Title: Emergence and Entanglement: A New World of Condensed Matter Physics

Date: Jan 30, 2014 09:30 AM

URL: http://pirsa.org/14010108
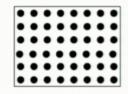
Abstract:

Condensed matter physics: Properties of matter

Why solids have shape and liquids have no shape
despite they are all formed by atoms?

Different organizations → different mech. properties

- Atoms in liquids have a random distribution
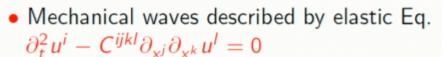 → cannot resist shear deformations (which do nothing)
  → liquids do not have shapes

- Atoms in solid have a ordered lattice distribution
  → can resist shear deformations
  → solids have shapes

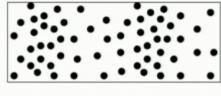- Mechanical waves described by Euler Eq.
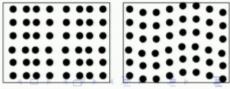 $$\partial_t^2 \rho - \partial_x^2 \rho = 0$$
  One longitudinal mode

- Mechanical waves described by elastic Eq.
  $$\partial_t^2 u^i - C^{ijkl} \partial_{x^j} \partial_{x^k} u^l = 0$$
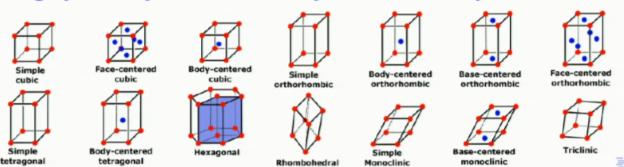 One long. mode and two trans. modes

A systematic theory of orders

Principle of emergence: Different orders → different wave equations → different physical properties of matter

Landau: **different orders have different symmetry breaking.**

- The Hamiltonian $H = \sum_i \frac{\hat{\mathbf{p}}_i^2}{2m} + \sum_{ij} V(\mathbf{r}_i - \mathbf{r}_j)$ of atoms have
  - continuous translation symmetries in x-, y-, z-directions
  - continuous $SO(3)$ spatial rotation symmetries
- The crystal ground states have lower symmetries:
  - discrete translation symmetries in x-, y-, z-dir.
  - discrete $SO(3)$ rotation symmetries

Using symmetry, we can classify all the 230 crystal orders

Simple cubic · Face-centered cubic · Body-centered cubic · Simple orthorhombic · Body-centered orthorhombic · Base-centered orthorhombic · Face-centered orthorhombic

Simple tetragonal · Body-centered tetragonal · Hexagonal · Rhombohedral · Simple Monoclinic · Base-centered monoclinic · Triclinic

Principle of emergence

Different orders $\rightarrow$ different wave equations
$\rightarrow$ different physical properties of matter
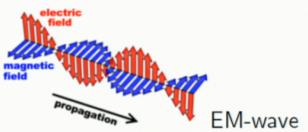
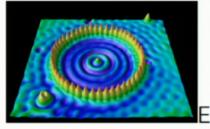**Do all waves and wave equations emerge from some orders?**

Two very important wave equations and waves

- Electromagnetic wave and Maxwell equation
$\partial \times \mathbf{E} + \partial_t \mathbf{B} = \partial \times \mathbf{B} - \partial_t \mathbf{E} = \partial \cdot \mathbf{E} = \partial \cdot \mathbf{B} = 0$
- Electron wave and Dirac equation: $[\partial_\mu \gamma^\mu + m_e]\psi = 0$
The amplitude of electron wave is anti commuting number
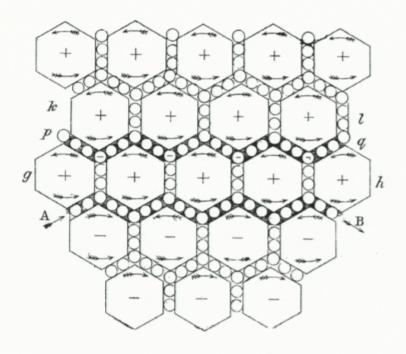

EM-wave

Electron-wave

What orders produce the above two waves? What are the origins of light (gauge bosons) and electrons (fermions)?
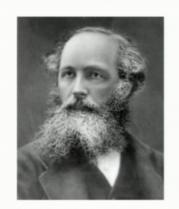
Maxwell's aether – a gear model

$$\partial \times B = \mathbf{j}$$

Elementary or emergent?

But none of the symmetry breaking orders can produce:
- electromagnetic wave satisfying the Maxwell equation
- electron wave satisfying the Dirac equation
  with anti-commuting amplitude (Fermi statistics).

Two choices:

- Declare that photons (EM waves) and electrons are elementary, and do not ask where do they come from.

- Declare that the symmetry breaking theory is incomplete. Maybe new orders beyond symmetry breaking can produce electromagnetic wave (the Maxwell equation) and electron wave (the Dirac equation and anti-commuting amplitude).

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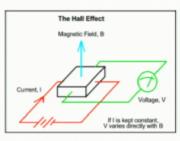
Xiao-Gang Wen, PI day, Jan. 30, 2014   Emergence and Entanglement – A New World of Condensed
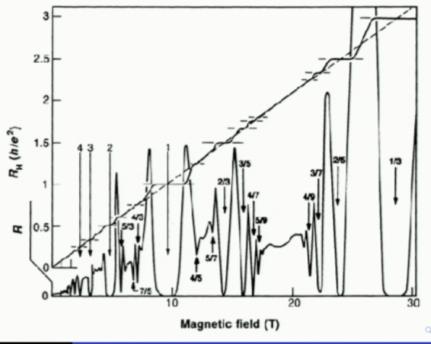
New order beyond symmetry breaking does exit

$$E_y = R_H j_x, \quad R_H = \frac{p}{q}\frac{h}{e^2}$$

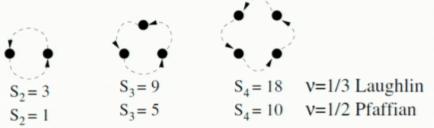- 2D electron gas in magnetic field has many **quantum Hall (QH) states**

Quantum dancing pattern: a picture of topological order

- Local dancing rules of a FQH liquid:
  (1) every electron dances around clock-wise
  (Φ_{FQH} only depends on $z = x + iy$)
  (2) takes exactly three steps to go around any others
  (Φ_{FQH}'s phase change 6π)
  $\rightarrow$ Global dancing pattern $\Phi_{FQH}(\{z_1, ..., z_N\}) = \prod(z_i - z_j)^3$
- A systematic theory of FQH state – **Pattern of zeros S_a:**
  a-electron cluster has a relative angular momentum S_a Wen-Wang 08

$S_2 = 3$ $S_3 = 9$ $S_4 = 18$ $\nu = 1/3$ Laughlin
$S_2 = 1$ $S_3 = 5$ $S_4 = 10$ $\nu = 1/2$ Pfaffian

- Local dancing rules are enforce by the Hamiltonian to lower energy.
- QH states do not produce Maxwell equation, but produce chiral edge wave: a) **perfect conducting** boundary; b) the boundary theory has **gravitational anomaly** $\leftarrow$ **Topological order**

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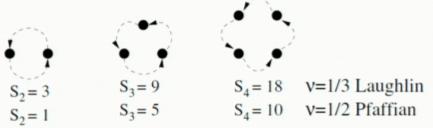
QH states in Graphene

Room temperature QH state

FQH states in graphene

Non-Abelian QH state in bilayer graphene

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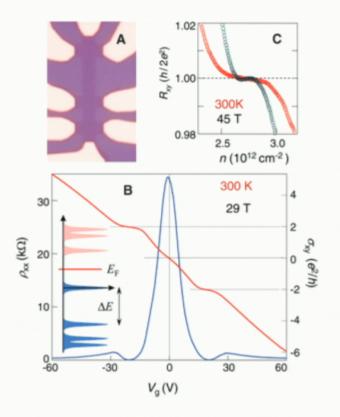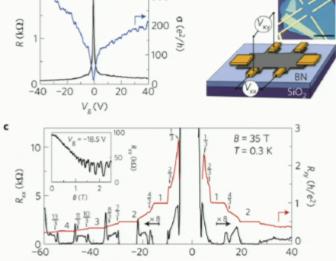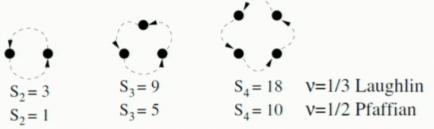
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- $= |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle... \to$ unentangled

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- $ = |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \otimes |\uparrow\rangle \otimes |\downarrow\rangle \ldots \rightarrow$ unentangled
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- Crystal order: $|\Phi_{\text{crystal}}\rangle = \left| \blacksquare \right\rangle = |0\rangle_{x_1} \otimes |1\rangle_{x_2} \otimes |0\rangle_{x_3} \ldots$
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- Particle condensation (superfluid)
  $|\Phi_{SF}\rangle = \sum_{\text{all conf.}} \left| \text{} \right\rangle$

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- Particle condensation (superfluid)
  $|\Phi_{\text{SF}}\rangle = \sum_{\text{all conf.}} \left|\,\rule{0pt}{2ex}\quad\right\rangle = (|0\rangle_{x_1} + |1\rangle_{x_1} + ..) \otimes (|0\rangle_{x_2} + |1\rangle_{x_2} + ..)...$
  = direct-product state → unentangled state (classical)

How to make long range entanglements (topo. orders)

To make topological order, we need to sum over different product states, but we should not sum over everything.

- Sum over a subset of the particle configurations, by first join the particles into strings, then sum over the loop states

→ string-net condensation (string liquid):

Levin-Wen 05

$$|\Phi_{\text{string}}\rangle = \sum_{\text{all closed-strings}} \left| \vphantom{\Big|} \right\rangle \quad \text{which is not a direct-product}$$

state and not a local deformation of direct-product states
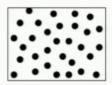→ non-trivial **topological orders** (for gapped states)

Long range entanglements
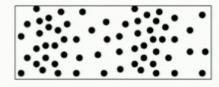→ emergence of electromagnetic waves (gauge fields)

- Wave in superfluid state $|\Phi_{SF}\rangle = \sum_{\text{all position conf.}} \left|\begin{smallmatrix} \cdots \end{smallmatrix}\right\rangle$:
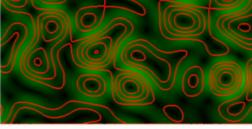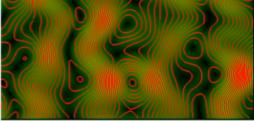
density fluctuations:
Euler eq.: $\partial_t^2 \rho - \partial_x^2 \rho = 0$
→ Longitudinal wave

- Wave in string liquid $|\Phi_{loop}\rangle = \sum_{\text{all loop conf.}} \left|\begin{smallmatrix} \cdots \end{smallmatrix}\right\rangle$:

String density $\mathbf{E(x)}$ fluctuations → waves in string condensed state.
Strings have no ends → $\partial \cdot \mathbf{E} = 0$ → only two transverse modes.
Equation of motion for string density → Maxwell equation:
$\dot{\mathbf{E}} - \partial \times \mathbf{B} = \dot{\mathbf{B}} + \partial \times \mathbf{E} = \partial \cdot \mathbf{B} = \partial \cdot \mathbf{E} = 0.$ ($\mathbf{E}$ electric field)

Xiao-Gang Wen, PI day, Jan. 30, 2014   Emergence and Entanglement – A New World of Condensed

Long range entanglements →
Emergence of Fermi statistics, fractional statistics, etc

In string condensed states, the ends of string are fractionalized excitations with fractional charge/statistics,...

- For string condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \left| \rule{0pt}{2em} \right\rangle$

  The end of strings are bosons.

- For string condensed state $|\Phi\rangle = \sum_{\text{all conf.}} \pm \left| \rule{0pt}{2em} \right\rangle$

  The end of strings are fermion. Levin-Wen 2003

  The different statistics of ends of strings is determined by the different kinds of string-net condensation.

- **Qubits organized as string-nets provide a way to unify gauge interactions and Fermi statistics in 3D**
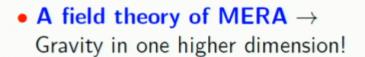
Can gravity emerge from qubits?

- **Multi-scale Entanglement Renormalization Ansatz (MERA)**
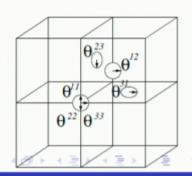 Use tensor network to encode a many-body wave function:
  $$\Phi(m_1, m_2, ...) = \mathrm{Tr}(TTT...)$$
  $$T \to T_{m_1 m_2 m_3 m_4}$$
  Discretized AdS/CFT

- **A field theory of MERA** $\to$
  Gravity in one higher dimension!

- **Quantum rotor model** $\to$
  emergent linearized Einstein equation.
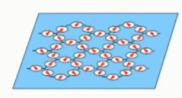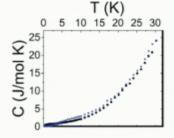 The ground state is not string liquid, nor membrane liquid, but something else.

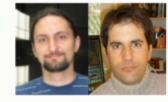# Where to find long-range entanglements: spin liquid

Herbertsmithsite: spin-1/2 on Kagome lattice $H = J\sum \mathbf{S}_i \cdot \mathbf{S}_j$.

- $J \sim 200K$, no phase trans. down to $50mK \rightarrow$ spin liquid Helton etal 06

  Frustrated spin systems with large quantum fluctuations

- Numerical calculations $\rightarrow Z_2$ topological order (emergence of Z_2 gauge theory) Misguich-Bernu-Lhuillier-Waldtmann 98; Jiang-Weng-Sheng 08; Yan-Huse-White 10

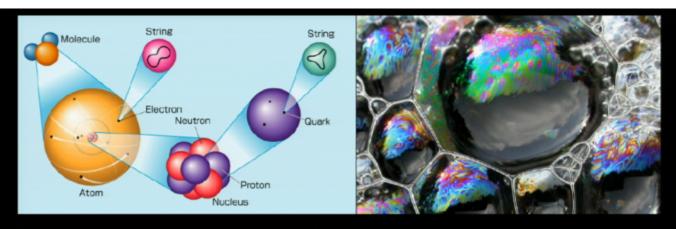- With next-neighbor coupling $\rightarrow$ chiral spin liquid with emergent semions! – tensor network approach

- Spin liquid on other lattice – Monte Carlo approaches

Long range entanglements are source of many wonders