = -\eta^{\mu\nu}$$

$$p^2 = -2$$





$$G_{\nu} = \eta_{\nu\nu} + \delta G_{\nu}$$

$$\delta G_{\nu}(p) \alpha_{-i}^{\uparrow} \hat{n}_{-i}^{\downarrow} |P\rangle$$

$$A_n^l = \frac{\partial \phi}{\partial x} e^{-\dots}$$

$$S^x + S^{bc} + \int dp \int d\omega \delta G_{\nu}(p) e^{iPx} \partial_x \bar{\chi} \chi$$

ccv ccv ccv ...

$$+ \lambda_{\nu\nu} + \delta G_{\nu}(p) e^{iPx} \partial_x \bar{\chi} \chi$$

$$A = \frac{\partial \ln \mathbb{E}[e^{pX}]}{\partial p} = S + S'' + \dots + \int_0^{\infty} \delta G_{\dots}(p) e^{pX} \partial X \bar{\delta} X^{\dots}$$

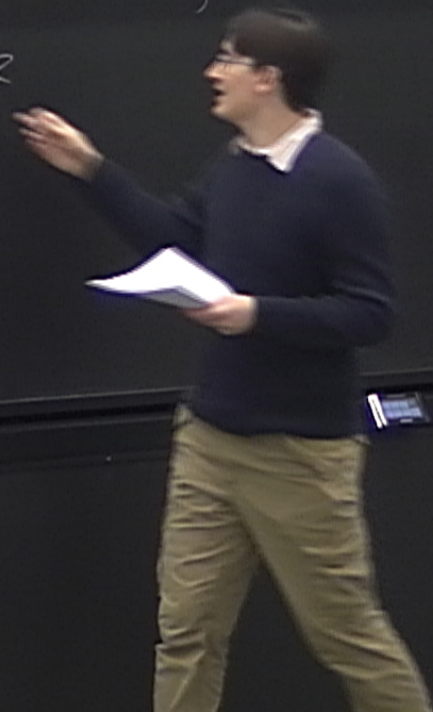
$$= A_0(0) + \lambda_{m+1} \delta G_{\dots}(p) e^{pX} \partial X \bar{\delta} X$$

ccV ccV cov ...

$$R^{1,2,4} \times S^{-1}$$

$$X^m \quad Y \quad Y \rightarrow Y + 2\pi R$$

$$\int d\theta \partial Y \bar{\partial} Y$$



$$A = \frac{\partial \ln \mathbb{E}[\exp(\lambda \cdot \int_0^T \sigma_t^2 dt)]}{\partial \lambda} = \int_0^T \sigma_t^2 dt + \dots$$

$$= A_0 + \lambda \int_0^T \sigma_t^2 dt + \dots$$

$$R^{1,2,4} \times S^1$$

$$Y$$

$$Y \rightarrow Y + 2\pi R$$

$$Y'$$

$$\int \sigma_t^2 dt$$

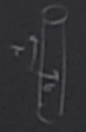
$$A = \frac{\partial \ln f(x)}{\partial x} = \left(\frac{\partial \ln f(x)}{\partial x} \right)_{x=0} + \int_0^x \frac{\partial^2 \ln f(x)}{\partial x^2} dx$$

$$= A_0 + \int_0^x \frac{\partial^2 \ln f(x)}{\partial x^2} dx$$

ccv ccv ccv ...

$$R^{1,2,4} \times S^{-1}$$

$$\int dx \rightarrow Y \rightarrow Y$$



$$Y \rightarrow Y + 2\pi R$$

$$Y(t) = y - 2ipT +$$

$$A = \frac{\partial \ln \mathbb{E} [e^{pX}]}{\partial p} \quad \text{with } S^1 + S^2 + \dots + S^N \text{ and } S_{G_{i,t}(p)} e^{pX} \partial X \bar{\partial} X^N$$

$$= A_0(p) + \lambda_{i,t} S_{G_{i,t}(p)} e^{pX} \partial X \bar{\partial} X$$

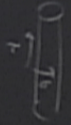
ccV ccV cov ...

$$R^{1,2,4} \times S^1$$

$$X^N \quad Y$$

$$Y \rightarrow Y + 2\pi R$$

$$\int d\theta \partial Y \bar{\partial} Y$$



$w \in \text{INTEGER}$

$$Y(\tau) = y - 2ip\tau + wR\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\tau+t)} \right]$$

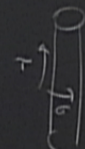


$$\mathbb{R}^{1,24} \times S^1$$

$$X^m \quad Y$$

$$\int d\sigma dY dY$$

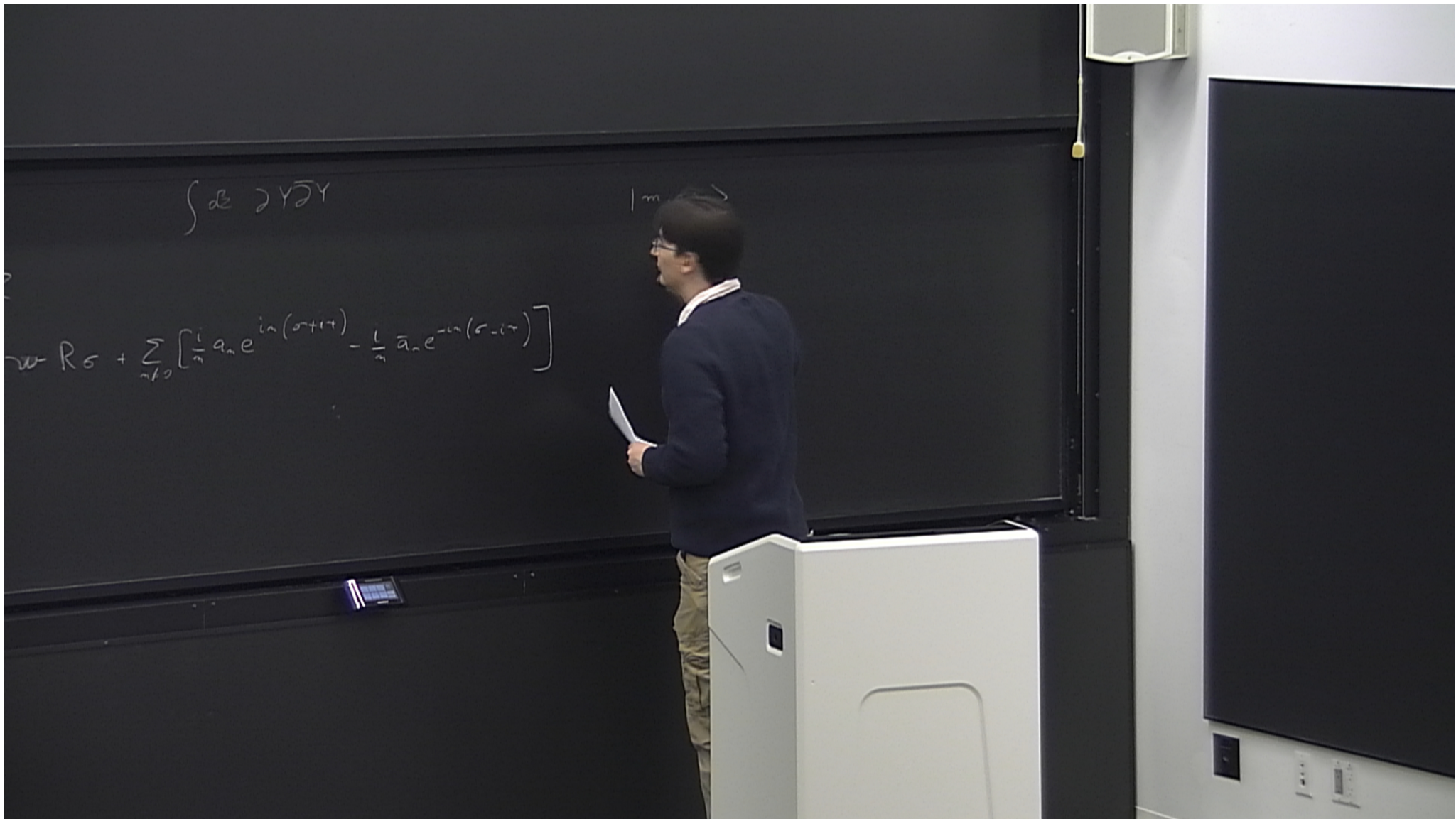
$$Y \rightarrow Y + 2\pi R$$



$$Y(\sigma, \tau) = y - 2i\pi\tau + w R \sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + \tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - \tau)} \right]$$

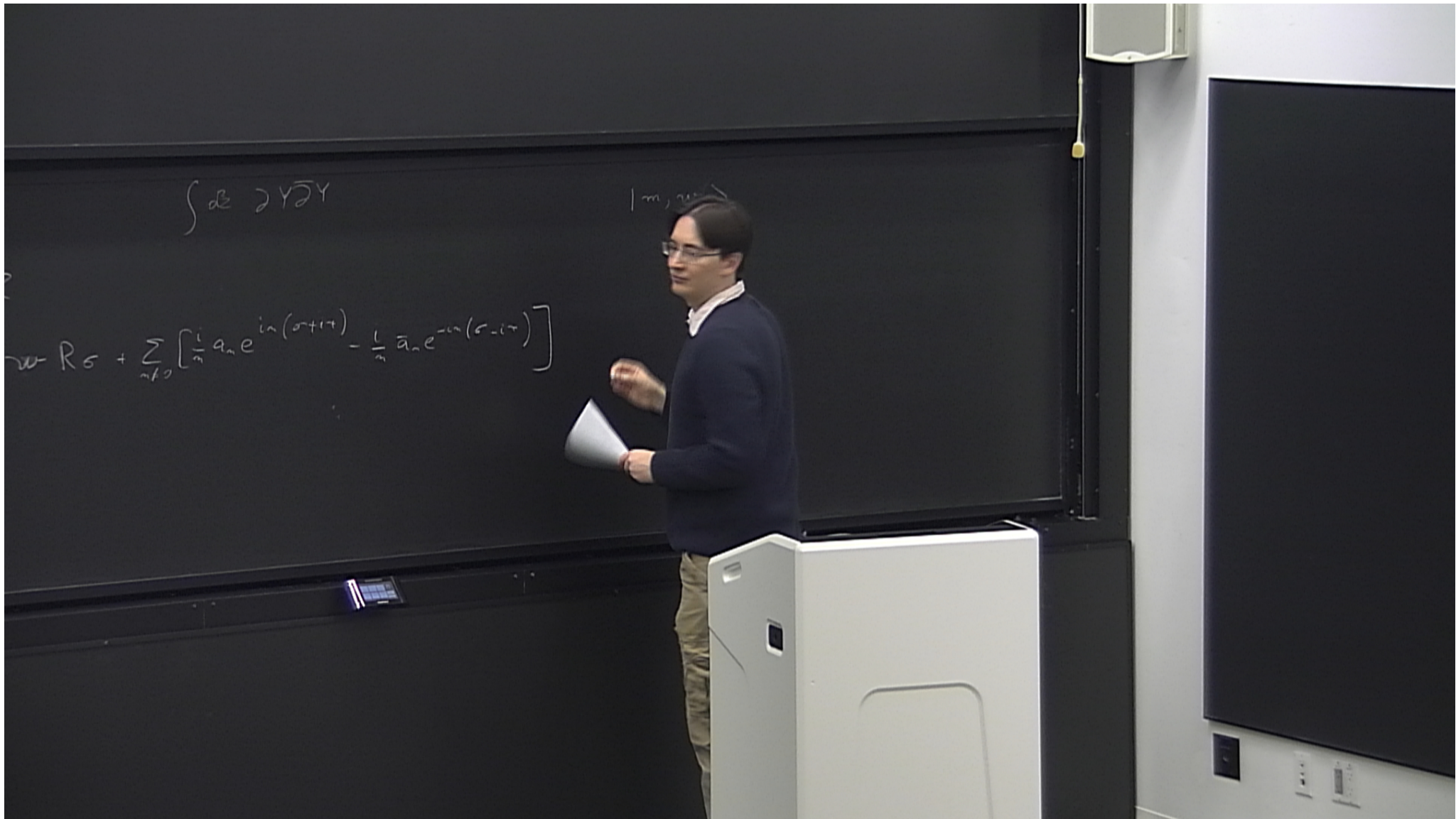
$$p = \frac{m}{R}$$

w : INTEGER
 m : INTEGER



$$\int_{\sigma} \gamma \gamma$$

$$w - R\sigma + \sum_{n=0}^{\infty} \left[\frac{i}{n} a_n e^{in(\sigma+i\tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma-i\tau)} \right]$$



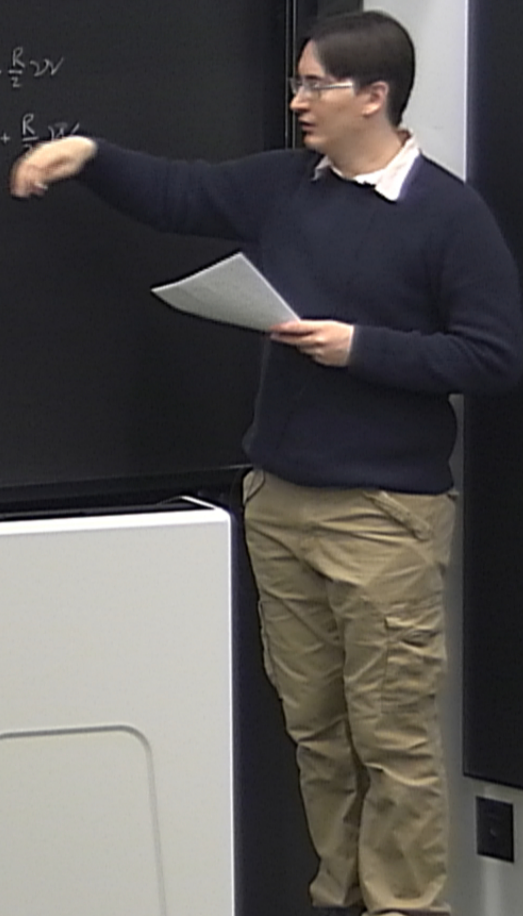
$$\int dz \partial Y \bar{\partial} Y$$

$a_n, \bar{a}_n \mid m, w \rangle$

$$\frac{\partial Y}{\partial Y}$$
$$a_0 = \frac{m}{R} - \frac{R}{2} \alpha V$$

$$\bar{a}_0 = \frac{m}{R} + \frac{R}{2} \alpha V$$

$$w - R\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - i\tau)} \right]$$



$$\int dz \partial Y \bar{\partial} Y$$

$$a_n, \bar{a}_n |m, w\rangle$$

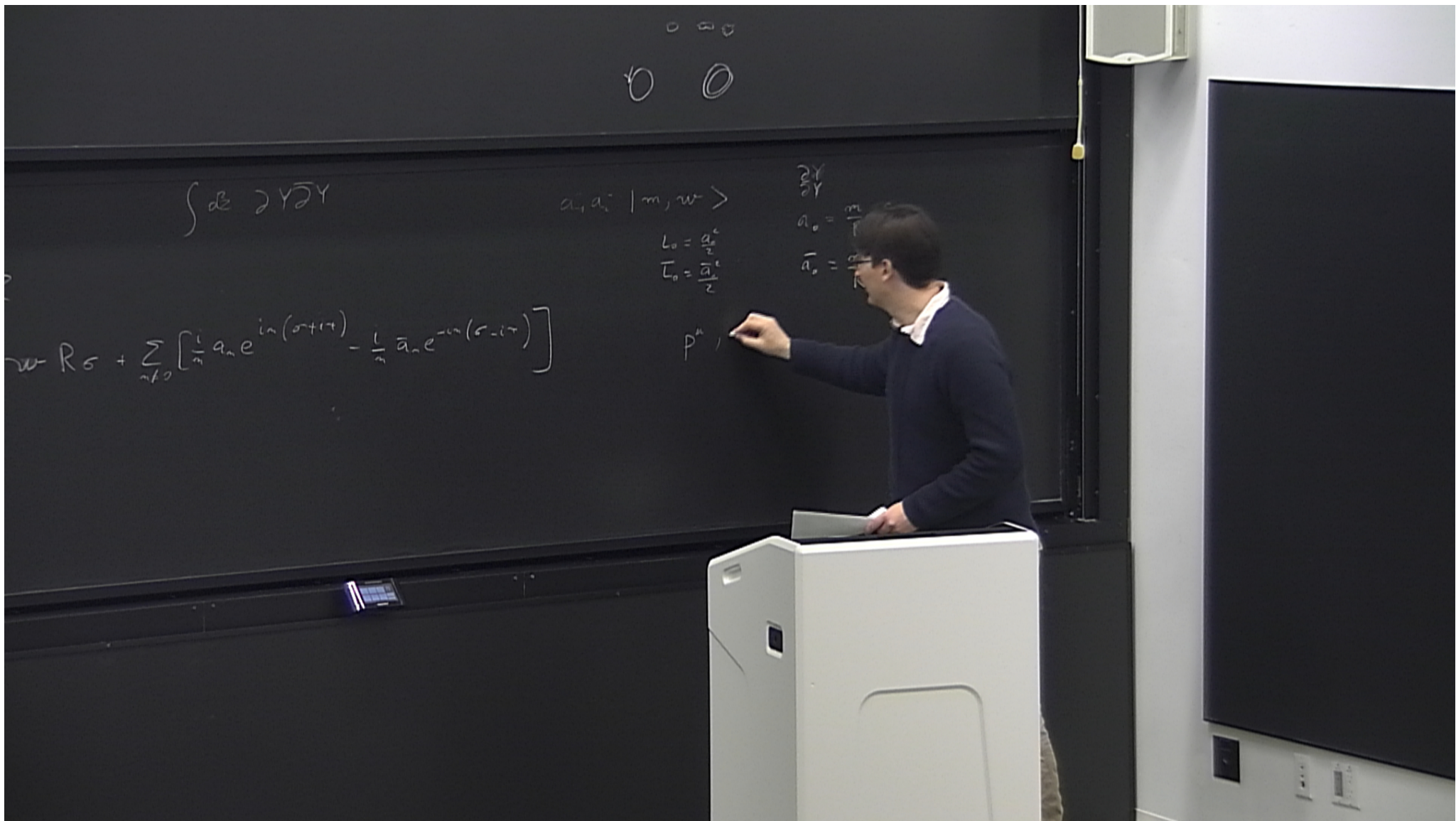
$$L_0 = \frac{\alpha_0^2}{2}$$
$$\bar{L}_0 = \frac{\bar{\alpha}_0^2}{2}$$

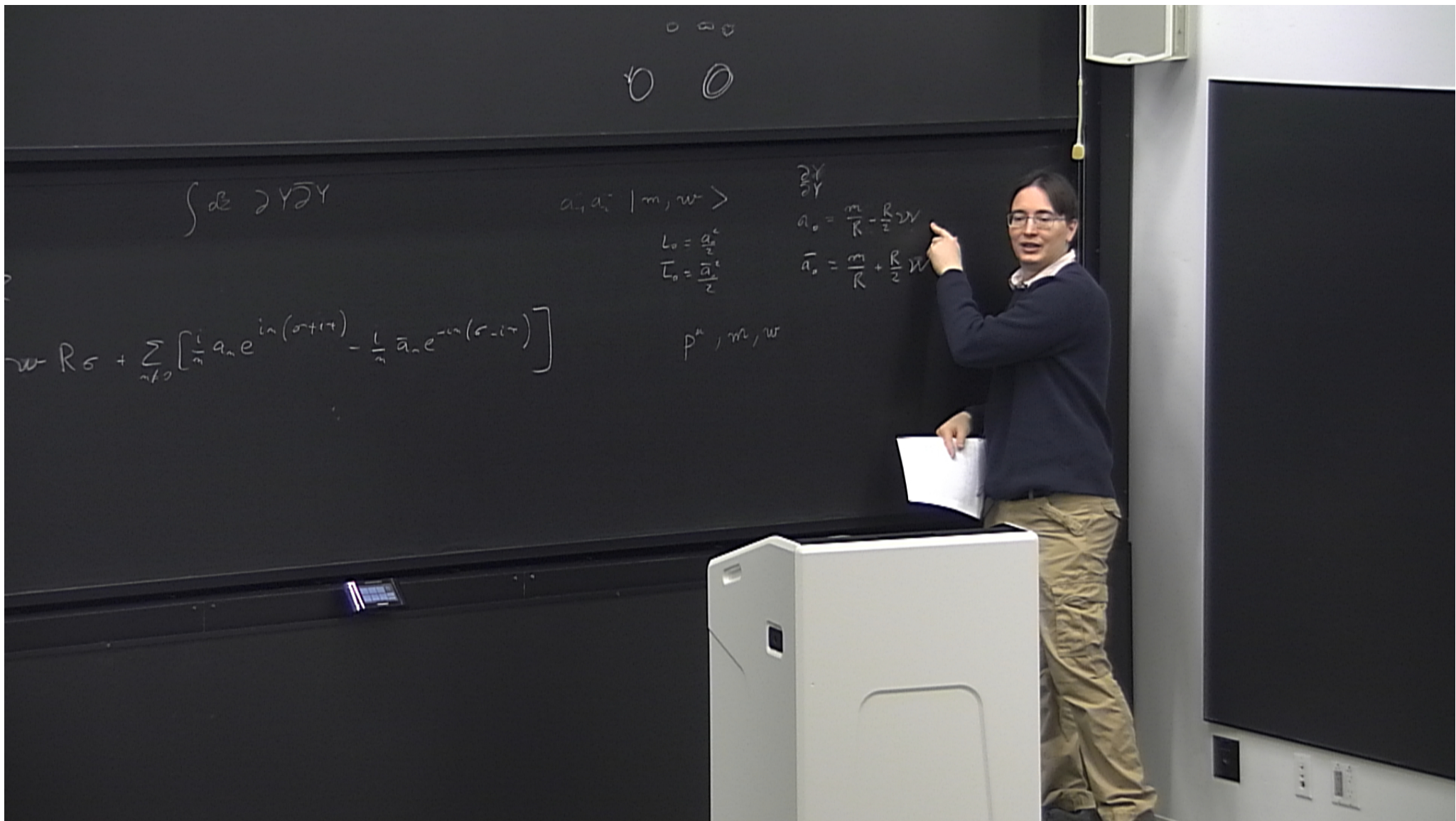
$$\frac{\partial Y}{\partial \tau}$$

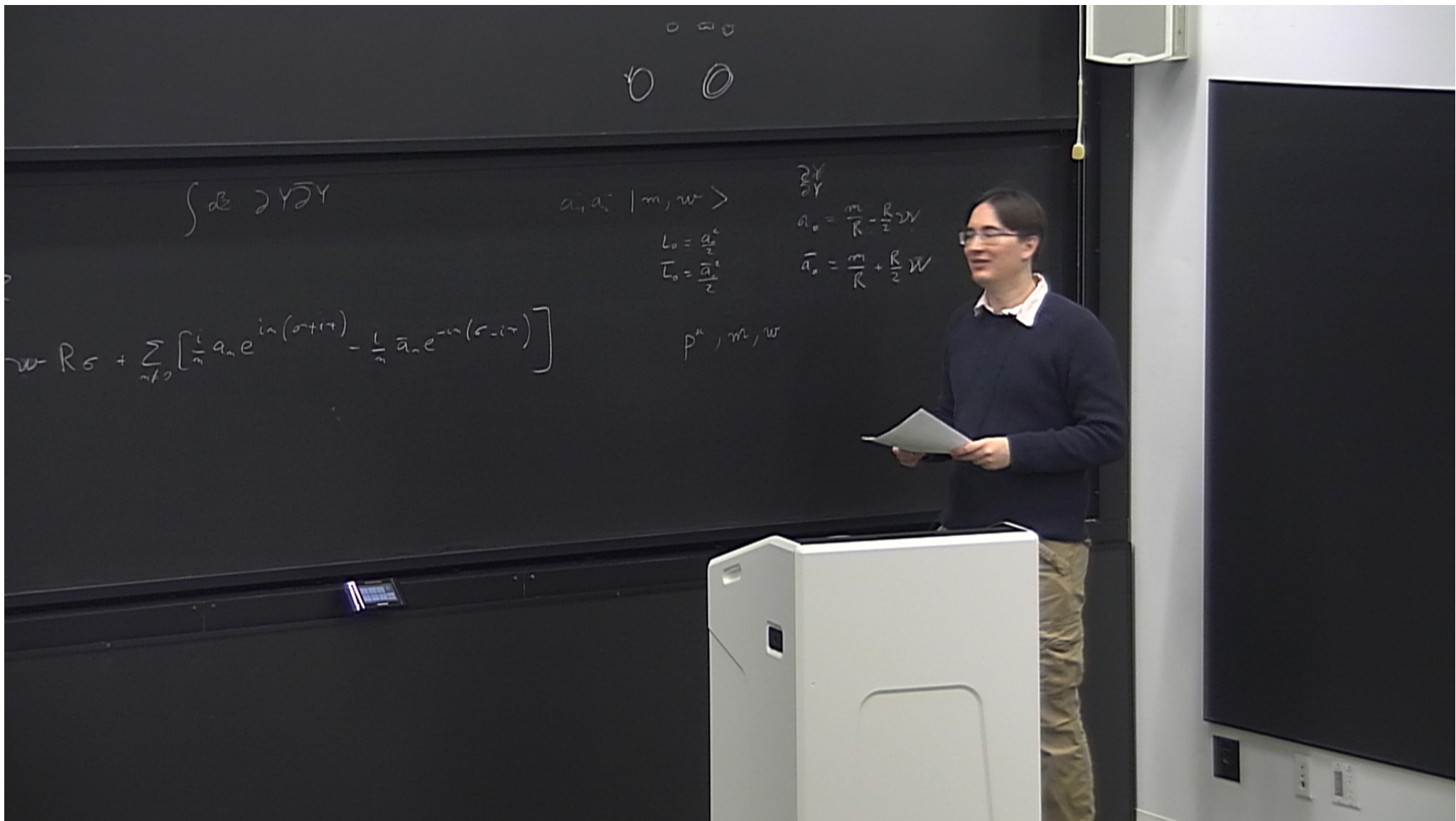
$$a_0 = \frac{m}{R} - \frac{R}{2} w$$

$$\bar{a}_0 = \frac{m}{R} + \frac{R}{2} w$$

$$w - R\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - i\tau)} \right]$$

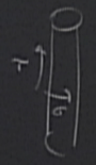






$$R^{1,2,4} \times S^1$$

$$X^m \quad Y$$



$\nu \in \mathbb{Z}$: INTEGER
 m : INTEGER

$$Y \rightarrow Y + 2\pi R$$

$$Y(\sigma, \tau) = y - 2ip\tau + w - R\sigma + \sum_{n \neq 0} \left[\alpha_n e^{-in(\sigma - i\tau)} - \frac{1}{n} \bar{\alpha}_n e^{-in(\sigma + i\tau)} \right]$$

$$P = \frac{m^2}{R}$$

$$\frac{m}{R} = \int \dots$$

$$\int d^2z \quad Y \partial Y$$

$$a_n, \bar{a}_n \quad |m, w\rangle$$

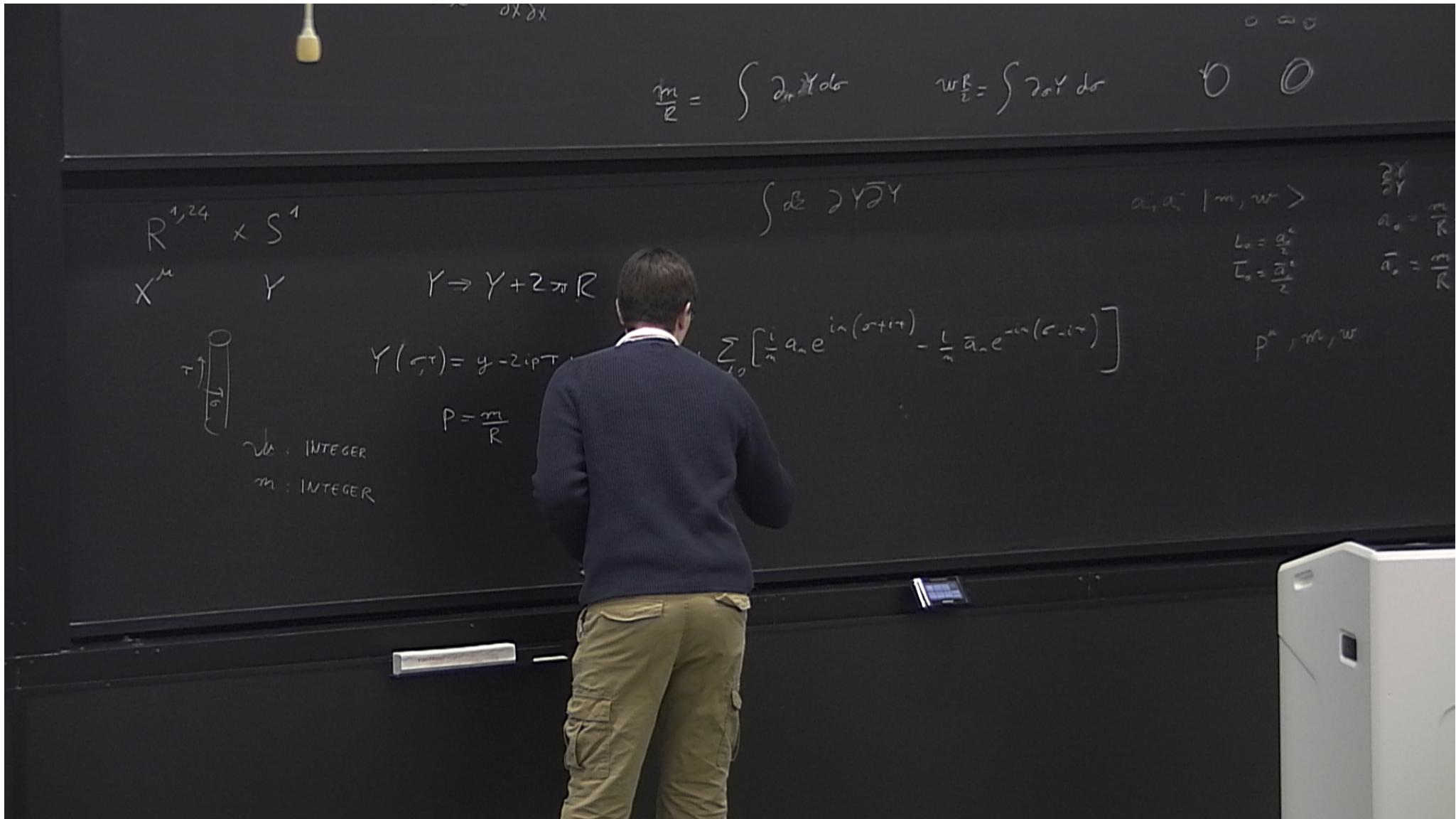
$$L_0 = \frac{\alpha_0^2}{2}$$

$$\bar{L}_0 = \frac{\bar{\alpha}_0^2}{2}$$

$$a_n = \frac{1}{\sqrt{2}} \alpha_n$$

$$\bar{a}_n = \frac{1}{\sqrt{2}} \bar{\alpha}_n$$

P^m, m, w



$\partial x \partial x$

$$\frac{m}{R} = \int \partial_r Y dr$$

$$\frac{wR}{z} = \int \partial_r Y dr$$

$\emptyset \quad \emptyset$

$$R^{1,24} \times S^1$$

$$X^m \quad Y$$

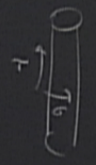
$$Y \rightarrow Y + 2\pi R$$

$$\int dz \partial Y \bar{\partial} Y$$

$a_n, \bar{a}_n \mid m, w \rangle$

$$L_0 = \frac{\alpha^2}{2}$$
$$\bar{L}_0 = \frac{\bar{\alpha}^2}{2}$$

$$a_n = \frac{2\pi}{R} n$$
$$\bar{a}_n = \frac{2\pi}{R} n$$



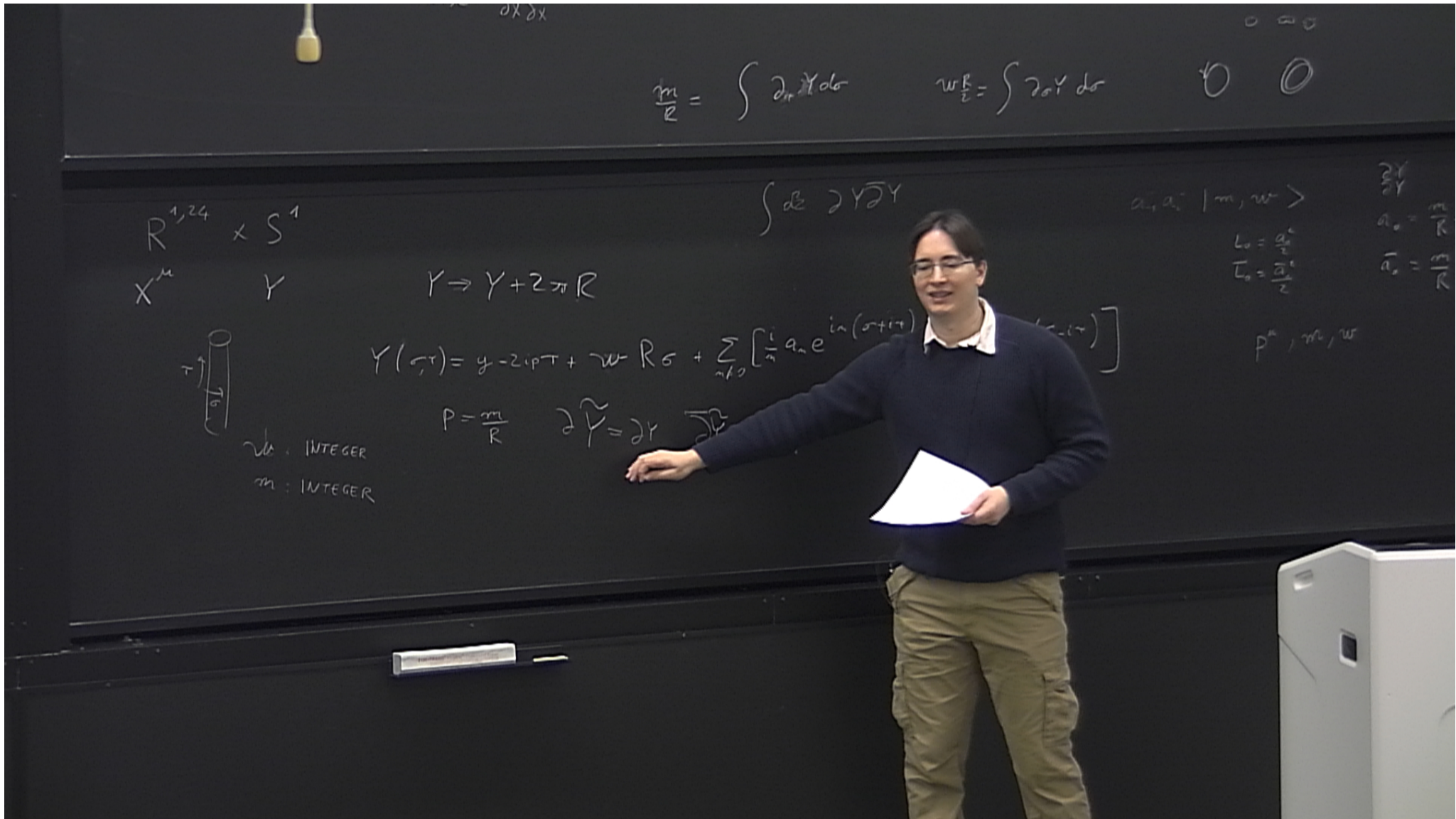
$$Y(\sigma, \tau) = y - 2ip\tau$$

$$\sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - i\tau)} \right]$$

p^m, m, w

$$p = \frac{m}{R}$$

$\sqrt{2\alpha'} : \text{INTEGER}$
 $m : \text{INTEGER}$



$$\frac{m}{R} = \int \partial_t Y dt$$

$$w \frac{R}{L} = \int \partial_t Y dt$$

0 0

$$R^{1,2,4} \times S^1$$

$$X^m \quad Y$$

$$Y \rightarrow Y + 2\pi R$$

$$\int dz \partial Y \bar{\partial} Y$$

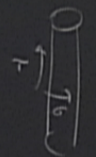
$$a_n, \bar{a}_n |m, w\rangle$$

$$L_0 = \frac{\alpha_n^2}{2}$$

$$\bar{L}_0 = \frac{\bar{\alpha}_n^2}{2}$$

$$a_n = \frac{2\pi}{R} n$$

$$\bar{a}_n = \frac{2\pi}{R} n$$



$$Y(\sigma, \tau) = y - 2ip\tau + w - R\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + i\tau)} + \frac{i}{n} \bar{a}_n e^{in(\sigma - i\tau)} \right]$$

$$p = \frac{m}{R} \quad \partial \tilde{Y} = \partial Y \quad \bar{\partial} \tilde{Y}$$

$w \in \mathbb{Z}$: INTEGER
 m : INTEGER

p^m, m, w

$$\frac{m}{R} = \int \partial_r Y dr$$

$$w \frac{R}{L} = \int \partial_r Y dr$$

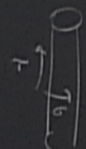
0 0

$$R^{1,2,4} \times S^1$$

$$X^m \quad Y$$

$$Y \rightarrow Y + 2\pi R$$

$$\int d^2 Y \partial Y$$



$$Y(\sigma, \tau) = y - 2ip\tau + w - R\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - i\tau)} \right]$$

$\nu \in \text{INTEGER}$
 $m : \text{INTEGER}$

$$p = \frac{m}{R}$$

$$\partial \tilde{Y} = \partial Y \quad \bar{\partial} \tilde{Y} = -\bar{\partial} Y$$

$$\tilde{Y} = \tilde{y} - 2p\sigma + i\nu R\tau + \sum \dots + \dots$$

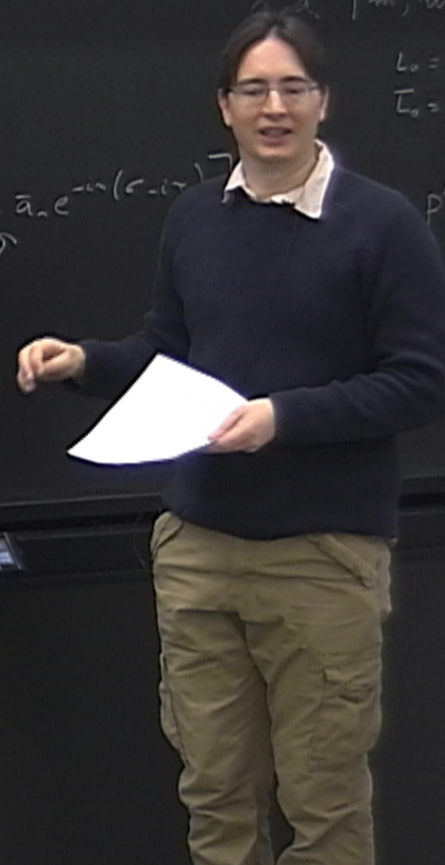
$$|m, w\rangle$$

$$L_0 = \frac{\alpha'}{2} p^2$$

$$\bar{L}_0 = \frac{\alpha'}{2} \bar{p}^2$$

$$a_n = \frac{2}{\alpha'} n$$

p^m, m, w



$$\frac{m}{R} = \int \partial_{\tau} Y d\sigma$$

$$\frac{wR}{2} = \int \partial_{\sigma} Y d\sigma$$

0 0

$$\int d\sigma \partial Y \partial Y$$

$a_{\pm}, a_{\pm}^{\dagger} |m, w\rangle$

$$L_0 = \frac{\alpha_0^2}{2}$$

$$\bar{L}_0 = \frac{\bar{\alpha}_0^2}{2}$$

$$\frac{\partial Y}{\partial \tau} = \frac{m}{R} - \frac{R}{2} \partial Y$$

$$\bar{\alpha}_0 = \frac{m}{R} + \frac{R}{2} W$$

$$R \leftrightarrow \frac{2}{R}$$

$$-R\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma+\tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma-\tau)} \right]$$

p^{μ}, m, w

$$= \partial \tau \partial \bar{Y} = -\partial \bar{\tau}$$

$$-2p^{\sigma} + i\alpha_0 R \tau + \sum \dots + \dots$$



$$\frac{m}{R} = \int \partial_r Y d\sigma$$

$$\frac{mR}{2} = \int \partial_\sigma Y d\sigma$$

0 0

$$\int d\sigma \partial Y \partial Y$$

$a_+, a_- |m, w\rangle$

$$L_0 = \frac{a_+^2}{2}$$

$$\bar{L}_0 = \frac{\bar{a}_+^2}{2}$$

$\frac{\partial Y}{\partial \sigma}$

$$a_+ = \frac{m}{R} - \frac{R}{2} w$$

$$\bar{a}_+ = \frac{m}{R} + \frac{R}{2} w$$

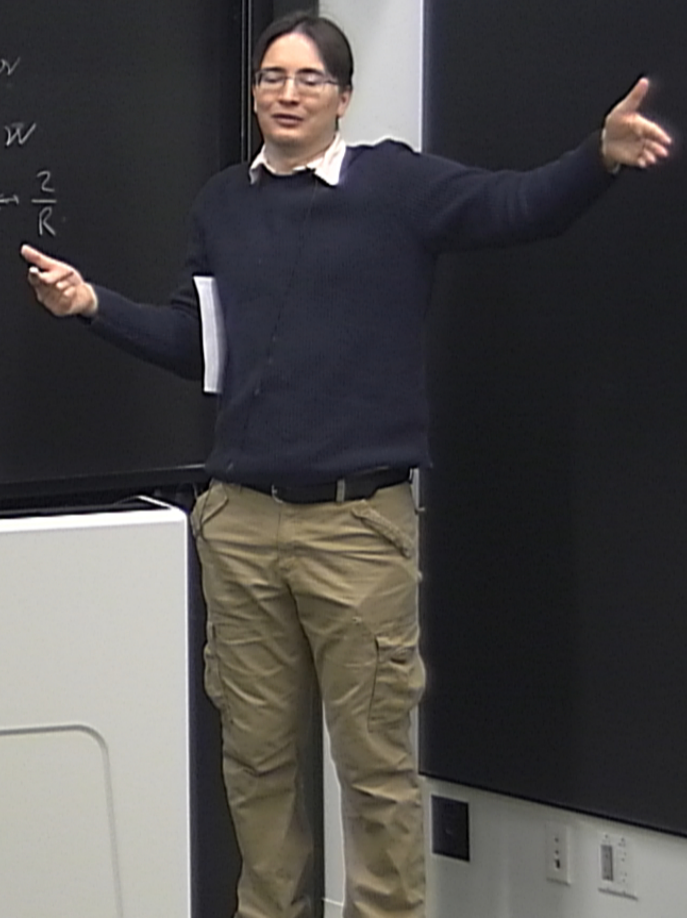
$$R \leftrightarrow \frac{2}{R}$$

p^+, m, w

$$+ R\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + \tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - \tau)} \right]$$

$$= \partial \tau \quad \partial \bar{\tau} = -\partial \tau$$

$$-2p^+ + i\omega R\tau + \sum \dots + \dots$$



$$\frac{m}{R} = \int \partial_r Y d\sigma$$

$$\frac{wR}{2} = \int \partial_\sigma Y d\sigma$$

0 0

$$\int d\sigma \partial Y \partial Y$$

$a_+, a_- |m, w\rangle$

$$L_0 = \frac{a_+^2}{2}$$

$$\bar{L}_0 = \frac{\bar{a}_+^2}{2}$$

$$\frac{\partial Y}{\partial \sigma} = \frac{m}{R} - \frac{R}{2} \omega$$

$$\bar{a}_+ = \frac{m}{R} + \frac{R}{2} \omega$$

$$R \leftrightarrow \frac{2}{R}$$

p^+, m, w

$$+ R\sigma + \sum_{n \neq 0} \left[\frac{i}{n} a_n e^{in(\sigma + \tau)} - \frac{i}{n} \bar{a}_n e^{-in(\sigma - \tau)} \right]$$

$$= \partial \tau \quad \partial \bar{\tau} = -\partial \tau$$

$$-2p^+ + i\omega R\tau + \sum \dots + \dots$$

