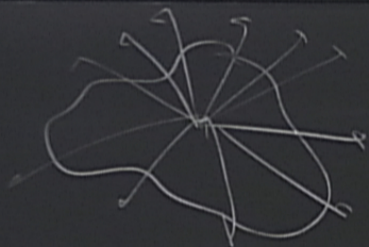


Title: 13/14 PSI - String Theory Review - Lecture 9

Date: Feb 07, 2014 10:15 AM

URL: <http://pirsa.org/14020027>

Abstract:



$$f(x, y) = 0$$

$$x = f(t)$$


$$y = g(t)$$

$$S(f) = \dots$$

$$f(x, y, t) = 0$$

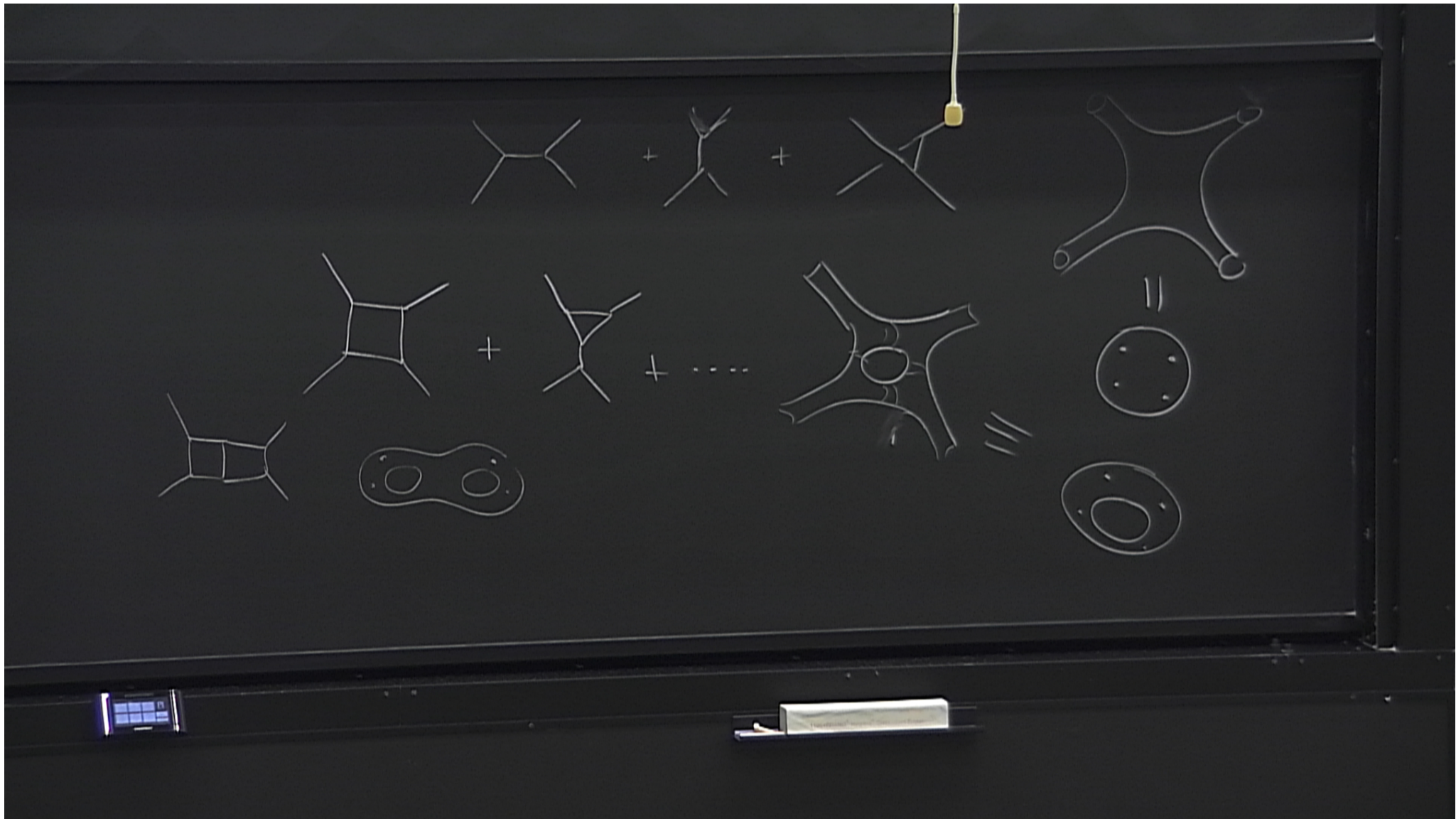
$$g(x, y, t) = 0$$

FOR SOME t

$$A = \sum_l \int_{\mathcal{D}_l} g^{i\bar{l}} \left(\int_{\mathcal{D}_l} \left[b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \mu^\nu} d\mu^\nu \right] \prod_i c_i \bar{c}_i V(z_i) \right) \tilde{h}_{ab}(\mu)$$


$$A = \sum_{\text{loops}} g^{2\text{loops}} \left\langle \prod_{\nu} \left[b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \mu^{\nu}} d\mu^{\nu} \right] \prod_i c_i \bar{c}_i V(z_i) \right\rangle_{\tilde{h}_{ab}(\mu)}$$

$h_{ab}(\mu)$



$$A = \sum_{\text{loops}} g^{2l_{\text{loops}}} \left\langle \prod_{\nu=1}^{\#\mu} \left[b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \mu^\nu} d\mu^\nu \right] \prod_i c_i \bar{c}_i V(z_i) \right\rangle$$

$$\tilde{h}_{ab}(\mu)$$



$$\#\mu = 2[3g - 3 + \text{no OF POINTS}]$$



A_{tree}

$$A_n^{tree} = \int \langle \dots \rangle$$

CONFORMAL GAUGE

$\mu \equiv$ POSITIONS OF PUNCTURES

$$A_n^{\text{tree}} = \int \left\langle c\bar{c}V_1(z_1) c\bar{c}V_2(z_2) c\bar{c}V_3(z_3) \prod_{k=4}^n V_{\alpha_k}(z_k) dz_k d\bar{z}_k \right\rangle_{\text{SPHERE}}$$

$$O_{\Delta, \bar{\Delta}}(z, \bar{z}) = \left(\frac{\partial \tilde{z}}{\partial z} \right)^{\Delta} \left(\frac{\partial \tilde{\bar{z}}}{\partial \bar{z}} \right)^{\bar{\Delta}} O(\tilde{z}, \tilde{\bar{z}})$$

$$[O(z) dz^{\Delta} d\bar{z}^{\bar{\Delta}}]$$

A_{tree}

$$A_n^{tree} = \int \langle c\bar{c}V_1(z_1) c\bar{c}V_2(z_2) c\bar{c}V_3(z_3) \dots \rangle$$

CONFORMAL GAUGE

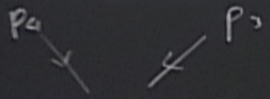
$\mu \equiv$ POSITIONS OF PUNCTURES

p_4 ↘ ↙ p_3

↗ ↘ p_1 p_2

$$\langle c\bar{c} c\bar{c} c\bar{c} \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{ip_1 X(z_1)} e^{ip_2 X(z_2)} e^{ip_3 X(z_3)} e^{ip_4 X(z_4)} \rangle = \int (\prod_i p_i) \prod_i |z_i - z_j|^{2p_i p_j}$$



$$\langle c\bar{c} c\bar{c} c\bar{c} \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_4|^2$$

$$\langle e^{ip_1 X(z_1)} e^{ip_2 X(z_2)} e^{ip_3 X(z_3)} e^{ip_4 X(z_4)} \rangle = \delta(\sum p_i) \prod |z_i - z_j|^{2p_i \cdot p_j}$$

$$\mathcal{A}_4^{\text{tree}} = \prod_{1 \leq i < j \leq 3} |z_i - z_j|^{2 + 2p_i \cdot p_j} \int dz \prod_{1 \leq i \leq 3} |z - z_i|^{2p_i \cdot p_i}$$

$$\int dz \frac{1}{|z|^\alpha} = \int \frac{z dz}{z^\alpha} \quad \alpha-1 < 1$$

$$z - z_j$$

$$\sum p_i = 0$$

$$p_i^2 = 2$$

$$\sim |z|^{2p_4 - \sum p_i} = |z|^{-2p_4} = |z|^{-4}$$

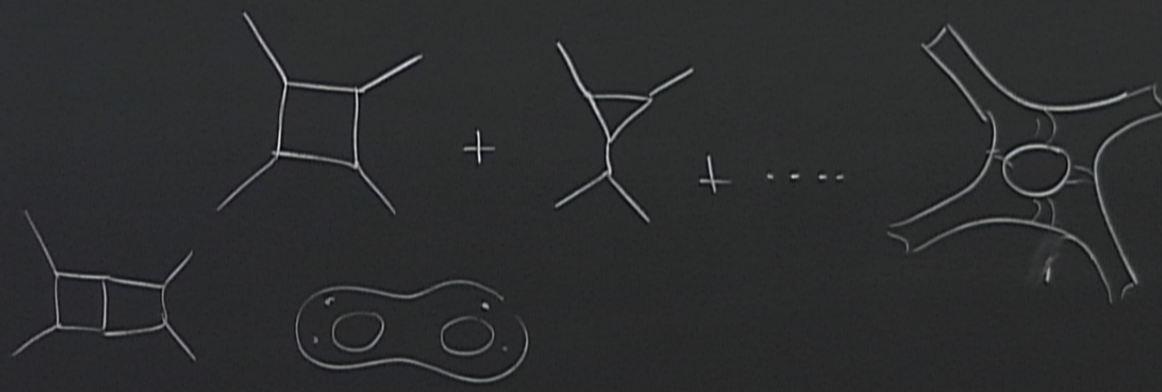
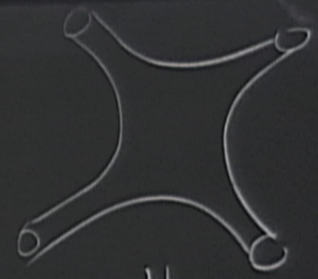
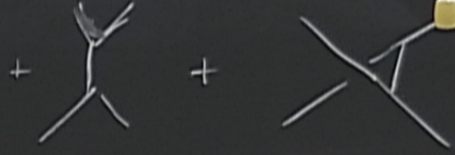
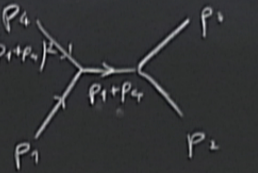
$$2p_4 - p_i \geq -2 \quad (p_4 + p_i)^2 \geq 2$$

$$= \int (\sum p_i) \prod |z_i - z_j|^{2p_i p_j}$$

$$\prod_{i < j} |z_i - z_j|^{2p_i p_j}$$

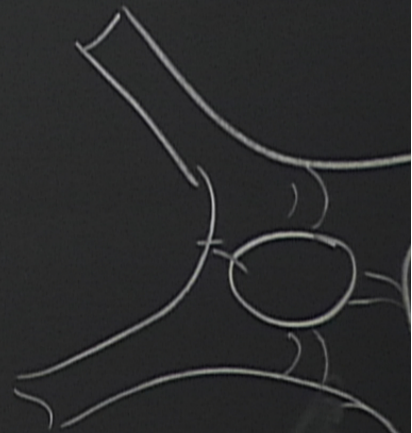
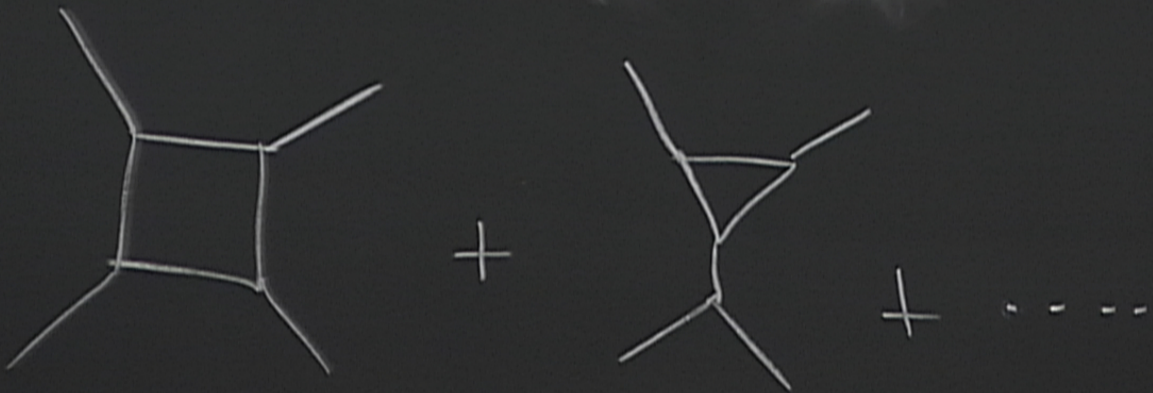
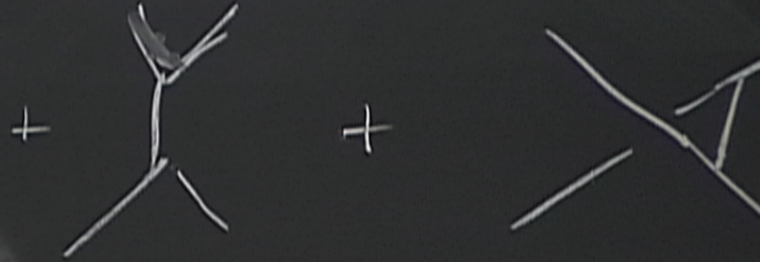
$$\frac{1}{(P_1 + P_2)^2} =$$

$$\int_0^\infty d\tau_{14} e^{-\tau_{14}(P_1 + P_2)^2}$$



\bar{z}^0

$$\frac{1}{(p_1 + p_2)^2 + m^2} = \int_0^\infty d\tau_{14} e^{-\tau_{14} (p_1 + p_2)^2 - m^2 \tau_{14}}$$

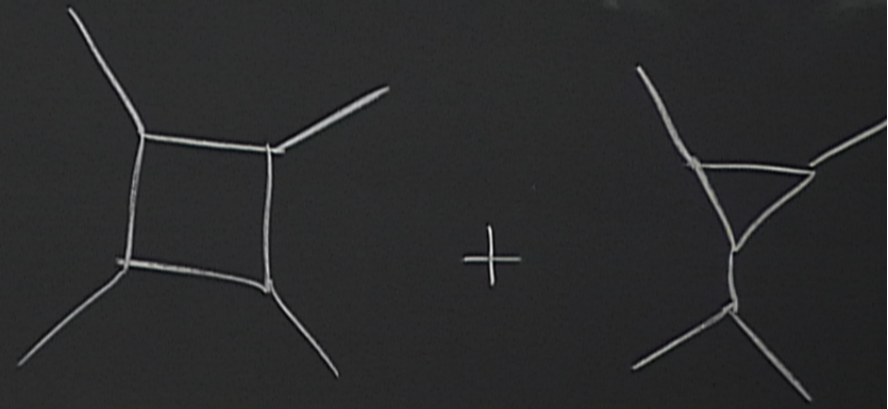


$$(p_1 + p_4)^2 + m^2 > 0$$

$$\frac{1}{(p_1 + p_4)^2 + m^2} = \int_0^{\infty} d\tau_{14} e^{-\tau_{14}((p_1 + p_4)^2 + m^2)}$$

0
 $\tilde{h}_{\mu\nu}(x)$

MC OF POINTS]



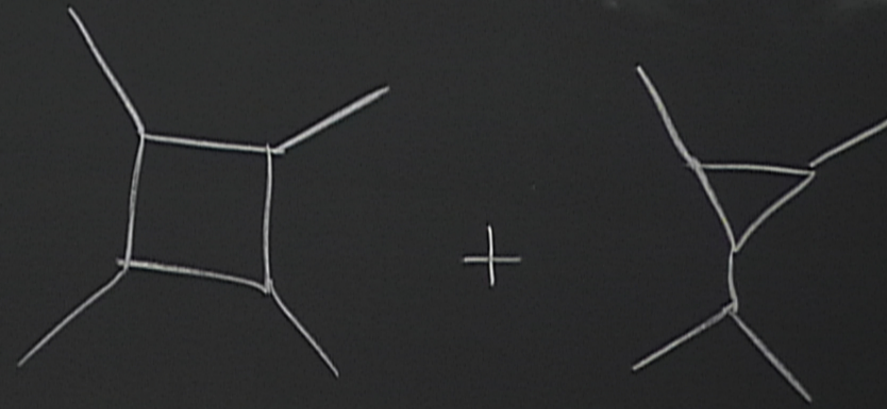
$$(P_1 + P_4)^2 + m^2 > 0$$

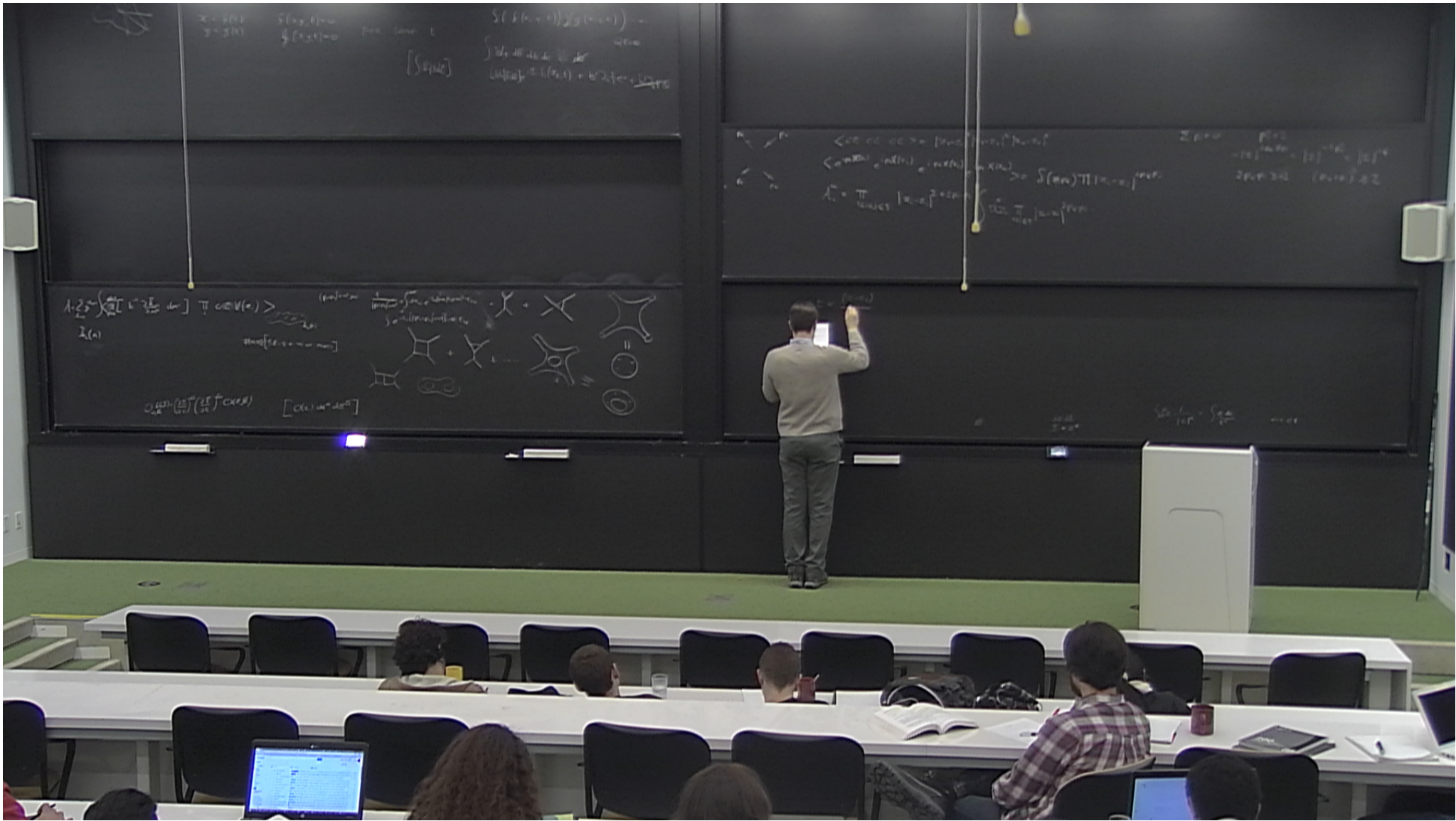
$$\frac{1}{(P_1 + P_4)^2 + m^2} = \int_0^\infty d\tau_{14} e^{-\tau_{14} [(P_1 + P_4)^2 + m^2]}$$

$$\int_0^\infty e^{it_{14} [(P_1 + P_4)^2 + m^2]} - \epsilon t_{14}$$

$\tilde{h}_{\mu\nu}(x)$

NO OF POINTS]





$$t = \frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$A_4 = \int dt |t|^{2p_4 - p_1} \dots$$

$$\int_{|z|<1} \frac{1}{|z|^k} = \int \frac{z dz}{z^k} \quad k-1 < 1$$

$$\bar{z} = z^*$$

$$t = \frac{(z-z_1)(z_2-z_1)}{(z-z_2)(z_1-z_1)}$$

$$h_z = \int dt |t|^{2p_1-p_2} |1-t|^{2p_2-p_1}$$

$$|z|^n = \int e^{-|z|^n}$$

$$\int_{|z|<1} \frac{1}{|z|^n} = \int \frac{r dr}{r^n} \quad n-1 < 1$$

$$t = \frac{(z-z_1)(z_2-z_1)}{(z-z_2)(z_1-z_1)}$$

$$h_2 = \int dt |t|^{2p_1-1} |1-t|^{2p_2-1} = \frac{\Gamma(1+p_1)\Gamma(1+p_2)}{\Gamma(1+p_1+p_2)}$$

$$|t|^\alpha = \int_0^1 e^{-st} t^\alpha e^{-s(1-t)} ds$$

$$\int_{|z|<1} \frac{1}{|z|^\alpha} = \int_0^1 \frac{r dr}{r^\alpha} \quad \alpha < 1$$

$$\frac{dz d\bar{z}}{z} = \frac{dz}{z}$$

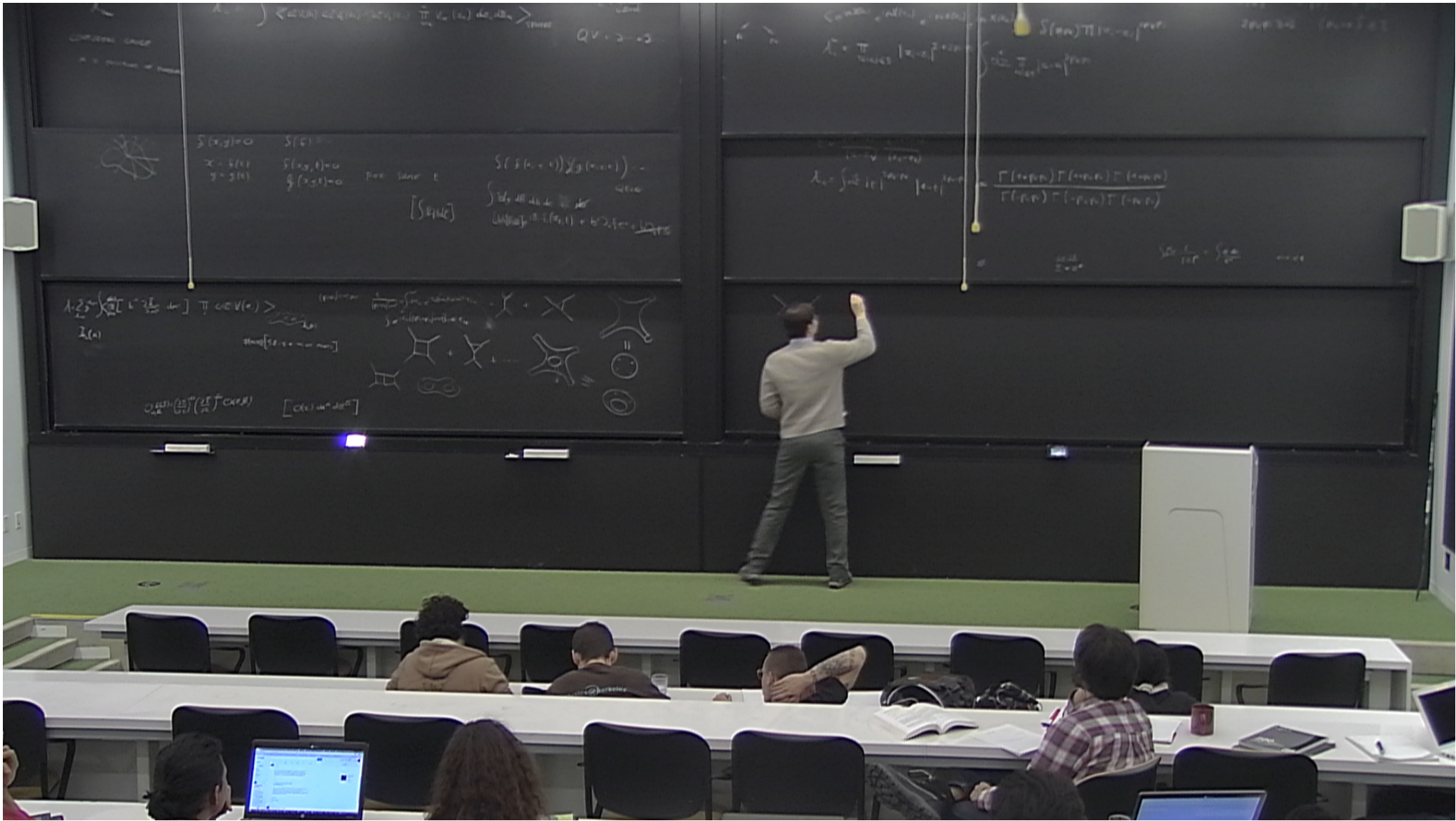
$$t = \frac{(z-z_1)(z_2-z_1)}{(z-z_2)(z_1-z_1)}$$

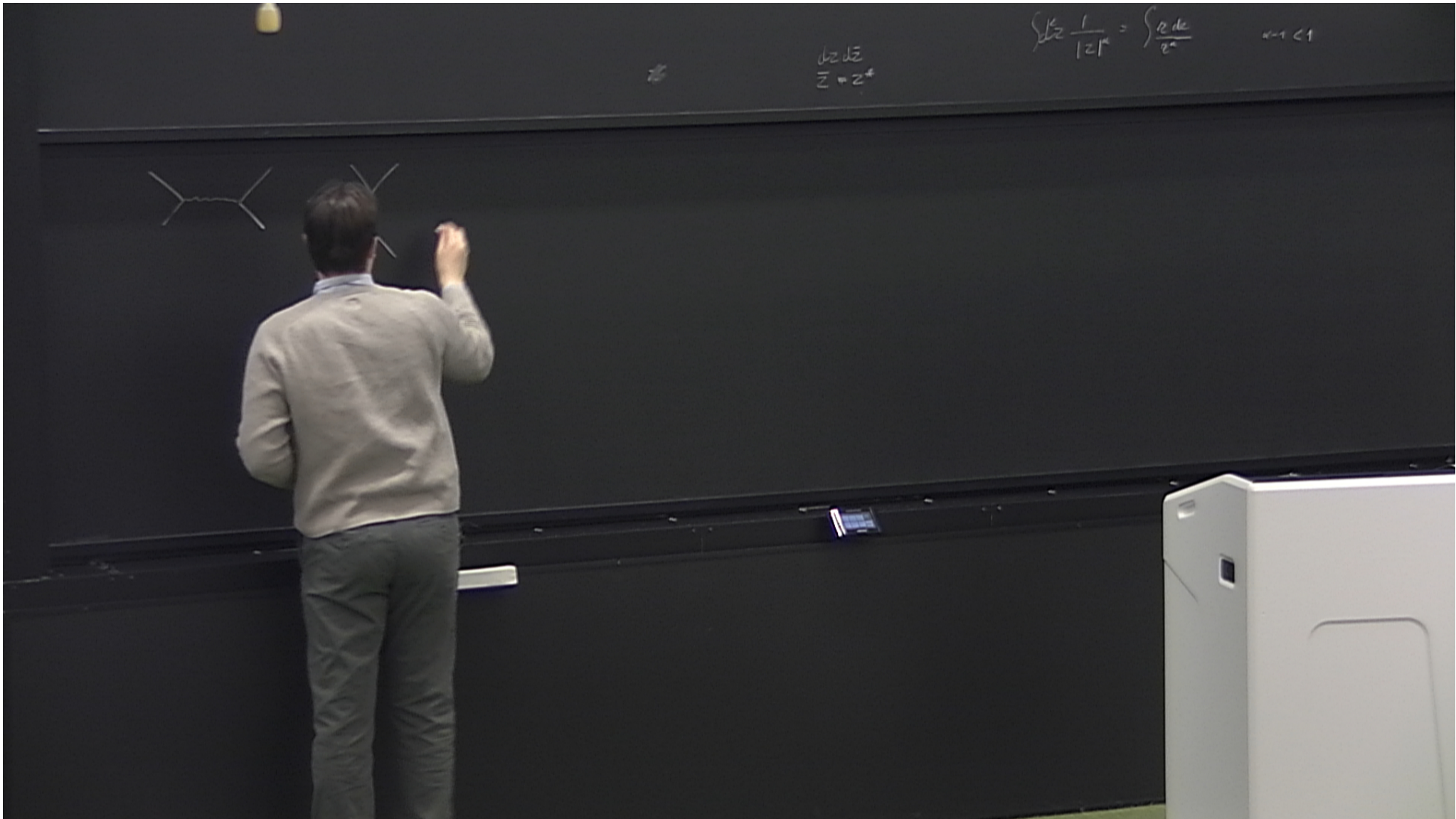
$$h_4 = \int dt |t|^{2p_1-p_1} |1-t|^{2p_2-p_2} = \frac{\Gamma(1+p_1+p_2) \Gamma(1+p_1-p_2) \Gamma(1+p_2-p_1)}{\Gamma(-p_1+p_2) \Gamma(-p_2+p_1)}$$

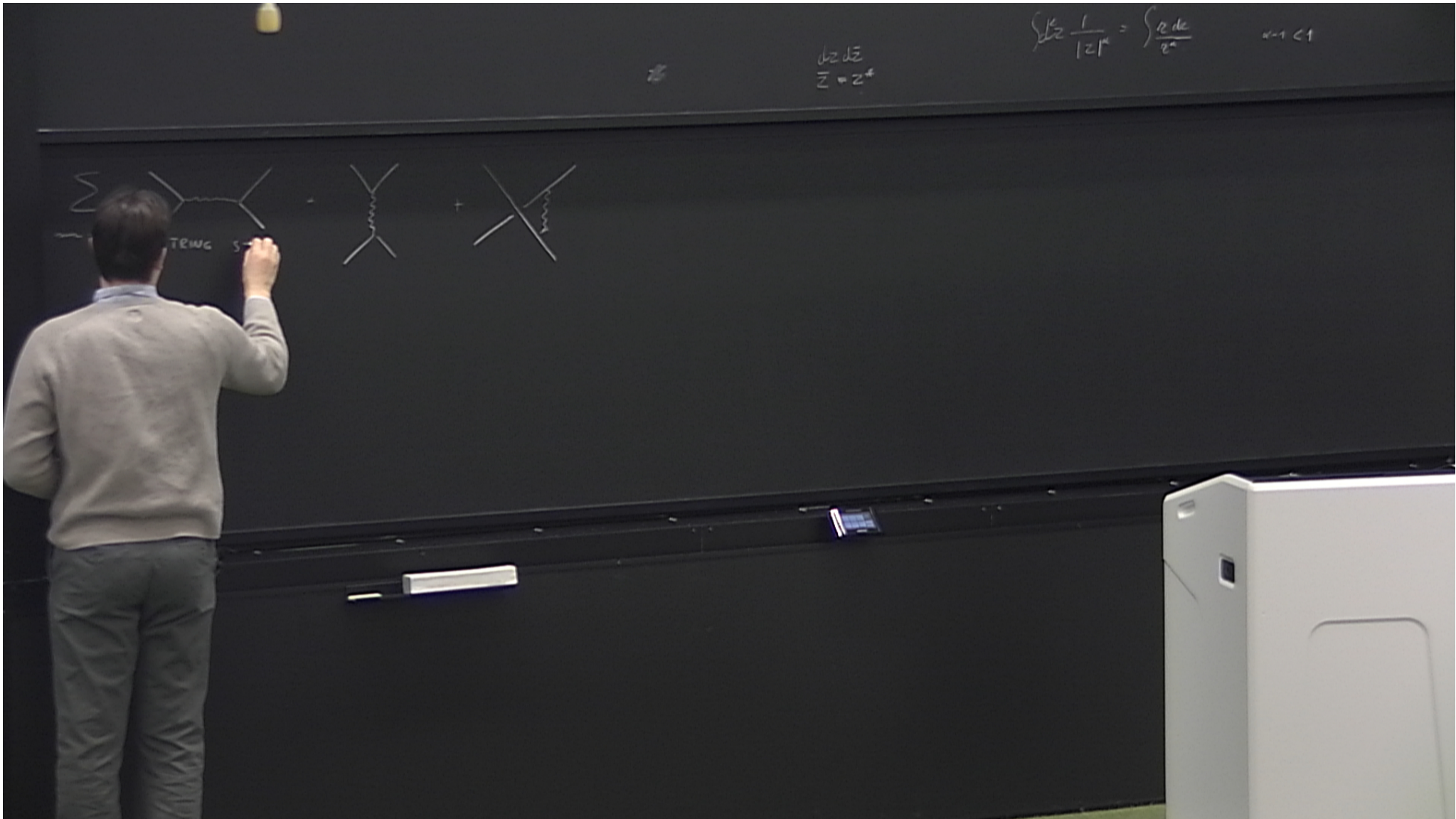
$$|z|^\alpha = \int e^{-st} t^\alpha s^{-\alpha-1} ds$$

$$\frac{dz d\bar{z}}{z} = \frac{dz}{z}$$

$$\frac{1}{|z|^\alpha} = \int \frac{z d\bar{z}}{z^\alpha} \quad \alpha > 1$$



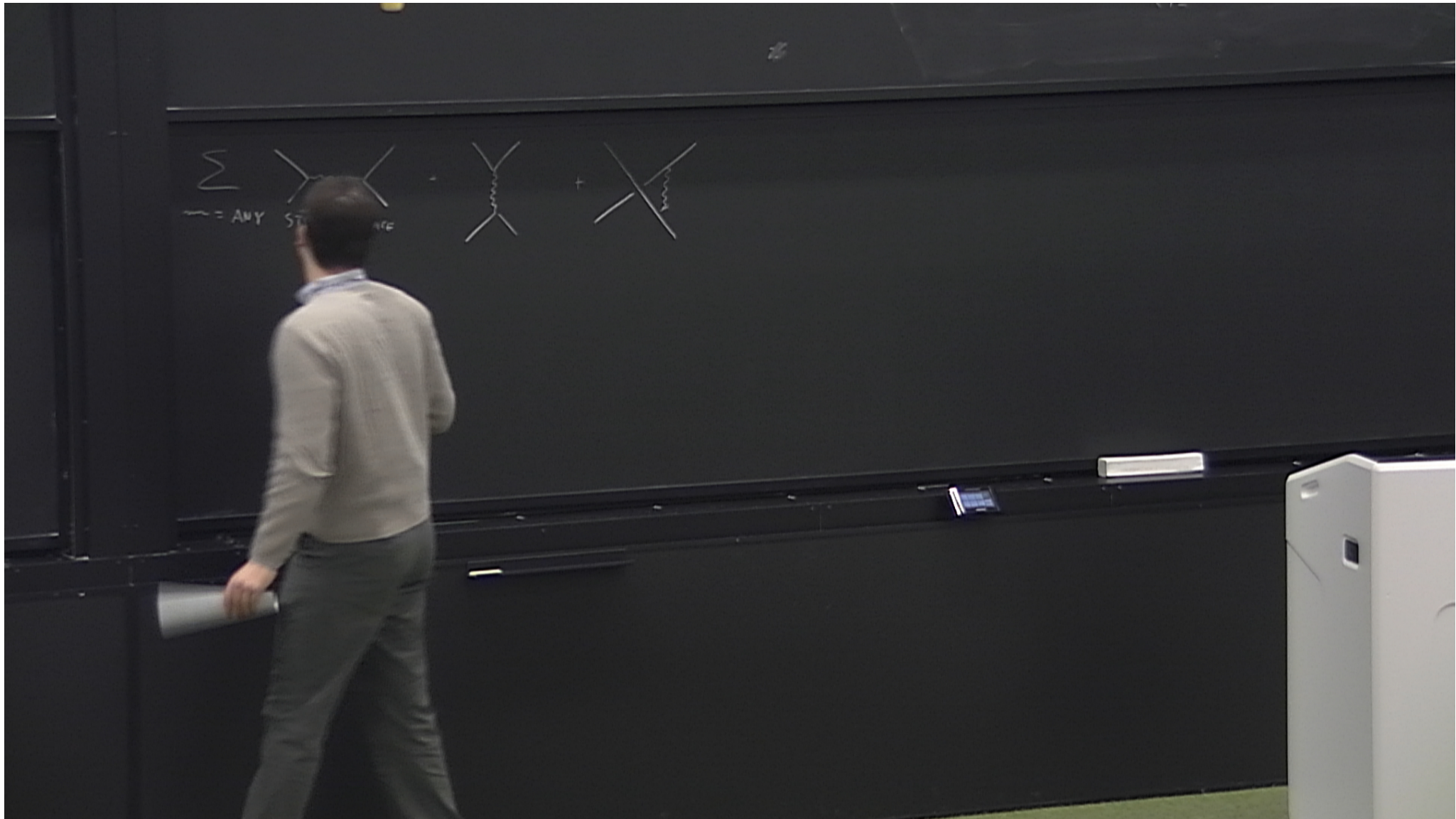


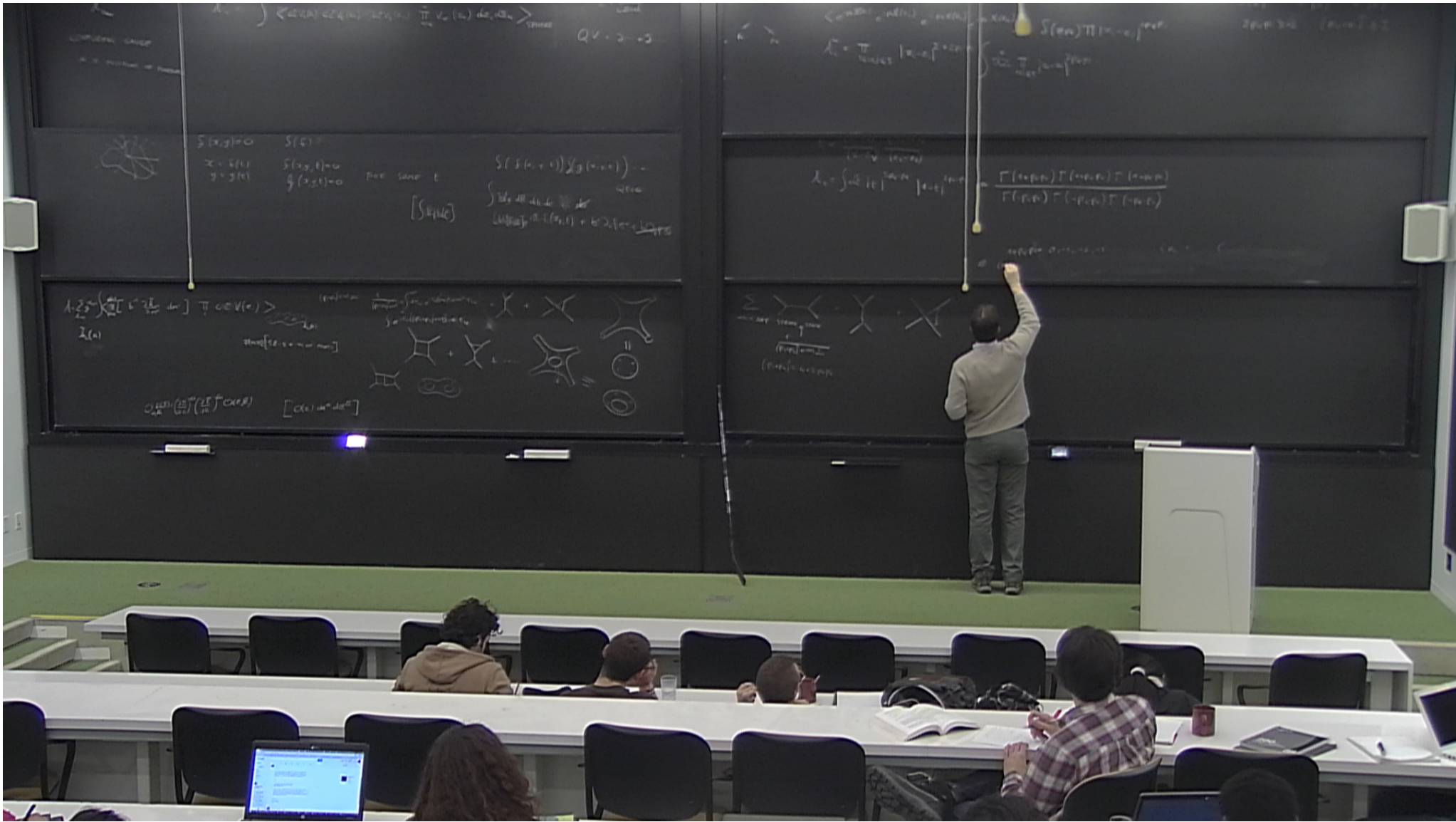




$$\bar{z} = z^*$$

$$\int \frac{dz}{|z|^k} = \int \frac{dz}{z^k}$$






①

$$1 + p_4 \cdot p_i = 0, -1, -2, -3, \dots$$

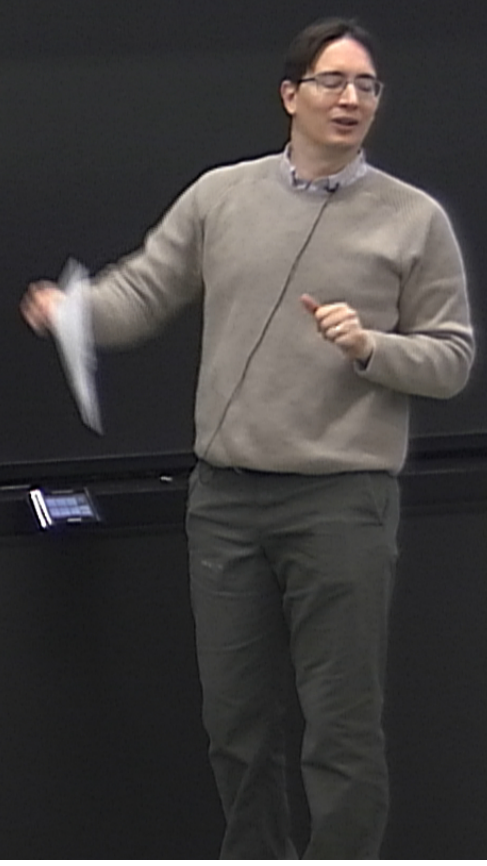
$$(p_1 + p_4)^2 = 2, 0, -2, -4, -6, \dots$$

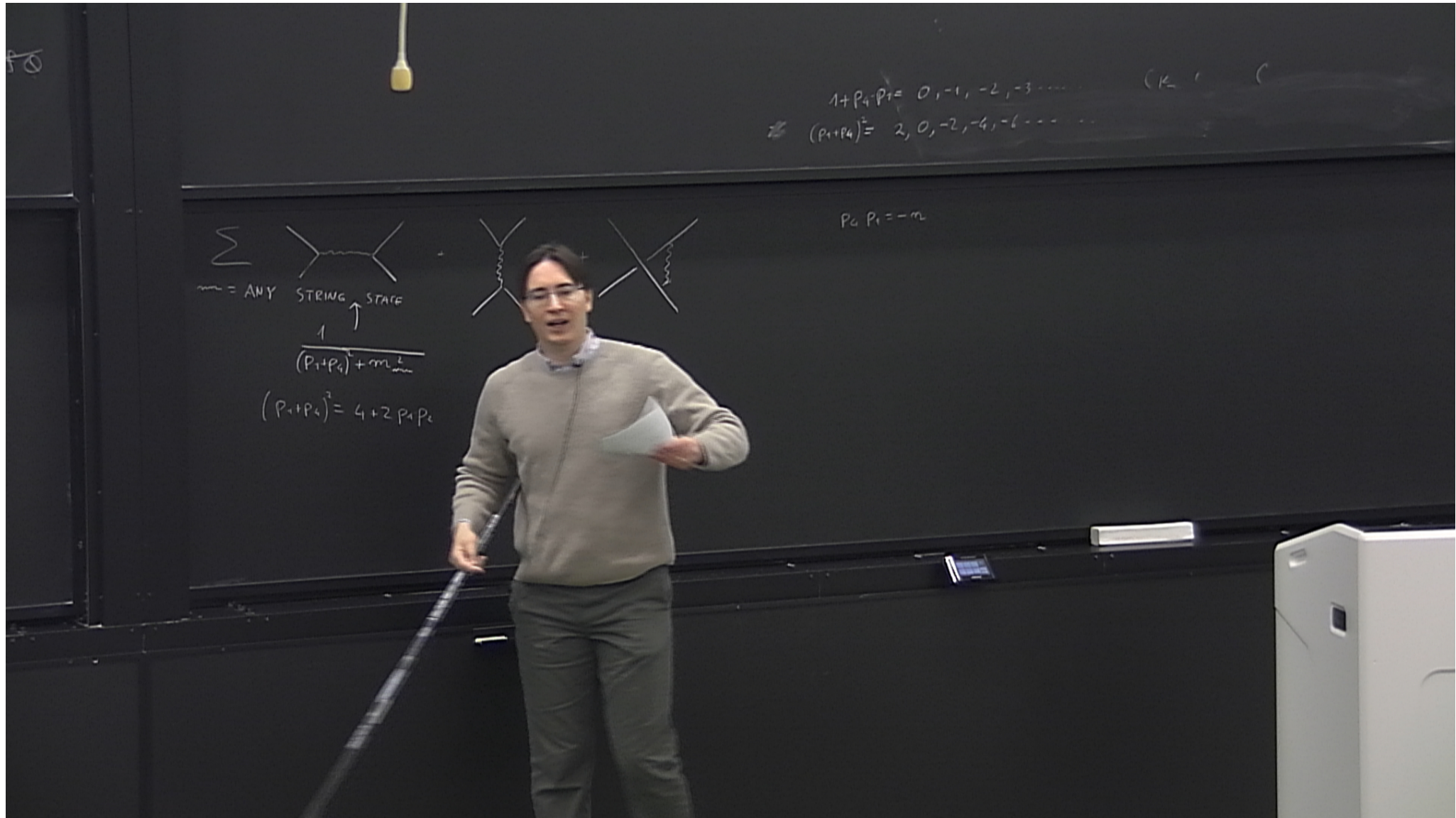
Σ 

$\sim =$ ANY STRING SPACE

$\frac{1}{(p_1 + p_4)^2 + m^2}$

$(p_1 + p_4)^2 = 4 + 2 p_1 \cdot p_4$






$$1 + P_4 P_1 = 0, -1, -2, -3, \dots$$

$$(P_1 + P_4)^2 = 2, 0, -2, -4, -6, \dots$$

$$P_0 P_1 = -m$$

Σ = ANY STRING SPACE
 (Diagram of a four-point vertex with a wavy internal line)
 $\frac{1}{(P_1 + P_4)^2 + m^2}$
 $(P_1 + P_4)^2 = 4 + 2P_1 P_2$

\sum 

 \sim ANY STRING SPACE

 $\frac{1}{(P_1+P_4)^2 + m^2}$

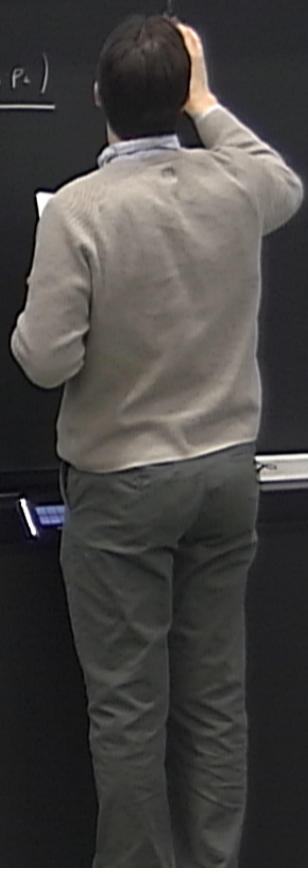
 $(P_1+P_4)^2 = 4 + 2P_1 \cdot P_4$

$1 + P_4 \cdot P_1 = 0, -1, -2, -3, \dots$

 $(P_1+P_4)^2 = 2, 0, -2, -4, -6, \dots$

$P_0 \cdot P_1 = -m$

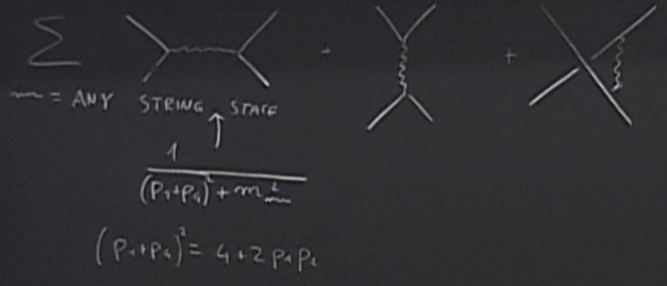
 $\sim \frac{1}{\alpha'} \frac{\Gamma(1 + P_4 \cdot P_1)}{\Gamma(\dots)}$



$$1 + p_4 \cdot p_1 = 0, -1, -2, -3, \dots \quad (k=1)$$

$$(p_1 + p_4)^2 = 2, 0, -2, -4, -6, \dots$$

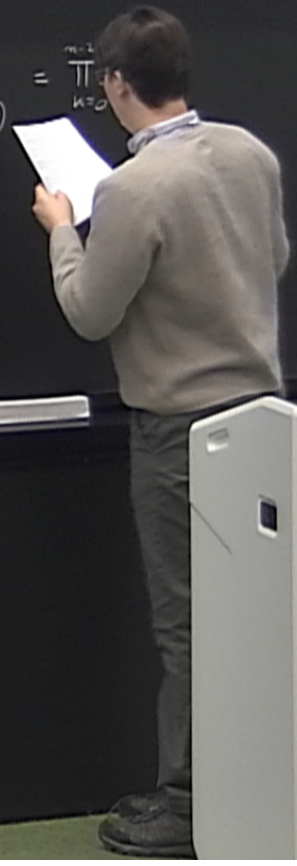
$$\Gamma(x+1) = x \Gamma(x)$$

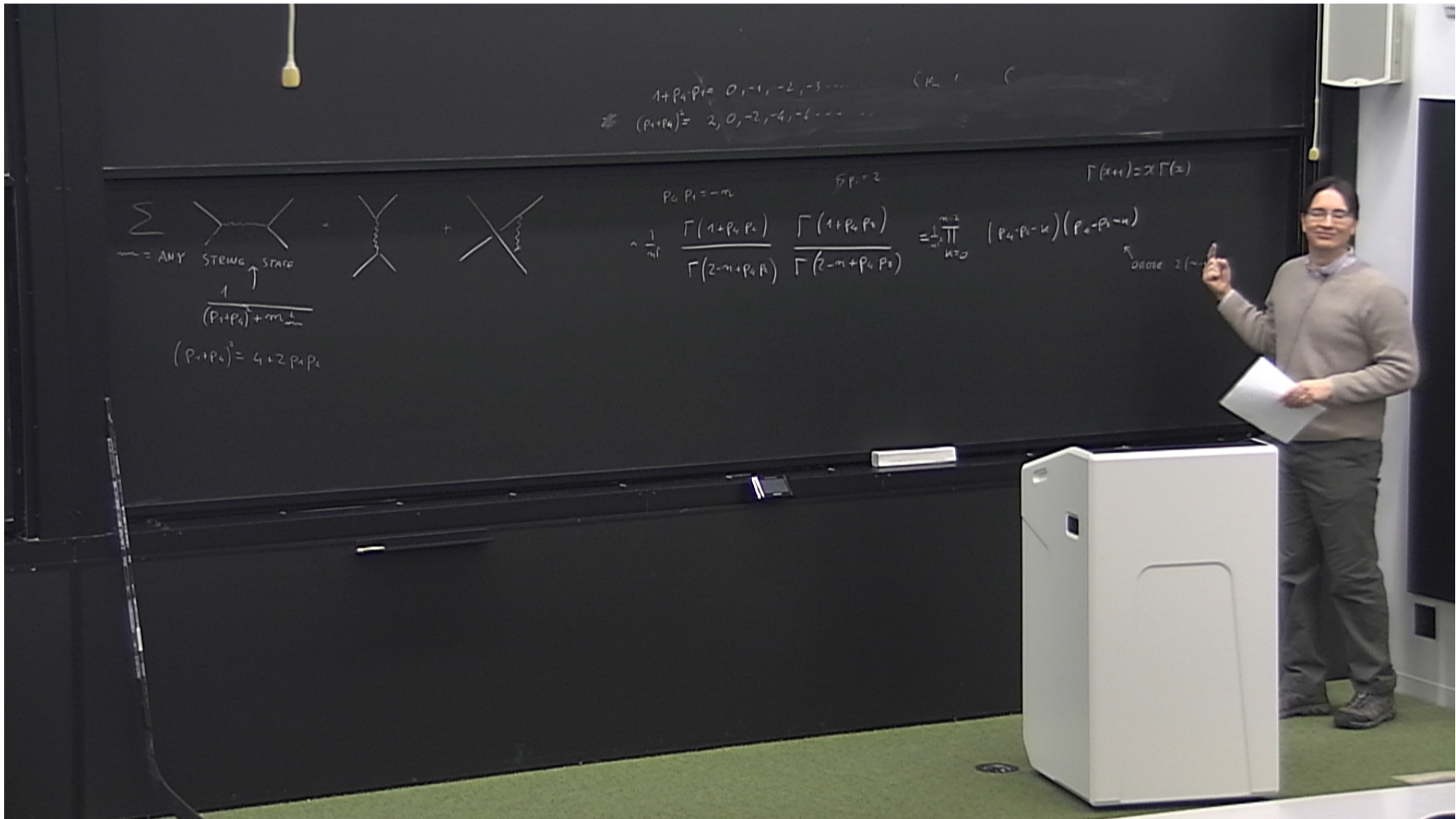


$$p_0 \cdot p_1 = -m$$

$$p_1 \cdot p_2 = 2$$

$$\frac{1}{4!} \frac{\Gamma(1 + p_4 \cdot p_1)}{\Gamma(2 - m + p_4 \cdot p_1)} \frac{\Gamma(1 + p_4 \cdot p_2)}{\Gamma(2 - m + p_4 \cdot p_2)} = \prod_{k=0}^{m-2} \dots$$





$$1 + p_0 \cdot p_1 = 0, -1, -2, -3, \dots \quad (K=1)$$

$$* (p_1 + p_4)^2 = 2, 0, -2, -4, -6, \dots$$

\sum = ANY STRING STATE
 $\frac{1}{(p_1 + p_4)^2 + m^2}$
 $(p_1 + p_4)^2 = 4 + 2 p_1 \cdot p_4$

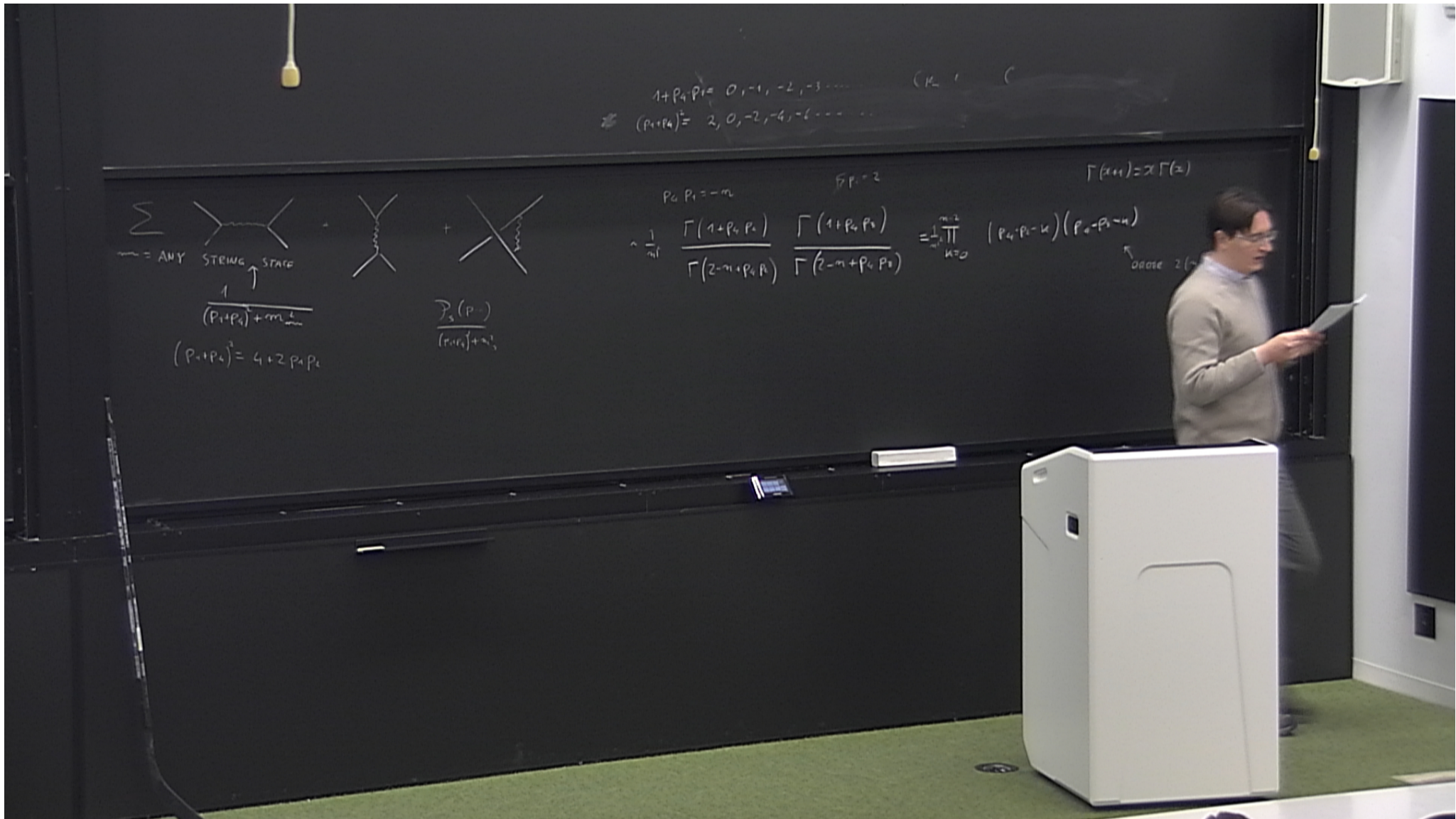
$$p_0 \cdot p_1 = -m$$

$$p_1 \cdot p_2 = 2$$

$$\frac{1}{m!} \frac{\Gamma(1 + p_0 \cdot p_1)}{\Gamma(2 - m + p_0 \cdot p_1)} \frac{\Gamma(1 + p_0 \cdot p_1)}{\Gamma(2 - m + p_0 \cdot p_1)} = \frac{1}{m!} \prod_{k=0}^{m-2} (p_4 \cdot p_1 - k)(p_4 \cdot p_3 - k)$$

$$\Gamma(x+1) = x \Gamma(x)$$

ORIGIN 2(-)



$$1 + p_0 p_1 = 0, -1, -2, -3, \dots \quad (K=1)$$

$$(p_0 + p_1)^2 = 2, 0, -2, -4, -6, \dots$$

\sum = ANY STRING STATE
 $\frac{1}{(p_0 + p_1)^2 + m^2}$
 $(p_0 + p_1)^2 = 4 + 2 p_0 p_1$

$\mathcal{P}_s(p_0)$
 $\frac{1}{(p_0 + p_1)^2 + m^2}$

$$p_0 p_1 = -m$$

$$\mathcal{P}_s(p_0) = \frac{\Gamma(1 + p_0 p_1)}{\Gamma(2 - m + p_0 p_1)}$$

$$\mathcal{P}_s(p_1) = \frac{\Gamma(1 + p_0 p_1)}{\Gamma(2 - m + p_0 p_1)}$$

$$= \frac{1}{m!} \prod_{k=0}^{m-1} (p_0 - p_1 - k) (p_0 - p_1 - k)$$

↑ degree 2(-m)

$$\Gamma(x+1) = x \Gamma(x)$$

