

Title: Quantum Fields and Strings

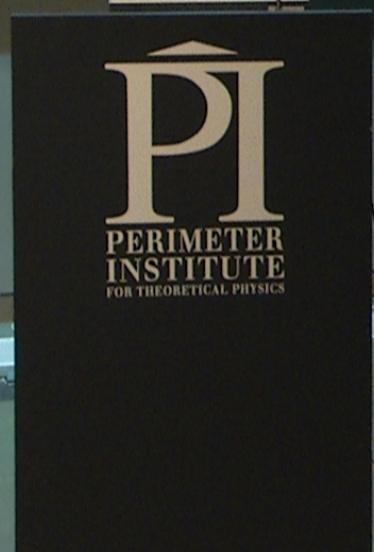
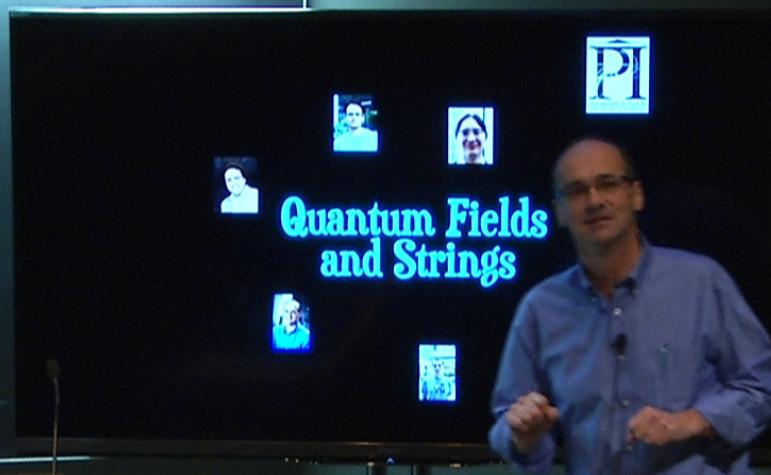
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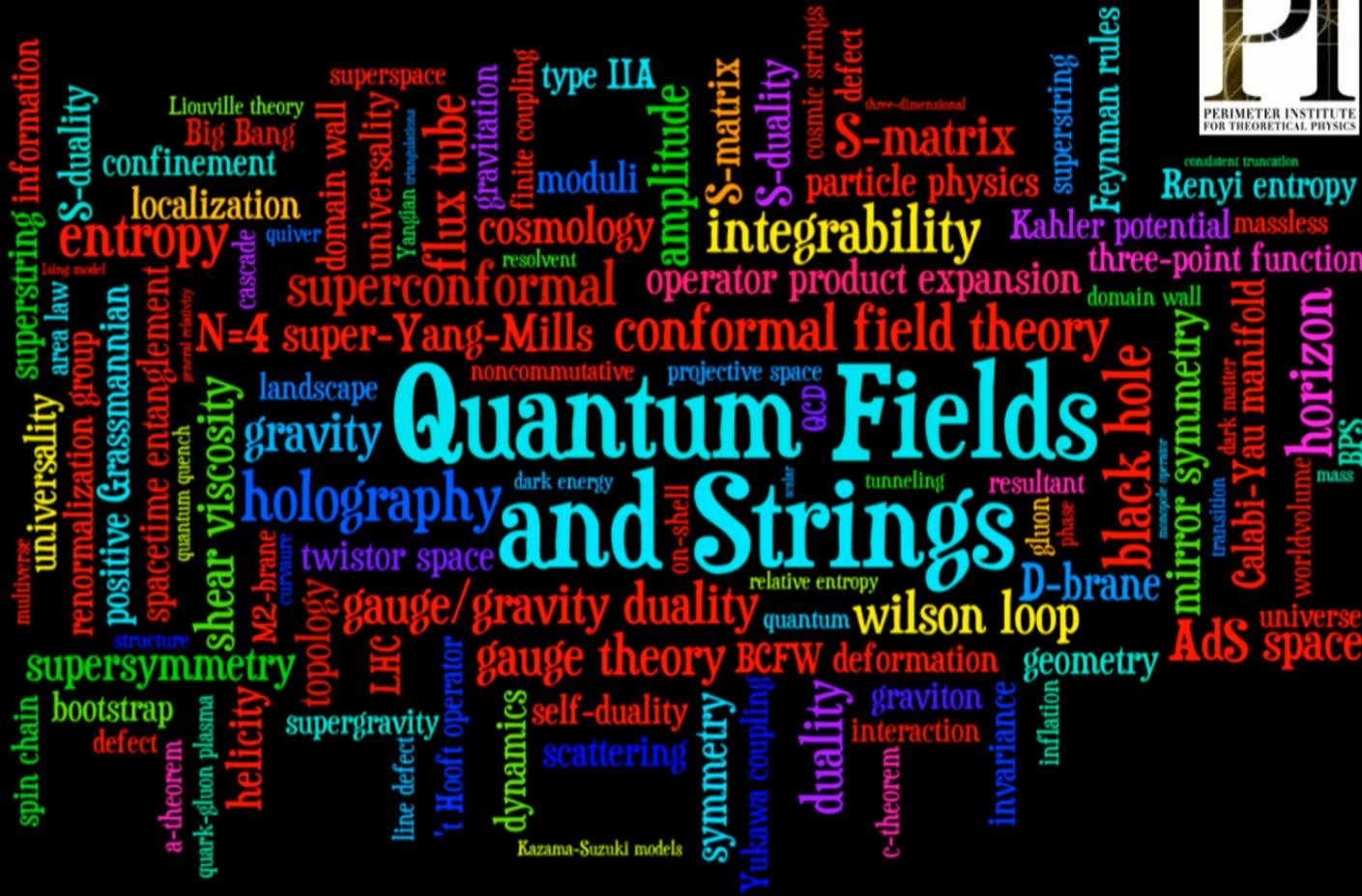
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Abstract:



Quantum Fields and Strings



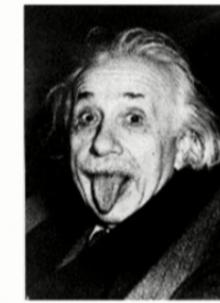


where?*

* according to relativity, where? includes when?

Einstein:

Gravity? It's all just straight lines!

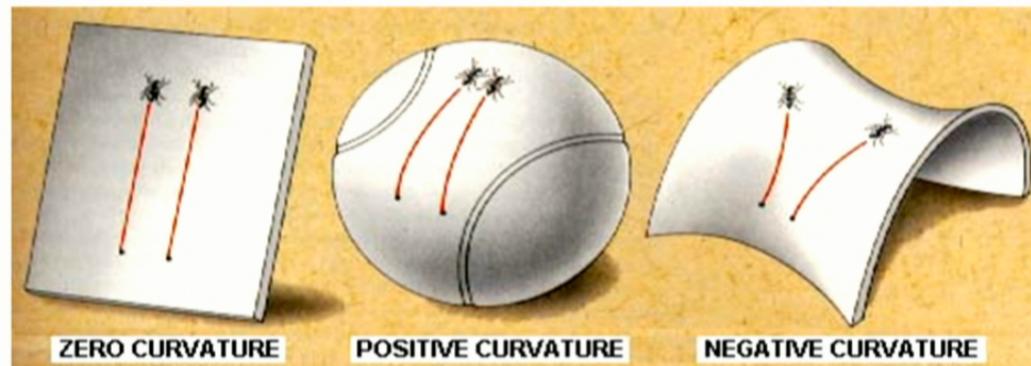


Special Relativity (1905):

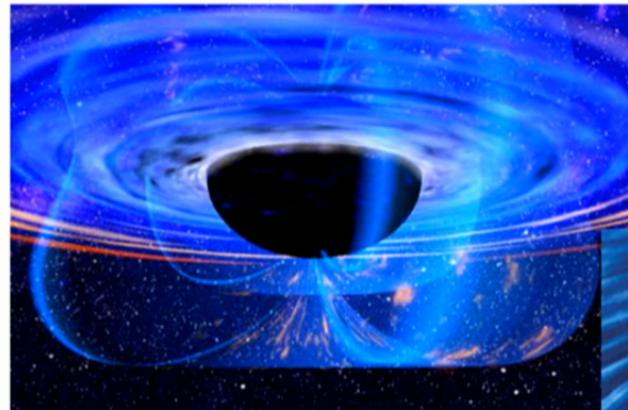
Space and time are inextricably linked → Spacetime

General Relativity (1915):

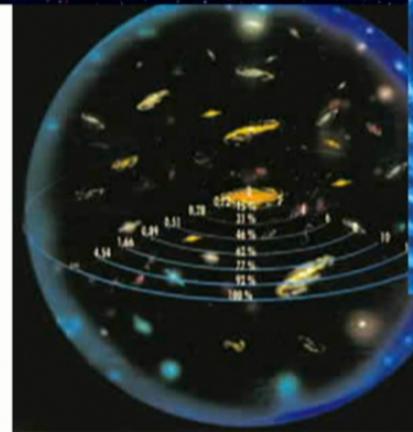
Gravity is a manifestation of spacetime curvature.



General Relativity: a rich source of new ideas



Black holes

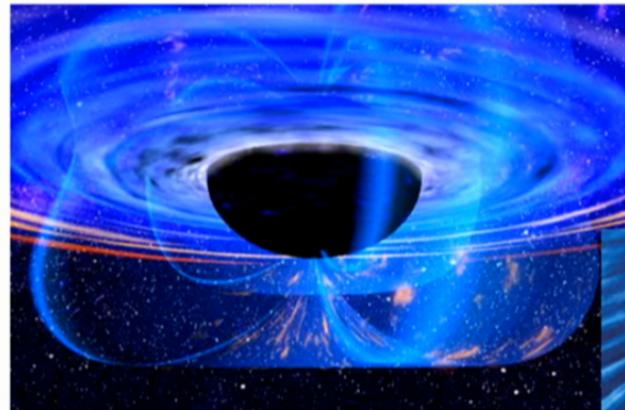


Gravitational waves

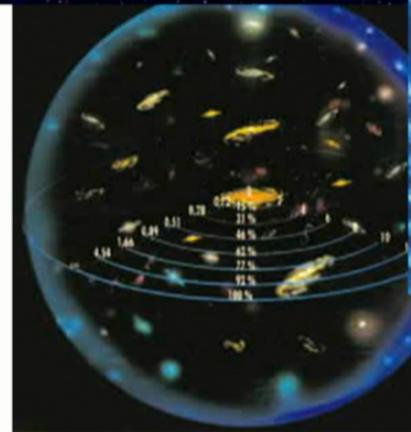


Expanding universe

General Relativity: a rich source of new ideas



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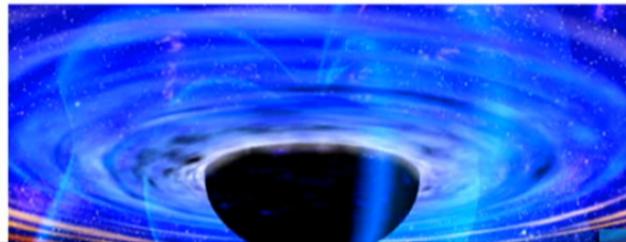


Gravitational waves



Expanding universe

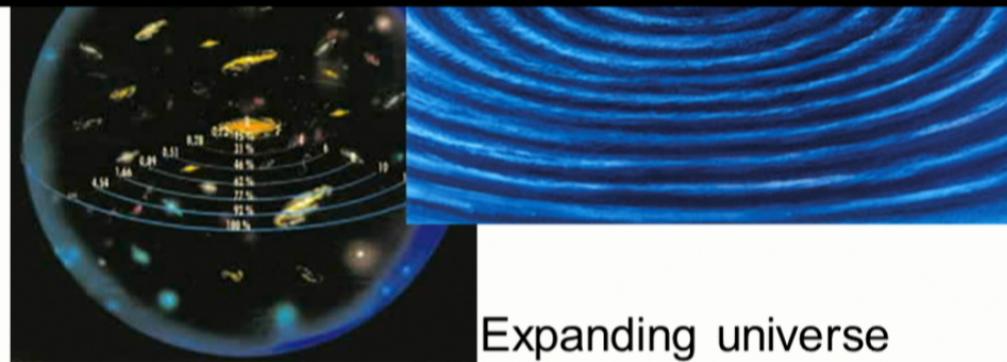
General Relativity: a rich source of new ideas



Black holes

Gravitational waves

Spacetime moves from providing a stage for physical phenomena, to being both the stage and an active player in the dynamics



Expanding universe

**What defines the quantum
structure of spacetime?**

Spacetime = Entanglement

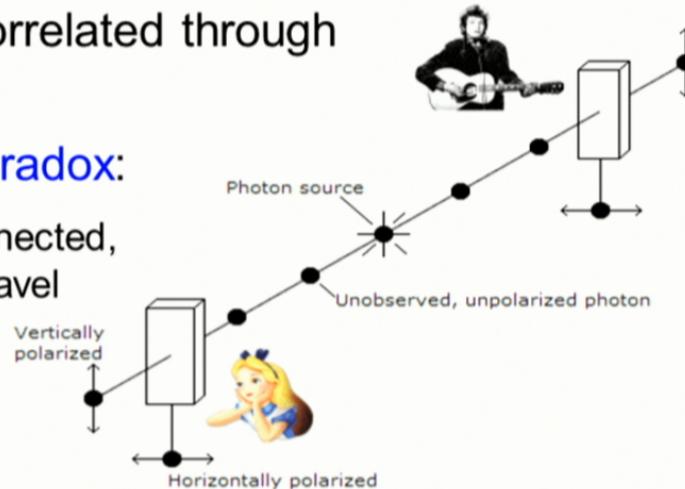
Quantum Entanglement

- different subsystems are correlated through global state of fully system

Einstein-Podolsky-Rosen Paradox:

- properties of pair of photons connected, no matter how far apart they travel

“*spukhafte Fernwirkung*” =
spooky action at a distance



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

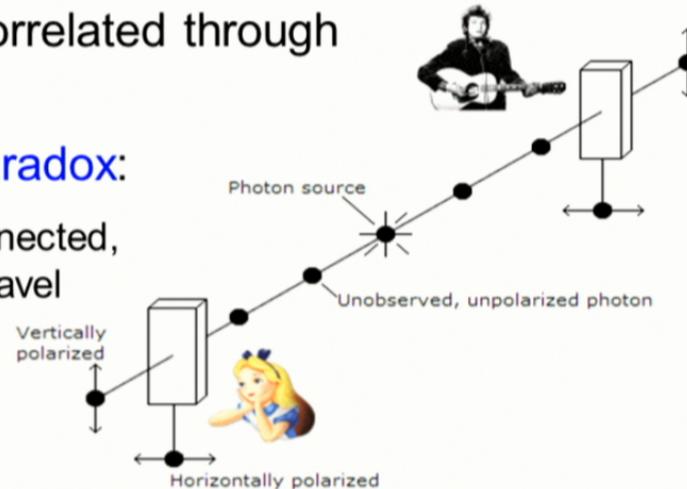
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Quantum Information: entanglement becomes a resource for (ultra)fast computations and (ultra)secure communications

Condensed Matter: key to “exotic” phases and phenomena, e.g., quantum Hall fluids, unconventional superconductors, quantum spin fluids,

Black Hole Entropy:

- Bekenstein and Hawking: event horizons have entropy!

$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{\mathcal{A}}{4G}$$

- extends to de Sitter horizons and Rindler horizons

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thermodynamics relativity geometry

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thermodynamics relativity geometry

quantum gravity

```
graph TD; A[thermodynamics] --> TopTerm[k_B c^3 / hbar]; B[relativity] --> BottomTerm[A / (4G)]; C[geometry] --> BottomTerm; D[quantum gravity] --> BottomTerm;
```

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- window into quantum gravity?!?

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graph TD; A[thermodynamics] --> BC[k_B c^3 / hbar]; B[relativity] --> BC; C[geometry] --> BC; BC --> D["S_BH = A / (4G)"]; D --- E[quantum gravity]
```

- extends to de Sitter horizons and Rindler horizons
- window into quantum gravity?!?
- quantum gravity provides a fundamental scale

$$\ell_P^2 = 8\pi G \hbar/c^3$$

Black Hole Entropy:

- Bekenstein and Hawking: event horizons have entropy!

$$S_{BH} = 2\pi \frac{\mathcal{A}}{\ell_P^2} + \dots$$

Where is the “Entanglement Entropy” in the black hole story?

- Sorkin: looking for origin of black hole entropy
 - in QFT, found leading term obeys “area law”: $S = c_0 \frac{\mathcal{A}_\Sigma}{\delta^2} + \dots$
- suggestive of Bekenstein-Hawking formula if $\delta \simeq \ell_P$

(Sorkin '84; Bombelli, Koul, Lee & Sorkin; Srednicki; Frolov & Novikov; ...)

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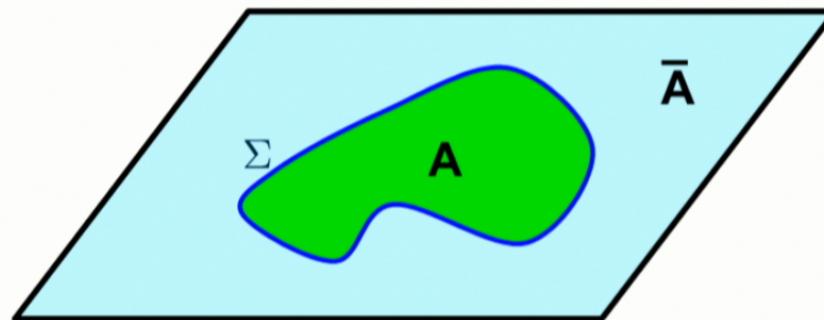
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- (Sorkin '84; Bombelli, Koul, Lee & Sorkin; Srednicki; Frolov & Novikov; ...)
- issues?: leading singularity not universal; regulator dependent
 - active topic in 90's but story left unresolved; recently, many are coming back to this idea with new perspectives (eg, Bianchi '12)

Proposal: Spacetime Entanglement

(Bianchi & RM)

- in a theory of quantum gravity, for any sufficiently large region A in a smooth background, consider entanglement entropy between dof describing A and \bar{A} ; contribution describing short-range entanglement is finite and described in terms of geometry of entangling surface with leading term:

$$S_{\text{EE}} = \frac{\mathcal{A}_\Sigma}{4G_N} + \dots$$



- higher order terms similar to Wald entropy (RM, Pourhasan & Smolkin)

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- evidence:
 1. holographic S_{EE} in AdS/CFT correspondence
 2. QFT renormalization of G_N
 3. induced gravity, eg, Randall-Sundrum 2 model
 4. Jacobson's "thermal origin" of gravity
 5. spin-foam approach to quantum gravity

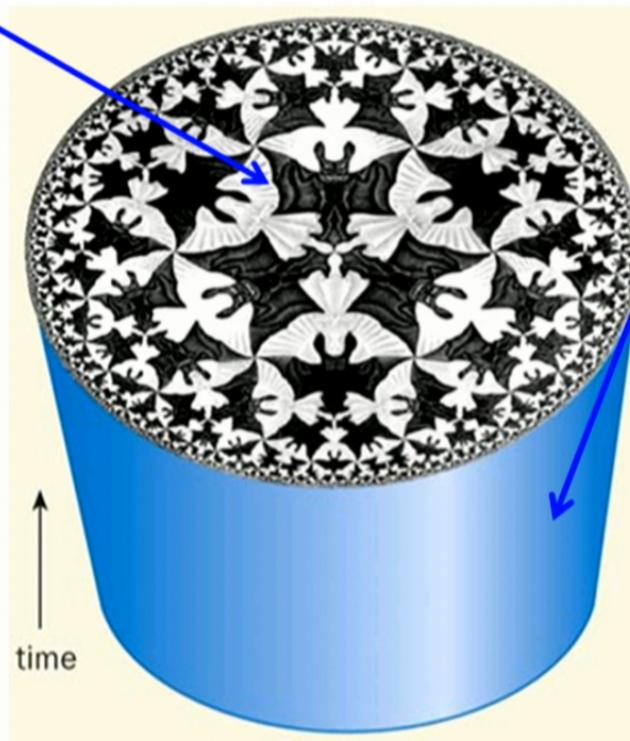
AdS/CFT Correspondence:

Bulk: gravity with negative Λ
in $d+1$ dimensions

anti-de Sitter
space

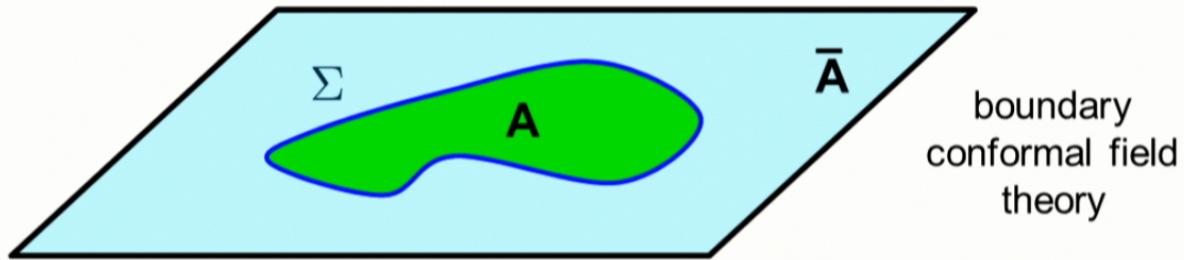
Boundary: quantum field theory
without intrinsic scales
in d dimensions

conformal
field theory



(Ryu & Takayanagi '06)

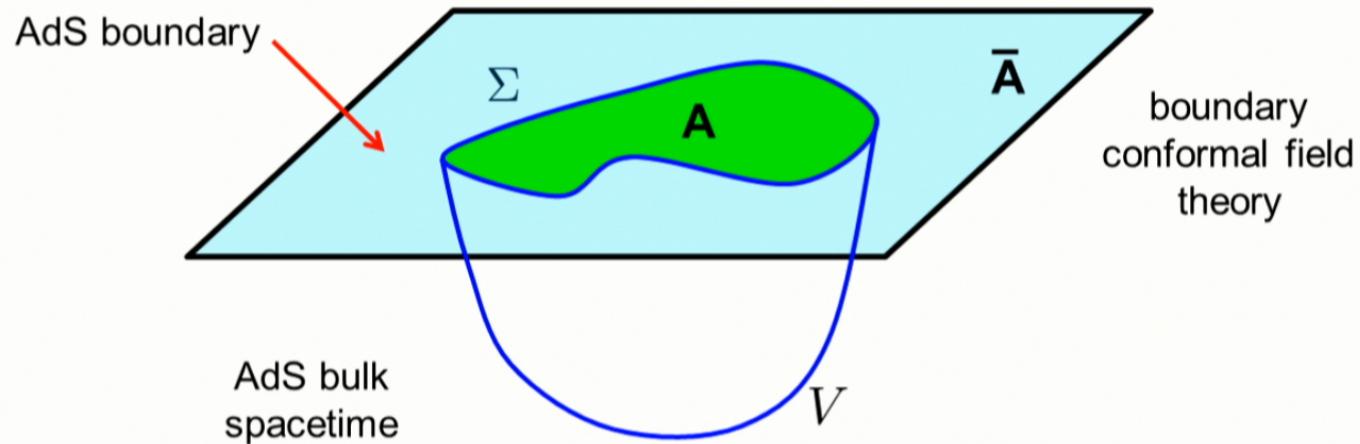
Holographic Entanglement Entropy:



$$S(A) = ???$$

(Ryu & Takayanagi '06)

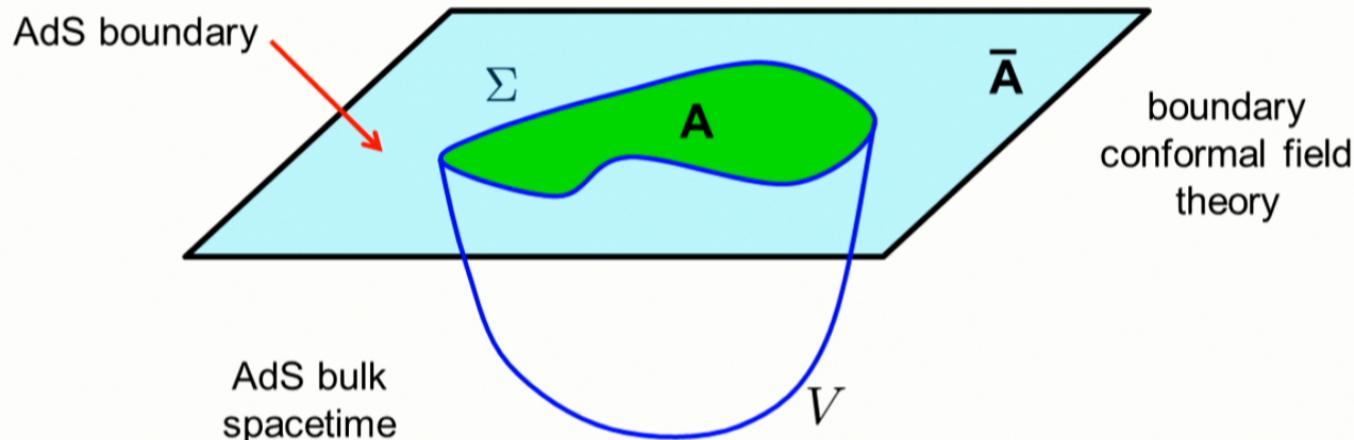
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$$S(A) = \min_{\partial V = \Sigma} \frac{A_V}{4G_N}$$

(Ryu & Takayanagi '06)

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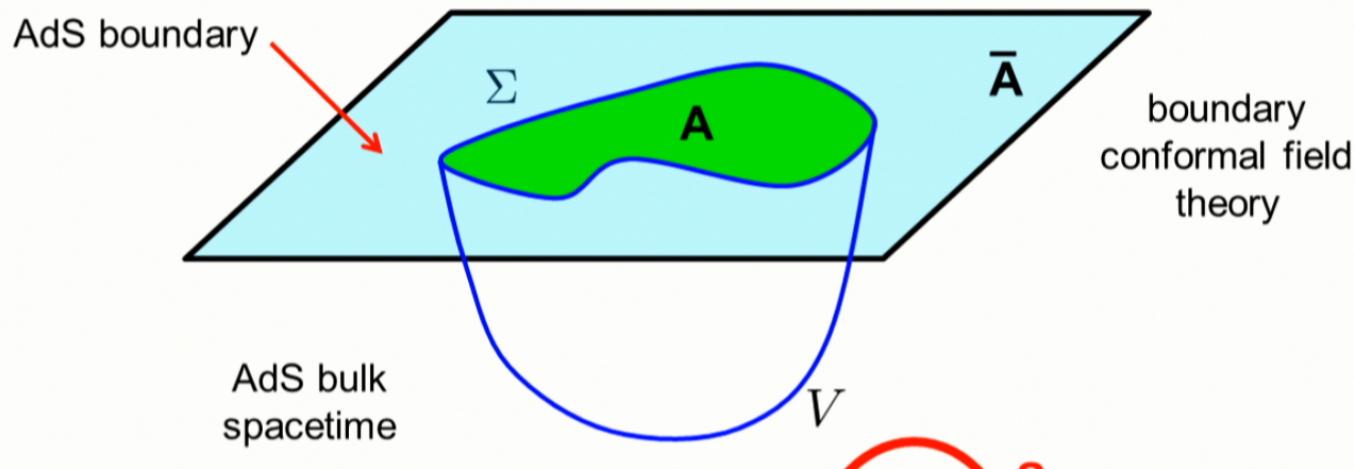


$$S(A) = \min_{\partial V = \Sigma} \frac{A_V}{4G_N}$$

- conjecture \longrightarrow many detailed consistency tests
(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Faulkner, . . .)
- **proof!!** \longrightarrow “generalized gravitational entropy”
(Lewkowycz & Maldacena)

(Ryu & Takayanagi '06)

Holographic Entanglement Entropy:



$$S(A) = \min_{\partial V = \Sigma} \frac{A_V}{4G_N} S_{BH}$$

- conjecture \longrightarrow many detailed consistency tests
(Ryu, Takayanagi, Headrick, Hung, Smolkin, RM, Faulkner, . . .)
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Lessons from Holographic EE:

$(\text{entanglement entropy})_{\text{boundary}}$

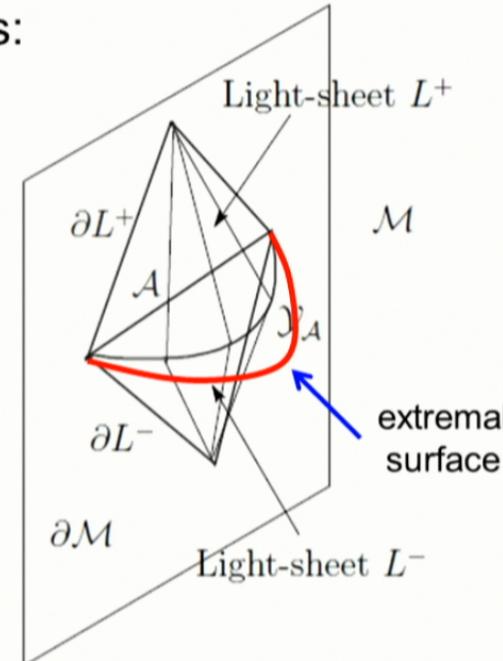
$= (\text{entropy associated with extremal surface})_{\text{bulk}}$

- R&T construction assigns entropy $S_{BH} = \mathcal{A}/(4G_N)$ to bulk regions with “unconventional” boundaries:

not black hole! not horizon!
not boundary of causal domain!

- indicates S_{BH} applies more broadly

→ spacetime entanglement proposal:
 S_{BH} defines an entropy in bulk
gravity for any surface



Lessons from Holographic EE:

$(\text{entanglement entropy})_{\text{boundary}}$

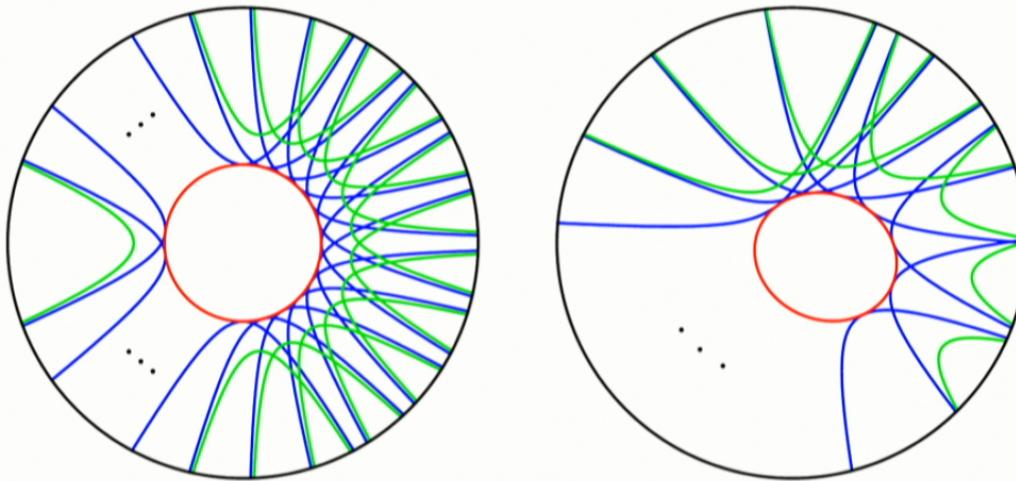
$= (\text{entropy associated with extremal surface})_{\text{bulk}}$

- R&T construction assigns entropy $S_{BH} = \mathcal{A}/(4G_N)$ to bulk regions with “unconventional” boundaries:
- with our proposal, S_{BH} assigns an entropy in bulk for any surface
- what about extremization?
 - needed to make precise match above
- S_{BH} on other surfaces already speculated to give other entropic measures of entanglement in boundary theory
 - entanglement between high and low scales
(Balasubramanian, McDermott & van Raamsdonk)
 - causal holographic information
(Hubeny & Rangamani; H, R & Tonni; Freivogel & Mosk; . . .)

“Hole-ography”:

- there are observables in the boundary CFT that measure the BH entropy of closed surfaces in the AdS bulk

$$\frac{\mathcal{A}}{4G_N} = \sum_{k=1}^{\infty} [S(I_k) - S(I_k \cap I_{k+1})]$$



D=3: Balasubramanian, Chowdhury, Czech, de Boer & Heller

higher D: RM, Rao, Wien

“Hole-ography”:

$$\mathcal{A}/(4G_N) = \sum_{k=1}^{\infty} S(I_k) - \sum_{k=1}^{\infty} S(I_k \cap I_{k+1})$$

- $S_{\text{BH}} = A/4G_N$ for arbitrary closed surfaces in bulk AdS is equivalent to a certain combination of (S_{EE})'s in bdry theory
→ microscopic prescription for “geometric entropy” appearing in spacetime entanglement conjecture

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$$\mathcal{A}/(4G_N) = \sum_{k=1}^{\infty} S(I_k) - \sum_{k=1}^{\infty} S(I_k \cap I_{k+1})$$

- $S_{\text{BH}} = A/4G_N$ for arbitrary closed surfaces in bulk AdS is equivalent to a certain combination of (S_{EE})’s in bdry theory
→ microscopic prescription for “geometric entropy” appearing in spacetime entanglement conjecture
- does \hat{S} have a direct interpretation in terms of bdry theory? 

Residual entropy?: $S_{\text{res}} = \max S(\rho) \mid \forall_i \text{Tr}_i \rho = \rho_i$

- related to “quantum marginal problem”

(Balasubramanian, Chowdhury, Czech, de Boer & Heller)
(see also: Kelly & Wall)

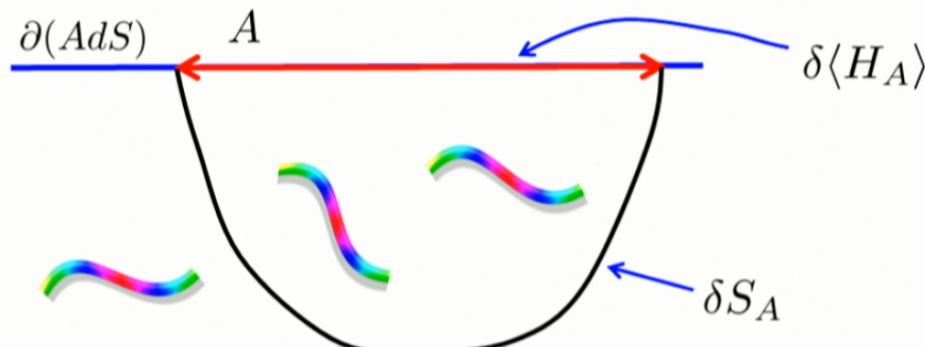
“Derivative” of entanglement entropy?: $\int d\theta \left. \frac{\partial S}{\partial \alpha} \right|_{\alpha(\theta)} \sim \frac{\mathcal{A}}{G_N}$

- seems related to Takayanagi’s “entanglement density”

(Nozaki, Numasawa & Takayanagi)

Gravitation from Entanglement:

- relative entropy: $S(\rho_1|\rho_0) = \text{tr}(\rho_1 \log \rho_1) - \text{tr}(\rho_1 \log \rho_0)$
- let: ρ_0 = reference state; ρ_1 = perturbed state
→ “1st law” of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$
- holographic realization: (Blanco, Casini, Hung & RM)



- apply 1st law for spheres of all sizes, positions and in all frames:

1st law of SEE \longleftrightarrow **bulk geometry satisfies linearized Einstein eq's**

(Lashkari, McDermott & Van Raamsdonk)
(Faulkner, Guica, Hartman, RM & Van Raamsdonk)

Conclusions:

- holographic S_{EE} suggests new perspectives
→ quantum information & entanglement are keys to fundamental issues in string theory/quantum gravity



- spacetime entanglement: S_{BH} applies for generic large regions
- “hole-ography” points to a precise definition in AdS/CFT context
- entanglement goes beyond geometry; knows gravity dynamics

Lots to explore!

