

Title: Cosmic Variance from Superhorizon Mode Coupling

Date: Jan 23, 2014 11:00 AM

URL: <http://pirsa.org/14010107>

Abstract: We observe a finite subvolume of the universe, so CMB and large scale structure data may give us either a representative or a biased sample of statistics in the larger universe. Mode coupling (non-Gaussianity) in the primordial perturbations can introduce a bias of parameters measured in any subvolume due to coupling to superhorizon background modes longer than the size of the subvolume. This leads to a "cosmic variance" of statistics on smaller scales, as the long-wavelength background modes vary around the global mean. We study this bias for local non-Gaussianity and quantify how observed statistics such as the power spectrum of the primordial perturbations, spectral index (scale-dependence in the power spectrum), amplitude of non-Gaussianity, dark matter halo power spectrum, and primordial tensor modes, can differ from the same quantities averaged throughout a volume much larger than the observable universe. More general kinds of mode coupling can change the relative sensitivity to different background modes. Finally, we consider what observations can tell us about the possibility of biasing from superhorizon modes.

Outline

1. Setup
2. Subsampling with Local Non-Gaussianity
 - A. Weak NG: biasing of observed parameters
 - B. Gaussian patches in a non-Gaussian universe
3. Subsampling with Scale-Dependence
 - A. Shift to observed spectral index
 - B. Shift to observed bispectral running
4. Beyond Local Non-Gaussianity; Tensor Modes
5. Summary, Outlook

Primordial Curvature Perturbation

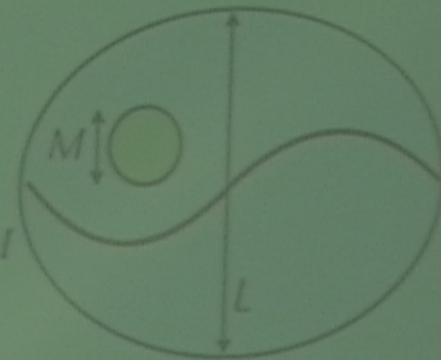
- ◆ Statistics of primordial curvature perturbation ζ carry information about inflation
- ◆ Assume statistics for ζ on a fixed spatial slice (at end of inflation)

Primordial Curvature Perturbation

- Statistics of primordial curvature perturbation ζ carry information about inflation
- Assume statistics for ζ on a fixed spatial slice (at end of inflation)

- Fluctuations in amount of expansion:

$$\begin{aligned}
 a(\mathbf{x}) &= \langle a \rangle_L (1 + \zeta(\mathbf{x})), \quad x \in \text{Vol}_L \\
 &= \langle a \rangle_M (1 + \zeta^{\text{obs}}(\mathbf{x})), \quad x \in \text{Vol}_M \\
 &= \langle a \rangle_L (1 + \zeta_l)
 \end{aligned}$$



- Long-short wavelength split: $\zeta(\mathbf{x}) = \zeta_l(\mathbf{x}) + \zeta_s(\mathbf{x})$
- Fluctuations around local background: $\zeta^{\text{obs}} = \zeta_s / (1 + \zeta_l)$

Primordial Non-Gaussianity (=NG)

- Gaussian fluctuations: observable modes uncorrelated to superhorizon modes, power spectrum completely describes statistics of fluctuations

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta(k)$$

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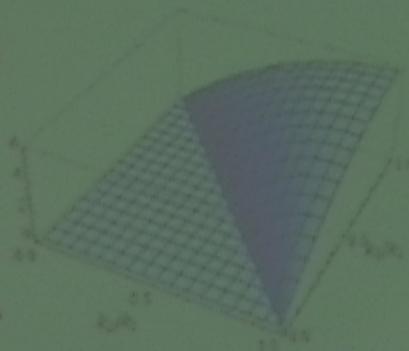
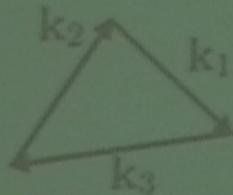
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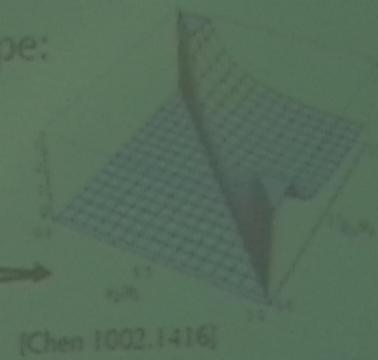
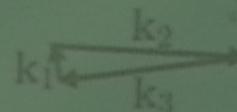
- Interactions \rightarrow mode coupling

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3)$$

Equilateral
Shape:



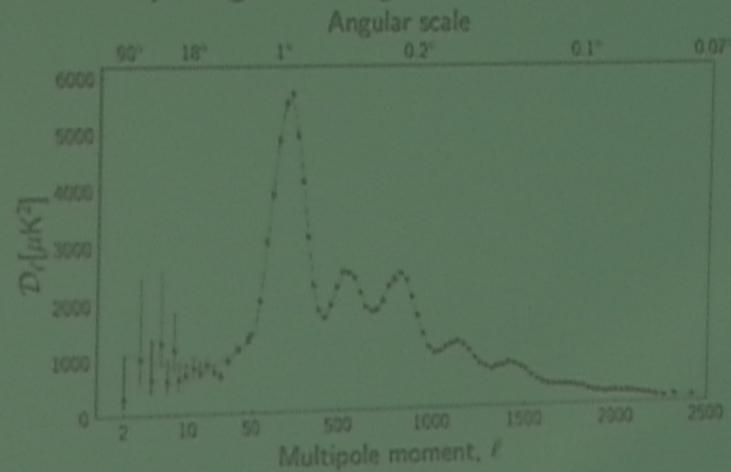
Local Shape:



- NG can increase or decrease amount of large scale structure
- Distinguish inflation models...but new cosmic variance uncertainty

Cosmic Variance

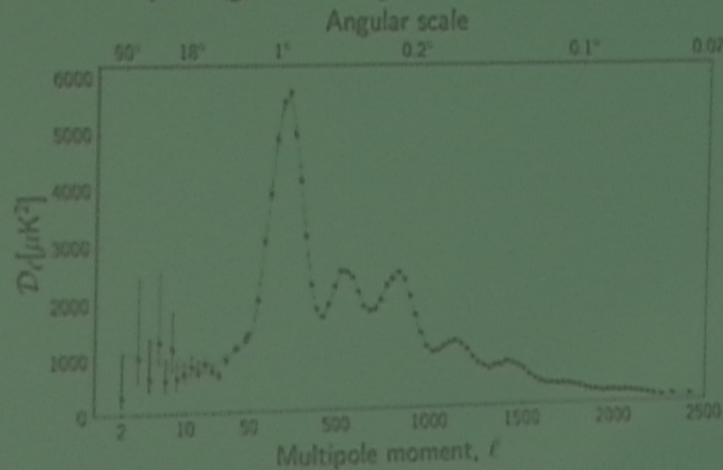
- Variance of sample mean = $\langle (\bar{X} - \langle X \rangle)^2 \rangle \sim \frac{\langle (\Delta X)^2 \rangle}{N}$
Sample mean \nearrow \bar{X} \nwarrow $\langle X \rangle$ Distribution mean
- Example: poor sampling for large-scale CMB modes:



Cosmic Variance

- $\text{Variance of sample mean} = \langle (\bar{X} - \langle X \rangle)^2 \rangle \sim \frac{\langle (\Delta X)^2 \rangle}{N}$

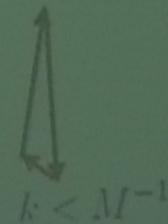
Sample mean \nearrow
Distribution mean \nwarrow
- Example: poor sampling for large-scale CMB modes:



- $N=1$ sample of superhorizon modes
 \rightarrow biased sample of subhorizon modes:

$$\langle \zeta_{k_1} \times \dots \times \zeta_{k_N} \rangle|_{\zeta_L} \neq \langle \zeta_{k_1} \times \dots \times \zeta_{k_N} \rangle|_0$$

Example: global 3-point affects observed 2-point



Local Ansatz: Weakly Non-Gaussian

- Local NG: $\zeta(\mathbf{x}) = f(\zeta_G(\mathbf{x})) - \langle f(\zeta_G) \rangle = f' \zeta_G + \frac{1}{2} f'' \zeta_G^2 + \dots$
- Nearly Gaussian: $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\text{NL}} (\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2 \rangle) + \frac{9}{25} g_{\text{NL}} \zeta_G^3(\mathbf{x}) + \dots$

- Long-short wavelength split:

$$\zeta_G \equiv \zeta_{Gs} + \zeta_{Gl} = \int_{L^{-1}}^{M^{-1}} \frac{d^3k}{(2\pi)^3} \zeta_G(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + \int_{M^{-1}}^{k_{\text{max}}} \frac{d^3k}{(2\pi)^3} \zeta_G(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

- Perturbations in subvolume:

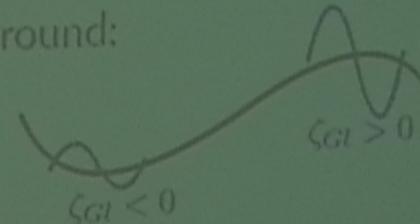
$$\zeta^{\text{obs}}(\mathbf{x}) = \zeta_G^{\text{obs}}(\mathbf{x}) + \frac{3}{5} f_{\text{NL}}^{\text{obs}} [\zeta_G^{\text{obs}}(\mathbf{x})^2 - \langle \zeta_G^{\text{obs}}(\mathbf{x})^2 \rangle] + \frac{9}{25} g_{\text{NL}}^{\text{obs}} \zeta_G^{\text{obs}}(\mathbf{x})^3 + \dots$$

$$(1 + \frac{6}{5} f_{\text{NL}} \zeta_{Gl}) \zeta_{Gs} + \dots$$

- Local statistics modulated by the local background:

$$P_{\zeta}^{\text{obs}}(k) = \left[1 + \frac{12}{5} f_{\text{NL}} \zeta_{Gl} + \dots \right] P_G(k)$$

$$f_{\text{NL}}^{\text{obs}} = f_{\text{NL}} - \frac{12}{5} f_{\text{NL}}^2 \zeta_{Gl} + \frac{9}{5} g_{\text{NL}} \zeta_{Gl} + \dots$$



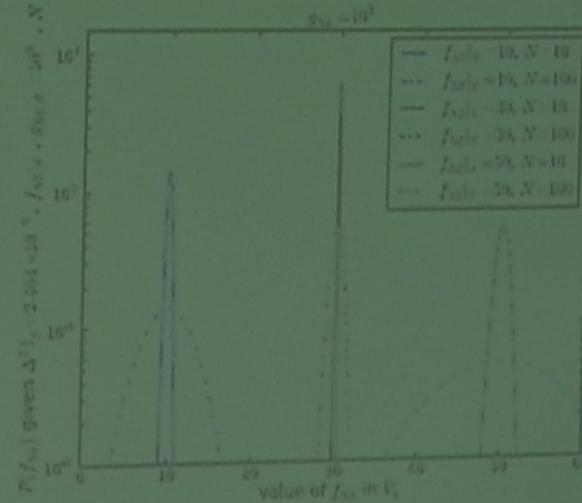
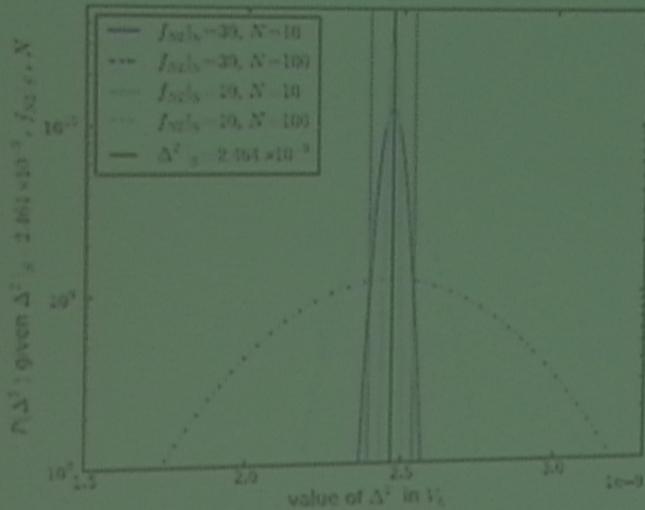
- Amount of bias \propto amount of global NG

[M. LoVerde, EN, & S. Shandera, 1303.3549]

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Local Ansatz: Weakly Non-Gaussian

Probability distributions for global quantities:



Statistics can vary significantly among subvolumes, even with globally small fluctuations, weak non-Gaussianity, and of order 10-100 extra e-folds

"Fine-tuned" example: Global g_{NL} will regenerate locally observed f_{NL} :
 [Numi+1301.3128, Byrnes+1306.2370]

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + g_{NL} \zeta_G^3(\mathbf{x})$$

$$\zeta^{\text{obs}} = (1 + 3g_{NL} \zeta_{GI}^2) \zeta_{Gs} + 3g_{NL} \zeta_{GI} \zeta_{Gs}^2 + g_{NL} \zeta_{Gs}^3$$

Local Ansatz: Strongly Non-Gaussian

$$\zeta(\mathbf{x}) = \zeta_G^p(\mathbf{x}) - \langle \zeta_G^p \rangle$$

Strongly NG field can appear nearly Gaussian:

$$\zeta^{\text{obs}}(\mathbf{x}) = p \zeta_{GI}^{p-1} \zeta_{Gs}(\mathbf{x}) + \frac{p!}{2!(p-2)!} \zeta_{GI}^{p-2} (\zeta_{Gs}^2(\mathbf{x}) - \langle \zeta_{Gs}^2 \rangle) + \dots$$

$$\zeta_{GI} \gg \sqrt{\langle \zeta_{Gs}^2 \rangle} \leftrightarrow \frac{L}{M} \gg k_{\text{max}} M \leftrightarrow N_{\text{superhorizon}} \gg N_{\text{subhorizon}}$$

→ Recover statistically natural, nearly Gaussian series, in typical subvolumes; series controlled by $\langle \zeta_{Gs}^2 \rangle^{1/2} / \zeta_{GI}$

[EN, S. Shandera, 1212.4550; LoVerde et. al., 1303.3549]

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Minimum level of NG in subvolumes:

$$\frac{3}{5}f_{\text{NL}}^{\text{obs}} \approx \frac{p-1}{2p\zeta_{Gl}^p} \left(\sim \frac{1}{\zeta_l} \right) \quad \text{Observed NG of 1 part in } 10^5 \text{ as background becomes nonperturbative}$$

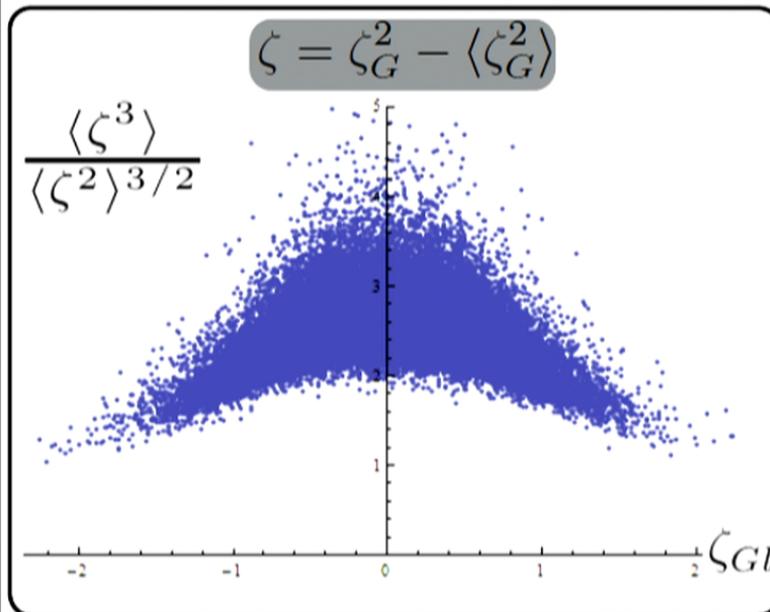
→ *Constrain superhorizon cosmic variance*

by tightening $f_{\text{NL}}^{\text{obs}}$ bounds

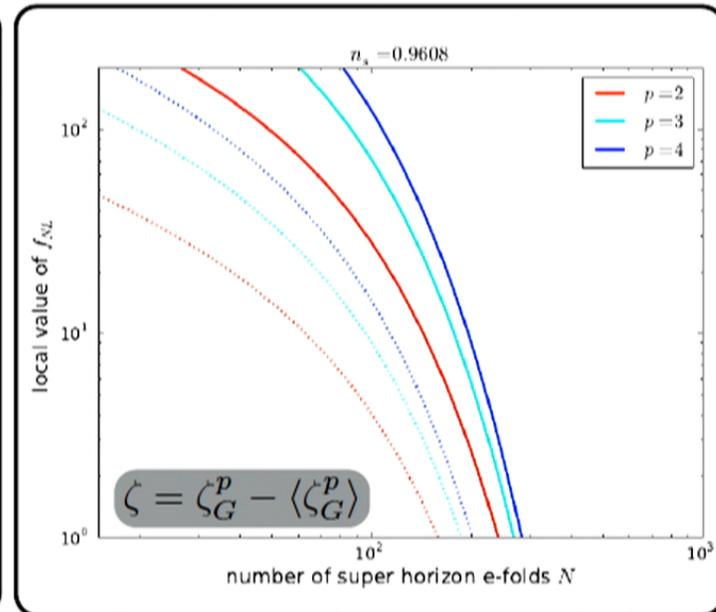
[EN, S. Shandera, 1212.4550; LoVerde et. al., 1303.3549]

Statistical Naturalness for Local NG

Numerical realizations:



Suppression of f_{NL}^{obs} :

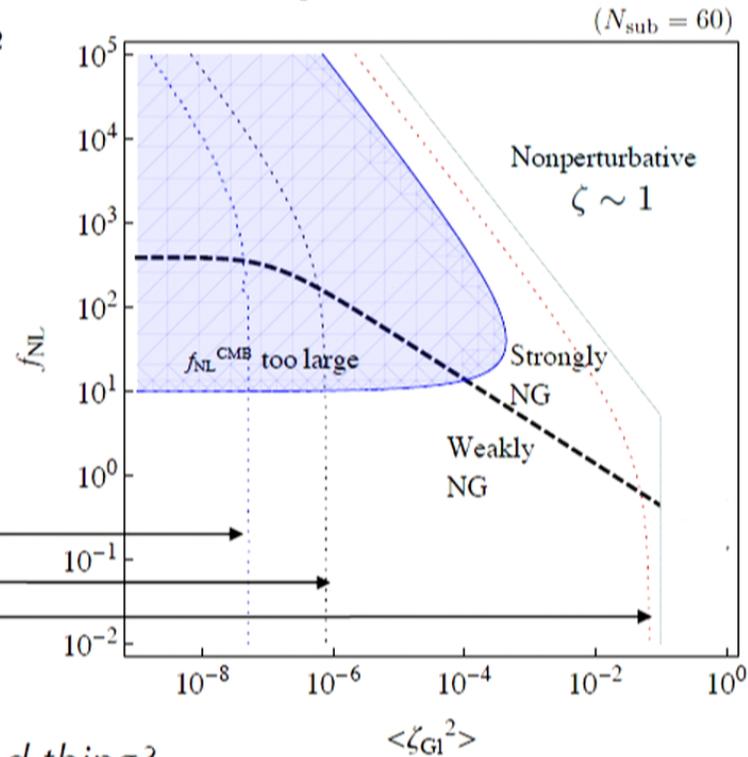


Local NG: Parameter Space

Single source, scale-invariant: $\zeta = \zeta_G + \frac{3}{5} f_{\text{NL}} (\zeta_G^2 - \langle \zeta_G^2 \rangle)$

$$f_{\text{NL}}^{\text{CMB}} = f_{\text{NL}} (1 + \frac{6}{5} f_{\text{NL}} \zeta_{\text{GI}})^{-2}$$

Constraints on $f_{\text{NL}}^{\text{CMB}}$ close to pushing strongly NG universe into nonperturbative regime



No f_{NL} detection...a good thing?

Running of Mode Coupling Constants

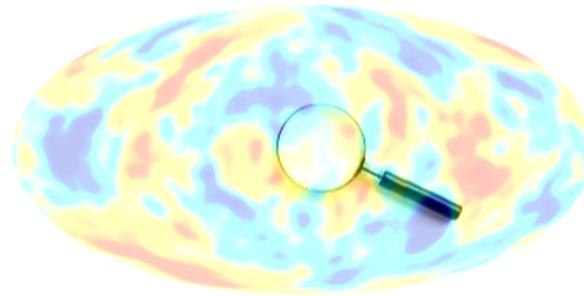
- Letting the subvolume scale M run, non-Gaussian couplings change with the local background:

[Boubekeur & Lyth, 0504046]
[Lyth, 0707.0361]

$$f_{\text{NL}}(\zeta_{G\ell}) = f_{\text{NL}}(\zeta_{G\ell}(M, \mathbf{x}))$$

$$g_{\text{NL}}(\zeta_{G\ell}) = g_{\text{NL}}(\zeta_{G\ell}(M, \mathbf{x}))$$

etc.



- Running towards weakly NG series with $1 \gg f_{\text{NL}} \sqrt{\langle \zeta_G^2 \rangle} \gg g_{\text{NL}} \langle \zeta_G^2 \rangle$
- Scaling of moments $\mathcal{M}_n \equiv \frac{\langle \zeta(\mathbf{x})^n \rangle}{\langle \zeta(\mathbf{x})^2 \rangle^{n/2}}$
runs towards hierarchical, $\mathcal{M}_n \sim \mathcal{M}_3^{n-2}$
- ...But local shapes (squeezed limits) are preserved

Scale-Dependent NG: $f_{NL}(k)$

- Generalized Local Ansatz:

$$\zeta(\mathbf{k}) = \phi_G(\mathbf{k}) + \sigma_G(\mathbf{k}) + \frac{3}{5} f_{NL}(k) [\sigma_G^2](\mathbf{k}) + \frac{9}{25} g_{NL}(k) [\sigma_G^3](\mathbf{k}) + \dots$$

Weak scale-dependence:

$$f_{NL}(k) = f_{NL}(k_p) \left(\frac{k}{k_p} \right)^{n_f} \quad \xi_m(k) \equiv \frac{P_{\sigma, NG}(k)}{P_\zeta(k)} = \xi_m(k_p) \left(\frac{k}{k_p} \right)^{n_f^{(m)}}$$

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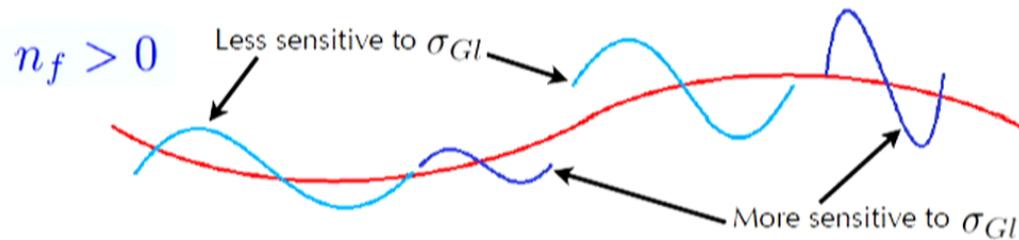
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- Running in global bispectrum \rightarrow observed runnings biased

$$P_\zeta^{\text{obs}}(k) = P_\zeta(k) \left[1 + \frac{12}{5} \xi_m(k) \frac{f_{NL}(k) \sigma_{GI} + \frac{3}{5} f_{NL}^2(k) (\sigma_{GI}^2 - \langle \sigma_{GI}^2 \rangle)}{1 + \frac{36}{25} f_{NL}^2(k) \langle \sigma_{GI}^2 \rangle} \right]$$

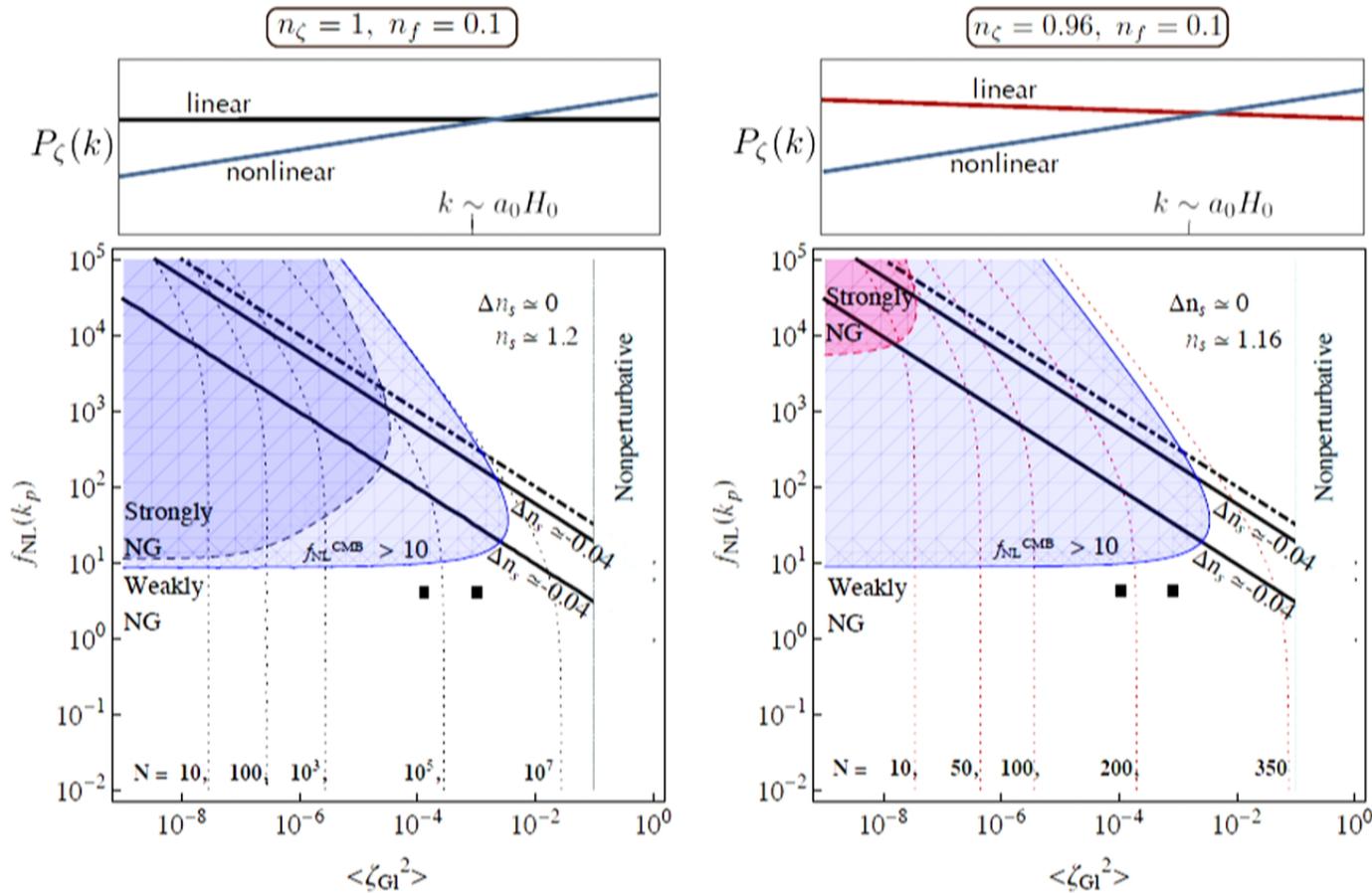


$$\Delta n_s(k) \approx \frac{12}{5} \xi_m(k) f_{NL}(k) \sigma_{GI} (n_f + n_f^{(m)}), \quad \frac{6}{5} f_{NL}(k) \sigma_{GI} \ll 1$$

How large can this be?

[J. Bramante, J. Kumar, EN, & S. Shandera, 1307.3549]

Scale-Dependent NG: Parameter Space



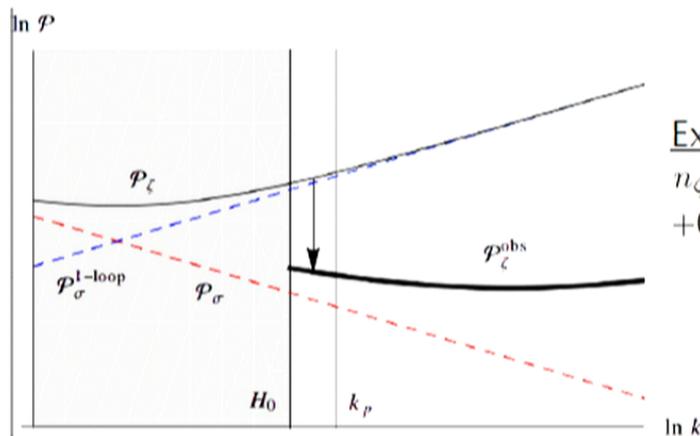
Scale-Dependent NG: $n_s^{\text{obs.}} \neq n_s$

Observed red tilt $n_s^{\text{obs.}} < 1$ consistent with globally flat or blue tilt $n_s > 1$
 Or, global red tilt can vary among Hubble patches

For $|\Delta n_s(k_{\text{obs}})| \gtrsim 0.04$ on observable scales, need

- Sufficient power for background modes: red tilt on larger scales
- Background ζ_l dominated by non-Gaussian source σ
- Blue tilt to f_{NL} for weakly NG, perturbative global statistics

→ Probe smaller scales observationally



Example of red-to-blue tilt

$$n_\zeta = 0.95, n_f = 0.05, f_{\text{NL}}(k_p) \langle \zeta_G^2 \rangle^{1/2} = 3 + 0.1\sigma \text{ background}$$

Non-Gaussian Effect on Halo Bias

Halo power spectrum affected by primordial bispectrum

$$\text{Constant } f_{\text{NL}}: B_{\zeta}(k_L, k_S, k_S) = \frac{12}{5} f_{\text{NL}} P_{\zeta}(k_L) P_{\zeta}(k_S)$$

$$\begin{aligned} \delta n(\mathbf{k}_L) &= \frac{\partial n}{\partial \delta} \delta(\mathbf{k}_L) + \frac{\partial n}{\partial \ln P_{\zeta}} \frac{12}{5} f_{\text{NL}} \zeta(\mathbf{k}_L) \\ &\sim \left(\frac{\partial n}{\partial \delta} + \frac{\partial n}{\partial \ln P_{\zeta}} \frac{12}{5} f_{\text{NL}} \frac{1}{k_L^2} \right) \delta(\mathbf{k}_L) \leftarrow \delta \sim k^2 \Phi \end{aligned}$$



[Dalal et. al., 0710.4560]

Shift to the Bispectrum: $n_{\text{sq.}} \neq n_{\text{sq.}}^{\text{obs.}}$

$$B_{\zeta}(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}(k_3) \xi_m(k_1) \xi_m(k_2) P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.} + \dots$$

$$B_{\zeta}^{\text{obs.}}(k_1, k_2, k_3) \approx \frac{6}{5} f_{\text{NL}}(k_3) \frac{\xi_m^{\text{obs.}}(k_1) P_{\zeta}^{\text{obs.}}(k_1)}{1 + \frac{6}{5} f_{\text{NL}}(k_1) \sigma_{\text{GI}}} \frac{\xi_m^{\text{obs.}}(k_2) P_{\zeta}^{\text{obs.}}(k_2)}{1 + \frac{6}{5} f_{\text{NL}}(k_2) \sigma_{\text{GI}}} + 2 \text{ perms.} + \dots$$

Additional k -dependence ($g_{\text{NL}} = 0$)

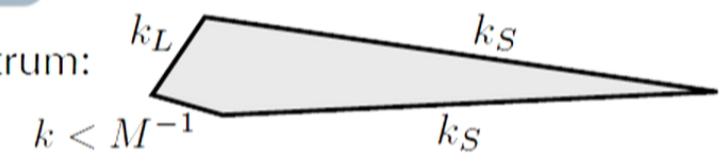
- Squeezed-limit ($k_1 \ll k_2, k_3$) running:

$$n_{\text{sq.}} \equiv \frac{d \ln B_{\zeta}(k_L, k_S, k_S)}{d \ln k_L} - (n_s - 1) = -3 + n_f^{(m)} \quad (\text{Large Volume})$$

- Infrared background breaks relation between B_{ζ} and P_{ζ}
- Single source can mimic multi-source statistics:

$$\Delta n_{\text{sq.}}(k) \approx -\frac{\frac{6}{5} f_{\text{NL}}(k) \zeta_{\text{GI}} n_f}{1 + \frac{6}{5} f_{\text{NL}}(k) \zeta_{\text{GI}}}$$

- Shift comes from global trispectrum:



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Halo power spectrum affected by primordial bispectrum

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[Dalal et. al., 0710.4560]

Two sources, scale-dependent:

$$B_{\zeta}(k_L, k_S, k_S) = \frac{12}{5} f_{\text{NL}}(k_S) \xi_m(k_L) \xi_m(k_S) P_{\zeta}(k_L) P_{\zeta}(k_S)$$

$$\longrightarrow \Delta \ln P_{\zeta}(k) = \frac{12}{5} f_{\text{NL}}(k) \xi_m(k) \xi_m(k_L) \zeta(\mathbf{k}_L)$$

$$\text{In halo power spectrum, } k^{-2} \longrightarrow k^{-2+n_f^{(m)}}$$

[Shandera et. al., 1010.3722]

Shift to the Bispectrum: $n_{\text{sq.}} \neq n_{\text{sq.}}^{\text{obs.}}$

$$B_{\zeta}(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}(k_3) \xi_m(k_1) \xi_m(k_2) P_{\zeta}(k_1) P_{\zeta}(k_2) + 2 \text{ perms.} + \dots$$

$$B_{\zeta}^{\text{obs}}(k_1, k_2, k_3) \approx \frac{6}{5} f_{\text{NL}}(k_3) \frac{\xi_m^{\text{obs}}(k_1) P_{\zeta}^{\text{obs}}(k_1)}{1 + \frac{6}{5} f_{\text{NL}}(k_1) \sigma_{\text{GI}}} \frac{\xi_m^{\text{obs}}(k_2) P_{\zeta}^{\text{obs}}(k_2)}{1 + \frac{6}{5} f_{\text{NL}}(k_2) \sigma_{\text{GI}}} + 2 \text{ perms.} + \dots$$

Additional k -dependence ($g_{\text{NL}} = 0$)

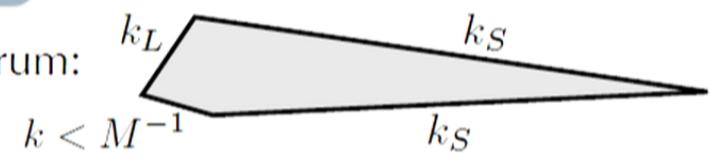
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- Shift comes from global trispectrum:



Non-local Mode Coupling: $\zeta = \zeta_G + F \star \zeta_G^2$

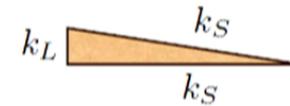
$$\zeta_{\mathbf{k}} = \zeta_{G,\mathbf{k}} + \int_{L^{-1}} \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} (2\pi)^3 \delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{k}) F(p_1, p_2, k) \zeta_{G,\mathbf{p}_1} \zeta_{G,\mathbf{p}_2} + \dots$$

Factorizable kernel with power laws; squeezed-limit bispectrum:

$$B_\zeta(k_L, k_S, k_S) = 4FP_\zeta(k_L)P_\zeta(k_S) \left(\frac{k_L}{k_p}\right)^{m_L} \left(\frac{k_S}{k_p}\right)^{m_S}$$

Shift to observed power spectrum,

$$P_\zeta^{\text{obs}} = P_\zeta \left[1 + a_{\text{NL}} \left(\frac{k}{k_p}\right)^{m_S} \zeta_{G_l}^{(m_L)} \right]^2$$



Weighted biasing from superhorizon modes:

$$\zeta_{G_l}^{(m_L)} \equiv \int_{L^{-1}} \frac{d^3 p}{(2\pi)^3} \zeta_{\mathbf{p}} \left(\frac{p}{k_p}\right)^{m_L}$$

- $m_L = 0$: local ansatz, equal sensitivity to all background modes
- $m_L > 0$: weaker coupling of long-to-short modes, cannot regenerate nearly Gaussian statistics
- $m_L < 0$: stronger long-to-short coupling than local NG [Schmidt & Hui, 1210.2965]

Non-local Mode Coupling: $\zeta = \zeta_G + F \star \zeta_G^2$

$$B_\zeta(k_L, k_S, k_S) = 4FP_\zeta(k_L)P_\zeta(k_S) \left(\frac{k_L}{k_p}\right)^{m_L} \left(\frac{k_S}{k_p}\right)^{m_S}$$

$$\zeta_{GI}^{(m_L)} \equiv \int_{L^{-1}}^{M^{-1}} \frac{d^3p}{(2\pi)^3} \zeta_P \left(\frac{p}{k_p}\right)^{m_L}$$

Equilateral Non-Gaussianity

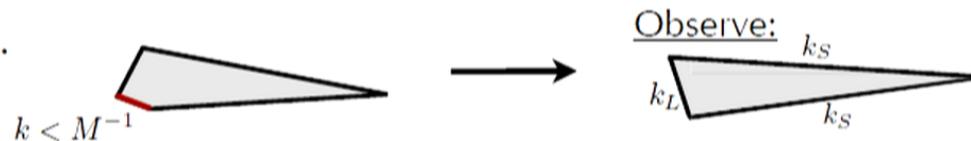
- $m_L = 2$, local power spectrum only sensitive to $\nabla^2 \zeta_{GI}$
- Single-field consistency relation:

$$B_\zeta(k_L, k_S, k_S) \simeq -(n_s - 1 + \mathcal{O}(k_L/k_S)^2) P_\zeta(k_L) P_\zeta(k_S) \quad [\text{Creminelli, 0407059}]$$

→ No sensitivity to background ζ_{GI} from
single-field inflation

- Equilateral squeezed-limit protected from background modes?

Eg.



Tensor Mode Coupling: $n_t^{\text{obs.}} \neq n_t$?

Sign of n_t to test inflation vs. alternatives? Red tilt $n_t < 0$ characteristic of inflation...could bias change sign? $\Delta n_t/n_t = \mathcal{O}(1)$

- $\langle \gamma_{\mathbf{k}_1} \gamma_{\mathbf{k}_2} \chi_{\mathbf{k}_3} \rangle$ squeezed-limit coupling from $\gamma = \gamma_G + f_{\gamma\gamma\chi} \gamma_G \chi_G$:

$$B_{\gamma\gamma\chi}(k_L, k_S, k_S) = 4f_{\gamma\gamma\chi} P_\chi(k_L) P_\gamma(k_S) \left(\frac{k_L}{k_p}\right)^{m_L} \left(\frac{k_S}{k_p}\right)^{m_S}$$

- Shift to observed tensor power,

$$P_\gamma^{\text{obs}} = P_\gamma \left[1 + f_{\gamma\gamma\chi} \left(\frac{k}{k_p}\right)^{m_S} \chi_{Gl}^{(m_L)} \right]^2$$

$$\text{with } \chi_{Gl}^{(m_L)} \equiv \int_{L^{-1}}^{M^{-1}} \frac{d^3 p}{(2\pi)^3} \chi_{\mathbf{p}} \left(\frac{p}{k_p}\right)^{m_L}$$

and running,

$$\Delta n_t(k) \approx 2f_{\gamma\gamma\chi}(k) \chi_{Gl}^{(m_L)} m_S$$

→ Possible for inflation models?

Summary

- ◆ Local statistics biased by superhorizon background modes
 - Constraints on global statistics (P_{ζ} , f_{NL} , n_s , n_t , n_{sq}) are probabilistic
 - Local and global statistics can be qualitatively different
 - Parameters from models need only render our observations typical among Hubble volumes
 - Bias grows with size of universe (\sim length of inflation)
- ◆ Nearly Gaussian Hubble volumes in a NG universe
 - Strong NG on superhorizon scales *with strong squeezed limit* can be consistent with observed Gaussianity
 - Statistical naturalness of weakly NG local ansatz
- ◆ Squeezed-limit behavior controls IR sensitivity
- ◆ Observations could tell us if this uncertainty is relevant to our interpretation of data
- ◆ Non-Gaussianity \longleftrightarrow Statistical Inhomogeneity [Byrnes et. al., 1111.2721]

Future Work

- ◆ Nonlocal mode coupling: *work in progress* [Soccimarro et. al., 1108.5512]
Statistical naturalness? Can local-type NG be induced in subvolumes?
Equilateral non-Gaussianity protected from infrared modes?
- ◆ Background mode biasing from models of inflation
[LoVerde 1310.5739], [Bartolo et. al., 1210.3257], [M. Thorsrud et. al., 1311.3302],
[Linde & Mukhanov, 0511736]
- ◆ Anomalies in CMB from superhorizon mode coupling?
[Schmidt & Hui, 1210.2965]
- ◆ Superhorizon bias at the level of the action...fixed classical configuration for superhorizon modes