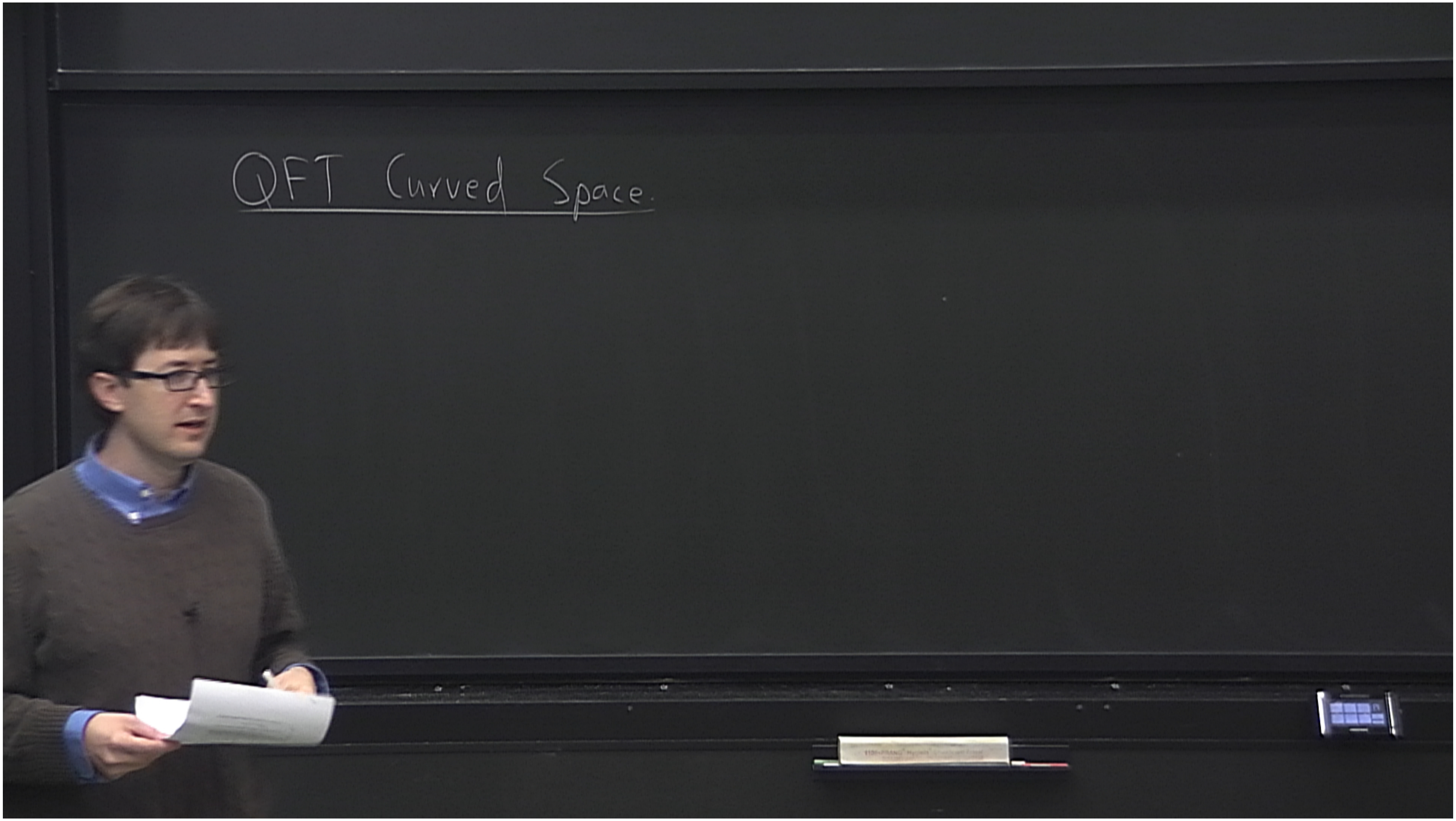
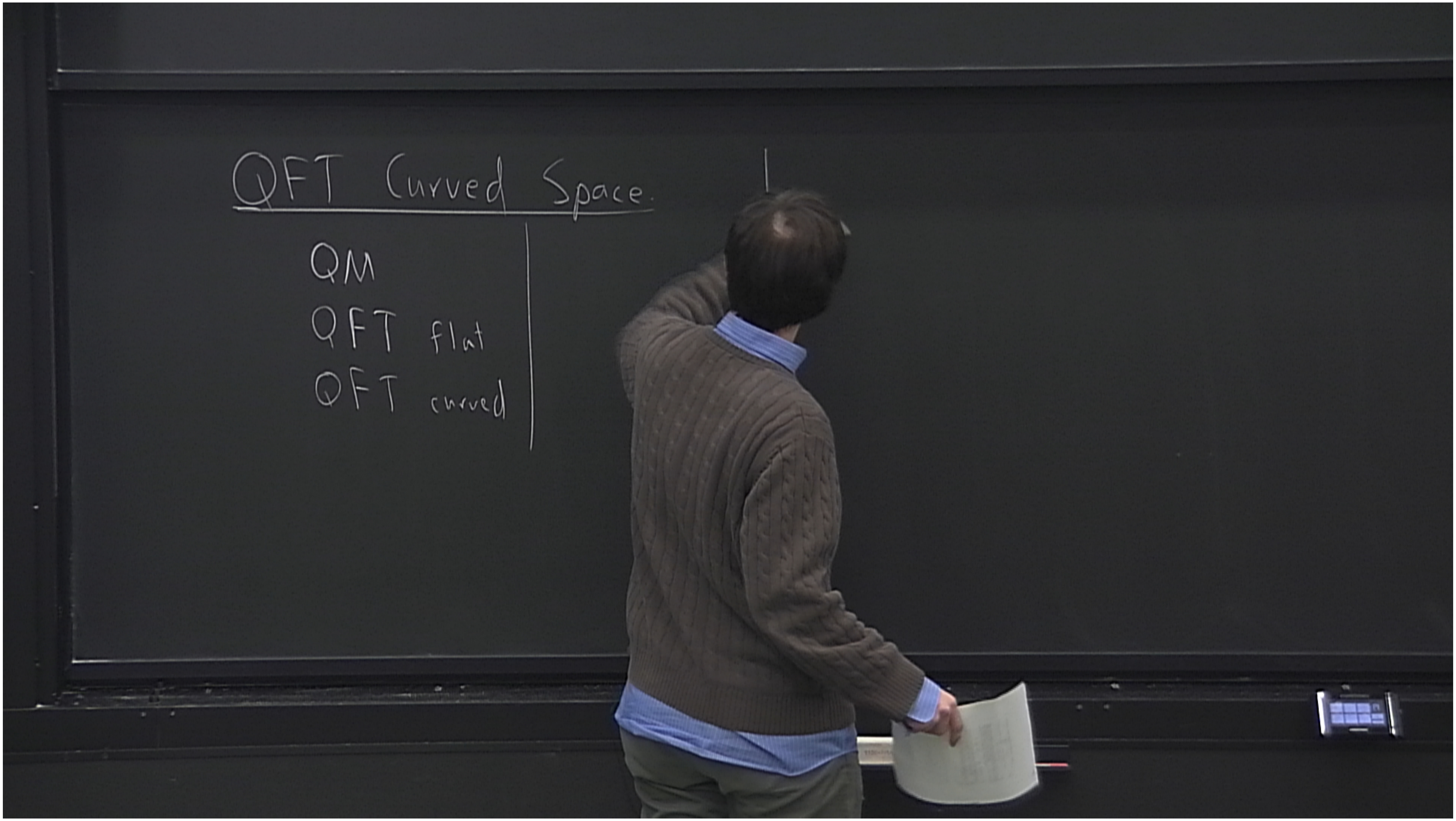
Title: 13/14 PSI - Cosmology Review - Lecture 11

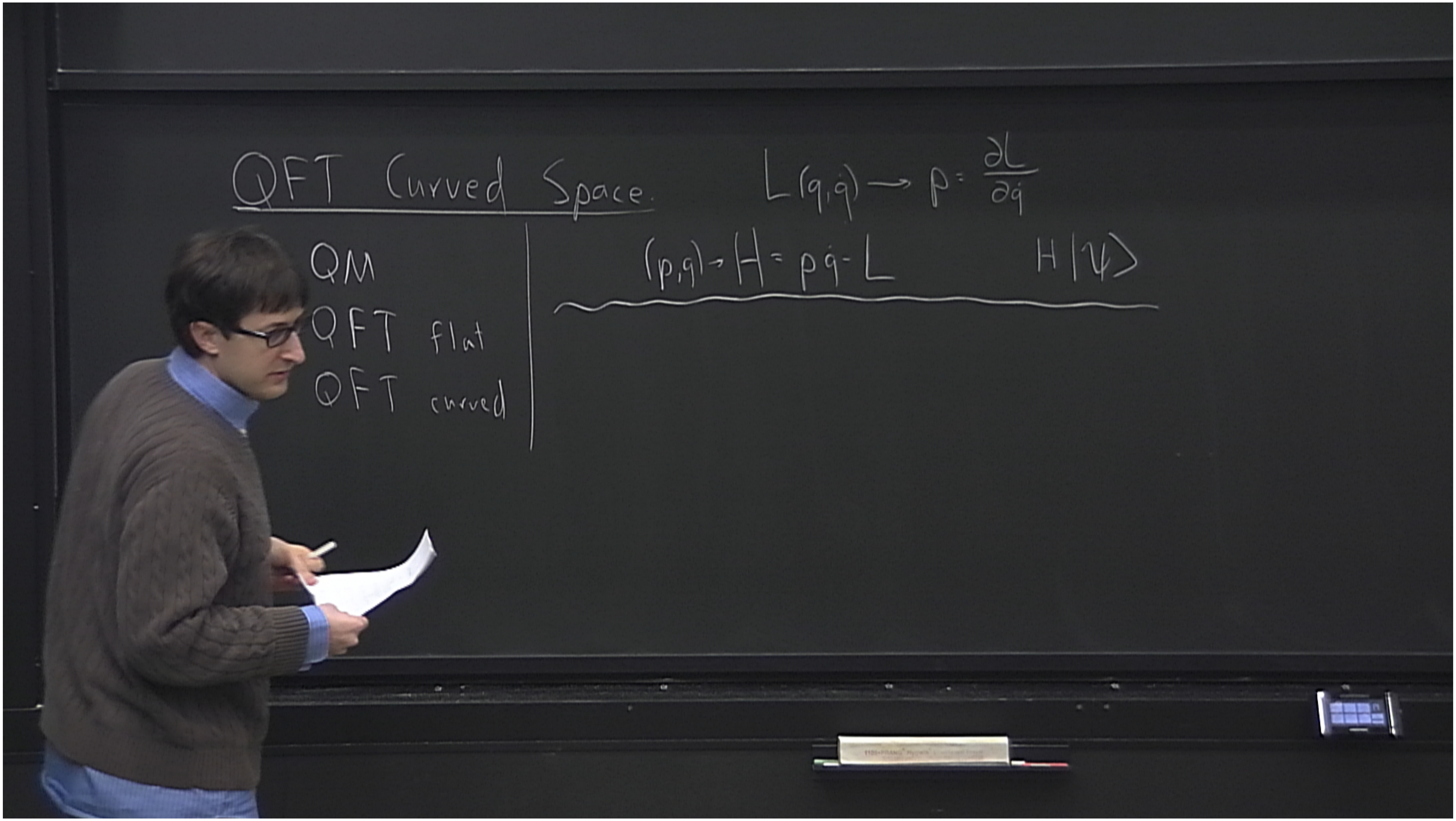
Date: Feb 12, 2014 11:30 AM

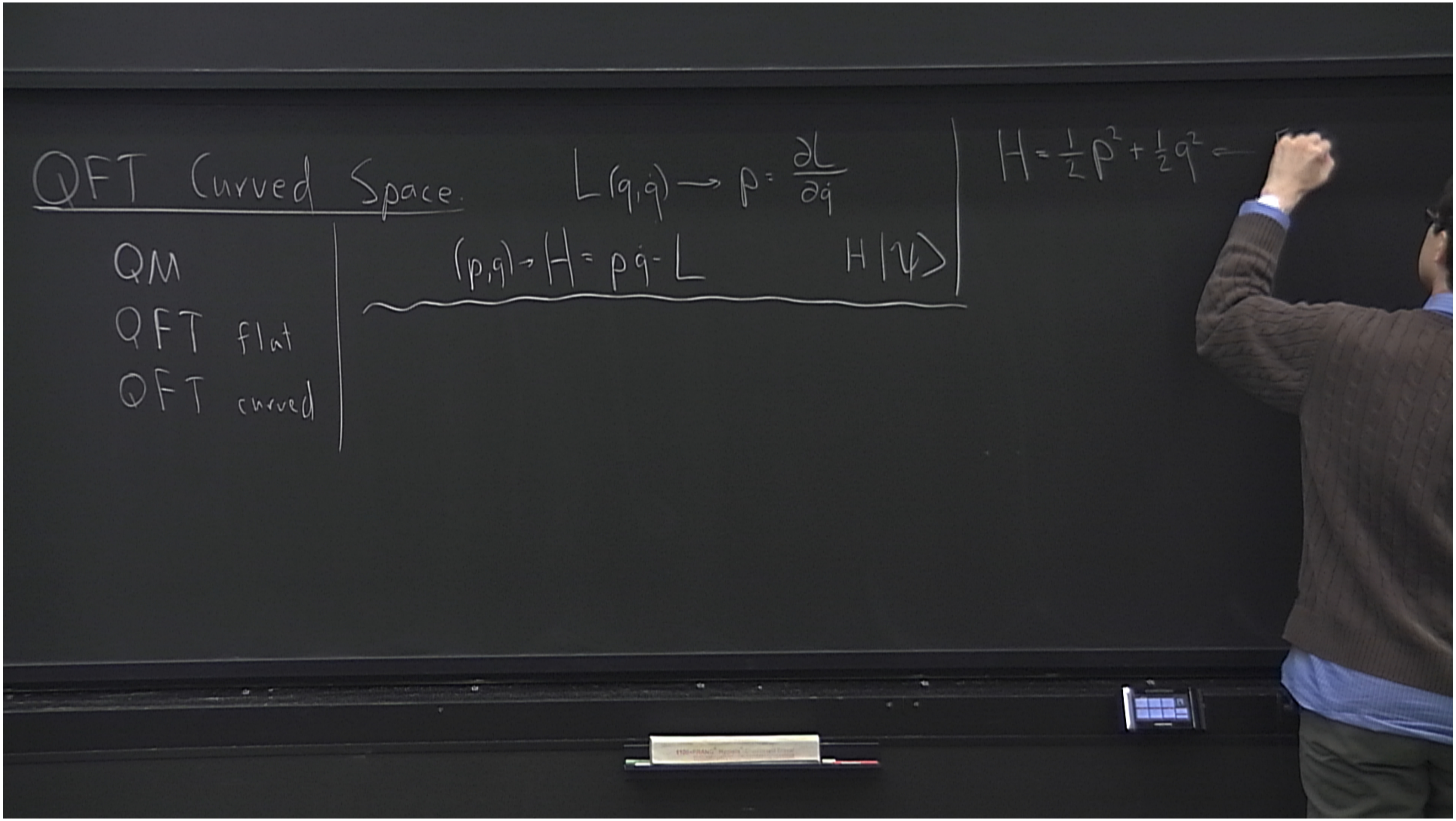
URL: http://pirsa.org/14020021

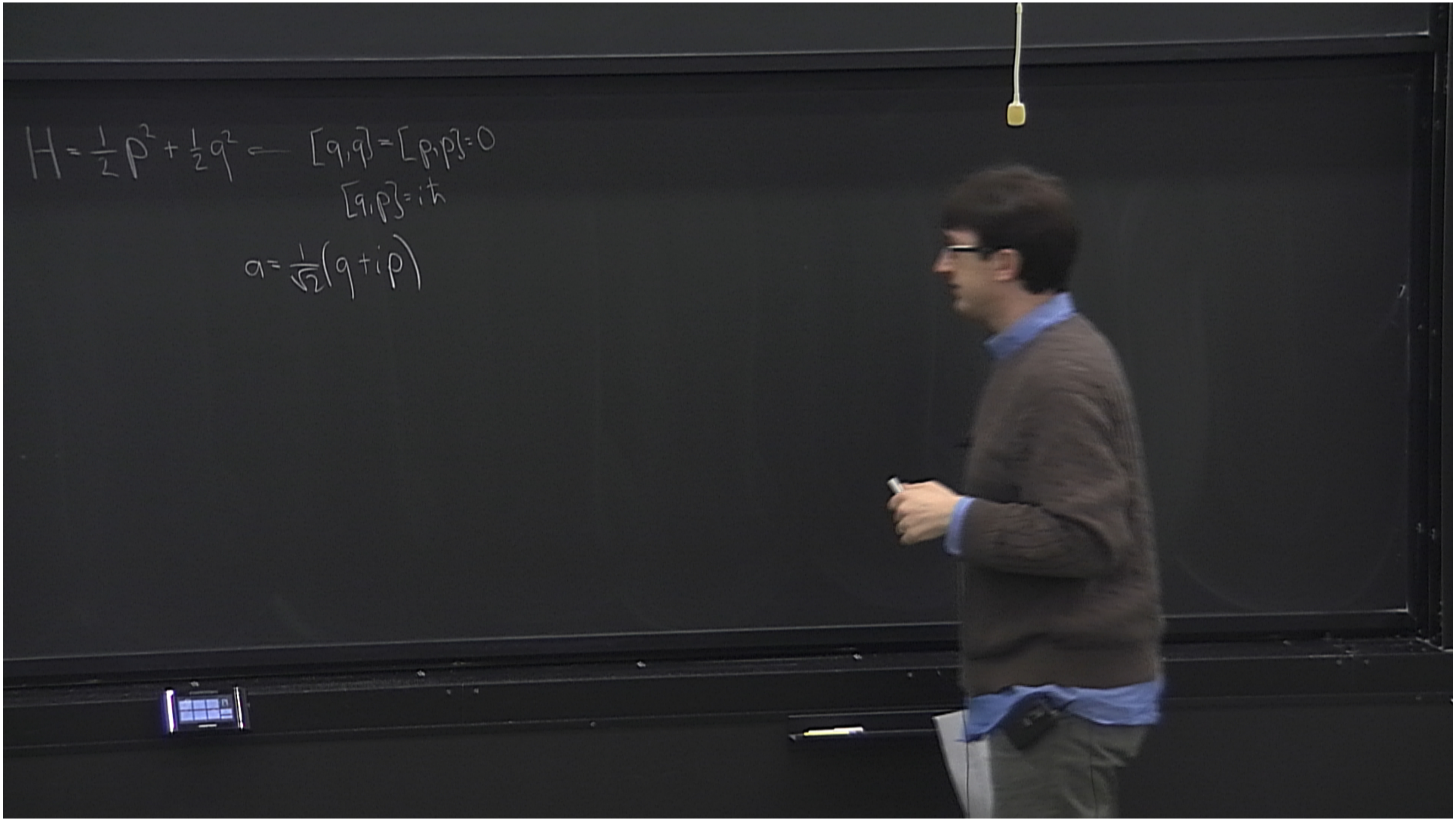
Abstract:

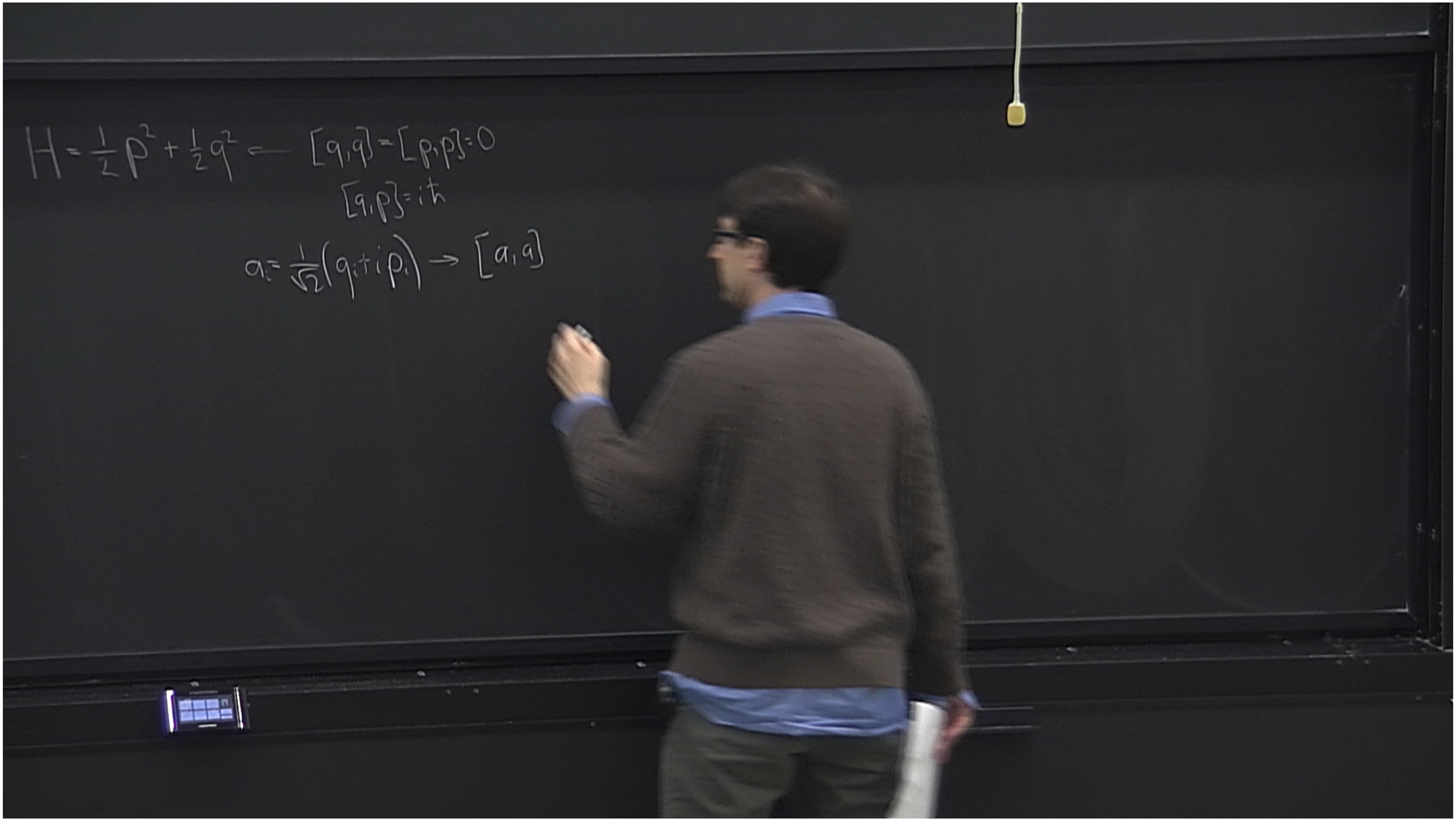

QFT Curved Space.

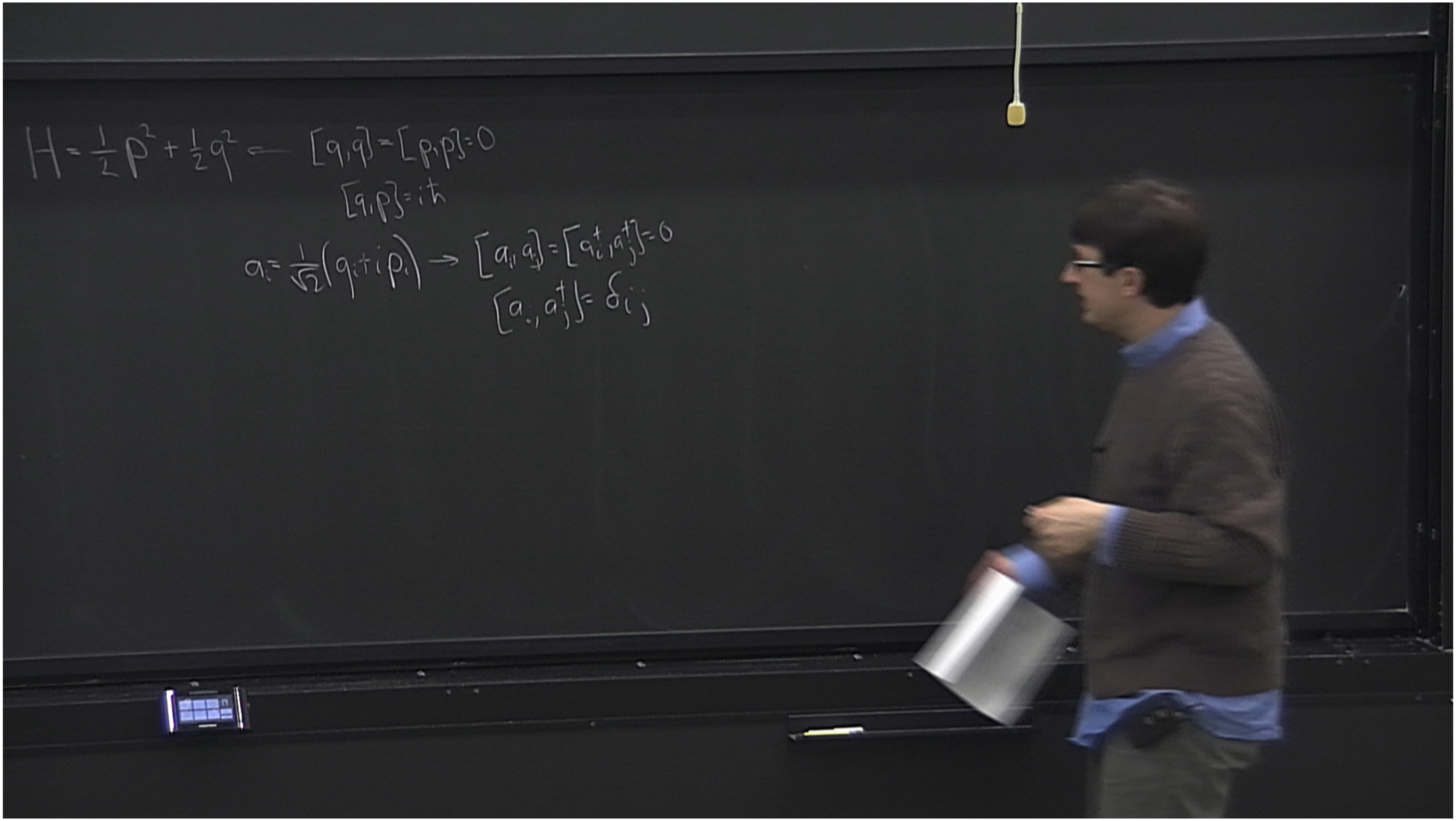

QFT Curved Space.
QM
QFT flat
QFT curved

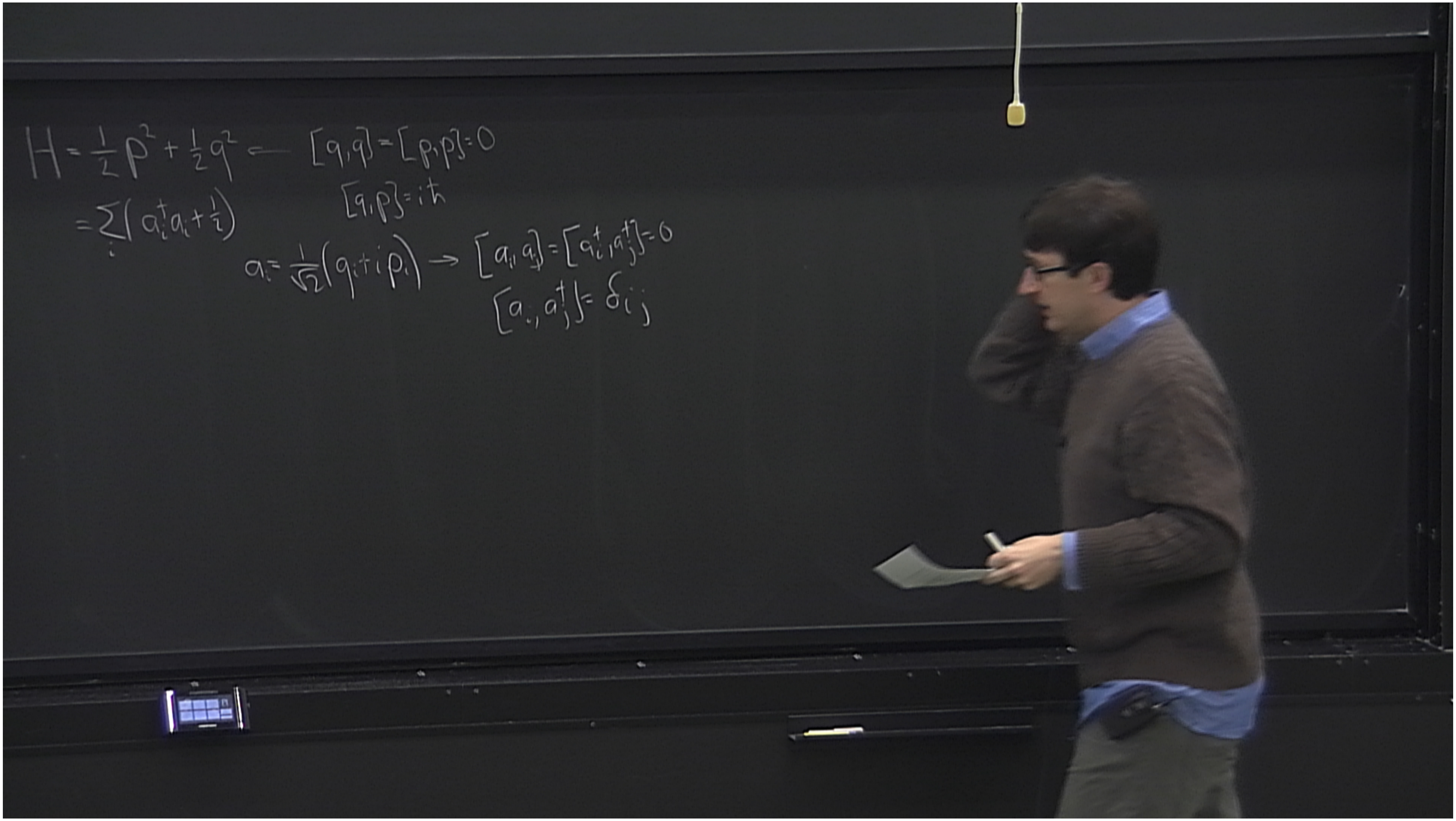

QFT Curved Space.
L(q,q̇) → p = ∂L/∂q̇
QM
QFT flat
QFT curved
(p,q) → H = pq̇ - L
H|ψ⟩

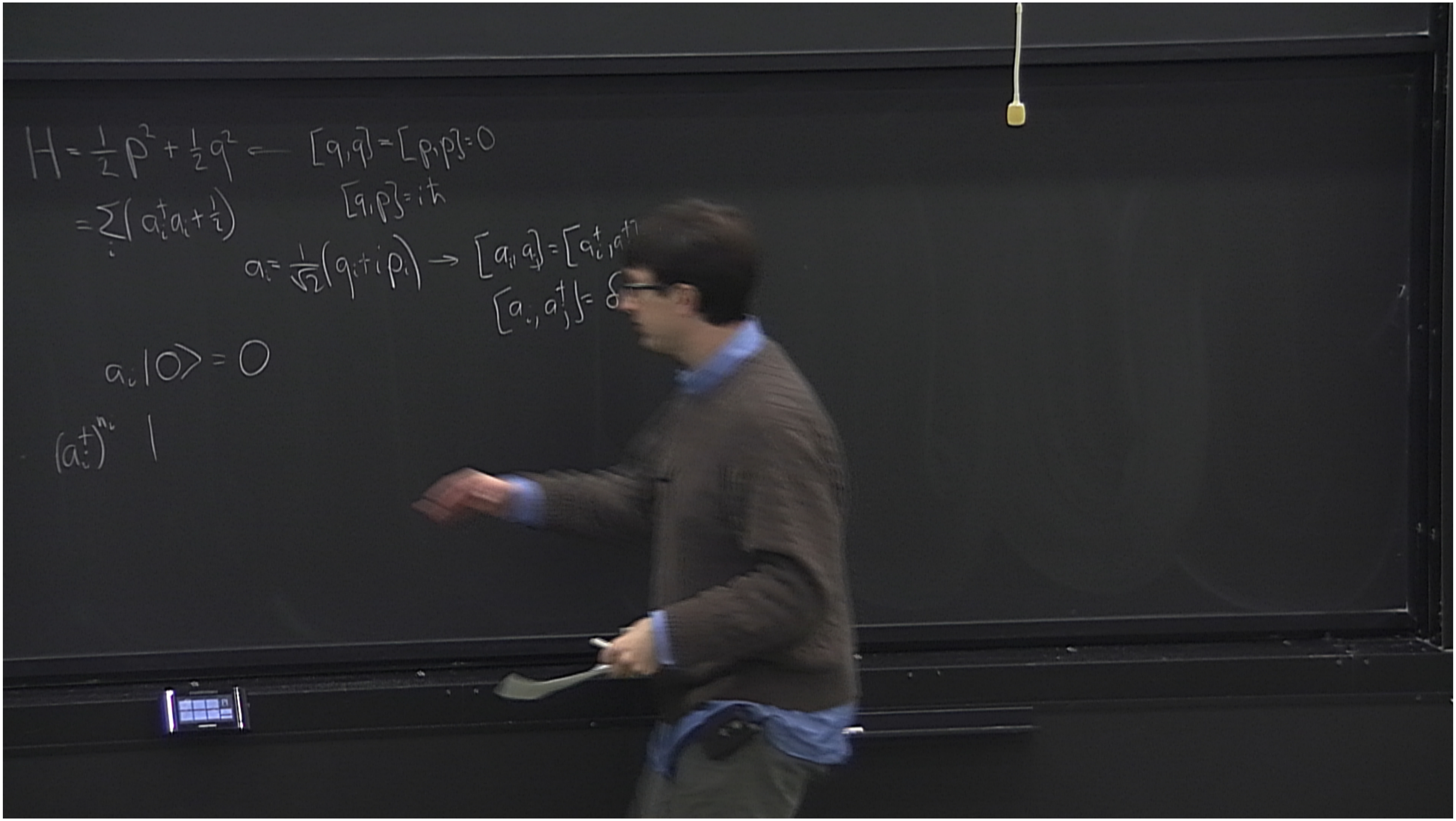

QFT Curved Space.
QM
QFT flat
QFT curved
L(q,q̇) → p = ∂L/∂q̇
(p,q) → H = pq̇ - L
H|ψ>
H = ½p² + ½q²

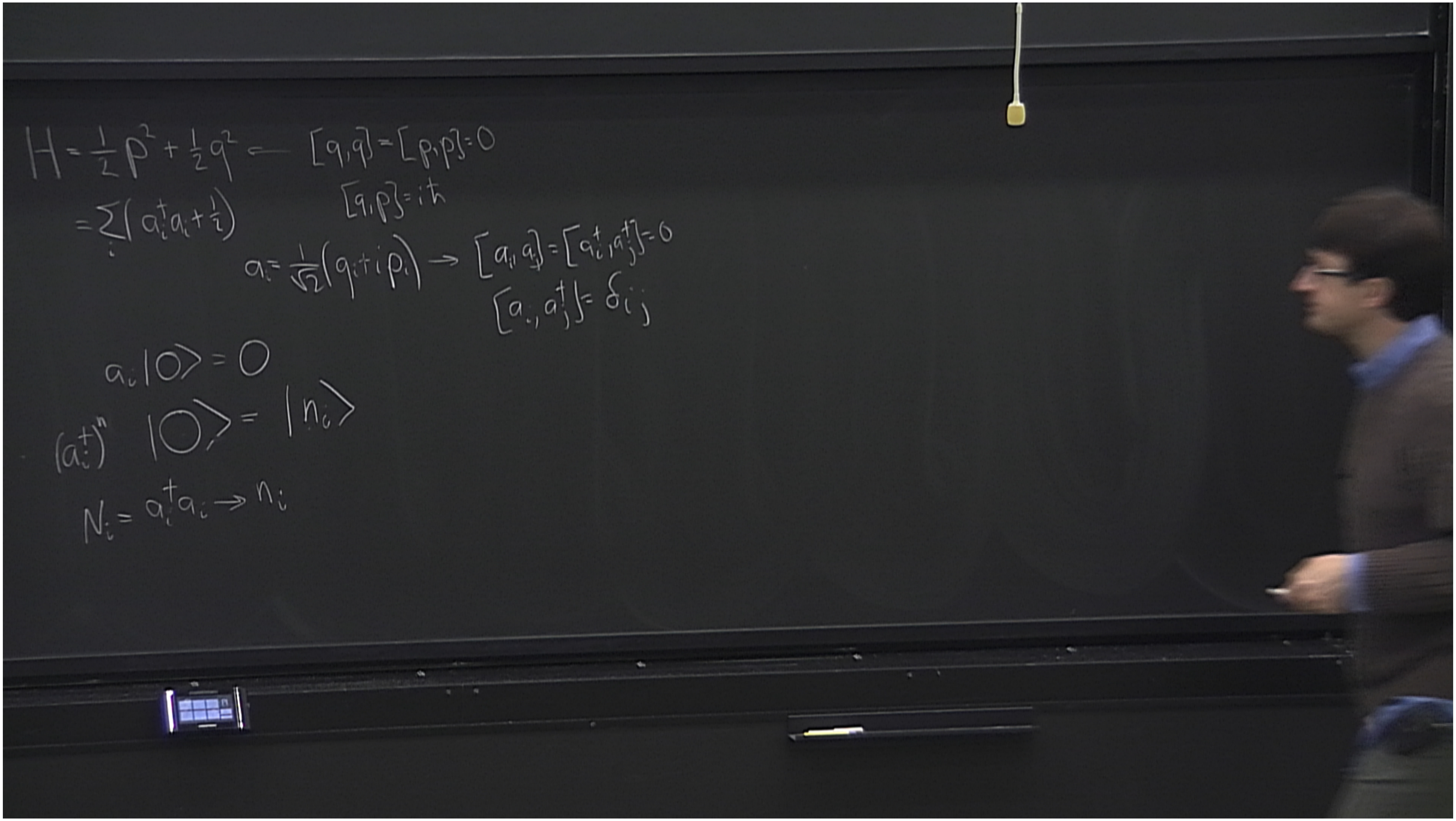

$H = \frac{1}{2}p^2 + \frac{1}{2}q^2$
$[q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$a = \frac{1}{\sqrt{2}}(q + ip)$

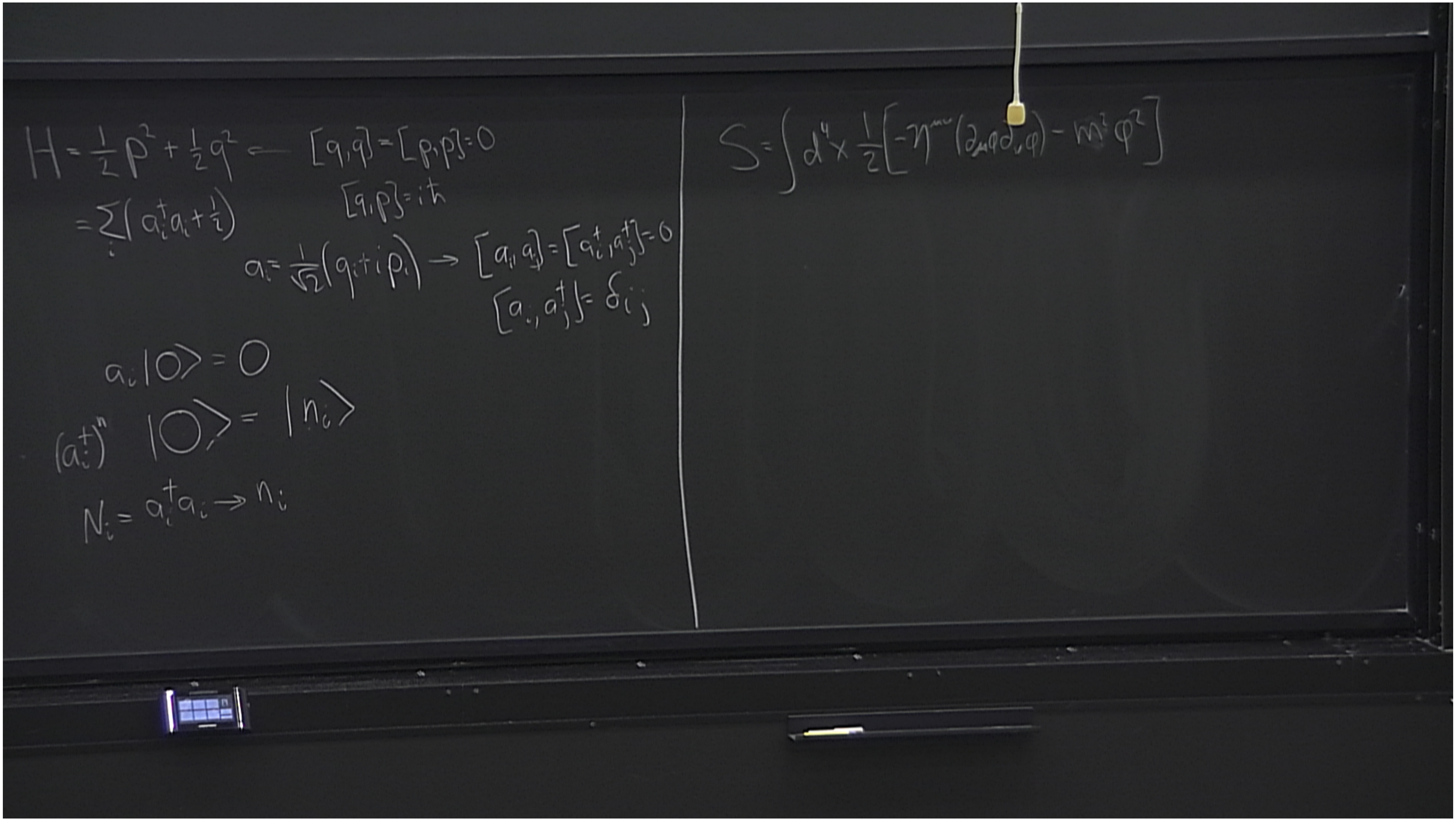

$H = \frac{1}{2}p^2 + \frac{1}{2}q^2 \leftarrow [q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$a_i = \frac{1}{\sqrt{2}}(q_i + i p_i) \rightarrow [a, a]$


$H = \frac{1}{2}p^2 + \frac{1}{2}q^2 \leftarrow [q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$a_i = \frac{1}{\sqrt{2}}(q_i + i p_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$
$[a_i, a_j^\dagger] = \delta_{ij}$

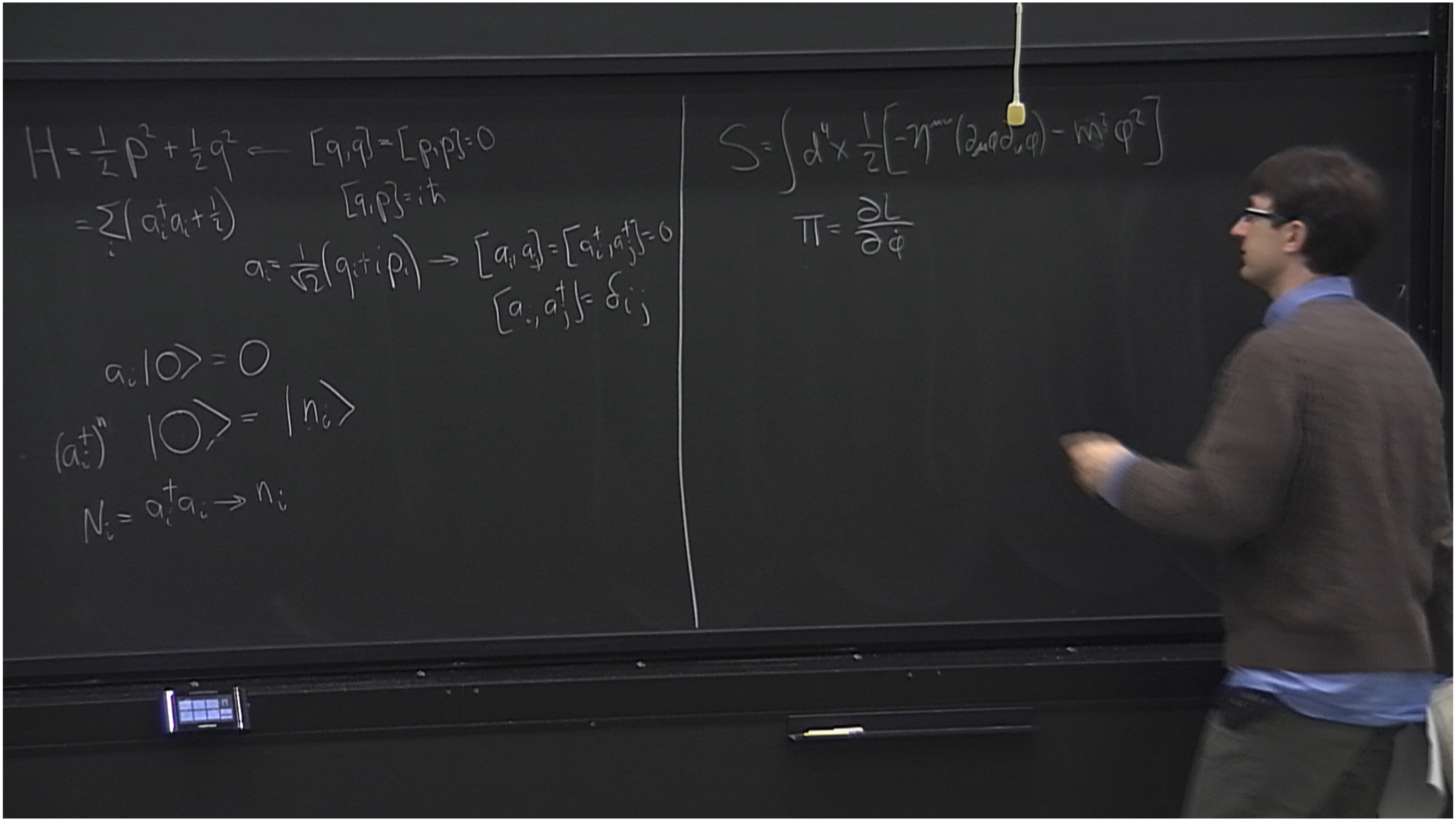

$H = \frac{1}{2}p^2 + \frac{1}{2}q^2$
$= \sum_i (a_i^\dagger a_i + \frac{1}{2})$
$[q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$a_i = \frac{1}{\sqrt{2}}(q_i + i p_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$
$[a_i, a_j^\dagger] = \delta_{ij}$

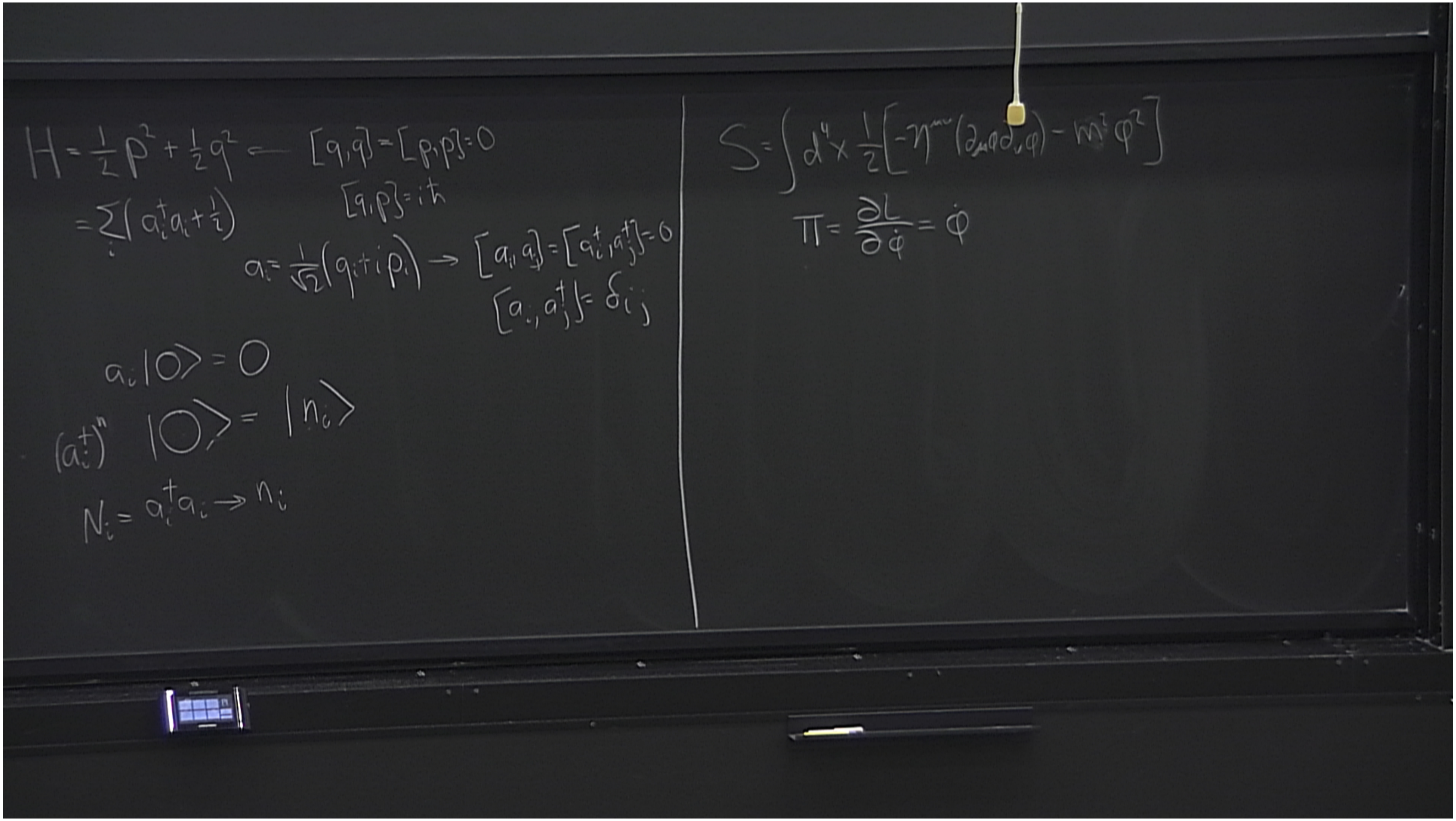

$H = \frac{1}{2}p^2 + \frac{1}{2}q^2 \leftarrow \quad [q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$= \sum_i \left(a_i^\dagger a_i + \frac{1}{2} \right)$
$a_i = \frac{1}{\sqrt{2}}(q_i + i p_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a$
$[a_i, a_j^\dagger] = \delta$
$a_i|0\rangle = 0$
$(a_i^\dagger)^{n_i} \,|$


H = ½p² + ½q² ← [q,q] = [p,p] = 0
= Σ_i (a_i† a_i + ½)
[q,p] = iħ
a_i = 1/√2 (q_i + i p_i) → [a_i, a_j] = [a_i†, a_j†] = 0
[a_i, a_j†] = δ_ij
a_i |0> = 0
(a_i†)^n |0> = |n_i>
N_i = a_i† a_i → n_i



$H = \frac{1}{2}p^2 + \frac{1}{2}q^2 \leftarrow [q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$= \sum_i (a_i^\dagger a_i + \frac{1}{2})$
$a_i = \frac{1}{\sqrt{2}}(q_i + ip_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$
$[a_i, a_j^\dagger] = \delta_{ij}$
$a_i|0\rangle = 0$
$(a_i^\dagger)^n |0\rangle = |n_i\rangle$
$N_i = a_i^\dagger a_i \rightarrow n_i$
$S = \int d^4x \frac{1}{2}[-\eta^{\mu\nu}(\partial_\mu\varphi\partial_\nu\varphi) - m^2\varphi^2]$


H = 1/2 p^2 + 1/2 q^2 ← [q,q] = [p,p] = 0
[q,p] = iħ
= Σ_i (a_i^† a_i + 1/2)
a_i = 1/√2 (q_i + i p_i) → [a_i, a_j] = [a_i^†, a_j^†] = 0
[a_i, a_j^†] = δ_ij
a_i |0> = 0
(a_i^†)^n |0> = |n_i>
N_i = a_i^† a_i → n_i
S = ∫ d^4x 1/2 [−η^{μν} (∂_μ φ ∂_ν φ) − m^2 φ^2]
π = ∂L/∂φ̇


$H = \frac{1}{2}p^2 + \frac{1}{2}q^2$
$[q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$= \sum_i (a_i^\dagger a_i + \frac{1}{2})$
$a_i = \frac{1}{\sqrt{2}}(q_i + i p_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$
$[a_i, a_j^\dagger] = \delta_{ij}$
$a_i |0\rangle = 0$
$(a_i^\dagger)^n |0\rangle = |n_i\rangle$
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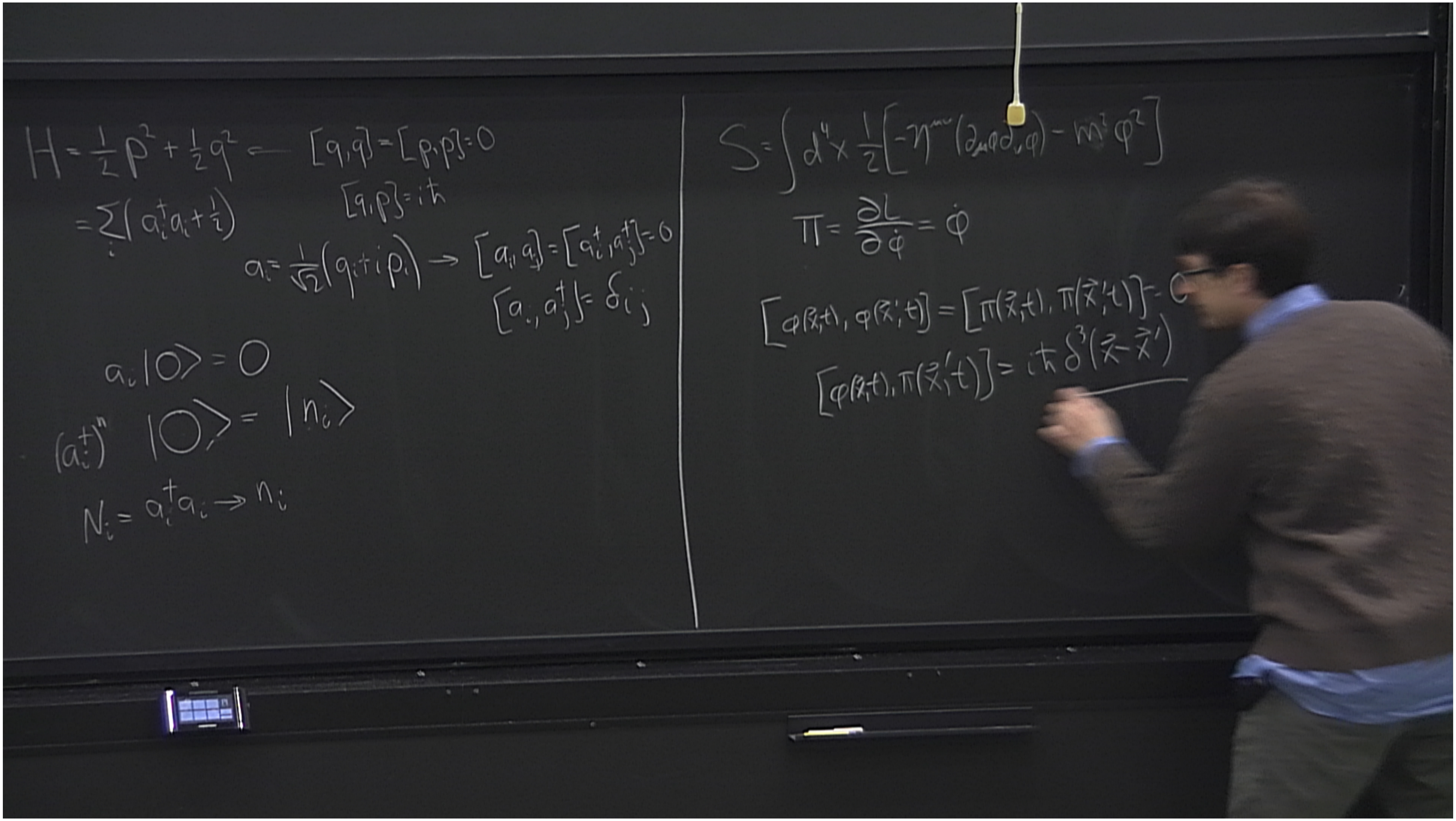
$S = \int d^4x \frac{1}{2}[-\eta^{\mu\nu}(\partial_\mu\varphi\partial_\nu\varphi) - m^2\varphi^2]$
$\pi = \frac{\partial L}{\partial \dot\varphi} = \dot\varphi$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$a_i = \frac{1}{\sqrt{2}}(q_i + ip_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$
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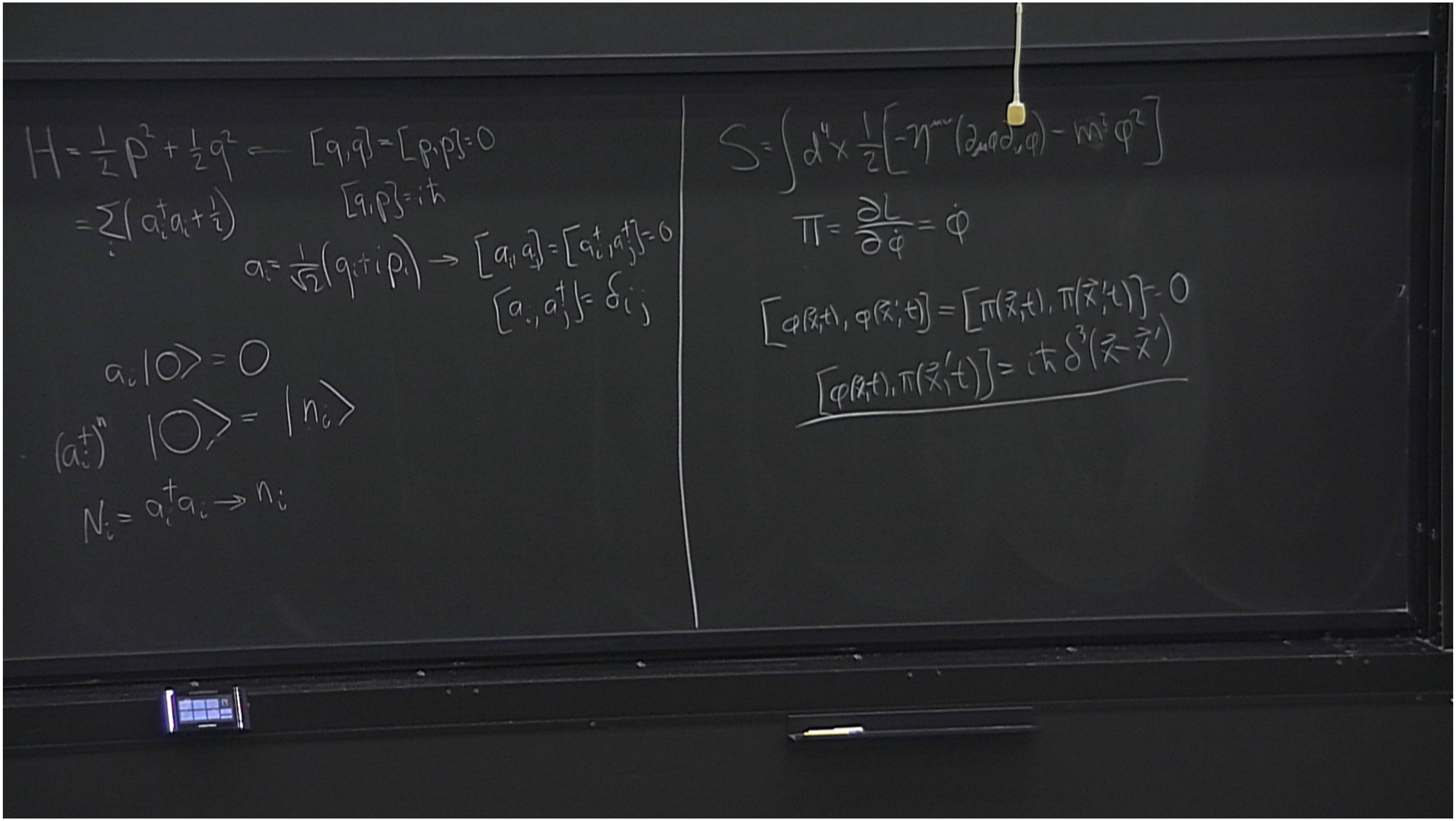
$\pi = \frac{\partial L}{\partial\dot\varphi} = \dot\varphi$
$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = [\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\hbar\delta^3(\vec{x}-\vec{x}')$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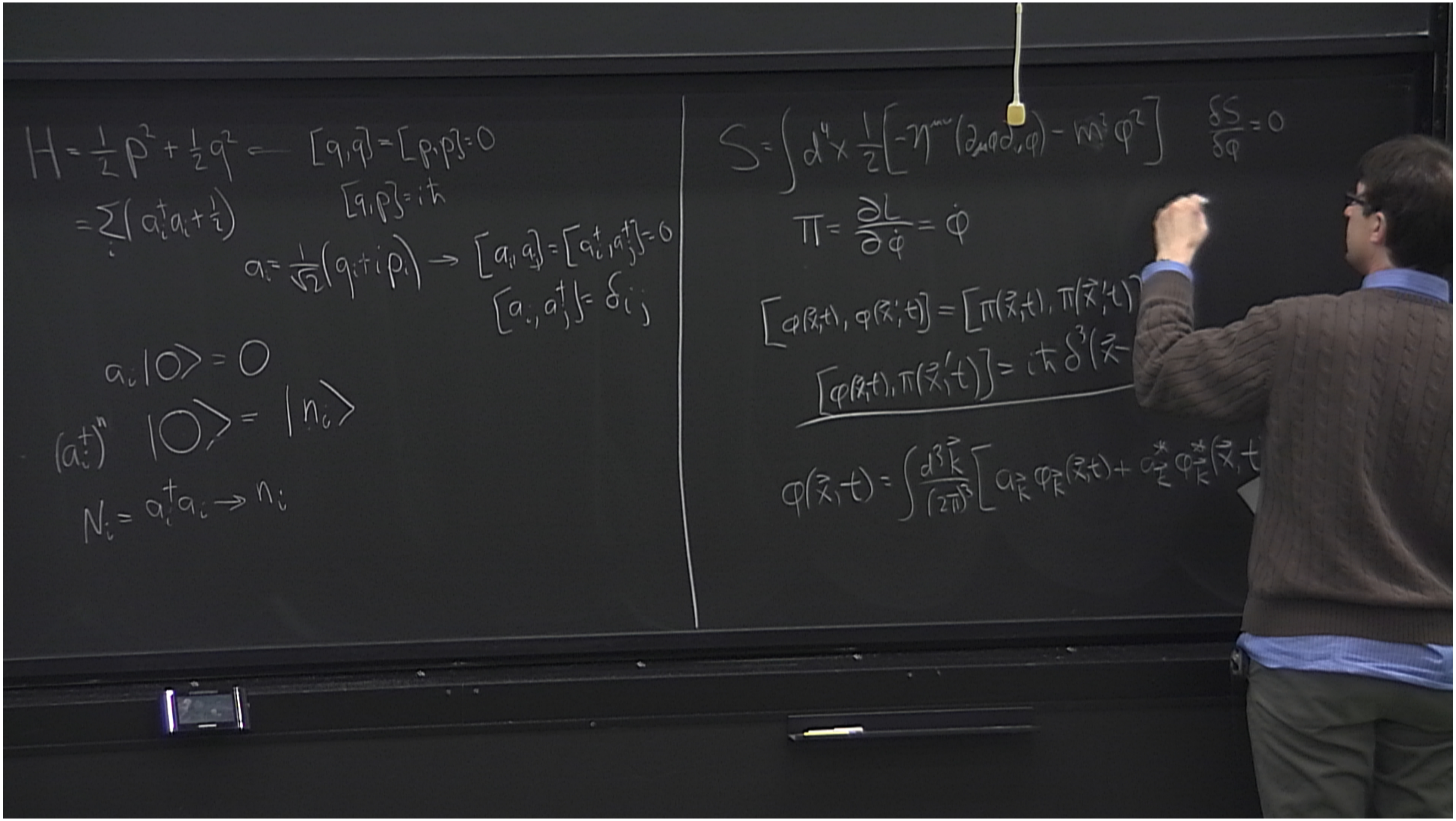
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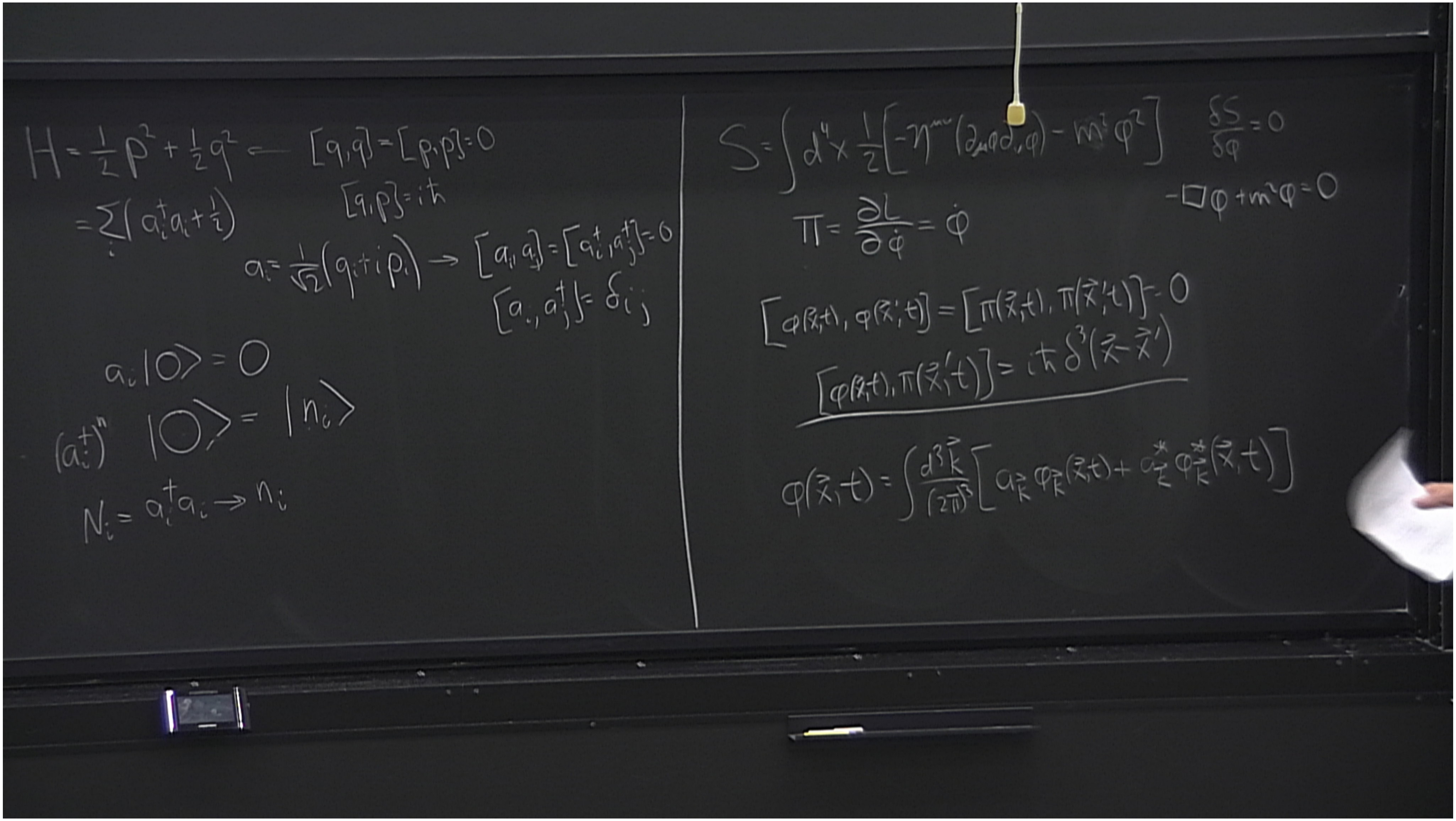
$(a_i^\dagger)^n |0\rangle = |n_i\rangle$
$N_i = a_i^\dagger a_i \rightarrow n_i$
$S = \int d^4x \frac{1}{2}[-\eta^{\mu\nu}(\partial_\mu\varphi\partial_\nu\varphi) - m^2\varphi^2]$
$\frac{\delta S}{\delta \varphi} = 0$
$\pi = \frac{\partial L}{\partial \dot{\varphi}} = \dot{\varphi}$
$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = [\pi(\vec{x},t), \pi(\vec{x}',t)]$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\hbar\delta^3(\vec{x}-$
$\varphi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^3}[a_{\vec{k}}\varphi_{\vec{k}}(\vec{x},t) + a_{\vec{k}}^*\varphi_{\vec{k}}^*(\vec{x},t$



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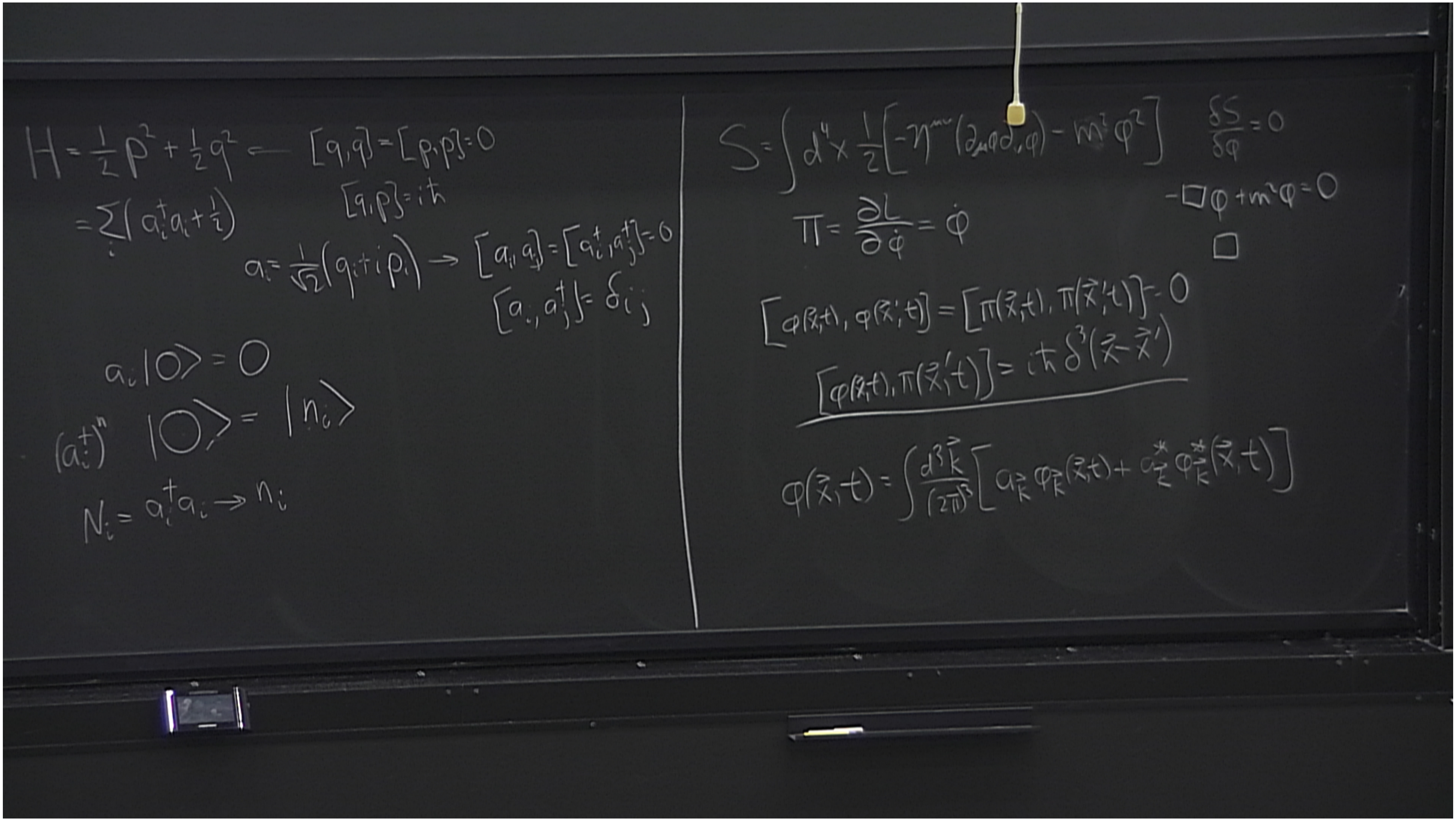
$[q,p] = i\hbar$
$= \sum_i (a_i^\dagger a_i + \frac{1}{2})$
$a_i = \frac{1}{\sqrt{2}}(q_i + i p_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$
$[a_i, a_j^\dagger] = \delta_{ij}$
$a_i |0\rangle = 0$
$(a_i^\dagger)^n |0\rangle = |n_i\rangle$
$N_i = a_i^\dagger a_i \rightarrow n_i$
$S = \int d^4x \frac{1}{2}[-\eta^{\mu\nu}(\partial_\mu\varphi\partial_\nu\varphi) - m^2\varphi^2]$
$\frac{\delta S}{\delta\varphi} = 0$
$-\Box\varphi + m^2\varphi = 0$
$\pi = \frac{\partial L}{\partial\dot\varphi} = \dot\varphi$
$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = [\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\hbar\delta^3(\vec{x}-\vec{x}')$
$\varphi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^3}[a_{\vec{k}}\varphi_{\vec{k}}(\vec{x},t) + a_{\vec{k}}^*\varphi_{\vec{k}}^*(\vec{x},t)]$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$[q,q] = [p,p] = 0$
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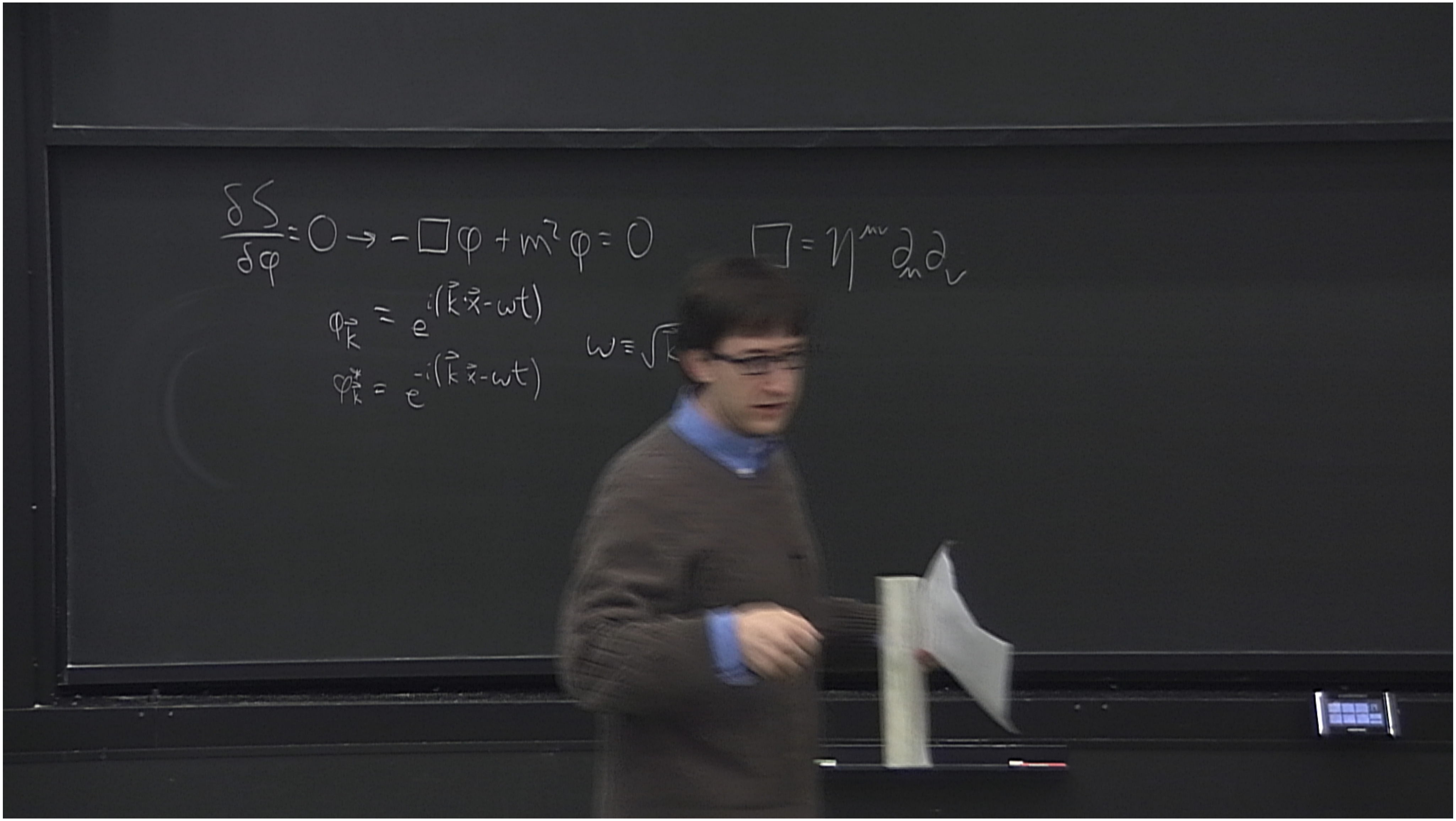
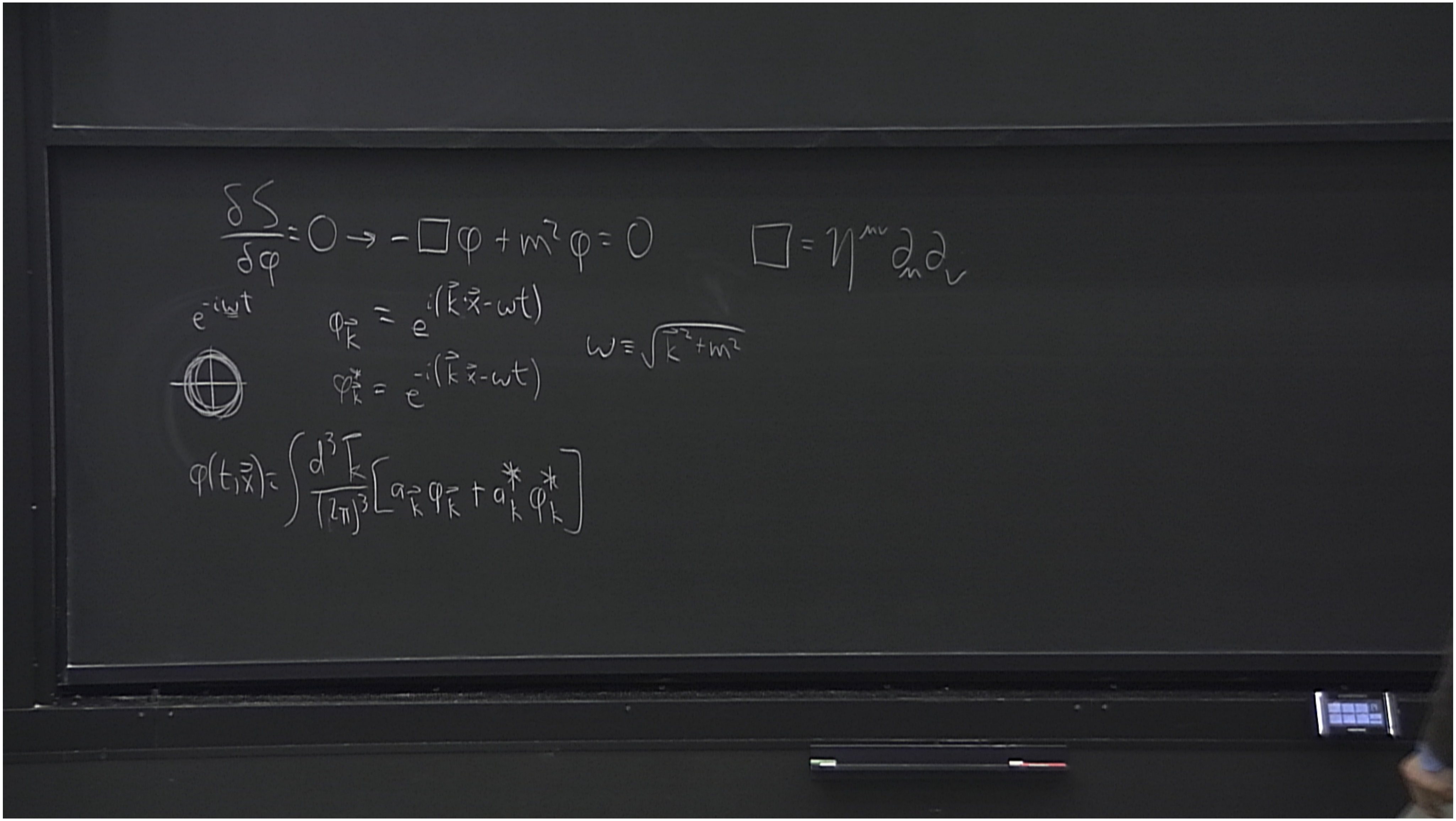
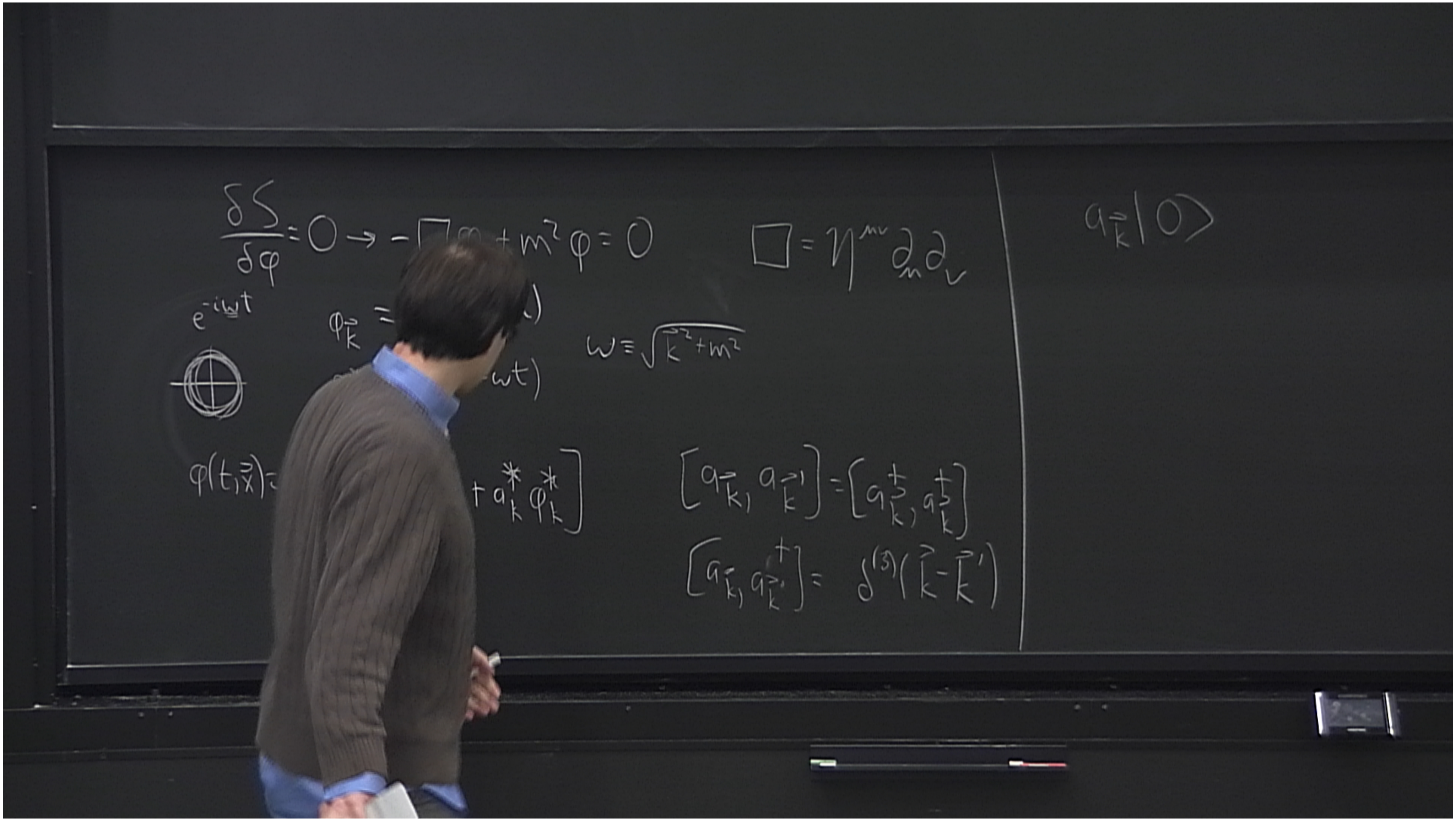
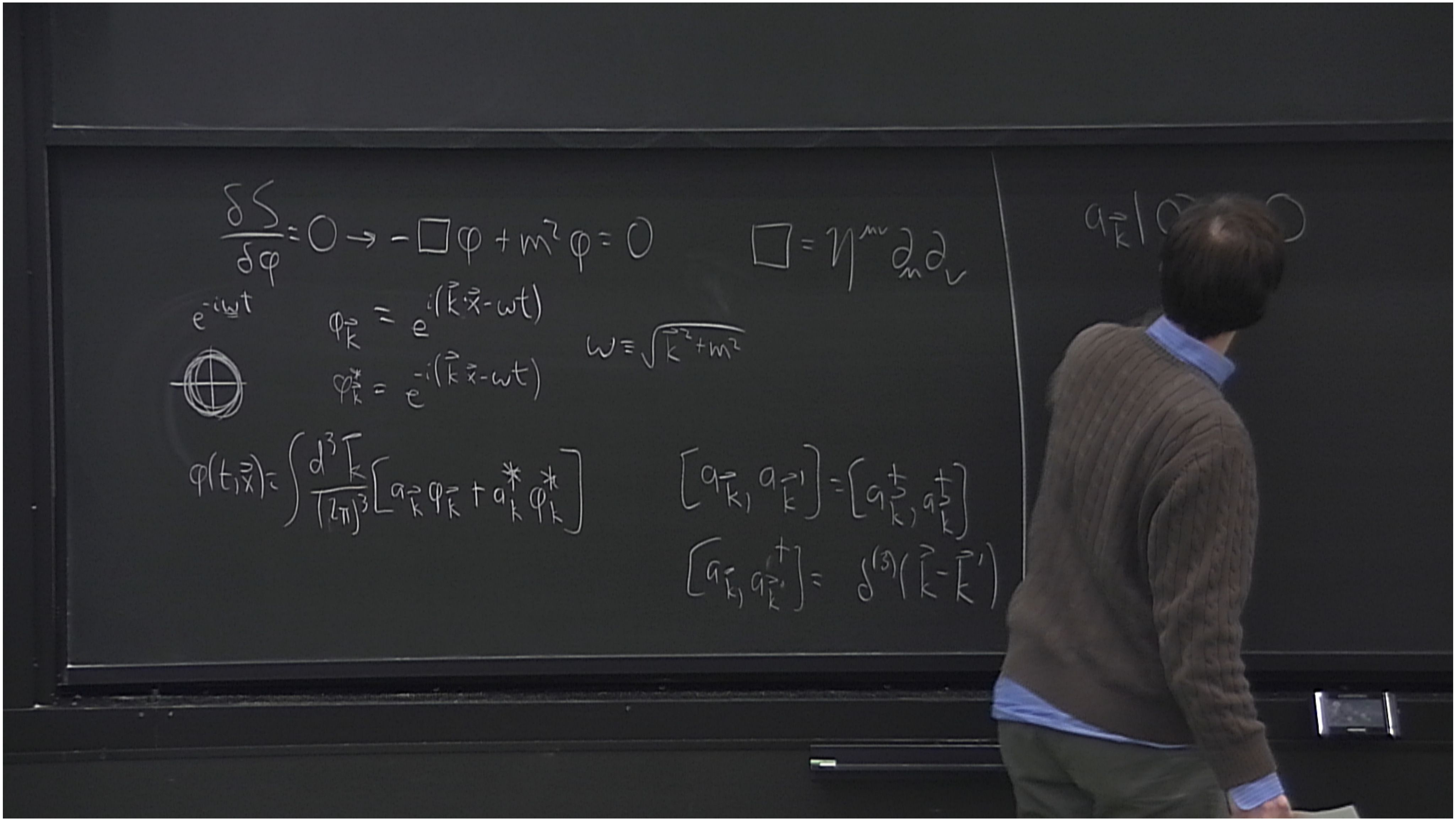
$a_i = \frac{1}{\sqrt{2}}(q_i + ip_i) \rightarrow [a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$
$[a_i, a_j^\dagger] = \delta_{ij}$
$a_i|0\rangle = 0$
$(a_i^\dagger)^n |0\rangle = |n_i\rangle$
$N_i = a_i^\dagger a_i \rightarrow n_i$
$S = \int d^4x \frac{1}{2}[-\eta^{\mu\nu}(\partial_\mu\varphi\partial_\nu\varphi) - m^2\varphi^2]$
$\frac{\delta S}{\delta\varphi} = 0$
$-\Box\varphi + m^2\varphi = 0$
$\Box$
$\pi = \frac{\partial L}{\partial\dot\varphi} = \dot\varphi$
$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = [\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\hbar\delta^3(\vec{x}-\vec{x}')$
$\varphi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^3}[a_{\vec{k}}\varphi_{\vec{k}}(\vec{x},t) + a_{\vec{k}}^*\varphi_{\vec{k}}^*(\vec{x},t)]$


$\frac{\delta S}{\delta \varphi} = 0 \rightarrow -\Box \varphi + m^2 \varphi = 0$
$\Box = \eta^{\mu\nu} \partial_\mu \partial_\nu$
$\varphi_{\vec{k}} = e^{i(\vec{k}\cdot\vec{x} - \omega t)}$
$\varphi^*_{\vec{k}} = e^{-i(\vec{k}\cdot\vec{x} - \omega t)}$
$\omega = \sqrt{k}$

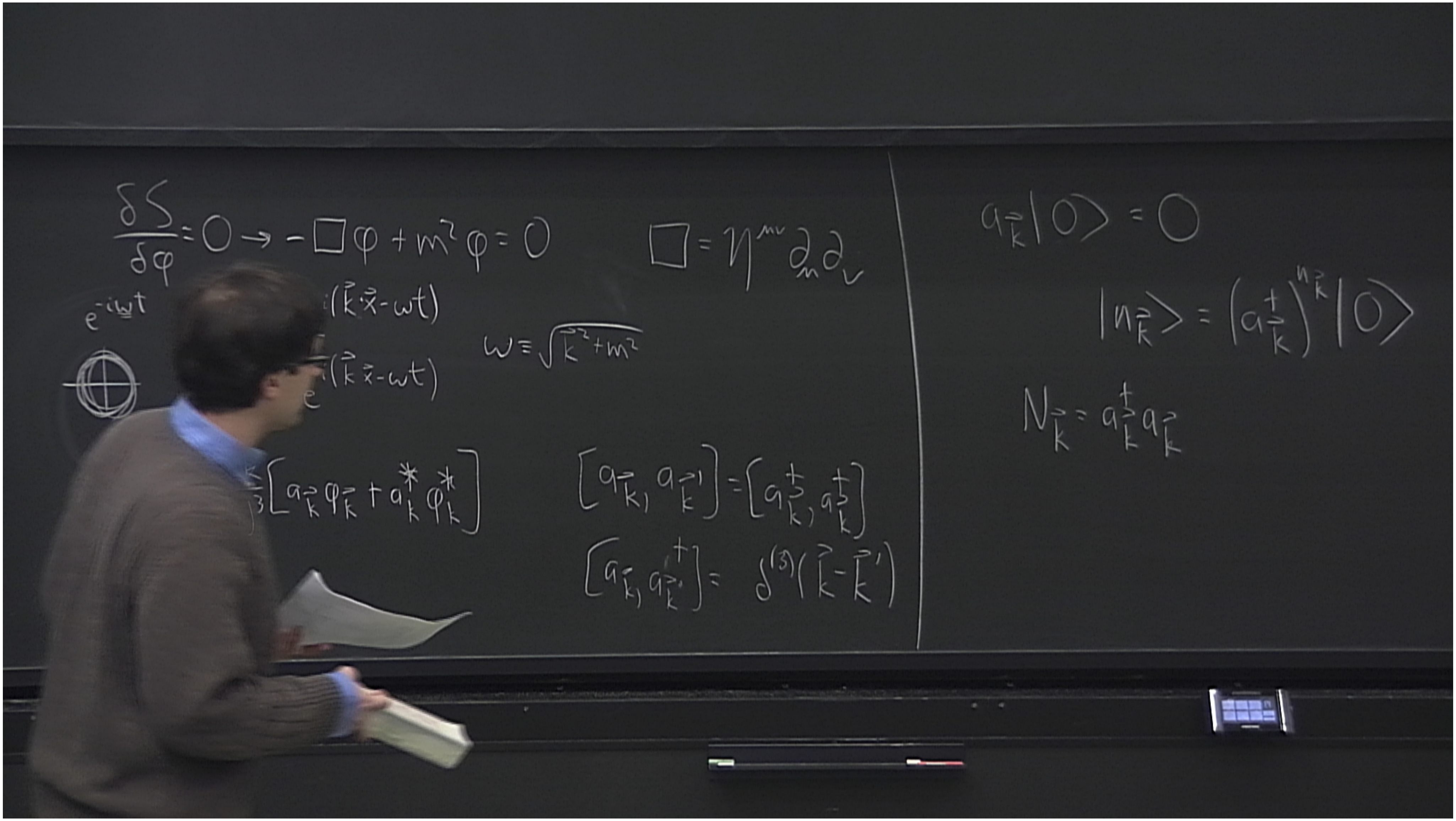

$\frac{\delta S}{\delta \varphi} = 0 \rightarrow -\Box\varphi + m^2\varphi = 0$
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$e^{-i\omega t}$
$\varphi_{\vec{k}} = e^{i(\vec{k}\cdot\vec{x} - \omega t)}$
$\omega = \sqrt{\vec{k}^2 + m^2}$
$\varphi^*_{\vec{k}} = e^{-i(\vec{k}\cdot\vec{x} - \omega t)}$
$\varphi(t,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\left[a_{\vec{k}}\varphi_{\vec{k}} + a^*_{\vec{k}}\varphi^*_{\vec{k}}\right]$

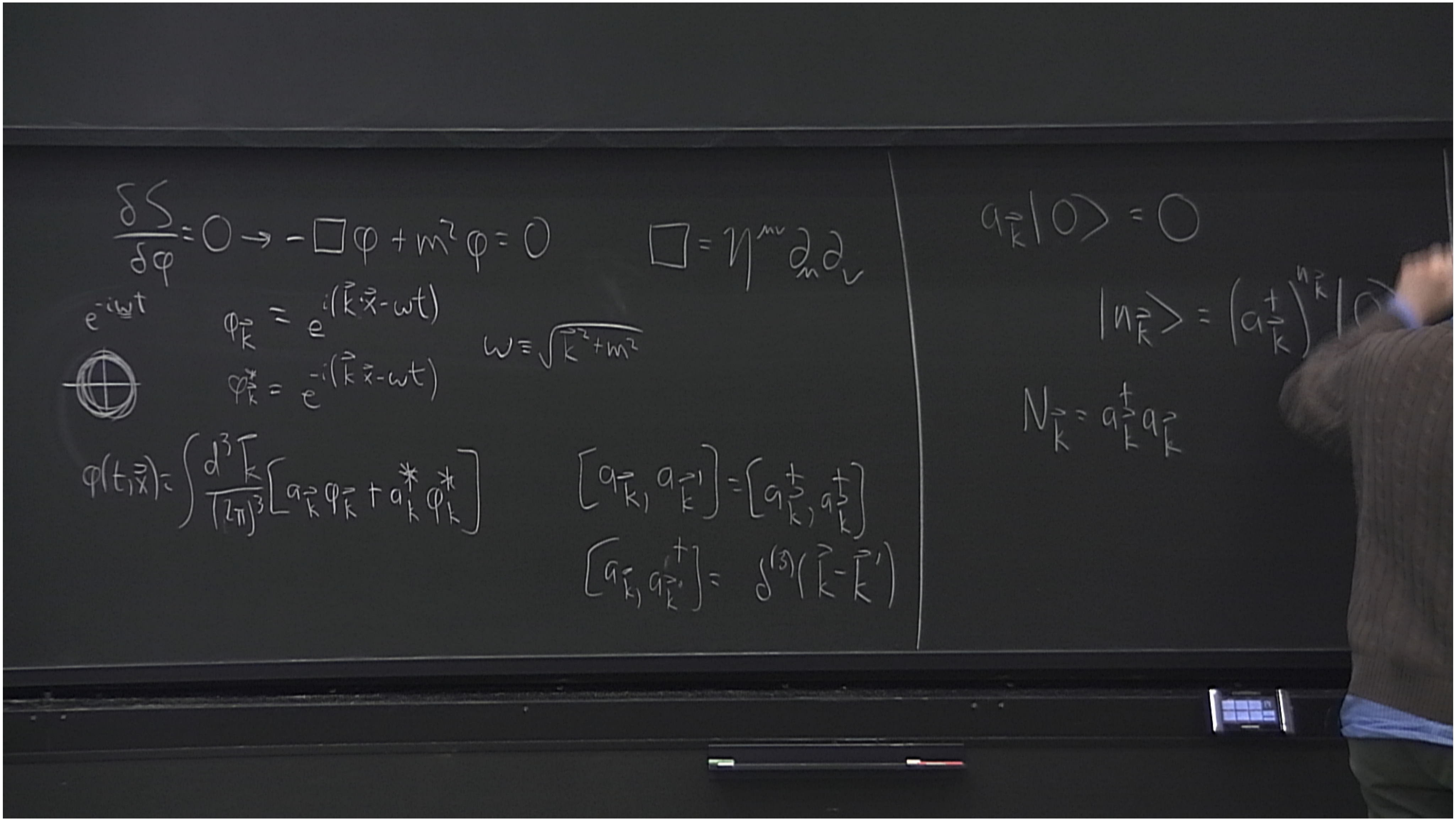

□ = η^{μν} ∂_μ ∂_ν
ω ≡ √(k² + m²)
[a_k, a_k'] = [a†_k, a†_k']
[a_k, a†_k'] = δ^(3)(k − k')
a_k |0⟩

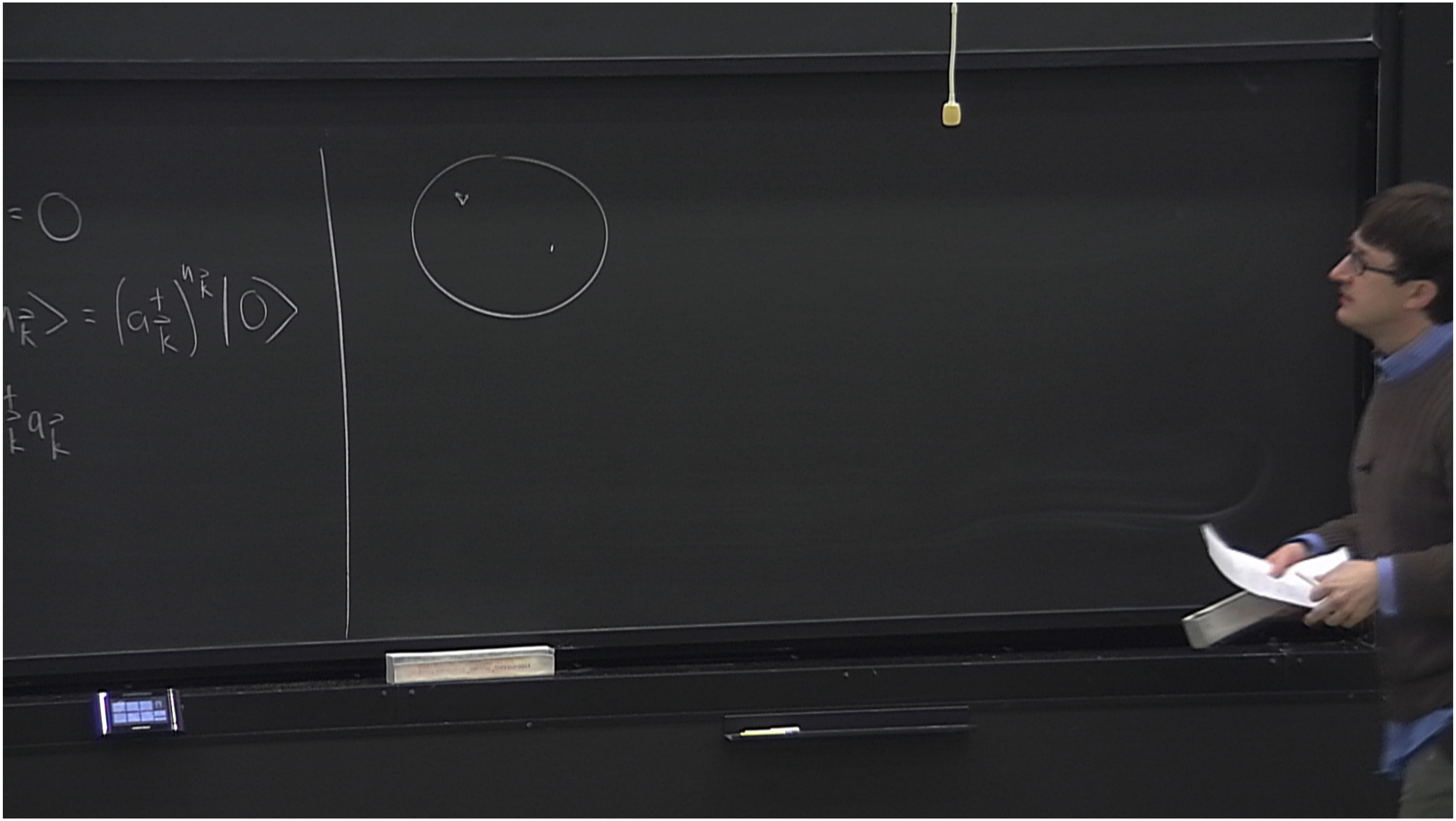

δS/δφ = 0 → −□φ + m²φ = 0
□ = η^μν ∂_μ ∂_ν
e^{-iωt}
φ_k = e^{i(k·x − ωt)}
φ*_k = e^{−i(k·x − ωt)}
ω = √(k² + m²)
φ(t, x) = ∫ d³k/(2π)³ [a_k φ_k + a*_k φ*_k]
[a_k, a_k'] = [a†_k, a†_k'] = 0
[a_k, a†_k'] = δ^(3)(k − k')
a_k |0⟩ = 0

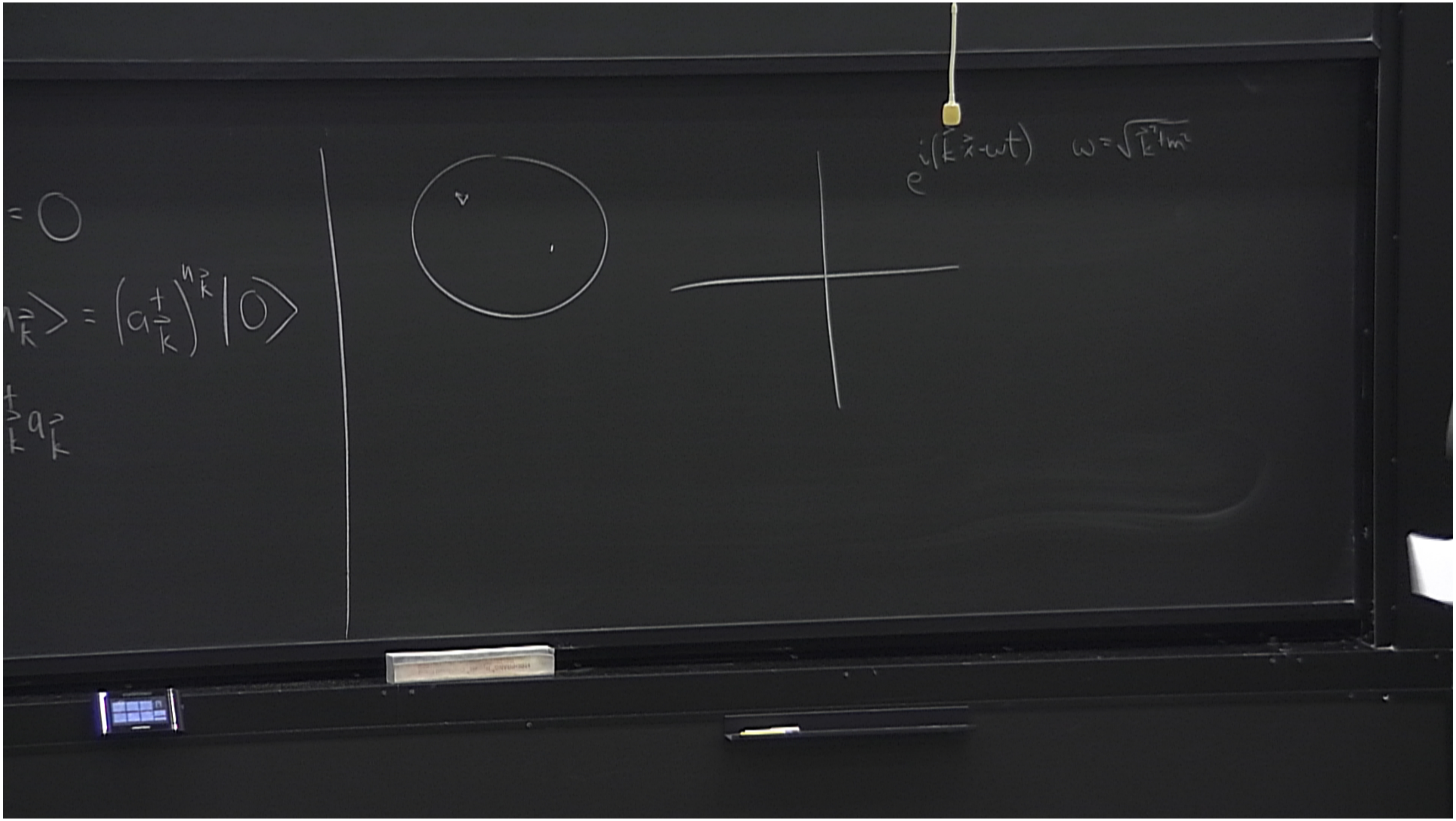
$$a_{\vec{k}}|0\rangle = 0$$

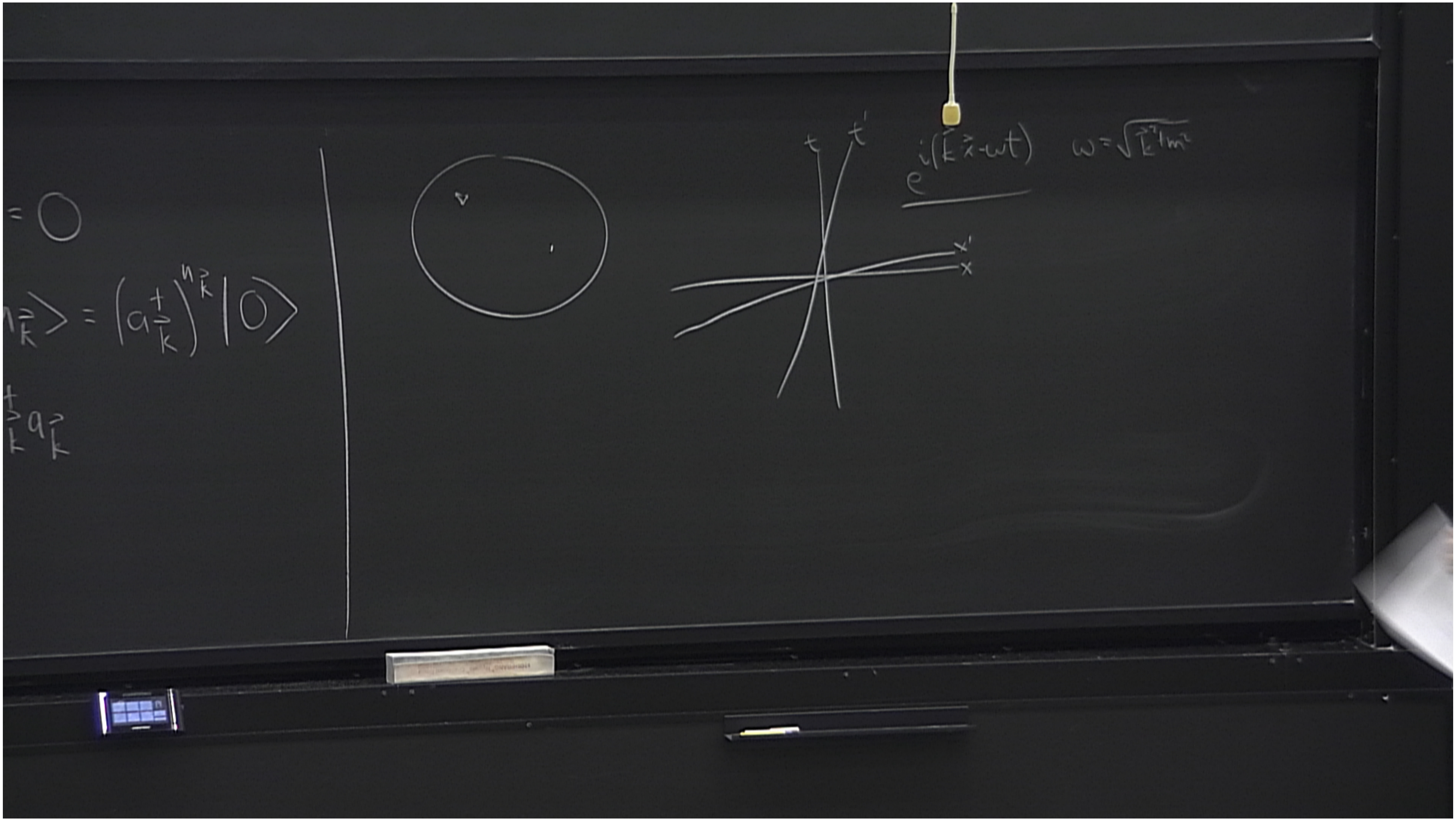
$$|n_{\vec{k}}\rangle = \left(a^{\dagger}_{\vec{k}}\right)^{n_{\vec{k}}}|0\rangle$$

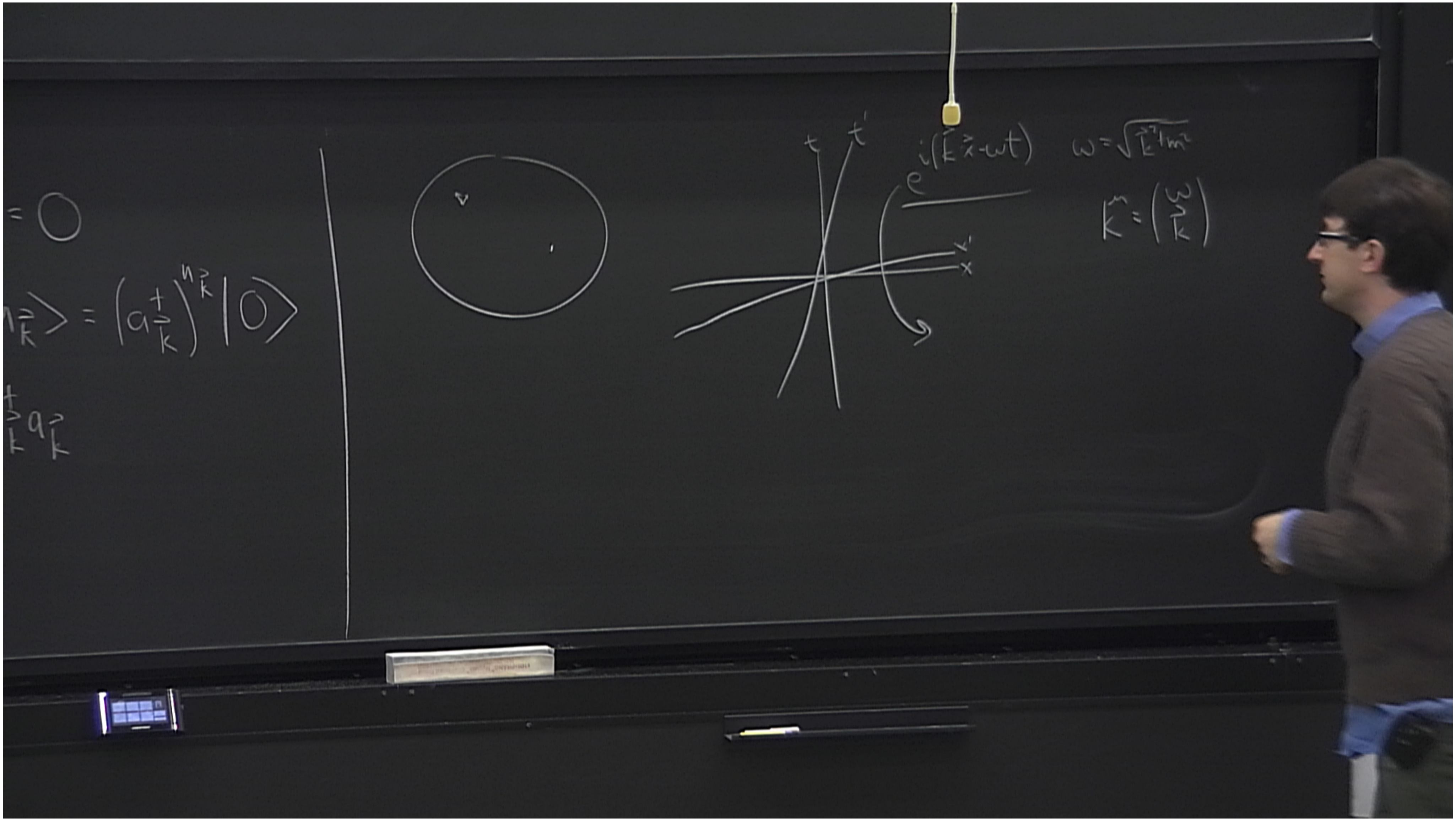

δS/δφ = 0 → −□φ + m²φ = 0
□ = η^{μν} ∂_μ ∂_ν
ω ≡ √(k² + m²)
[a_k, a_k'] = [a†_k, a†_k']
[a_k, a†_k'] = δ^(3)(k − k')
a_k |0> = 0
|n_k> = (a†_k)^{n_k} |0>
N_k = a†_k a_k

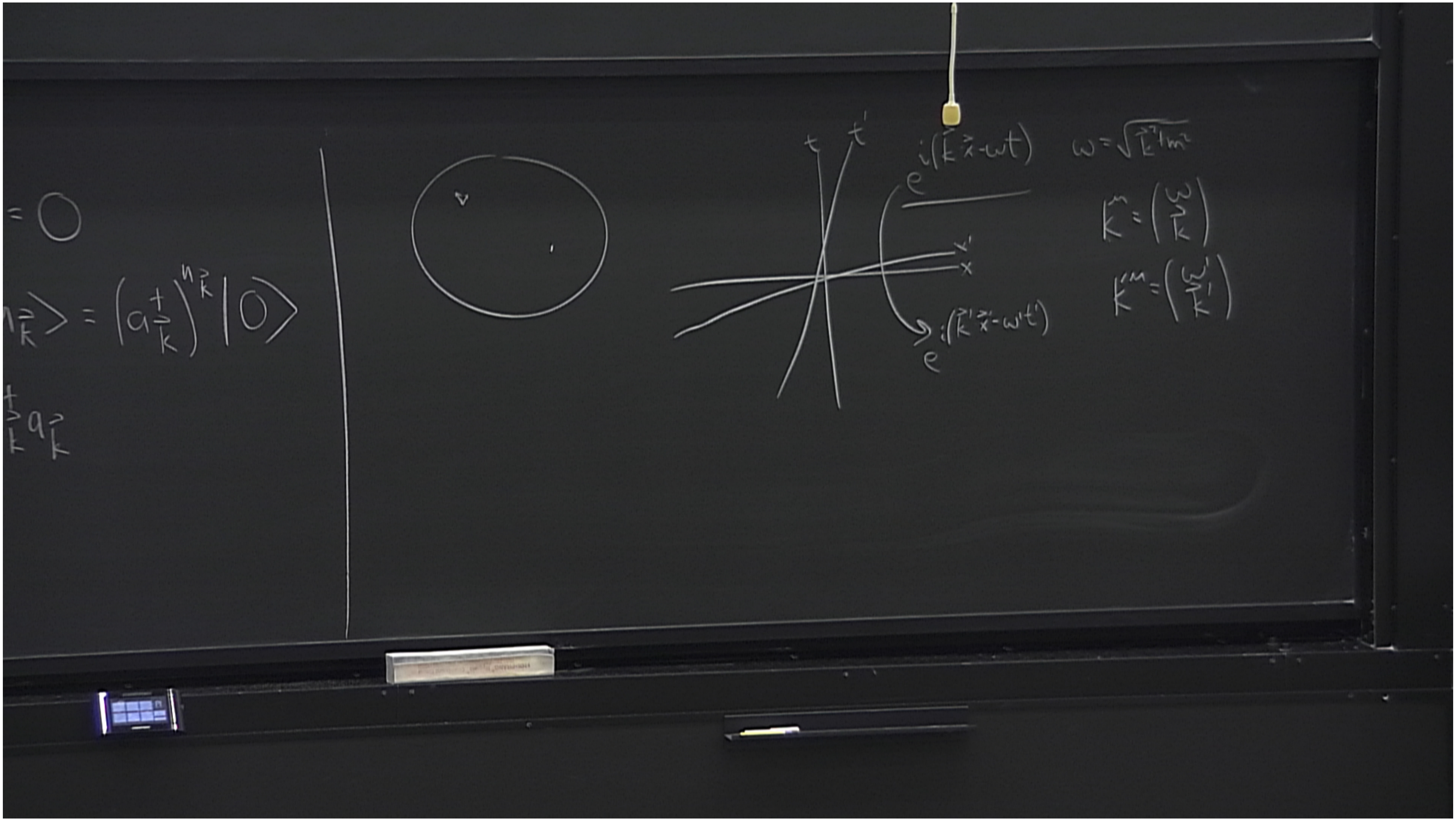

δS/δφ = 0 → −□φ + m²φ = 0
□ = η^{μν} ∂_μ ∂_ν
e^{-iωt}
φ_k = e^{i(k·x − ωt)}
φ*_k = e^{−i(k·x − ωt)}
ω ≡ √(k² + m²)
φ(t, x) = ∫ d³k/(2π)³ [a_k φ_k + a*_k φ*_k]
[a_k, a_k'] = [a†_k, a†_k'] = 0
[a_k, a†_k'] = δ^{(3)}(k − k')
a_k |0> = 0
|n_k> = (a†_k)^{n_k} |0>
N_k = a†_k a_k

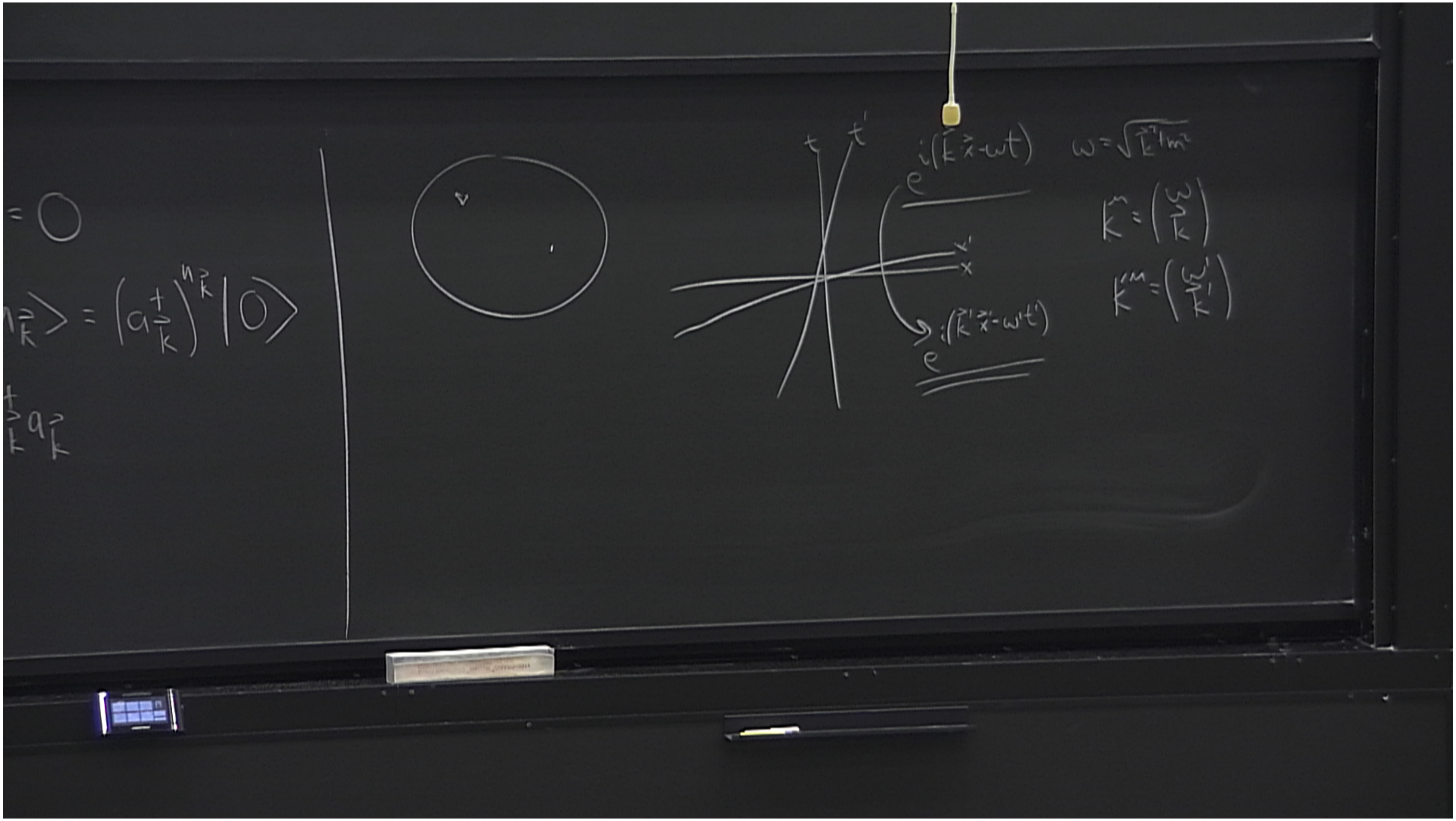

= 0
n_k> = (a†_k)^{n_k} |0>
a†_k a_k

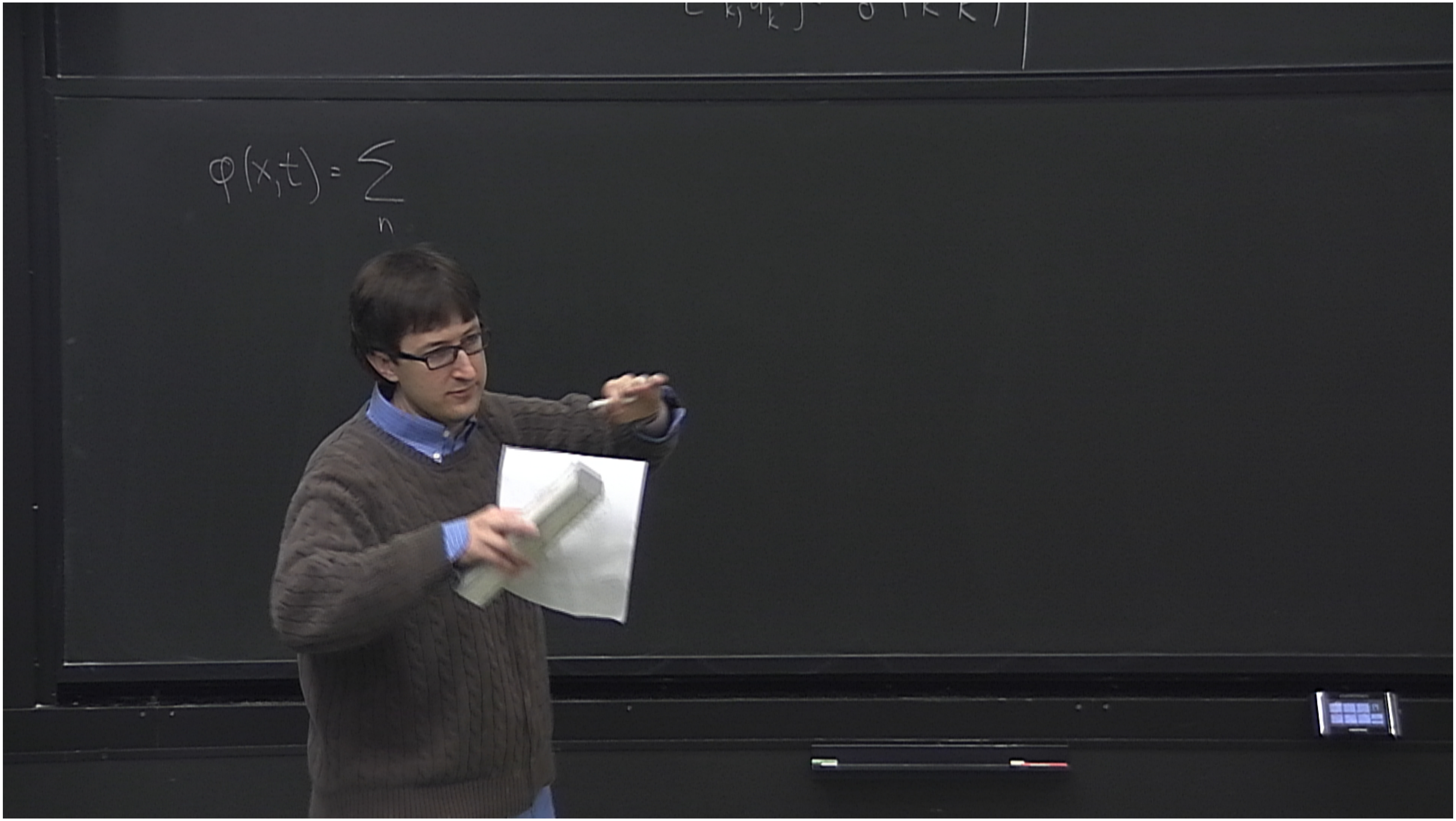

$e^{i(\vec{k}\cdot\vec{x}-\omega t)}$
$\omega=\sqrt{\vec{k}^2+m^2}$

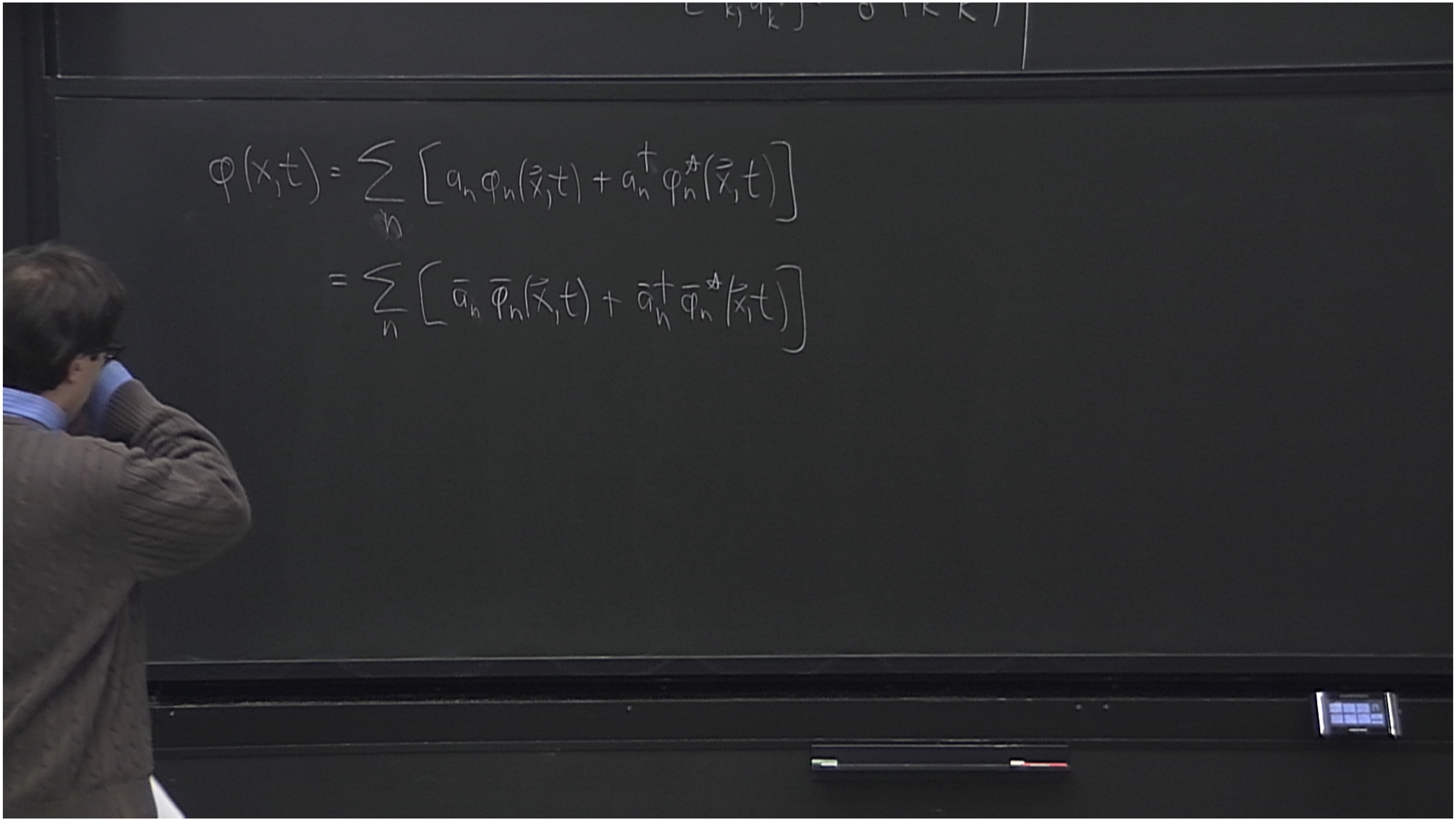

= 0
> = (a†)|0>
t
t'
x'
x
e^{i(k·x-ωt)}
ω=√(k²+m²)

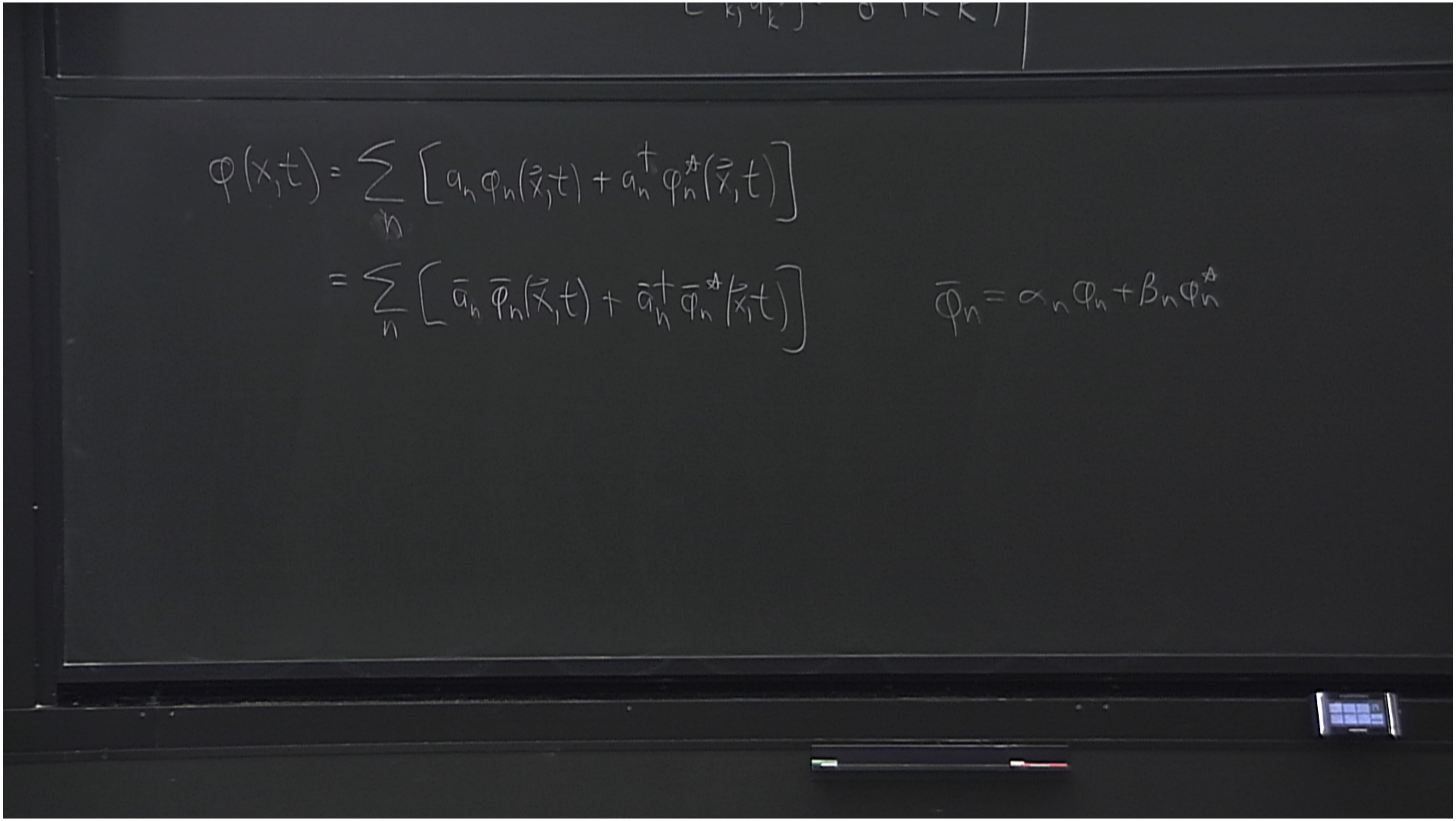

t
t'
x'
x
e^{i(k x - ωt)}
ω = √(k²+m²)
k^μ = (ω, k)

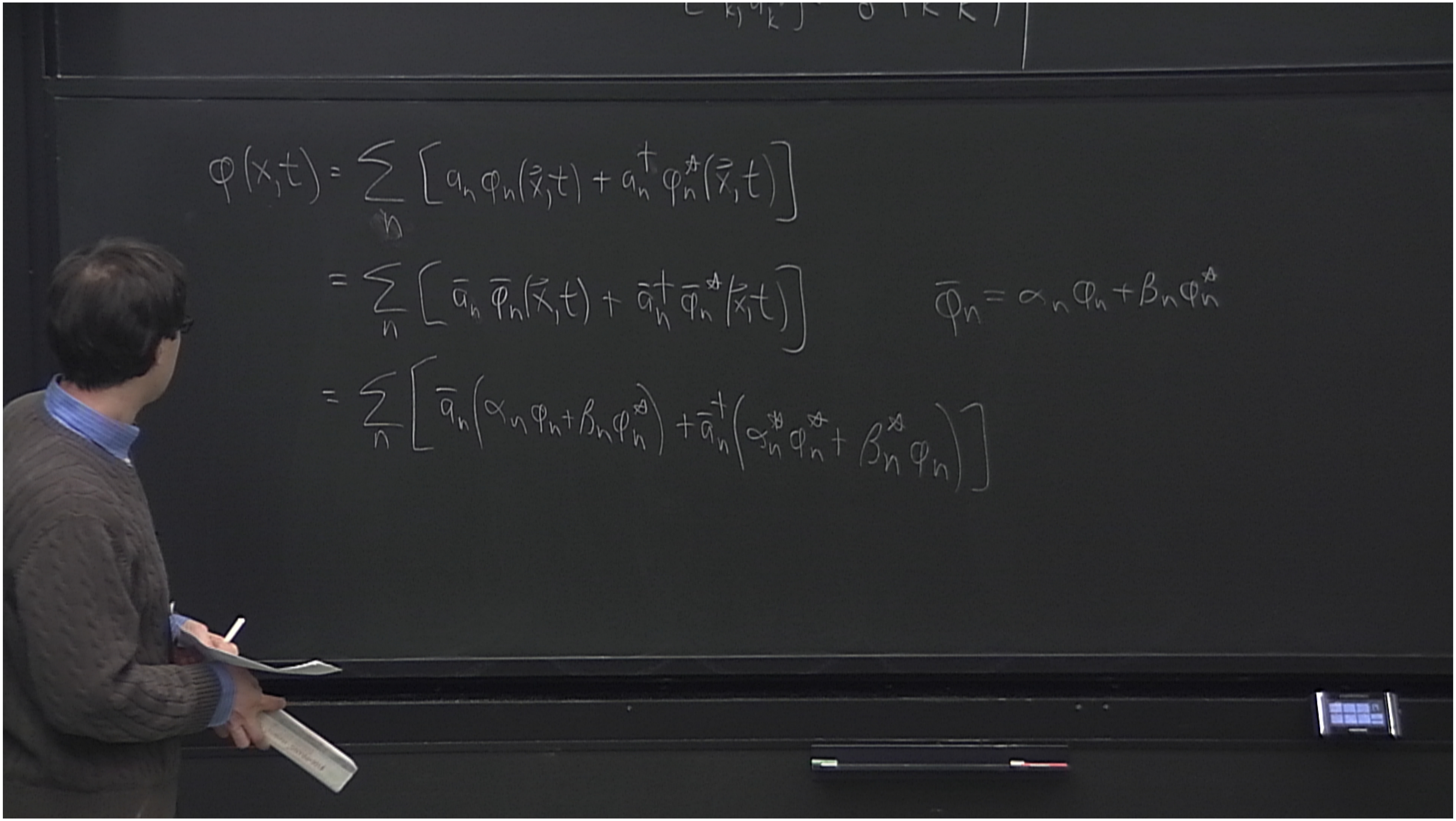

t
t'
x'
x
ω = √(k²+m²)

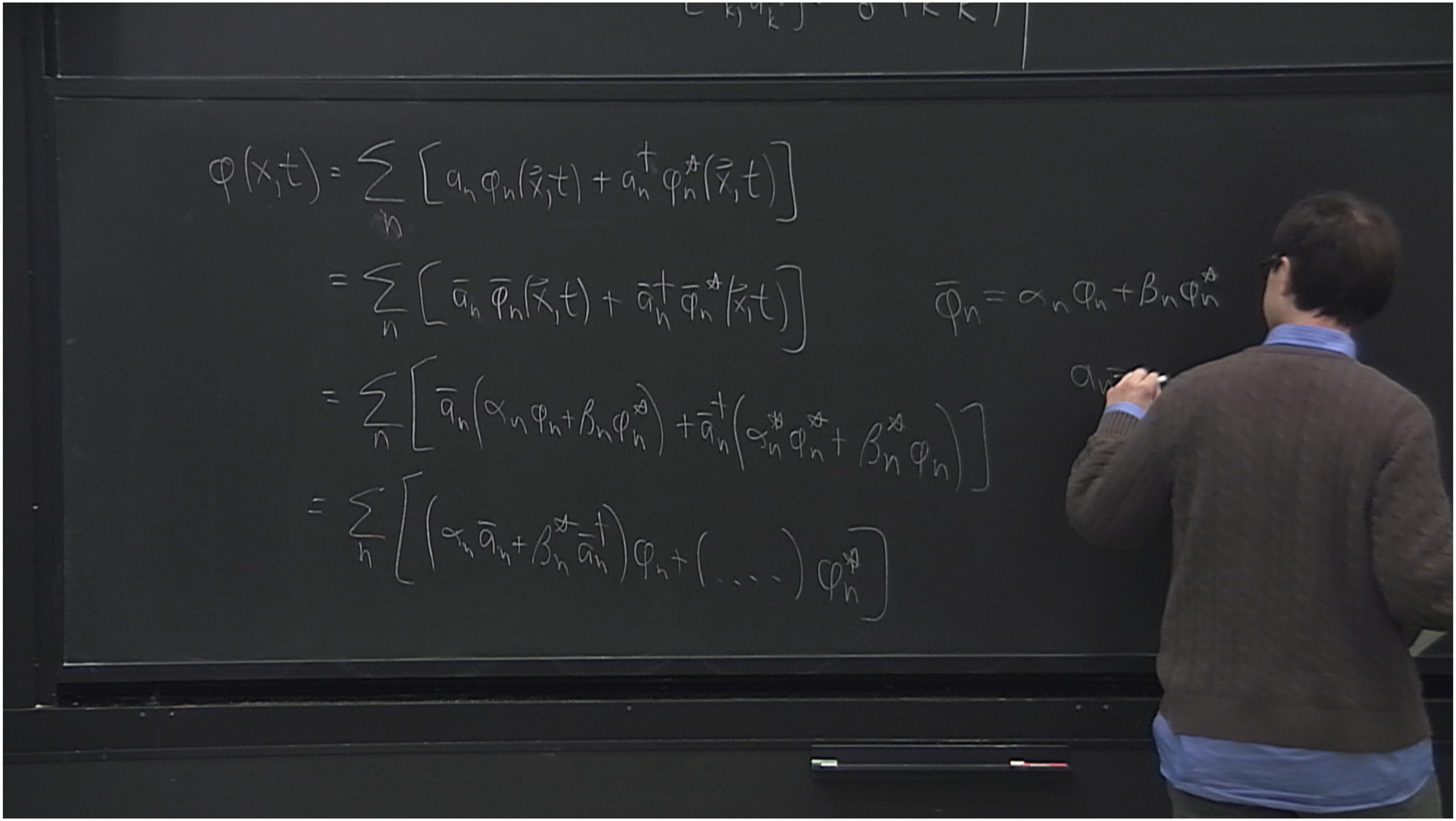

t
t'
x'
x
ω = √(k² + m²)


$\varphi(x,t) = \sum_{n}$


$\varphi(x,t) = \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$
$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$


$\varphi(x,t) = \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$
$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$
$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$


$\varphi(x,t) = \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$
$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$
$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$
$= \sum_n \left[\bar{a}_n (\alpha_n \varphi_n + \beta_n \varphi_n^*) + \bar{a}_n^\dagger (\alpha_n^* \varphi_n^* + \beta_n^* \varphi_n) \right]$


$\varphi(x,t) = \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$
$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$
$= \sum_n \left[\bar{a}_n (\alpha_n \varphi_n + \beta_n \varphi_n^*) + \bar{a}_n^\dagger (\alpha_n^* \varphi_n^* + \beta_n^* \varphi_n) \right]$
$= \sum_n \left[(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger) \varphi_n + (\ldots) \varphi_n^* \right]$
$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$

$$\varphi(x,t) = \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$$

$$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$$

$$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$$

$$a_n = \alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger$$

$$= \sum_n \left[\bar{a}_n \left(\alpha_n \varphi_n + \beta_n \varphi_n^* \right) + \bar{a}_n^\dagger \left(\alpha_n^* \varphi_n^* + \beta_n^* \varphi_n \right) \right]$$

$$= \sum_n \left[\left(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger \right) \varphi_n + \left(\ldots \right) \varphi_n^* \right]$$

$$\varphi(x,t) = \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$$

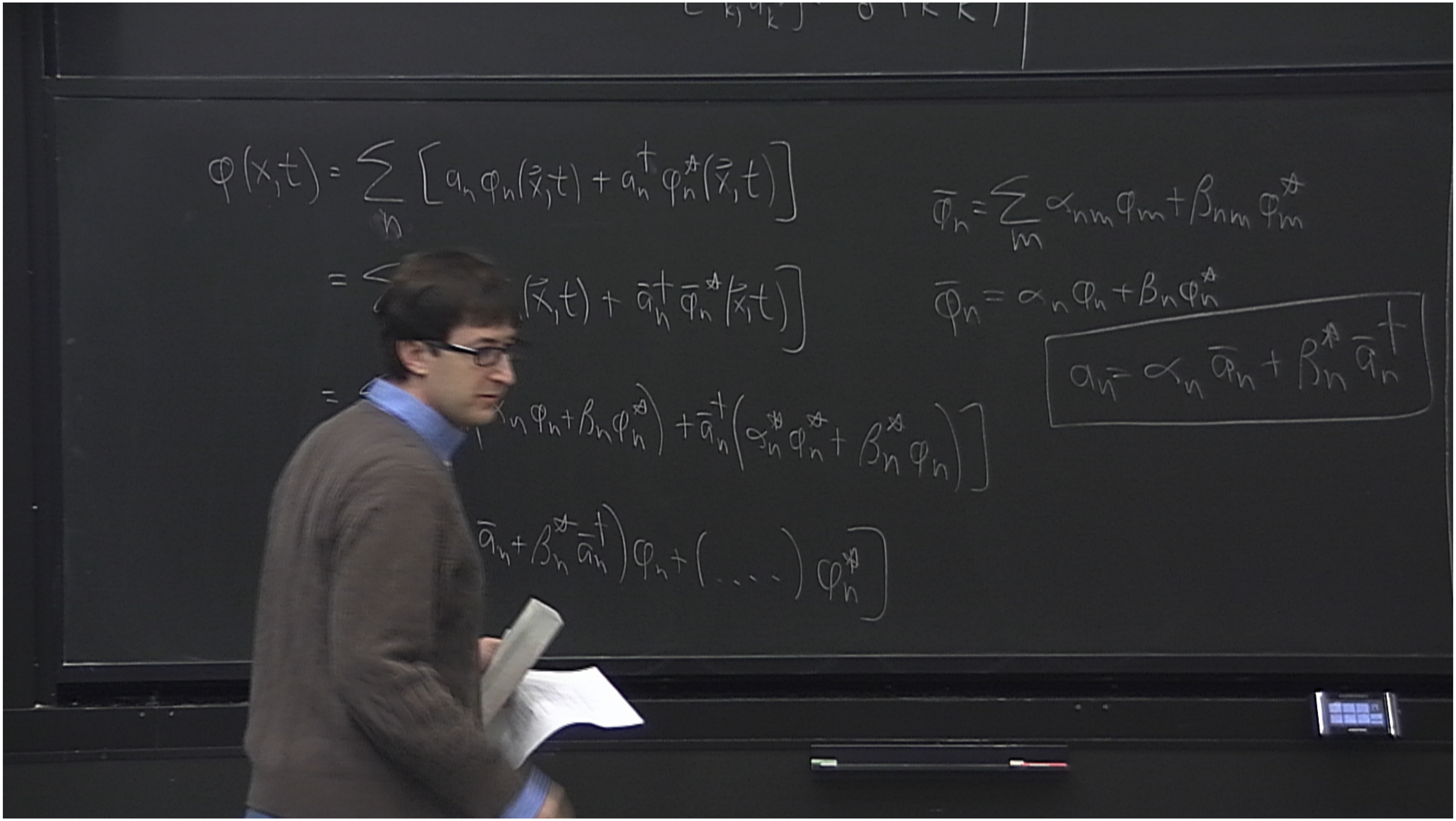
$$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$$

$$= \sum_n \left[\bar{a}_n (\alpha_n \varphi_n + \beta_n \varphi_n^*) + \bar{a}_n^\dagger (\alpha_n^* \varphi_n^* + \beta_n^* \varphi_n) \right]$$

$$= \sum_n \left[(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger) \varphi_n + (\dots) \varphi_n^* \right]$$

$$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$$

$$\boxed{a_n = \alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger}$$


φ(x,t) = Σ_n [a_n φ_n(x,t) + a_n^† φ_n^*(x,t)]
φ̄_n = Σ_m α_nm φ_m + β_nm φ_m^*
φ̄_n = α_n φ_n + β_n φ_n^*
a_n = α_n ā_n + β_n^* ā_n^†

$$\varphi(x,t) = \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$$

$$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$$

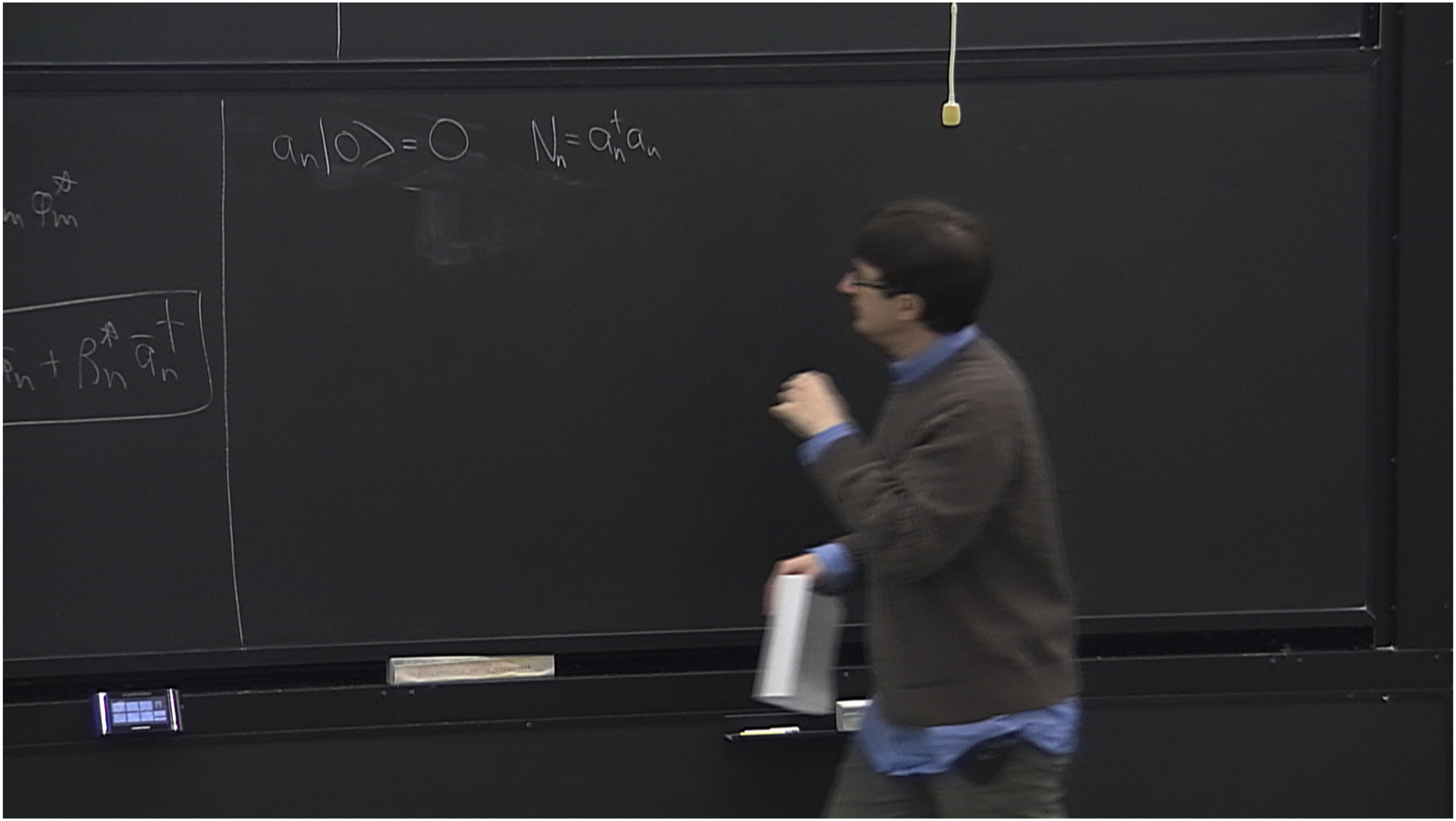
$$= \sum_n \left[\bar{a}_n (\alpha_n \varphi_n + \beta_n \varphi_n^*) + \bar{a}_n^\dagger (\alpha_n^* \varphi_n^* + \beta_n^* \varphi_n) \right]$$

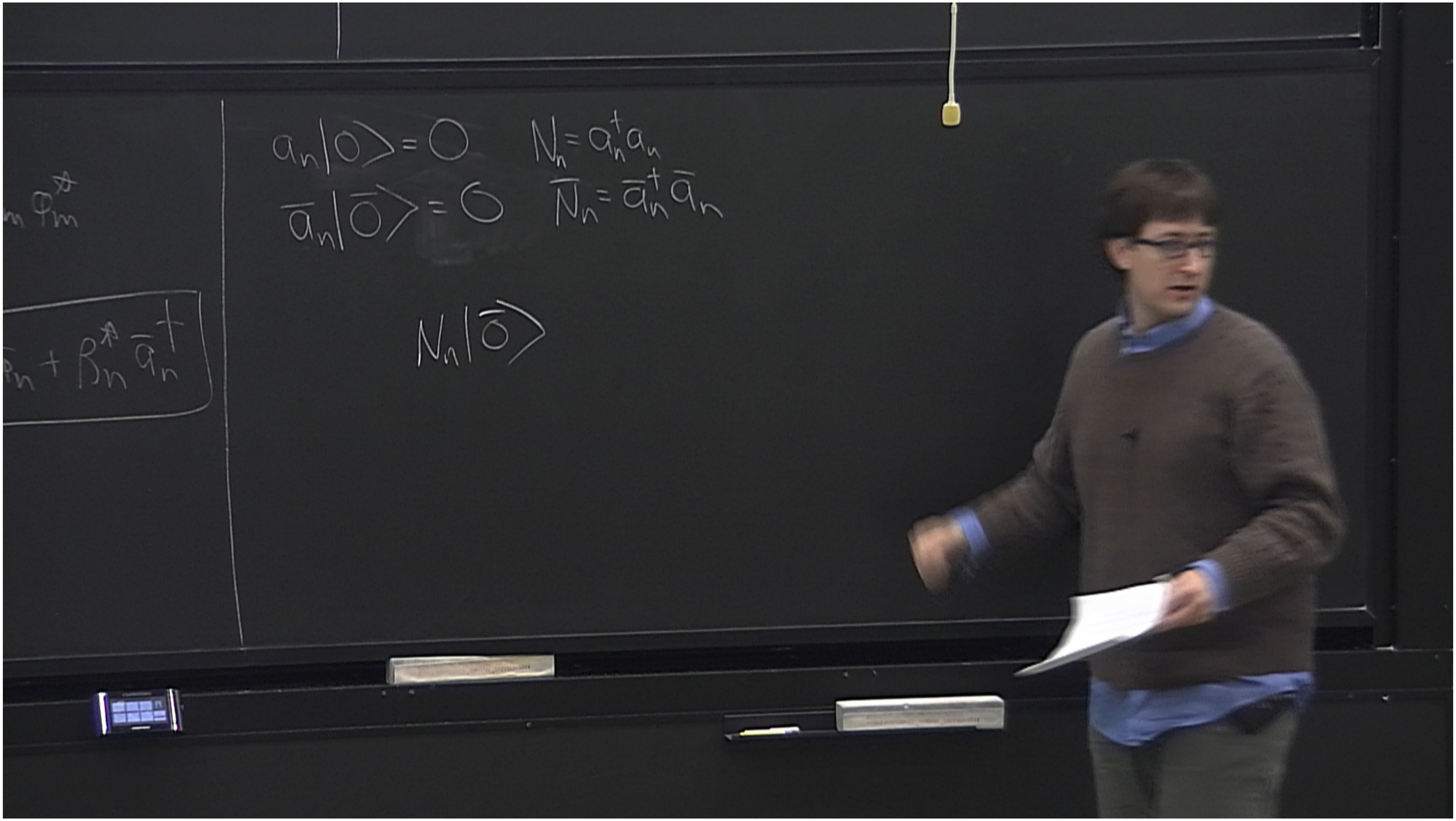
$$= \sum_n \left[(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger) \varphi_n + (\ldots) \varphi_n^* \right]$$

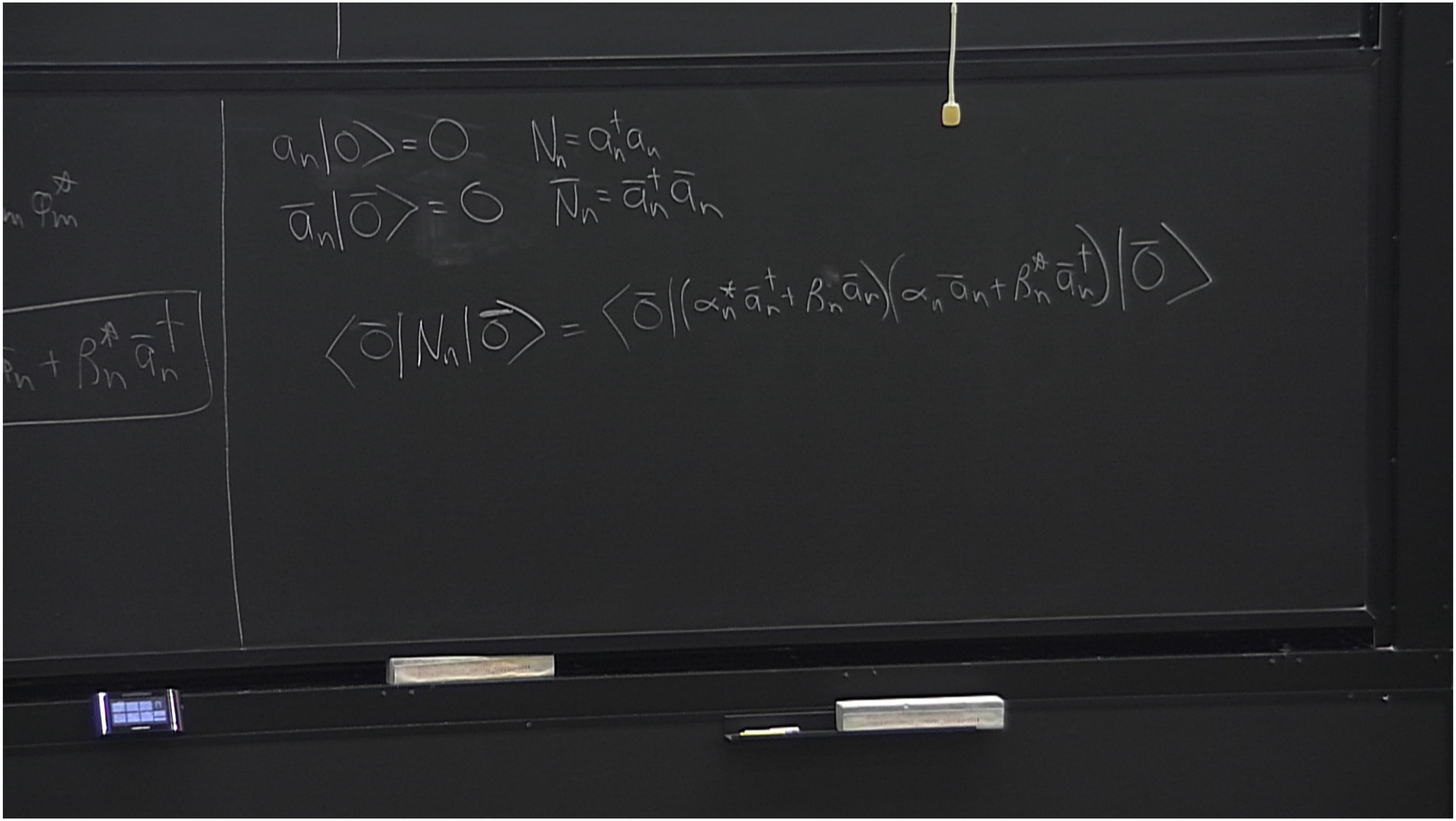
$$\bar{\varphi}_n = \sum_m \alpha_{nm} \varphi_m + \beta_{nm} \varphi_m^*$$

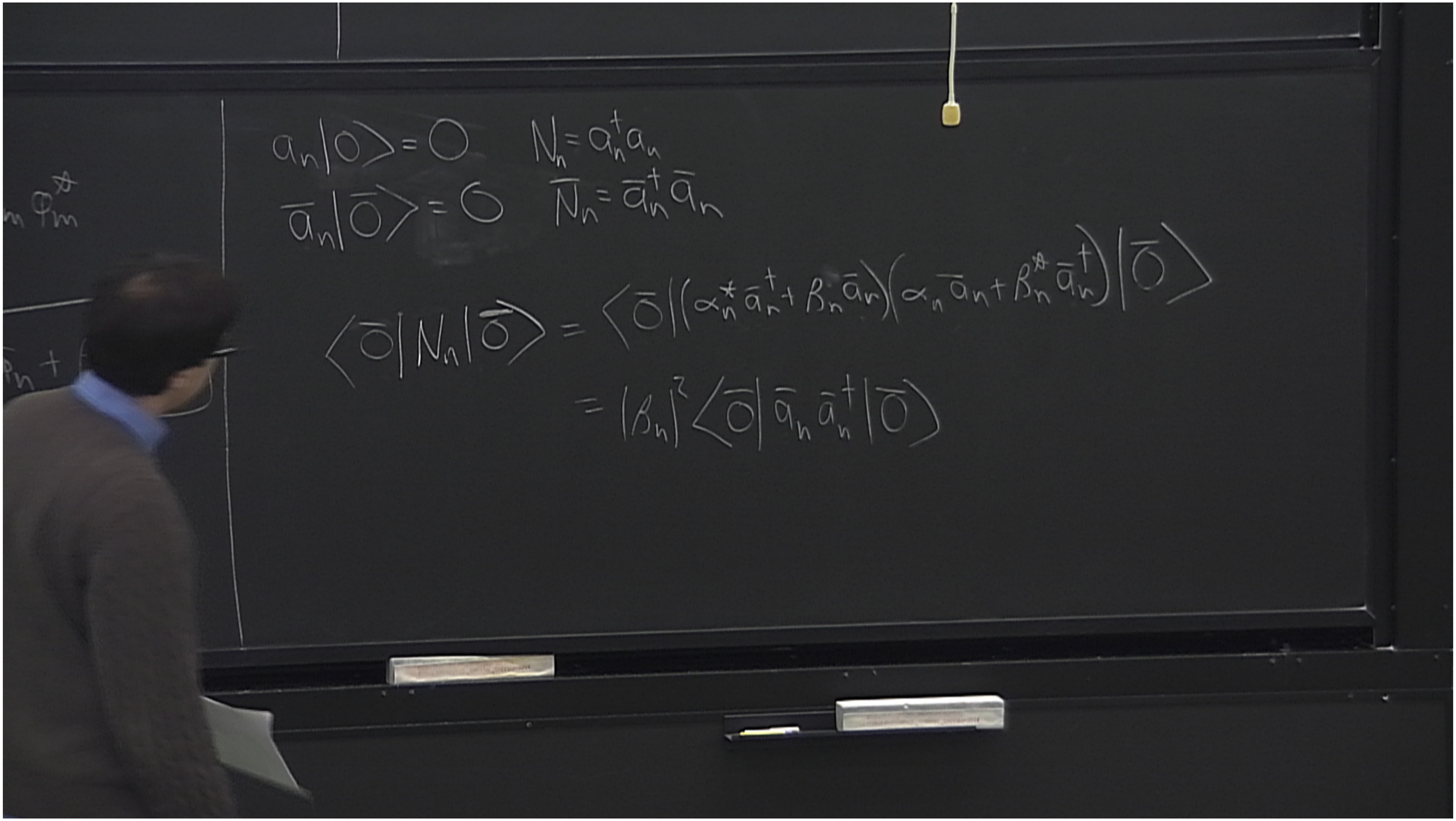
$$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$$

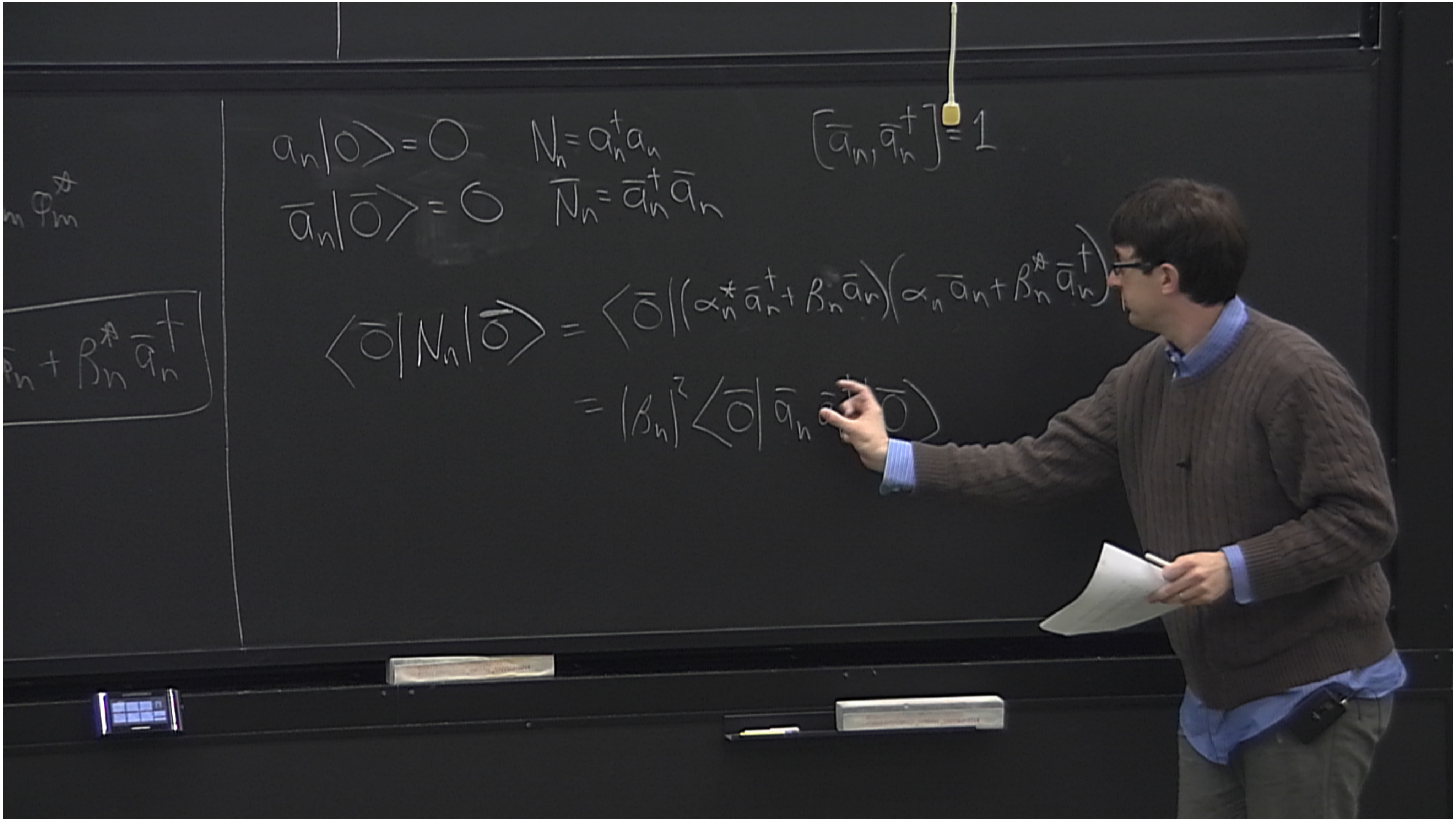
$$\boxed{a_n = \alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger}$$

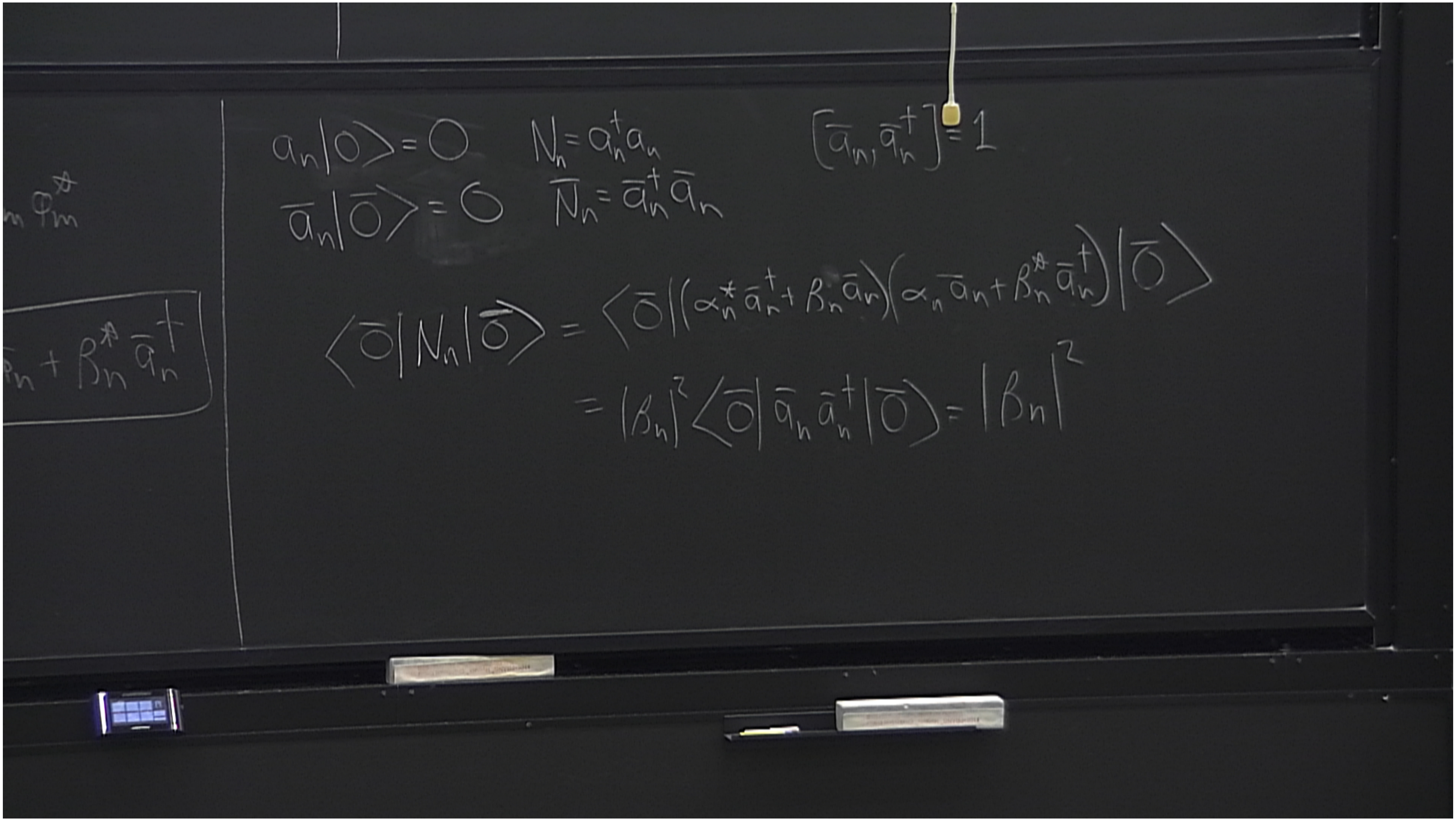

$a_n|0\rangle = 0$
$N_n = a_n^\dagger a_n$


a_n|0> = 0   N_n = a_n† a_n
ā_n|0̄> = 0   N̄_n = ā_n† ā_n
N_n|0̄>

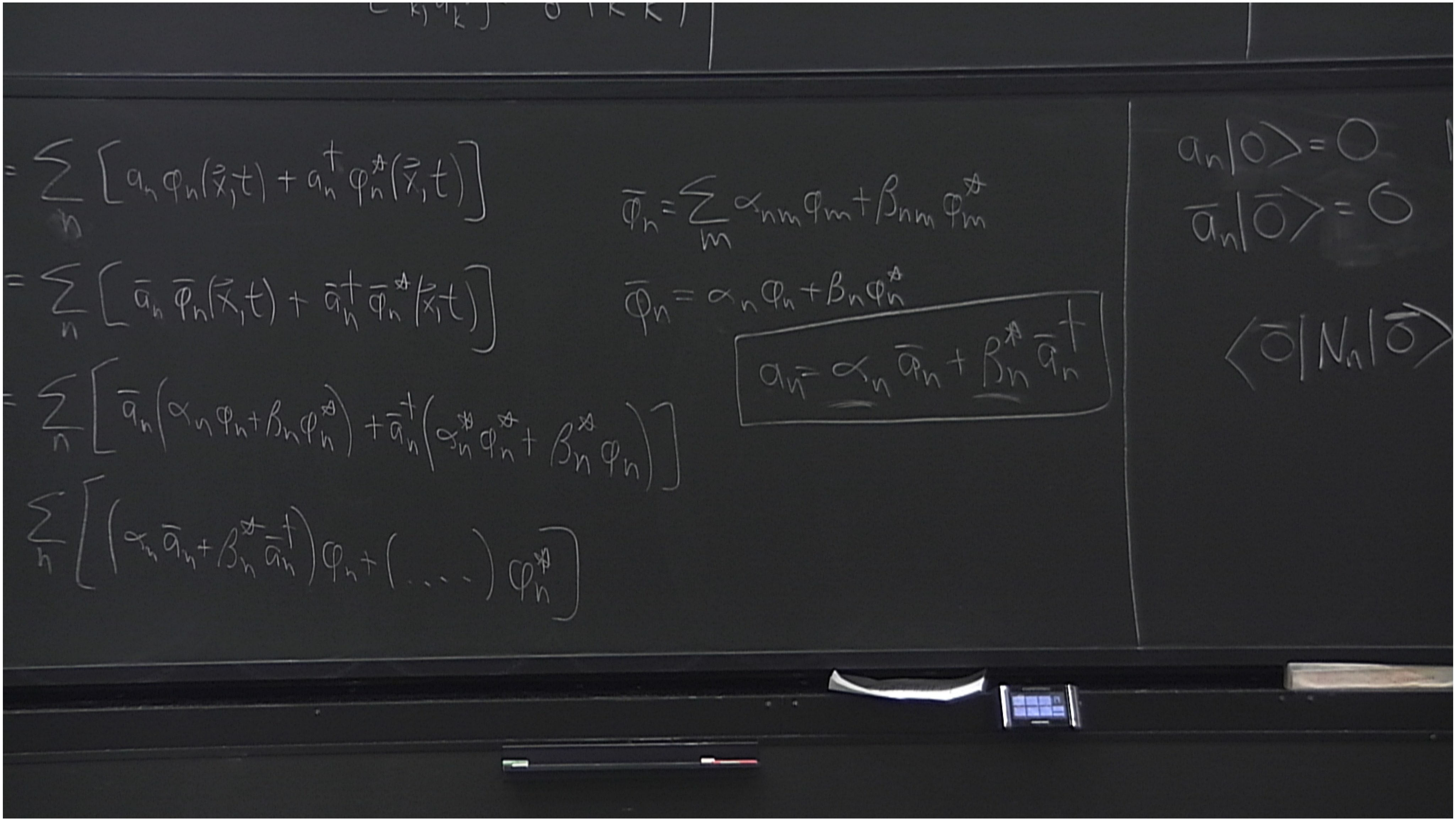

$a_n|0\rangle = 0 \quad N_n = a_n^\dagger a_n$
$\bar{a}_n|\bar{0}\rangle = 0 \quad \bar{N}_n = \bar{a}_n^\dagger \bar{a}_n$
$\langle\bar{0}|N_n|\bar{0}\rangle = \langle\bar{0}|(\alpha_n^* \bar{a}_n^\dagger + \beta_n \bar{a}_n)(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger)|\bar{0}\rangle$

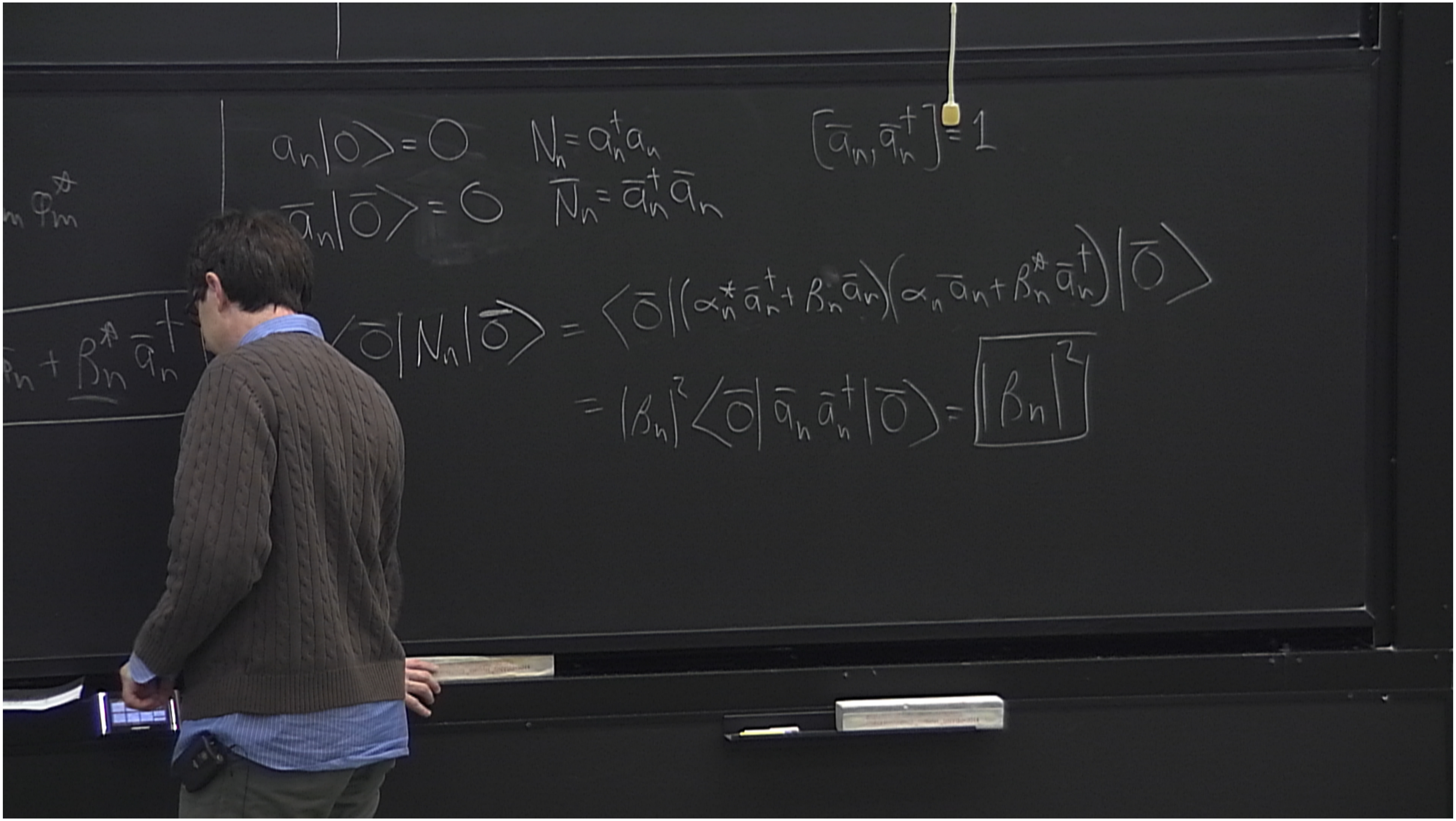

$a_n|0\rangle = 0$ $N_n = a_n^\dagger a_n$
$\bar{a}_n|\bar{0}\rangle = 0$ $\bar{N}_n = \bar{a}_n^\dagger \bar{a}_n$
$\langle\bar{0}|N_n|\bar{0}\rangle = \langle\bar{0}|(\alpha_n^* \bar{a}_n^\dagger + \beta_n \bar{a}_n)(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger)|\bar{0}\rangle$
$= |\beta_n|^2 \langle\bar{0}|\bar{a}_n \bar{a}_n^\dagger|\bar{0}\rangle$

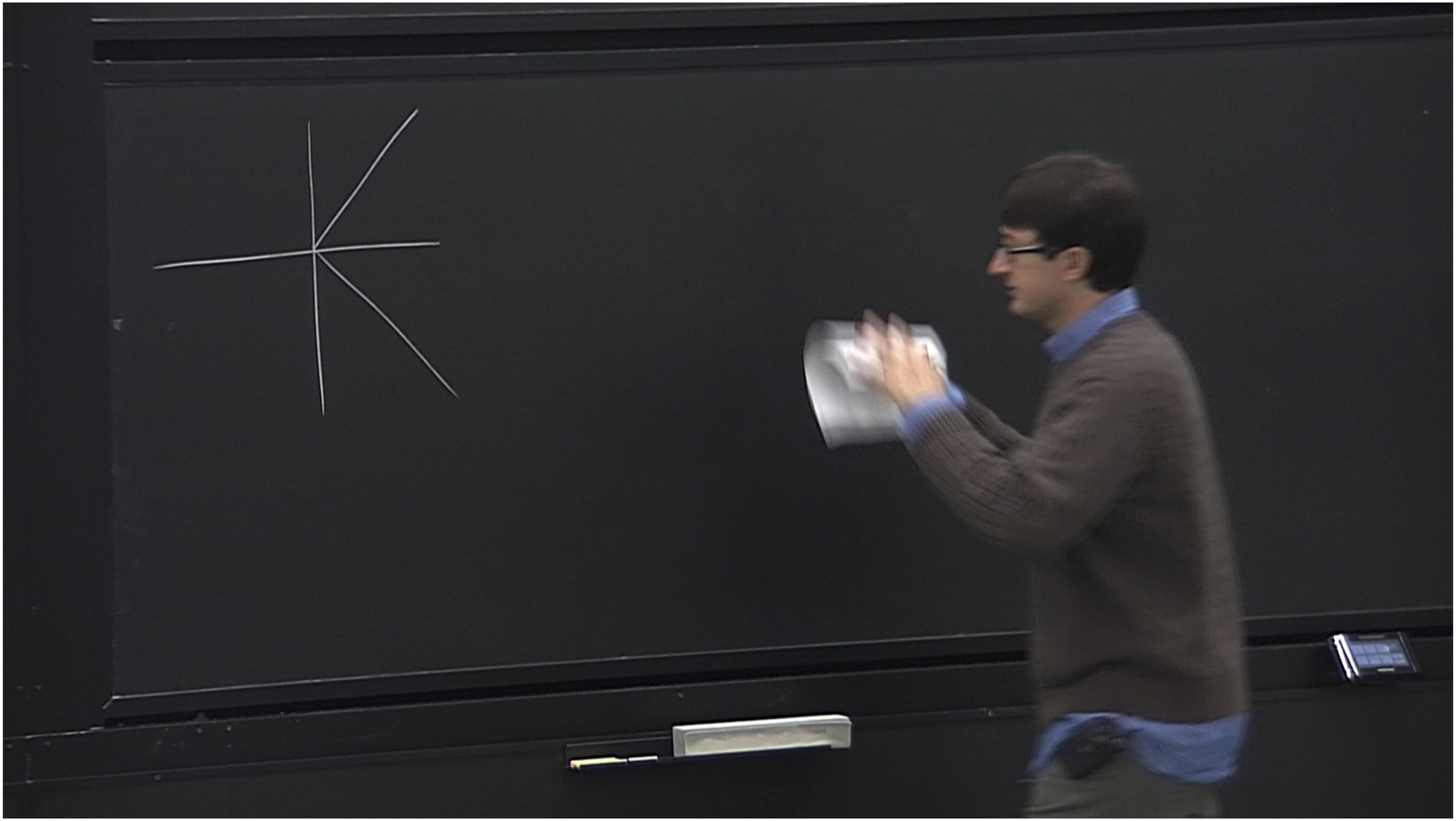

$a_n|0\rangle = 0$
$N_n = a_n^\dagger a_n$
$\bar{a}_n|\bar{0}\rangle = 0$
$\bar{N}_n = \bar{a}_n^\dagger \bar{a}_n$
$[\bar{a}_n, \bar{a}_n^\dagger] = 1$
$\langle \bar{0}|N_n|\bar{0}\rangle = \langle \bar{0}|(\alpha_n^* \bar{a}_n^\dagger + \beta_n \bar{a}_n)(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger)$
$= |\beta_n|^2 \langle \bar{0}|\bar{a}_n$

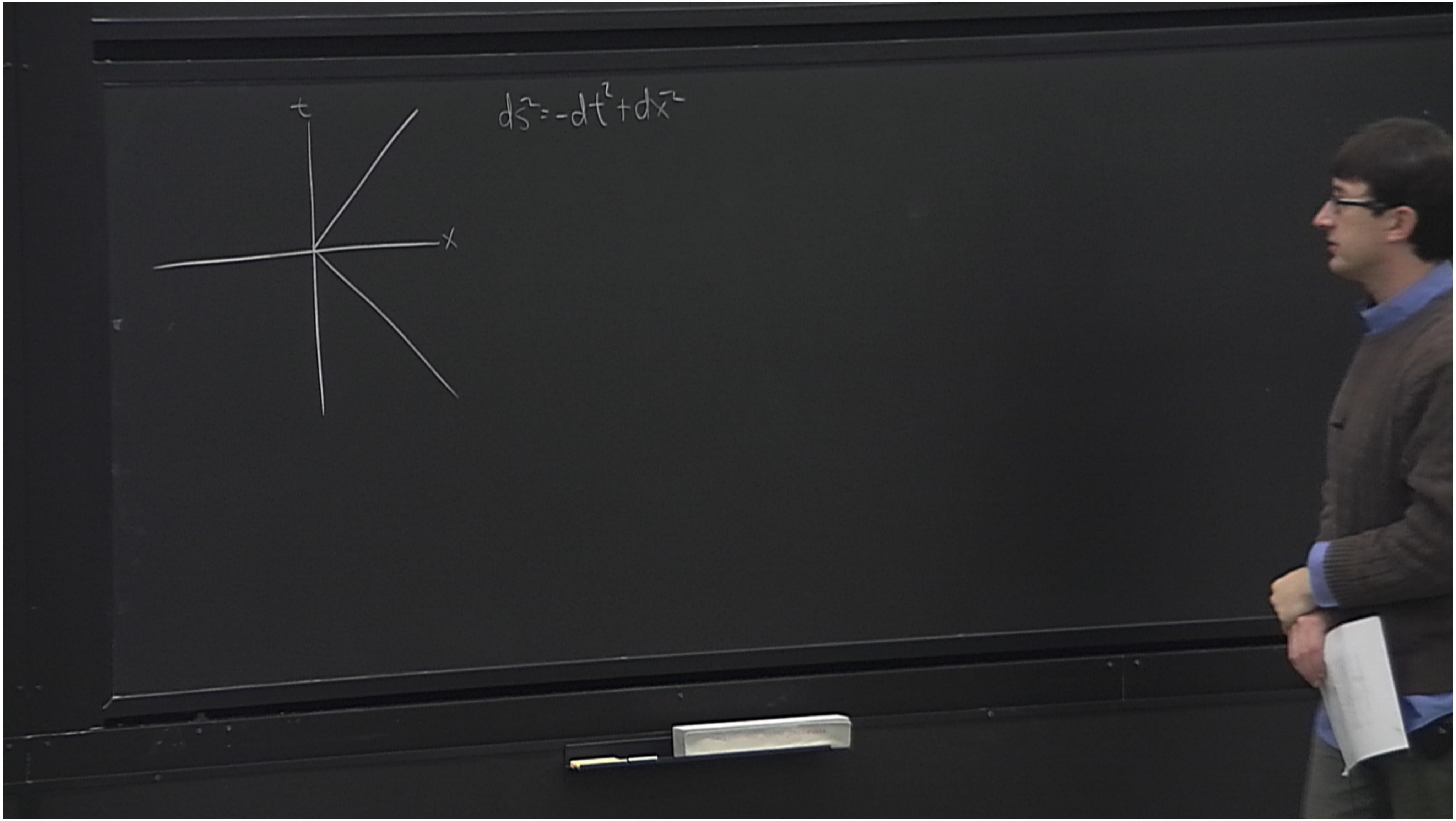


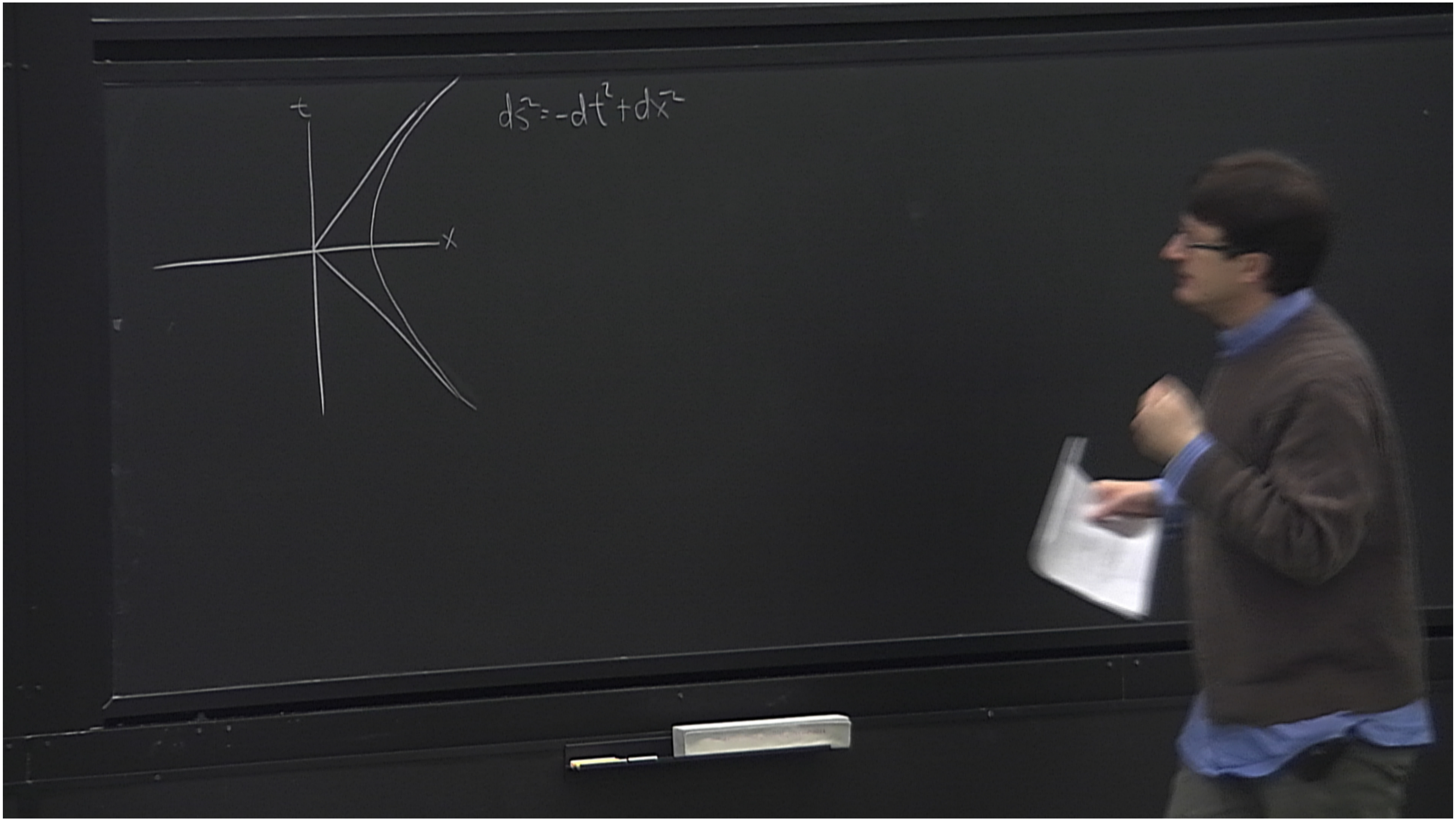
$a_n|0\rangle = 0$ $N_n = a_n^\dagger a_n$ $[\bar{a}_n, \bar{a}_n^\dagger] = 1$
$\bar{a}_n|\bar{0}\rangle = 0$ $\bar{N}_n = \bar{a}_n^\dagger \bar{a}_n$
$\langle \bar{0}|N_n|\bar{0}\rangle = \langle \bar{0}|(\alpha_n^* \bar{a}_n^\dagger + \beta_n \bar{a}_n)(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger)|\bar{0}\rangle$
$= |\beta_n|^2 \langle \bar{0}|\bar{a}_n \bar{a}_n^\dagger|\bar{0}\rangle = |\beta_n|^2$

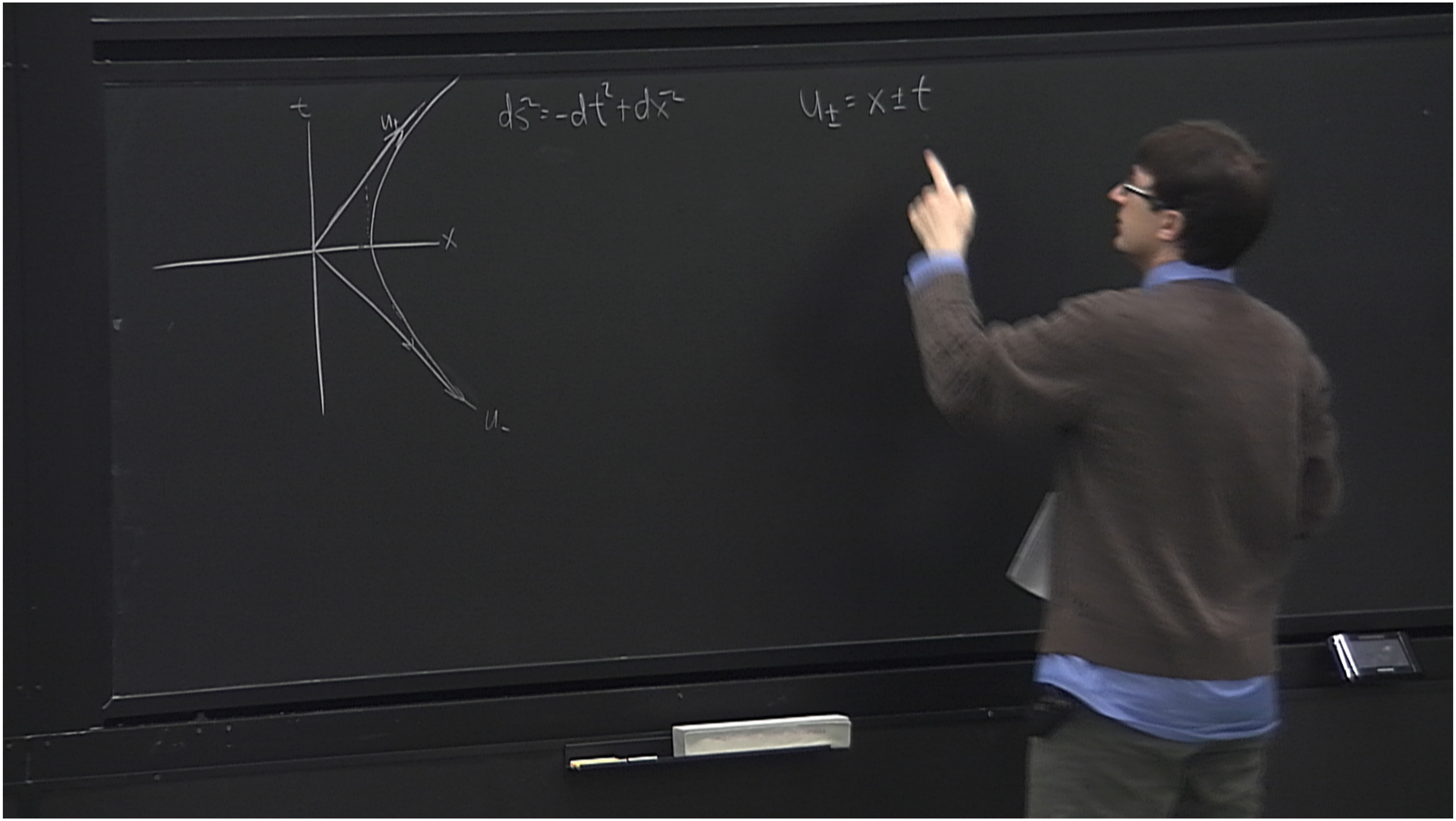

$= \sum_n \left[a_n \varphi_n(\vec{x},t) + a_n^\dagger \varphi_n^*(\vec{x},t) \right]$
$= \sum_n \left[\bar{a}_n \bar{\varphi}_n(\vec{x},t) + \bar{a}_n^\dagger \bar{\varphi}_n^*(\vec{x},t) \right]$
$= \sum_n \left[\bar{a}_n (\alpha_n \varphi_n + \beta_n \varphi_n^*) + \bar{a}_n^\dagger (\alpha_n^* \varphi_n^* + \beta_n^* \varphi_n) \right]$
$\sum_n \left[(\alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger) \varphi_n + (\dots) \varphi_n^* \right]$
$\bar{\varphi}_n = \sum_m \alpha_{nm} \varphi_m + \beta_{nm} \varphi_m^*$
$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$
$a_n = \alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger$
$a_n|0\rangle = 0$
$\bar{a}_n|\bar{0}\rangle = 0$
$\langle \bar{0}|N_n|\bar{0}\rangle$

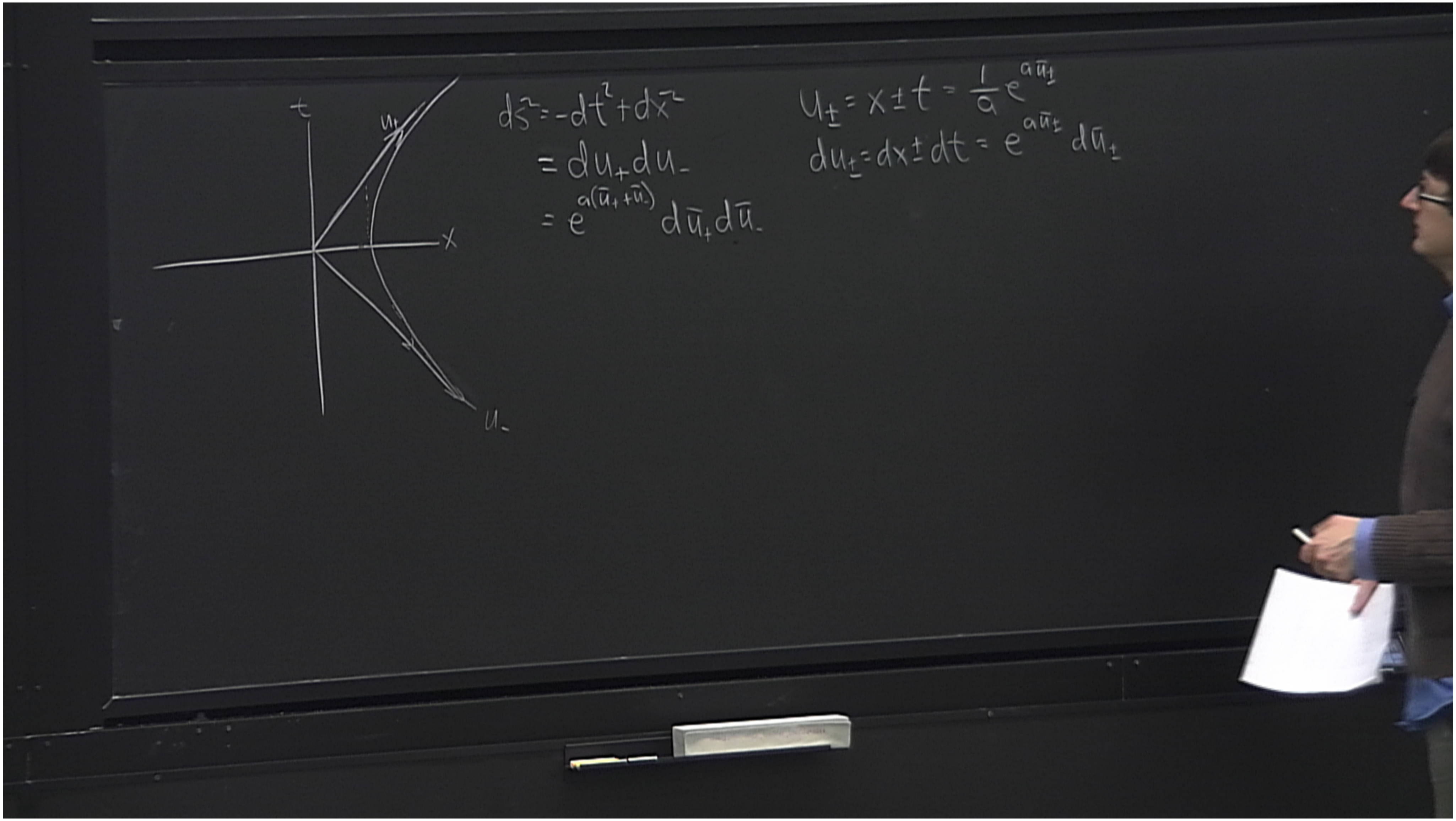

a_n|0> = 0   N_n = a_n† a_n   [ā_n, ā_n†] = 1
ā_n|0̄> = 0   N̄_n = ā_n† ā_n
<0̄|N_n|0̄> = <0̄|(α_n* ā_n† + β_n ā_n)(α_n ā_n + β_n* ā_n†)|0̄>
= |β_n|² <0̄|ā_n ā_n†|0̄> = |β_n|²

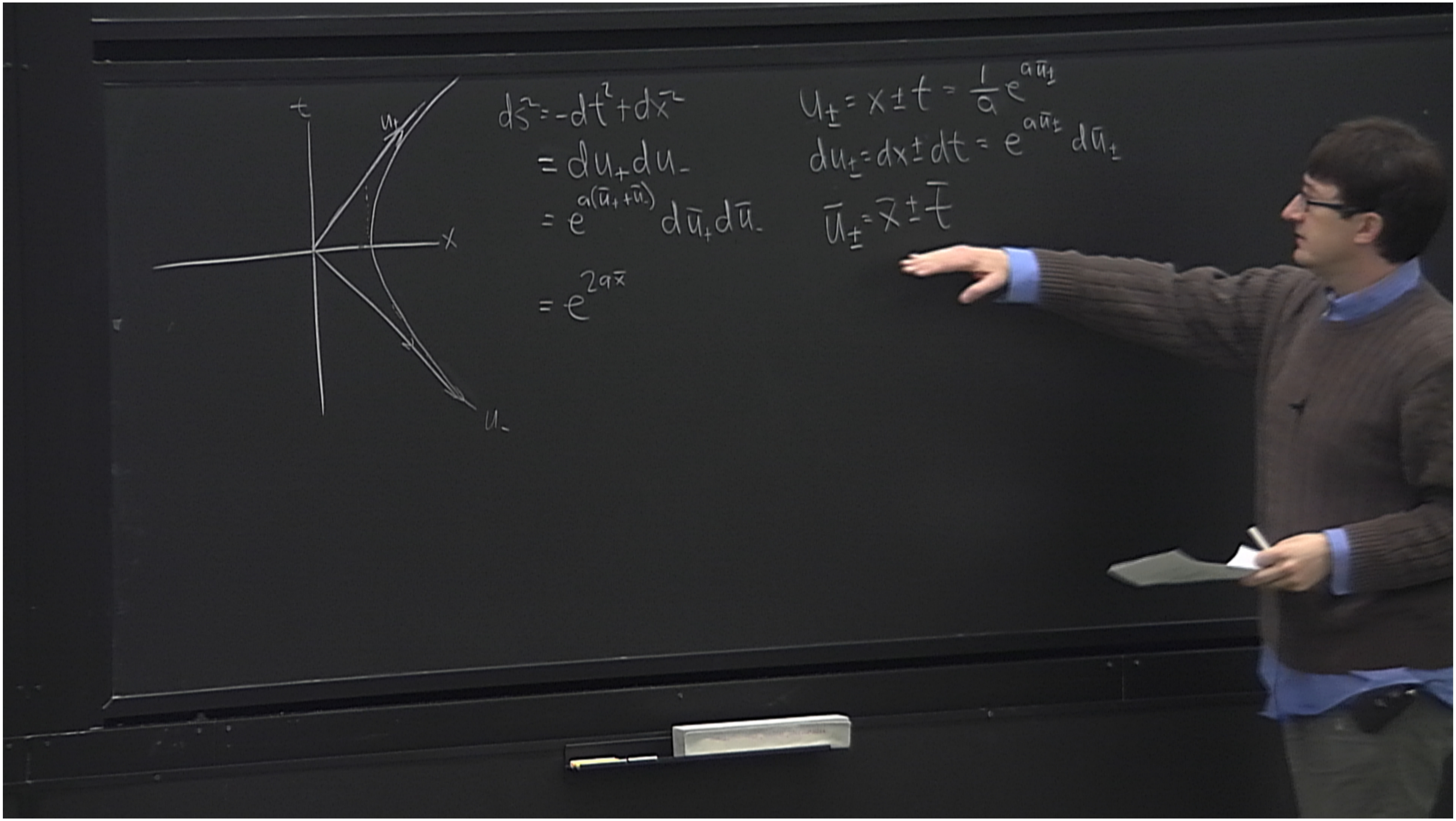


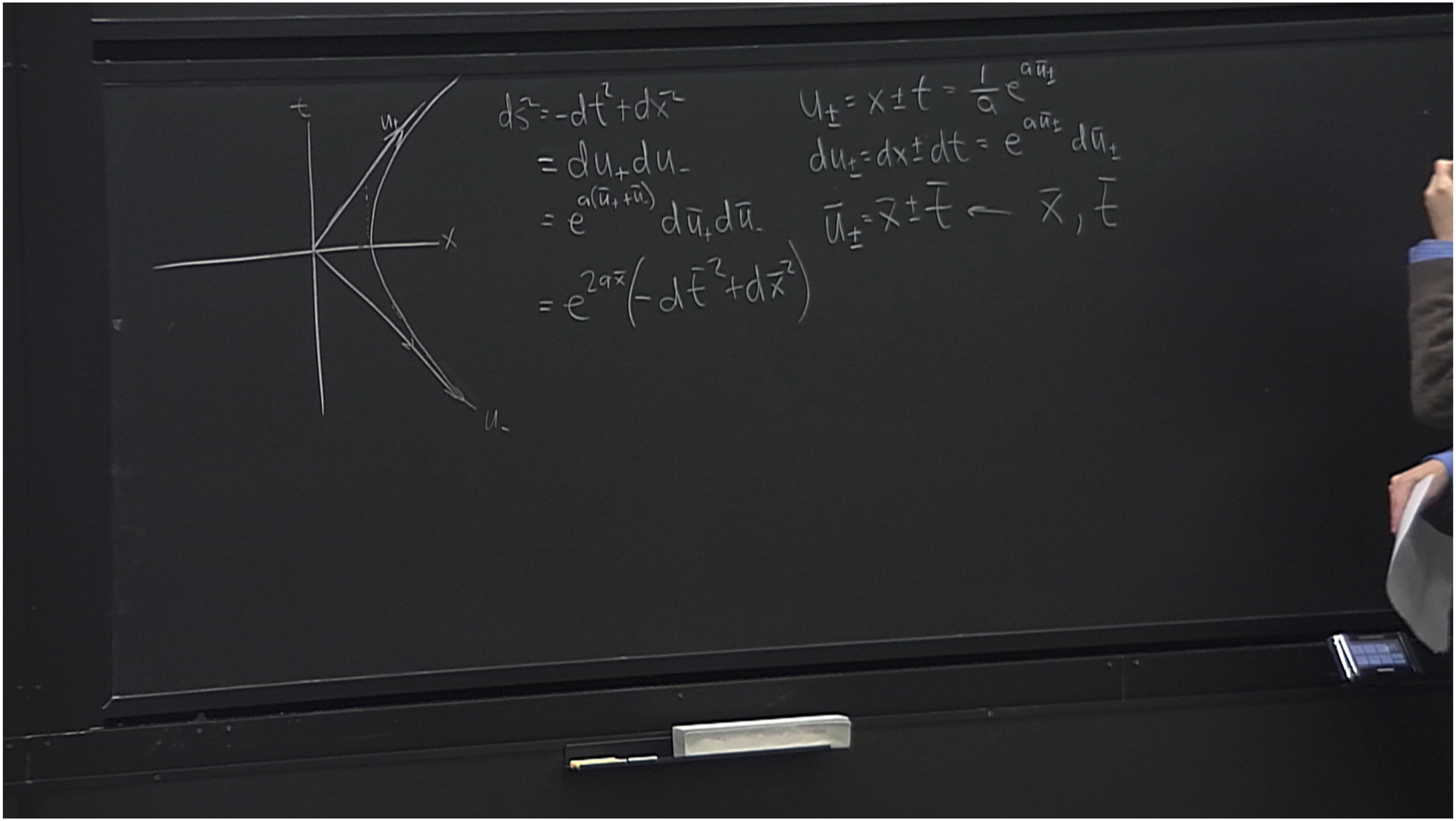

t
x
ds² = -dt² + dx²


t
x
ds² = -dt² + dx²

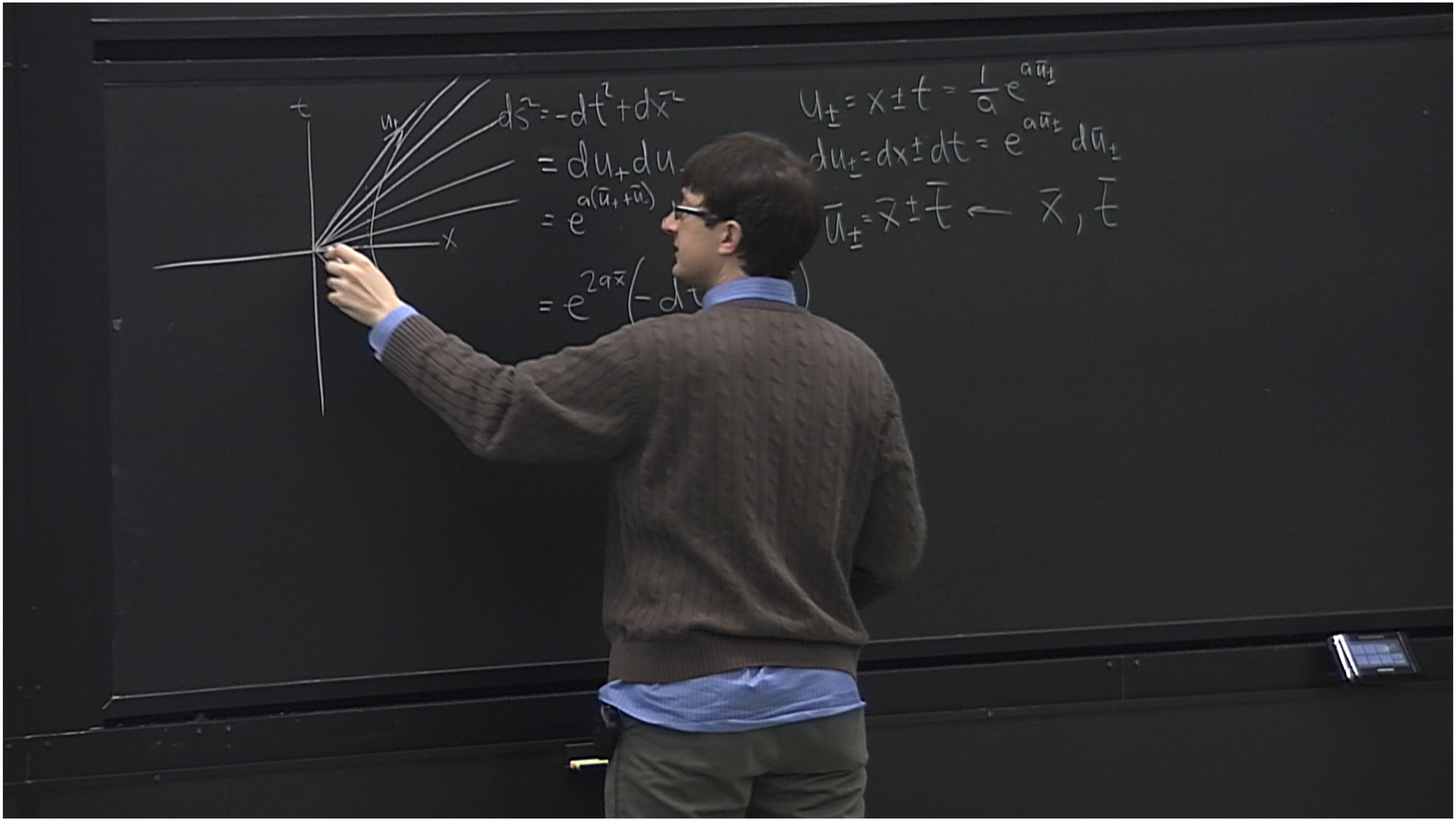

t
x
u_+
u_-
$ds^2 = -dt^2 + dx^2$
$u_\pm = x \pm t$

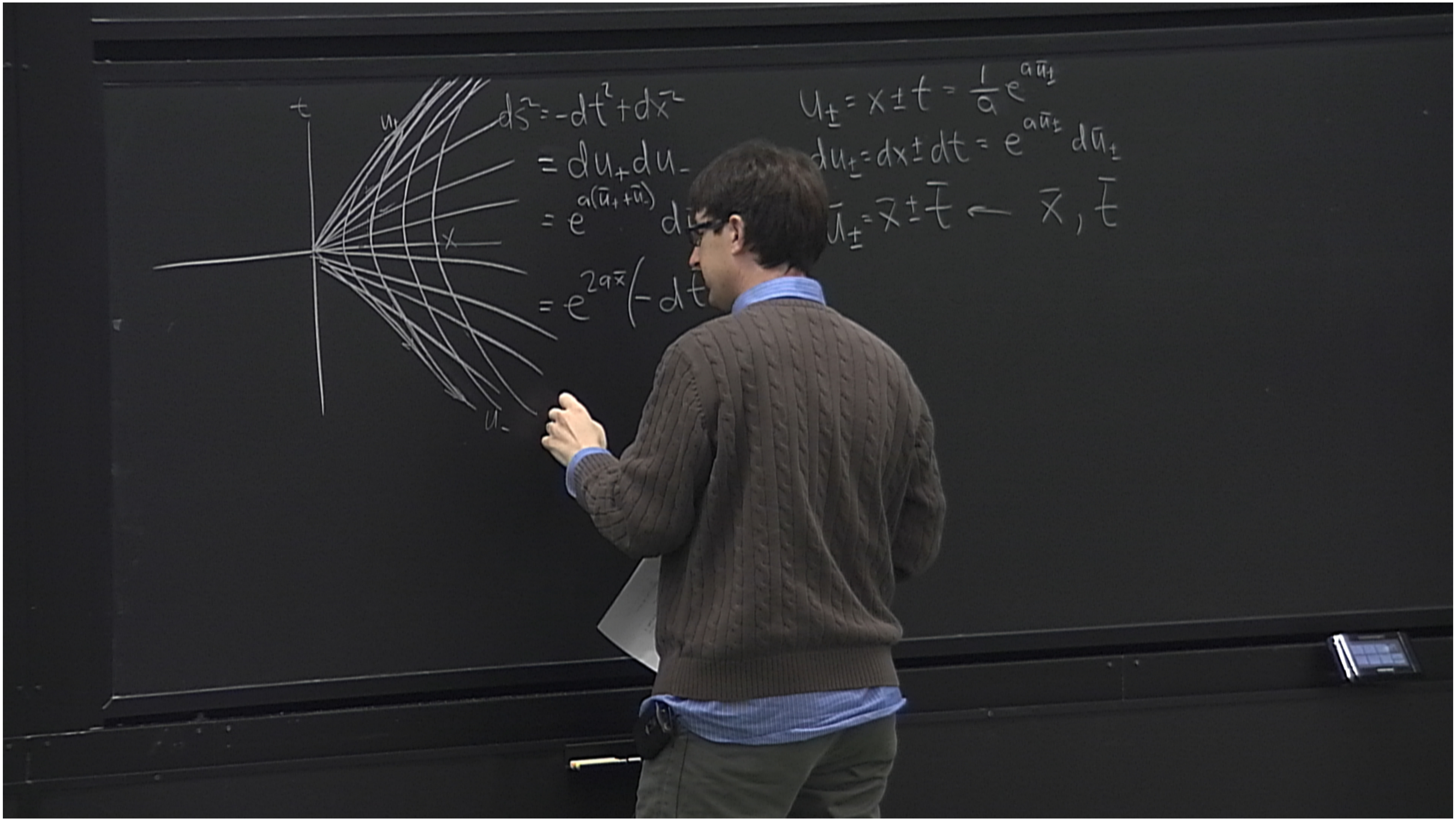

t
x
u_+
u_-
$ds^2 = -dt^2 + dx^2$
$= du_+ du_-$
$= e^{a(\bar{u}_+ + \bar{u}_-)} d\bar{u}_+ d\bar{u}_-$
$u_\pm = x \pm t = \frac{1}{a} e^{a\bar{u}_\pm}$
$du_\pm = dx \pm dt = e^{a\bar{u}_\pm} d\bar{u}_\pm$

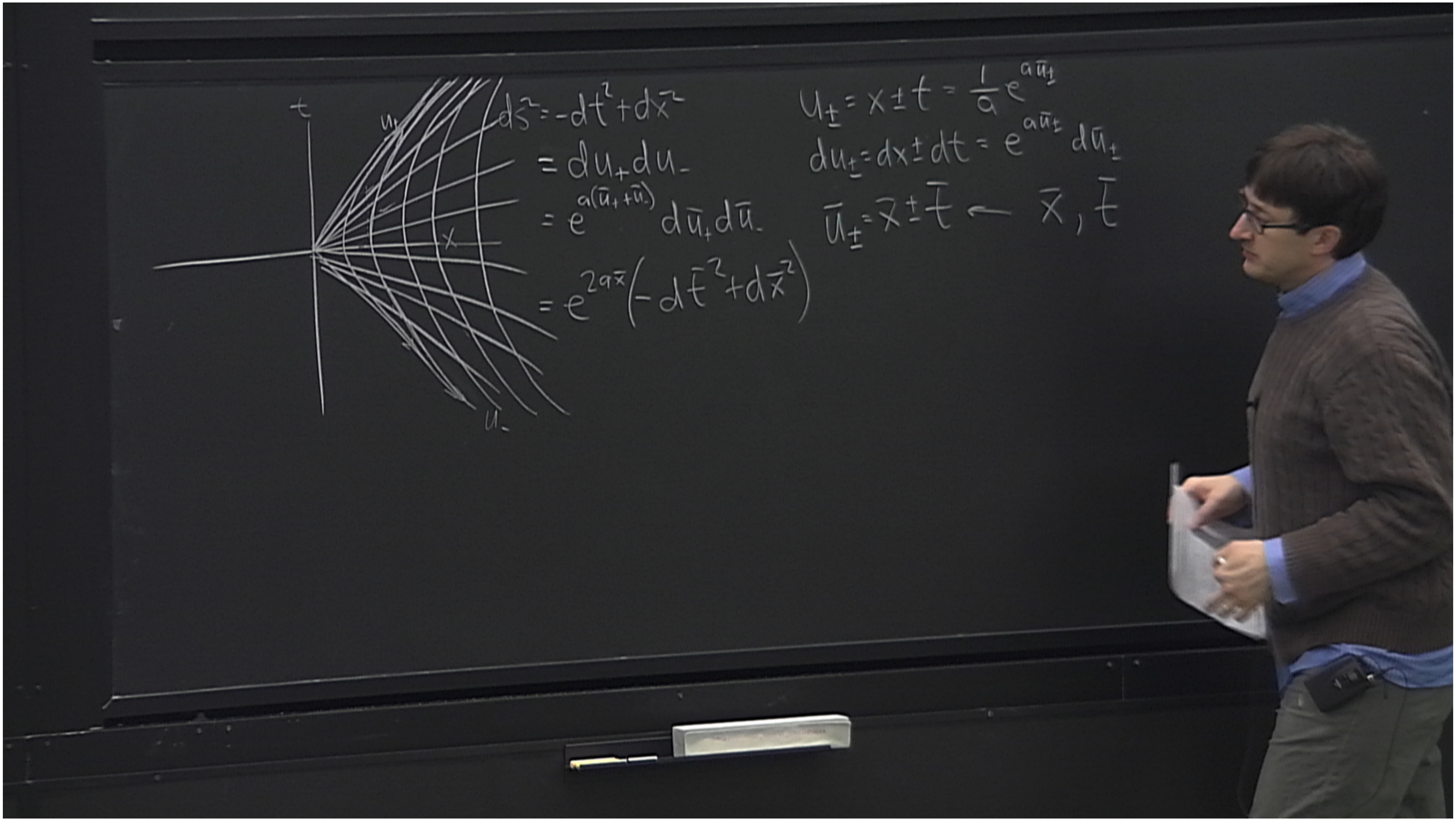

t
x
u_+
u_-
$ds^2 = -dt^2 + dx^2$
$= du_+ du_-$
$= e^{a(\bar{u}_+ + \bar{u}_-)} d\bar{u}_+ d\bar{u}_-$
$= e^{2a\bar{x}}$
$u_\pm = x \pm t = \frac{1}{a} e^{a\bar{u}_\pm}$
$du_\pm = dx \pm dt = e^{a\bar{u}_\pm} d\bar{u}_\pm$
$\bar{u}_\pm = \bar{x} \pm \bar{t}$

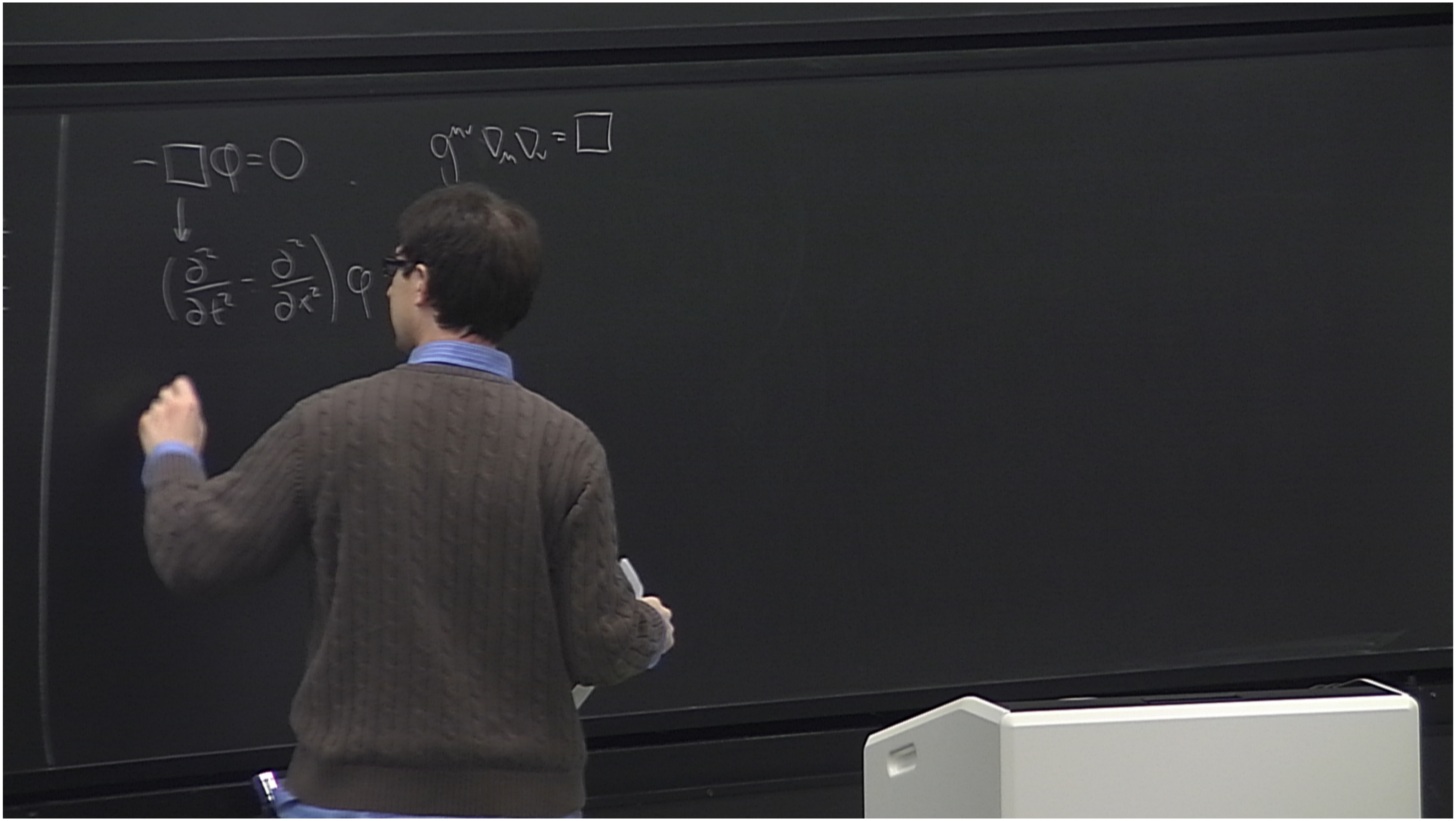


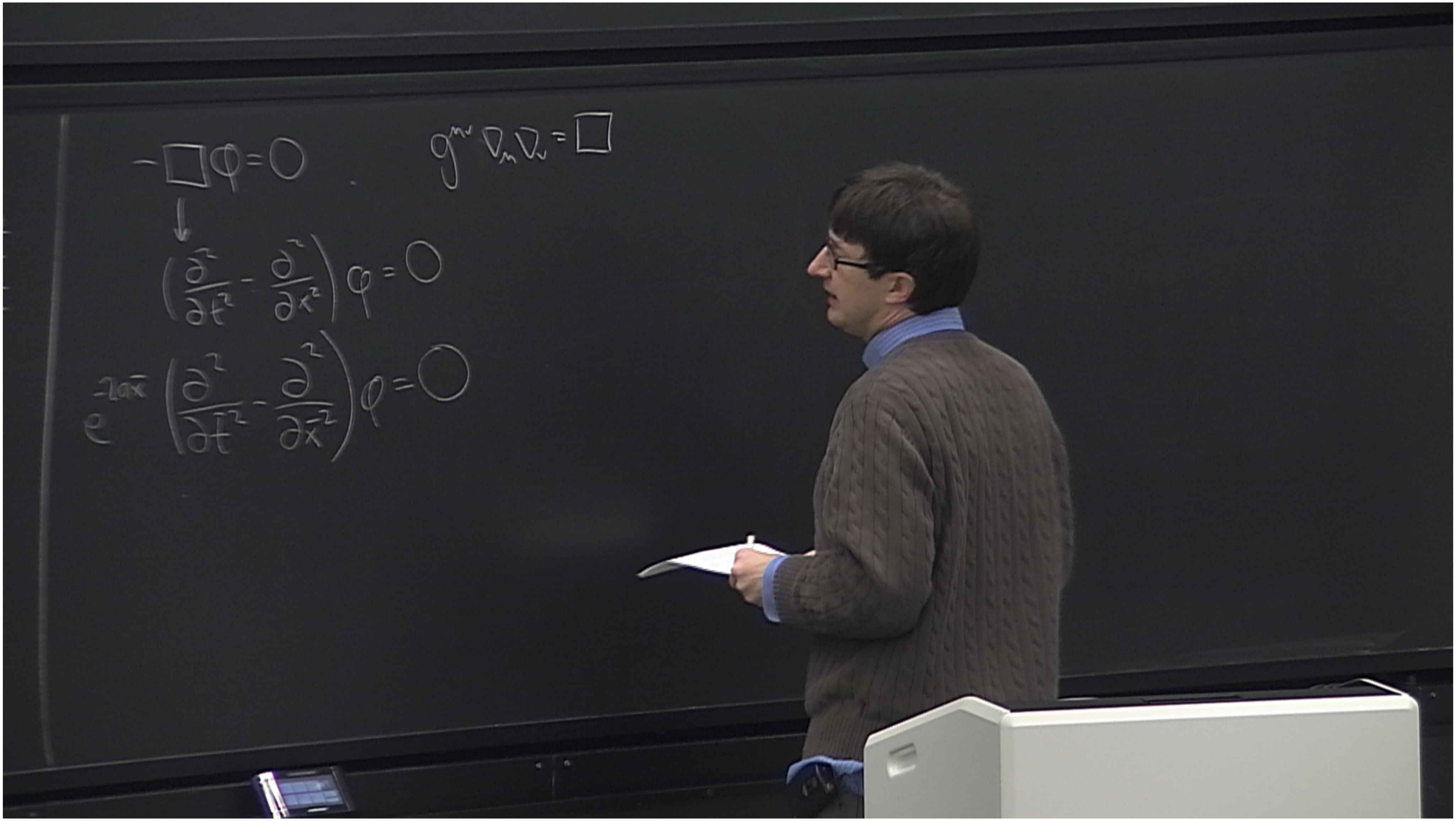
t
x
u_+
u_-
$ds^2 = -dt^2 + dx^2$
$= du_+ du_-$
$= e^{a(\bar{u}_+ + \bar{u}_-)} d\bar{u}_+ d\bar{u}_-$
$= e^{2a\bar{x}}(-d\bar{t}^2 + d\bar{x}^2)$
$u_\pm = x \pm t = \frac{1}{a} e^{a\bar{u}_\pm}$
$du_\pm = dx \pm dt = e^{a\bar{u}_\pm} d\bar{u}_\pm$
$\bar{u}_\pm = \bar{x} \pm \bar{t} \leftarrow \bar{x}, \bar{t}$

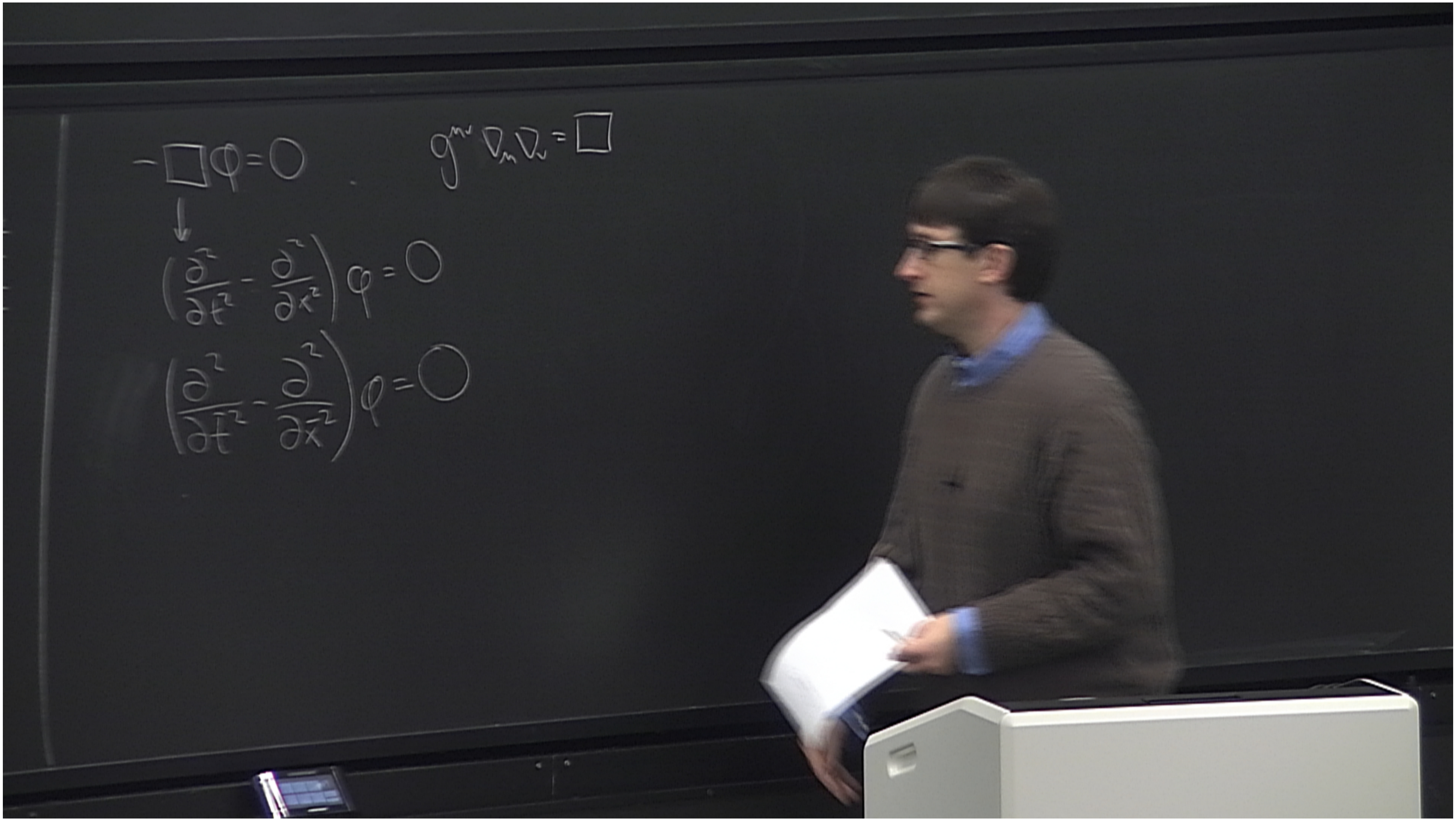

t
x
u+
ds̄² = -dt̄² + dx̄²
= du+du-
u± = x ± t = (1/a) e^{aū±}
du± = dx ± dt = e^{aū±} dū±
ū± = x̄ ± t̄ ← x̄, t̄

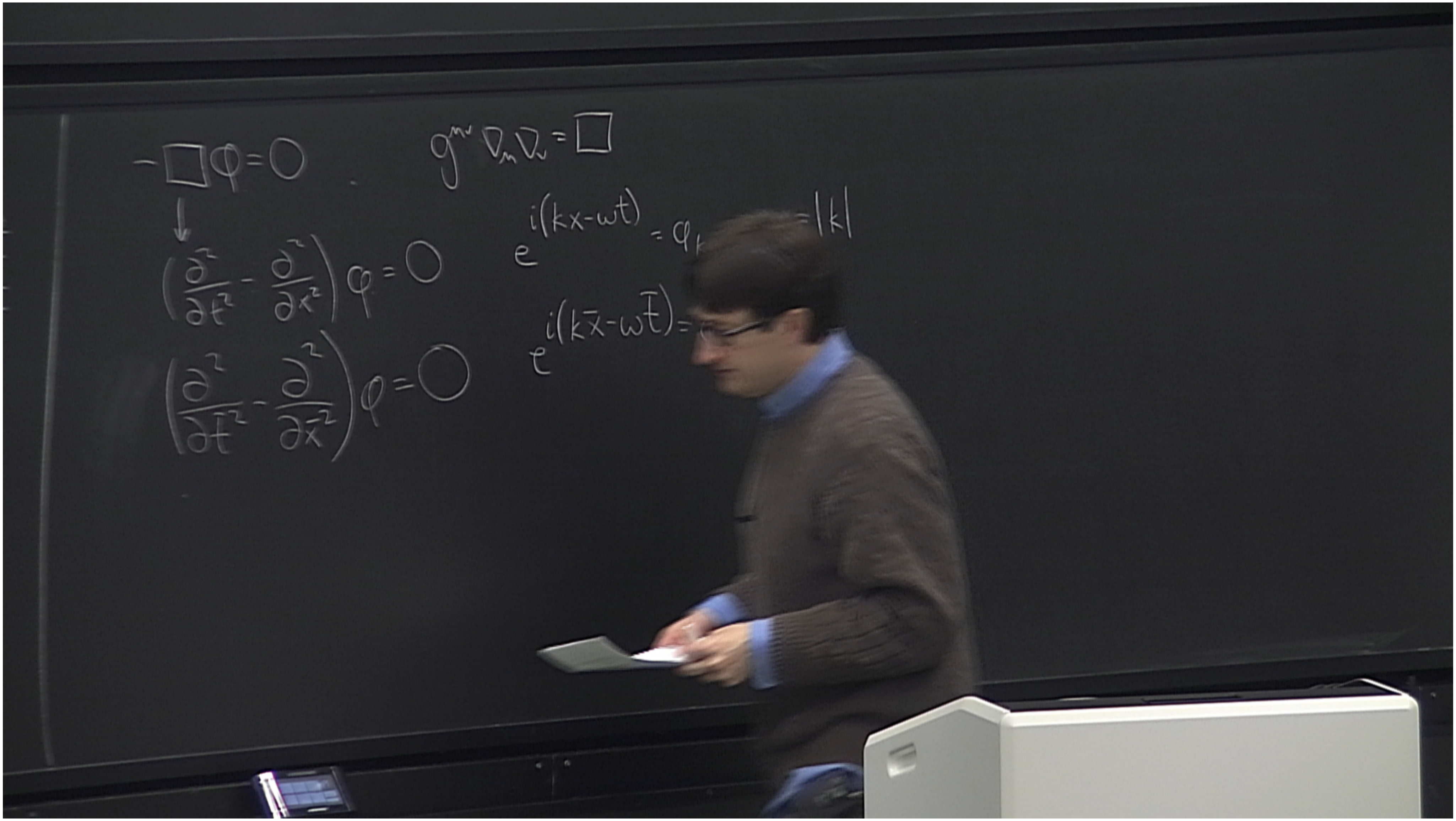

t
x
u+
u-
ds² = -dt² + dx²
= du+ du-
= e^{a(ū+ + ū-)} d
= e^{2ax̄}(-dt̄
u± = x ± t = (1/a) e^{aū±}
du± = dx ± dt = e^{aū±} dū±
ū± = x̄ ± t̄ ← x̄, t̄


t
u_+
x
u_-
$ds^2 = -dt^2 + dx^2$
$= du_+ du_-$
$= e^{a(\bar{u}_+ + \bar{u}_-)} d\bar{u}_+ d\bar{u}_-$
$= e^{2a\bar{x}}(-d\bar{t}^2 + d\bar{x}^2)$
$u_\pm = x \pm t = \frac{1}{a} e^{a\bar{u}_\pm}$
$du_\pm = dx \pm dt = e^{a\bar{u}_\pm} d\bar{u}_\pm$
$\bar{u}_\pm = \bar{x} \pm \bar{t} \leftarrow \bar{x}, \bar{t}$

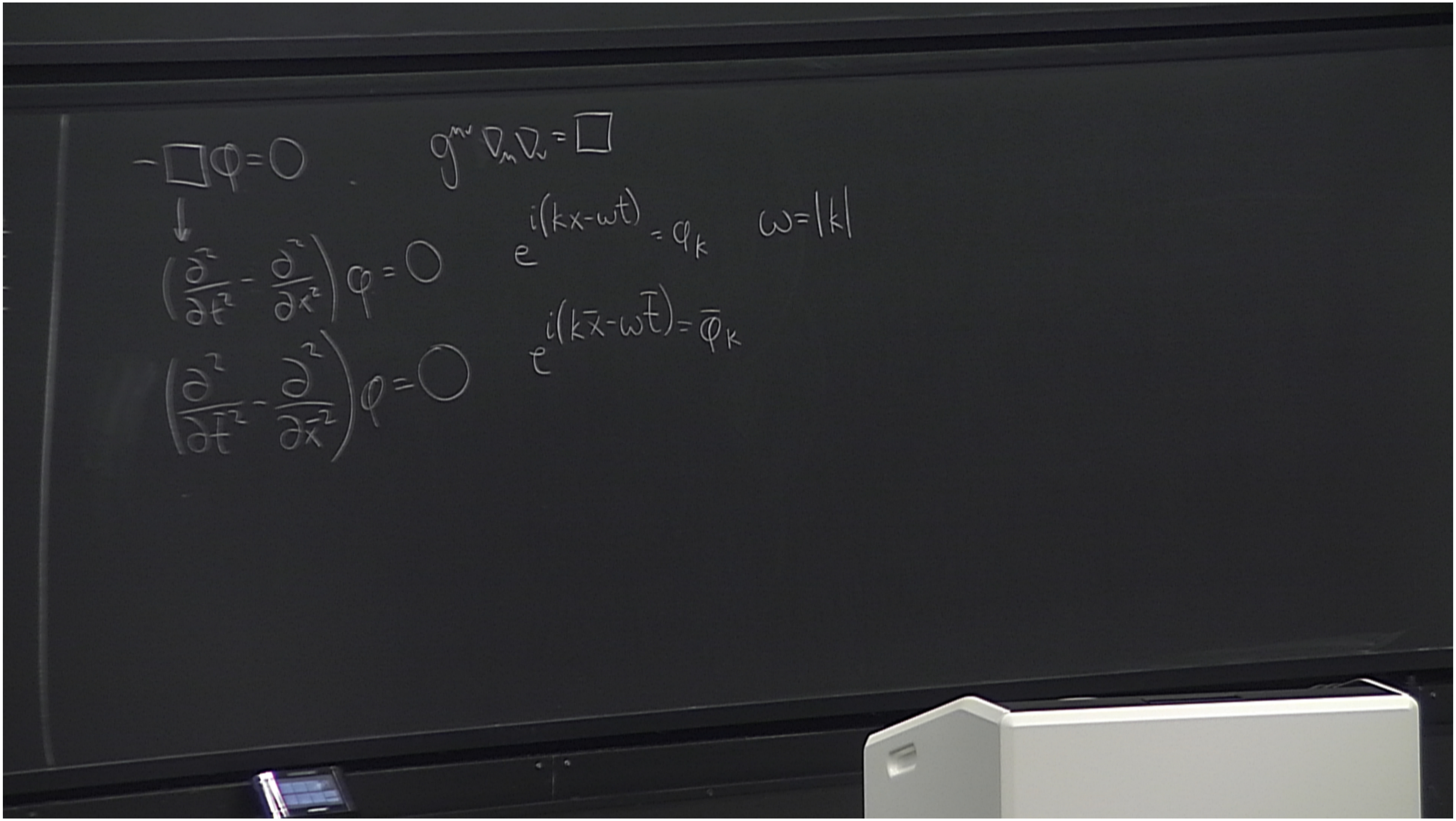

$-\Box\varphi = 0$
$g^{\mu\nu}\nabla_\mu\nabla_\nu = \Box$
$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\varphi$

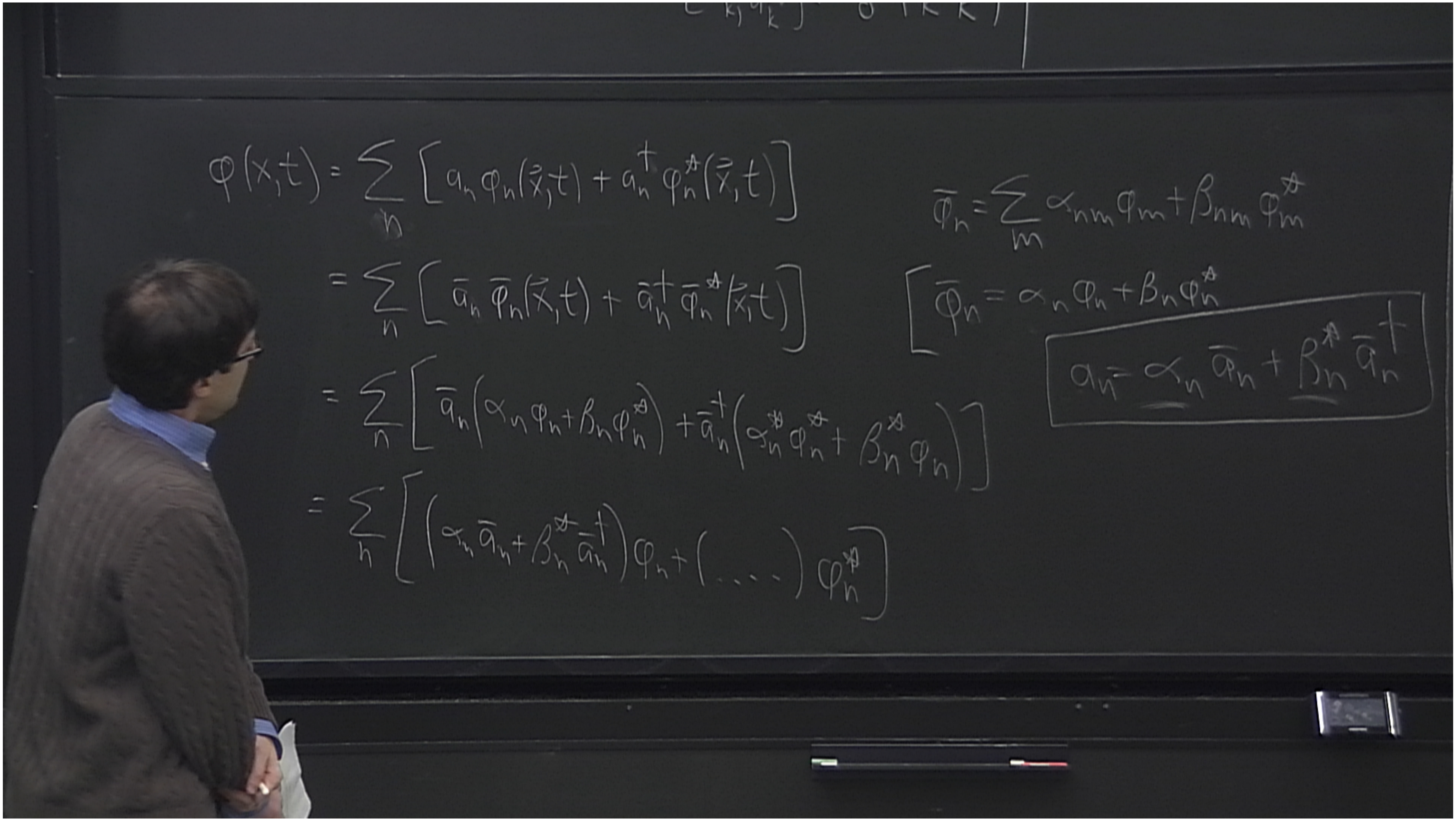

$-\Box\varphi = 0$
$g^{\mu\nu}\nabla_\mu\nabla_\nu = \Box$
$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\varphi = 0$
$e^{-2a\bar{x}}\left(\frac{\partial^2}{\partial \bar{t}^2} - \frac{\partial^2}{\partial \bar{x}^2}\right)\varphi = 0$

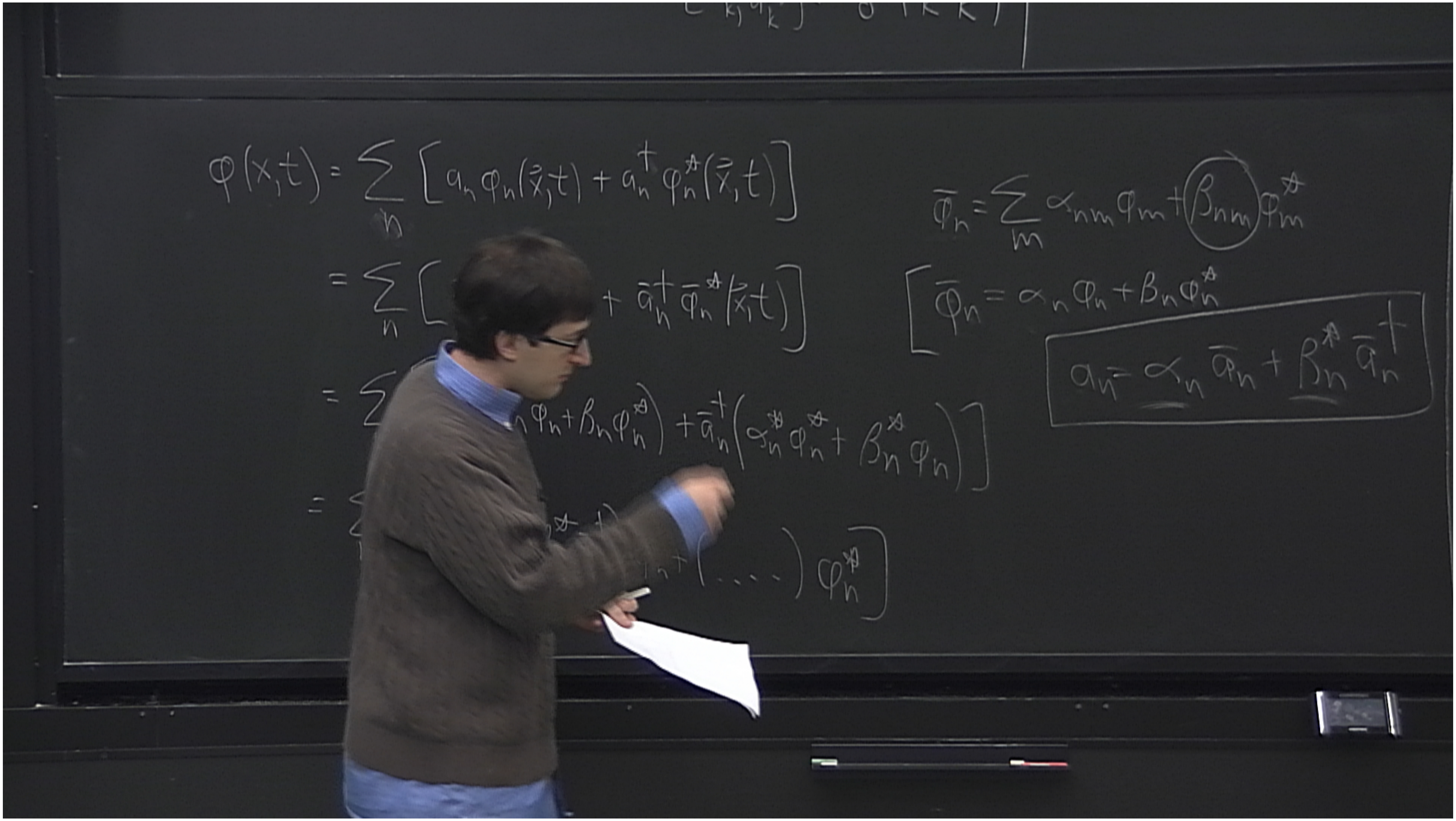

$-\Box\varphi=0$
$g^{\mu\nu}\nabla_\mu\nabla_\nu=\Box$
$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)\varphi=0$
$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right)\varphi=0$



−□φ=0
g^{μν}∇_μ∇_ν=□
(∂²/∂t² − ∂²/∂x²)φ=0
(∂²/∂t̄² − ∂²/∂x̄²)φ=0
e^{i(kx−ωt)}
e^{i(kx̄−ωt̄)}


$-\Box\varphi = 0$
$g^{\mu\nu}\nabla_\mu\nabla_\nu = \Box$
$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\varphi = 0$
$e^{i(kx-\omega t)} = \varphi_k$
$\omega = |k|$
$e^{i(k\bar{x}-\omega\bar{t})} = \bar{\varphi}_k$
$\left(\frac{\partial^2}{\partial \bar{t}^2} - \frac{\partial^2}{\partial \bar{x}^2}\right)\varphi = 0$


φ(x,t) = Σ_n [a_n φ_n(x,t) + a_n† φ_n*(x,t)]
= Σ_n [ā_n φ̄_n(x,t) + ā_n† φ̄_n*(x,t)]
= Σ_n [ā_n(α_n φ_n + β_n φ_n*) + ā_n†(α_n* φ_n* + β_n* φ_n)]
= Σ_n [(α_n ā_n + β_n* ā_n†) φ_n + (. . . .) φ_n*]
φ̄_n = Σ_m α_nm φ_m + β_nm φ_m*
[φ̄_n = α_n φ_n + β_n φ_n*
a_n = α_n ā_n + β_n* ā_n†


$\varphi(x,t) = \sum_n [a_n \varphi_n(\bar{x},t) + a_n^\dagger \varphi_n^*(\bar{x},t)]$
$\bar{\varphi}_n = \sum_m \alpha_{nm}\varphi_m + \beta_{nm}\varphi_m^*$
$\bar{\varphi}_n = \alpha_n \varphi_n + \beta_n \varphi_n^*$
$a_n = \alpha_n \bar{a}_n + \beta_n^* \bar{a}_n^\dagger$

$$-\Box\varphi = 0 \qquad g^{\mu\nu}\nabla_\mu\nabla_\nu = \Box$$

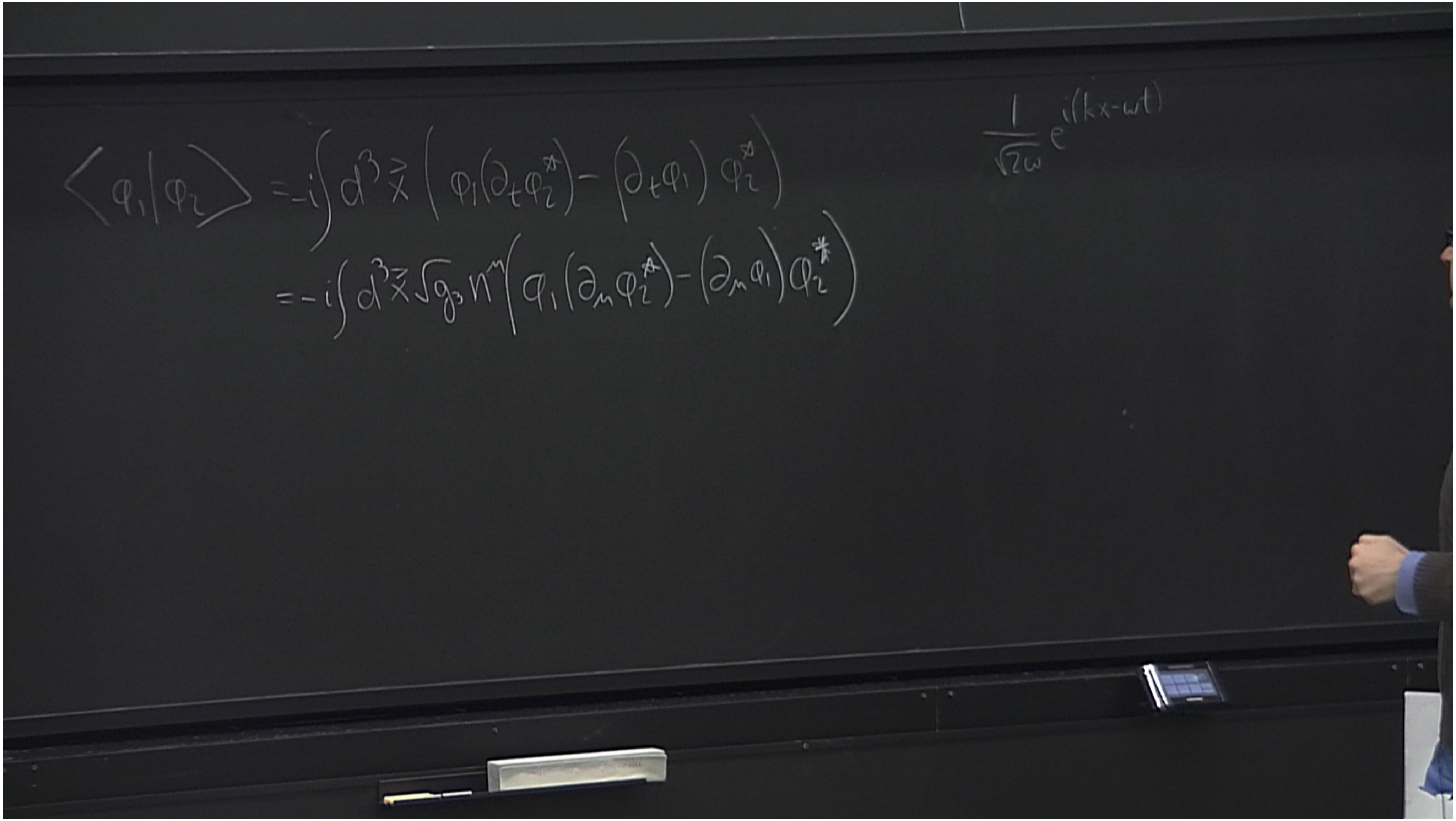
$\downarrow$

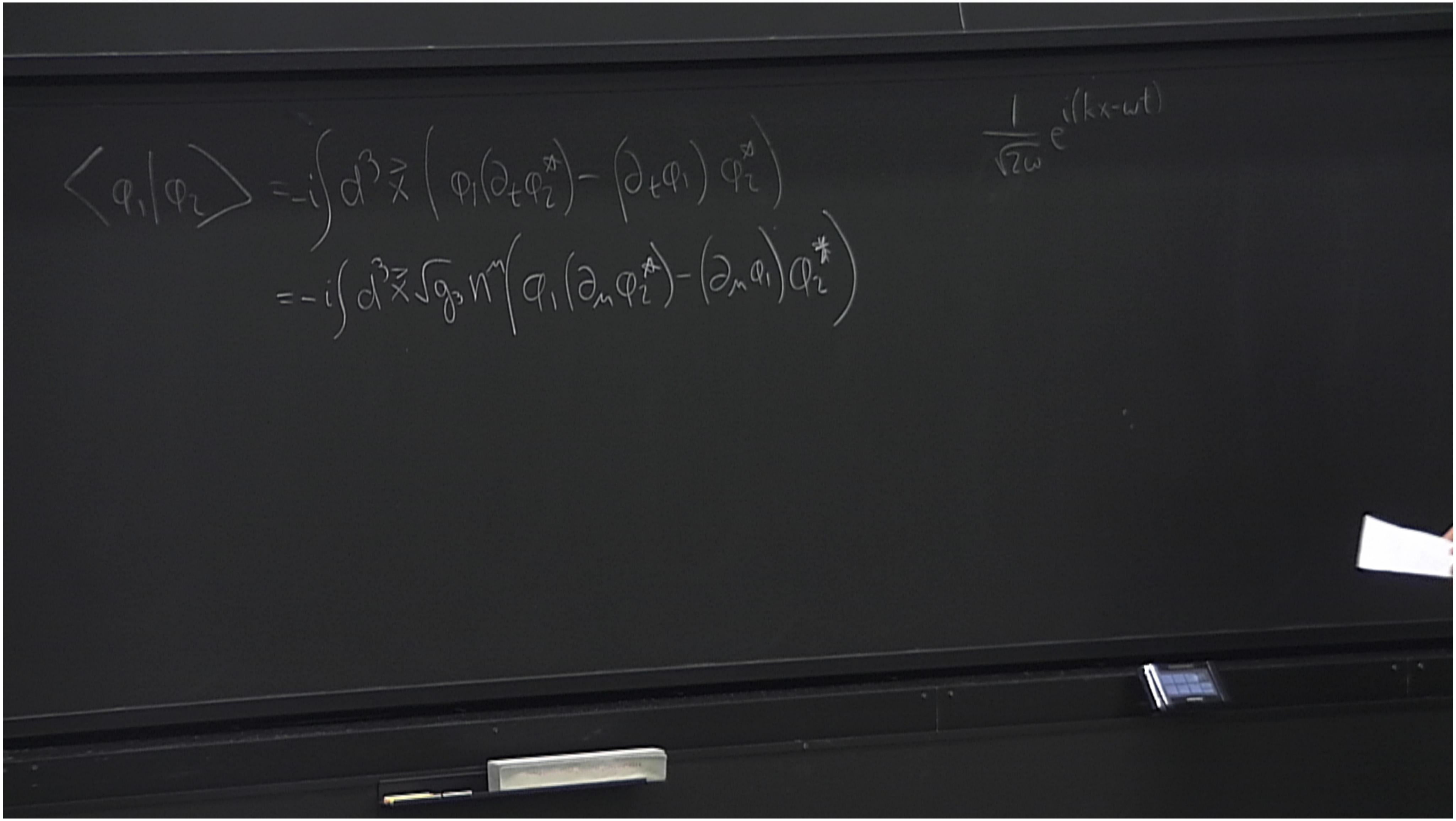
$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\varphi = 0 \qquad e^{i(kx-\omega t)} = \varphi_k \qquad \omega = |k|$$

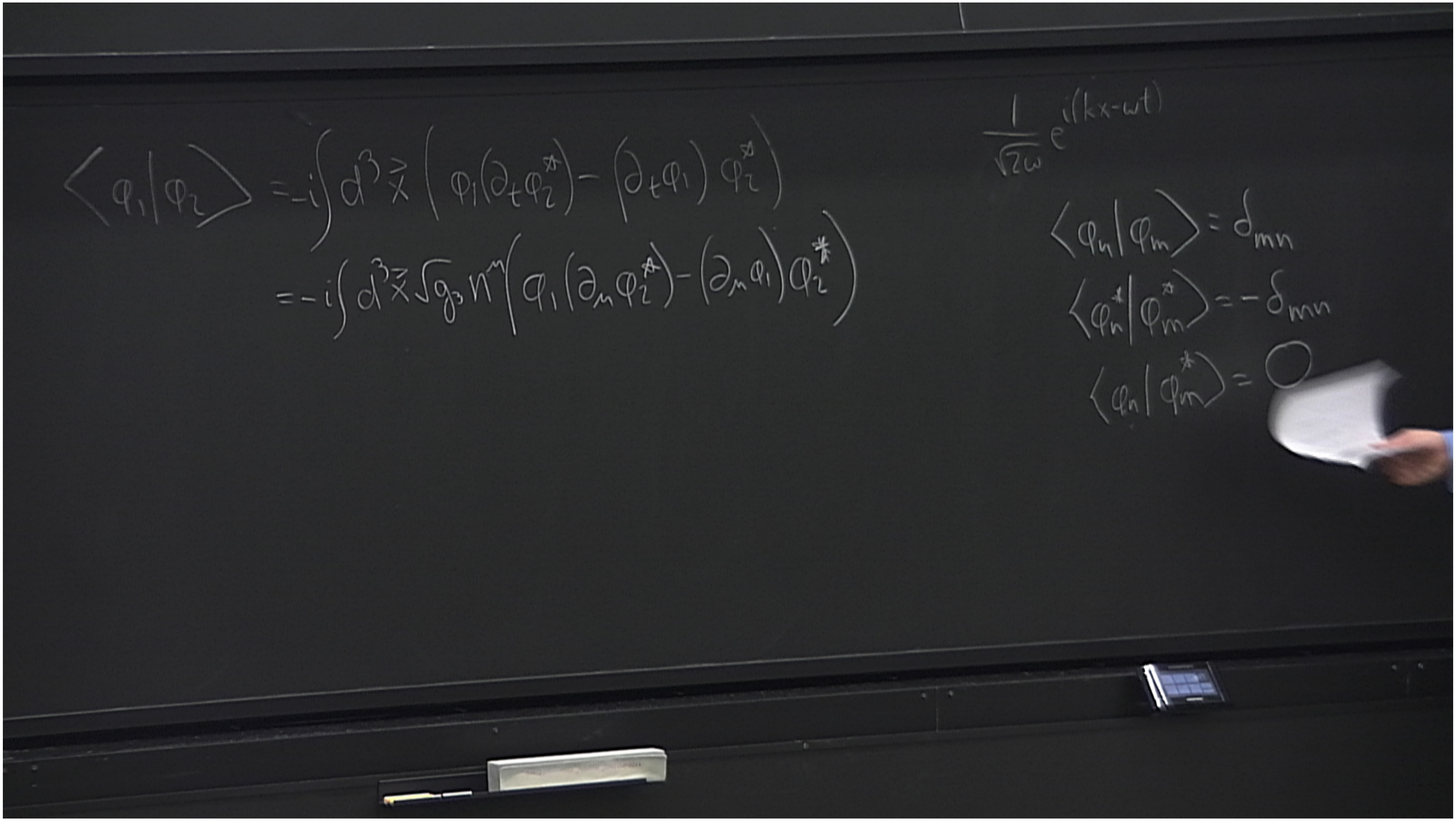
$$\left(\frac{\partial^2}{\partial \bar{t}^2} - \frac{\partial^2}{\partial \bar{x}^2}\right)\varphi = 0 \qquad e^{i(k\bar{x}-\omega \bar{t})} = \bar{\varphi}_k$$

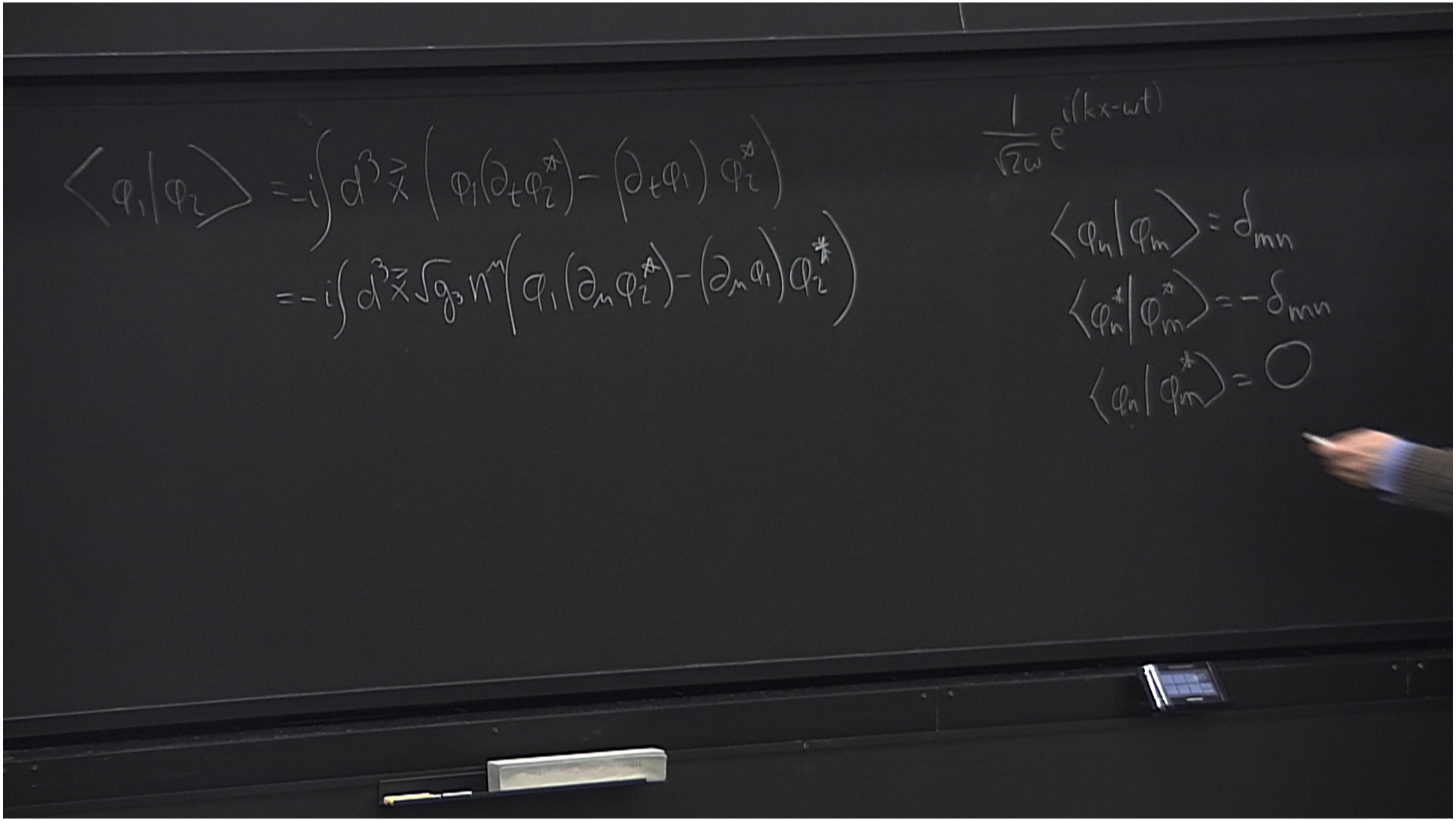
$$\langle \varphi_1 | \varphi_2 \rangle = -i \int d^3\vec{x} \left(\varphi_1 (\partial_t \varphi_2^*) - (\partial_t \varphi_1) \varphi_2^* \right)$$

$$= -i \int d^3\vec{x} \sqrt{g_3}\, n^\mu \left(\varphi_1 (\partial_\mu \varphi_2^*) - (\partial_\mu \varphi_1) \varphi_2^* \right)$$


$\langle \varphi_1 | \varphi_2 \rangle = -i \int d^3\vec{x} \left(\varphi_1 (\partial_t \varphi_2^*) - (\partial_t \varphi_1) \varphi_2^* \right)$
$= -i \int d^3\vec{x} \sqrt{g_3}\, n^\mu \left(\varphi_1 (\partial_\mu \varphi_2^*) - (\partial_\mu \varphi_1) \varphi_2^* \right)$
$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$


$\langle \varphi_1 | \varphi_2 \rangle = -i \int d^3\vec{x} \left(\varphi_1 (\partial_t \varphi_2^*) - (\partial_t \varphi_1) \varphi_2^* \right)$
$= -i \int d^3\vec{x} \sqrt{g_3}\, n^\mu \left(\varphi_1 (\partial_\mu \varphi_2^*) - (\partial_\mu \varphi_1) \varphi_2^* \right)$
$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$


$\langle \varphi_1 | \varphi_2 \rangle = -i \int d^3x \left(\varphi_1 (\partial_t \varphi_2^*) - (\partial_t \varphi_1) \varphi_2^* \right)$
$= -i \int d^3x \sqrt{g_3}\, n^\mu \left(\varphi_1 (\partial_\mu \varphi_2^*) - (\partial_\mu \varphi_1) \varphi_2^* \right)$
$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_n | \varphi_m \rangle = \delta_{mn}$
$\langle \varphi_n^* | \varphi_m^* \rangle = -\delta_{mn}$
$\langle \varphi_n | \varphi_m^* \rangle = 0$



$\langle \varphi_1 | \varphi_2 \rangle = -i \int d^3\vec{x} \left(\varphi_1 (\partial_t \varphi_2^*) - (\partial_t \varphi_1) \varphi_2^* \right)$
$= -i \int d^3\vec{x} \sqrt{g_3}\, n^\mu \left(\varphi_1 (\partial_\mu \varphi_2^*) - (\partial_\mu \varphi_1) \varphi_2^* \right)$
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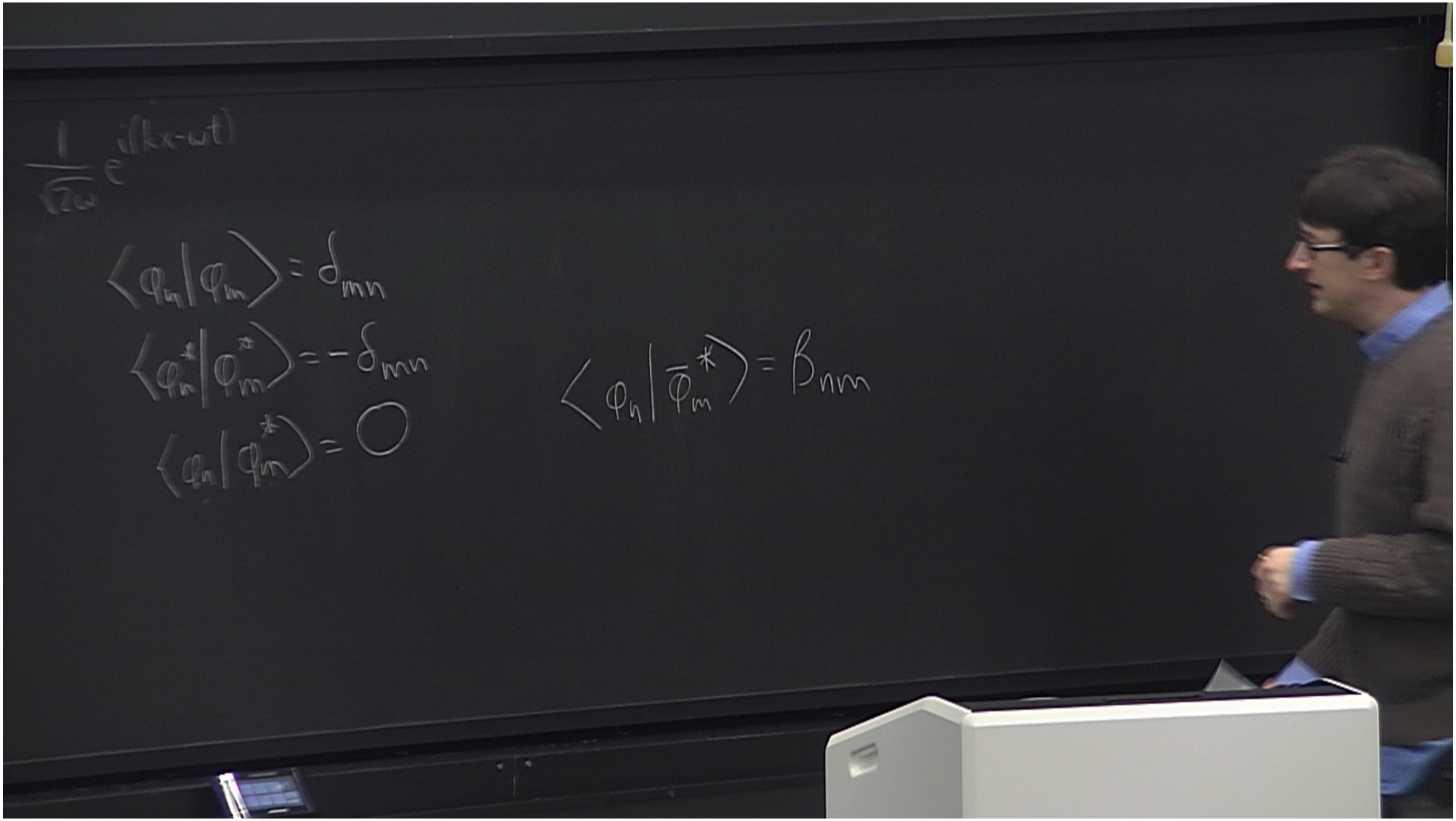
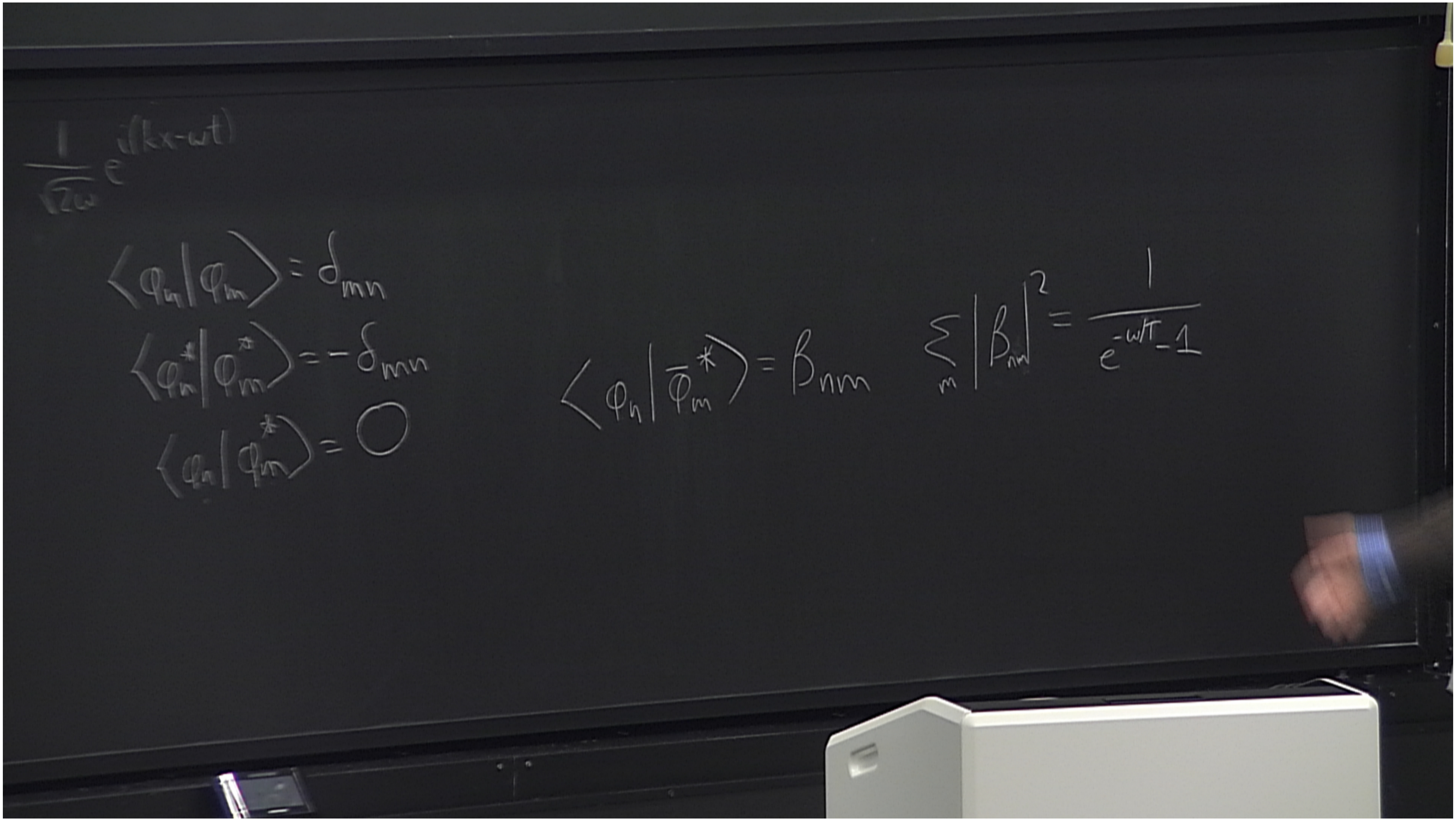
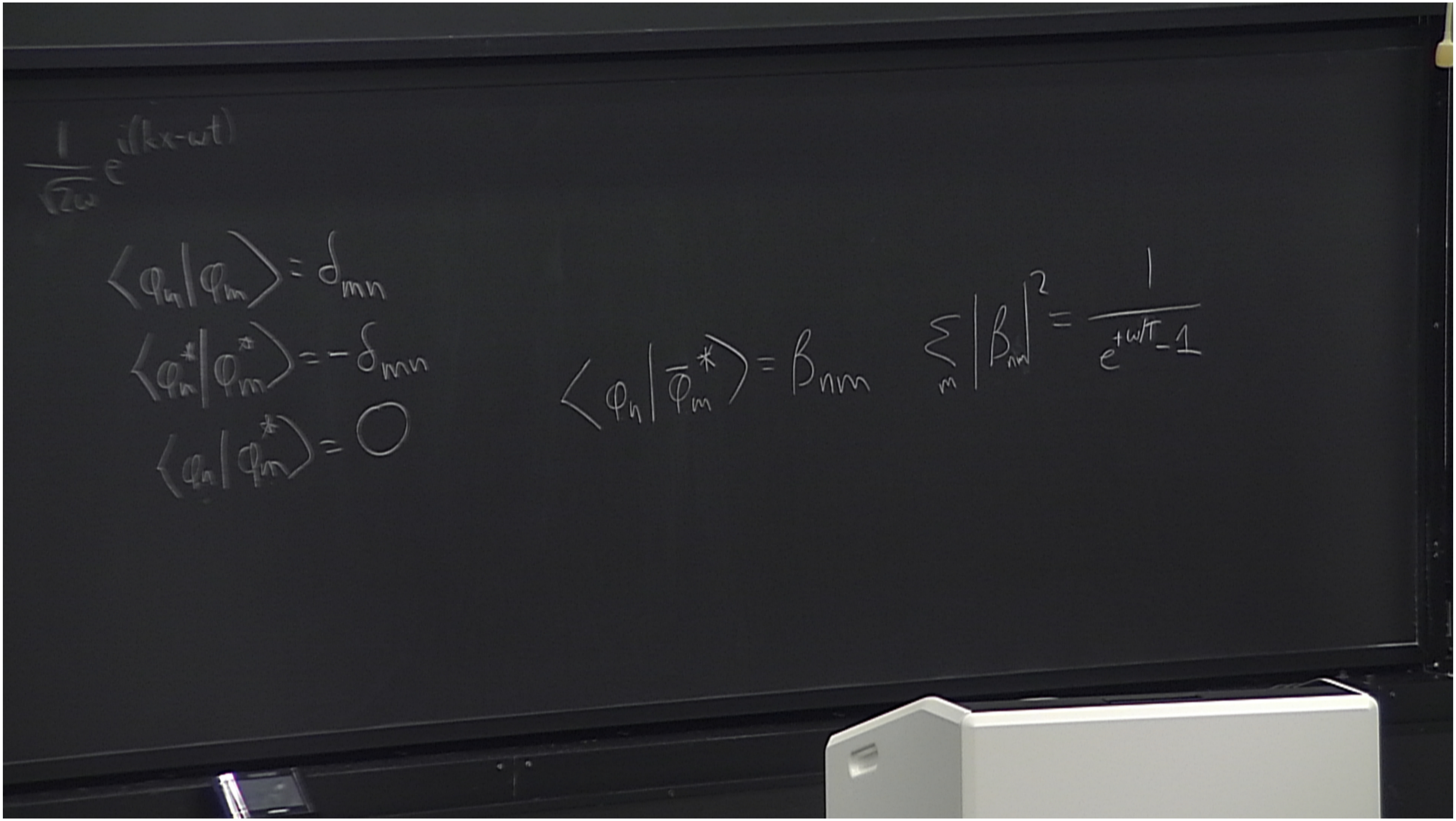
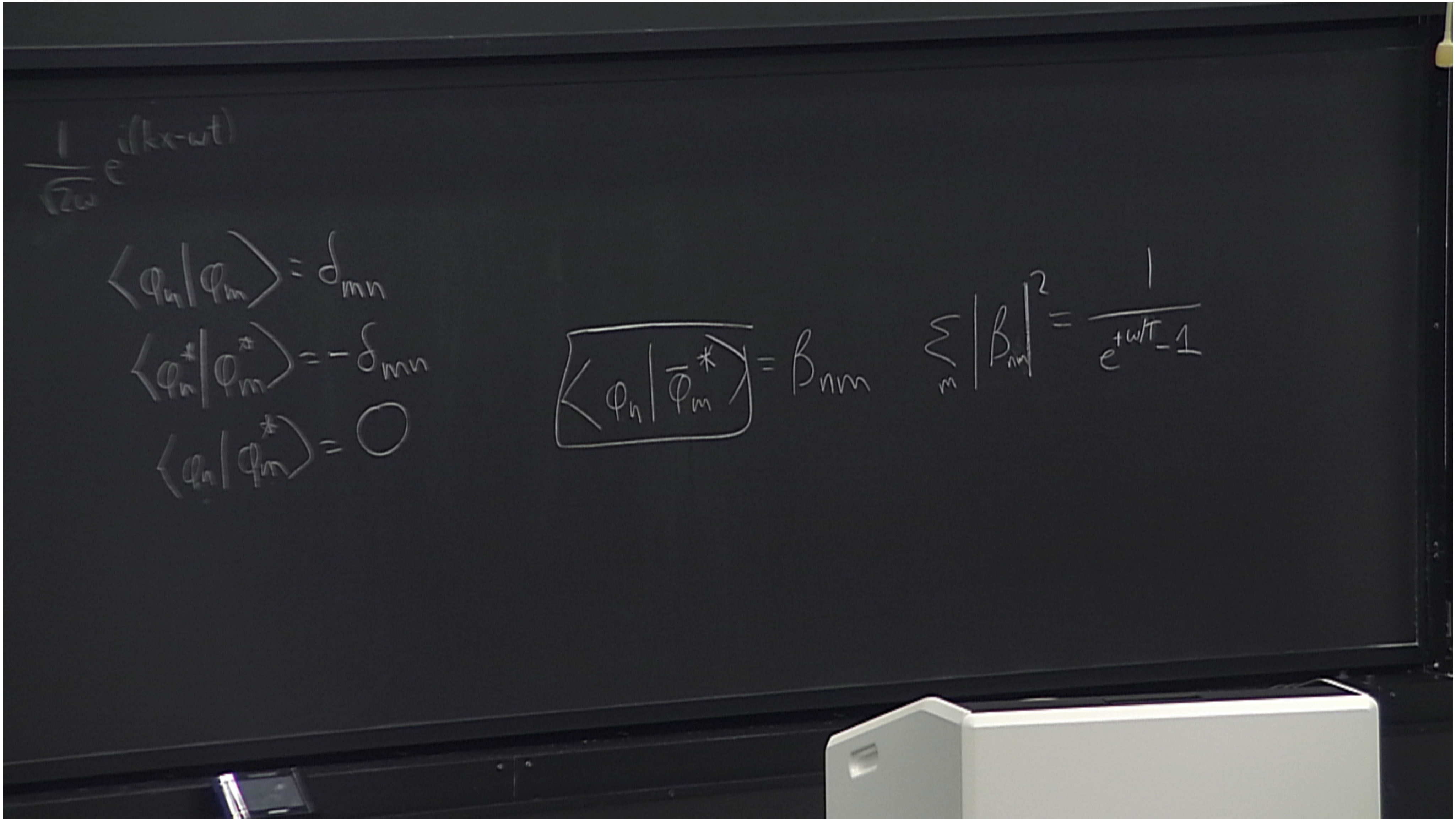
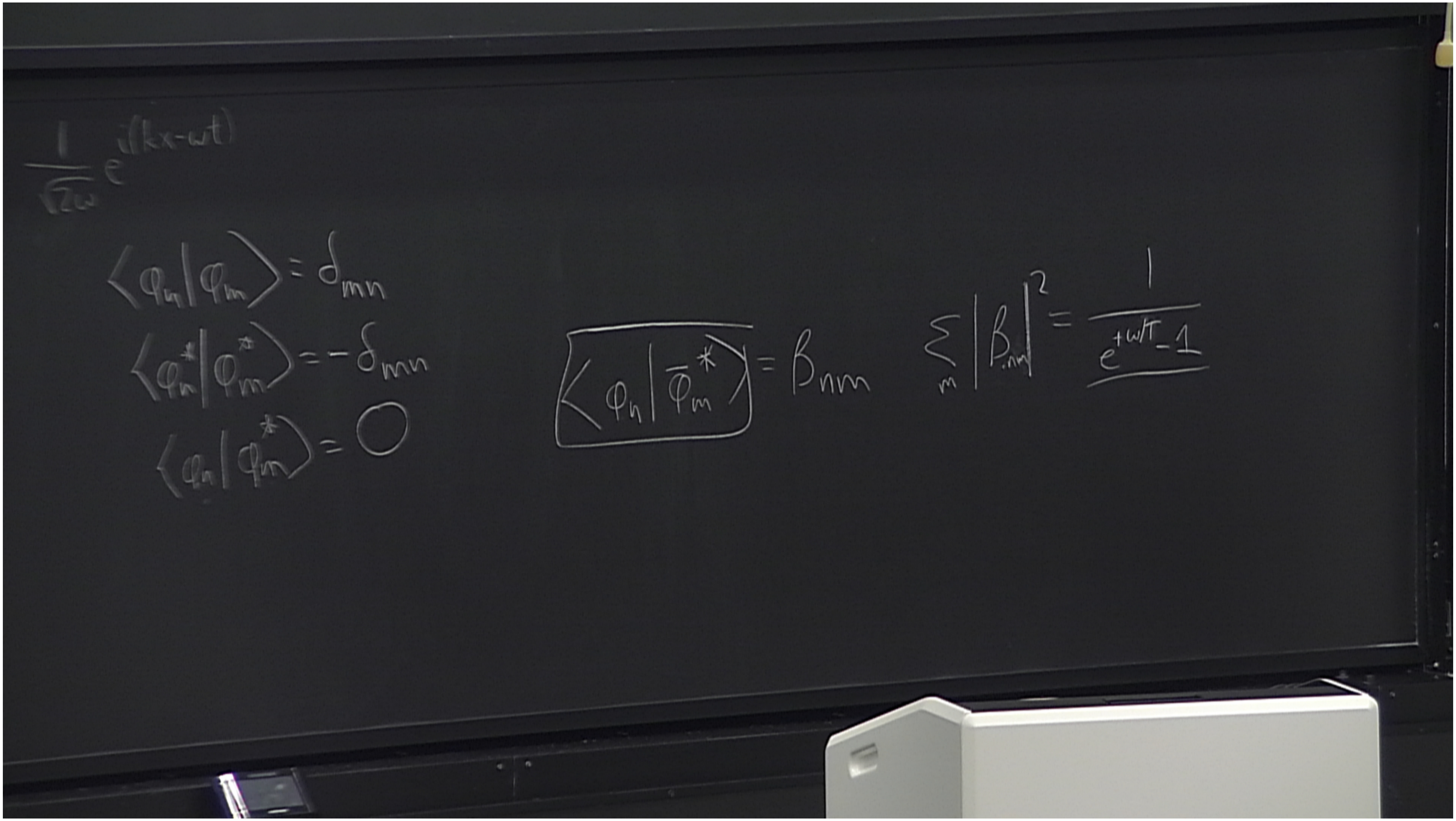
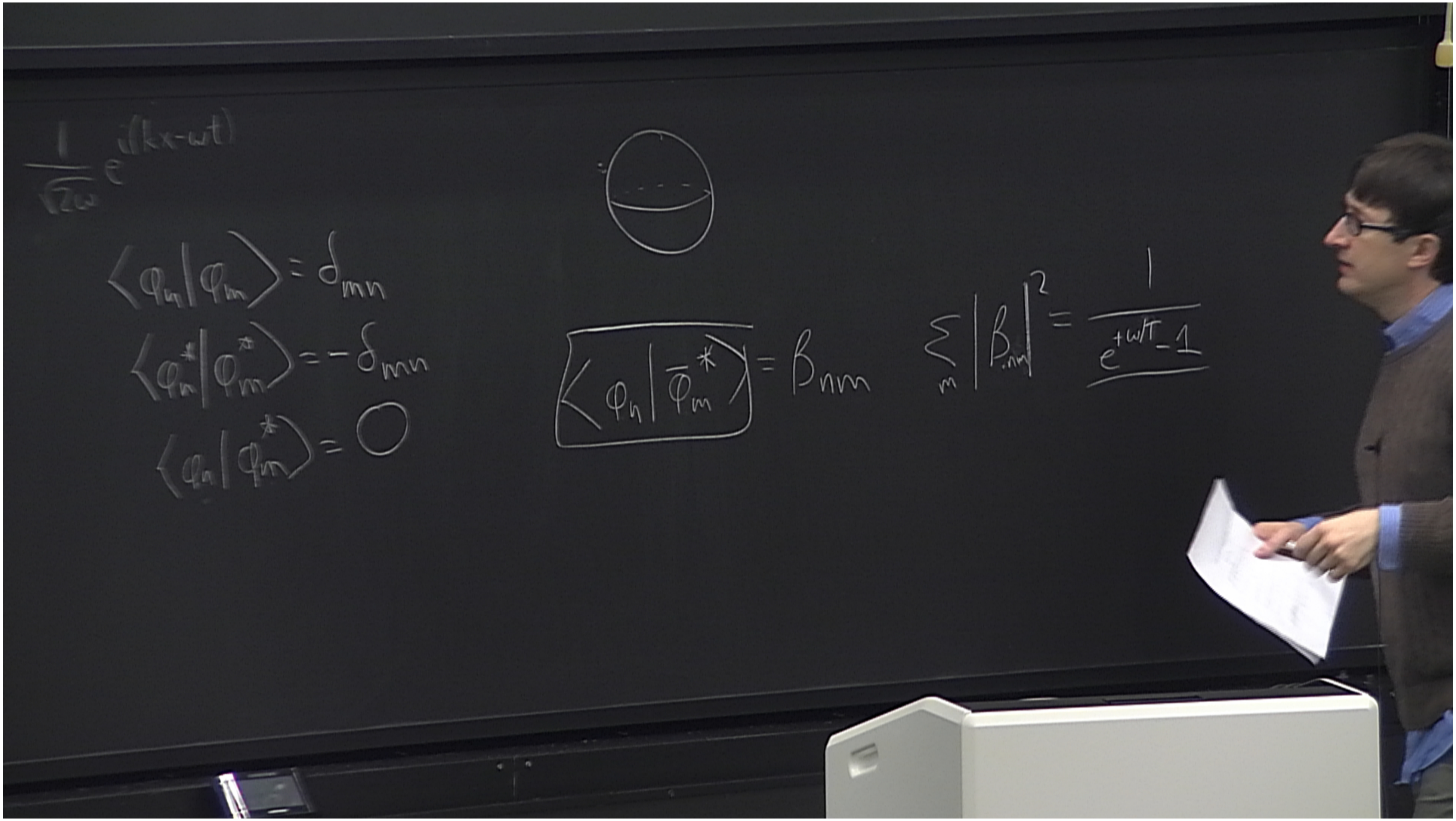

$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_n | \varphi_m \rangle = \delta_{mn}$
$\langle \varphi_n^* | \varphi_m^* \rangle = -\delta_{mn}$
$\langle \varphi_n | \varphi_m^* \rangle = 0$
$\langle \varphi_n | \bar{\varphi}_m^* \rangle = \beta_{nm}$

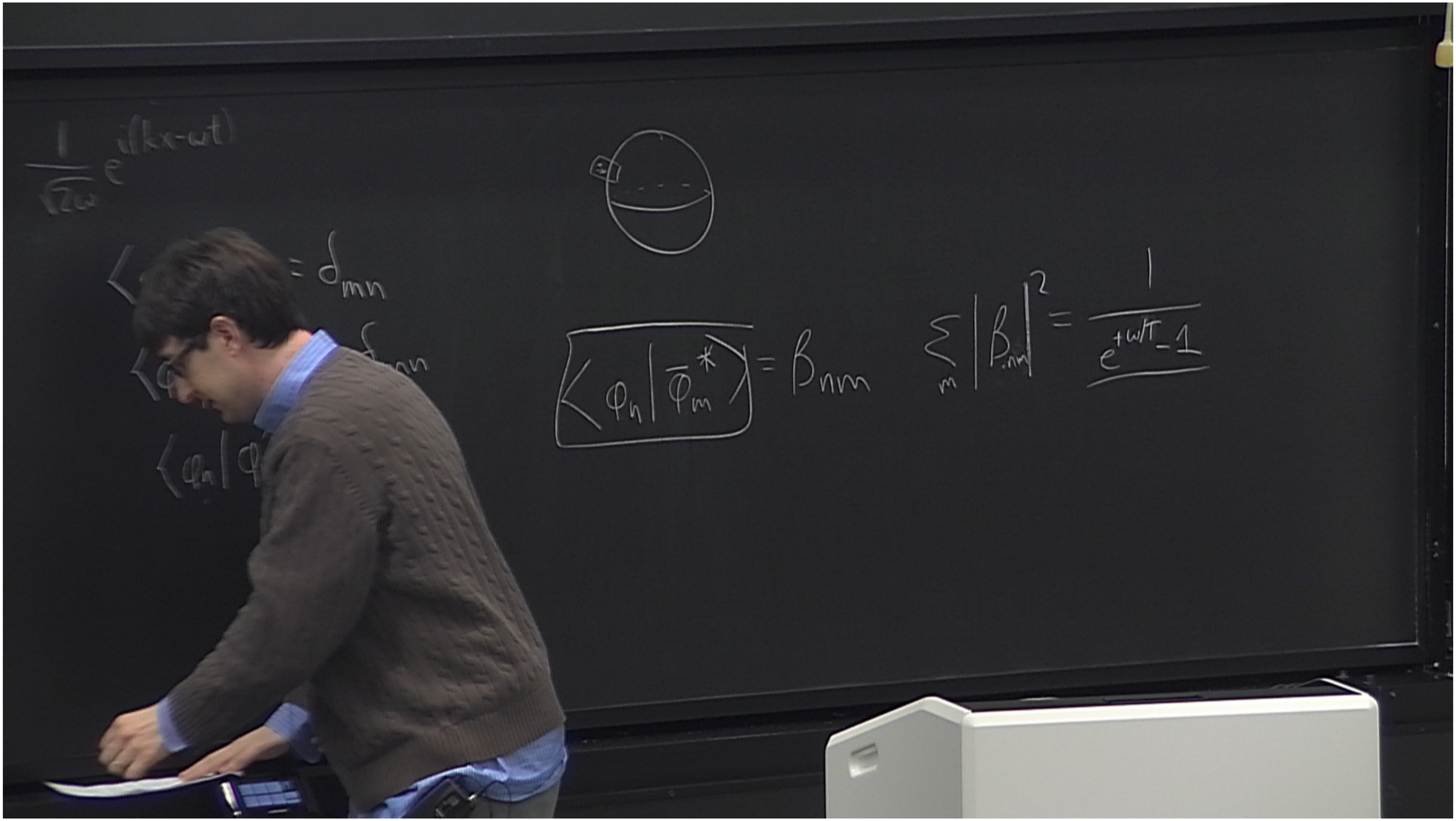

$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_n | \varphi_m \rangle = \delta_{mn}$
$\langle \varphi_m^* | \varphi_m^* \rangle = -\delta_{mn}$
$\langle \varphi_n | \varphi_m^* \rangle = 0$
$\langle \varphi_n | \bar{\varphi}_m^* \rangle = \beta_{nm}$
$\sum_m |\beta_{nm}|^2 = \frac{1}{e^{-\omega/T} - 1}$

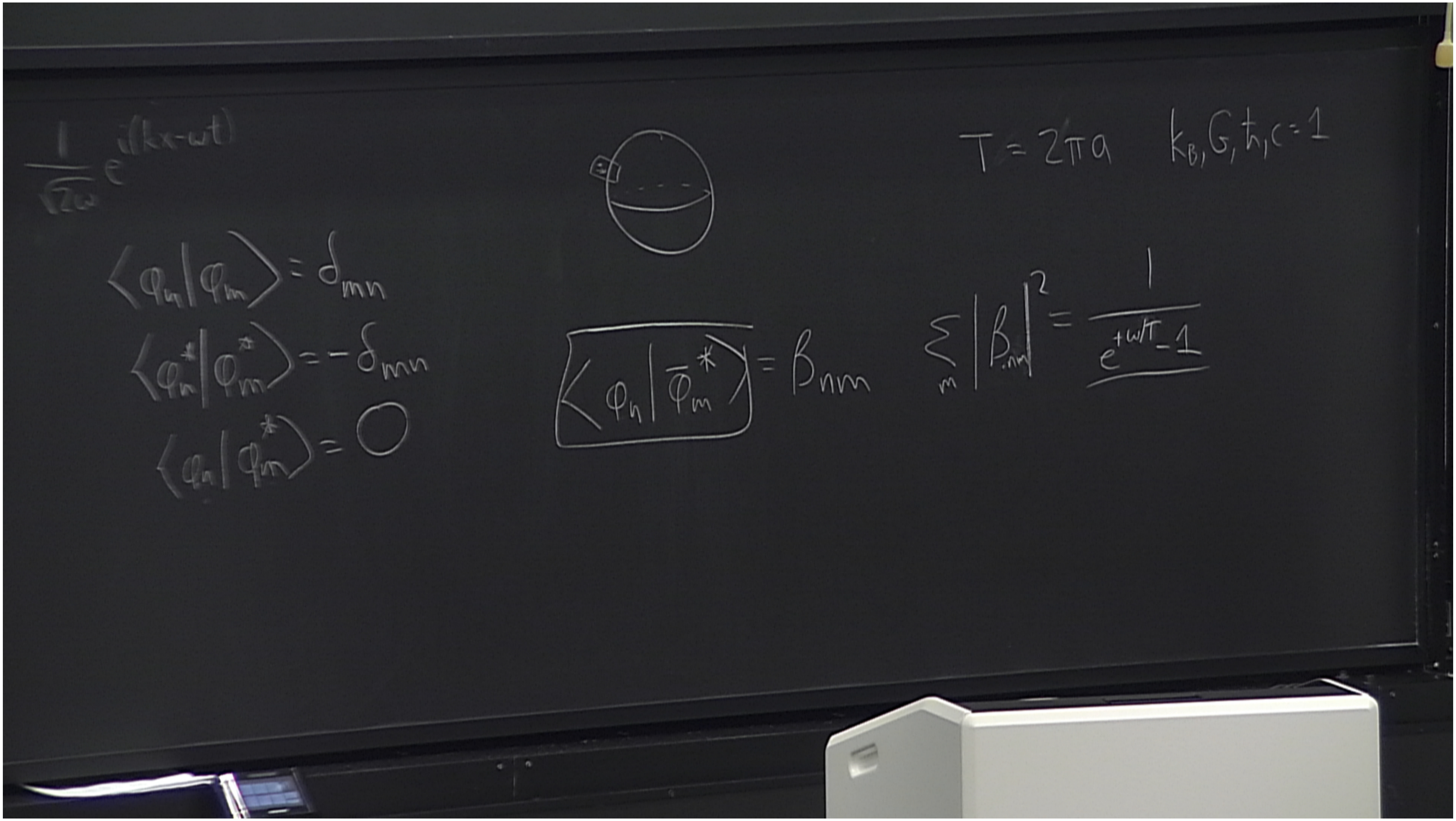

$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_n | \varphi_m \rangle = \delta_{mn}$
$\langle \varphi_m^* | \varphi_m^* \rangle = -\delta_{mn}$
$\langle \varphi_n | \varphi_m^* \rangle = 0$
$\langle \varphi_n | \bar{\varphi}_m^* \rangle = \beta_{nm}$
$\sum_m |\beta_{nm}|^2 = \frac{1}{e^{+\omega/T}-1}$

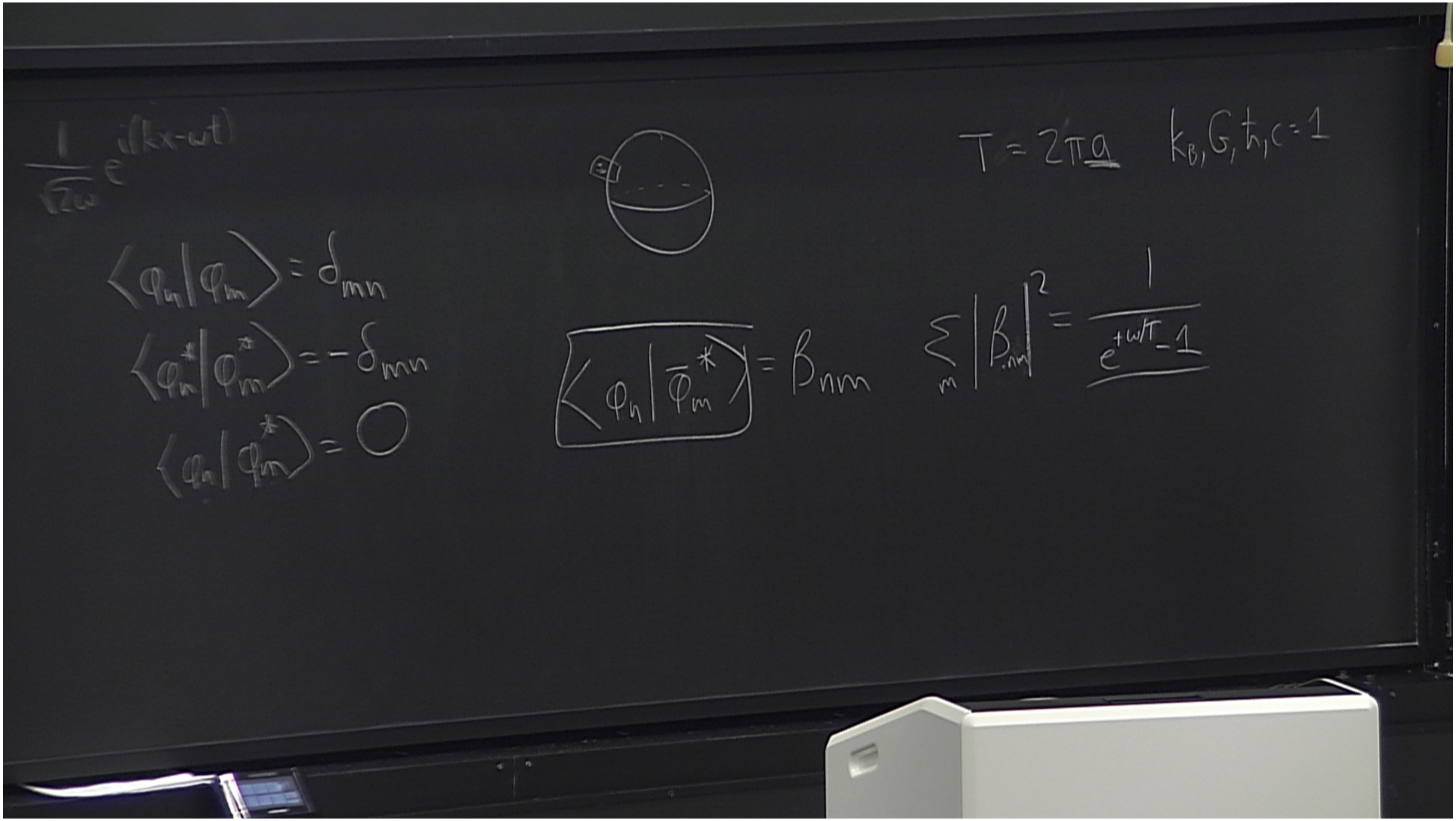

$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_n | \varphi_m \rangle = \delta_{mn}$
$\langle \varphi_n^* | \varphi_m^* \rangle = -\delta_{mn}$
$\langle \varphi_n | \varphi_m^* \rangle = 0$
$\langle \varphi_n | \bar{\varphi}_m^* \rangle = \beta_{nm}$
$\sum_m |\beta_{nm}|^2 = \frac{1}{e^{+\omega/T} - 1}$

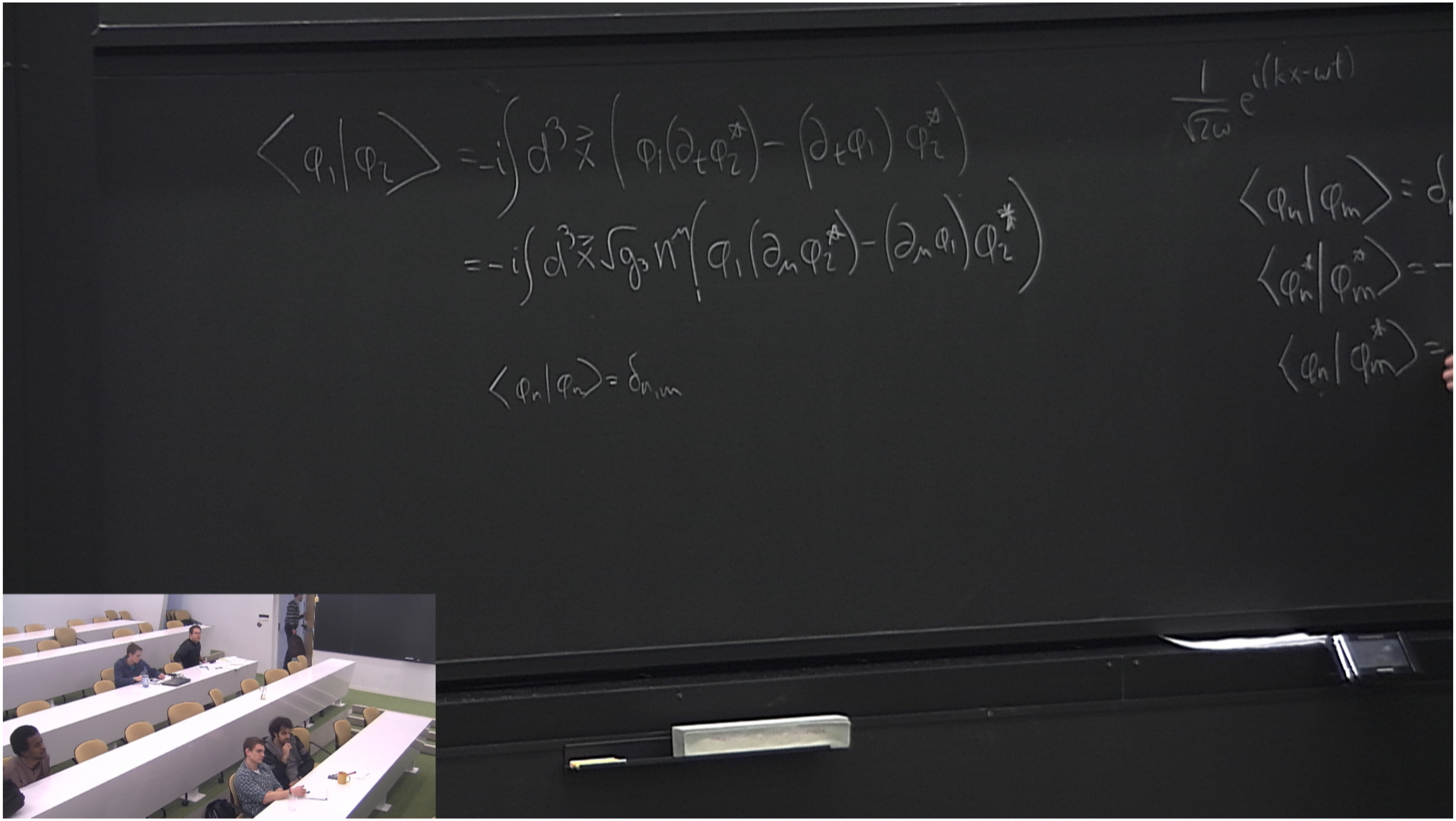

$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_n | \varphi_m \rangle = \delta_{mn}$
$\langle \varphi_m^* | \varphi_m^* \rangle = -\delta_{mn}$
$\langle \varphi_n | \varphi_m^* \rangle = 0$
$\langle \varphi_n | \bar{\varphi}_m^* \rangle = \beta_{nm}$
$\sum_m |\beta_{nm}|^2 = \frac{1}{e^{+\omega/T} - 1}$


1/√2ω e^{i(kx-ωt)}
⟨φn|φm⟩ = δmn
⟨φn*|φm*⟩ = −δmn
⟨φn|φm*⟩ = 0
⟨φn|φ̄m*⟩ = βnm
Σm |βnm|² = 1/(e^{+ω/T} − 1)

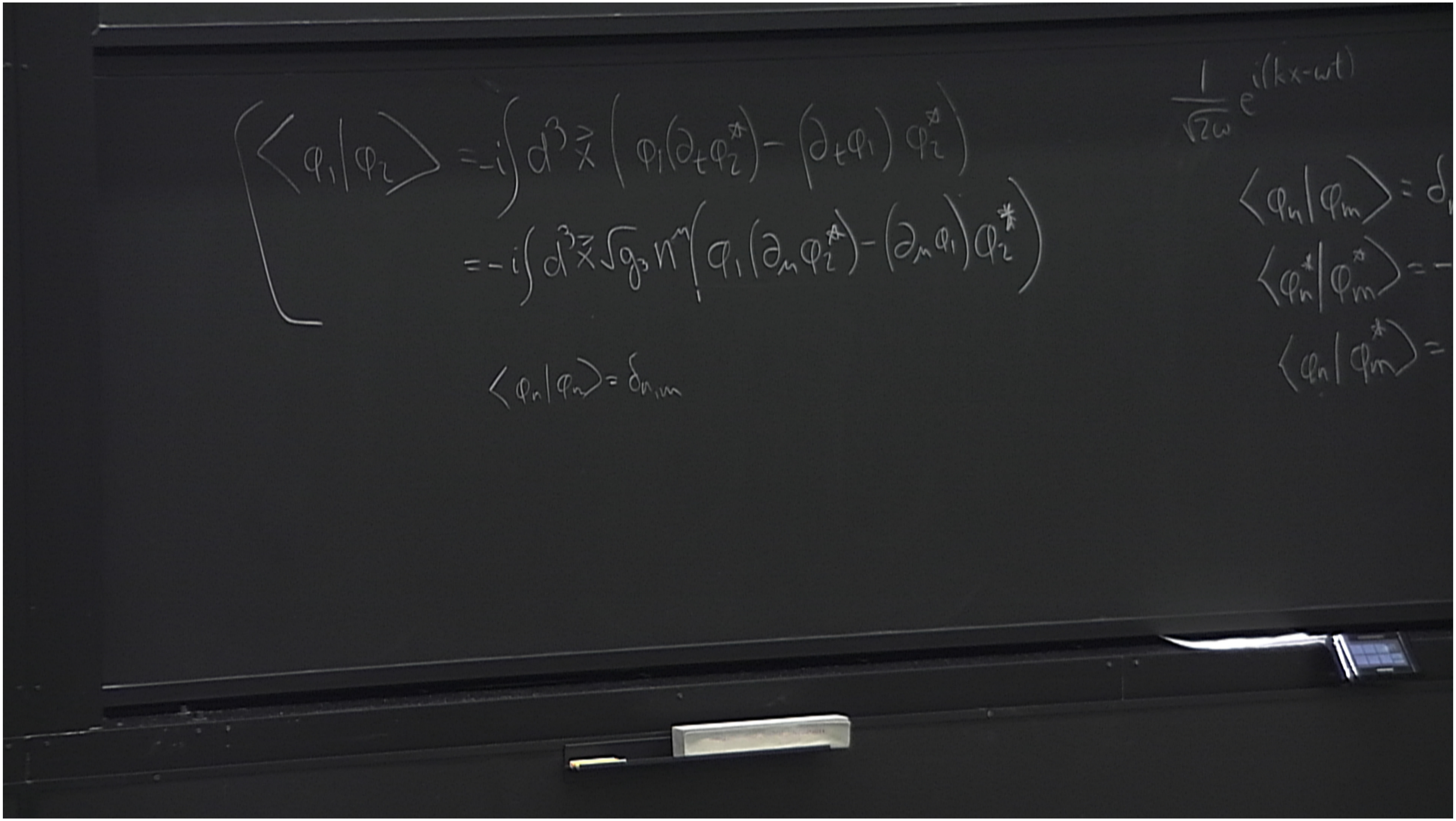

1/√2ω e^{i(kx-ωt)}
= δ_mn
⟨φ_n | φ̄_m^*⟩ = β_nm
Σ_m |β_nm|² = 1/(e^{+ω/T} - 1)


1/√2ω e^{i(kx-ωt)}
T = 2πa
k_B, G, ħ, c = 1
⟨φ_n|φ_m⟩ = δ_mn
⟨φ_m^*|φ_m^*⟩ = -δ_mn
⟨φ_n|φ_m^*⟩ = 0
⟨φ_n|φ̄_m^*⟩ = β_nm
Σ_m |β_nm|^2 = 1/(e^{+ω/T} - 1)


$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$T = 2\pi a$
$k_B, G, \hbar, c = 1$
$\langle \varphi_n | \varphi_m \rangle = \delta_{mn}$
$\langle \varphi_m^* | \varphi_m^* \rangle = -\delta_{mn}$
$\langle \varphi_n | \varphi_m^* \rangle = 0$
$\langle \varphi_n | \bar{\varphi}_m^* \rangle = \beta_{nm}$
$\sum_m |\beta_{nm}|^2 = \frac{1}{e^{+\omega/T} - 1}$



$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_1 | \varphi_2 \rangle = -i \int d^3\vec{x} \left(\varphi_1 (\partial_t \varphi_2^*) - (\partial_t \varphi_1) \varphi_2^* \right)$
$= -i \int d^3\vec{x} \sqrt{g_3}\, n^\mu \left(\varphi_1 (\partial_\mu \varphi_2^*) - (\partial_\mu \varphi_1) \varphi_2^* \right)$
$\langle \varphi_n | \varphi_m \rangle = \delta_{n,m}$
$\langle \varphi_n | \varphi_m \rangle = \delta$
$\langle \varphi_n^* | \varphi_m^* \rangle = -$
$\langle \varphi_n | \varphi_m^* \rangle =$


$\langle \varphi_1 | \varphi_2 \rangle = -i \int d^3\vec{x} \left(\varphi_1 (\partial_t \varphi_2^*) - (\partial_t \varphi_1) \varphi_2^* \right)$
$= -i \int d^3\vec{x} \sqrt{g_3}\, n^\mu \left(\varphi_1 (\partial_\mu \varphi_2^*) - (\partial_\mu \varphi_1) \varphi_2^* \right)$
$\langle \varphi_n | \varphi_m \rangle = \delta_{n,m}$
$\frac{1}{\sqrt{2\omega}} e^{i(kx-\omega t)}$
$\langle \varphi_n | \varphi_m \rangle =$
$\langle \varphi_n^* | \varphi_m^* \rangle =$
$\langle \varphi_n | \varphi_m^* \rangle =$