Title: Does the Quantum Particle know its own Energy?

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Abstract: If a wave function does not describe microscopic reality then what does? Reformulating quantum mechanics in path-integral terms leads to a notion of ``precluded event'' and thence to the proposal that quantal reality differs from classical reality in the same way as a set of worldlines differs from a single worldline. One can then ask, for example, which sets of electron trajectories correspond to a Hydrogen atom in its ground state and how they differ from those of an excited state. We address the analogous questions for simple model that replaces the electron by a particle hopping (in discrete time) on a circular lattice.

Does a quantum particle know its own energy?

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work with

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David Rideout, Sumati Surya, Petros Wallden,

To what physical reality does the quantum formalism refer?

The answer that I will propose springs from thinking of quantum mechanics as a theory of histories based on the path integral, and from putting aside questions of probability in order to grapple first with the logical paradoxes of quantum mechanics.

The resulting framework identifies physical reality with a kind of generalized history described by an anhomomorphic coevent.

I will illustrate this proposal in the context of a very simple toy-universe, that of a particle hopping on a lattice, and I will present an elementary calculation that I hope will indicate how the proposal is meant to go more generally.

Because we work with histories (trajectories) and not with wave functions, concepts like position will have straightforward meanings but ones like momentum or energy will not. We then have to ask, for example, how an electron in a hydrogen atom can know when it is in its ground state or perhaps in some excited state. Is there something about its trajectory that carries this information? Hence the title of this talk.

A simple unitary model — the n-site hopper

Particle hops on a circular lattice $\mathbb{Z}_n = (0, 1, 2 \cdots n-1)$

Both space **and** time are discrete

Transfer matrix is discrete analog of non-relativistic free particle on line

Amplitude to hop in one step from $x \in \mathbb{Z}_n$ to x' depends on whether n is even or odd.

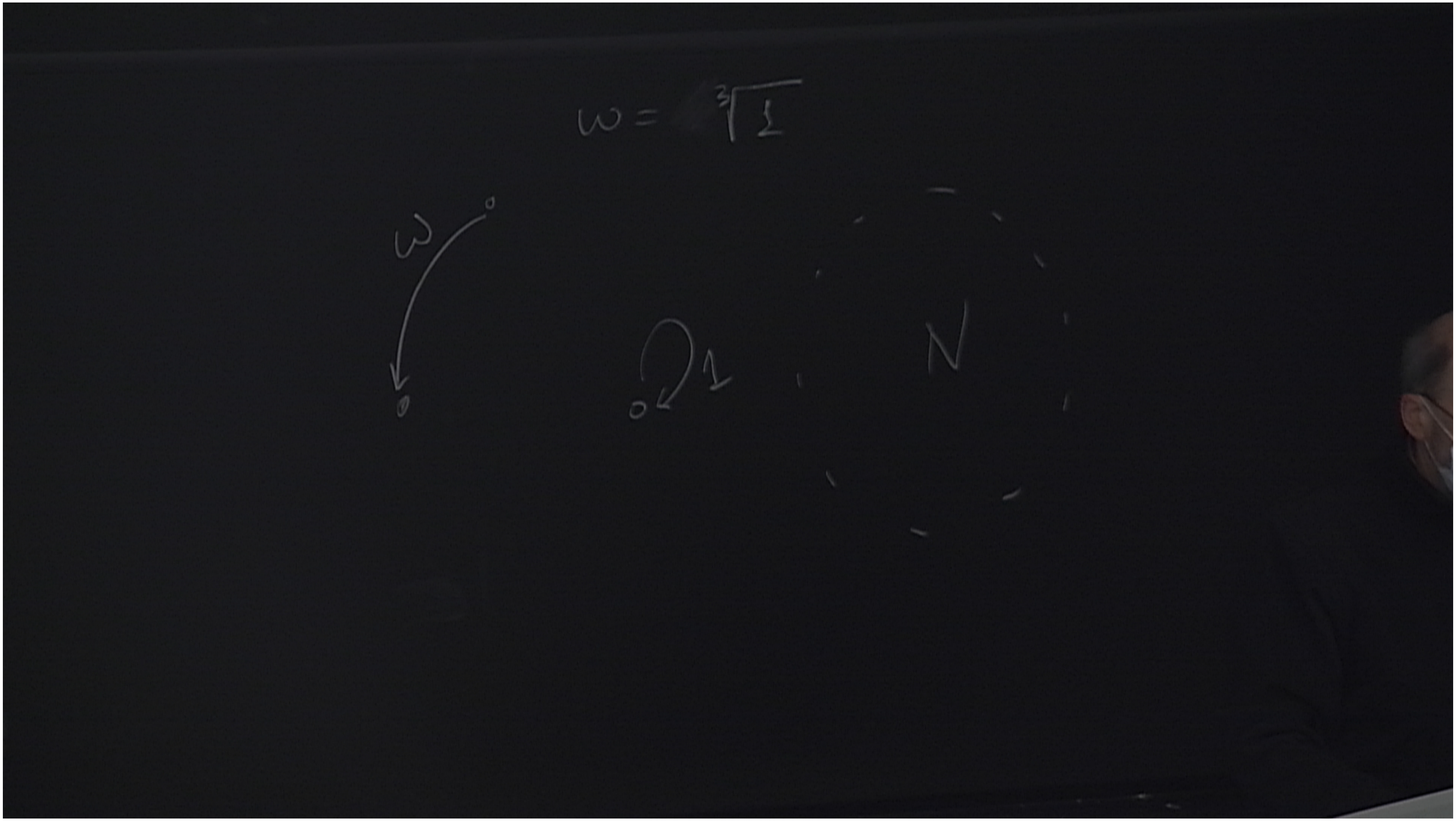
(Let $\mathbf{1} = e^{2\pi i}$)

$$\frac{1}{\sqrt{n}} \quad \mathbf{1}^{(x-x')^2/\mathbf{n}} \qquad n \text{ odd}$$

$$\frac{1}{\sqrt{n}} \quad \mathbf{1}^{(x-x')^2/\mathbf{2n}} \qquad n \text{ even}$$

Special cases, $n = 2$ and $n = 3$:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \omega & \omega \\ \omega & 1 & \omega \\ \omega & \omega & 1 \end{pmatrix} \qquad \omega = 1^{1/3}$$


ω = ∛1
ω
1
N

Review of anhomomorphic coevents and the "Multiplicative Scheme" (MSk)

Histories are fundamental, not the wave-function

Physical reality — a "possible world" — is described by a **coevent** ϕ

It says which **events** A happen and which don't

Dynamics takes the form of a path integral (or path sum)

ψ enters only as an initial condition (an approximate summary of past)

What does the path-integral compute?

The quantal measure of an event $\mu(A)$ (cf. decoherence functional)

How compute $\mu(A)$ for a unitary system: propagate ψ forward via A and form $||\psi_A||^2$

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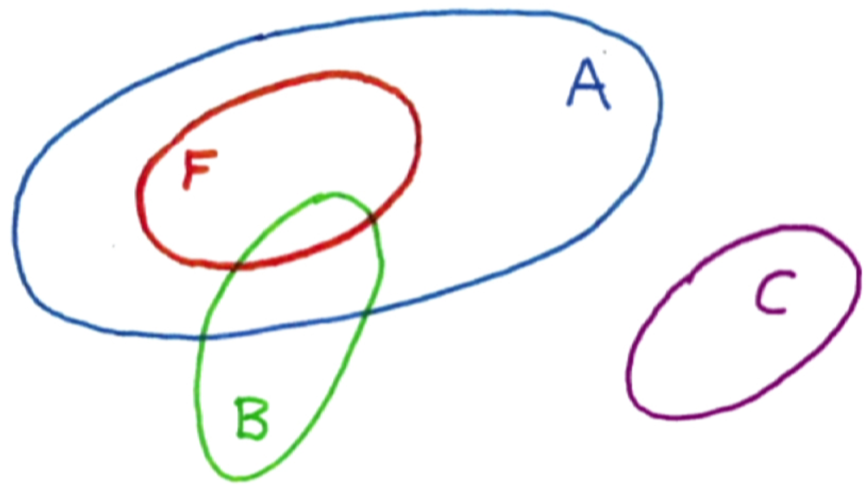
Preclusion arises naturally out of path integral

Hard to imagine that $\mu = 0$ doesn't mean preclusion (e.g. instruments)

To be **primitive** *F* must be **minimal** consistent with the preclusions



Reality ⟷ a set of histories

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Preclusion **partly** determines ϕ: it says certain events don't happen

To complete the story we need more input: **primitivity** ("minimality")

How to define primitivity is a question . . .

MSk is perhaps the simplest answer

Suitably phrased, it coincides precisely with the corresponding classical criterion.

Classically a single history describes reality. Quantally, **interference** produces

non-classical patterns of preclusion which require a different description ...

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The event A happens iff $F \subseteq A$

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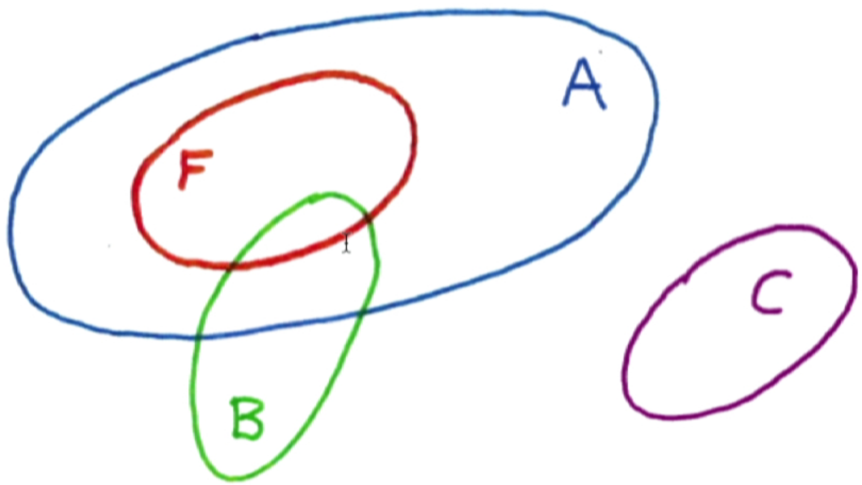

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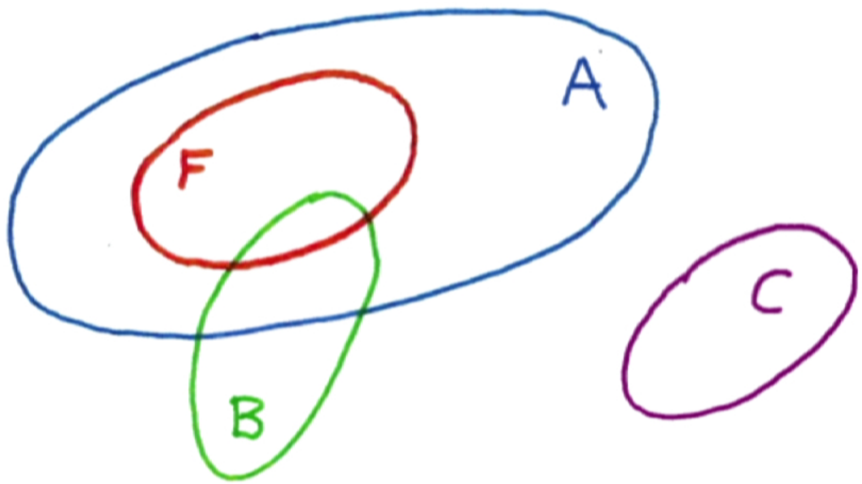

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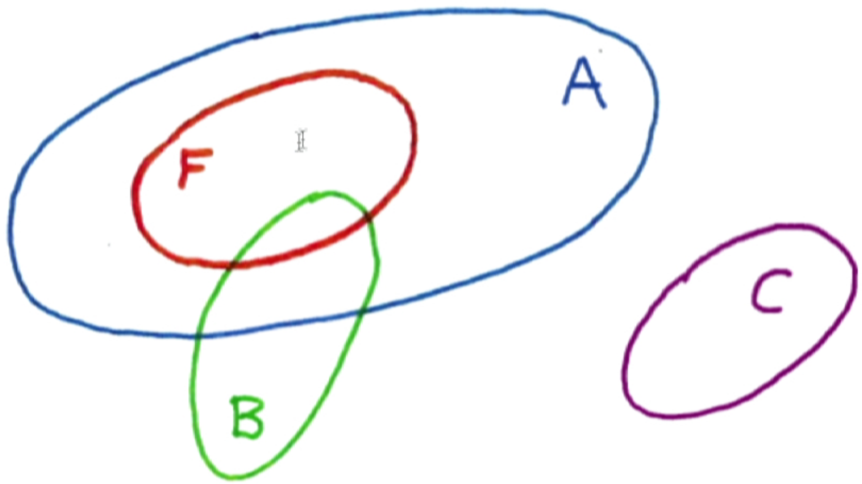

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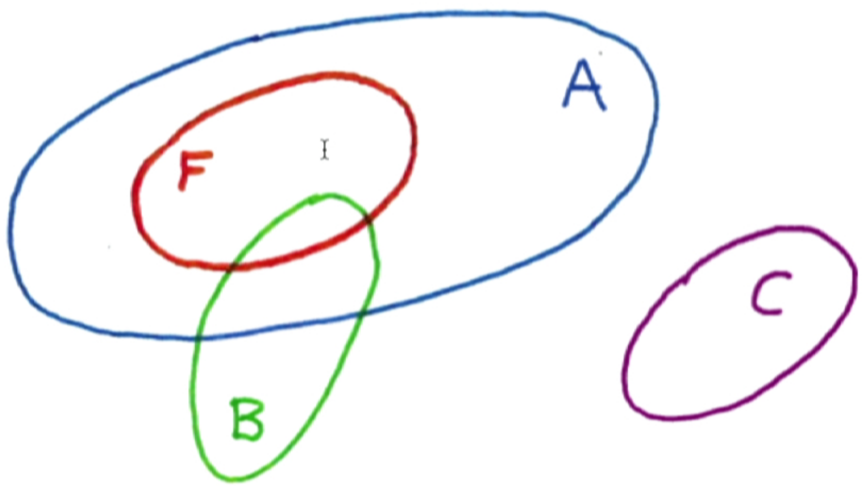

Reality $\longleftrightarrow$ a set of histories

Much more could be said about this scheme and its consequences, but

I want to focus on a very concrete example and on our specific question:

Does the particle know its own energy?

In simple cases, preclusion is decided by whether the sum of the amplitudes vanishes

$\Longrightarrow$ It will be easy to find the primitive supports ...

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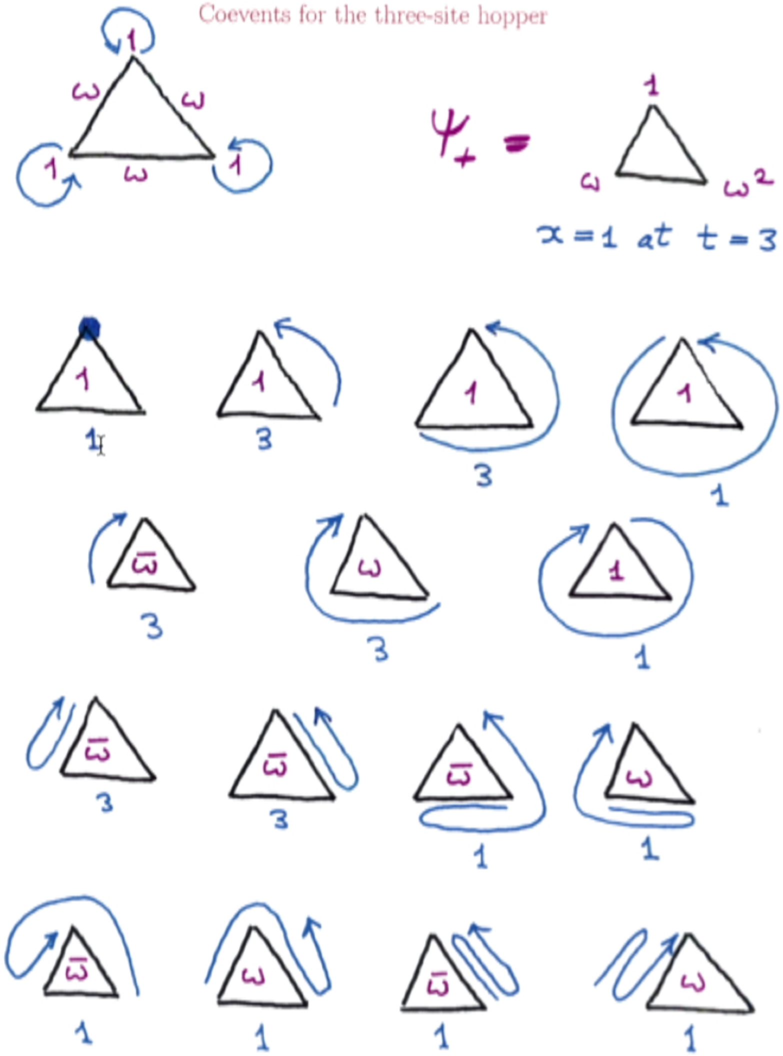
Coevents for the three-site hopper



Histories and their amplitudes for the three-site hopper

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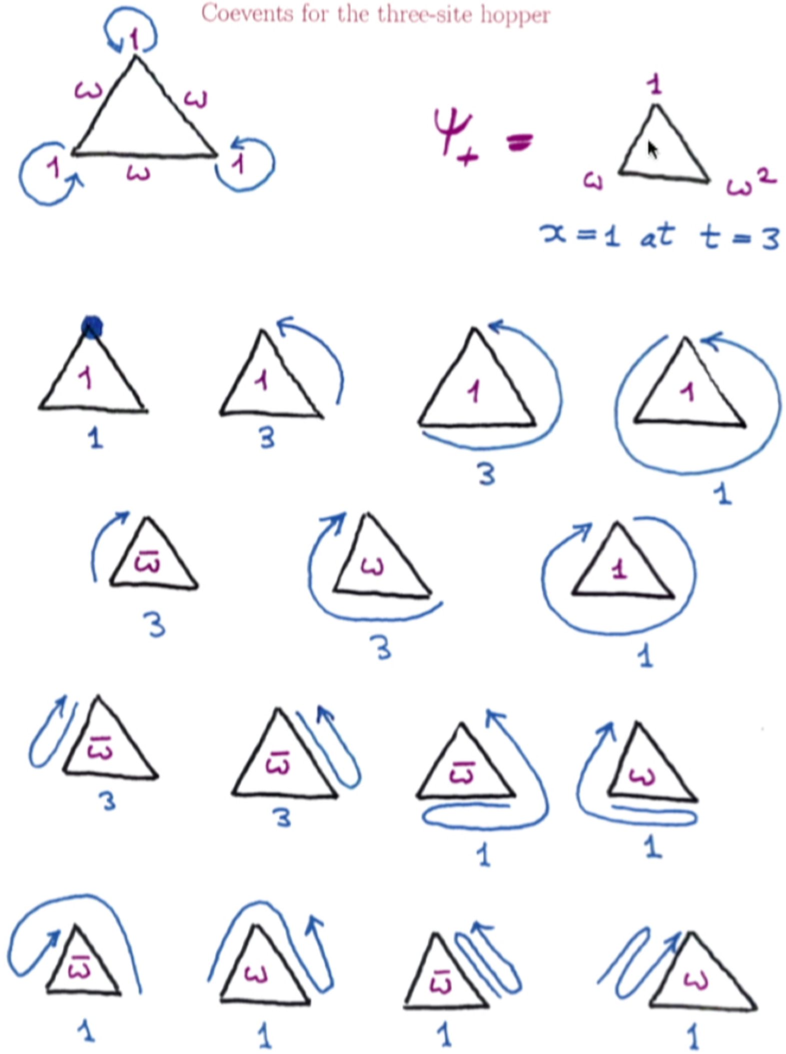
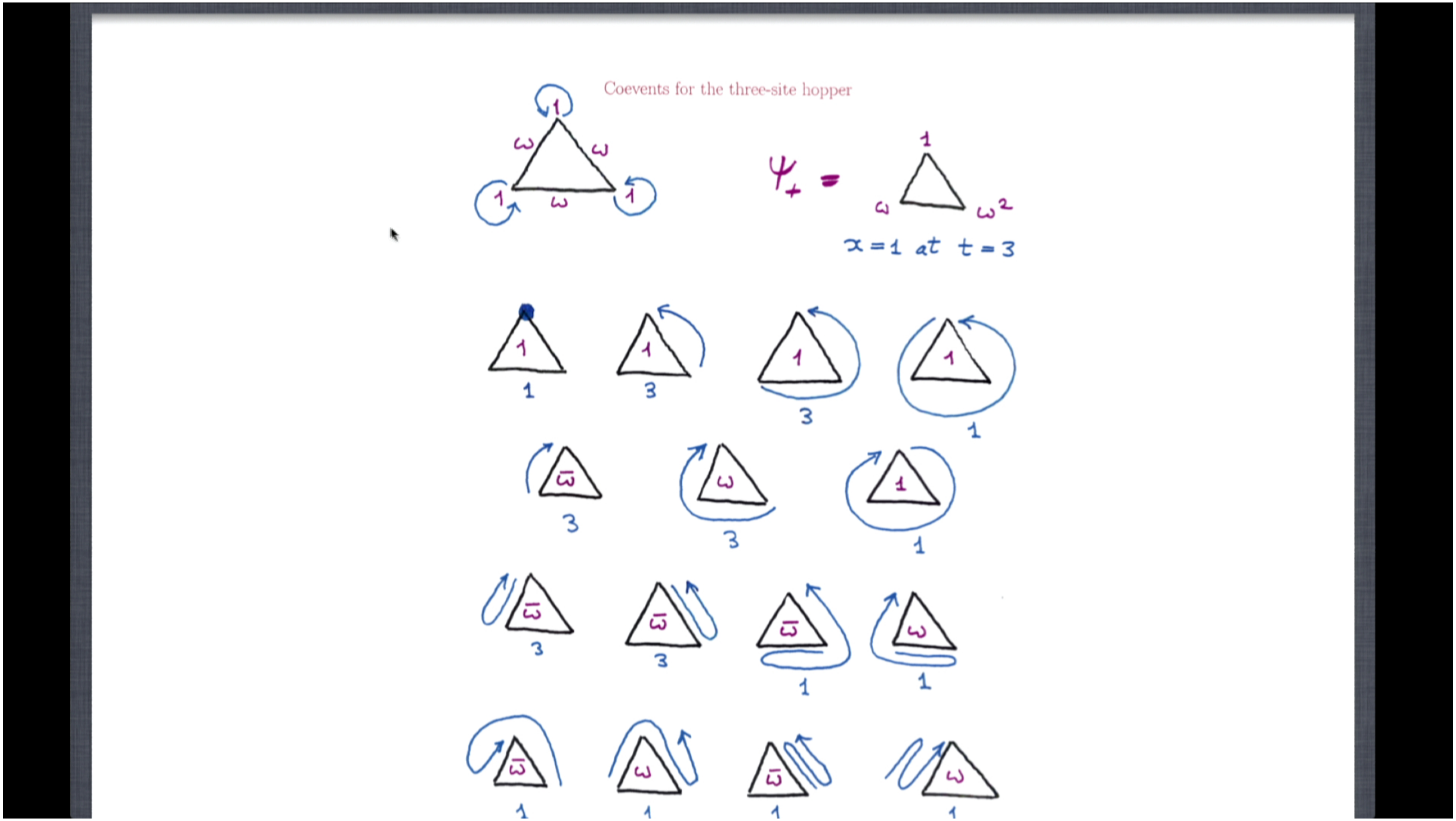
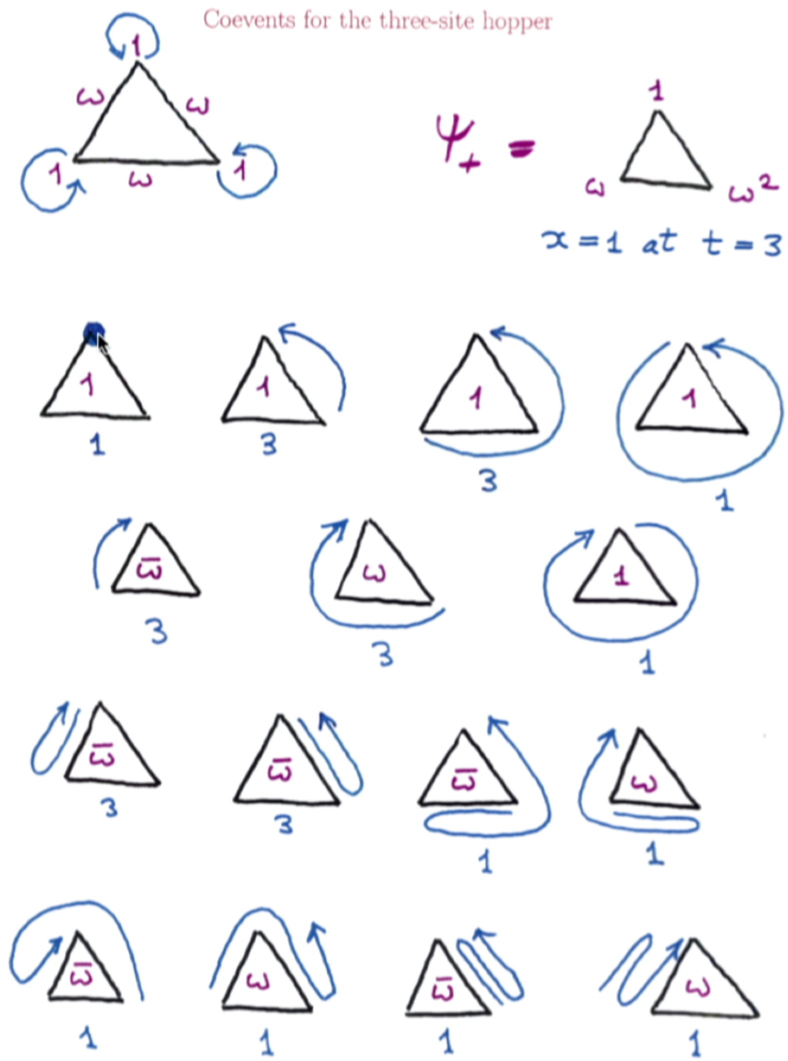



Coevents for the three-site hopper
$\psi_+ =$
$x = 1$ at $t = 3$

Coevents for the three-site hopper



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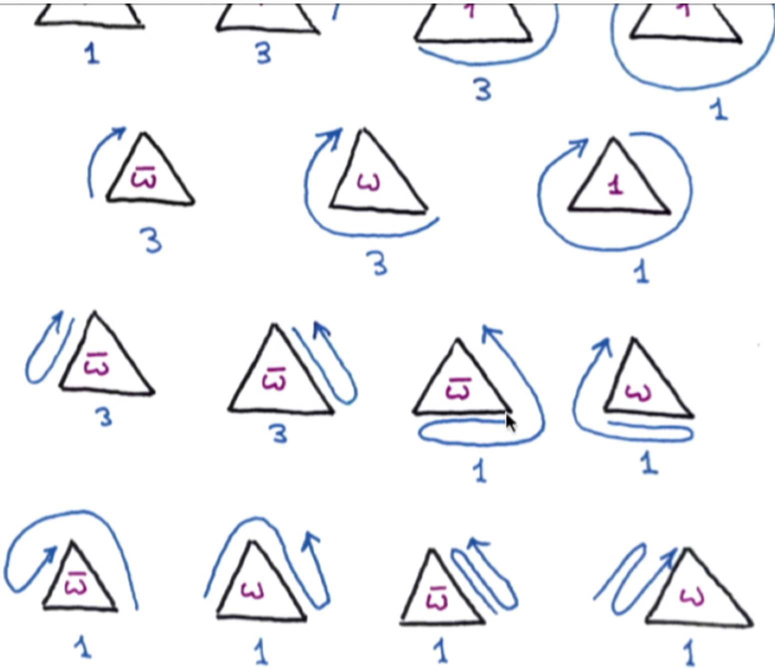

Histories and their amplitudes for the three-site hopper

The primitive coevents for $\psi = \psi_+$

$$\overline{\omega}^{\{12\}} \qquad \mathbf{1}^{\{9\}} \qquad \omega^{\{6\}}$$

Which combinations yield primitive coevents?

$$\mathbf{1} + \omega + \overline{\omega} = \mathbf{0}$$

So 7 copies of 1 or seven copies of $\overline{\omega}$ will work

$$1^{\{7\}} \implies \binom{9}{7} = 36 \quad \text{primitive coevents}$$

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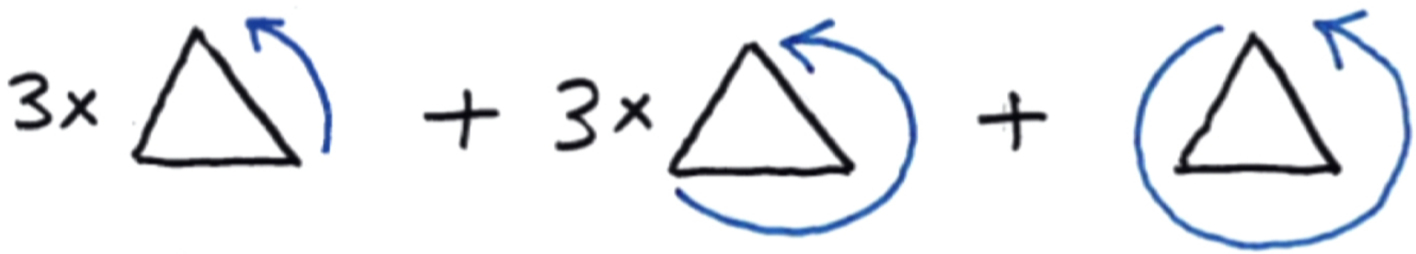
A total of 828 primitive coevents in all. For example this support:



This coevent has a net circulation of $3 \times 1 + 3 \times 2 + 1 \times 3 = 12$

(The coevents of type $\varpi^{\{7\}}$ contribute no net circulation on average)

Average net circulation over all primitive coevents = $7/23 > 0$

The particle "tends to circulate positively"

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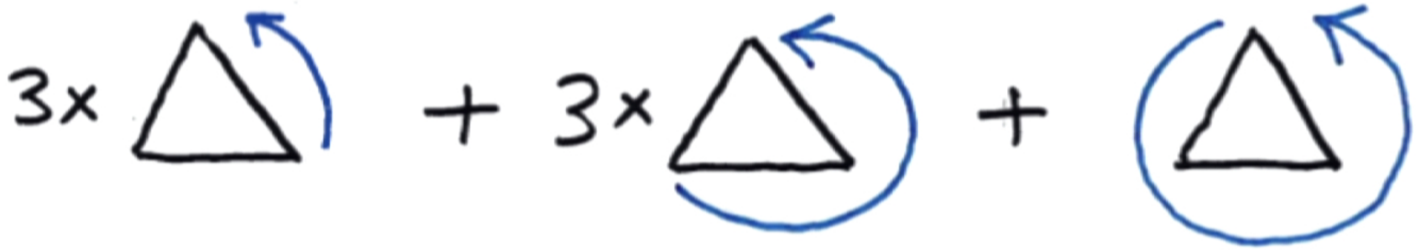

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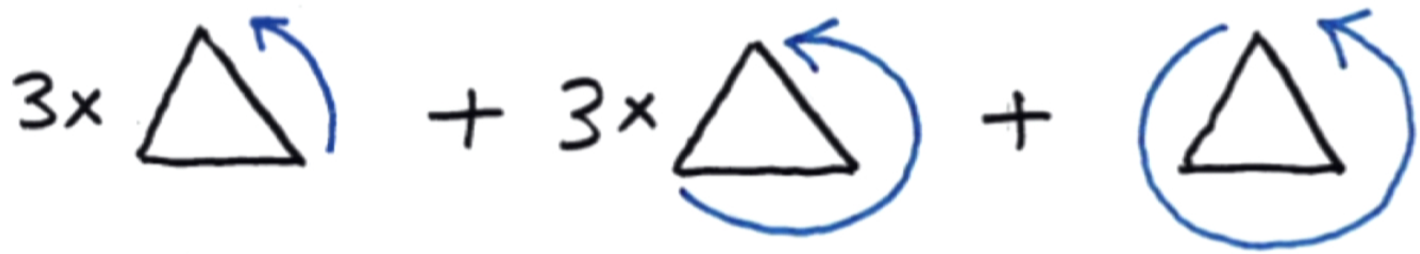

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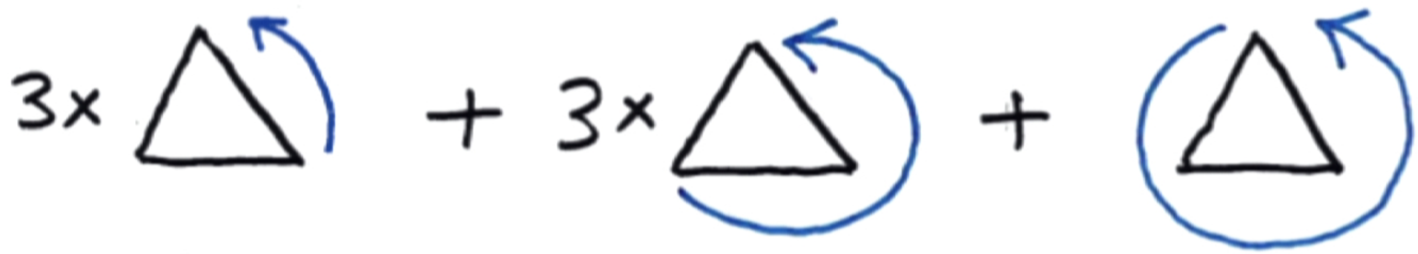

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The particle "tends to circulate positively"

How peripatetic is the ground state particle?

The event that the hopper never rests is affirmed by eight primitive coevents.

That event happens in the reality described by those coevents

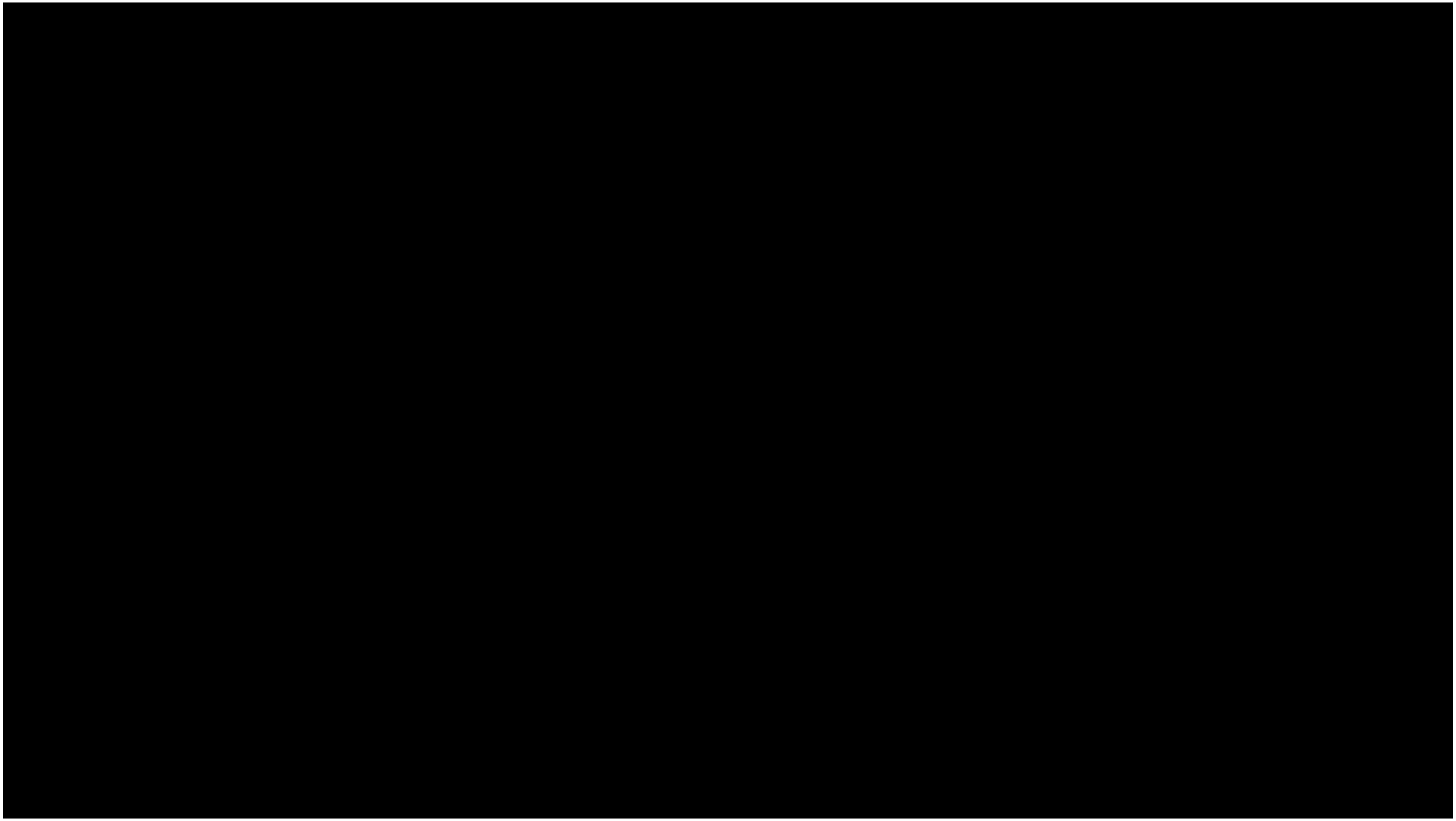
In all we find:

8 coevents: the hopper never rests

28 coevents: the hopper either never rests (6 paths) or never moves (1 path)

792 coevents: the hopper hops twice and rests once

Very different from Bohmian answer!



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Events Coevents Preclusion Primitivity

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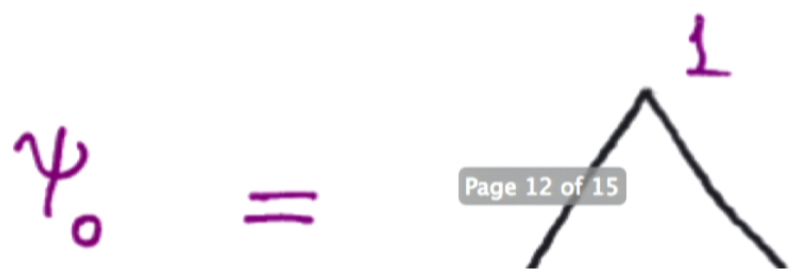

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Does the particle know its energy?

Well, can it distinguish between the two initial-states ψ_0 and ψ_+ ?

There are 828 coevents in each case

No coevent is common to both cases

Knowing the coevent $\Longrightarrow$ knowing the energy

(given that ψ is a stationary state so that "energy" has meaning)

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(given that ψ is a stationary state so that “energy” has meaning)

Moreover, various simple events can distinguish between the two cases:

If the particle **never rests** then $\psi = \psi_0$

If the particle **moves only counterclockwise** then $\psi = \psi_+$

If the particle **rests exactly once** then $\psi = \psi_+$

etcetera

In this sense . . .

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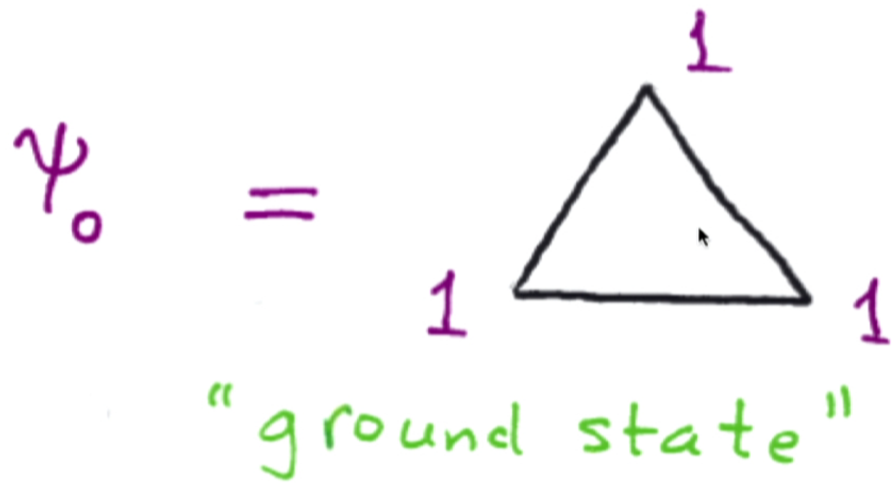

Same set of paths, different amplitude assignments

The multiset of amplitudes happens to be the same: $1^{\{9\}} + \omega^{\{6\}} + \overline{\omega}^{\{12\}}$

Hence 828 coevents again

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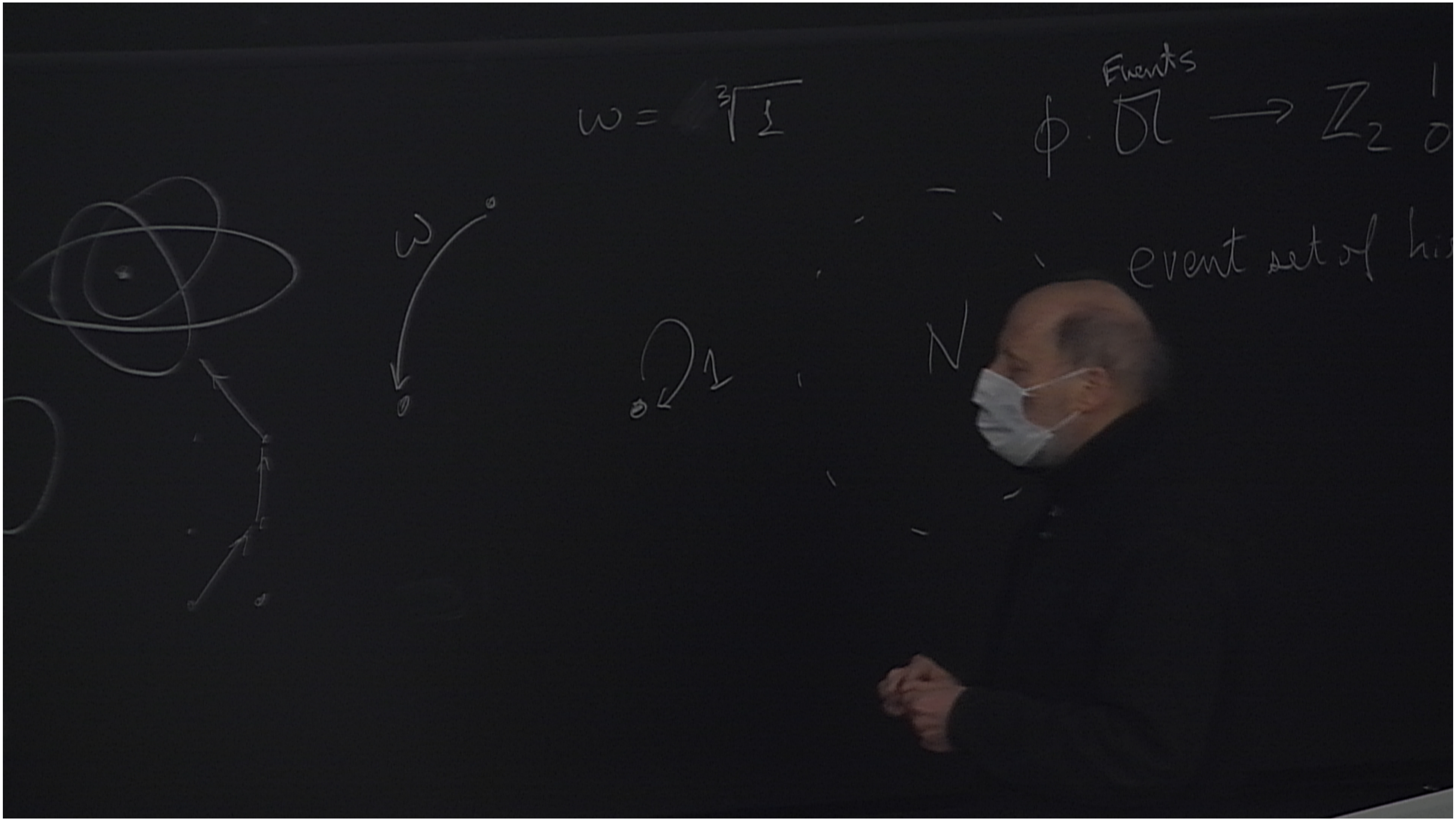

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