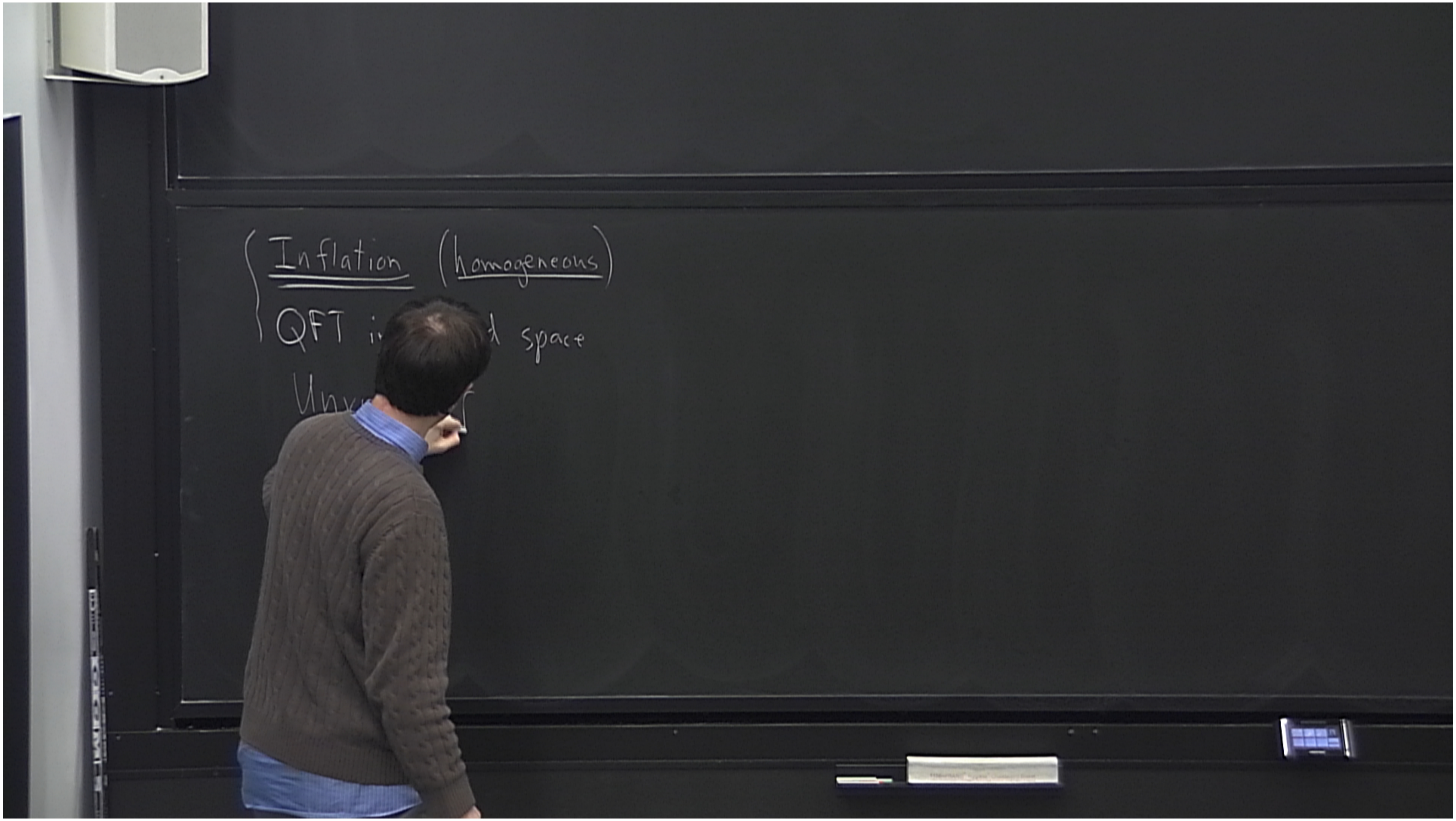
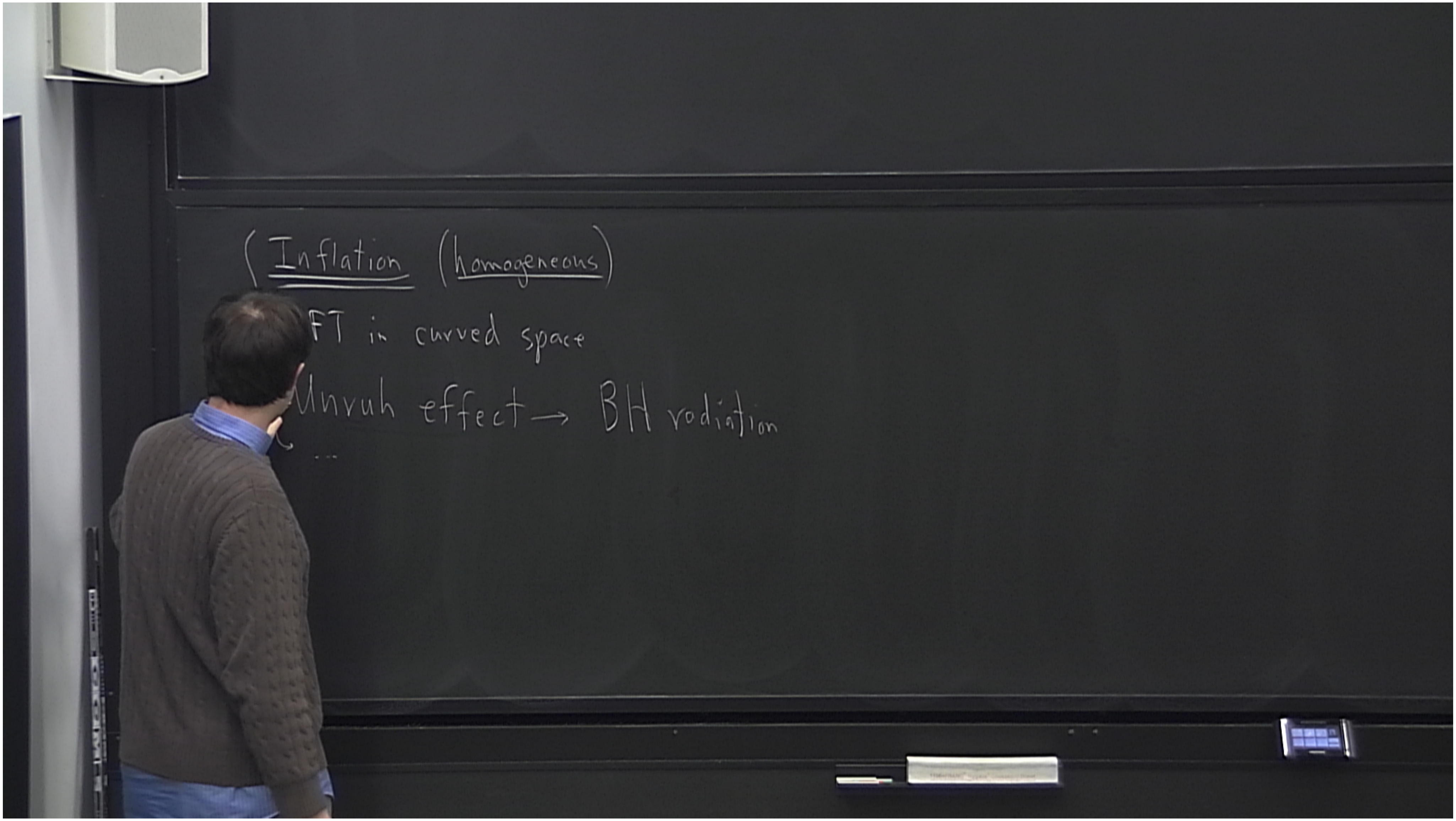
Title: 13/14 PSI - Cosmology Review - Lecture 10

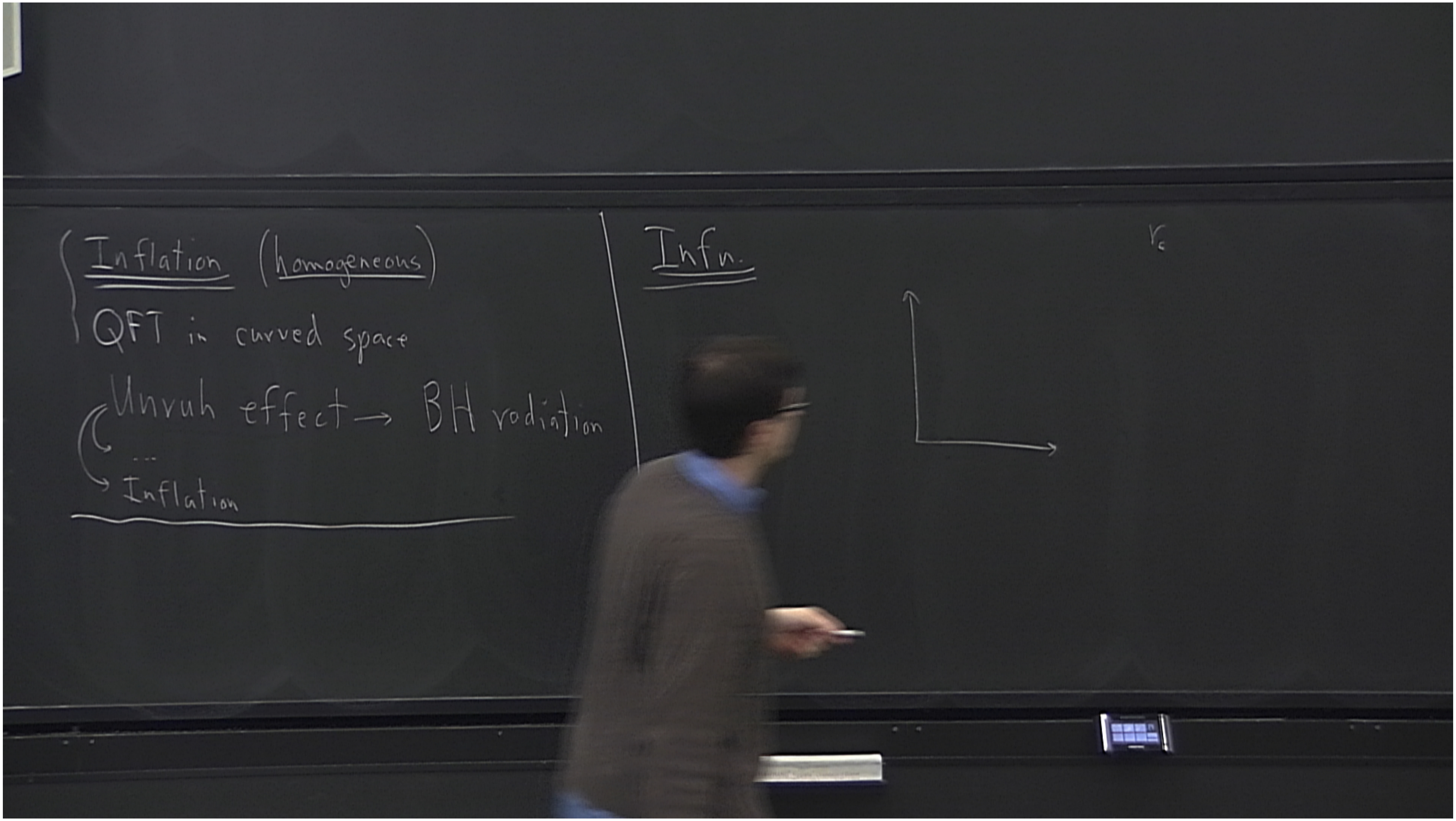
Date: Feb 11, 2014 11:30 AM

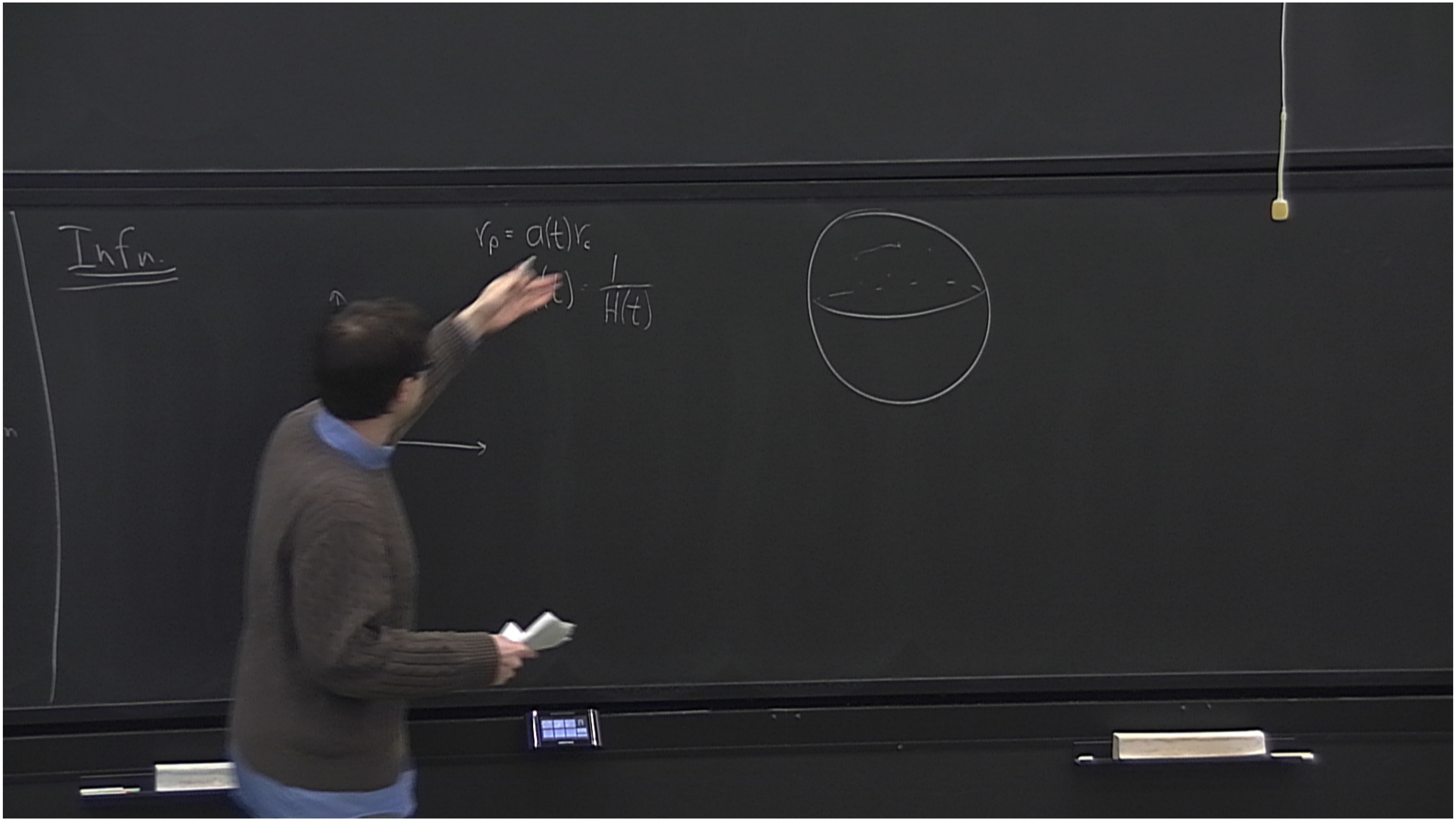
URL: http://pirsa.org/14020018

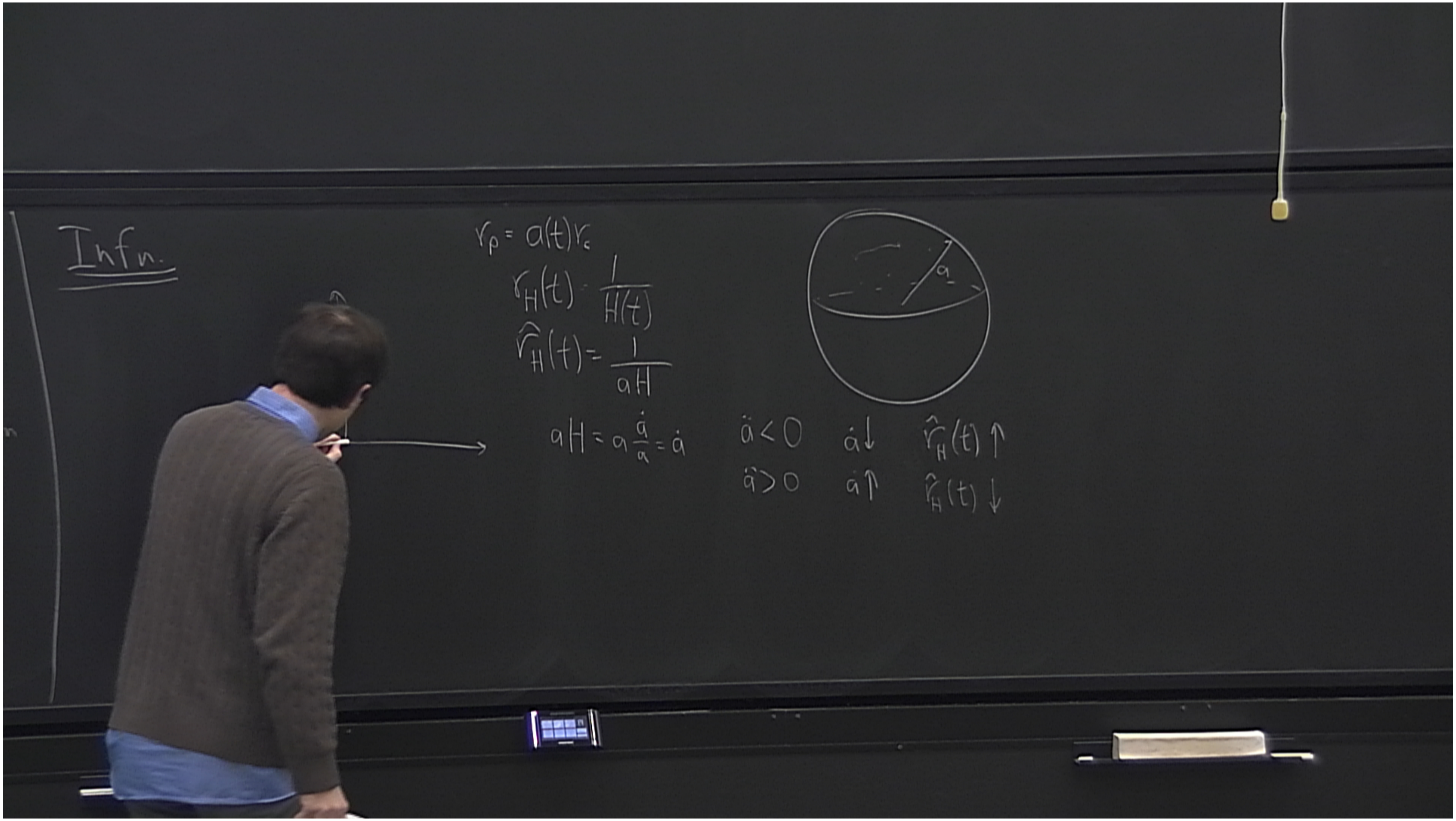
Abstract:

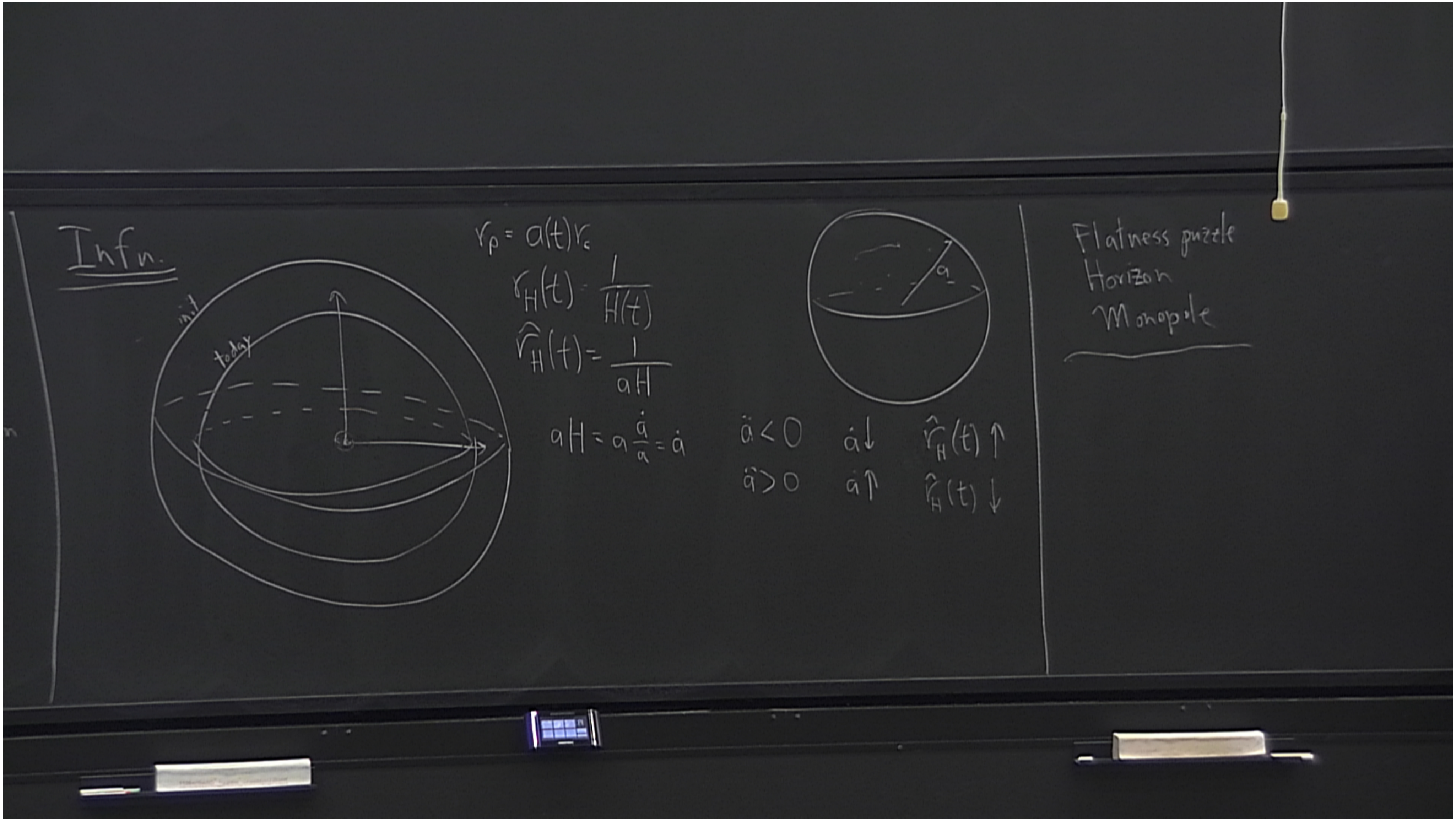

Inflation (homogeneous)
QFT
space

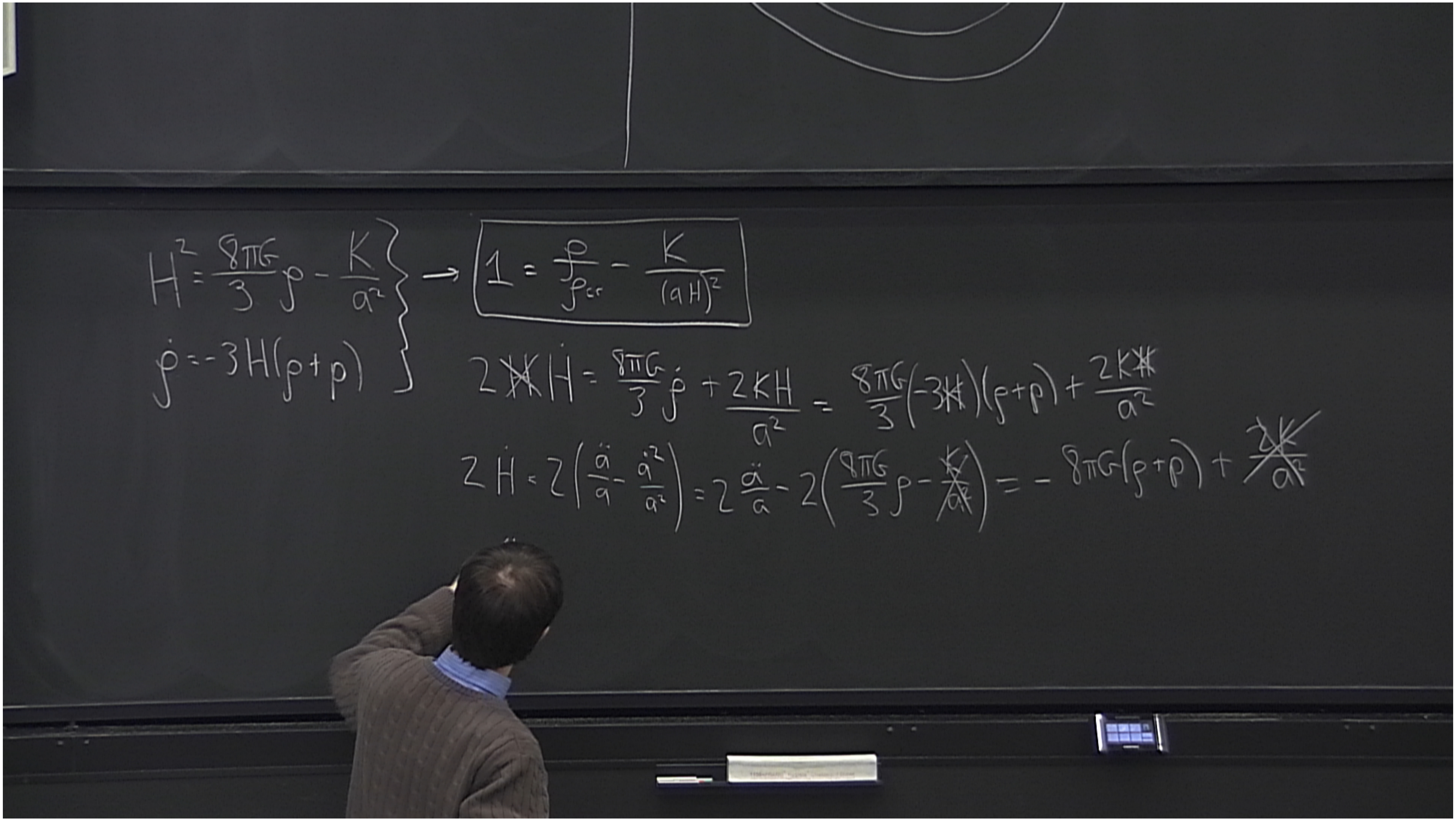

Inflation (homogeneous)
FT in curved space
Unruh effect → BH radiation

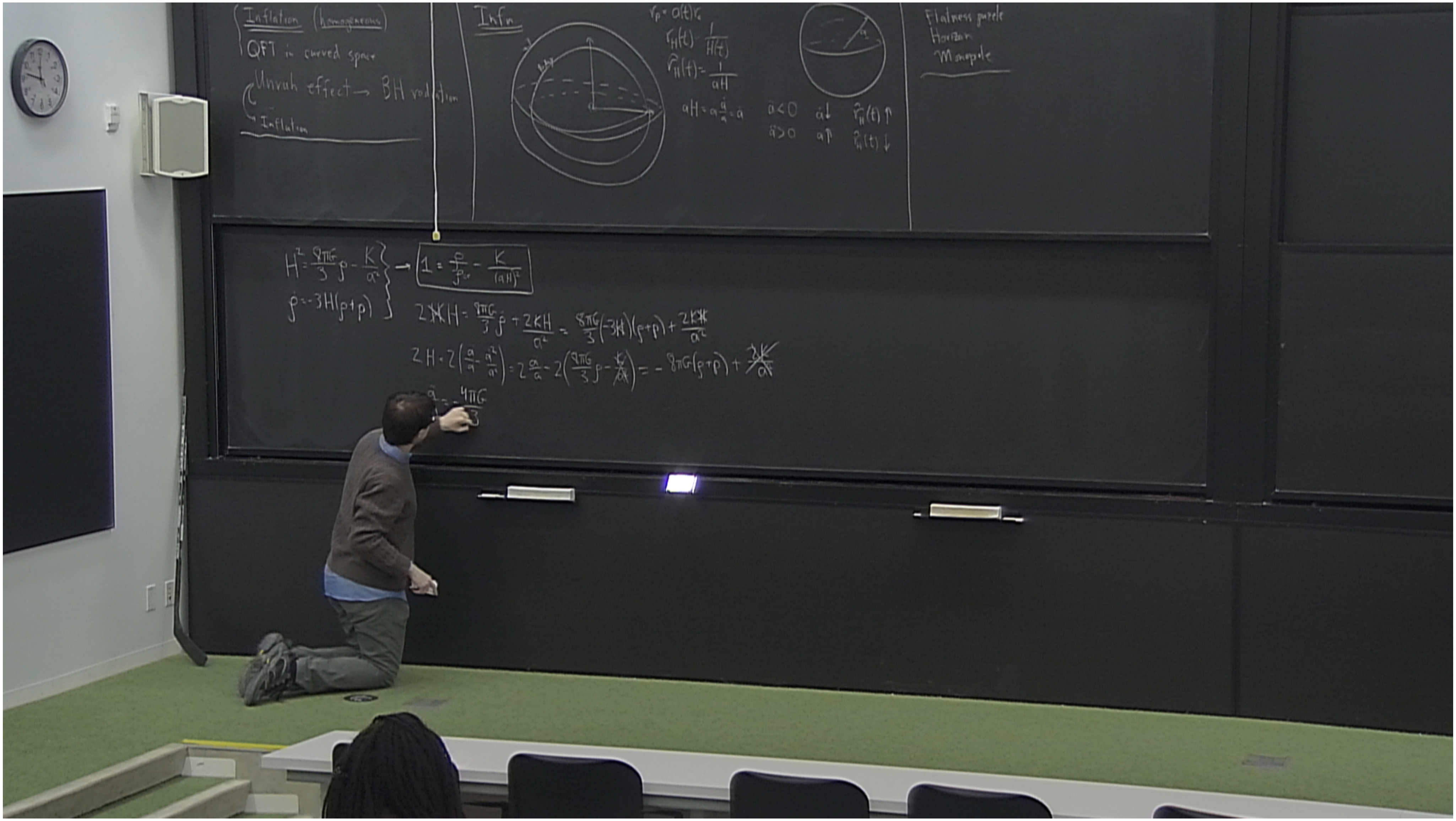

Inflation (homogeneous)
QFT in curved space
Unruh effect → BH radiation
...
Inflation
Infn.

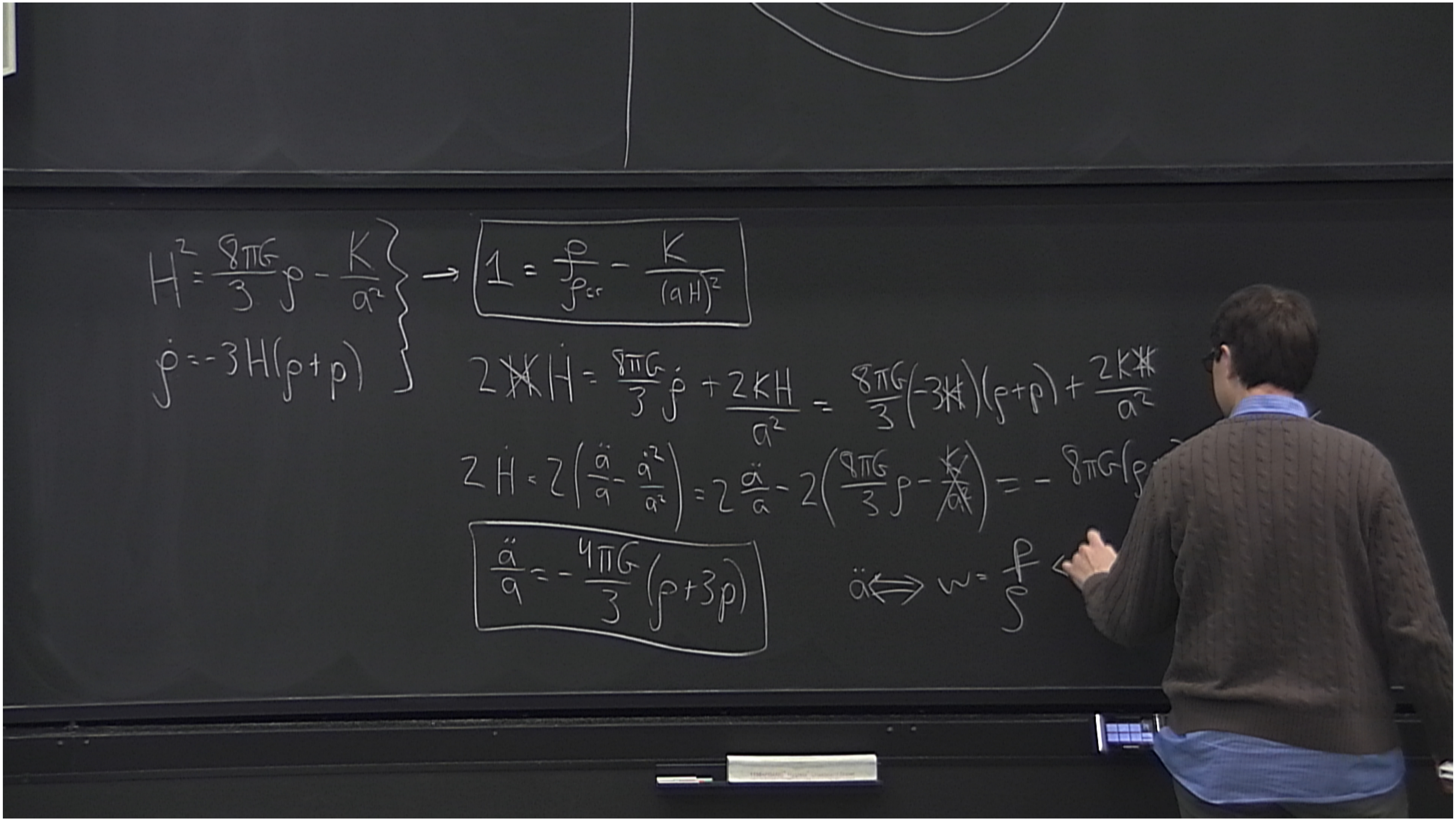

Infn.
rp = a(t)rc
1
H(t)

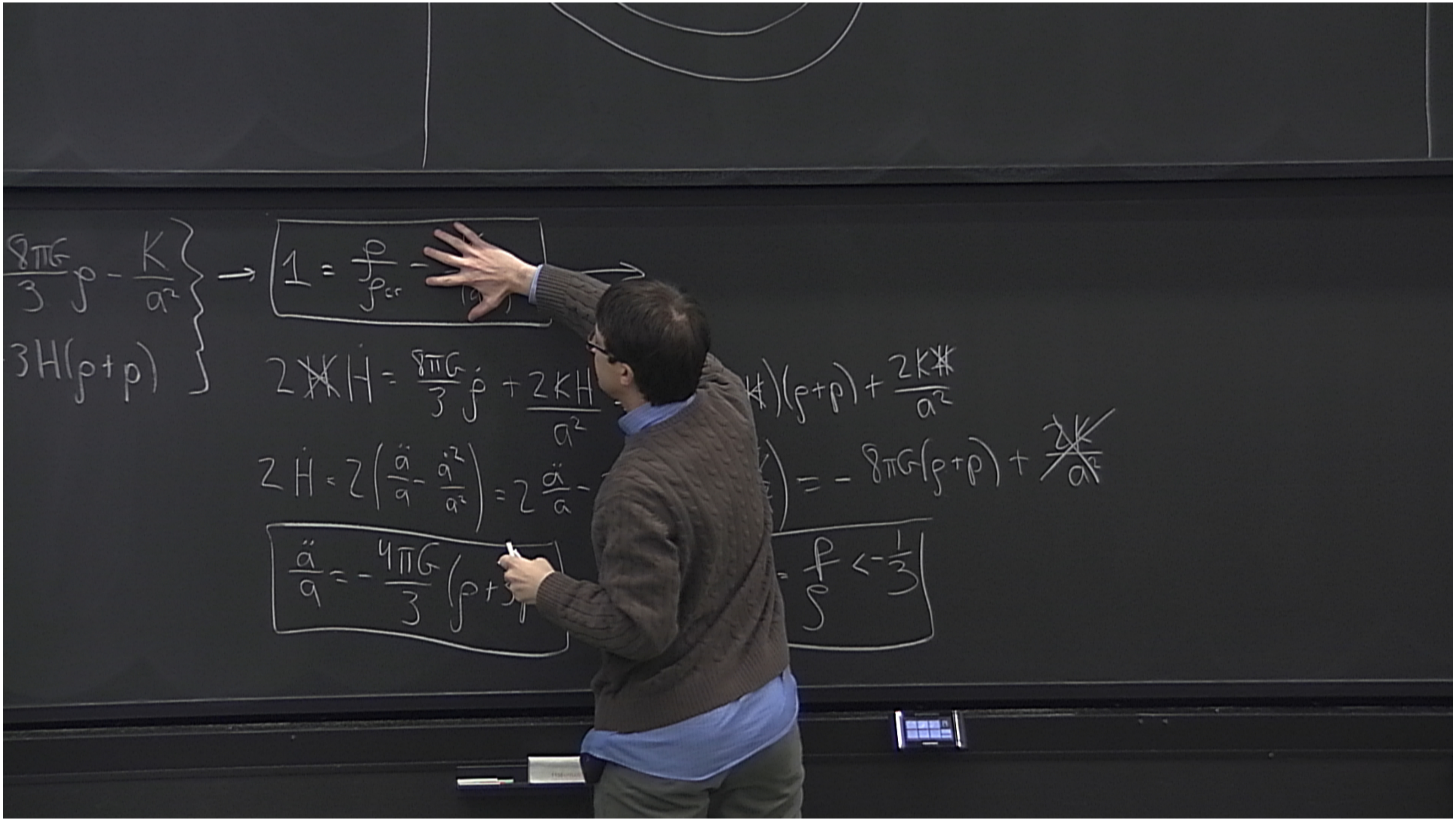

Infn.
r_p = a(t) r_c
r_H(t) = 1/H(t)
r̂_H(t) = 1/aH
aH = a ȧ/a = ȧ
ä < 0 ȧ↓ r̂_H(t)↑
ä > 0 ȧ↑ r̂_H(t)↓
a

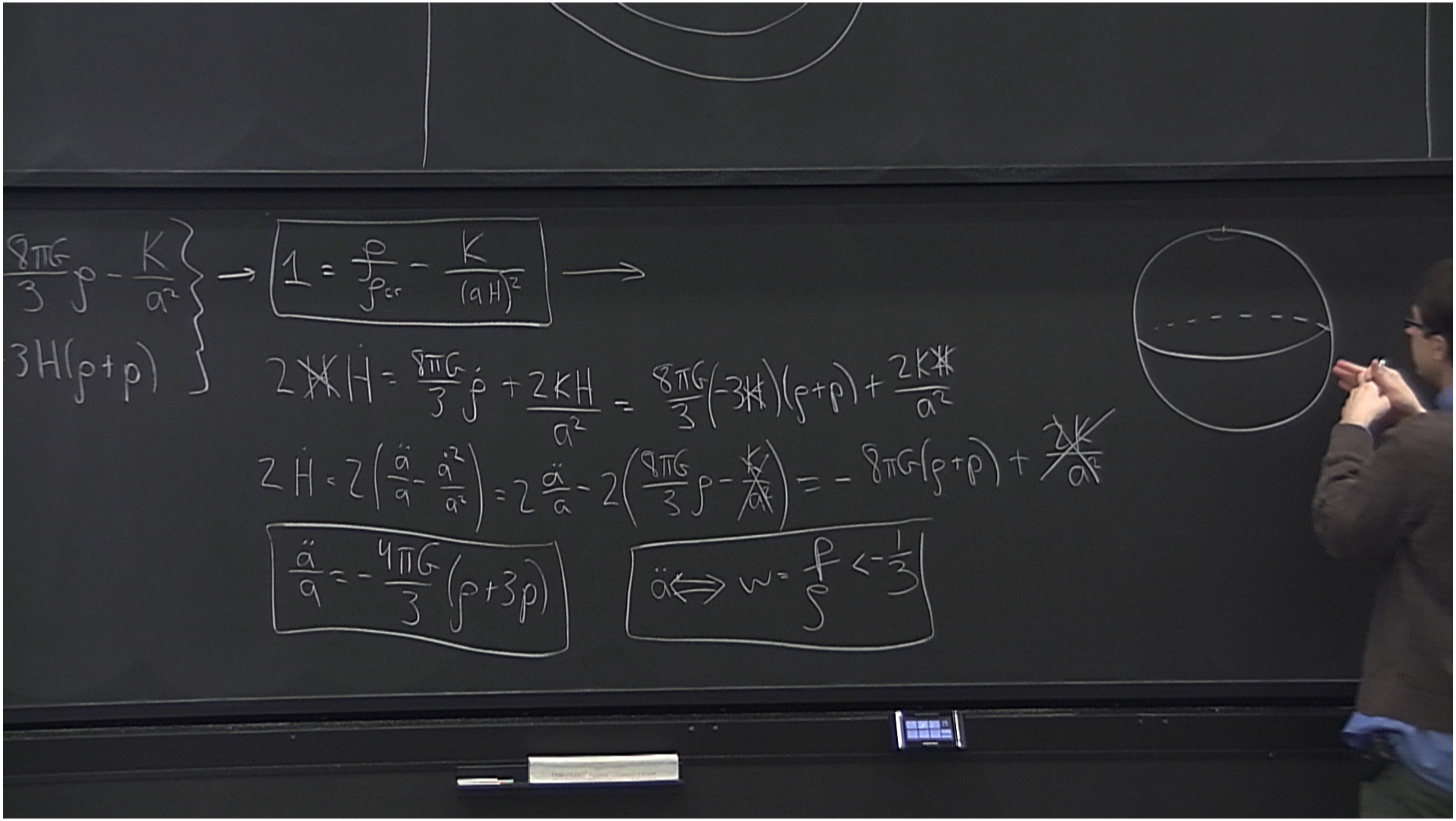

Infn.
init
today
r_p = a(t) r_c
r_H(t) = 1/H(t)
r̂_H(t) = 1/aH
aH = a ȧ/a = ȧ
a
ä < 0   ȧ↓   r̂_H(t) ↑
ä > 0   ȧ↑   r̂_H(t) ↓
Flatness puzzle
Horizon
Monopole

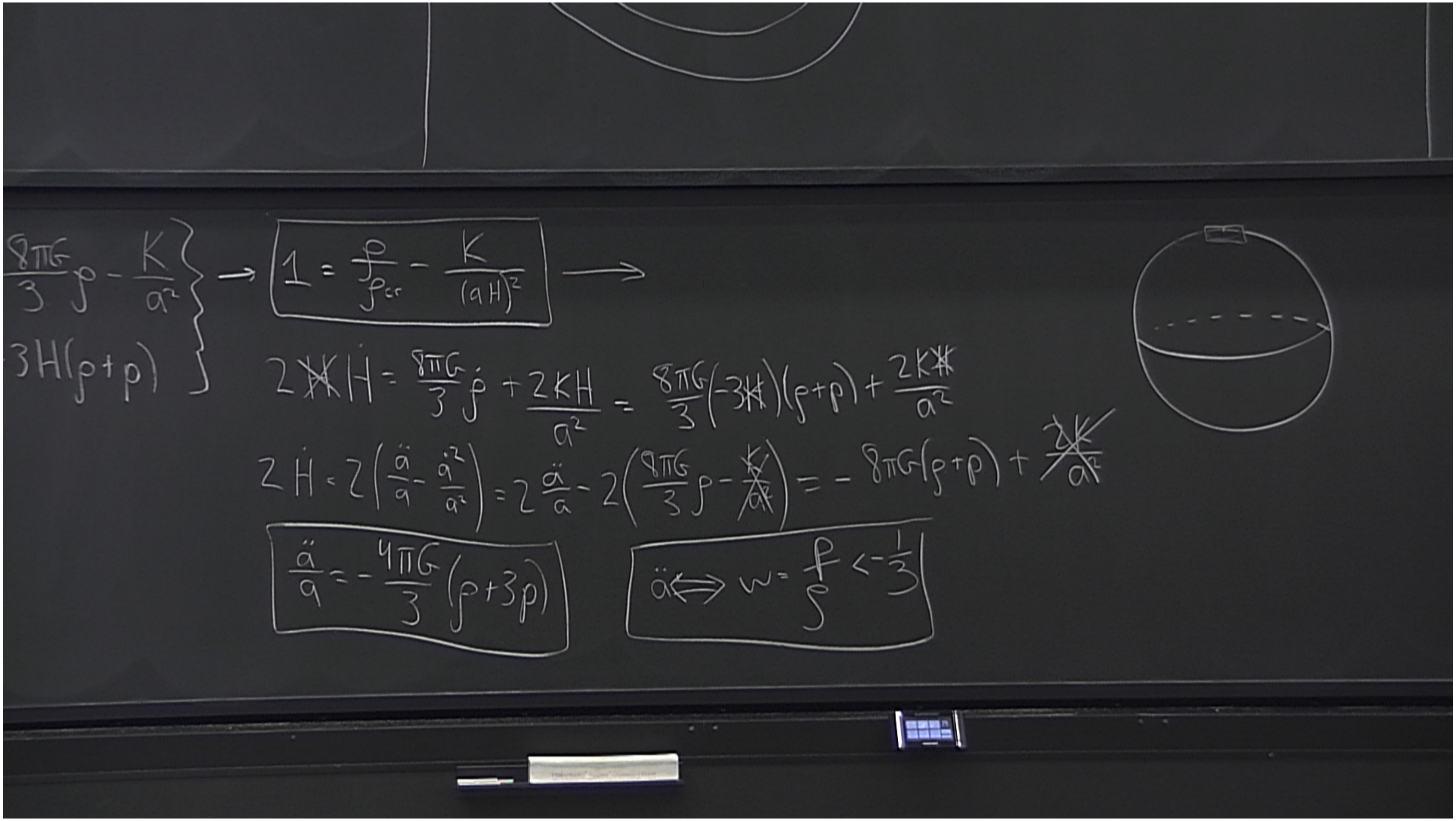

$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$\dot\rho = -3H(\rho + p)$
$1 = \frac{\rho}{\rho_{cr}} - \frac{K}{(aH)^2}$
$2H\dot H = \frac{8\pi G}{3}\dot\rho + \frac{2KH}{a^2} = \frac{8\pi G}{3}(-3H)(\rho+p) + \frac{2KH}{a^2}$
$2\dot H = 2\left(\frac{\ddot a}{a} - \frac{\dot a^2}{a^2}\right) = 2\frac{\ddot a}{a} - 2\left(\frac{8\pi G}{3}\rho - \frac{K}{a^2}\right) = -8\pi G(\rho+p) + \frac{2K}{a^2}$


Inflation (homogeneous)
QFT in curved space
Unruh effect → BH radiation
Inflation
Flatness puzzle
Horizon
Monopole


H² = 8πG/3 ρ − K/a²
ρ̇ = −3H(ρ+p)
1 = ρ/ρcr − K/(aH)²
2HḢ = 8πG/3 ρ̇ + 2KH/a² = 8πG/3 (−3H)(ρ+p) + 2KH/a²
2Ḣ = 2(ä/a − ȧ²/a²) = 2 ä/a − 2(8πG/3 ρ − K/a²) = − 8πG(ρ
ä/a = − 4πG/3 (ρ+3p)
ä ⟺ w = p/ρ




$\frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$3H(\rho+p)$
$1 = \frac{\rho}{\rho_{cr}} - \frac{K}{(aH)^2}$
$2H\dot{H} = \frac{8\pi G}{3}\dot{\rho} + \frac{2KH}{a^2} = \frac{8\pi G}{3}(-3H)(\rho+p) + \frac{2KH}{a^2}$
$2\dot{H} = 2\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) = 2\frac{\ddot{a}}{a} - 2\left(\frac{8\pi G}{3}\rho - \frac{K}{a^2}\right) = -8\pi G(\rho+p) + \frac{2K}{a^2}$
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho+3p)$
$\ddot{a} \Leftrightarrow w = \frac{p}{\rho} < -\frac{1}{3}$


$\frac{8\pi G}{3}\rho - \frac{K}{a^2}$ → $1 = \frac{\rho}{\rho_{cr}} - \frac{K}{(aH)^2}$ →
$-3H(\rho+p)$
$2H\dot{H} = \frac{8\pi G}{3}\dot{\rho} + \frac{2KH}{a^2} = \frac{8\pi G}{3}(-3H)(\rho+p) + \frac{2KH}{a^2}$
$2\dot{H} = 2\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) = 2\frac{\ddot{a}}{a} - 2\left(\frac{8\pi G}{3}\rho - \frac{K}{a^2}\right) = -8\pi G(\rho+p) + \frac{2K}{a^2}$
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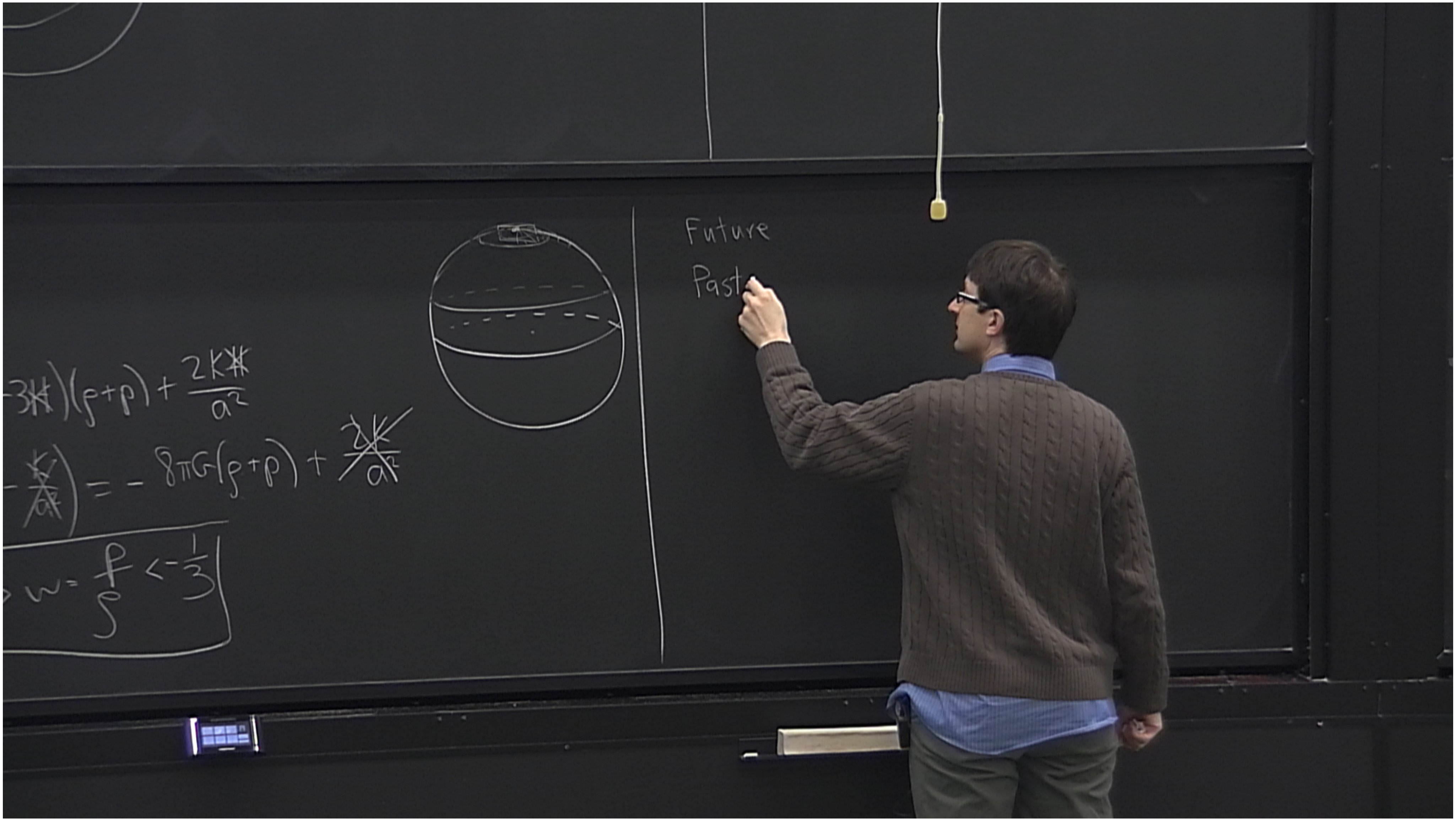
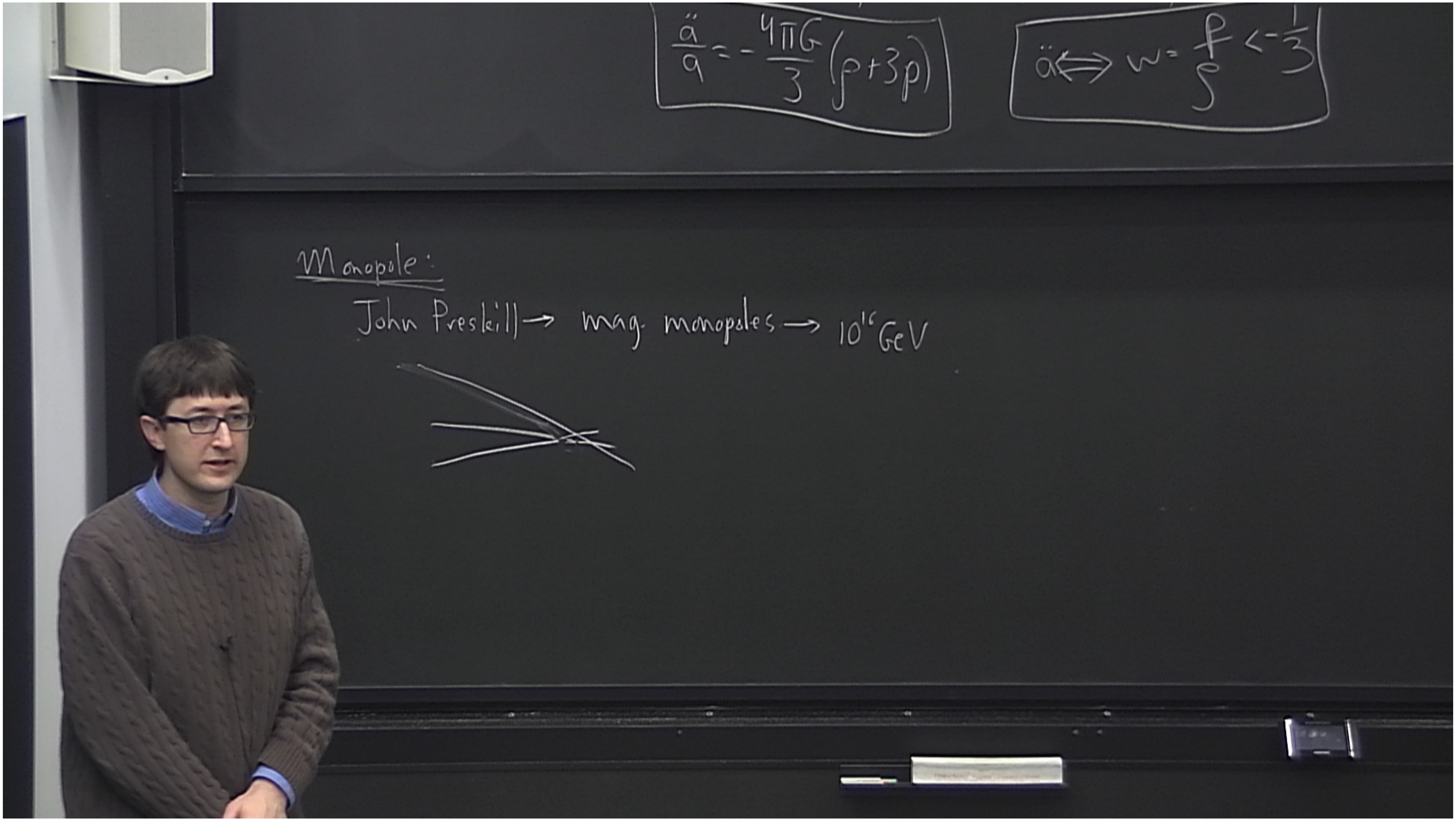
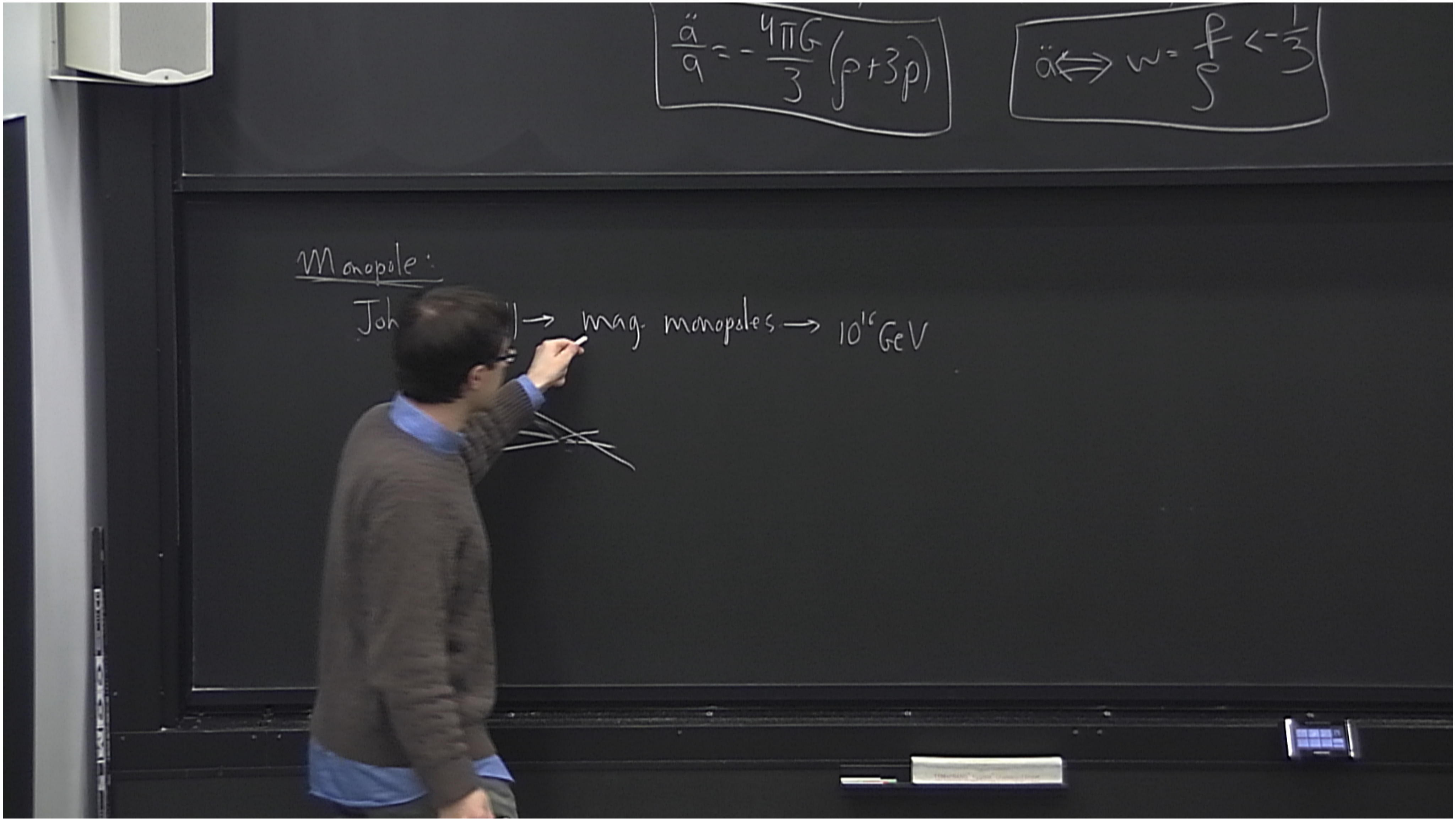
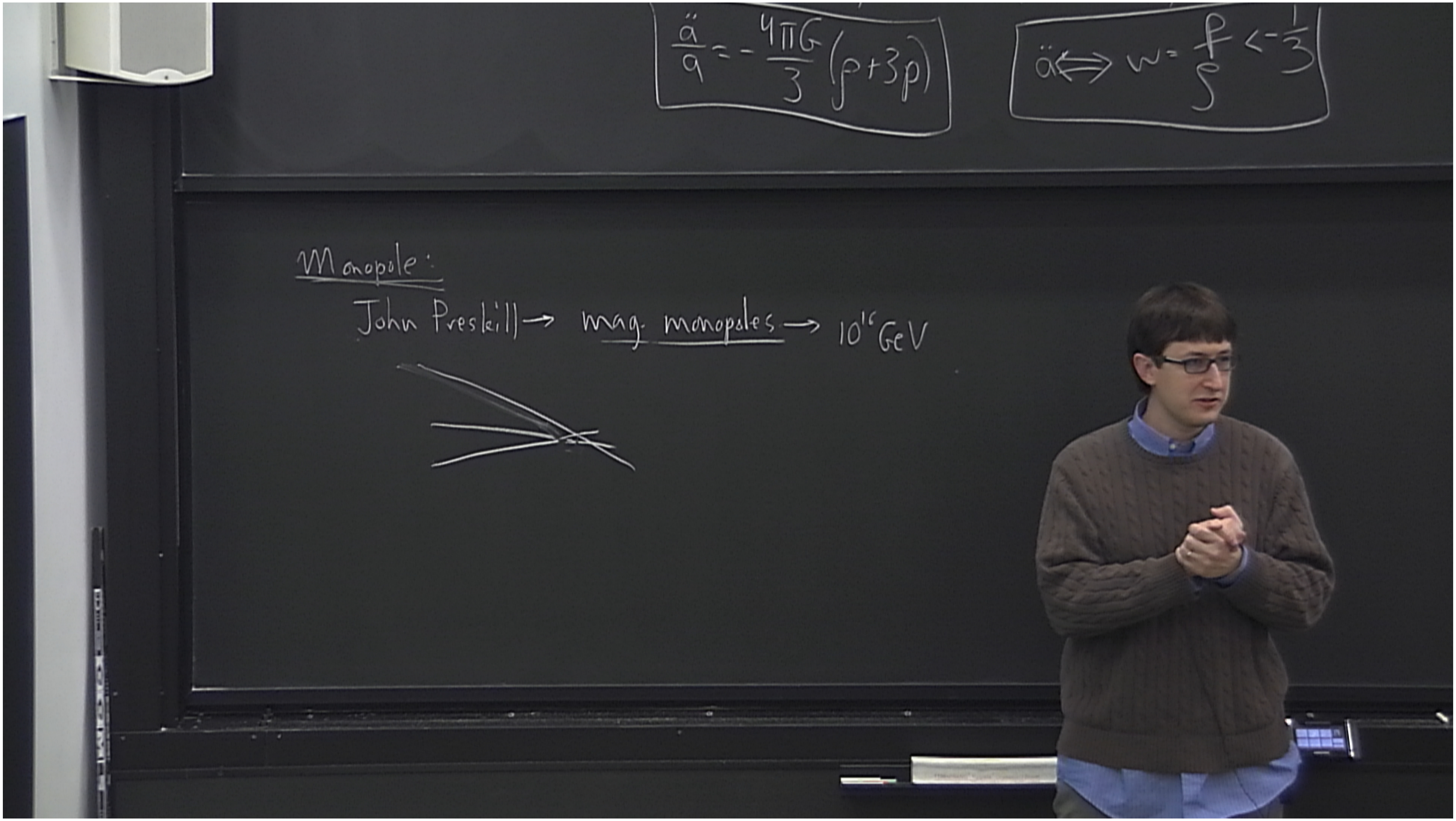


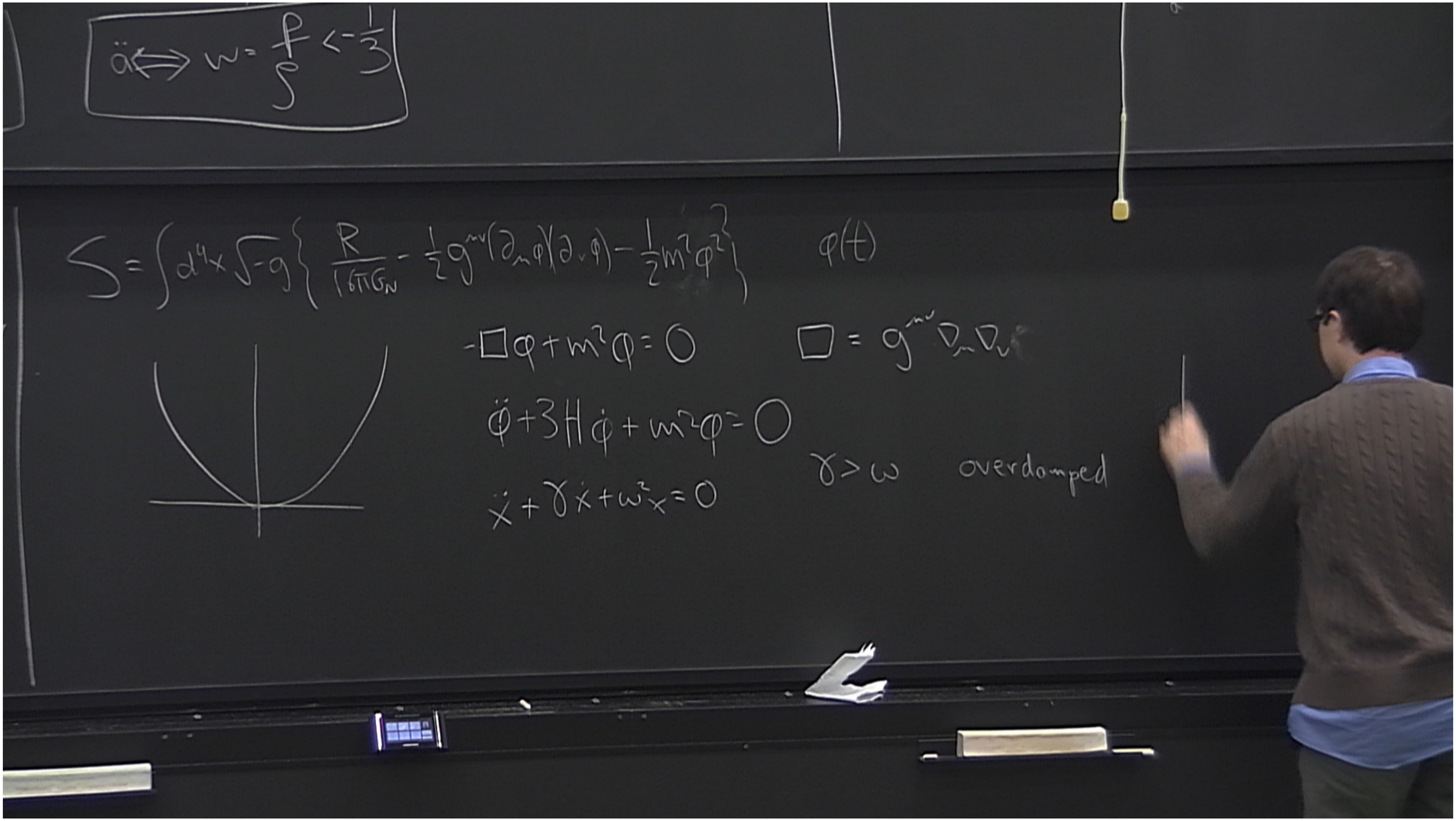
Future
Past

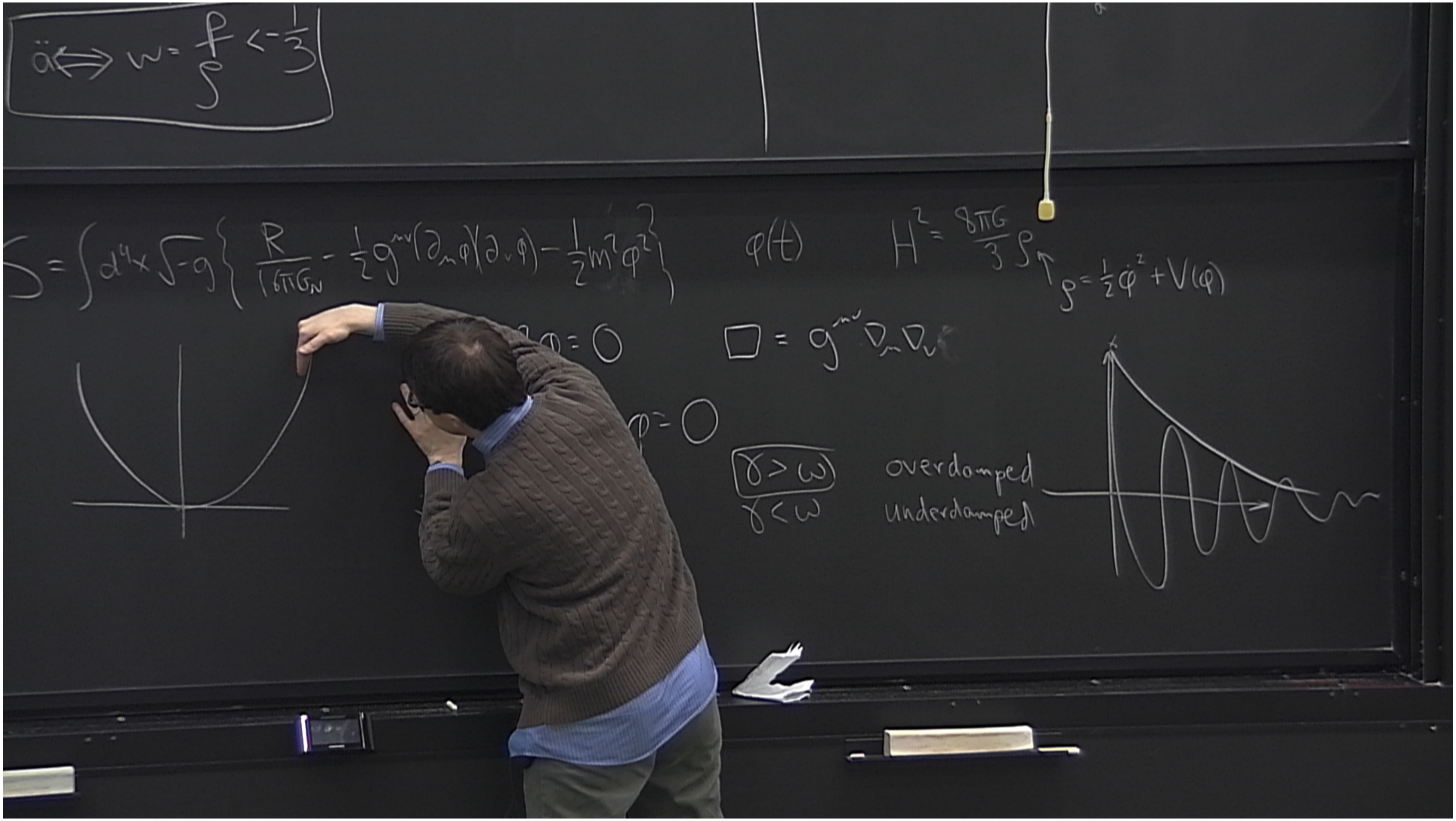

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
$\ddot{a} \Longleftrightarrow w = \frac{p}{\rho} < -\frac{1}{3}$
Monopole:
John Preskill → mag. monopoles → 10^{16} GeV

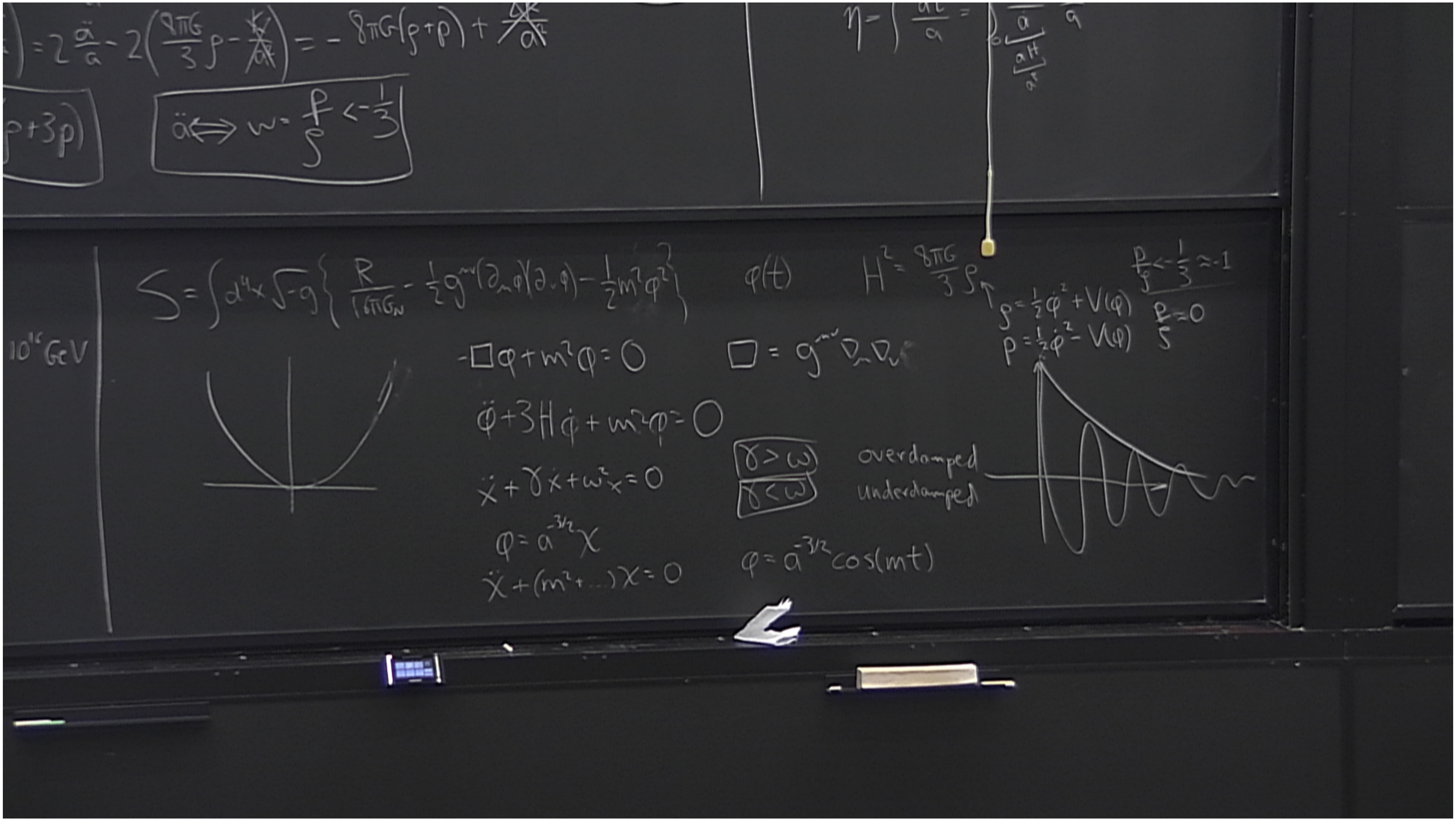


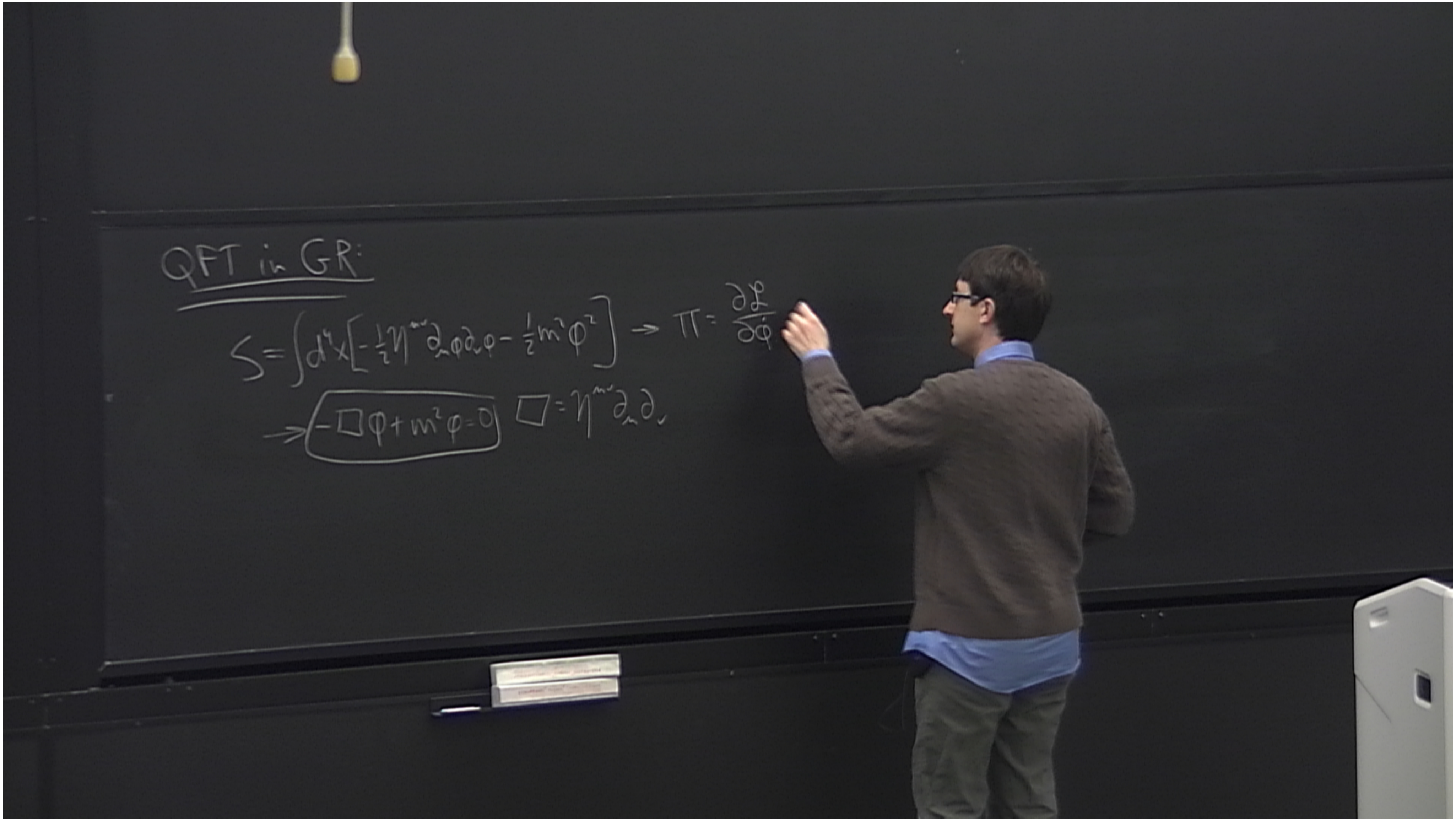
ä/a = - 4πG/3 (ρ+3p)
ä ⟺ w = p/ρ < -1/3
Monopole:
mag. monopoles → 10^16 GeV

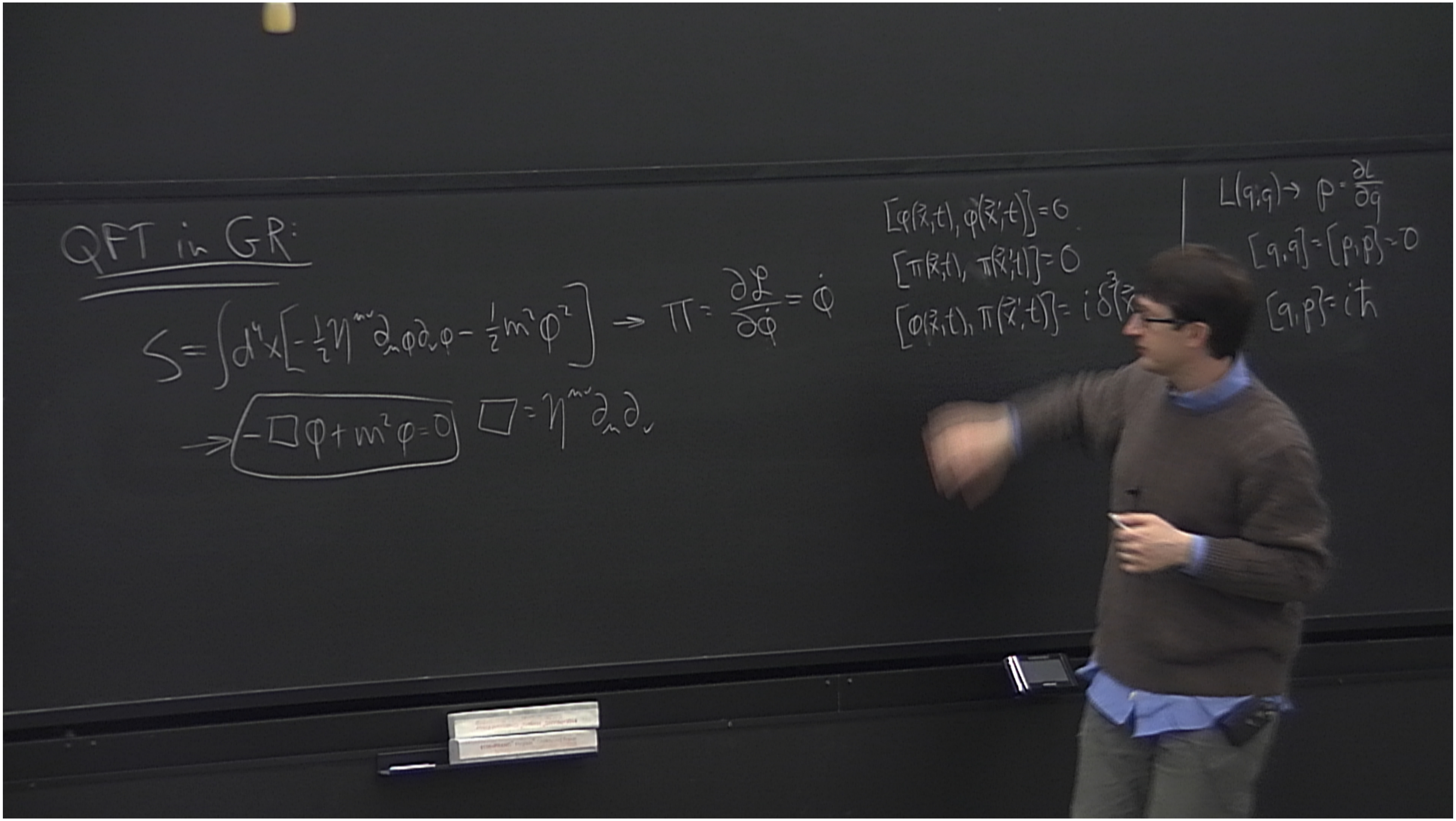

ä/a = - 4πG/3 (ρ+3p)
ä ⟺ w = p/ρ < -1/3
Monopole:
John Preskill → mag. monopoles → 10^16 GeV

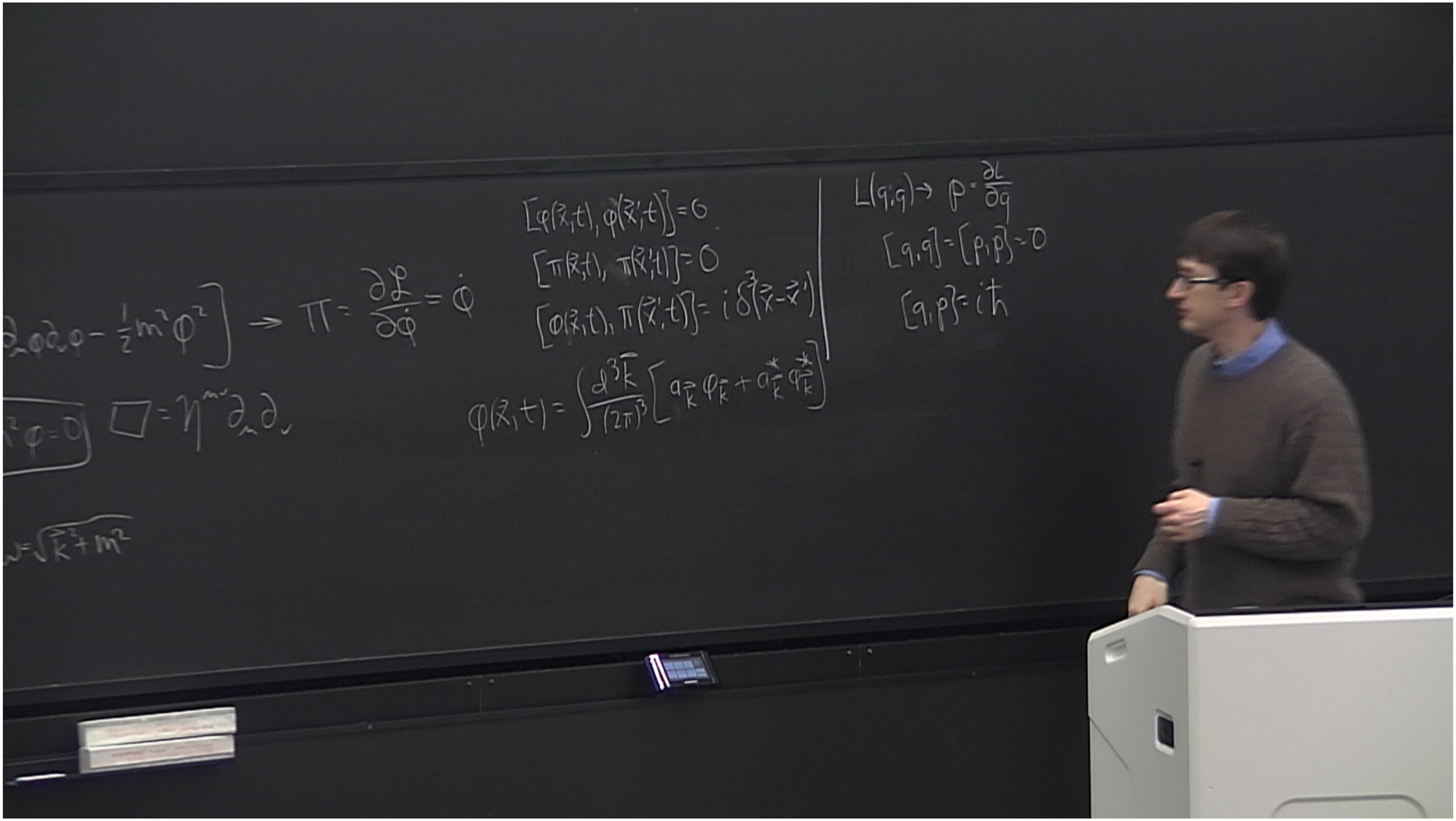

ä ⟺ w = p/ρ < −1/3
S = ∫d⁴x √−g { R/(16πG_N) − ½ g^{μν}(∂_μφ)(∂_νφ) − ½ m²φ² }
φ(t)
−□φ + m²φ = 0
□ = g^{μν} ∇_μ ∇_ν
φ̈ + 3Hφ̇ + m²φ = 0
ẍ + γẋ + ω²x = 0
γ > ω
overdamped

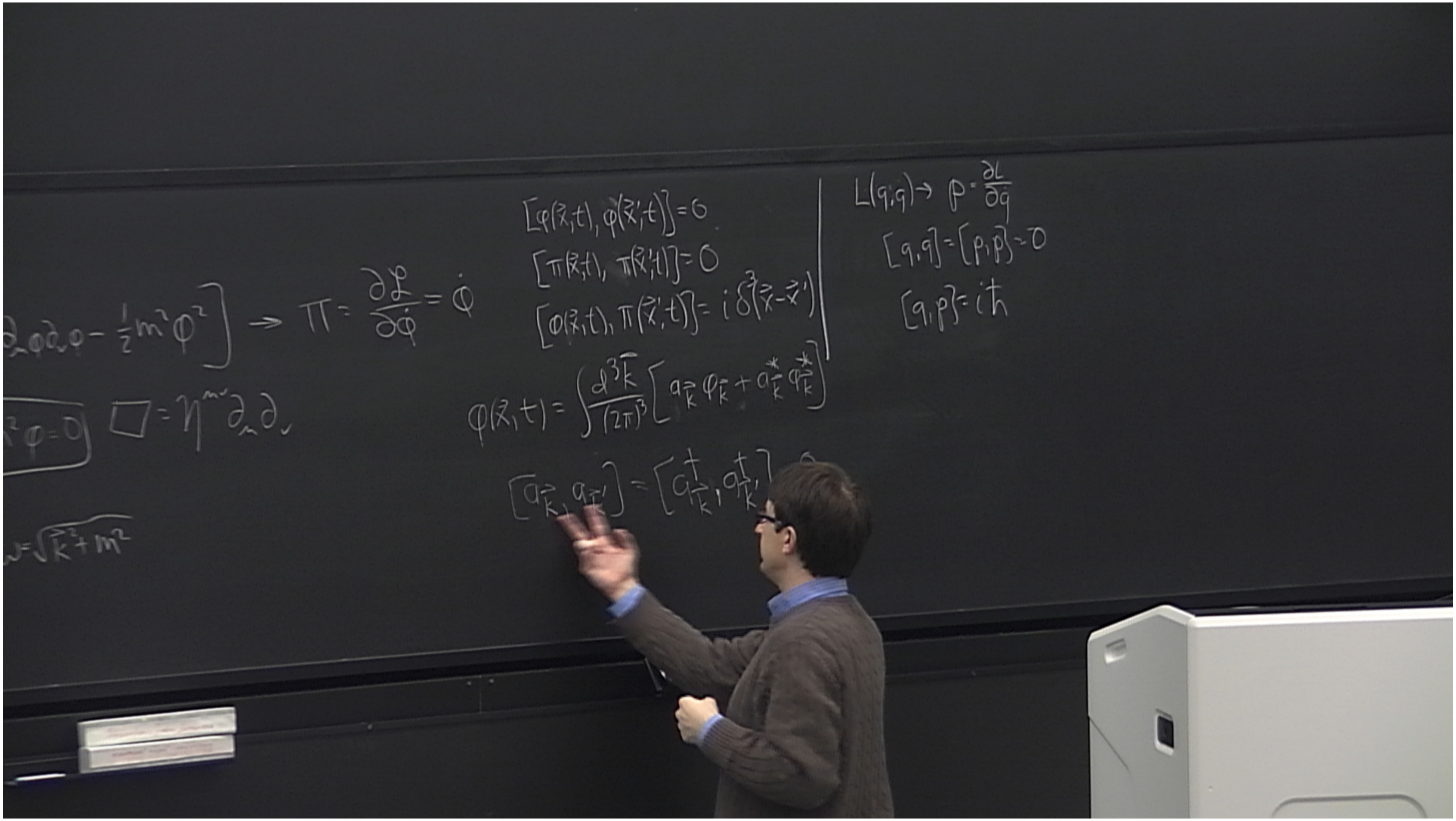

ä ⟺ w = p/ρ < −1/3
S = ∫d⁴x √−g { R/16πG_N − ½ g^μν (∂_μ φ)(∂_ν φ) − ½ m² φ² }
φ(t)
H² = 8πG/3 ρ
ρ = ½ φ̇² + V(φ)
□ = g^μν ∇_μ ∇_ν
γ > ω overdamped
γ < ω underdamped


$= 2\frac{\ddot{a}}{a} - 2\left(\frac{8\pi G}{3}\rho - \frac{k}{a^2}\right) = -8\pi G(\rho + p) + \frac{k}{a^2}$
$\rho + 3p$
$\ddot{a} \Leftrightarrow w = \frac{p}{\rho} < -\frac{1}{3}$
$\eta = \int \frac{dt}{a}$
$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G_N} - \frac{1}{2} g^{\mu\nu}(\partial_\mu\varphi)(\partial_\nu\varphi) - \frac{1}{2}m^2\varphi^2 \right\}$
$\varphi(t)$
$H^2 = \frac{8\pi G}{3}\rho$
$\frac{p}{\rho} < -\frac{1}{3} \approx -1$
$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$
$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$
$\frac{p}{\rho} \approx 0$
10^{16} GeV
$-\Box\varphi + m^2\varphi = 0$
$\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$
$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$
$\ddot{x} + \gamma\dot{x} + \omega^2 x = 0$
$\gamma > \omega$ overdamped
$\gamma < \omega$ underdamped
$\varphi = a^{-3/2}\chi$
$\ddot{\chi} + (m^2 + \ldots)\chi = 0$
$\varphi = a^{-3/2}\cos(mt)$

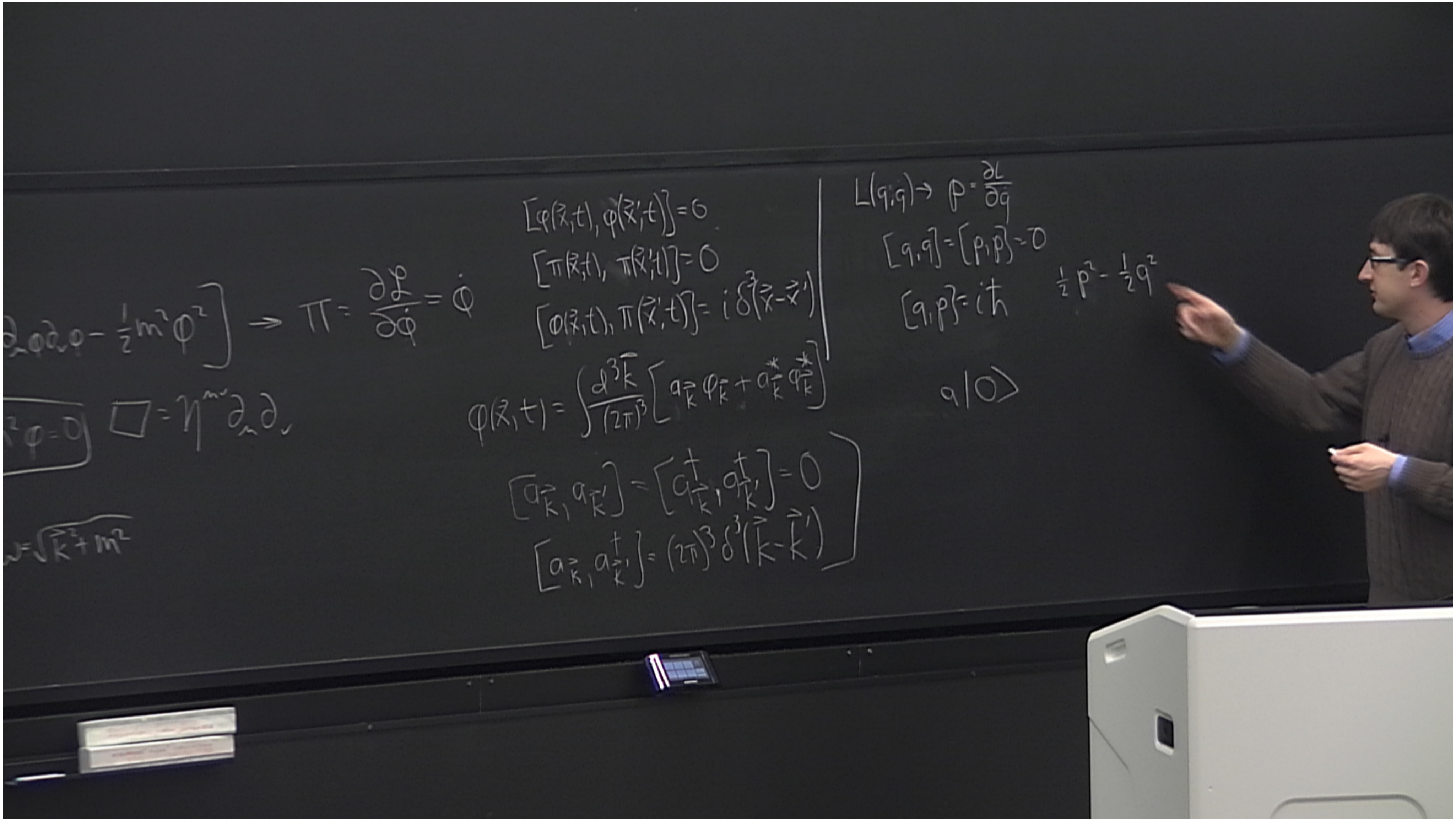

QFT in GR:
$S = \int d^4x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 \right] \rightarrow \pi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}}$
$\rightarrow -\Box\varphi + m^2\varphi = 0$
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$

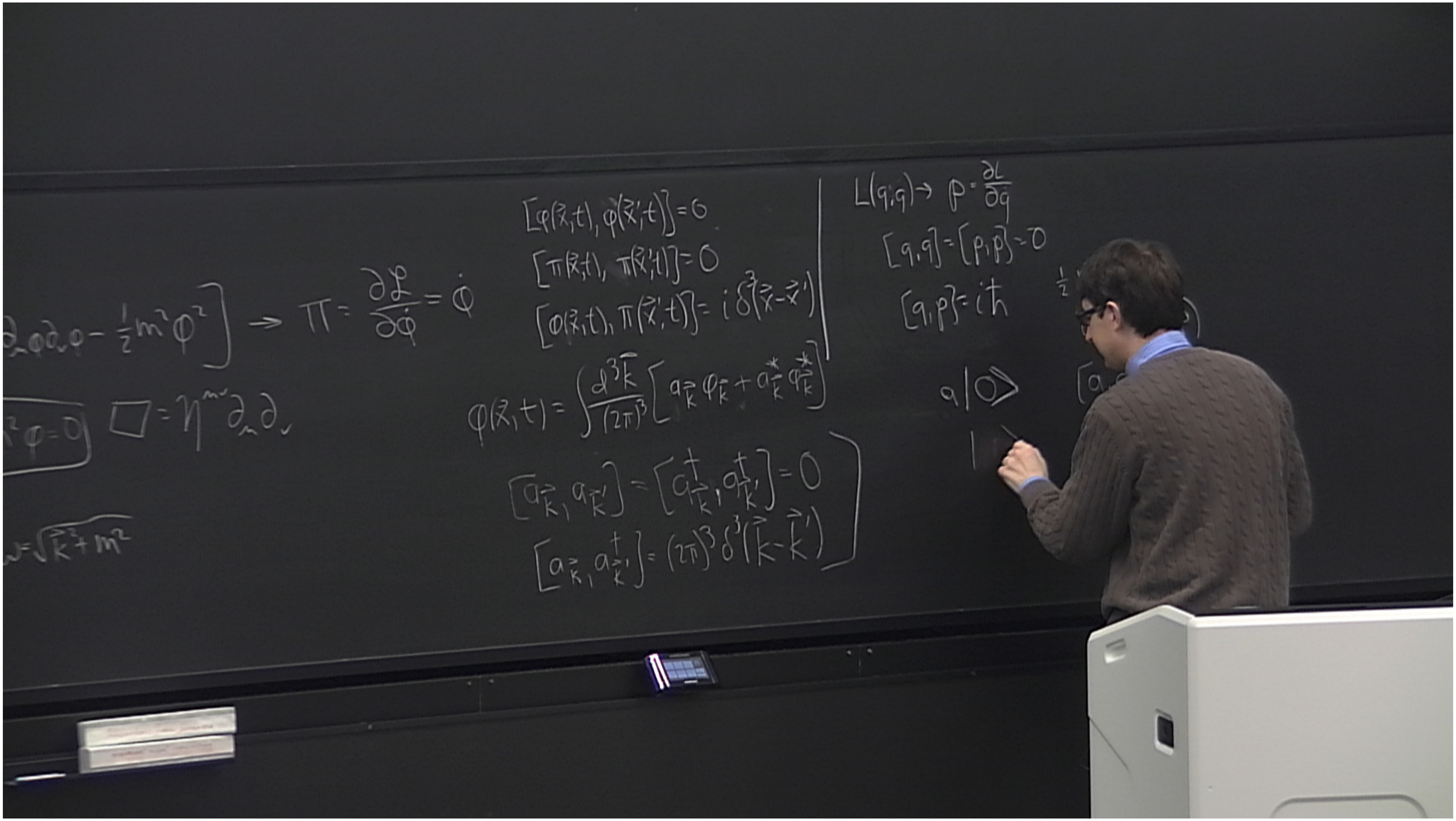

QFT in GR:
$S = \int d^4x \left[-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2 \right] \rightarrow \pi = \frac{\partial \mathcal{L}}{\partial \dot\varphi} = \dot\varphi$
$-\Box\varphi + m^2\varphi = 0$
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = 0$
$[\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\delta^3$
$L(q,\dot q) \rightarrow p = \frac{\partial L}{\partial \dot q}$
$[q,q] = [p,p] = 0$
$[q,p] = i\hbar$


[φ(x,t), φ(x',t)] = 0
[π(x,t), π(x',t)] = 0
[φ(x,t), π(x',t)] = i δ³(x − x')
L(q, q̇) → p = ∂L/∂q̇
[q, q] = [p, p] = 0
[q, p] = iħ
π = ∂ℒ/∂φ̇ = φ̇
□ = η^{μν} ∂_μ ∂_ν
φ(x,t) = ∫ d³k/(2π)³ [a_k φ_k + a*_k φ*_k]
ω = √(k² + m²)



$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = 0$
$[\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\delta^3(\vec{x}-\vec{x}')$
$L(q,\dot{q}) \to p = \frac{\partial L}{\partial \dot{q}}$
$[q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2] \to \pi = \frac{\partial \mathcal{L}}{\partial\dot{\varphi}} = \dot{\varphi}$
$\varphi = 0$ $\square = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$\varphi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^3}[a_{\vec{k}}\varphi_{\vec{k}} + a^*_{\vec{k}}\varphi^*_{\vec{k}}]$
$[a_{\vec{k}}, a_{\vec{k}'}] = [a^\dagger_{\vec{k}}, a^\dagger_{\vec{k}'}]$
$\omega = \sqrt{\vec{k}^2 + m^2}$


$\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}m^2\varphi^2] \rightarrow \pi = \frac{\partial\mathcal{L}}{\partial\dot\varphi} = \dot\varphi$
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$\omega = \sqrt{\vec{k}^2 + m^2}$
$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = 0$
$[\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\delta^3(\vec{x}-\vec{x}')$
$\varphi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^3}[a_{\vec{k}}\varphi_{\vec{k}} + a^*_{\vec{k}}\varphi^*_{\vec{k}}]$
$[a_{\vec{k}}, a_{\vec{k}'}] = [a^\dagger_{\vec{k}}, a^\dagger_{\vec{k}'}] = 0$
$[a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta^3(\vec{k}-\vec{k}')$
$L(q,\dot q) \rightarrow p = \frac{\partial L}{\partial \dot q}$
$[q,q] = [p,p] = 0$
$[q,p] = i\hbar$
$\frac{1}{2}p^2 - \frac{1}{2}q^2$
$a|0\rangle$



$[\varphi(\vec{x},t), \varphi(\vec{x}',t)] = 0$
$[\pi(\vec{x},t), \pi(\vec{x}',t)] = 0$
$[\varphi(\vec{x},t), \pi(\vec{x}',t)] = i\delta^3(\vec{x}-\vec{x}')$
$\rightarrow \pi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \dot{\varphi}$
$\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$
$\varphi(\vec{x},t) = \int \frac{d^3\vec{k}}{(2\pi)^3}\left[a_{\vec{k}}\varphi_{\vec{k}} + a^*_{\vec{k}}\varphi^*_{\vec{k}}\right]$
$[a_{\vec{k}}, a_{\vec{k}'}] = [a^\dagger_{\vec{k}}, a^\dagger_{\vec{k}'}] = 0$
$[a_{\vec{k}}, a^\dagger_{\vec{k}'}] = (2\pi)^3\delta^3(\vec{k}-\vec{k}')$
$L(q,\dot{q}) \rightarrow p = \frac{\partial L}{\partial \dot{q}}$
$[q,q] = [p,p] = 0$
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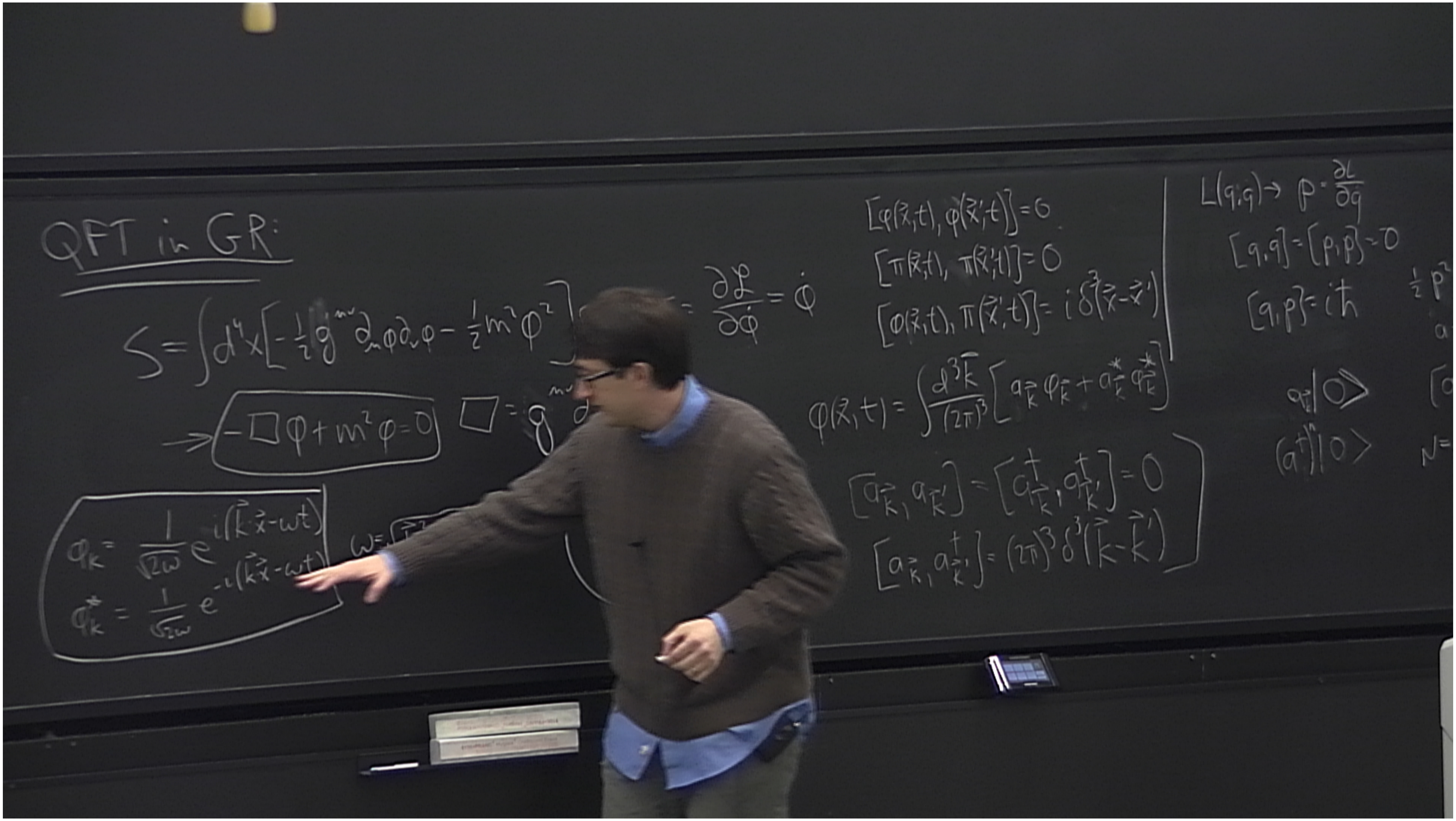
$a|0\rangle$


QFT in GR: