

Title: Introduction to Quantum Field Theory for Cosmology - Lecture 16

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Abstract:

QFT for Cosmology, Achim Kempf, Winter 14, Lecture 16

Note Title

Recall:

- Using different choices of mode functions, $v_k(\eta)$, $\tilde{v}_k(\eta)$, we can write $\hat{\mathcal{X}}_k(\eta)$ in different ways:

$$\begin{aligned}\hat{\mathcal{X}}_k(\eta) &= \frac{1}{\sqrt{2}} (v_k^*(\eta) a_k + v_k(\eta) a_{-k}^\dagger) \\ &= \frac{1}{\sqrt{2}} (\tilde{v}_k^*(\eta) \tilde{a}_k + \tilde{v}_k(\eta) \tilde{a}_{-k}^\dagger)\end{aligned}\tag{A}$$

- △ Since for each k the space of possible mode functions is $\overset{\text{complex}}{2}$ -dimensional, there exist complex d_k, f_k so that:

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(Recall: Because $\tilde{v}_k(\eta)$ must obey the Wronskian condition α_k and β_k must obey $|\alpha_k|^2 - |\beta_k|^2 = 1$)

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△ From (A) and (B) we obtain (exercise):

$$a_k = d_k^* \tilde{a}_k + \beta_k \tilde{a}_{-k}^*$$

△ Thus, $a_k |0\rangle = 0$ becomes $(d_k^* \tilde{a}_k + \beta_k \tilde{a}_{-k}^*) |0\rangle = 0$, which yields:

$$|0\rangle = \left[\prod_k \frac{1}{\sqrt{2\pi}} e^{-\frac{\beta_k}{2d_k^*} \tilde{a}_k^* \tilde{a}_k} \right] |0\rangle \quad (T)$$

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needed for normalization

We can now express all basis vectors $|0\rangle, a_k^+ |0\rangle, a_k^+ a_{k'}^+ |0\rangle \dots$
in terms of the basis vectors $|\tilde{0}\rangle, \tilde{a}_k^+ |\tilde{0}\rangle, \tilde{a}_k^+ \tilde{a}_{k'}^+ |\tilde{0}\rangle \dots$

Example scenario:

* Assume $v_k(\eta), \tilde{v}_k(\eta)$ chosen so that $|0\rangle, |\tilde{0}\rangle$ are vacuum at η_1, η_2

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- * Assume $u_k(\eta), \tilde{v}_k(\eta)$ chosen so that $|0\rangle, |\bar{0}\rangle$ are vacuum at η_1, η_2 .
- * Assume system is in vacuum state at η_1 , i.e. $|\Omega\rangle = |0\rangle$.
- * Then system's state $|\Omega\rangle$ at η_2 is an excited state, i.e., a state with particles!

The extent of particle creation?

- Eqn. (T) shows that there is a finite probability amplitude for finding arbitrarily many particles at time t_2 . Does that mean ∞ many get created (at ∞ energy expense and thus halting the expansion?)

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- Let us calculate the expected number of created particles:

* Definition (QM):

$\hat{N} := a^\dagger a$ is called a "Number operator"

* Why? It is a self-adjoint observable with eigenbasis:

$$\hat{N}(a^\dagger)^n |0\rangle = n(a^\dagger)^n |0\rangle$$

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* Exercise: verify.

* Definition (QFT): $\hat{N}_k := a_k^\dagger a_k$

Interpretation of \hat{N}_k in QFT

* Assume that at some time, η , the state $|0\rangle$ is the vacuum.

* Thus, at η , for example the state $(a_k^\dagger)^n |0\rangle$ is a state with n particles of momentum k .

* Now assume that at η the system is in an arbitrary state $|\Omega\rangle$.

* Then, at η , the expected number of particles of momentum k is:

Interpretation of \hat{N}_k in QFT

- * Assume that at some time, η , the state $|0\rangle$ is the vacuum.
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- * Now assume that at η the system is in an arbitrary state $|\Omega\rangle$.
- * Then, at η , the expected number of particles of momentum k is:

$$\bar{N}_k = \langle \Omega | \hat{N}_k | \Omega \rangle$$

Calculation in the above scenario for $\tilde{N}_k := \tilde{a}_k^\dagger \tilde{a}_k$ at time τ_2

$$\begin{aligned} \bar{N}_k &= \langle \Omega | \hat{N}_k | \Omega \rangle \\ &= \langle 0 | \tilde{a}_k^\dagger \tilde{a}_k | 0 \rangle \end{aligned}$$

Now use that $a_k = \alpha_k^\dagger \tilde{a}_k + \beta_k \tilde{a}_{-k}^\dagger$, i.e.

also, that $\tilde{a}_k = \tilde{\alpha}_k^\dagger a_k + \tilde{\beta}_k a_{-k}^\dagger$

Exercise: Calculate $\tilde{\alpha}_k, \tilde{\beta}_k$ in terms of α_k, β_k .

$$\begin{aligned} &= \langle 0 | (\tilde{\alpha}_k a_k^\dagger + \tilde{\beta}_k^\dagger a_{-k}) (\tilde{\alpha}_k^\dagger a_k + \tilde{\beta}_k a_{-k}^\dagger) | 0 \rangle \\ &= \langle 0 | \tilde{\beta}_k^\dagger \tilde{\beta}_k a_{-k} a_{-k}^\dagger + \cancel{a_k^\dagger a_k} + \cancel{a_k^\dagger a_{-k}^\dagger} + \cancel{a_k a_k} | 0 \rangle \\ &= \tilde{\beta}_k^\dagger \tilde{\beta}_k \langle 0 | a_{-k}^\dagger a_{-k} + 1 | 0 \rangle \end{aligned}$$

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(using infrared regularization we have $[\tilde{\alpha}_k, \tilde{\alpha}_{k'}^\dagger] = \delta_{k,k'}$)

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Total particle number:

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$$\bar{N} = \sum_{\mathbf{k}} \langle \Omega | \hat{N}_{\mathbf{k}} | \Omega \rangle = \sum_{\mathbf{k}} \tilde{\beta}_{\mathbf{k}}^* \tilde{\beta}_{\mathbf{k}}$$

□ Note:

- * We assumed here an infrared, i.e., a box regularization. (Else the number of created particles can only be 0 or ∞)
← Exercise: Why?
- * Else, \bar{N} may come out infinite, but that can be ok.
- * This happens even for photon creation through moving charges.
- * R. Stueckelberg suggested to use a "similarity" regularization.

□ The expected total number of particles at time η_2 is then:

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- * Else, \bar{N} may come out infinite, but that can be ok.
- * This happens even for photon creation through moving charges.
- * But we always must have of course finite "energy":

$$\langle \Omega | \hat{H}(\eta) | \Omega \rangle < \infty$$

Identification of the vacuum state

How can we identify, at any arbitrary fixed time, η , that Hilbert space vector, say $|\text{vacuum at } \eta\rangle$, which describes the vacuum, i.e., the no particle state, at that time, η ?

Q: Is $|\text{vacuum at } \eta\rangle$ one of the (infinitely many) states

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that come with choices of mode functions

$$v_k, \tilde{v}_k, \hat{v}_k, \dots$$

through $a_k |0\rangle = 0, \tilde{a}_k |\hat{\delta}\rangle = 0, \hat{a}_k |\hat{\delta}\rangle = 0, \dots ?$

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Yes, if or when |vacuum at η > exists at all,

then there exist suitable mode functions, V_k ,

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specifying at time η that $v_k(\eta) = \tau_k, v_k'(\eta) = s_k$
for a suitable choice of $\tau_k, s_k \in \mathbb{C}$.

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□ Ansatz:

Let us try to define the vacuum state at a time η as that Hilbert space vector (up to a phase) which at time η minimizes the Hamiltonian, $H^{(0)}(\eta)$.

□ To this end, we will choose $\tau_k, s_k \in \mathbb{C}$ suitably, so that $v_k(\eta) = \tau_k$, $v_k'(\eta) = s_k$ define that mode

□ To this end, we will choose $\tau_k, s_k \in \mathbb{C}$ suitably, so that $V_k(\eta) = \tau_k$, $V_k'(\eta) = s_k$ define that mode function v_k so that its $|0\rangle$ is the lowest energy state.

Calculation of the lowest energy state at some arbitrary fixed time, η_1 .

$$\langle 0 | \hat{H}^{(x)}(\eta_1) | 0 \rangle = \langle 0 | \frac{1}{2} \int_{\text{box}} \hat{\chi}'^2(\eta_1, x) + \sum_{i=1}^3 \hat{\chi}_{ii}^2(\eta_1, x) + \left(m^2 a^2(\eta_1) - \frac{a''(\eta_1)}{a(\eta_1)} \right) \hat{\chi}^2(\eta_1, x) d^3x | 0 \rangle$$

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Exercise:

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$$\hat{x}_k(\eta,1) = \frac{1}{\sqrt{2}} (v_k^+(\eta,1) a_k + v_k(\eta,1) a_{-k}^+)$$

to evaluate this energy expectation value.

Result:

$$\langle 0 | \hat{H}^{(x)}(\eta,1) | 0 \rangle = \langle 0 | \frac{1}{4} \sum_k (v_k^2(\eta,1) + \omega_k^2(\eta,1) v_k^2(\eta,1)) a_k^+ a_k^+ \rangle$$

Result:

$$\begin{aligned}\langle 0 | \hat{H}^{(x)}(\gamma, t) | 0 \rangle &= \langle 0 | \frac{1}{4} \sum_{\mathbf{k}} (v_{\mathbf{k}}^{\prime 2}(\gamma, t) + \omega_{\mathbf{k}}^2(\gamma, t) v_{\mathbf{k}}^2(\gamma, t)) a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}^{\dagger} \\ &\quad + \frac{1}{4} \sum_{\mathbf{k}} (v_{\mathbf{k}}^{\prime \dagger 2}(\gamma, t) + \omega_{\mathbf{k}}^2(\gamma, t) v_{\mathbf{k}}^{\dagger 2}(\gamma, t)) a_{\mathbf{k}} a_{-\mathbf{k}} \\ &\quad + \frac{1}{2} \sum_{\mathbf{k}} (|v_{\mathbf{k}}^{\prime}(\gamma, t)|^2 + \omega_{\mathbf{k}}^2(\gamma, t) |v_{\mathbf{k}}(\gamma, t)|^2) (a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + \frac{1}{2}) | 0 \rangle \\ &= \frac{1}{4} \sum_{\mathbf{k}} |v_{\mathbf{k}}^{\prime}(\gamma, t)|^2 + \omega_{\mathbf{k}}^2(\gamma, t) |v_{\mathbf{k}}(\gamma, t)|^2\end{aligned}$$

Here: the time-dependent frequency reads: $\omega_{\mathbf{k}}^2(\gamma) := k^2 + m^2 a^2(\gamma) - \frac{a'(\gamma)^2}{a(\gamma)}$

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Here: the time-dependent frequency reads: $\omega_k^2(\gamma) := k^2 + \hbar^2 a^2(\gamma) - \frac{a'(\gamma)}{a(\gamma)}$

Note: We assume $\omega_k^2(\gamma) > 0$ because, else, the potential is inverted. 12 / 27

$$\begin{aligned}
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$$+\frac{1}{4} \sum_{\mathbf{k}} (v_{\mathbf{k}}^{\prime 2}(\gamma) + \omega_{\mathbf{k}}^2(\gamma) v_{\mathbf{k}}^{\prime 2}(\gamma)) a_{\mathbf{k}} a_{-\mathbf{k}}$$

$$+\frac{1}{2} \sum_{\mathbf{k}} (|v_{\mathbf{k}}^{\prime}(\gamma)|^2 + \omega_{\mathbf{k}}^2(\gamma) |v_{\mathbf{k}}(\gamma)|^2) (a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + \frac{1}{2}) |0\rangle$$

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Recall:

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Recall:

- * We defined $r_k := V_k(\gamma_1)$, $s_k := V_k'(\gamma_1)$
- * We need to determine $r_k, s_k \in \mathbb{C}$
- * This will determine a full mode function V_k with its a_k
- * This determines a corresponding $|0\rangle$ obeying $a_k |0\rangle = 0$

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- * This determines a corresponding $|0\rangle$ obeying $a_k |0\rangle = 0$
- * Our ansatz is then that:

$$|\text{vacuum at } \gamma_1\rangle = |0\rangle$$

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* Using the definitions $r_k = v_k(\eta_1)$, $s_k = v_k'(\eta_1)$:

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* We want to minimize this expression, subject to the Wronskian condition

Concretely:

* From above, the energy at η_1 is:

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* We want to minimize this expression, subject to the Wronskian condition

$$V_k'(\gamma_1) V_k^*(\gamma_1) - V_k(\gamma_1) V_k'^*(\gamma_1) = 2i$$

i.e., subject to the constraint:

$$s_k \tau_k^* - \tau_k s_k^* = 2i \quad (C)$$

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∂S

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* Exercise:

Show that the solution is:

$$v_k = \frac{1}{\sqrt{\omega_k}} e^{i\theta}$$

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define a mode function v_k for all η so that

$$\hat{x}_k(\eta_1) = \frac{1}{\sqrt{2}} (v_k^*(\eta_1) a_k + v_k(\eta_1) a_{-k}^\dagger)$$

and the corresponding state $|0\rangle$ obeying $a_k |0\rangle = 0$

is the lowest energy state of the Hamiltonian $\hat{H}^{(k)}(\eta_1)$

Special case: Minkowski space

□ Minkowski space is the special case $a(\eta) = 1$ for all η .

Then, $\omega_k^2(\eta) = \vec{k}^2 + m^2$ is a constant. Also: $\eta = t$.

□ We conclude that $|0\rangle$ is the state of lowest energy at a time η , if we choose the mode functions which obey these conditions:

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- * Verify that the state $|0\rangle$ that we have found for Minkowski space agrees with the state that we identified as the Minkowski space vacuum at the beginning of the course.
- * Show that, if we, similarly, determine the lowest energy state at another time, η_2 , then we obtain the same mode function v_k (up to an irrelevant phase).
- * This means that the same vector $|0\rangle$ minimizes

for Minkowski space agrees with the state that we identified as the Minkowski space vacuum at the beginning of the course.

- * Show that, if we, similarly, determine the lowest energy state at another time, η_2 , then we obtain the same mode function v_k (up to an irrelevant phase).
- * This means that the same vector $|0\rangle$ minimizes the energy at all times, on Minkowski space, (which had to come out because of time translation symmetry).

Back to our ansatz, namely the assumption:

At an arbitrary time η , the vacuum (no particles) state is that state which is the lowest energy state $|0\rangle$ at time η :

$$|\text{vacuum at } \eta_i\rangle = |0\rangle$$

▢ Implied prediction:

Universe expands $\Rightarrow \dot{H}^{(x)}(\eta_1) \neq \dot{H}^{(x)}(\eta_2)$

\Rightarrow expect particle production, in general.

and much higher in the fast^{ly} expanding early universe

▴ Concretely: current production rate $\approx 10 \frac{\text{particles}}{(\text{km})^3 \text{year} \cdot \text{species}}$!

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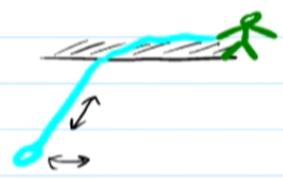
Concretely: current production rate $\approx 10 \frac{\text{particles}}{(\text{km})^3 \text{year} \cdot \text{species}}$!

Experiment: That's much too high! We only have $\approx 10^9 \frac{\text{particles with mass}}{(\text{km})^3}$ ⚡

- Reconsider:
- Recall that any quantum system does not get excited (or only very little), if we change its parameters (e.g. the $\omega_k(\eta)$) "slowly".
 - For the oscillator, "slow", is slow compared to the natural frequency of the oscillator.

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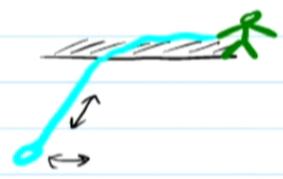


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Only changes in length which occur fast compared to the oscillator's frequency can parametrically excite the oscillator.

- Since the universe presently expands slowly, we should expect essentially no particle production, and indeed we don't see any, experimentally.

→ How to improve our ansatz for vacuum identification?

Idea:

△ Consider models where the universe is initially Minkowski and then undergoes an expansion whose parameter change (of $\omega_k(\eta)$) is slow, i.e., adiabatic.

↑ Note: the overall change may still be large!

⇒ We expect essentially no particle creation.

⇒ The vacuum state (i.e. no particle state) should always stay essentially the same Hilbert space vector.

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21

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- ⇒ We expect essentially, no particle creation.
- ⇒ The vacuum state (i.e. no particle state) should always stay essentially, the same Hilbert space vector.
- ⇒ Since there is only one vacuum state, $|0\rangle$, for all time, there is one mode function, v_k , whose $|0\rangle$ is the vacuum at all time.

How can we find this mode function v_k ?

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□ Easy: We know $v_k(\eta)$ at very early times, when the universe was still Minkowski:

$$v_k(\eta) = \frac{1}{\sqrt{\omega_k}} e^{i\omega_k(\eta - \eta_0)}$$

↑ arbitrary reference time

Then: the K.G. eqn. yields $v_k(\eta)$ at all time!

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We say that a mode k evolves **adiabatically slow**, if:

Intuition:

$\frac{\omega'}{\omega^2}$ and $\frac{\omega''}{\omega^3}$ are rate of change of frequency compared to the frequency, and also rate of acceleration of frequency compared to the frequency.

$$\frac{\omega'_k(\gamma)}{\omega_k^2(\gamma)} \ll 1 \quad \text{and} \quad \frac{\omega''_k(\gamma)}{\omega_k^3(\gamma)} \ll 1 \quad (AC)$$

Note: The denominators are chosen so that the quotients are unitless, because only pure numbers can reasonably be said to be small or large.

□ **Exercise:** Prove the proposition.

Is initial Minkowski period really necessary?

- * Try to identify the v_k whose $|o\rangle$ is the adiabatically defined vacuum without referring to what v_k would look like in an earlier Minkowski period of the universe.
- * Namely, try to identify v_k by a characteristic property that it has at all time.
- * Indeed, we notice: (Exercise: check this)

Our v_k of (5) above satisfies at all times:

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Our v_k of (5) above satisfies at all times:

$$v_k(\eta) = e^{i\theta} \frac{1}{\sqrt{\omega_k(\eta)}}, \quad v_k'(\eta) = \left(i\omega_k(\eta) - \frac{1}{2} \frac{\omega_k'(\eta)}{\omega_k(\eta)} \right) \frac{e^{i\theta}}{\sqrt{\omega_k(\eta)}} \quad (AV)$$

"The general adiabatic vacuum identification"

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- * Consider an arbitrary time η_1 .
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- * We then identify that state as the vacuum $|0\rangle$ (i.e. as the no particle state) at η_1 , whose mode function v_k is specified by the conditions (AV) at η_1 :

$$v_k(\eta) = e^{i\theta} \frac{1}{\omega_k(\eta)}, \quad v_k'(\eta) = (i\omega_k(\eta) - \frac{1}{2}\omega_k'(\eta)) \frac{e^{i\theta}}{\omega_k(\eta)}$$

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- * We call this $|0\rangle$ the "adiabatic vacuum" at η_1 .

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Remarks:

□ Recall that the criteria for choosing v_k so that its $|0\rangle$ is the lowest energy vacuum at time η_1 , are:

$$v_k(\eta_1) = \frac{1}{\sqrt{\omega_k(\eta_1)}} e^{i\theta} , \quad v_k'(\eta_1) = i\sqrt{\omega_k(\eta_1)} e^{i\theta} \quad (E_{25/27})$$

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Remarks:

□ Recall that the criteria for choosing v_k so that its $|\langle 0 \rangle|$ is the lowest energy vacuum at time η , are:

$$v_k(\eta, i) = \frac{1}{\sqrt{\omega_k(\eta, i)}} e^{i\theta}, \quad v_k'(\eta, i) = i\sqrt{\omega_k(\eta, i)} e^{i\theta} \quad (EV)$$

□ Note that **AV** and **EV** generally differ!

⇒ The adiabatically-defined vacuum is generally not the lowest energy state!

□ Note that the adiabatic vacuum criterion should only be applied when the evolution

of the system is considered in all directions

$$v_0(\eta_1) = e^{-\frac{i}{\sqrt{\omega_0(\eta_1)}}}, \quad v_k(\eta_1) = \left(i\omega_0(\eta_1) - \frac{1}{2} \frac{\omega_0'(\eta_1)}{\omega_0(\eta_1)} \right) \frac{e^{-\frac{i}{\sqrt{\omega_0(\eta_1)}}}}{\sqrt{\omega_0(\eta_1)}} \quad (AV)$$

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View in the Schrödinger picture:

- Now, $|4(\eta)\rangle$ evolves in time.
- Also, at every time, a different vector $|0\eta\rangle$ is the vacuum.
- If the evolution is adiabatic, we have that if the system starts in the vacuum $|4(\eta_0)\rangle = |0\eta_0\rangle$, then it stays in the vacuum:

$$|4(t)\rangle = |0\eta\rangle \quad (\text{And stays in } |n\rangle \text{ if starts in } |n\rangle)$$

- Caution, however: As we saw, for this to be true, $|0\eta\rangle$ is not the lowest energy state at η .

Now, $|1(\eta)\rangle$ evolves in time.

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If the evolution is adiabatic, we have that if the system starts in the vacuum $|1(\eta_0)\rangle = |0\eta_0\rangle$, then it stays in the vacuum:

$|1(t)\rangle = |0\eta_t\rangle$ (And stays in $|n\rangle$ if starts in $|n\rangle$)

Caution, however: As we saw, for this to be true, $|0\eta_t\rangle$ is not the lowest energy state at η .

Note: When the parameters stop changing, the adiabatic vacuum becomes the lowest energy state, because then: (AV) becomes $(E \dots)$