Title: Whatâ€™s Happening in Cosmology?

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Abstract:

What's happening in cosmology?

Kendrick Smith and Latham Boyle
PI Day, January 2014

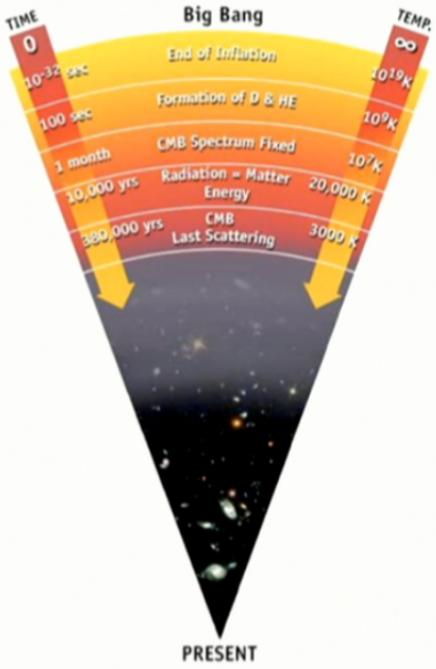
Outline

Standard cosmological model

CMB: a cosmology success story

What's next?

Standard cosmological model



Thermal history: early universe is hot dense plasma with small ($\sim 10^{-5}$) perturbations propagating as sound waves

At z ~ 1100, temperature is 3000 Kelvin. Free protons and electrons combine to form neutral hydrogen ("recombination"); universe becomes transparent.

Small perturbations grow via gravitational instability, to order-one perturbations at z~0. Photons which have been freestreaming since z=1100 are observed as the CMB.

Standard cosmological model

Expansion history: universe (neglecting perturbations) is described by FRW metric

$$ds^2 = -dt^2 + a(t)^2 dx^2$$

where the scale factor a(t) evolves via Friedmann's equation

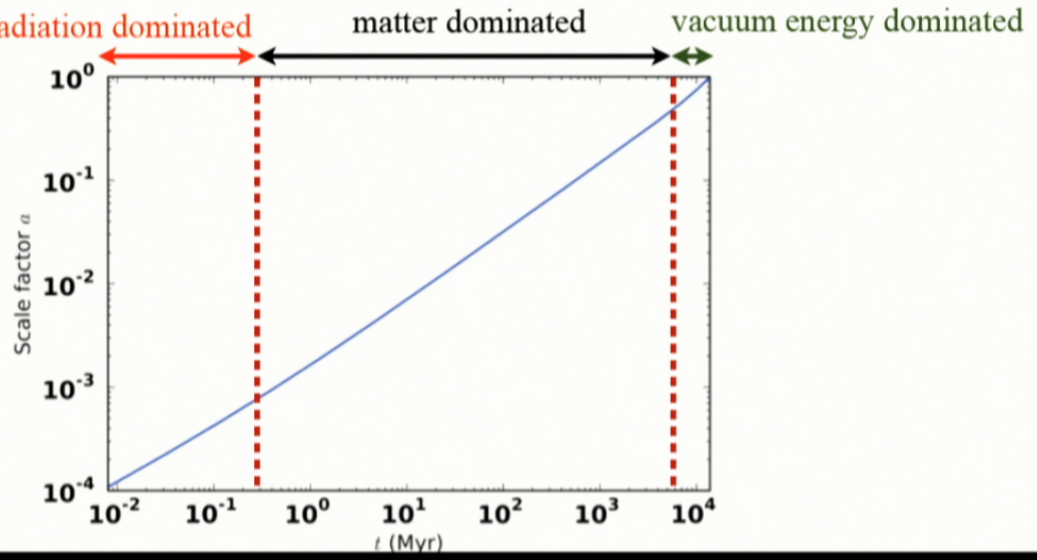
$$H(a) = \frac{d\log a}{dt} = \left(\frac{8\pi G}{3}\rho_{\rm tot}\right)^{1/2}$$

(Energy density in radiation) $\propto a^{-4}$
(Energy density in matter) $\propto a^{-3}$
(Cosmological constant) $\propto a^{0}$

Standard cosmological model

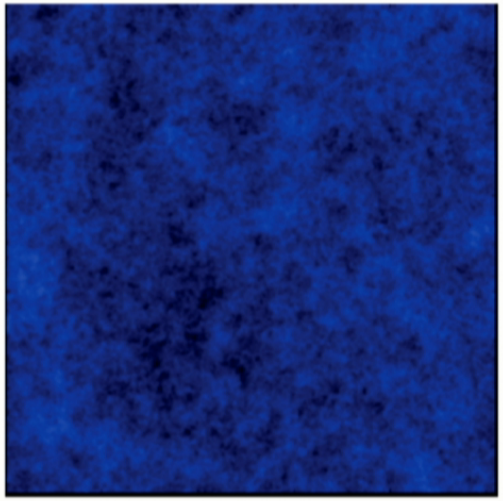
Cosmologists' parametrization:
(3 parameters since $\Omega_b + \Omega_c + \Omega_\Lambda = 1$)

$$\begin{cases} H_0 = \text{Hubble parameter at } z = 0 \\ \Omega_\Lambda = \rho_\Lambda / \rho_{\rm tot} \text{ at } z = 0 \\ \Omega_c = \rho_c / \rho_{\rm tot} \text{ at } z = 0 \\ \Omega_b = \rho_b / \rho_{\rm tot} \text{ at } z = 0 \end{cases}$$



Perturbations: initial perturbations are "adiabatic"

ζ = time delay between constant density and spatially flat slicings



ζ is a nearly scale-invariant Gaussian field

Each Fourier mode $\zeta(\mathbf{k})$ is an independent Gaussian random variable with variance

$$\langle \zeta(\mathbf{k})\zeta(\mathbf{k}')^* \rangle = A_\zeta k^{n_s-4}(2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')$$

(Scale-invariant corresponds to $n_s = 1$)

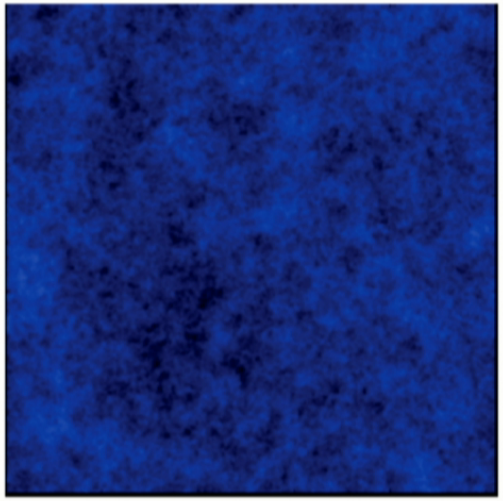
In this talk, "standard cosmological model" means 6 parameters:

$$\{H_0, \Omega_c, \Omega_b, A_\zeta, n_s, z_{\rm rei}\}$$

nuisance parameter: redshift of reionization

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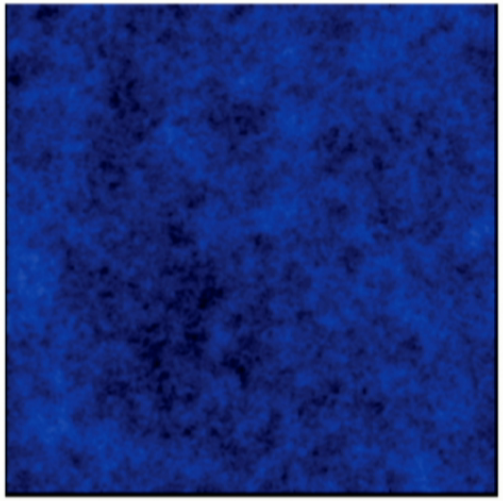
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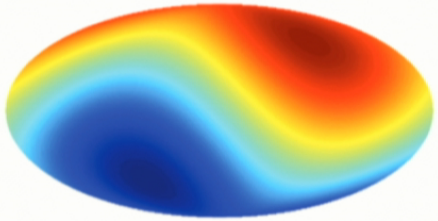
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What's next?

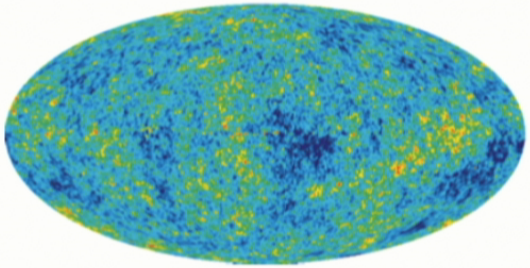
CMB sky



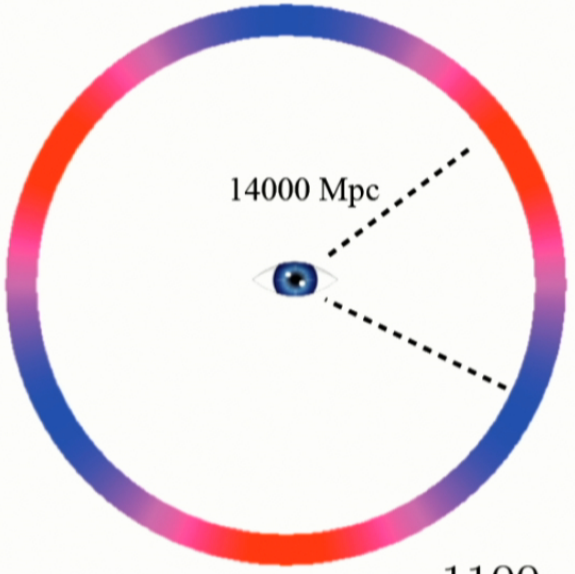
monopole: blackbody, 2.7255 K



dipole: 3 mK, from motion of Earth



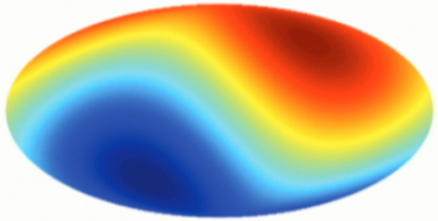
higher anisotropy: 100 μK, "snapshot of universe at z=1100"



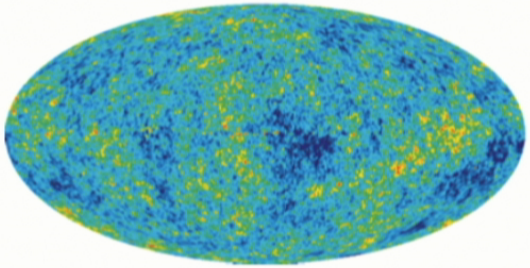
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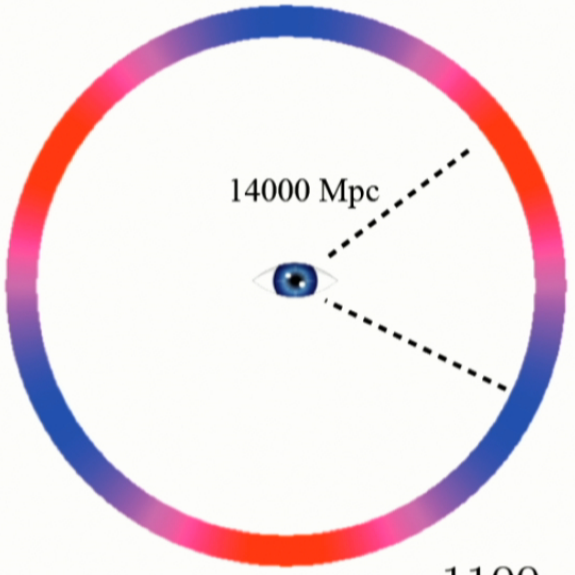
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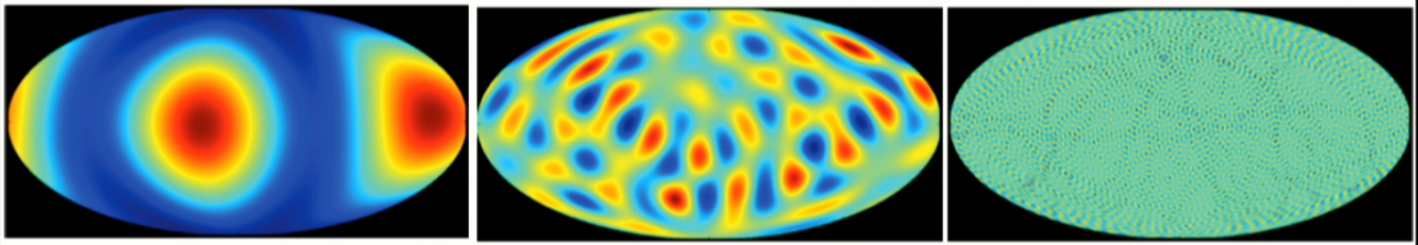
Spherical harmonic representation

Represent full-sky temperature in spherical harmonic basis:

$$\Delta T(\theta,\phi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta,\phi)$$

$(\Delta T) \rightarrow a_{\ell m}$ is analog of Fourier transform on the sphere

ℓ = "wavelengths per 360 degrees on sky"



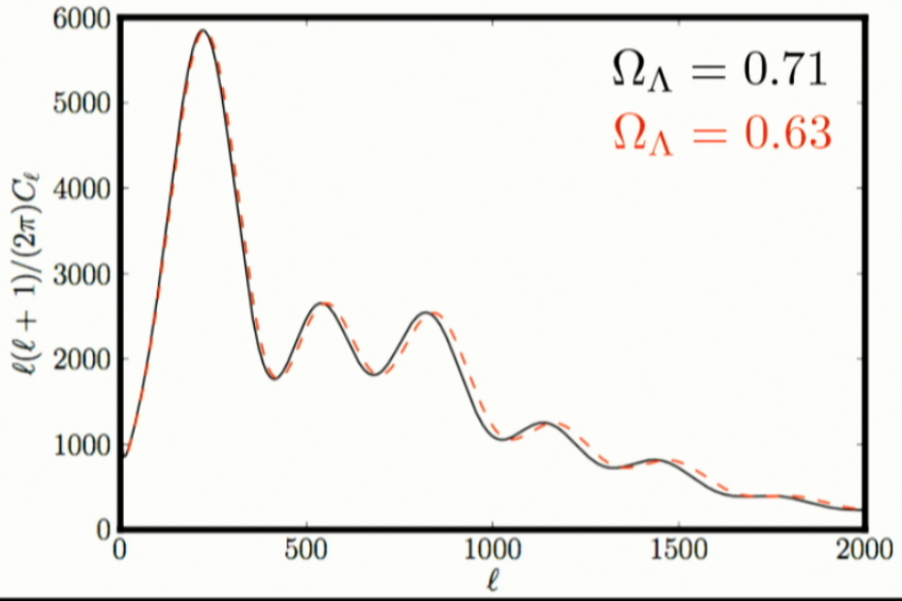
$\ell = 2$ $\ell = 10$ $\ell = 100$

Power spectrum

Standard cosmological model predicts: each $a_{\ell m}$ is an independent Gaussian random variable with ℓ-dependent variance

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \, \delta_{\ell \ell'} \delta_{m m'}$$

The power spectrum C_ℓ depends on cosmological parameters

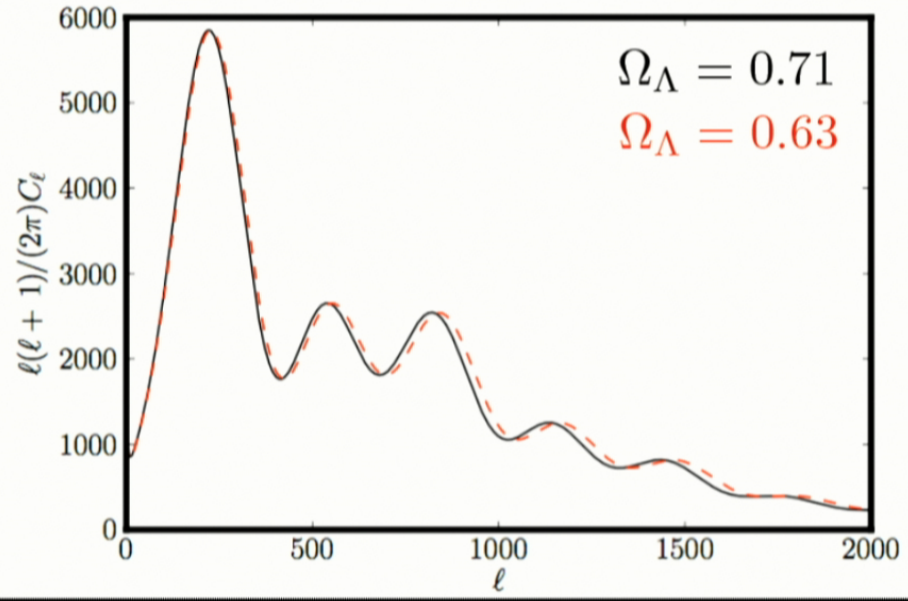


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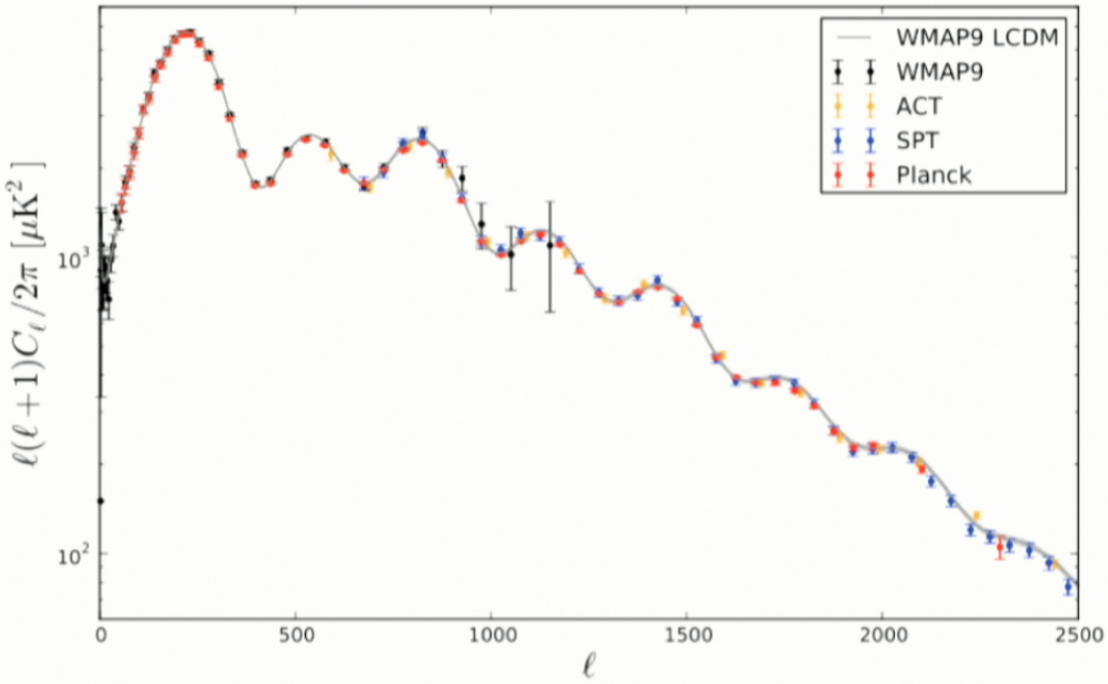
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The power spectrum C_ℓ depends on cosmological parameters



Current measurements of the CMB power spectrum are fully consistent with the standard model



Standard model constraints

$$\Omega_b h^2 = 0.02205 \pm 0.00028$$
$$\Omega_c h^2 = 0.1199 \pm 0.0027$$
$$\Omega_\Lambda = 0.685 \pm 0.017$$
$$A_s = (2.1955 \pm 0.0549) \times 10^{-9}$$
$$n_s = 0.9603 \pm 0.0073$$
$$\tau = 0.089 \pm 0.013$$

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$$\Omega_b h^2 = 0.02205 \pm 0.00028$$ baryon asymmetry!

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Although the model is simple, it contains
some of the deepest mysteries in theoretical physics

Deviations from standard model

Some 1-parameter extensions to the standard model
In all cases, the 95% confidence region includes the SM value

Curvature	$-0.0071 < \Omega_K < 0.0060$
Neutrino mass	$\sum m_\nu < 0.230$ eV
No. of neutrino species	$2.79 < N_{\rm eff} < 3.84$
Primordial gravity waves	$r < 0.111$
Running spectral index	$-0.031 < dn_s/(d\log k) < 0.002$
Dark energy equation of state	$-1.38 < w < -0.90$
Primordial non-Gaussianity	$-8.9 < f_{NL}^{\rm loc} < 14.3$
	$-192 < f_{NL}^{\rm equil} < 108$
	$-103 < f_{NL}^{\rm ortho} < 53$

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Future experiments will need to solve many hard problems (hardware, modeling, and statistical). It's easy to be pessimistic in the face of hard problems. However, historical evidence suggests a perspective of cautious optimism…

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- Peebles: "I did not continue (with computation of CMB anisotropy), in part because I had trouble imagining that such tiny disturbances could be observed"
- Sunyaev: "I did not think that the acoustic oscillation would ever be observed"
- Mukhanov: "I thought it would take 1000 years to detect the logarithmic dependence of the power spectrum"

What's next?

Observational cosmology is largely concerned with enumerating possible extensions of the 6-parameter model and improving associated parameter constraints:

$$\left\{ \sum m_\nu, r, \frac{dn_s}{d\log k}, \Omega_K, N_\nu, w, f_{NL}, \ \ldots \right\}$$

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…but dark matter and cosmological constant were also surprises…

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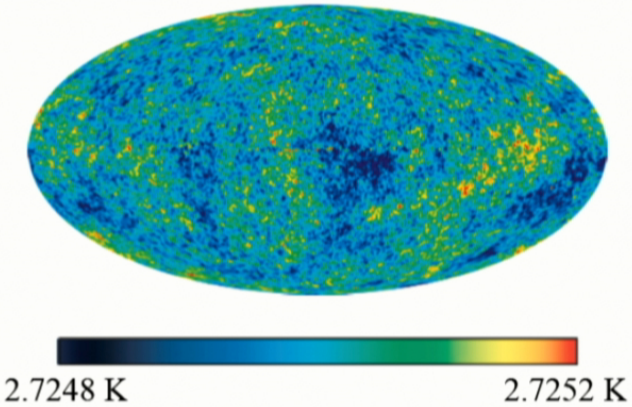
unclear what to expect!
very interesting in near future

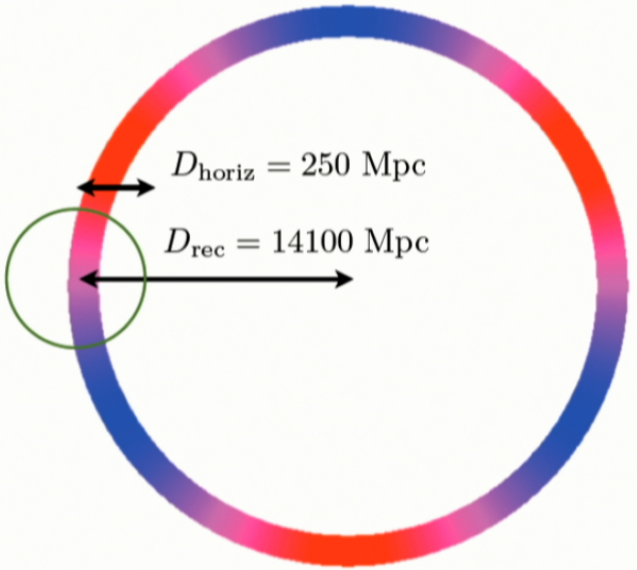
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Inflation: lightning review

Inflation was originally proposed as a solution to the horizon problem: CMB is nearly isothermal, suggesting that all parts of the last scattering surface were once in causal contact

However, the causal horizon at last scattering is much smaller: points separated by $> 1^\circ$ have never been in causal contact





Inflation: lightning review

Simplest model: single-field slow-roll inflation

Radiation dominated expansion is preceded by a stage in which the energy density of the universe is dominated by a scalar field slowly rolling down a potential V(ϕ).

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2}(\partial\phi)^2 - V(\phi) \right)$$

If the potential is flat, in the sense that

$$\epsilon = \frac{M_{\rm Pl}^2}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2$$

$$\eta = M_{\rm Pl}^2 \left(\frac{V''(\phi)}{V(\phi)} \right)$$

are both small, then horizon is large at early times

Inflation: lightning review

Generation of perturbations: a famous calculation shows that quantum mechanical perturbations are generated during inflation with the correct statistical properties to be the initial conditions for our universe!

$$P_\zeta(k) = A_\zeta \left(\frac{k}{k_0}\right)^{n_s-4} \qquad n_s - 1 = 2\eta - 6\epsilon$$

Inflation also predicts a Gaussian background of gravity waves

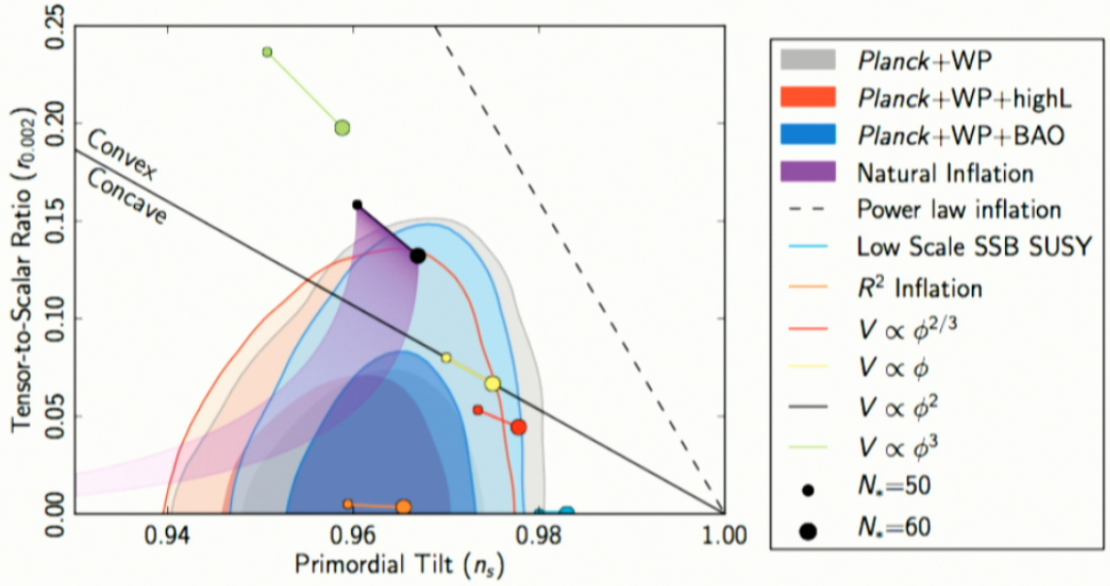
$$P_{\rm gw}(k) = r A_\zeta \left(\frac{k}{k_0}\right)^{n_t-3} \qquad r = 16\epsilon \qquad n_t = 2\epsilon$$

This calculation assumes a classical slow-roll trajectory and a quantum-mechanical initial state with no particles (Bunch-Davies vacuum). Does this solve the initial conditions problem?

CMB constraints

Many inflationary models can be compared to CMB data by simply locating them in the (n_s, r) plane

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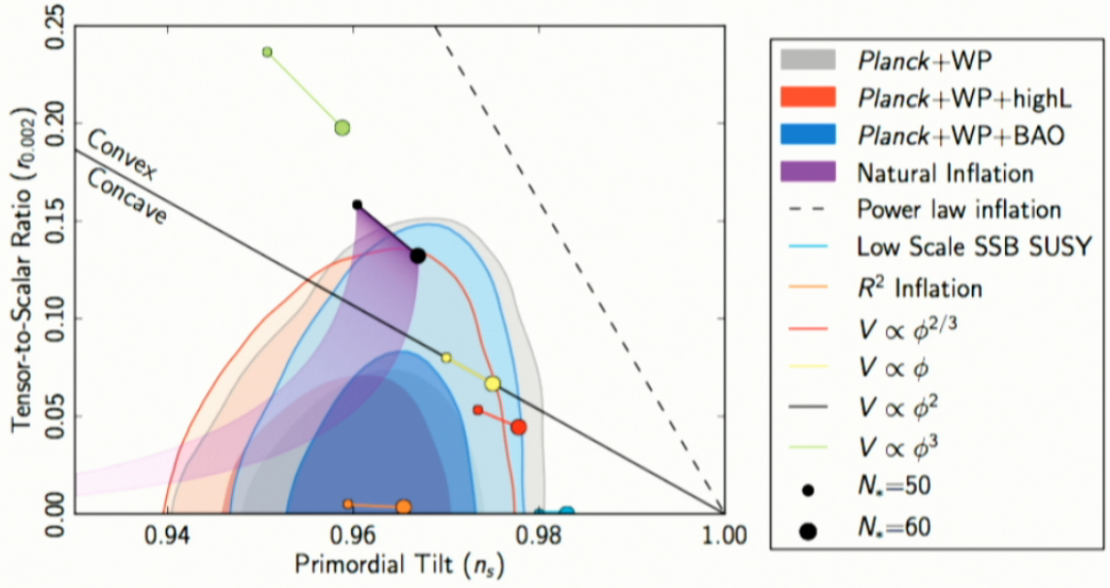
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Large improvement in r is possible

In the future, high-sensitivity measurements of CMB polarization will dramatically improve upper limits on r

Now: $r < 0.11$ (CMB temperature)
$r < 0.72$ (CMB polarization)

Ultimate limit: $r < 0.11$ (CMB temperature)
$r \lesssim 10^{-6}$ (CMB polarization!)

A detection of primordial gravity waves would be amazing!
Do we expect to see a signal?

Do we expect to see gravity waves?

It has been argued that inflation "generically" predicts r ~ 0.1

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$$V^{1/4} = (3 \times 10^{16}\ {\rm GeV}) \times r^{1/4}$$

Ekpyrotic/cyclic models unambiguously predict tiny r

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Mode-counting

For many cosmological parameters, large-scale structure will ultimately be much more powerful than the CMB, essentially by mode-counting

CMB: 2d field, l_{max}=2000
$\Rightarrow$ (4 x 10^6 modes)

Large-scale structure: 3d field, k_{max}=0.5 Mpc^{-1}, z_{max}=1.0
$\Rightarrow$ (2 x 10^8 modes)

Ultimate futuristic scenario: large-scale structure at high redshift, measured via 21-cm line. In this case there is essentially no theoretical limit to the number of modes which might ultimately be measured (number of modes larger than the Jeans scale is 10^{18})

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Example: neutrino mass

Neutrino oscillation experiments measure Δm^2_ν between species

Current analysis of world data:
$$\Delta m^2_{31} = (0.049 \pm 0.0012 \text{ eV})^2$$
$$\Delta m^2_{21} = (0.0087 \pm 0.00013 \text{ eV})^2$$

Cosmology is complementary: LSS is mainly sensitive to $\sum_\nu m_\nu$

Cosmological upper limit (CMB + BAO):

$$\sum m_\nu < 0.230 \text{ eV} \qquad (95\% \text{ CL})$$

Minimum value allowed by oscillation experiments is ~0.06 eV

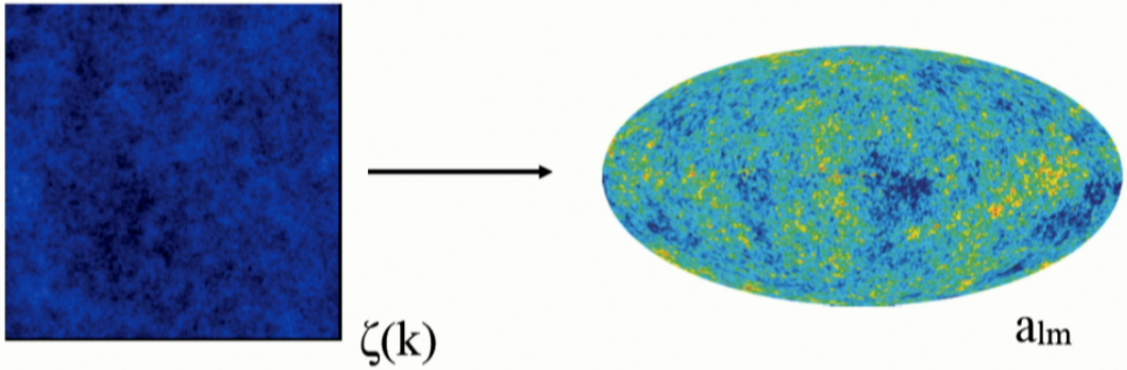
Next-generation large-scale structure experiments should easily reach this level (e.g. Euclid/LSST/DESI each forecast $\sigma(m_\nu) \sim 0.02$ eV)

Statistical challenges

For data analysis purposes, CMB can be treated as a complicated but well-understood "linear black box": observed a_{lm}'s are linear in the initial perturbation $\zeta(k)$

$$a_{lm} = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \Delta_\ell(k) Y^*_{\ell m}(\hat{\mathbf{k}}) \zeta(\mathbf{k})$$

transfer function (computed numerically given cosmological parameters)



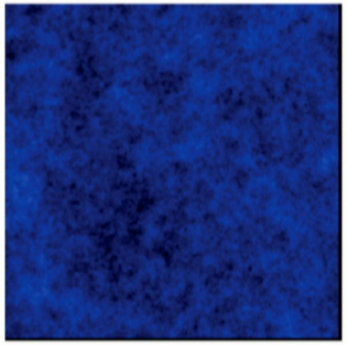
$\zeta(k)$ a_{lm}

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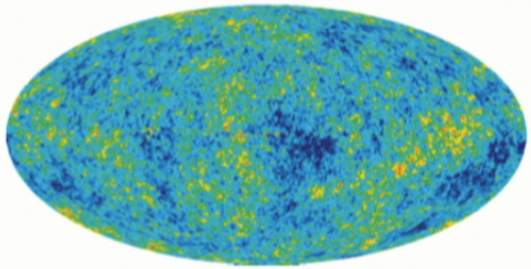
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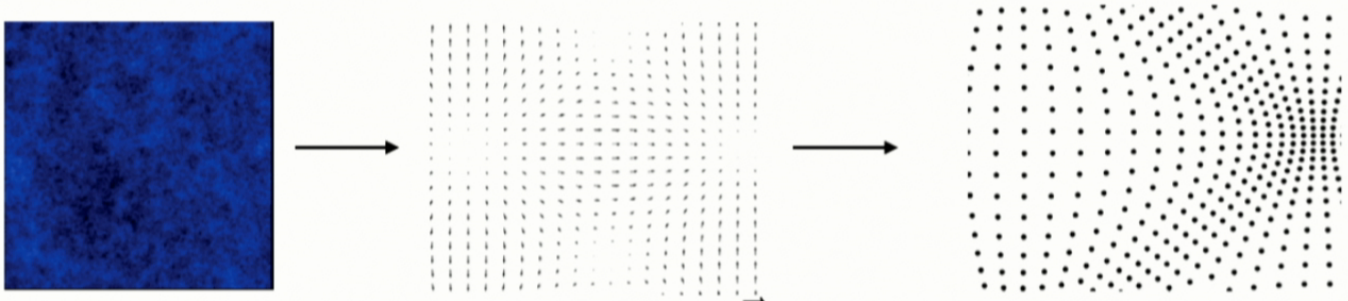
a_{lm}

Statistical challenges

Recent developments ("effective field theory of large-scale structure") suggest that a simple picture is possible

E.g. a toy model which is surprisingly accurate: displace a uniform grid of particles by a vector field of the form

$$\vec{S}(x) = \vec{\nabla} \int \frac{d^3k}{(2\pi)^3} A(k)\zeta(k)e^{ik\cdot x}$$

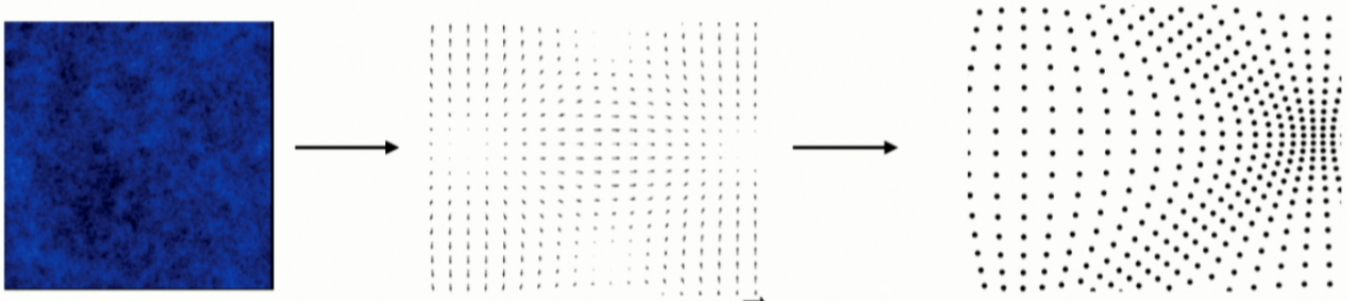


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Recent developments ("effective field theory of large-scale structure") suggest that a simple picture is possible

E.g. a toy model which is surprisingly accurate: displace a uniform grid of particles by a vector field of the form

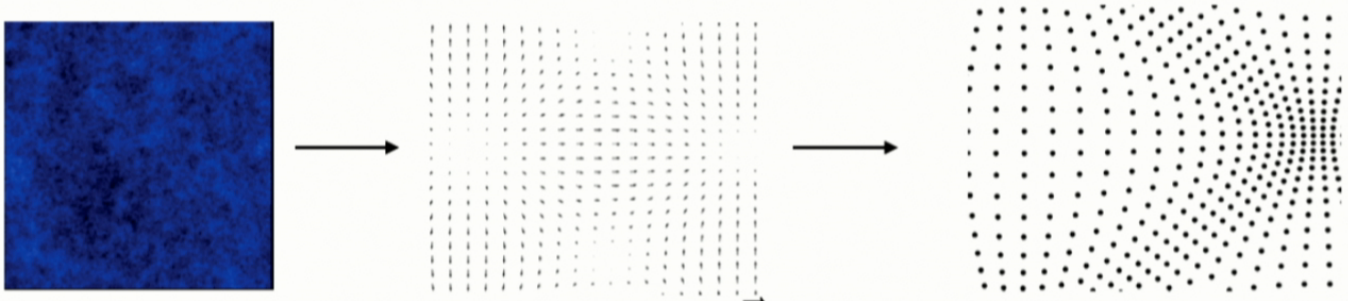
$$\vec{S}(x) = \vec{\nabla} \int \frac{d^3k}{(2\pi)^3} A(k)\zeta(k)e^{ik\cdot x}$$



Statistical challenges

Example problem: If this toy model were exact, and we observe the final density field with some statistical noise, how do we estimate the initial ζ(k), and how do we characterize uncertainty in the estimate?

$$\vec{S}(x) = \vec{\nabla} \int \frac{d^3k}{(2\pi)^3} A(k)\zeta(k)e^{ik\cdot x}$$



Conclusions

- Current observations are consistent with a 6-parameter model $\{H_0, \Omega_c, \Omega_b, A_\zeta, n_s, z_{\rm rei}\}$
- Although the model is simple, it encapsulates deep mysteries (baryon asymmetry, dark matter, dark energy / c.c., origin of perturbations)
- We are still in an era where new experiments are rapidly improving statistical errors on key parameters
- Many theoretical, statistical and computational problems must be solved in order to keep making progress!