Title: Simulating the Universe(s)

Date: Jan 14, 2014 11:00 AM

URL: http://pirsa.org/14010101

Abstract: The theory of eternal inflation in an inflaton potential with multiple vacua predicts that our universe is one of many bubble universes nucleating and growing inside an ever-expanding false vacuum. The collision of our bubble with another could provide an important observational signature to test this scenario. In this talk I will describe an algorithm for accurately computing the cosmological observables arising from bubble collisions directly from the Lagrangian of a single scalar field. This represents the first fully-relativistic set of predictions from an ensemble of scalar field models giving rise to eternal inflation, and I will describe on-going phenomenological studies and observational searches.

Simulating the Universe(s)

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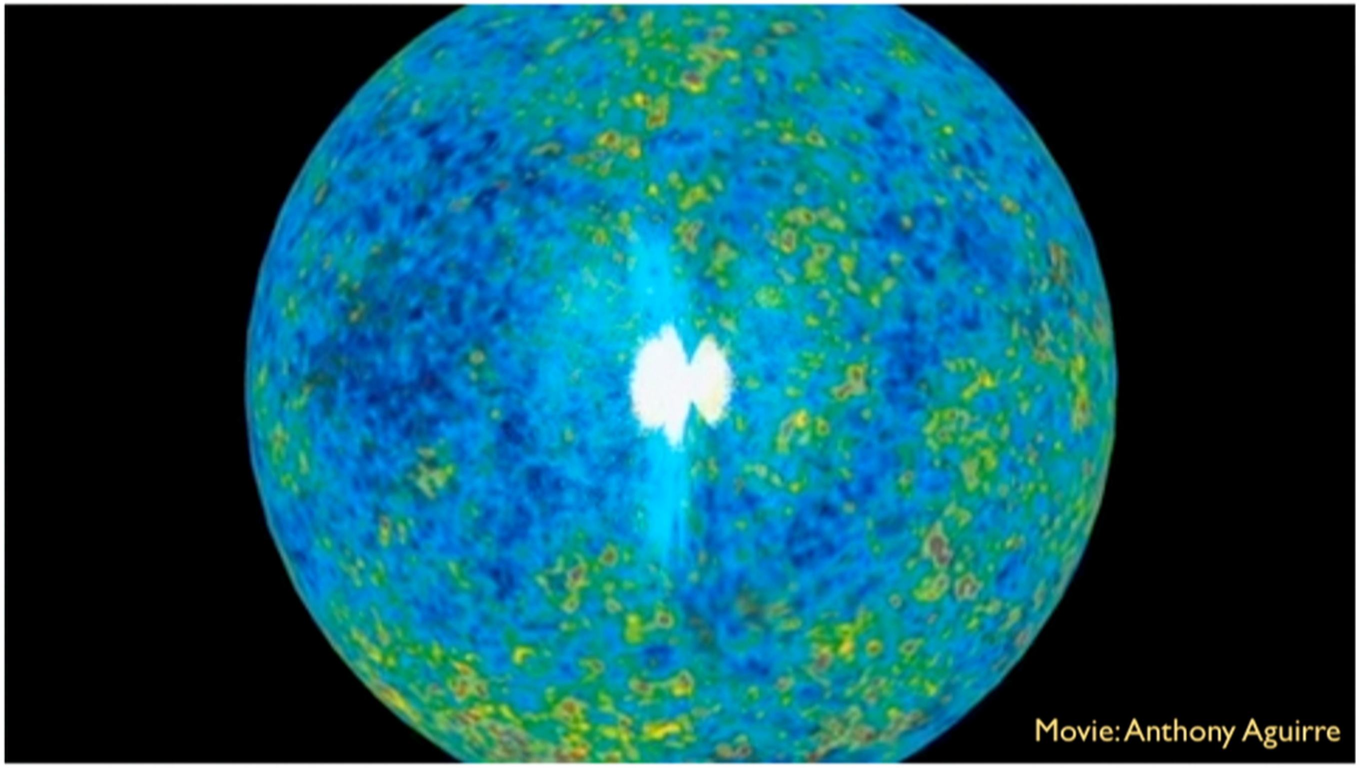
L. Lehner, S. Leibling

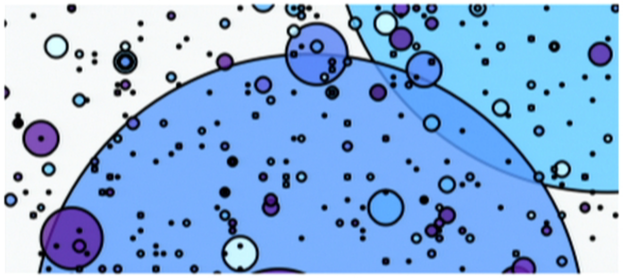
Eternal Inflation: is this our universe?



Observational Tests of Eternal Inflation

- But is eternal inflation experimentally verifiable?

Our bubble does not evolve in isolation....

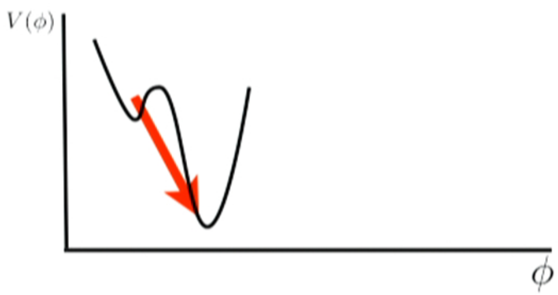


The collision of our bubble with others provides an observational test of eternal inflation.

Aguirre, MCJ, Shomer

Open Inflation

- How does our observable universe fit into this picture?



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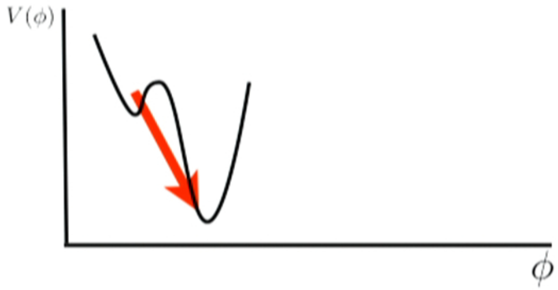

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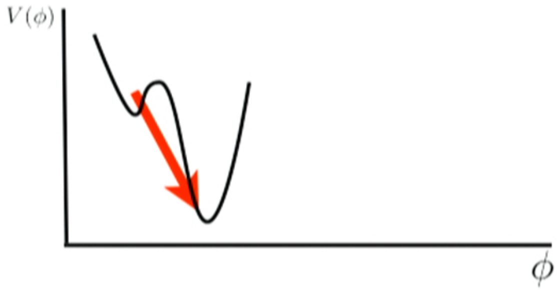

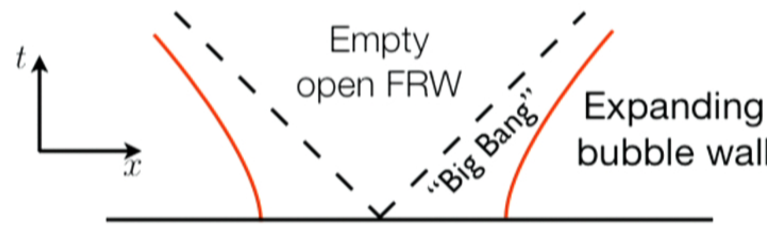

- Vacuum bubbles are open and empty.

Open Inflation

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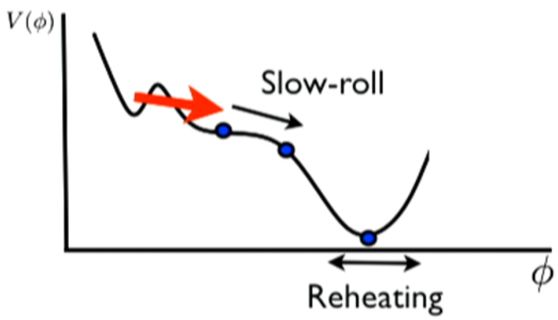

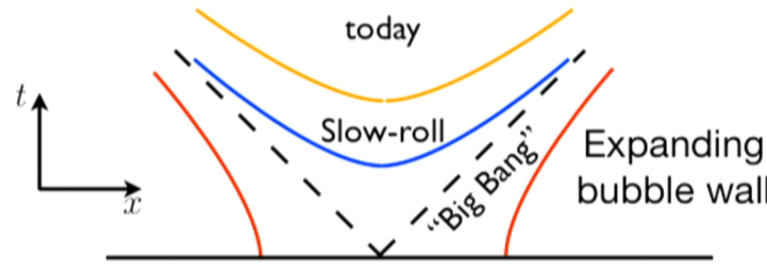

- Adding an epoch of slow-roll inflation inside the bubble makes a viable cosmology.

"Open Inflation" - Bucher, Goldhaber, Turok; Gott

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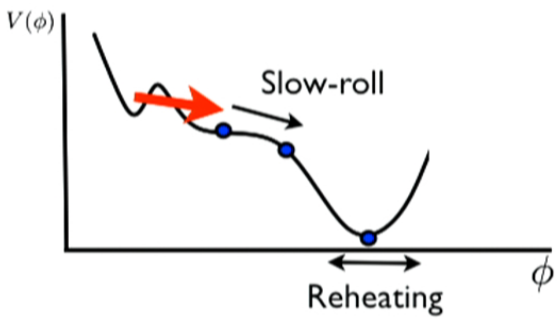

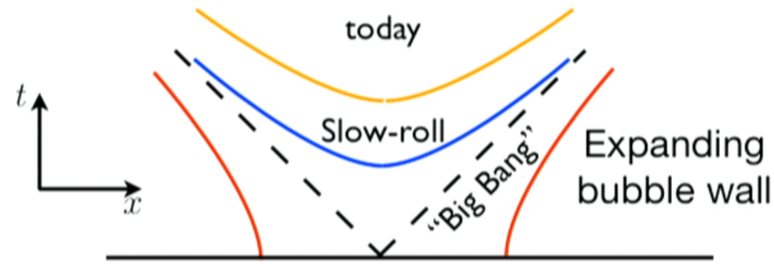

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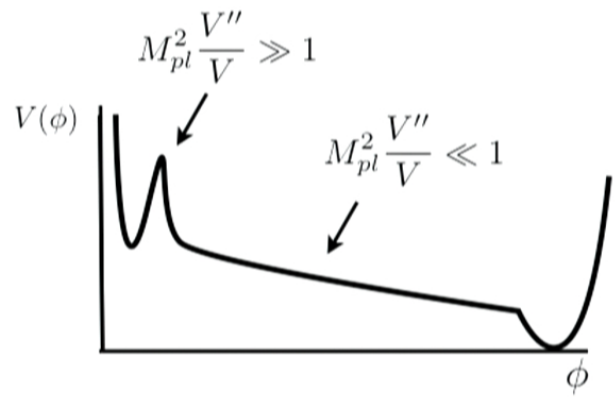
- The condition to form bubbles via the CDL instanton is at odds with the condition for slow roll inflation:



Open inflation requires a hierarchy in scales.

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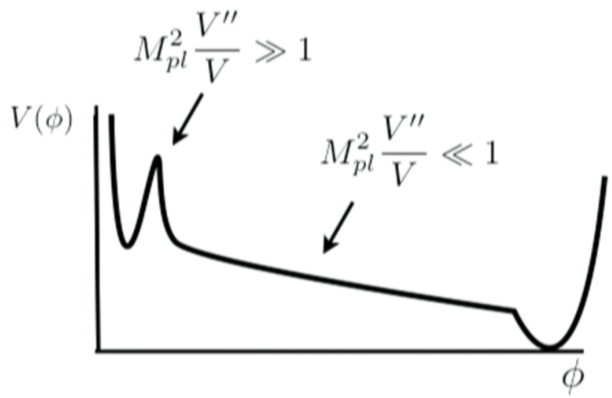

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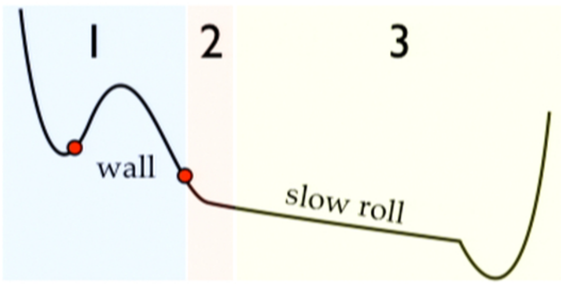
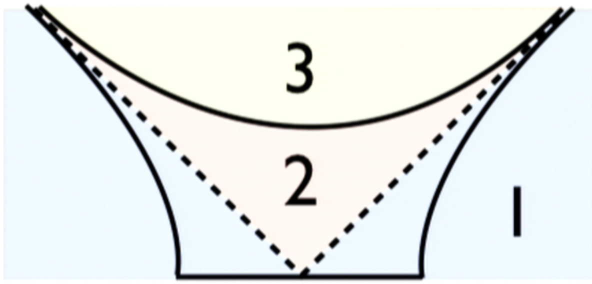


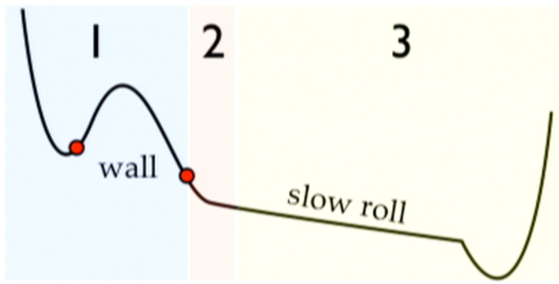


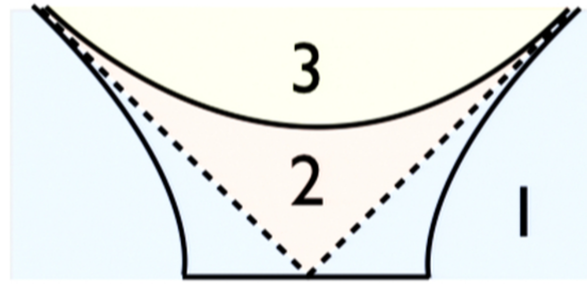
- Three regions on the potential affect three different features of the single bubble spacetime.

Bubble phenomenology: embed different models of inflation inside different bubbles.

A ***calculable*** theory of statistically isotropic initial conditions for your favorite model of inflation.

Open Inflation





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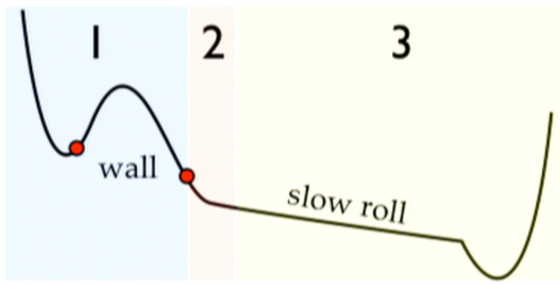

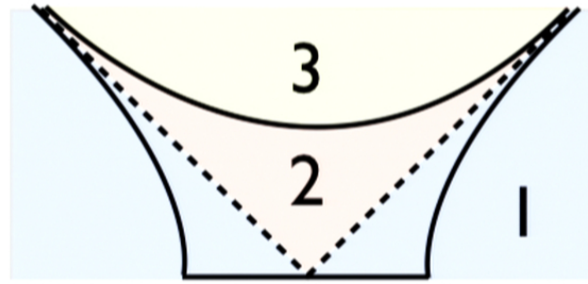

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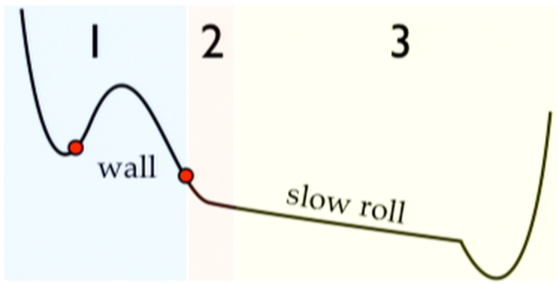

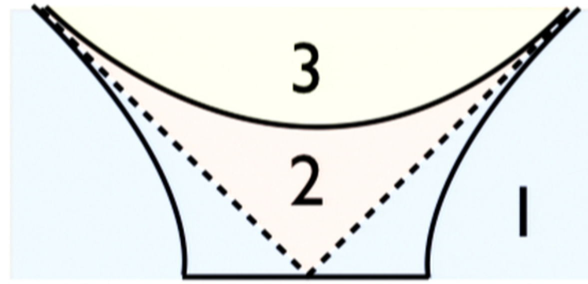

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Open Inflation

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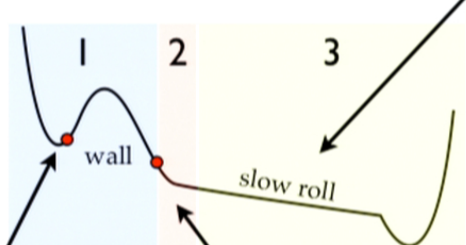
$$\Omega_k \neq 0 \quad (\Omega_k < 10^{-2}) \longleftrightarrow N_e \text{ (+ reheating)}$$



Modified vacuum
extra tensors
supercurvature modes

features in the potential

Open Inflation

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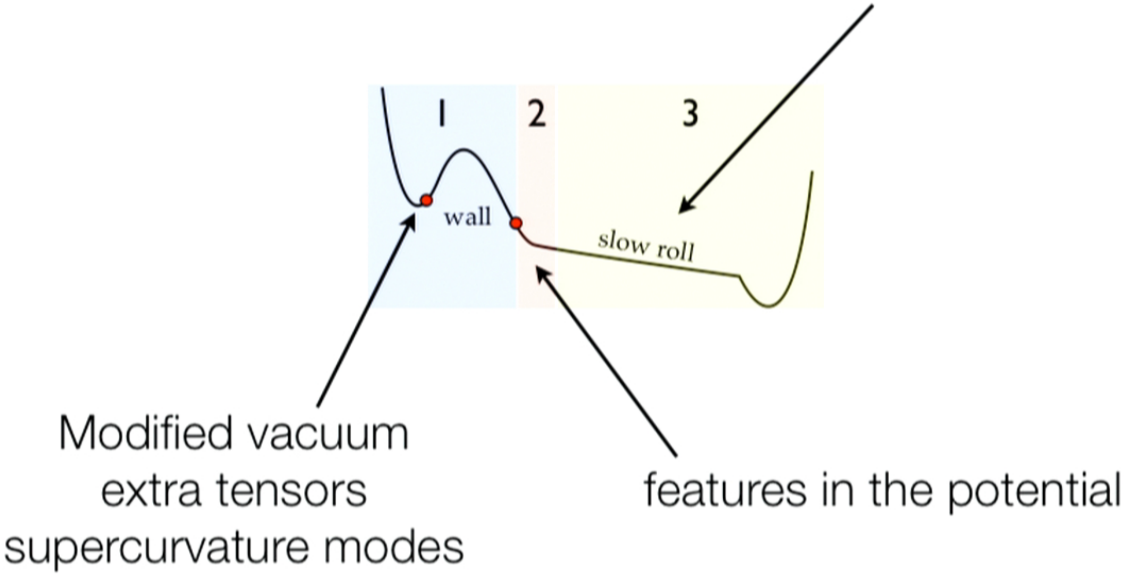
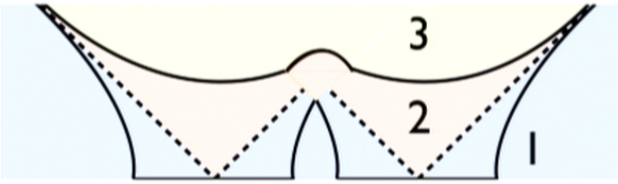


Collisions

- Observational effects:

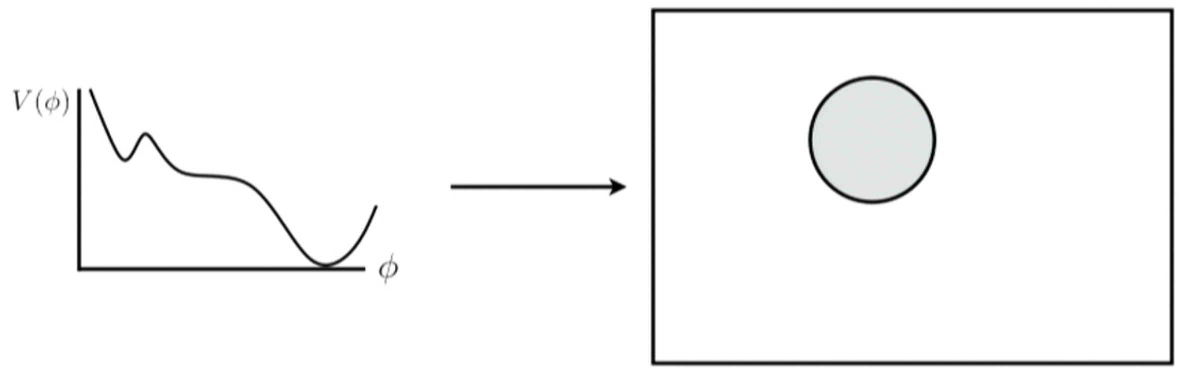


- Collisions are always in our past.
- The outcome is fixed by the potential and kinematics.
- Bubble nucleation is a stochastic process.

A ***calculable*** theory of inhomogeneous initial conditions for your favorite inflationary model.

Making predictions

- How does one go about making a prediction?



Properties of a single bubble.

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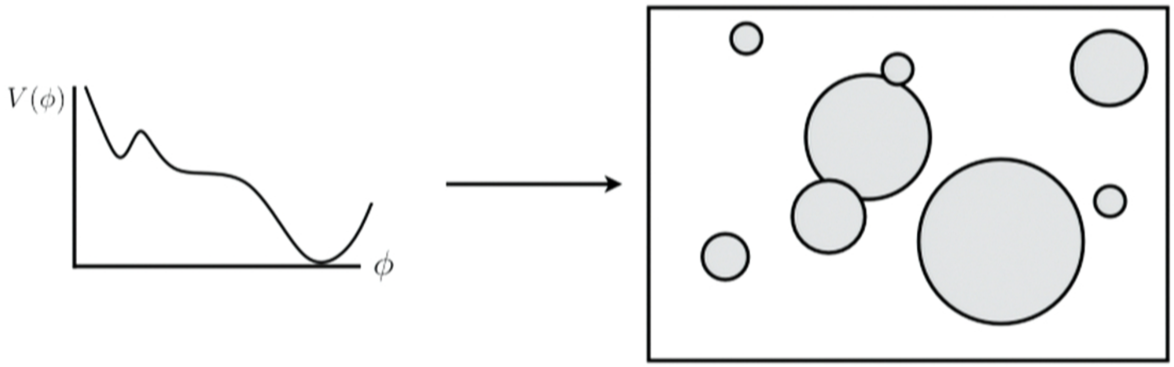

Properties of a single bubble.

Properties of the eternally inflating spacetime (eg probability of bubble nucleation).

Making predictions

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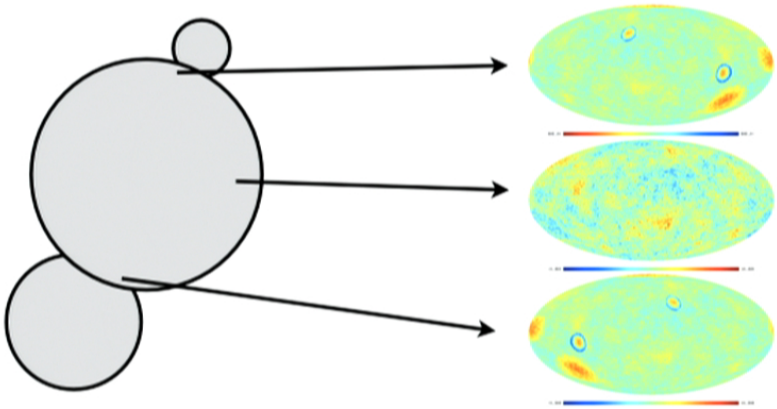

Ensemble of possible observational signatures and probability distribution over possibilities.

(Ensemble - stochastic nucleation + observer location)

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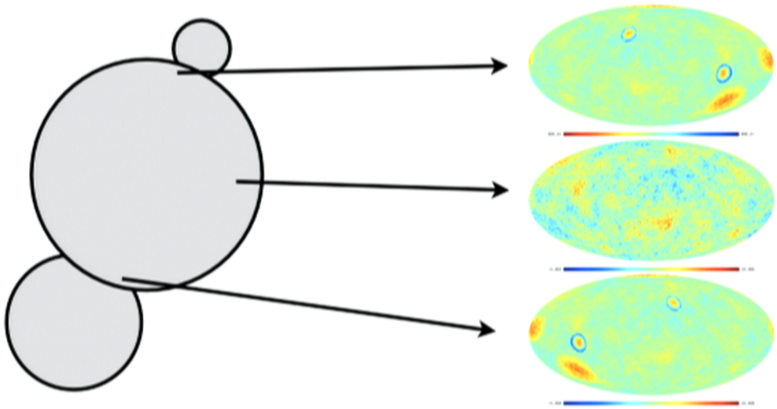

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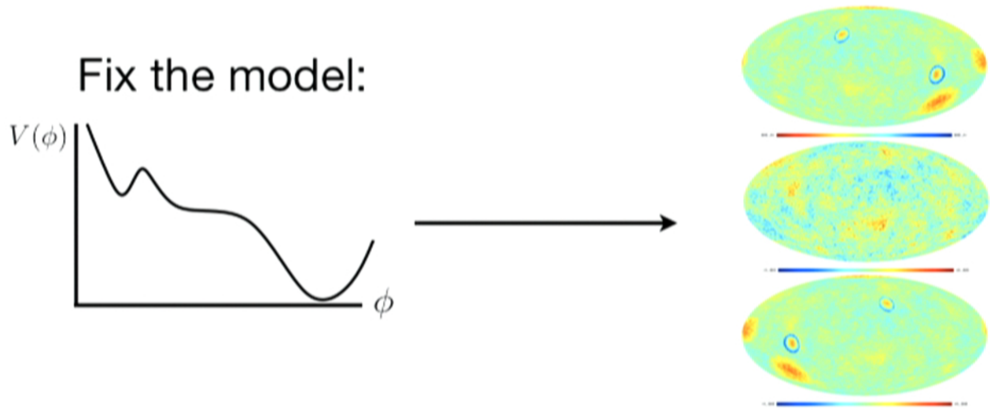
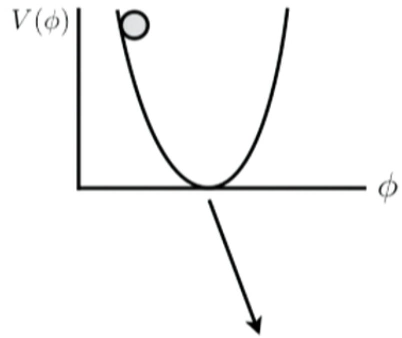
What constitutes a set of predictions?

Fix the model:

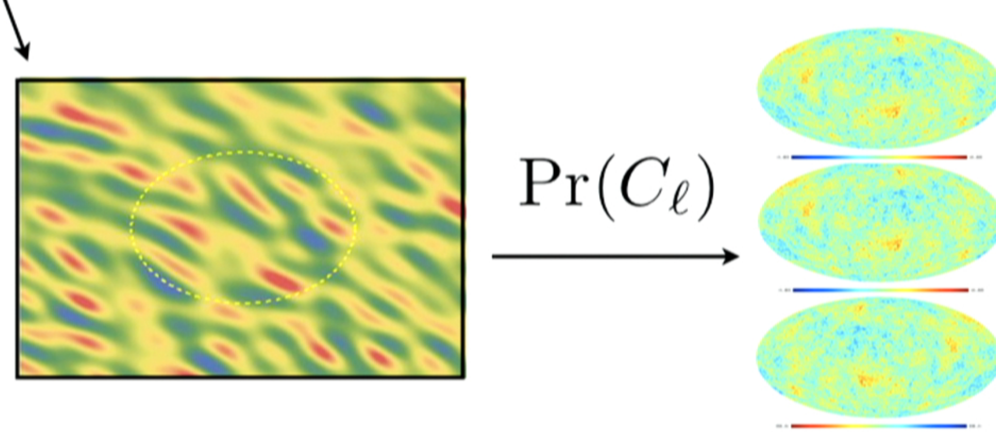


$\bar{N}_s$	expected number of collisions
$\mathbf{m}$	observables characterizing each collision
$\Pr(N_s, \mathbf{m} \mid \bar{N}_s)$	How many of each type do I expect to find?

What constitutes a set of predictions?

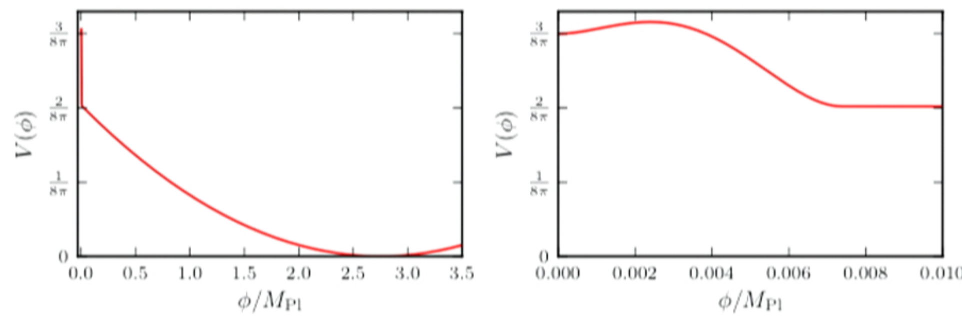


- Initial conditions
- Dynamics
- Map to observables
- Probability measure



(Ensemble - stochastic fluctuations + observer location)

Properties of a single model



- Potential: match barrier suitable for CDL instanton to chaotic inflation.
- Vary the matching point to vary the duration of inflation.
- Re-scale the potential to vary the nucleation rate.
- Phenomenologically viable models of single bubble open inflation.

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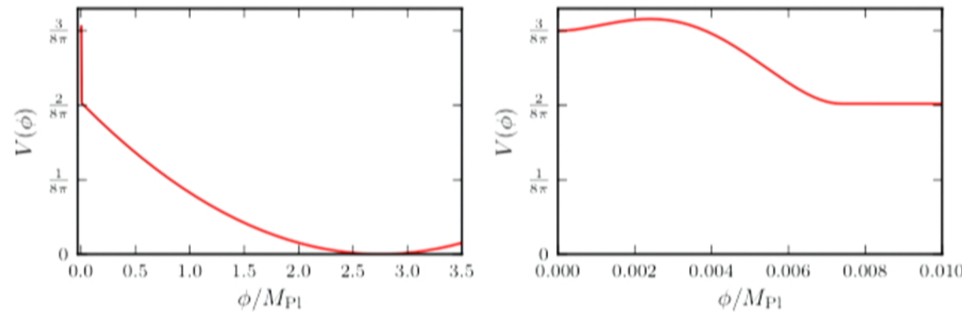

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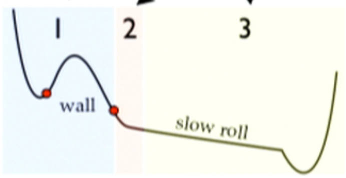
Properties of the eternally inflating spacetime

- Total number of causally accessible collisions:

$$\bar{N}_s = \frac{16\pi\lambda}{3H_F^2} \times \frac{\Omega_k^{1/2}}{H_I^2}$$



- Can be used as a (not necessarily unique) proxy for models.

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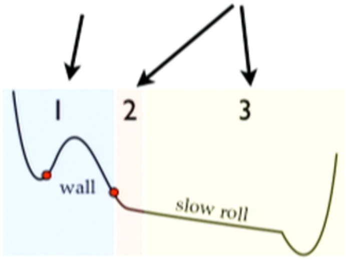

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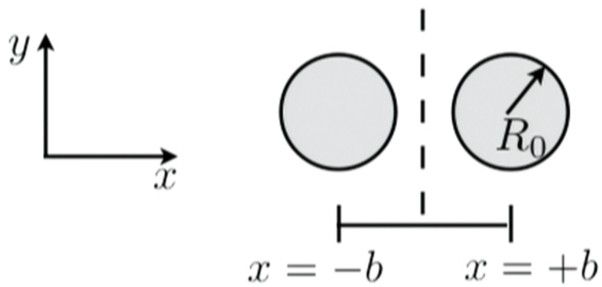
Properties of the collision spacetimes

- Collisions are rare: consider them individually.
- Symmetries of the collision spacetime:



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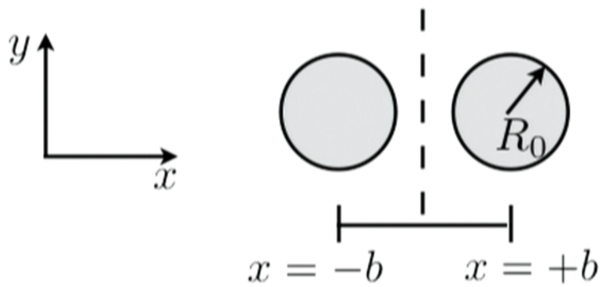

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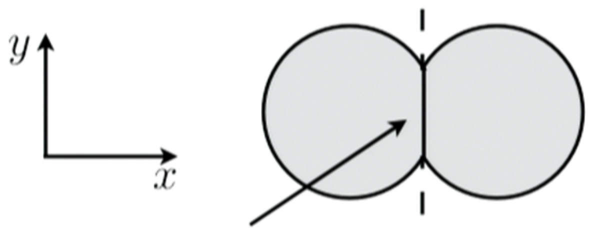

$$(x \pm b)^2 + y^2 + w^2 - t^2 = R_0^2$$

$y^2 + w^2 - t^2 = (R_0^2 - b^2)$ ⟶ Collision spacetime has SO(2,1) symmetry

Hawking, Moss, Stewart

- There are enough symmetries to reduce the problem to 1 space and 1 time dimension.

Determining the collision spacetime

- To definitively study what happens, need full GR and numerics.
  - We want to find the post-collision cosmology: GR.
  - Huge center of mass energy in the collision.
  - Non-linear potential, non-linear field equations.
- Evolution code written in Python and c incorporating:
  - 4th order convergence.
  - Adaptive mesh refinement (AMR).
  - Adaptive simulation boundaries.

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Evolution Equations

$$H_F^2 ds^2 = -\alpha^2 dN^2 + \cosh^2 N a^2 dx^2 + \sinh^2 N (d\chi^2 + \sinh^2 \chi d\phi^2)$$

Evolution:

$$\frac{d\alpha}{dN} = \alpha(A+B)$$

$$\frac{da}{dN} = a(-A+B)$$

$$\frac{d\Pi}{dN} = -\left(\tanh(N) + \frac{2}{\tanh(N)}\right) + \frac{d}{dx}\left(\frac{\alpha\phi'}{a\cosh^2(N)}\right) - \alpha a \partial_\phi V.$$

$$\Pi \equiv \frac{a}{\alpha}\frac{d\phi}{dN}$$

$$A \equiv \tanh(N) + \frac{1}{2\tanh N} - \frac{\alpha^2}{2}\left(\frac{1}{\cosh(N)\ \sinh(N)} + 8\pi \tanh(N)\ V(\phi)\right)$$

$$B \equiv 2\pi\ \tanh(N)\ \frac{\alpha^2}{a^2}\left(\frac{\phi'^2}{\cosh^2(N)} + \Pi^2\right),$$

Constraint:

$$\frac{d\alpha}{dx} = \frac{4\pi\tanh(N)\ \alpha^2\phi'\Pi}{a}$$

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Initial Conditions

- Taking the limit as N goes to 0 in the evolution equations:

$$\begin{aligned} \alpha &= 1 - \alpha_2(x) N^2 \\ a &= 1 + a_2(x) N^2 \\ \phi &= 1 + \phi_2(x) N^2 \\ \Pi &= 2\phi_2 N. \end{aligned}$$

$$\begin{aligned} \alpha_2 &= -\frac{1}{2} + \frac{2\pi}{3}\left(2V(\phi_0) - \phi_0'^2\right) \\ a_2 &= -\frac{1}{2} + \frac{4\pi}{3}\left(V(\phi_0) + \phi_0'^2\right) \\ \phi_2 &= \frac{1}{6}\left(\phi_0'' - \partial_\phi V|_{\phi_0}\right), \end{aligned}$$

- The initial field profile $\phi_0(x)$ is determined by adding two widely separated CDL instantons constructed using the CosmoTransitions code (Wainwright).

http://chasm.uchicago.edu/cosmotransitions/

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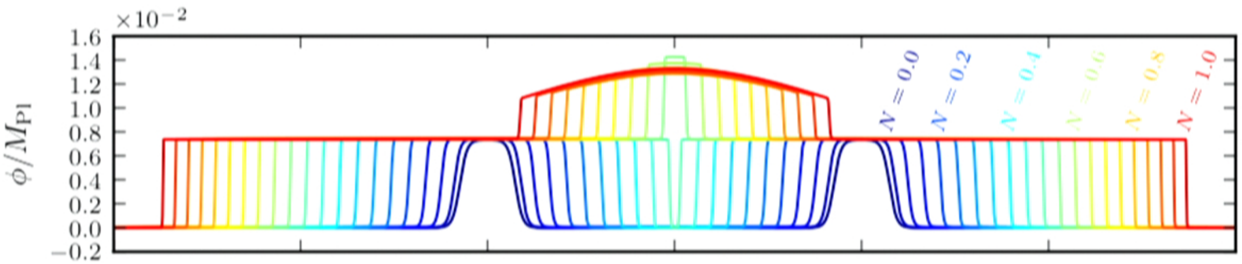
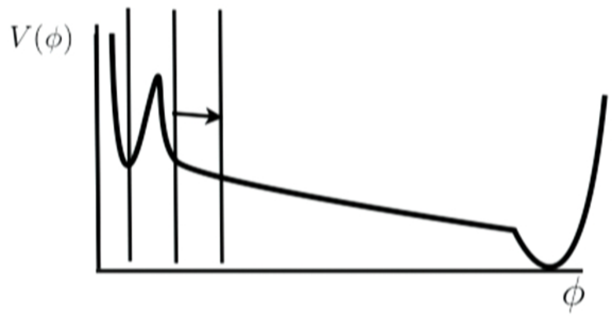
Example



- Field profiles superpose in the immediate aftermath of collision.
- Leads to an advance of the inflaton down the slope by a distance of order the barrier width in a growing region of spacetime.



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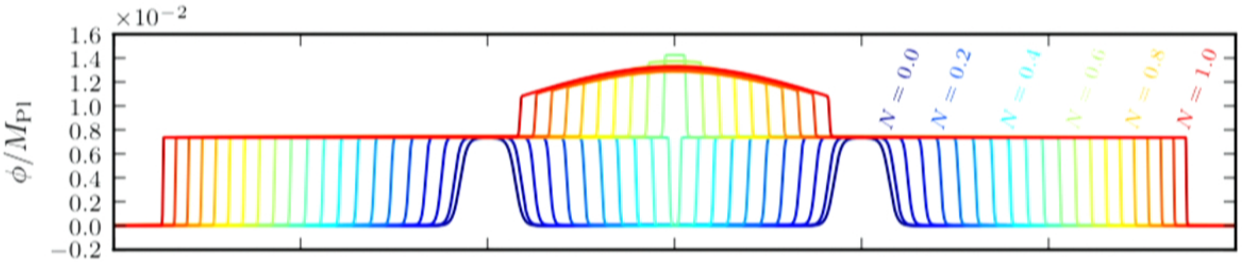

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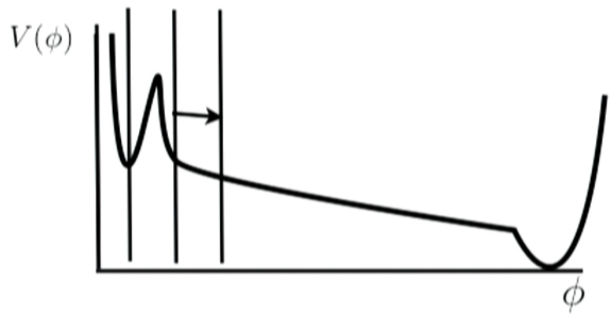
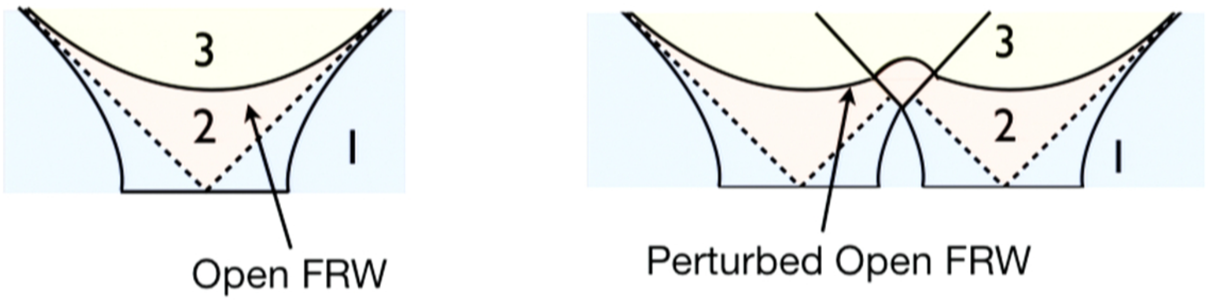
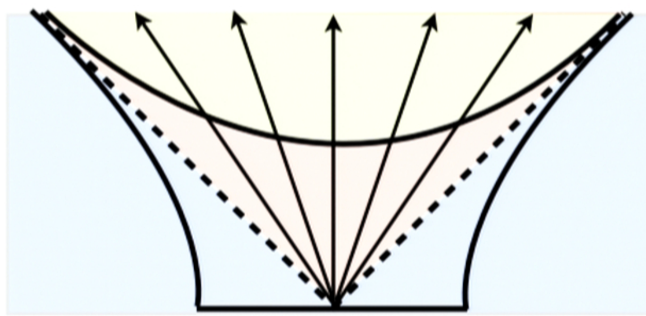


Metric inside the bubble



- We have the global metric.
- We want to find the perturbed open FRW metric in one bubble.
- Different possible approaches:
  - Covariant approach. Xue et. al.
  - Local expansion (like Fermi coordinates).
  - Geodesic shooting.

Metric inside the bubble



- At early times all non-singular metrics approach:

$$H_F^2 ds^2 = -d\tau^2 + \left[\frac{\tau}{1-\frac{R^2}{4}}\right]^2 \left(dX^2 + dY^2 + dZ^2\right) \quad \text{Milne}$$

- Positions label geodesics in Minkowski space with rapidity

$$\eta = 2\ \mathrm{arctanh}\left[\frac{\sqrt{X^2+Y^2+Z^2}}{2}\right]$$

- Comoving geodesics don't evolve, so these are good coordinate labels for all times.

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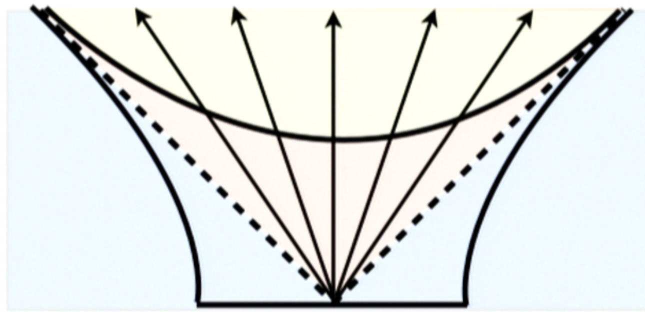

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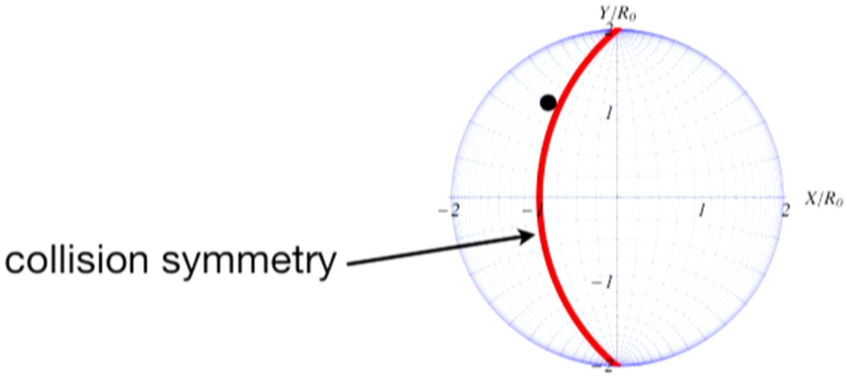
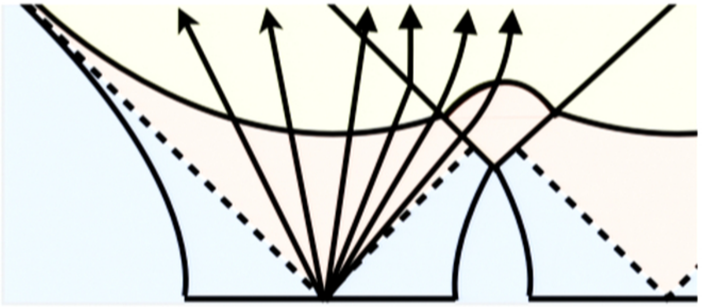


Constant time hypersurface

$$H_F^2 ds^2 = -d\tau^2 + \left[\frac{\tau}{1-\frac{R^2}{4}}\right]^2 \left(dX^2 + dY^2 + dZ^2\right)$$

$$H_F^2 ds^2 = -d\tau^2 + \tau^2 \left[d\xi^2 + \cosh^2\xi \left(d\rho^2 + \sinh^2\rho \; d\varphi^2\right)\right]$$

Metric inside the bubble

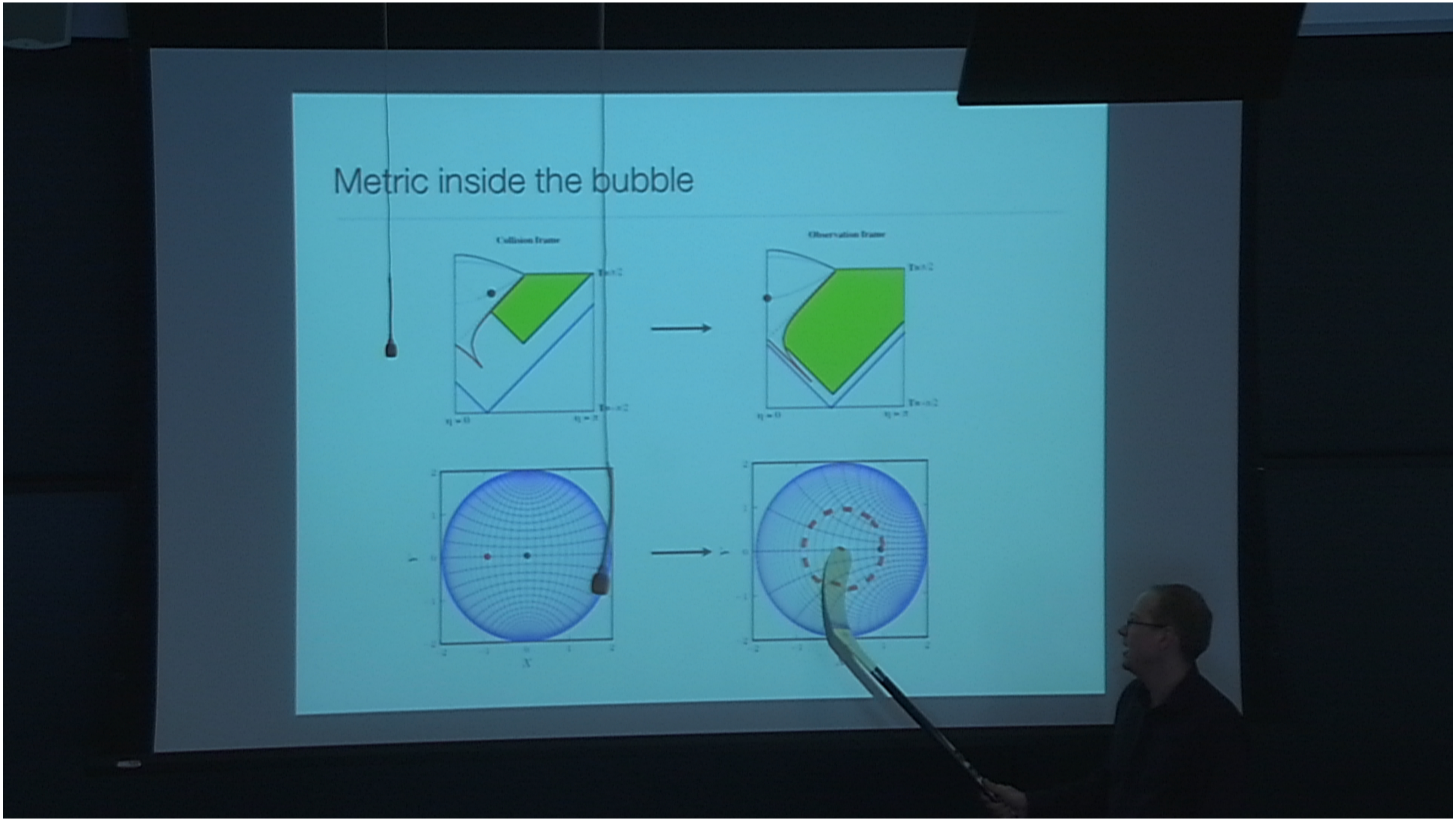


- We evolve the geodesics through the simulation metric:

$$\left.\frac{dN}{d\tau}\right|_{\tau=0} = \cosh(\xi)\,, \quad N(\tau=0)=0,$$
$$\left.\frac{dx}{d\tau}\right|_{\tau=0} = \sinh(\xi)\,, \quad x(\tau=0)=0,$$
$$\left.\frac{d\rho}{d\tau}\right|_{\tau=0} = 0, \quad \rho(\tau=0)=\rho,$$
$$\left.\frac{d\varphi}{d\tau}\right|_{\tau=0} = 0, \quad \varphi(\tau=0)=\varphi.$$

$$\frac{d^2N}{d\tau^2} + \Gamma^N_{NN}\left(\frac{dN}{d\tau}\right)^2 + 2\Gamma^N_{Nx}\frac{dN}{d\tau}\frac{dx}{d\tau} + \Gamma^N_{xx}\left(\frac{dx}{d\tau}\right)^2 = 0,$$
$$\frac{d^2x}{d\tau^2} + \Gamma^x_{NN}\left(\frac{dN}{d\tau}\right)^2 + 2\Gamma^x_{Nx}\frac{dN}{d\tau}\frac{dx}{d\tau} + \Gamma^x_{xx}\left(\frac{dx}{d\tau}\right)^2 = 0,$$

$$\longrightarrow N(\xi,\tau) \quad x(\xi,\tau)$$


Metric inside the bubble
Collision frame
Observation frame

Metric inside the bubble

- Finally, passing through a sequence of coordinate transformations, we apply:

$$g_{\mu\nu}[X] = \frac{dx^\alpha}{dX^\mu}\frac{dx^\beta}{dX^\nu} g_{\alpha\beta}[x(X)]$$

Perturbed Open FRW   Simulation metric

- We want to map onto the scalar perturbations:

$$\delta g_{ij} \equiv -2D^{(\mathrm{syn})}(\vec{X},\tau)\delta_{ij} + E_{ij}^{(\mathrm{syn})}(\vec{X},\tau)$$

trace   trace-free, symmetric

- Vectors small empirically, tensors zero by symmetry.

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Metric inside the bubble

- We define:

$$\delta g_{ij} = \left(g_{ij}^{(\mathrm{coll})} - g_{ij}^{(\mathrm{no\ coll})} \right) \frac{\left(1 - \frac{R^2}{4}\right)^2}{a^2(\tau)}$$

- And extract:

$$D^{(\mathrm{syn})} = -\frac{1}{6}\mathrm{Tr}\left(\delta g_{ij}\right) \qquad E_{ij}^{(\mathrm{syn})} = \delta g_{ij} + 2D^{(\mathrm{syn})}\delta_{ij}$$

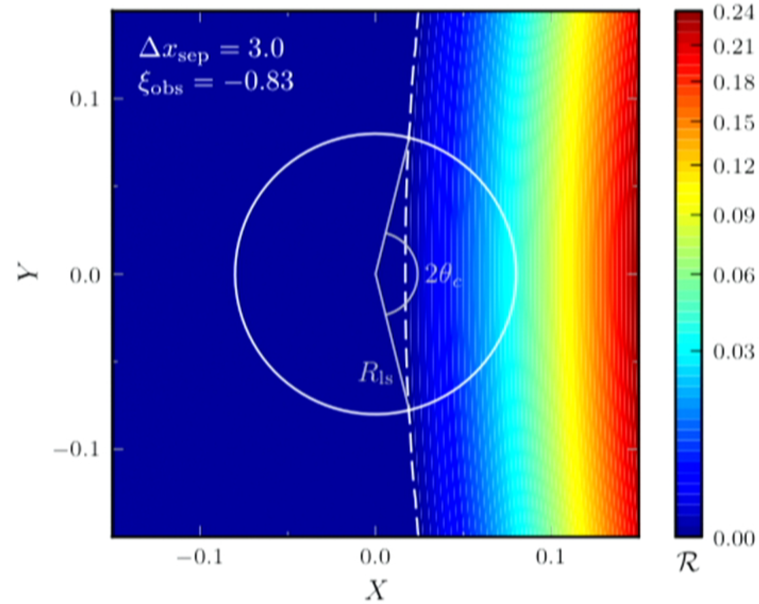
- This gives a non-linear generalization of the perturbed FRW metric in synchronous gauge.

- Convenient to extract the comoving curvature perturbation by performing a linear gauge transformation:

$$\mathcal{R} = D^{(\mathrm{syn})} + \frac{1}{4}E_{\mathrm{xx}} + H\frac{\delta\phi}{\partial_\tau \phi_0}$$

subject to assumptions...

Metric inside the bubble

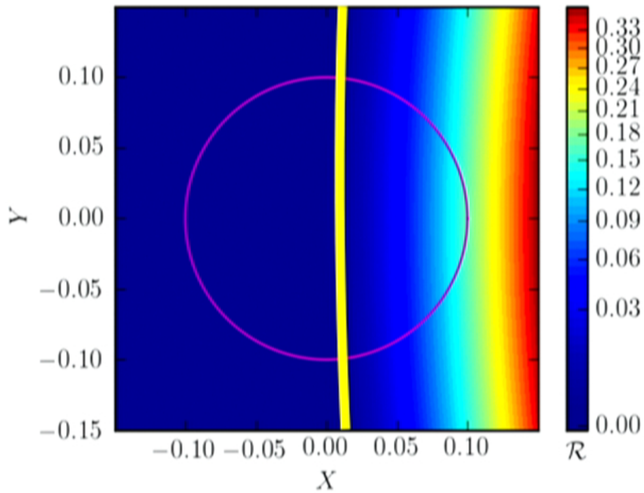


Perturbation freezes in

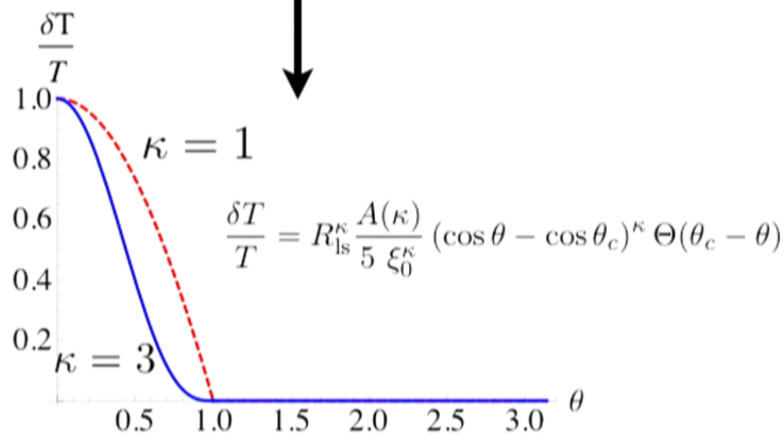
Nearly planar on scales smaller than the curvature radius

Linear assumption holds in some window

CMB Signature



$$\frac{\delta T}{T} = \frac{\mathcal{R}(x_{\rm ls})}{5}$$



(To do: beyond Sachs-Wolfe)

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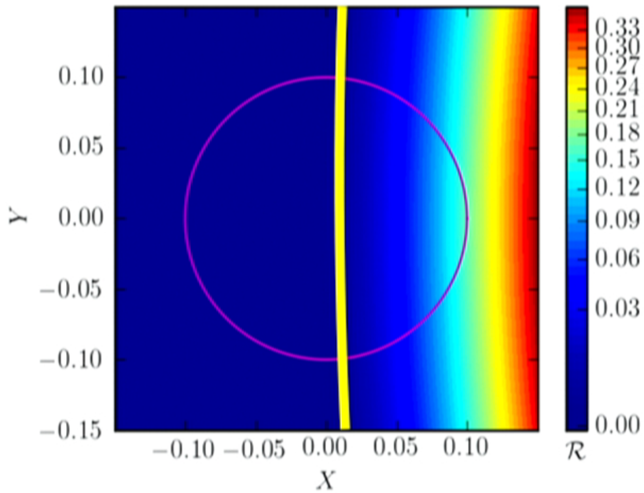

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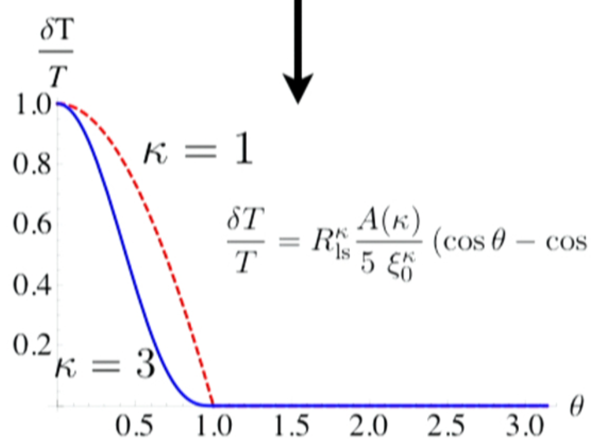

$$\frac{\delta T}{T} = R_{\rm ls}^{\kappa} \frac{A(\kappa)}{5\ \xi_0^{\kappa}} \left(\cos\theta - \cos\theta_c\right)^{\kappa} \Theta(\theta_c - \theta)$$

(To do: beyond Sachs-Wolfe)

CMB Signature

- Some properties to note:
  - Profile never linear in $\cos\theta$ as in previous studies.
  - Hotspot because inflaton is advanced by the collision.
  - Larger collisions have larger amplitudes for fixed kinematics.
  - May need small curvature to have small signal ($\Omega_k < 2 \times 10^{-7}$)

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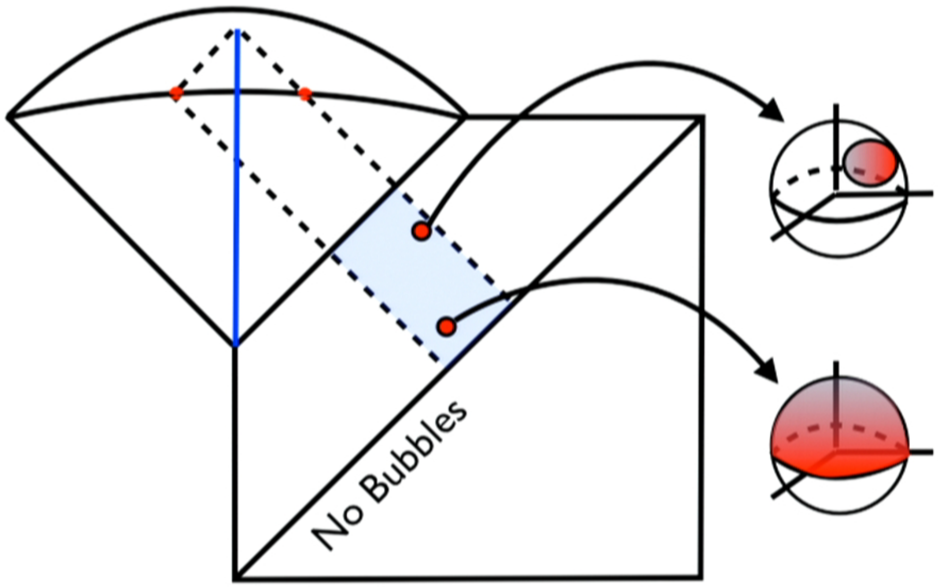
Calculating the prior



$$dN = \lambda dV_4^{\mathrm{plc}}$$

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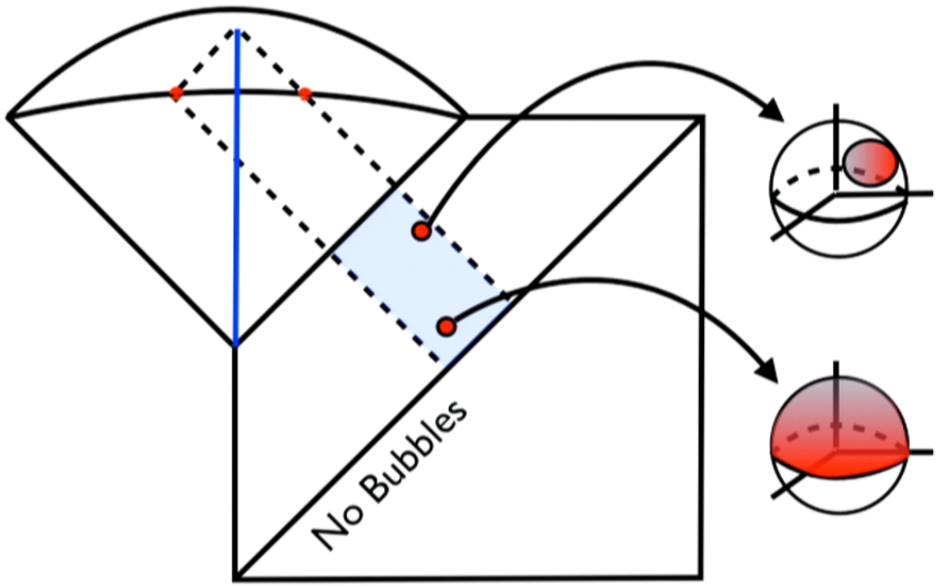

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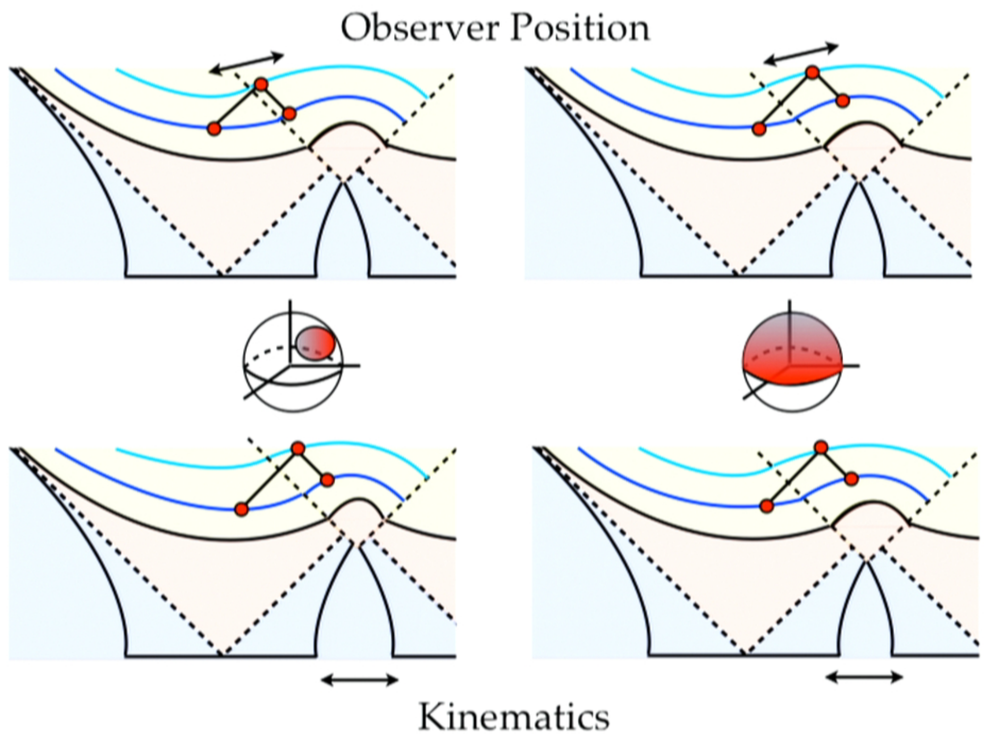
Two sources of variability



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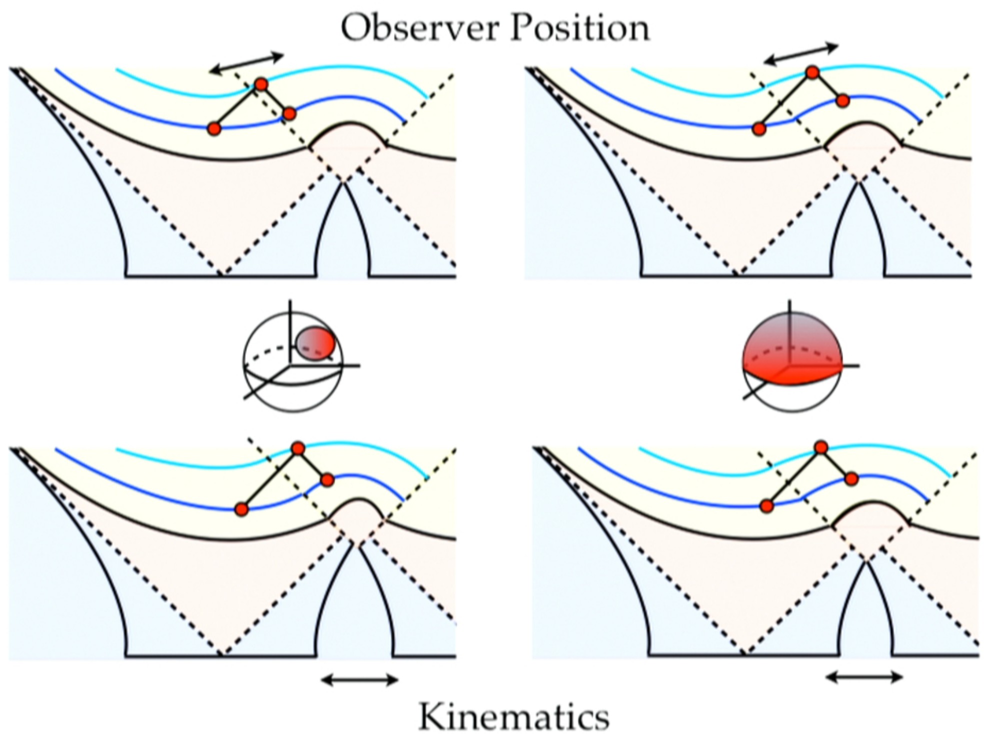
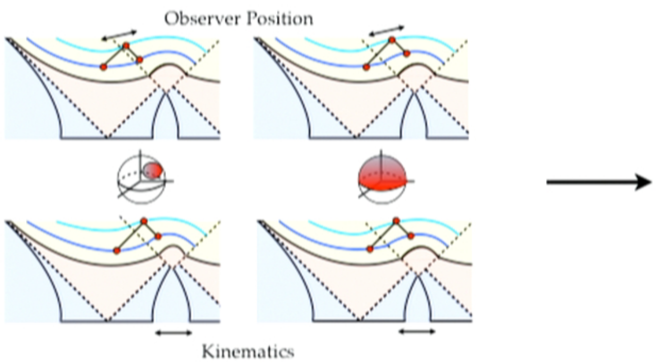


Prior over CMB Signature



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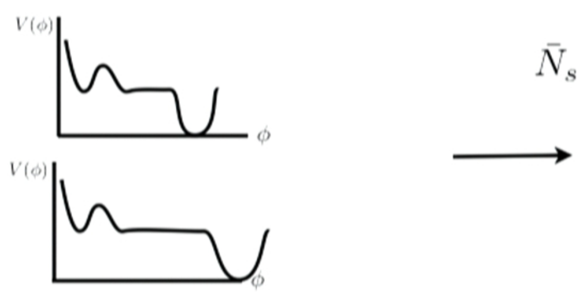
$$\theta_c,\ \kappa$$



$$\bar{N}_s = \frac{16\pi\lambda}{3H_F^2} \times \frac{\Omega_k^{1/2}}{H_I^2} \rightarrow N_0 e^{-B/\beta} R_{\rm ls}$$

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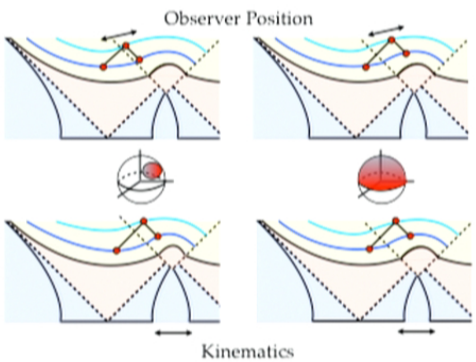

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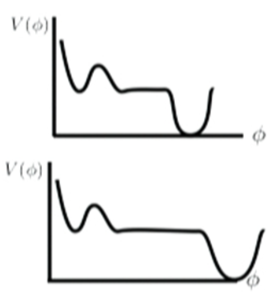

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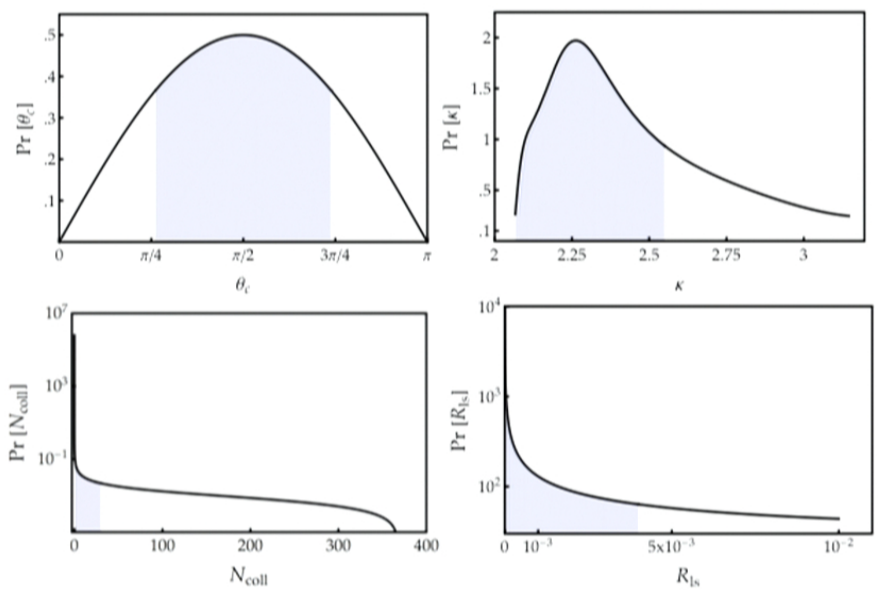
Marginalized Prior

$$\Pr(N_{\text{coll}}, R_{\text{ls}}, \kappa, \theta_c) = \Pr(\beta, R_{\text{ls}}, \Delta x_{\text{sep}}, \xi_{\text{obs}}) \left| \frac{dN_{\text{coll}}\ dR_{\text{ls}}\ d\kappa\ d\theta_c}{d\beta\ dR_{\text{ls}}\ d\Delta x_{\text{sep}}\ d\xi_{\text{obs}}} \right|^{-1}$$



$$\Pr(\beta) = \frac{1}{\Delta\beta} \qquad \Pr(N_e) \propto N_e^{-m}$$

What's next?

- Phenomenology - how universal is the signature?
  - Single field.
  - Multi-field.
  - Variations in the cosmology.
- Full set of cosmological signatures:
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  - LSS

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Properties of the eternally inflating spacetime

- Total number of causally accessible collisions:

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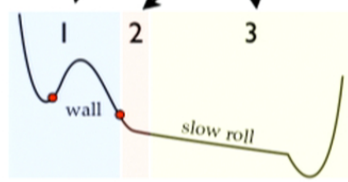

- Can be used as a (not necessarily unique) proxy for models.

$\bar{N}_s \sim 1, \bar{N}_s \gg 1$	$\bar{N}_s \ll 1$
testable	untestable