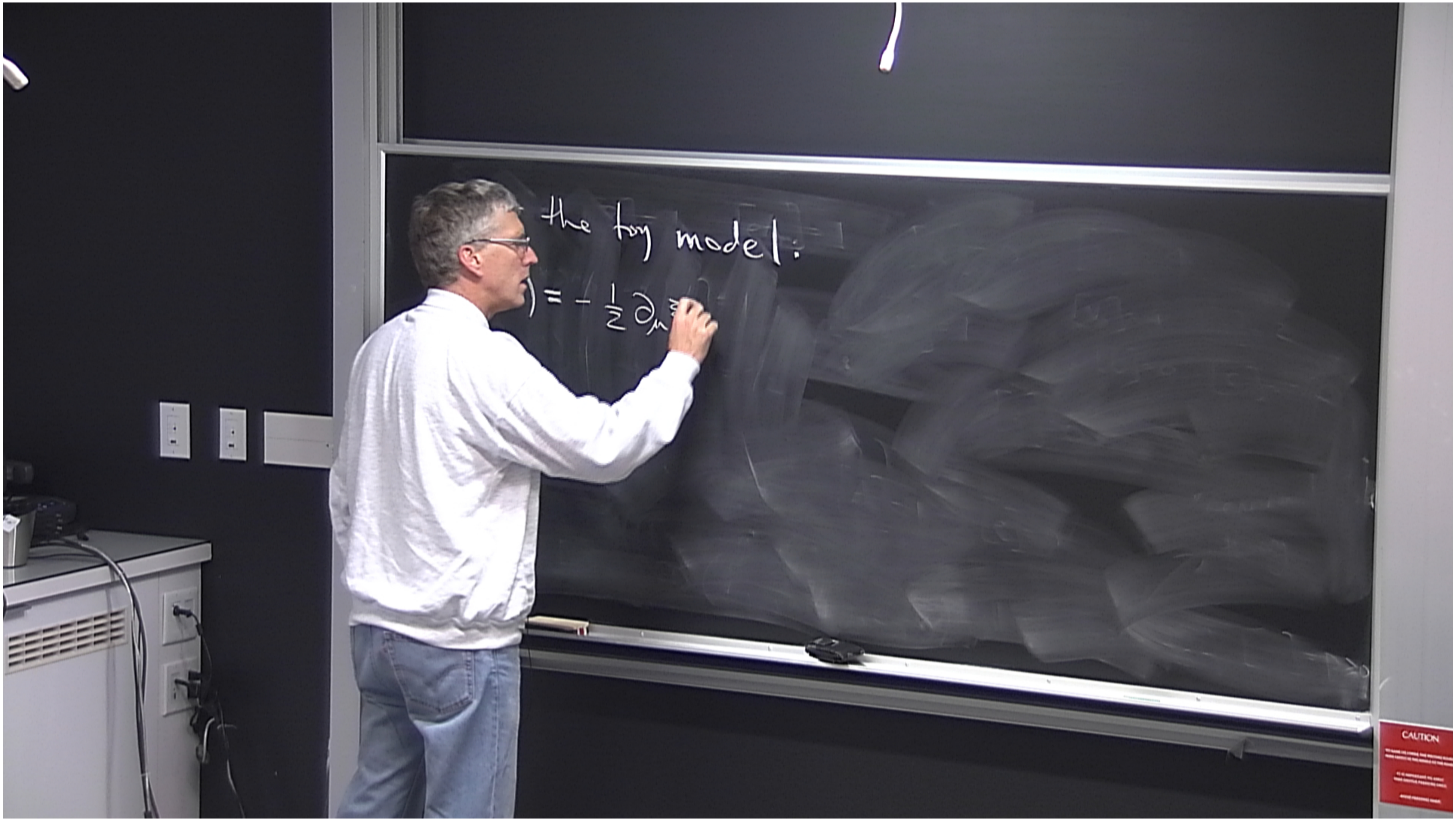
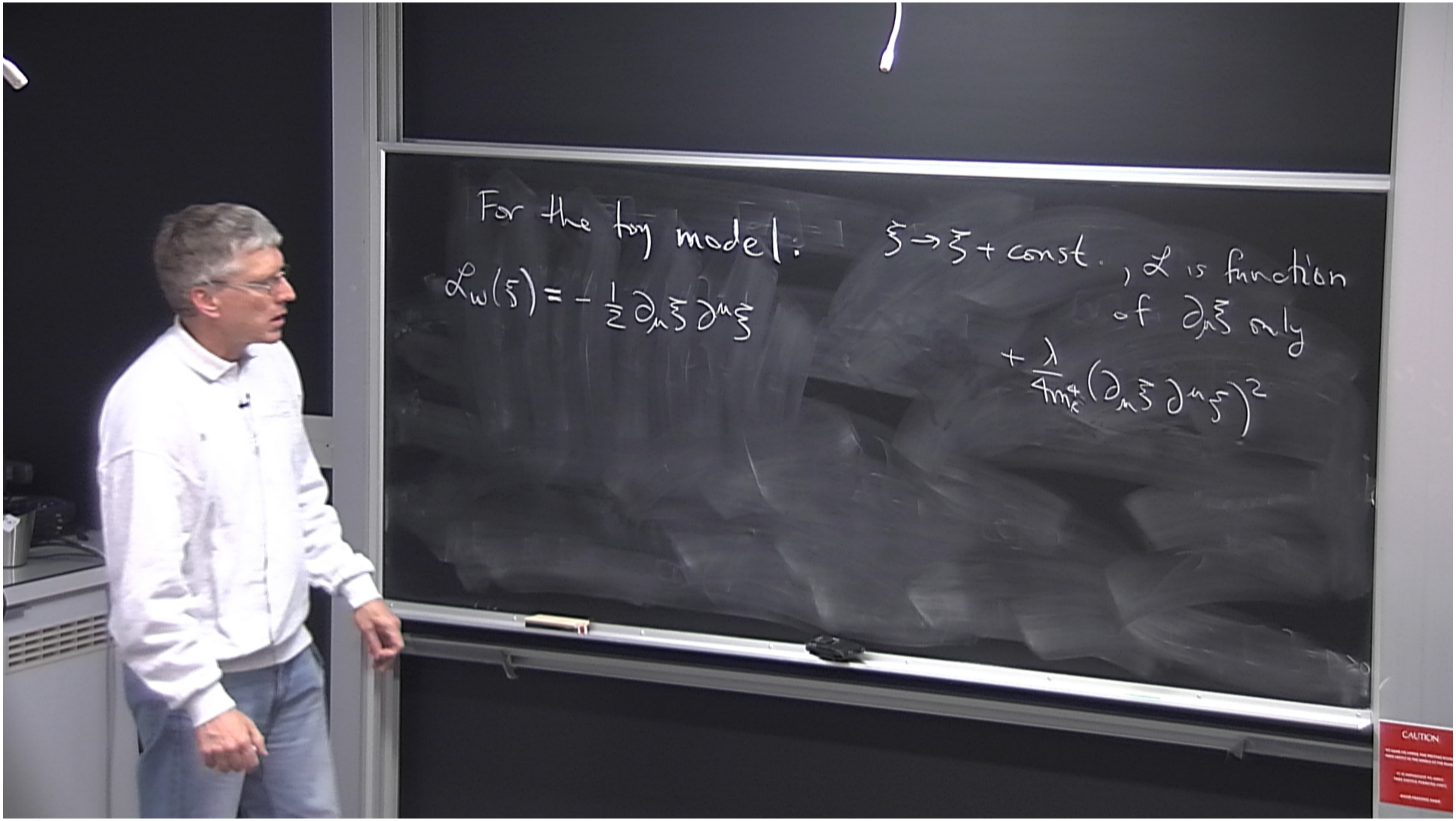
Title: Introduction to Effective Field Theories - Lecture 8

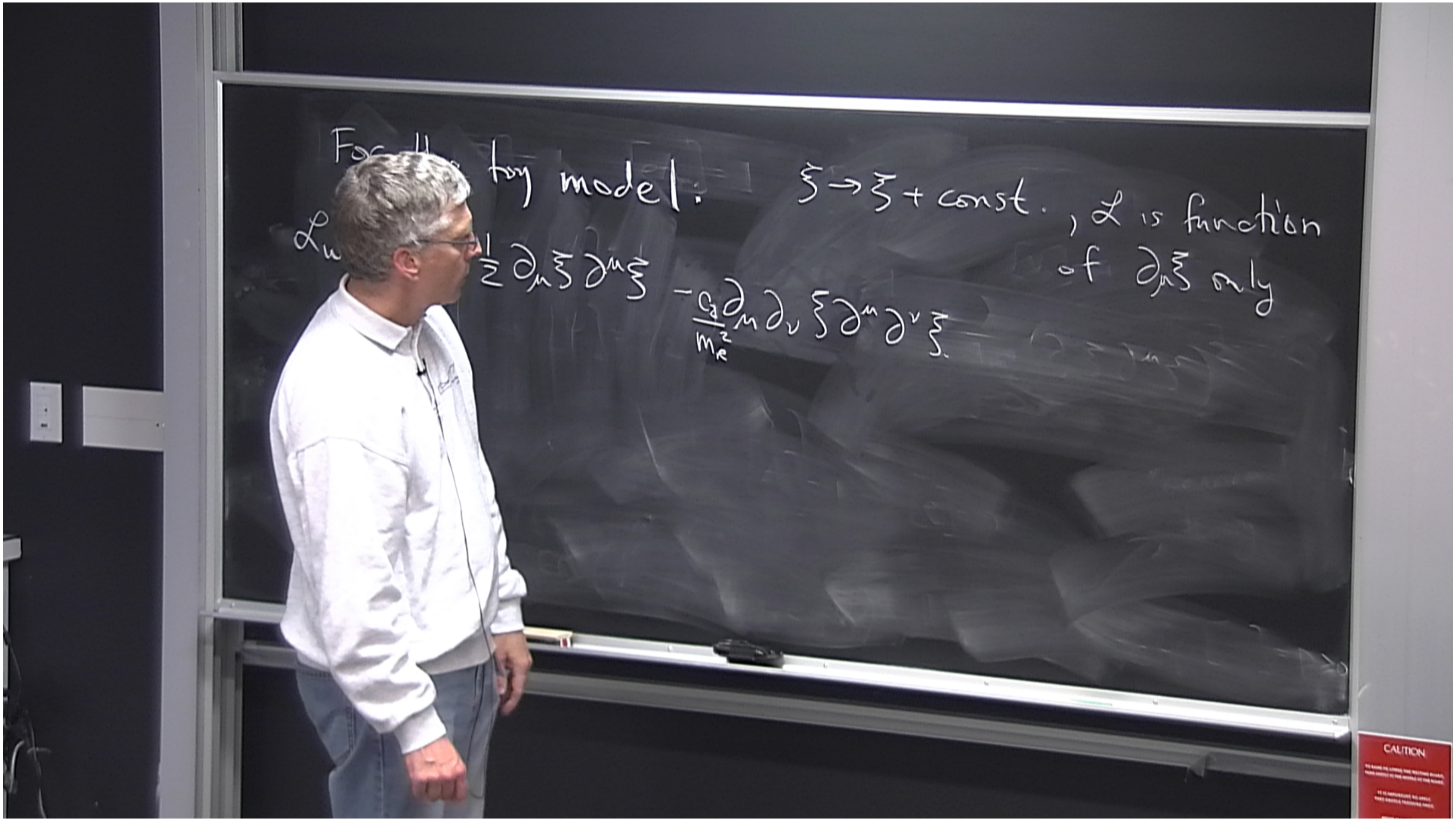
Date: Jan 31, 2014 02:30 PM

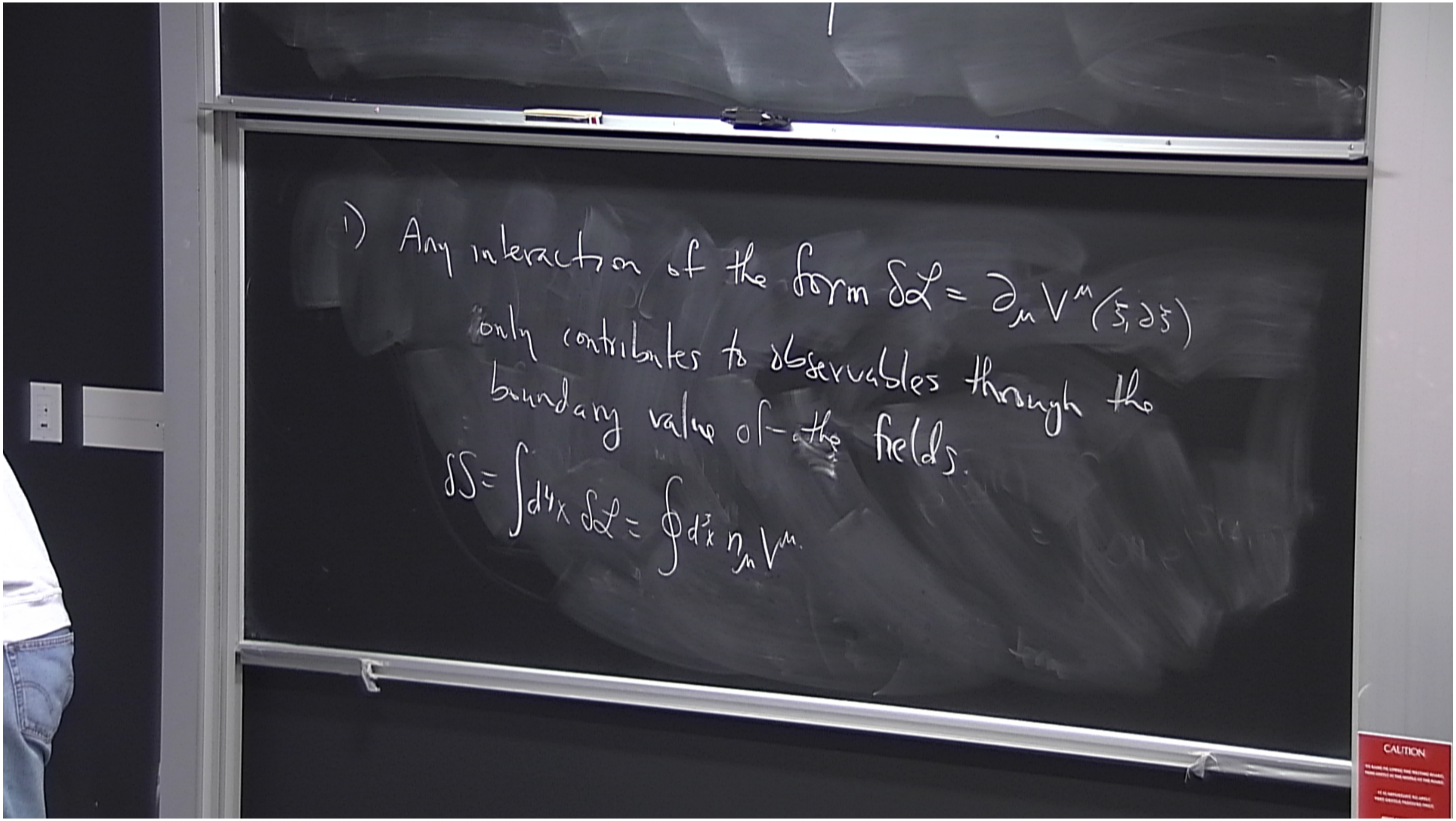
URL: http://pirsa.org/14010094

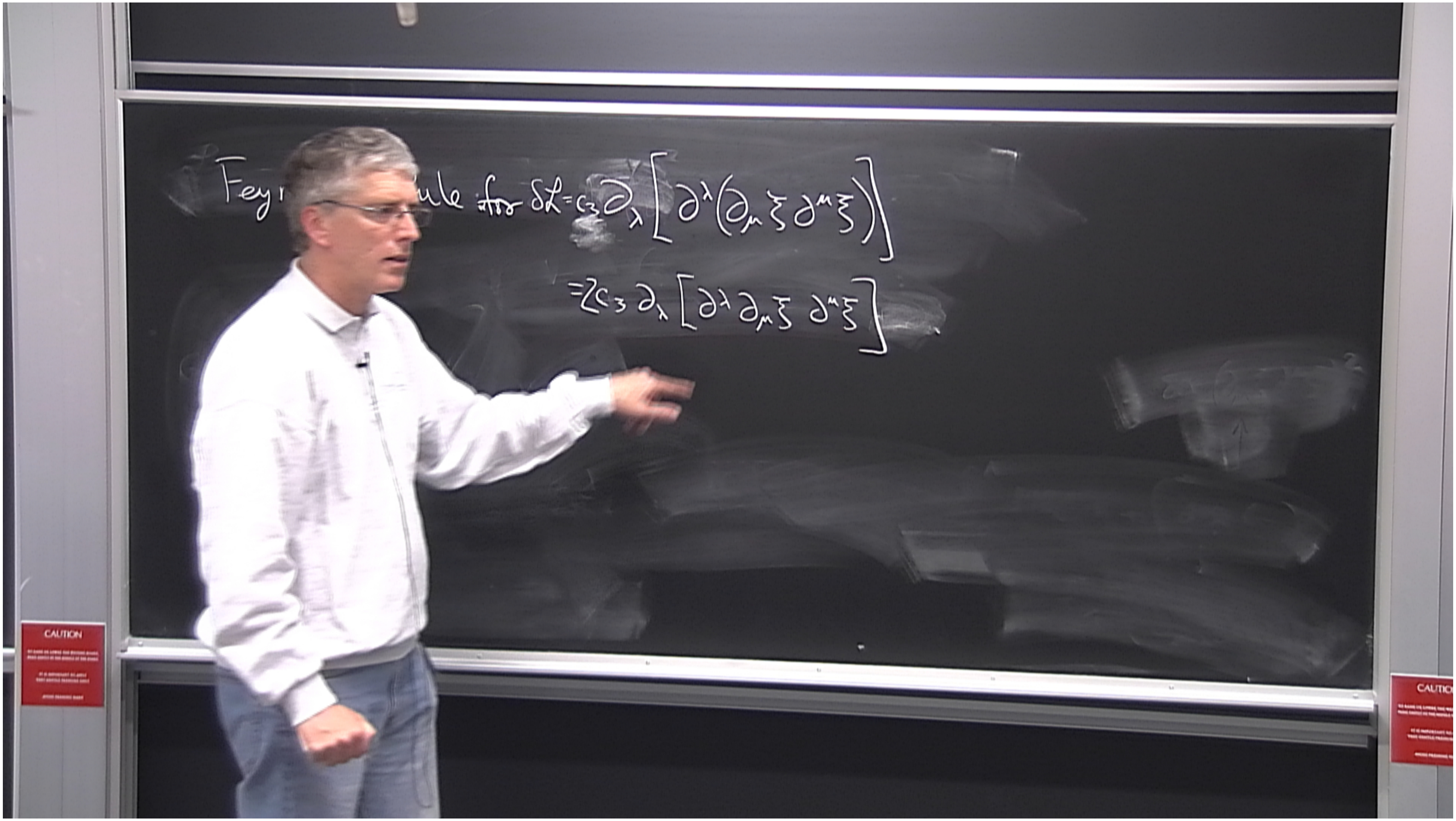
Abstract:


the toy model:
CAUTION


For the toy model:
$\mathcal{L}_w(\xi) = -\frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi$
$\xi \to \xi + \text{const.}$, $\mathcal{L}$ is function of $\partial_\mu\xi$ only
$+ \frac{\lambda}{4m^4}(\partial_\mu\xi\,\partial^\mu\xi)^2$
CAUTION


toy model:
ξ → ξ + const. , ℒ is function of ∂μξ only
½ ∂μξ ∂μξ
− c/m² ∂μ ∂ν ξ ∂μ ∂ν ξ
CAUTION


1) Any interaction of the form $\delta\mathcal{L} = \partial_\mu V^\mu(\xi, \partial\xi)$
only contributes to observables through the
boundary value of the fields.
$\delta S = \int d^4x \, \delta\mathcal{L} = \oint d^3x \, n_\mu V^\mu$.
CAUTION


Fey ule for $\delta\mathcal{L} = c_3 \partial_\lambda \left[\partial^\lambda (\partial_\mu \xi \, \partial^\mu \xi) \right]$
$= 2c_3 \partial_\lambda \left[\partial^\lambda \partial_\mu \xi \, \partial^\mu \xi \right]$
CAUTION
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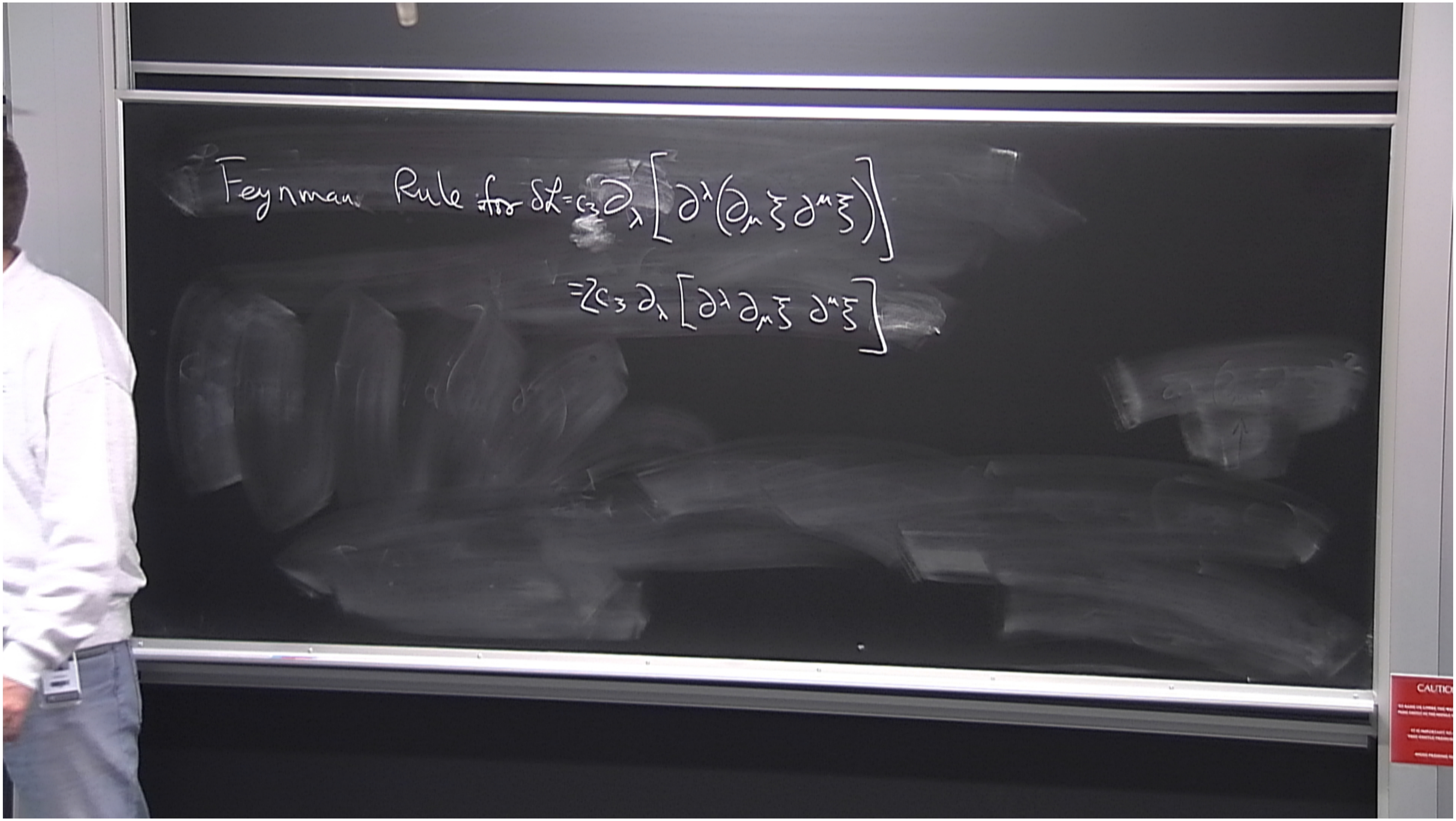
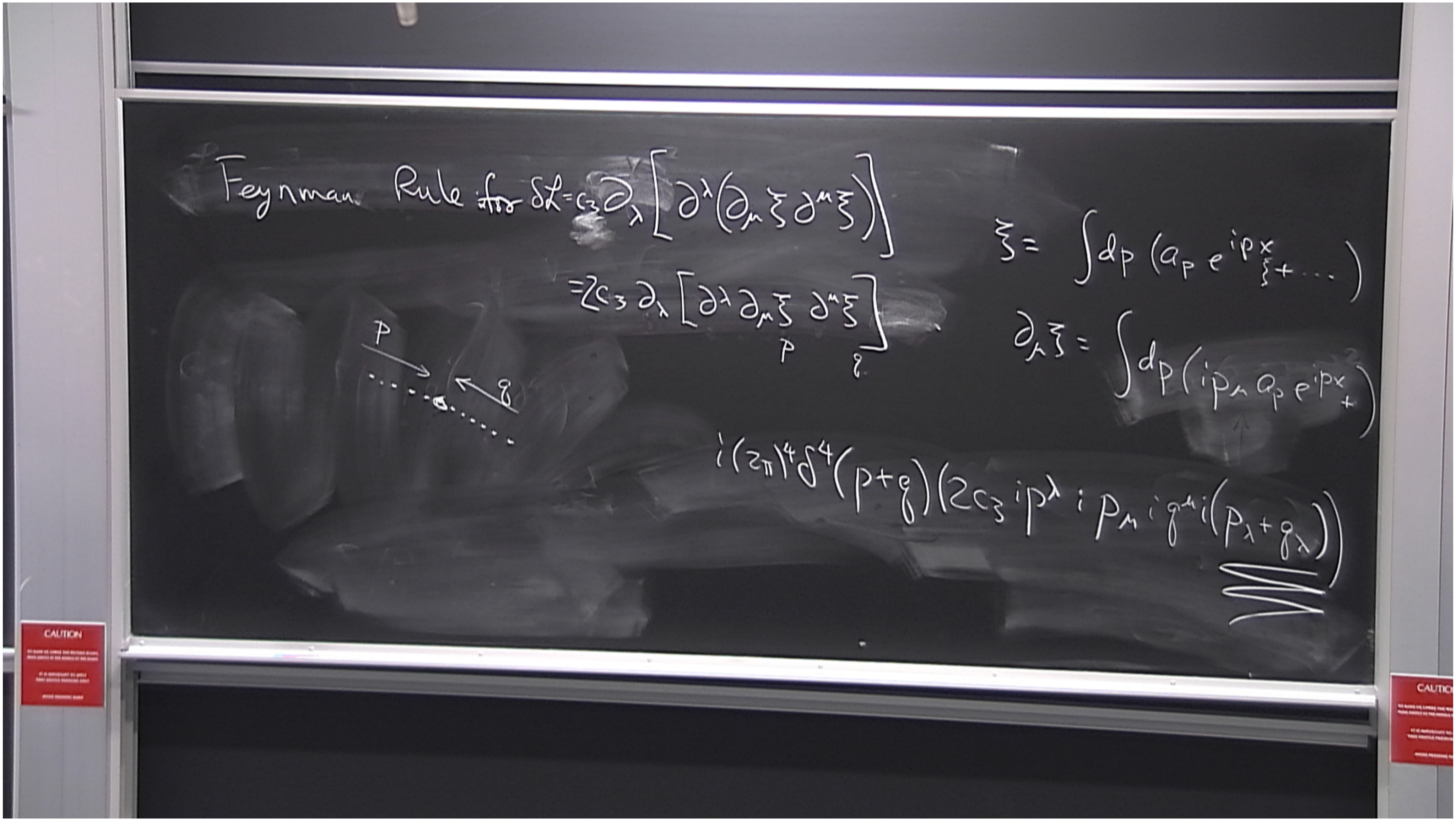
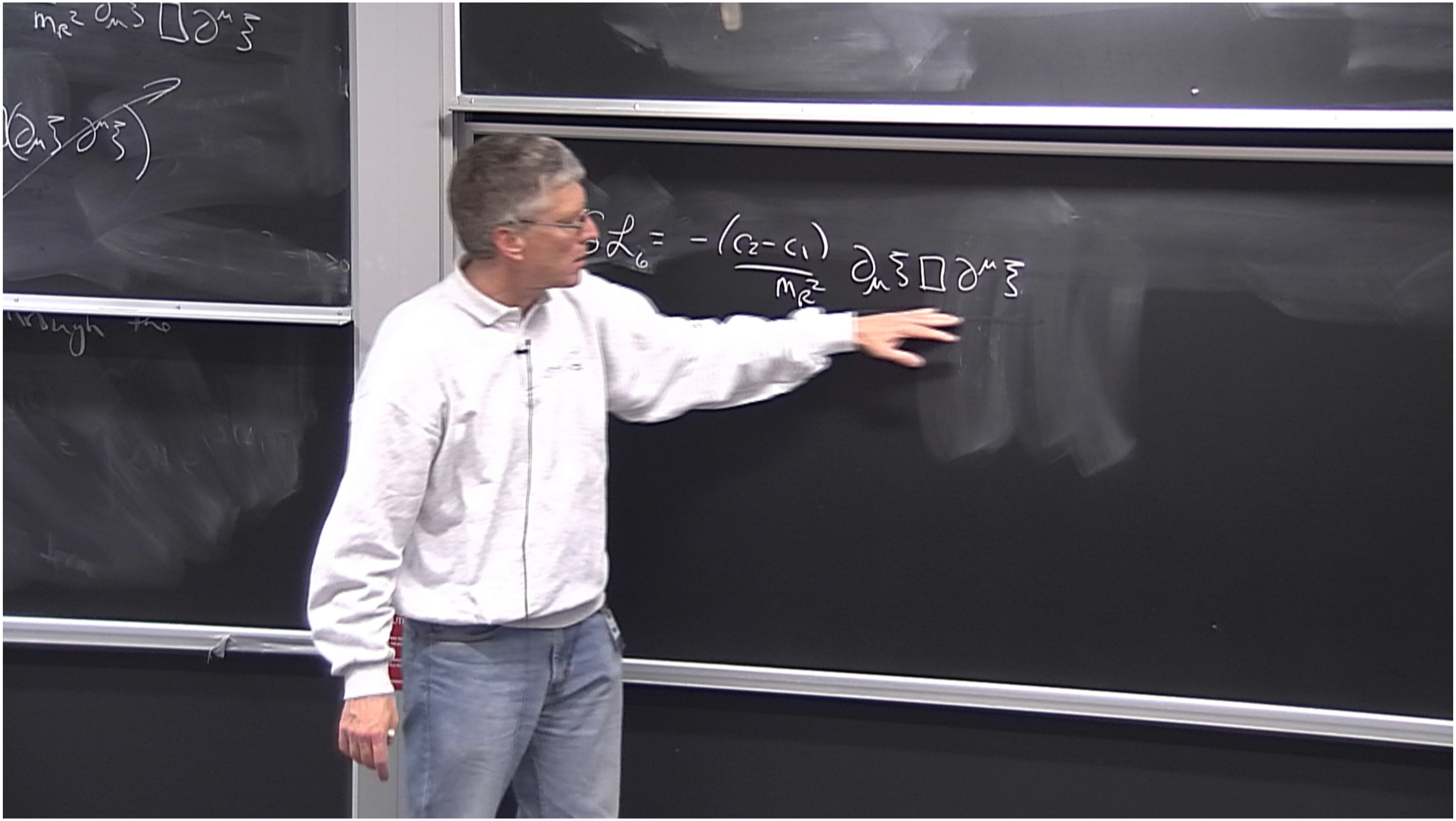
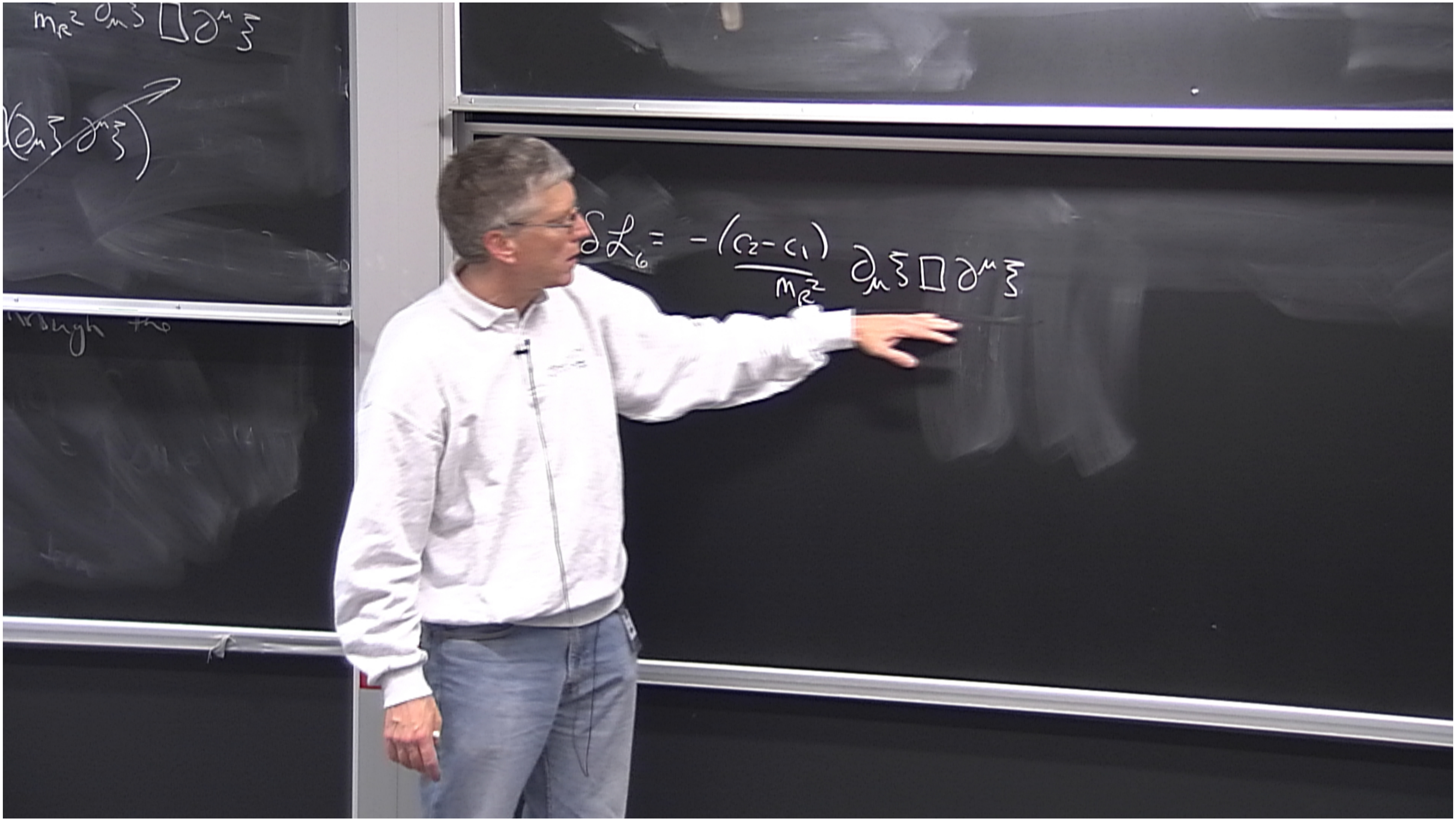
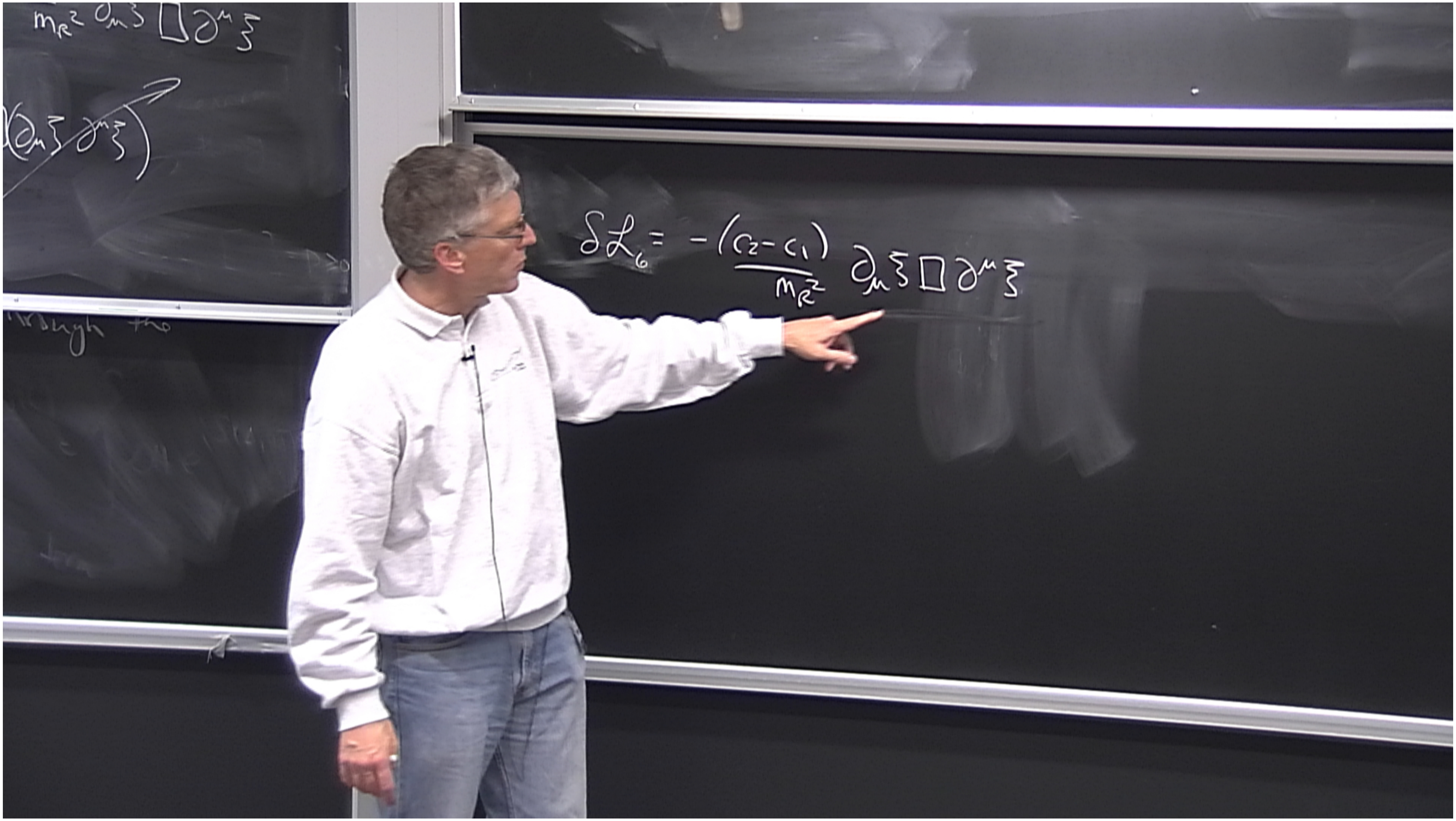
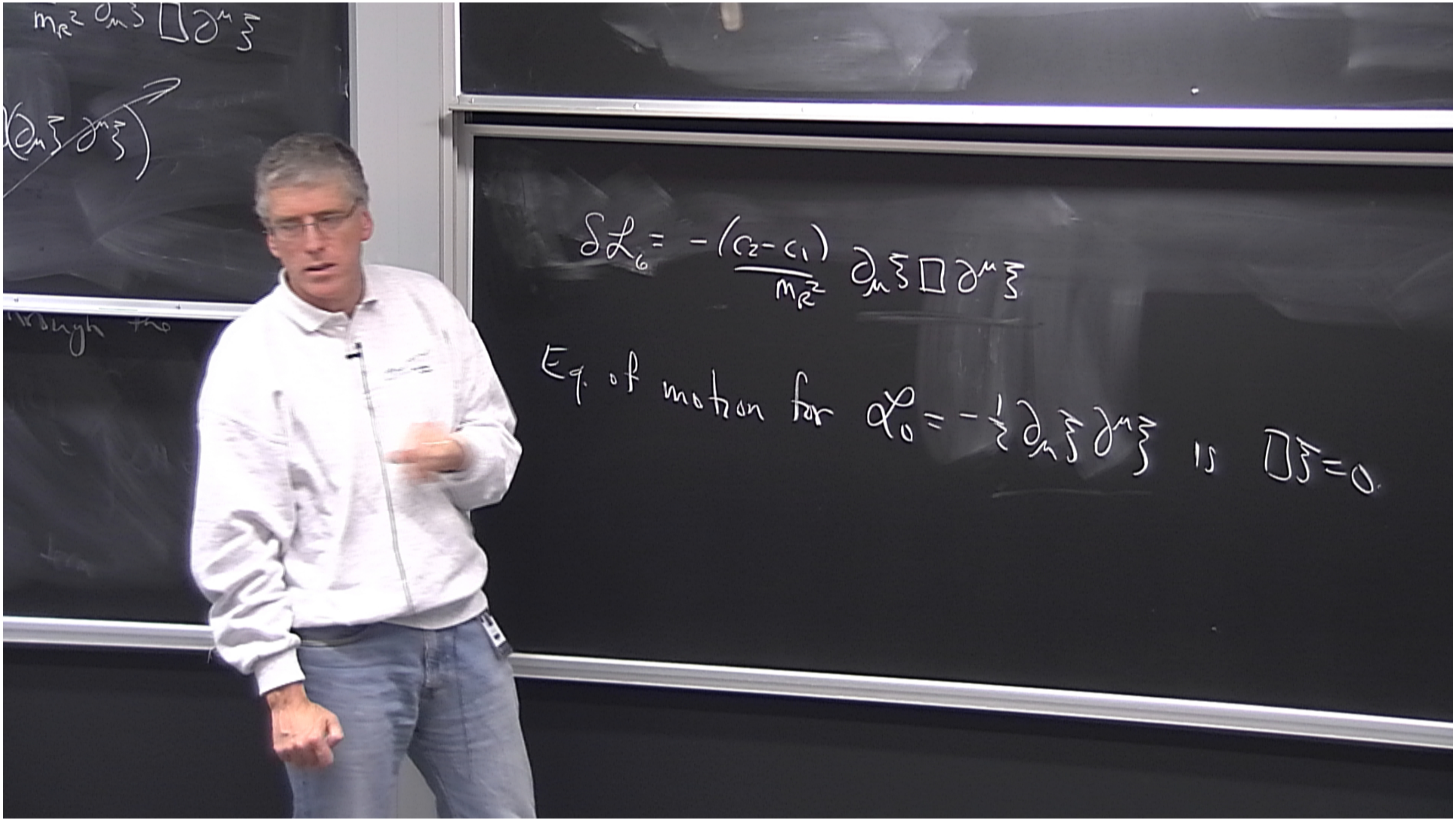

Feynman Rule for $\delta\mathcal{L} = c_3 \partial_\lambda \left[\partial^\lambda (\partial_\mu \xi \, \partial^\mu \xi) \right]$
$= 2 c_3 \partial_\lambda \left[\partial^\lambda \partial_\mu \xi \, \partial^\mu \xi \right]$
CAUTION

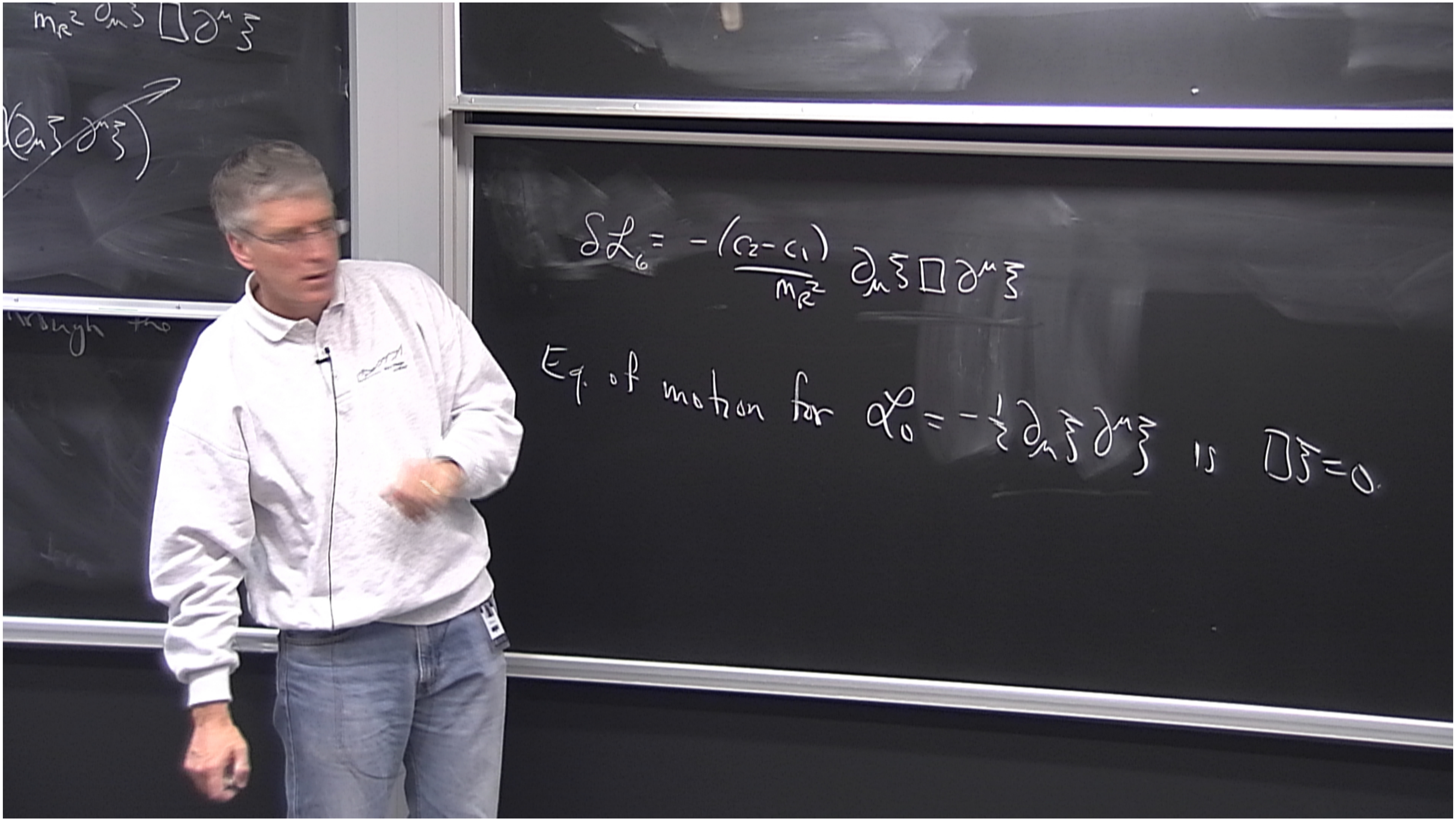

Feynman Rule for $\delta\mathcal{L} = c_3 \partial_\lambda \left[\partial^\lambda (\partial_\mu \xi \partial^\mu \xi) \right]$
$= 2c_3 \partial_\lambda \left[\partial^\lambda \partial_\mu \xi \, \partial^\mu \xi \right]$
p
q
$\xi = \int dp \, (a_p e^{ipx} + \dots)$
$\partial_\mu \xi = \int dp \, (i p_\mu a_p e^{ipx} +$
$i(2\pi)^4 \delta^4(p+q)(2c_3 \, i p^\lambda \, i p_\mu \, i q^\mu \, i(p_\lambda + q_\lambda))$

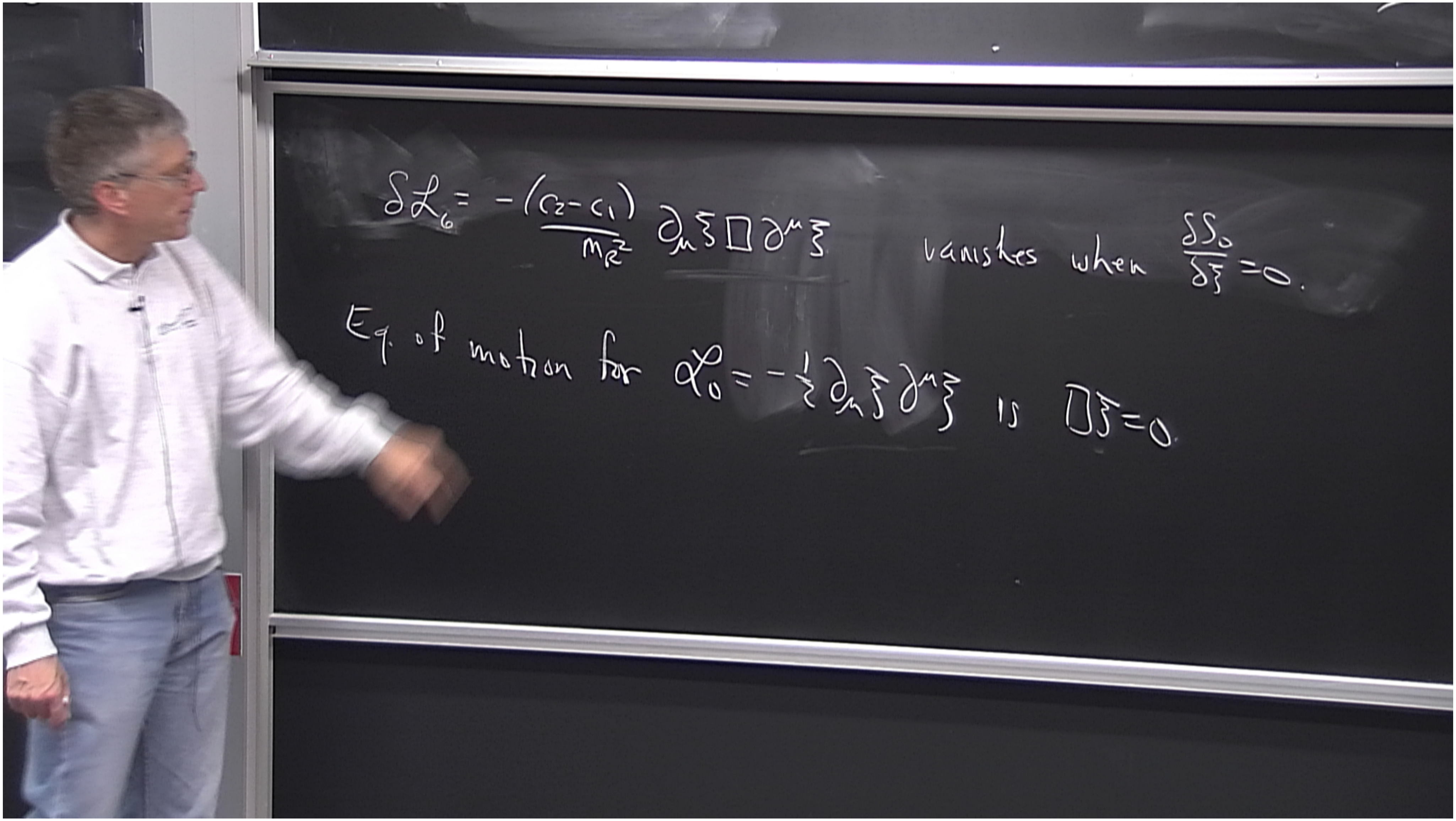

δL₆ = −(c₂ − c₁)/m_R² ∂_μ ξ □ ∂^μ ξ

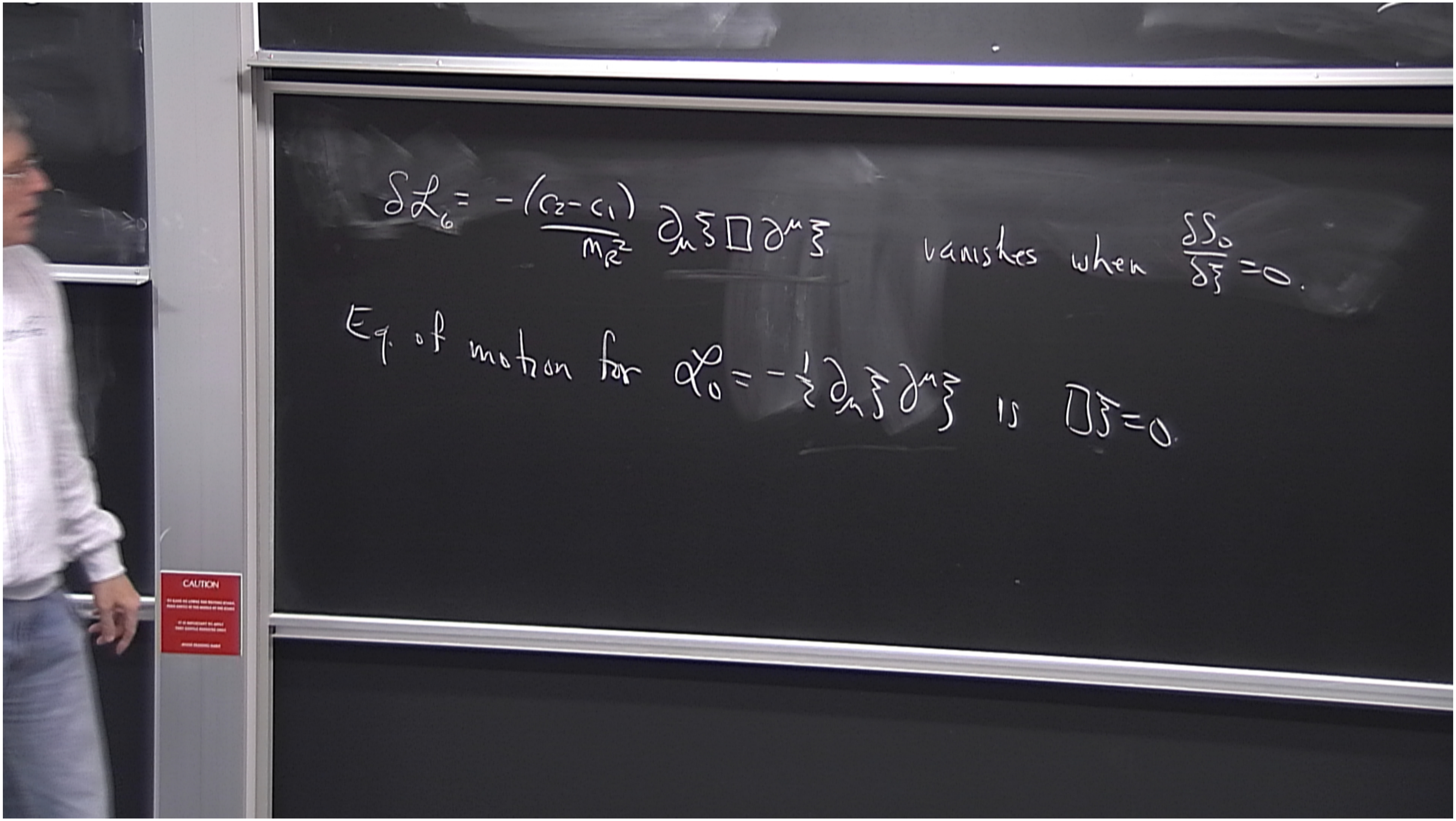

$\delta \mathcal{L}_6 = -\frac{(c_2 - c_1)}{m_R^2} \partial_\mu \xi \Box \partial^\mu \xi$


δL_6 = −(c_2 − c_1)/m_R^2 ∂_μ ξ □ ∂^μ ξ
m_R^2 ∂_μ ξ □ ∂^μ ξ


$\delta \mathcal{L}_6 = -\frac{(c_2 - c_1)}{m_R^2} \partial_\mu \xi \Box \partial^\mu \xi$
Eq. of motion for $\mathcal{L}_0 = -\frac{1}{2} \partial_\mu \xi \partial^\mu \xi$ is $\Box \xi = 0$.


δL₆ = −(c₂ − c₁)/m_R² ∂_μ ξ □ ∂^μ ξ
Eq. of motion for L₀ = −½ ∂_μ ξ ∂^μ ξ is □ξ = 0


$\delta \mathcal{L}_6 = -\frac{(c_2 - c_1)}{m_R^2} \partial_\mu \xi \Box \partial^\mu \xi$ vanishes when $\frac{\delta S_0}{\delta \xi} = 0$.
Eq. of motion for $\mathcal{L}_0 = -\frac{1}{2} \partial_\mu \xi \partial^\mu \xi$ is $\Box \xi = 0$.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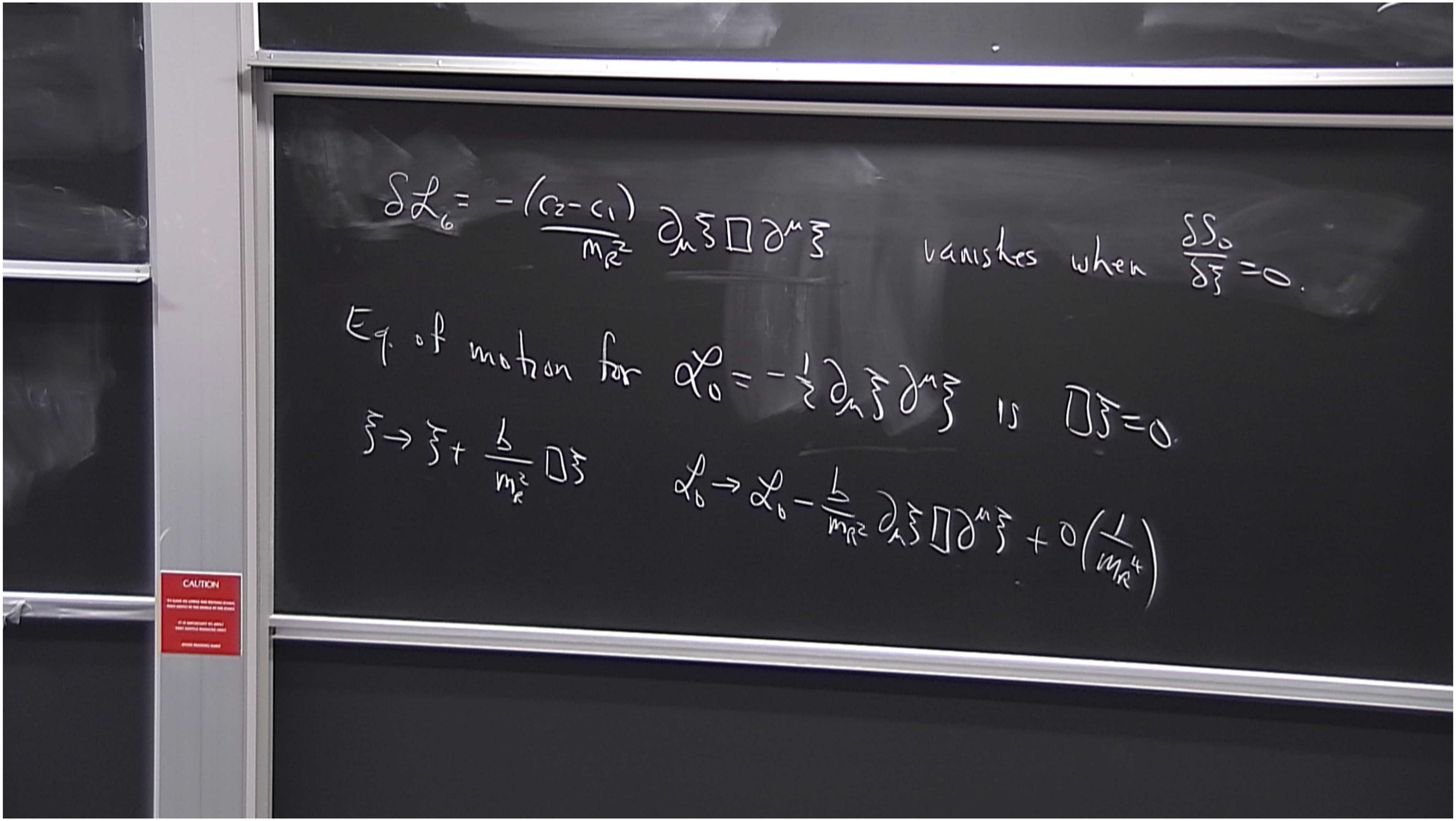
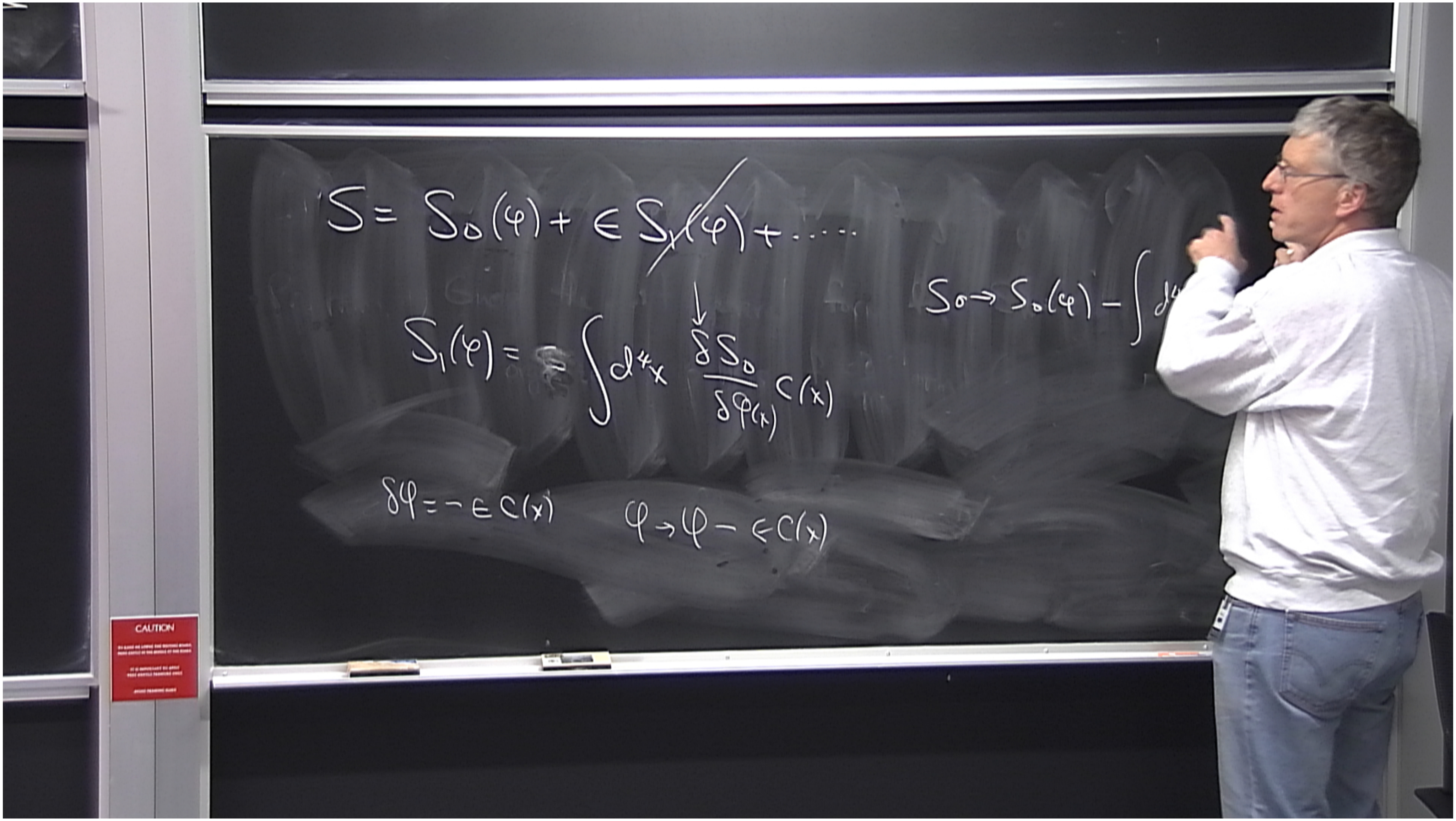

$\delta \mathcal{L}_6 = -\frac{(c_2 - c_1)}{m_R^2} \partial_\mu \xi \Box \partial^\mu \xi$ vanishes when $\frac{\delta S_0}{\delta \xi} = 0$.
Eq. of motion for $\mathcal{L}_0 = -\frac{1}{2} \partial_\mu \xi \partial^\mu \xi$ is $\Box \xi = 0$.
$\xi \to \xi + \frac{b}{m_R^2} \Box \xi$
$\mathcal{L}_0 \to \mathcal{L}_0 - \frac{b}{m_R^2} \partial_\mu \xi \Box \partial^\mu \xi + O\left(\frac{1}{m_R^4}\right)$

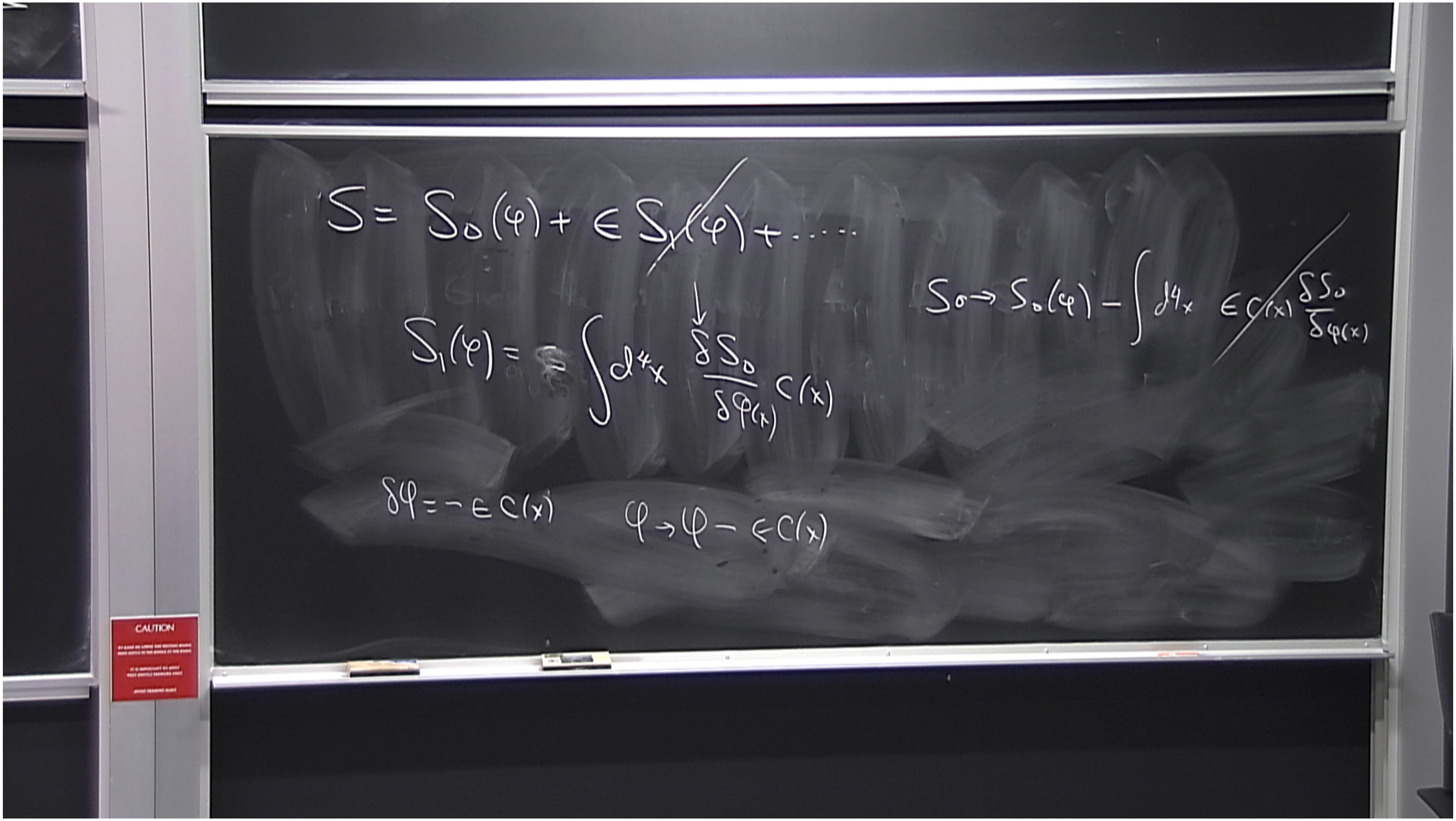
$$\delta \mathcal{L}_6 = -\frac{(c_2 - c_1)}{m_R^2} \partial_\mu \xi \, \Box \, \partial^\mu \xi \quad \text{vanishes when } \frac{\delta S_0}{\delta \xi} = 0.$$

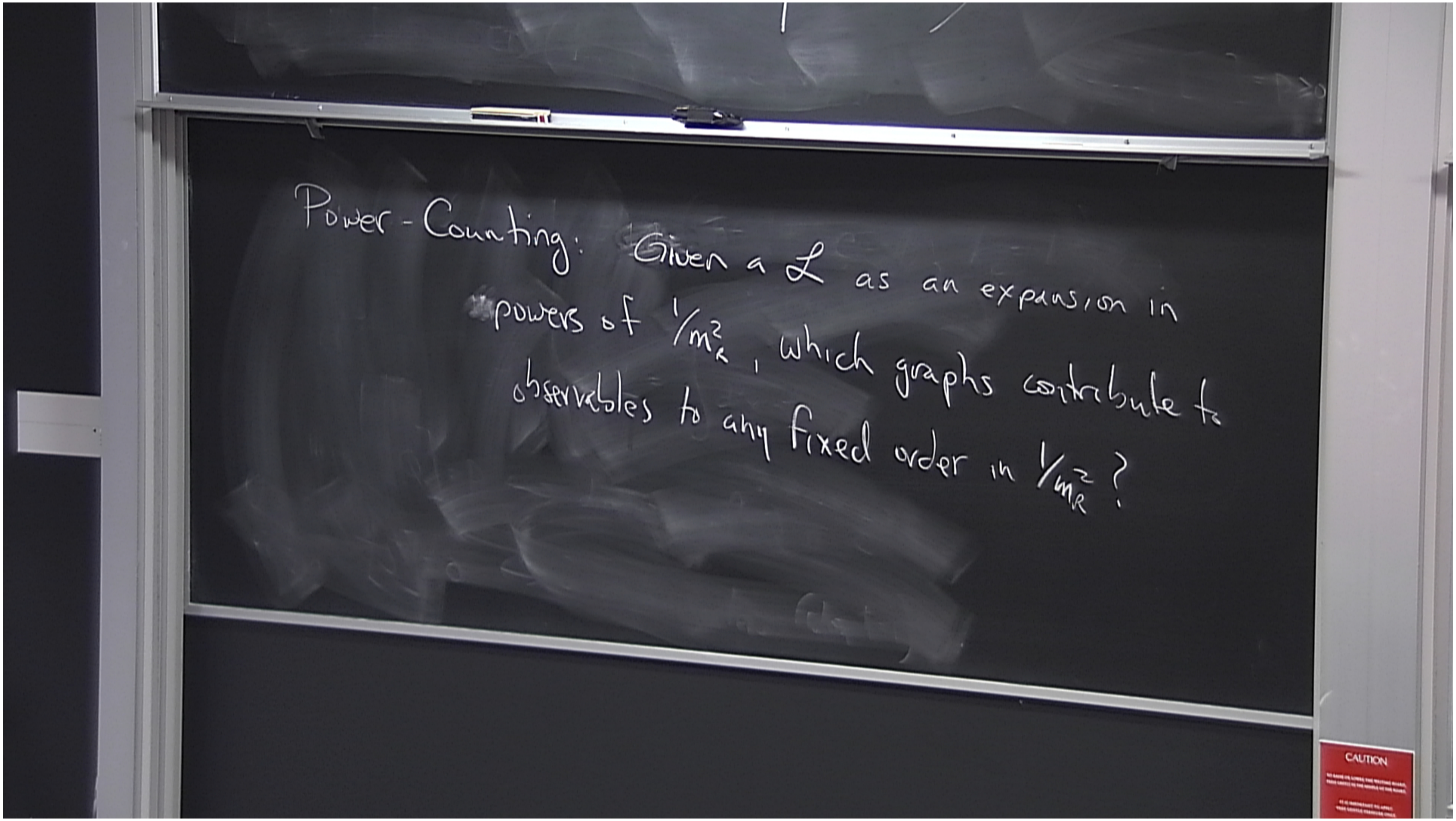
Eq. of motion for $\mathcal{L}_0 = -\frac{1}{2} \partial_\mu \xi \, \partial^\mu \xi$ is $\Box \xi = 0$.

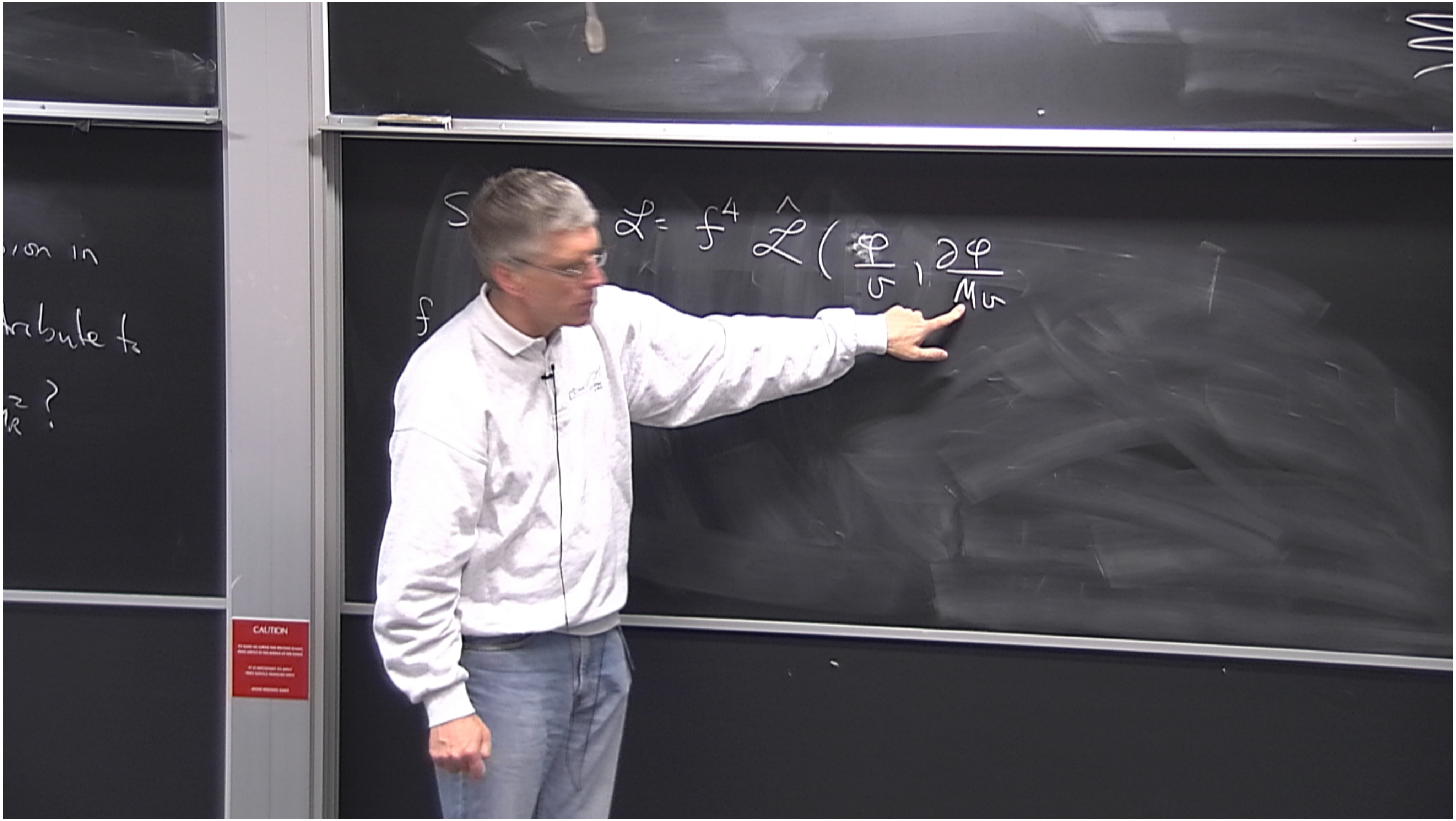
$$\xi \to \xi + \frac{b}{m_R^2} \Box \xi \qquad \mathcal{L}_0 \to \mathcal{L}_0 - \frac{b}{m_R^2} \partial_\mu \xi \, \Box \, \partial^\mu \xi + O\left(\frac{1}{m_R^4}\right)$$

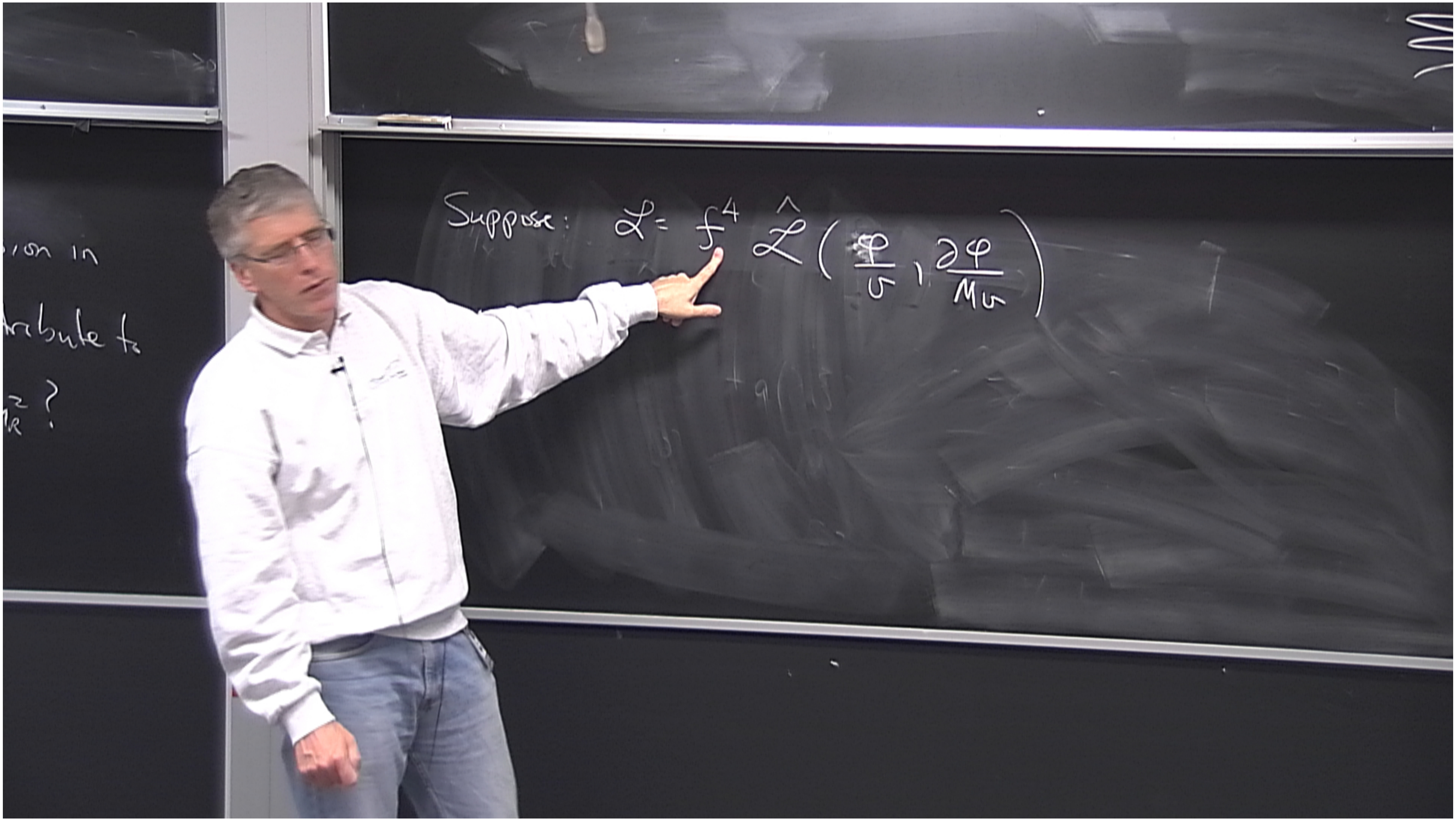
So $b = -c_1$ eliminates the $\partial \xi \Box \partial \xi$ term from $\mathcal{L}$ to order $1/m_R^2$.

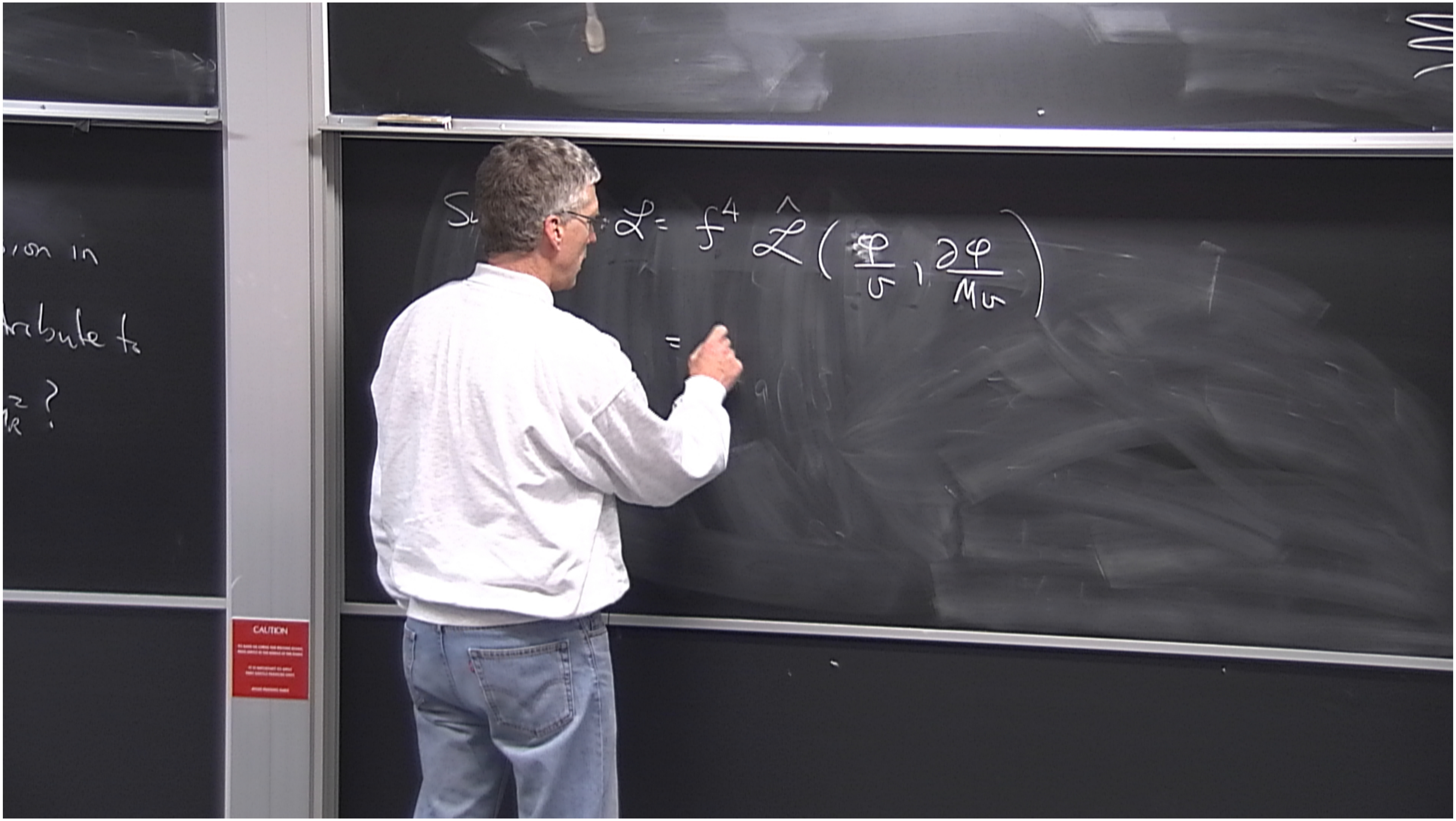

$S = S_0(\varphi) + \epsilon S_1(\varphi) + \ldots$
$S_0 \rightarrow S_0(\varphi) - \int d^4$
$S_1(\varphi) = \int d^4x \frac{\delta S_0}{\delta \varphi(x)} c(x)$
$\delta\varphi = -\epsilon c(x)$
$\varphi \rightarrow \varphi - \epsilon c(x)$
CAUTION

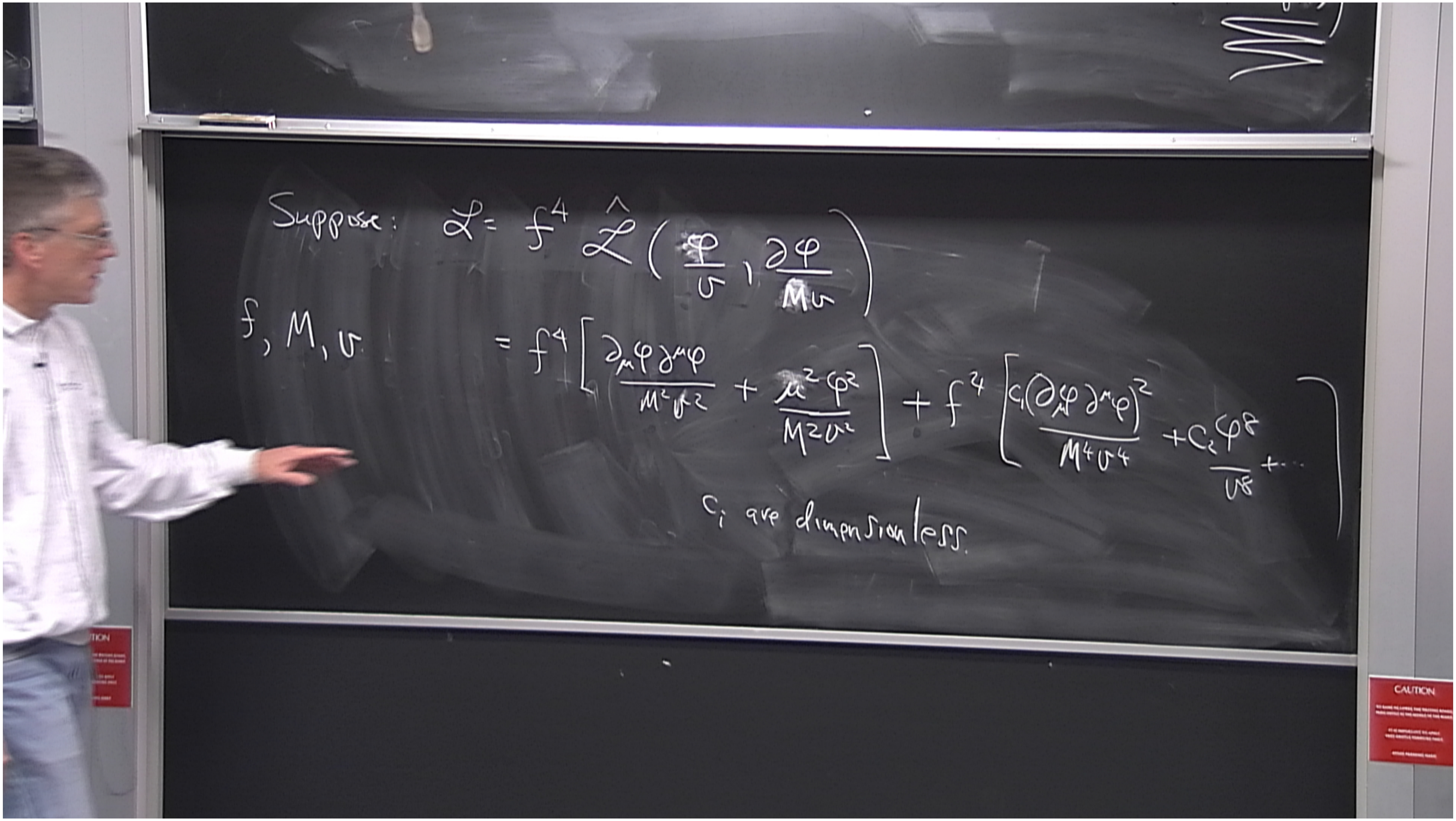

$S = S_0(\varphi) + \epsilon S_1(\varphi) + \ldots$
$S_0 \to S_0(\varphi) - \int d^4x \, \epsilon C(x) \frac{\delta S_0}{\delta \varphi(x)}$
$S_1(\varphi) = \int d^4x \, \frac{\delta S_0}{\delta \varphi(x)} C(x)$
$\delta\varphi = -\epsilon C(x)$
$\varphi \to \varphi - \epsilon C(x)$
CAUTION

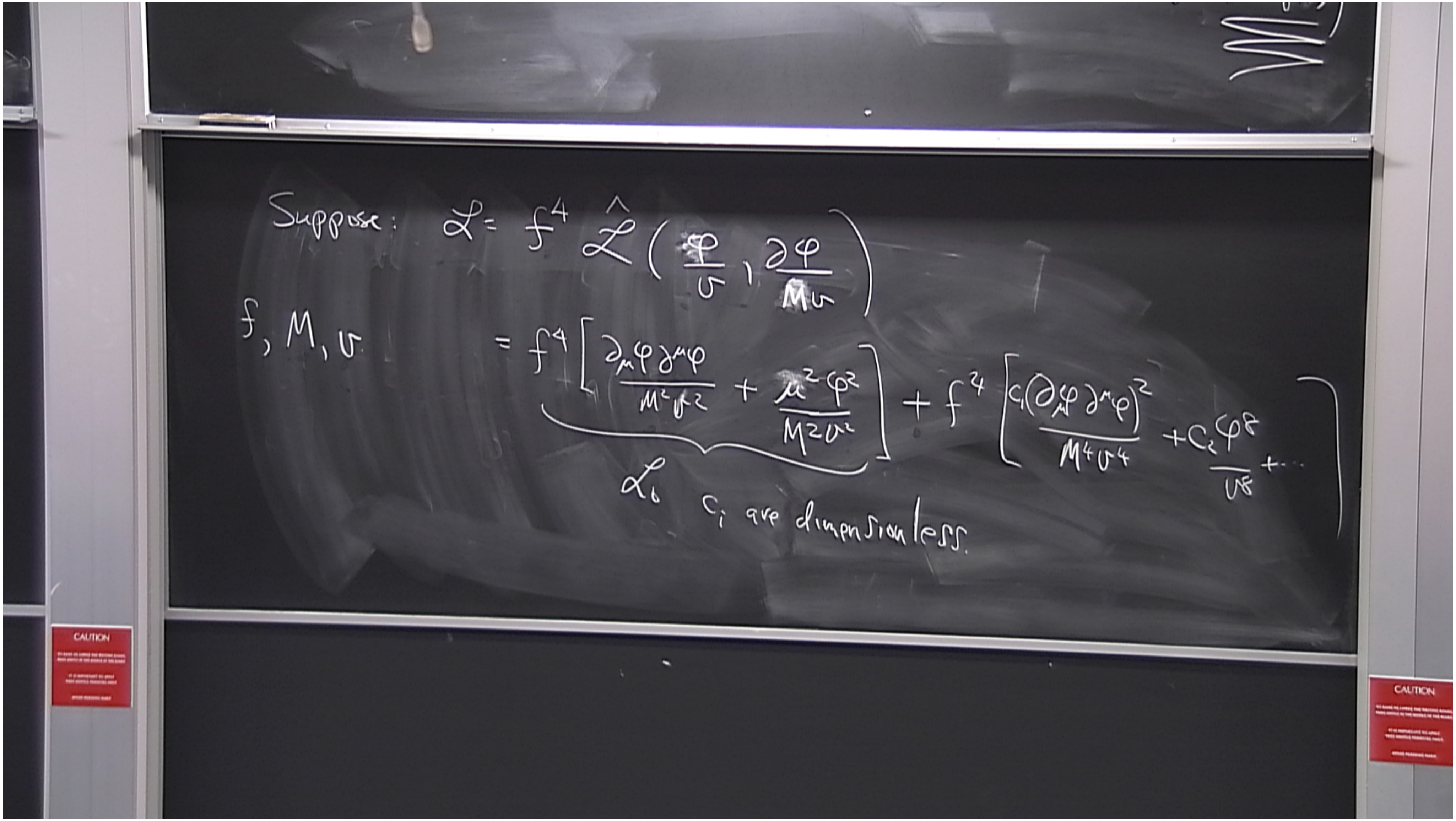

Power-Counting: Given a $\mathcal{L}$ as an expansion in
powers of $1/m_R^2$, which graphs contribute to
observables to any fixed order in $1/m_R^2$?

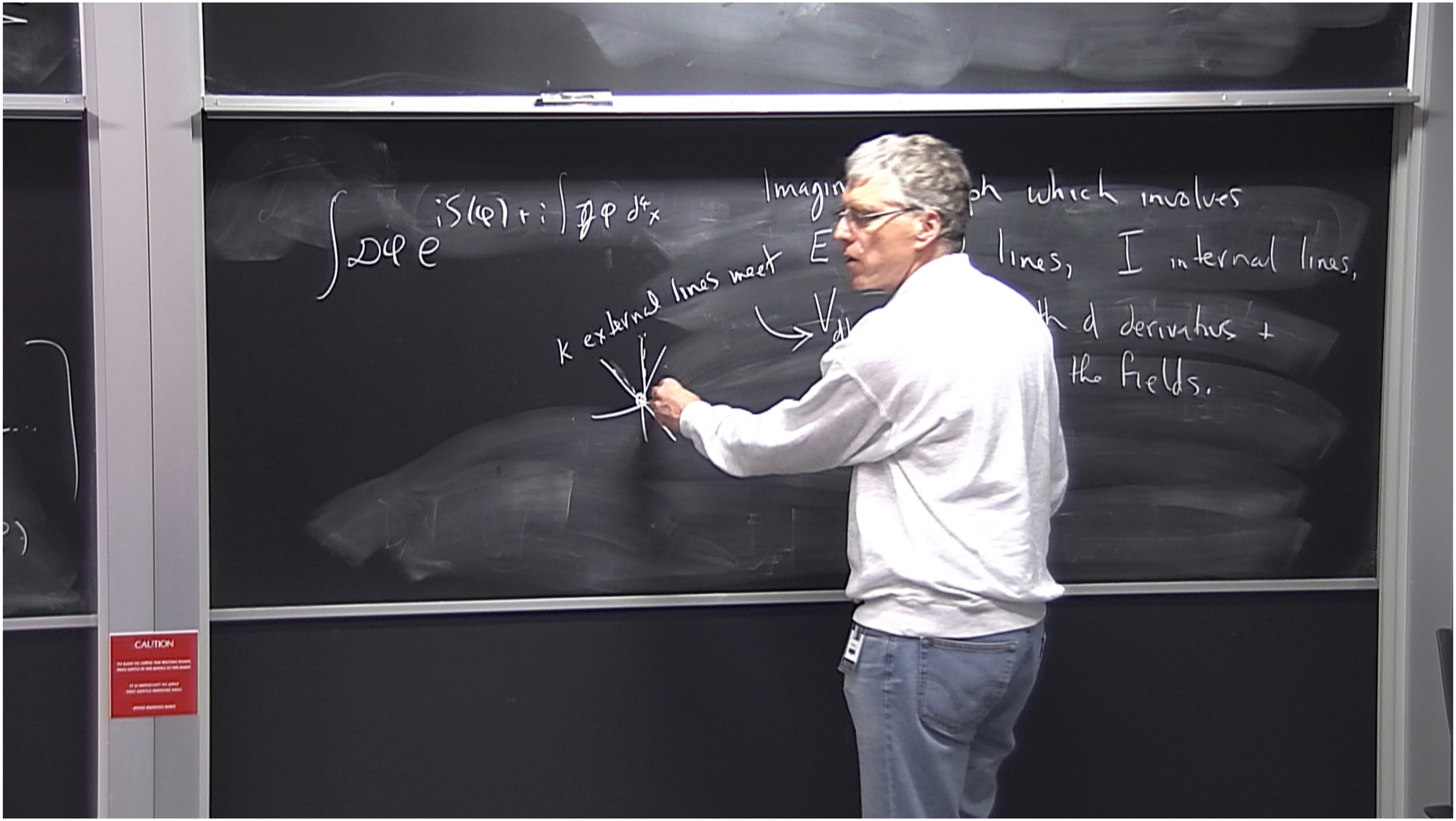

$\mathcal{L} = f^4 \hat{\mathcal{L}}\left(\frac{\varphi}{v}, \frac{\partial\varphi}{Mv}\right.$
CAUTION

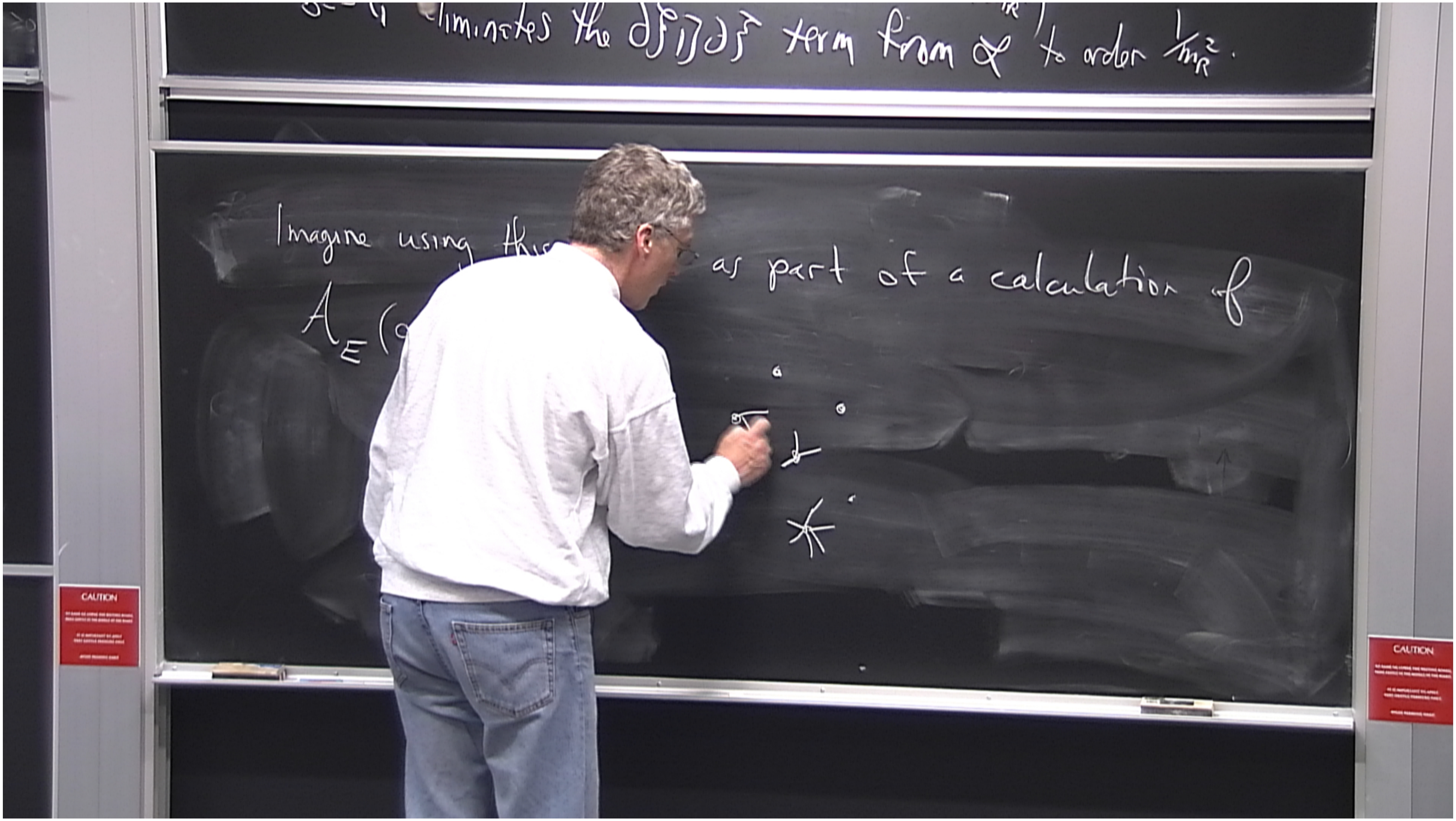

Suppose: $\mathcal{L} = f^4 \hat{\mathcal{L}}\left(\frac{\varphi}{v}, \frac{\partial\varphi}{Mv}\right)$

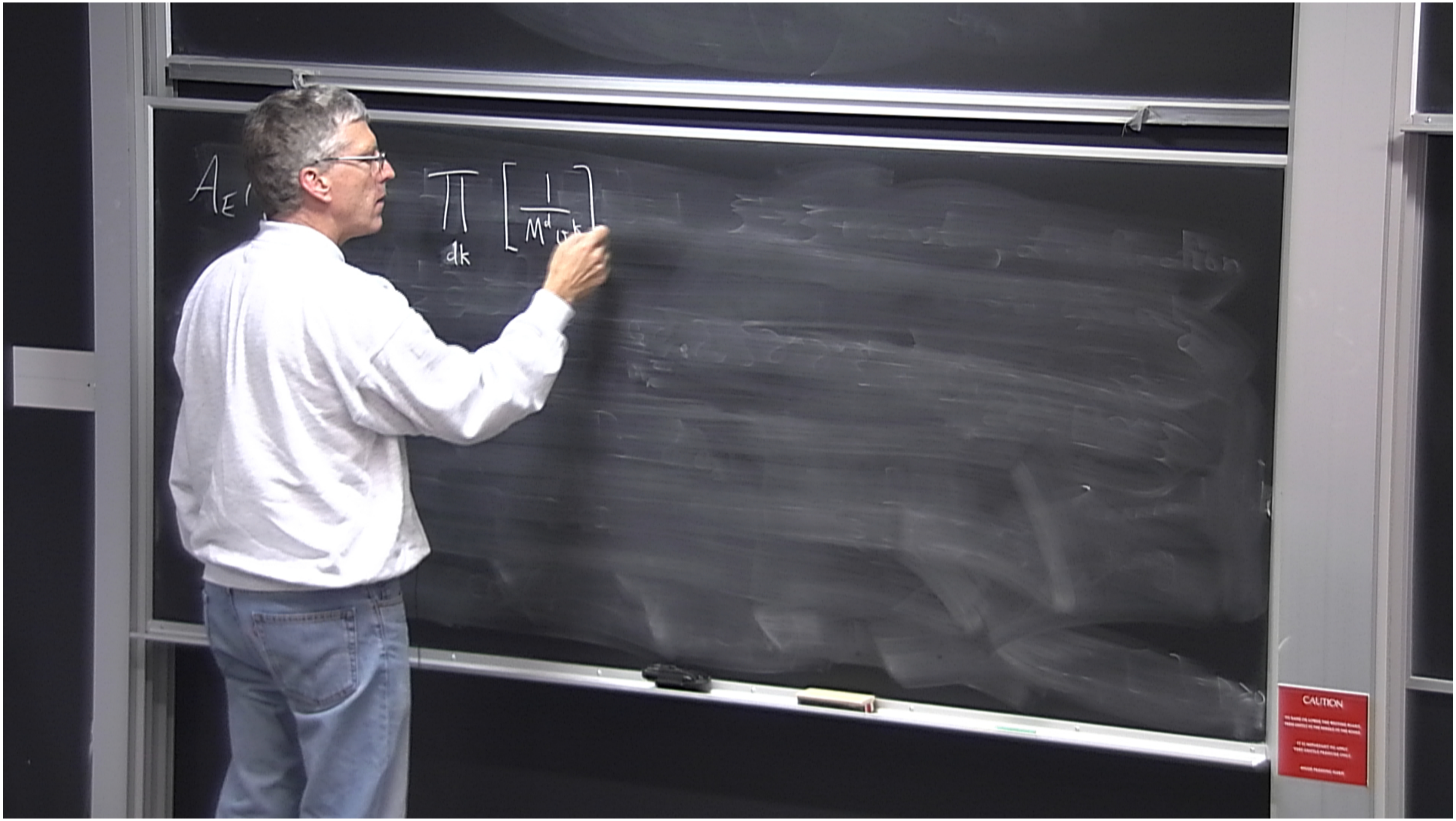

$\mathcal{L} = f^4 \hat{\mathcal{L}}\left(\frac{\varphi}{v}, \frac{\partial\varphi}{Mv}\right)$

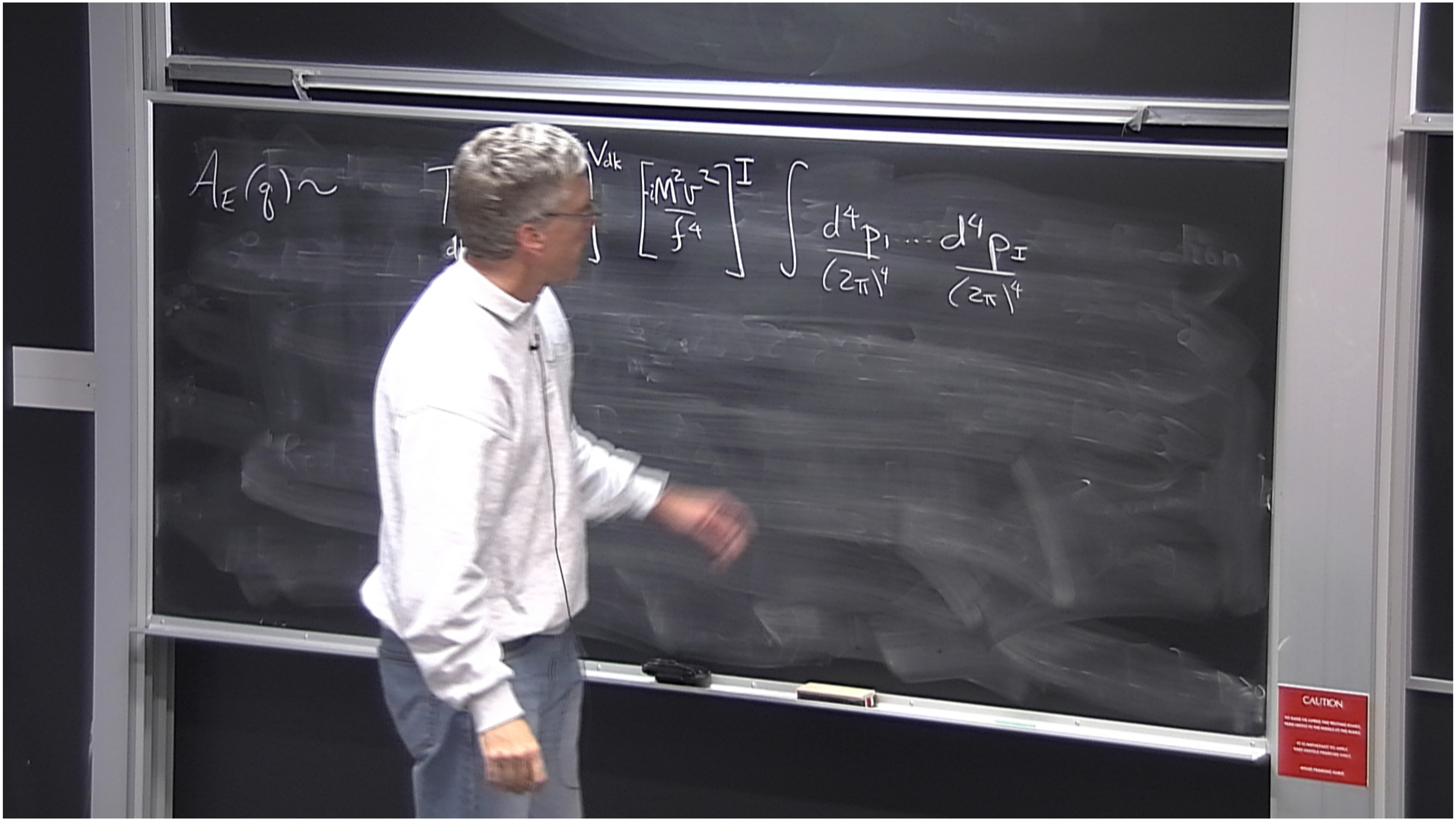

Suppose: $\mathcal{L} = f^4 \hat{\mathcal{L}}\left(\frac{\varphi}{v}, \frac{\partial\varphi}{Mv}\right)$
f, M, v
$= f^4\left[\frac{\partial_\mu\varphi\partial^\mu\varphi}{M^2v^2} + \frac{\mu^2\varphi^2}{M^2v^2}\right] + f^4\left[c_1\frac{(\partial_\mu\varphi\partial^\mu\varphi)^2}{M^4v^4} + c_2\frac{\varphi^8}{v^8} + \dots\right]$
c_i are dimensionless.

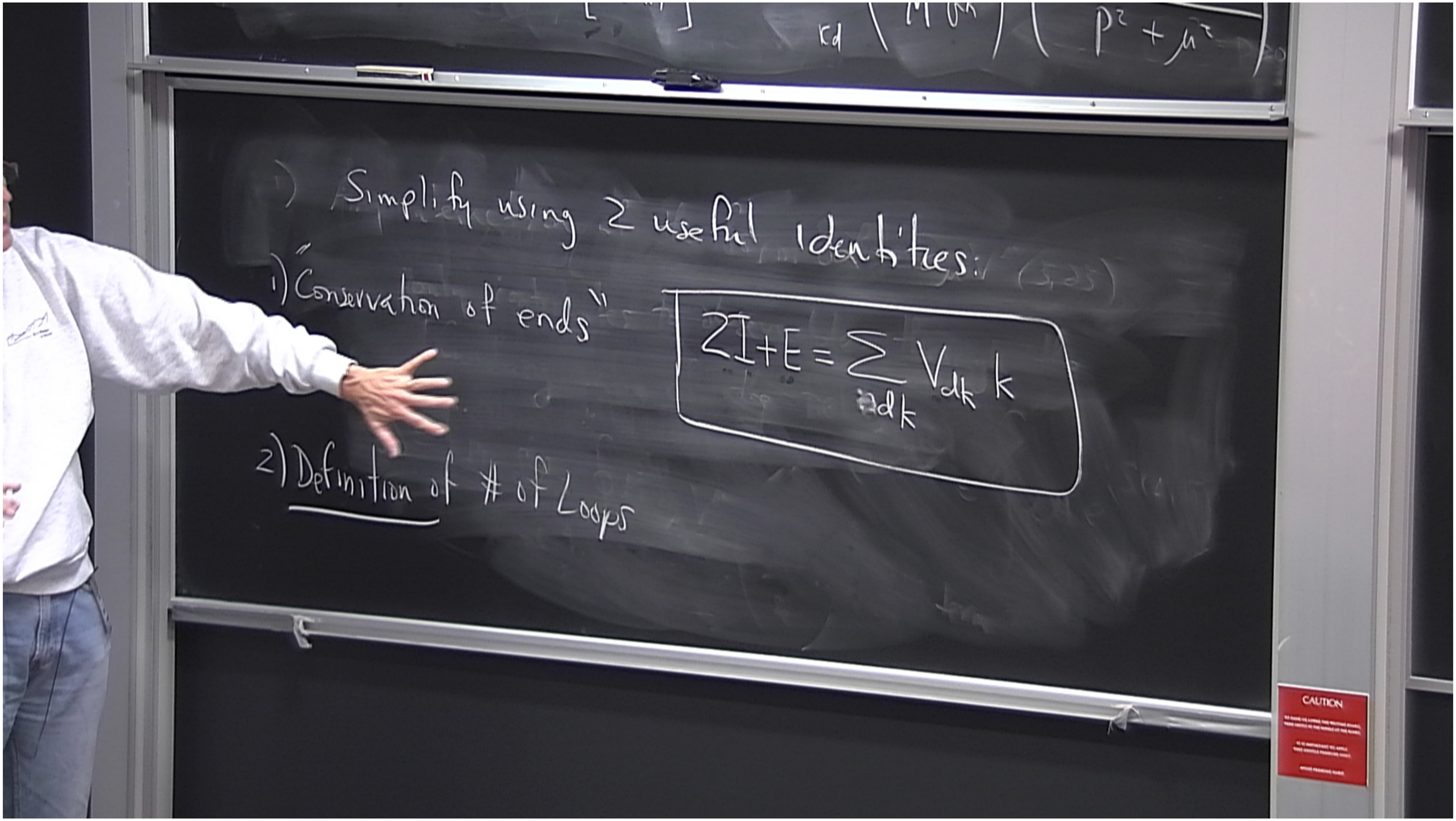

Suppose: $\mathcal{L} = f^4 \hat{\mathcal{L}}\left(\frac{\varphi}{v}, \frac{\partial\varphi}{Mv}\right)$
f, M, v
$= f^4\left[\frac{\partial_\mu\varphi\partial^\mu\varphi}{M^2v^2} + \frac{\mu^2\varphi^2}{M^2v^2}\right] + f^4\left[c_1\frac{(\partial_\mu\varphi\partial^\mu\varphi)^2}{M^4v^4} + c_2\frac{\varphi^8}{v^8} + \dots\right]$
$\mathcal{L}_0$
c_i are dimensionless.

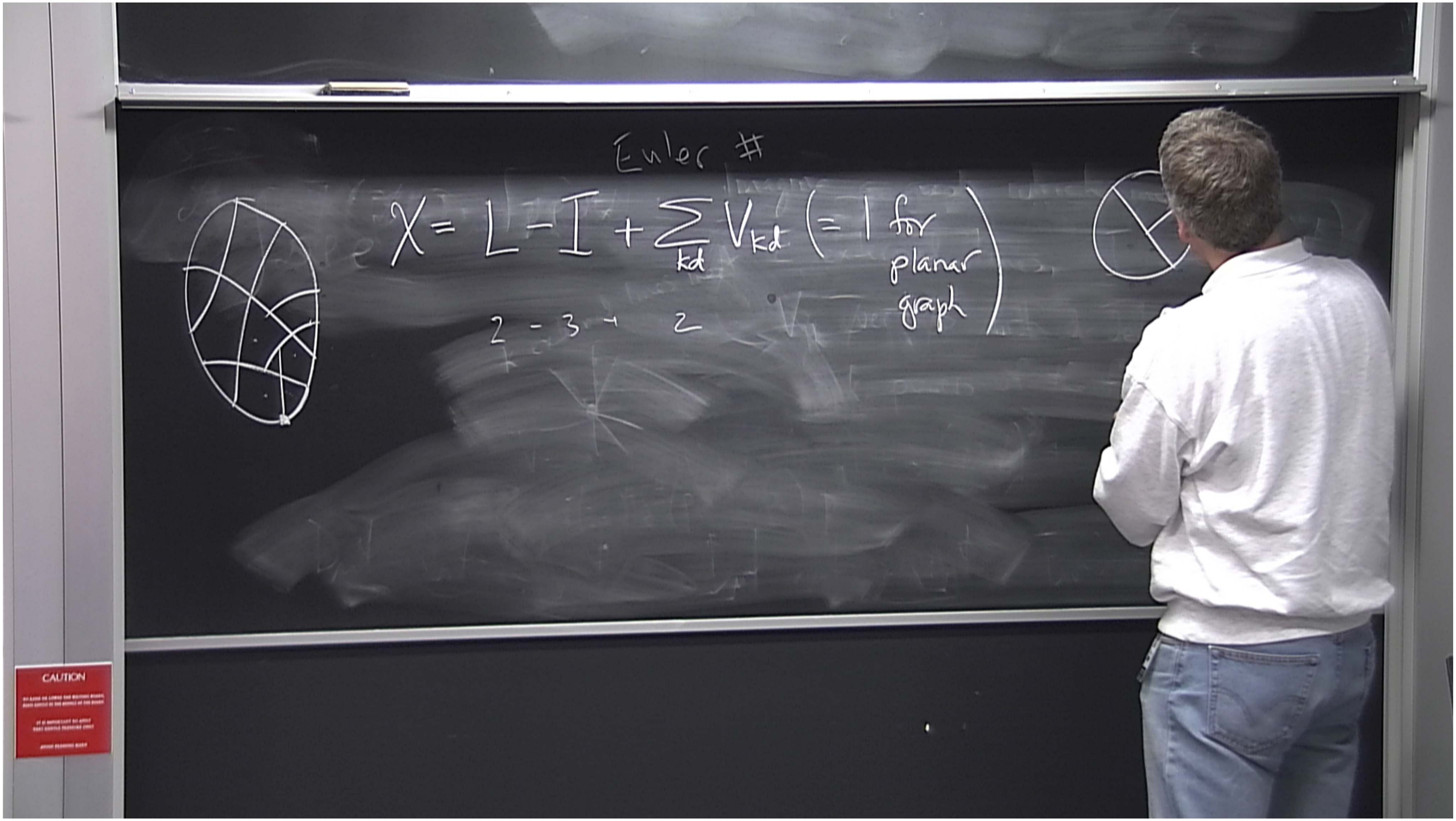

which involves
lines, I internal lines,
k external lines meet
d derivatives +
the fields.
CAUTION

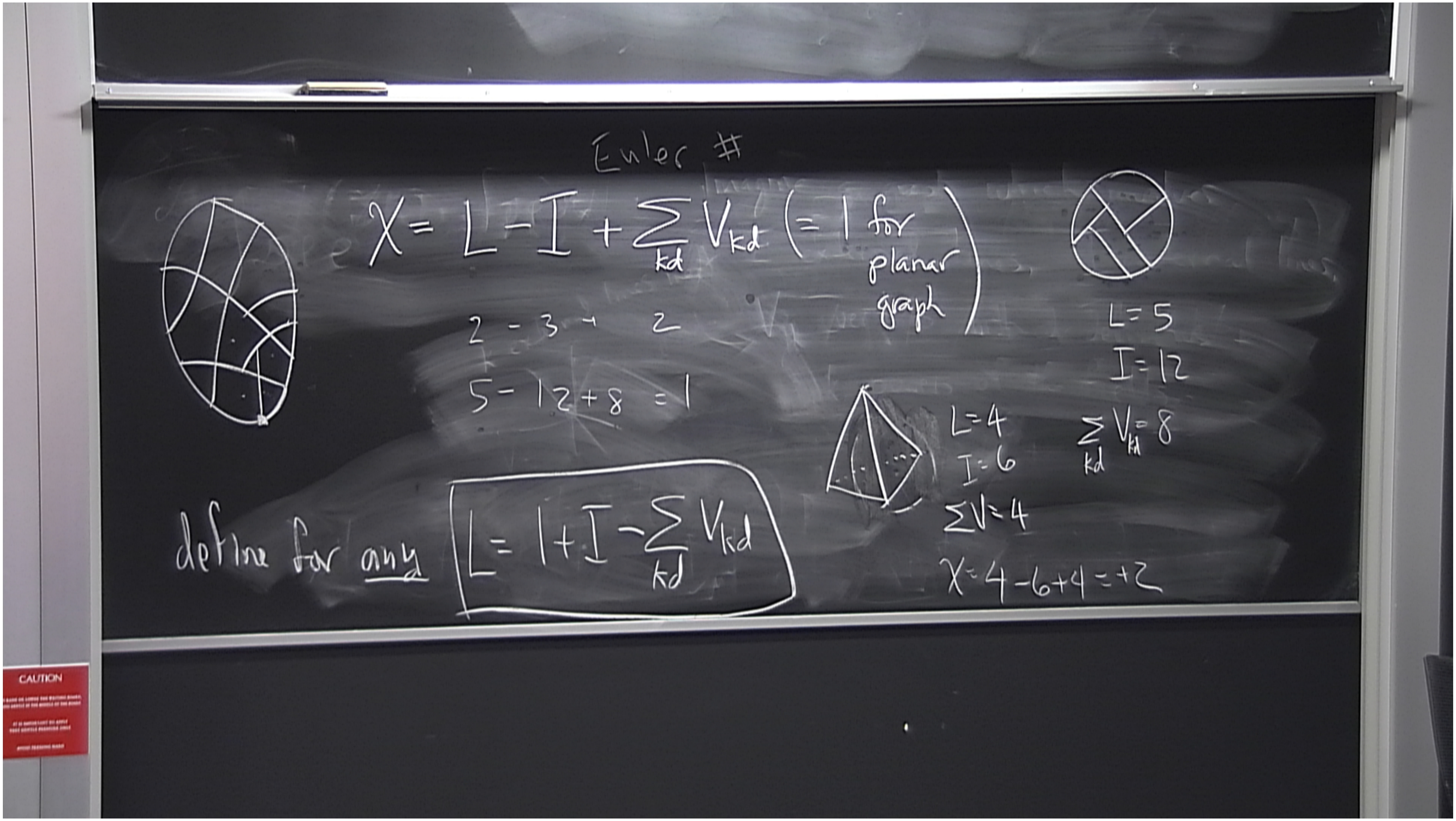

term from
Imagine using
as part of a calculation of

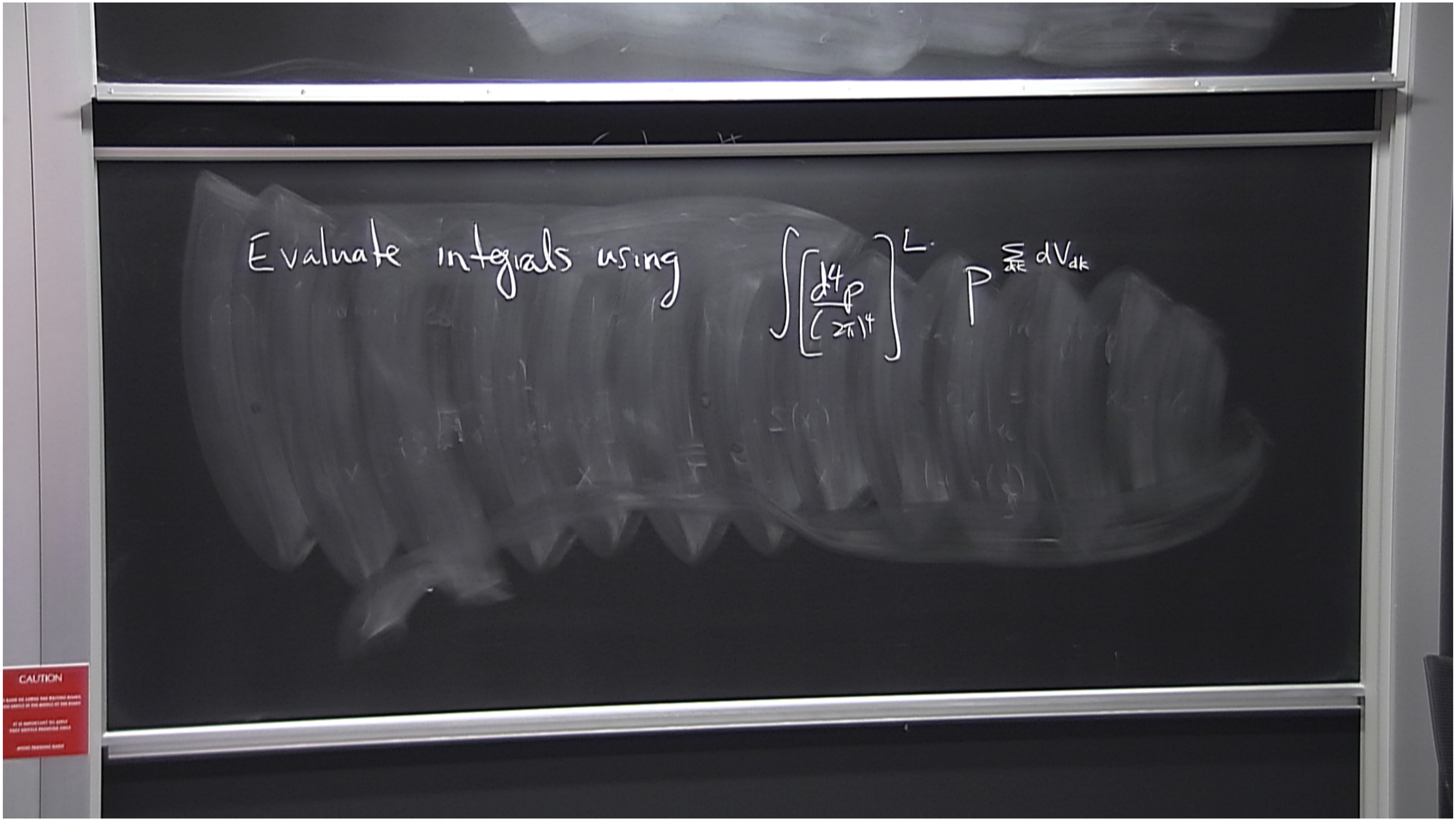

CAUTION

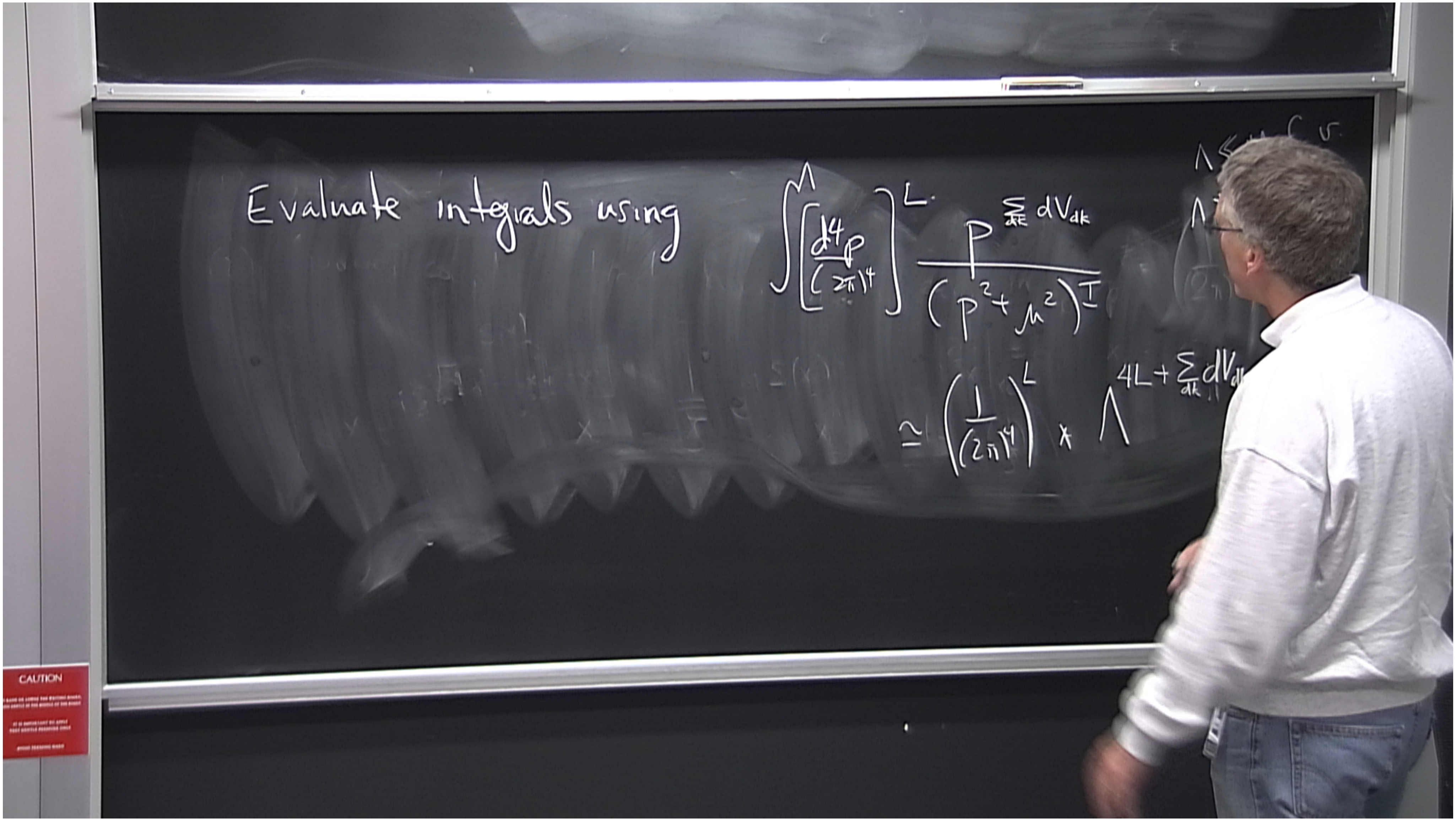

$A_E(q) \sim$
$\left[\frac{iM^2 v^{-2}}{f^4}\right]^I \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_I}{(2\pi)^4}$
CAUTION

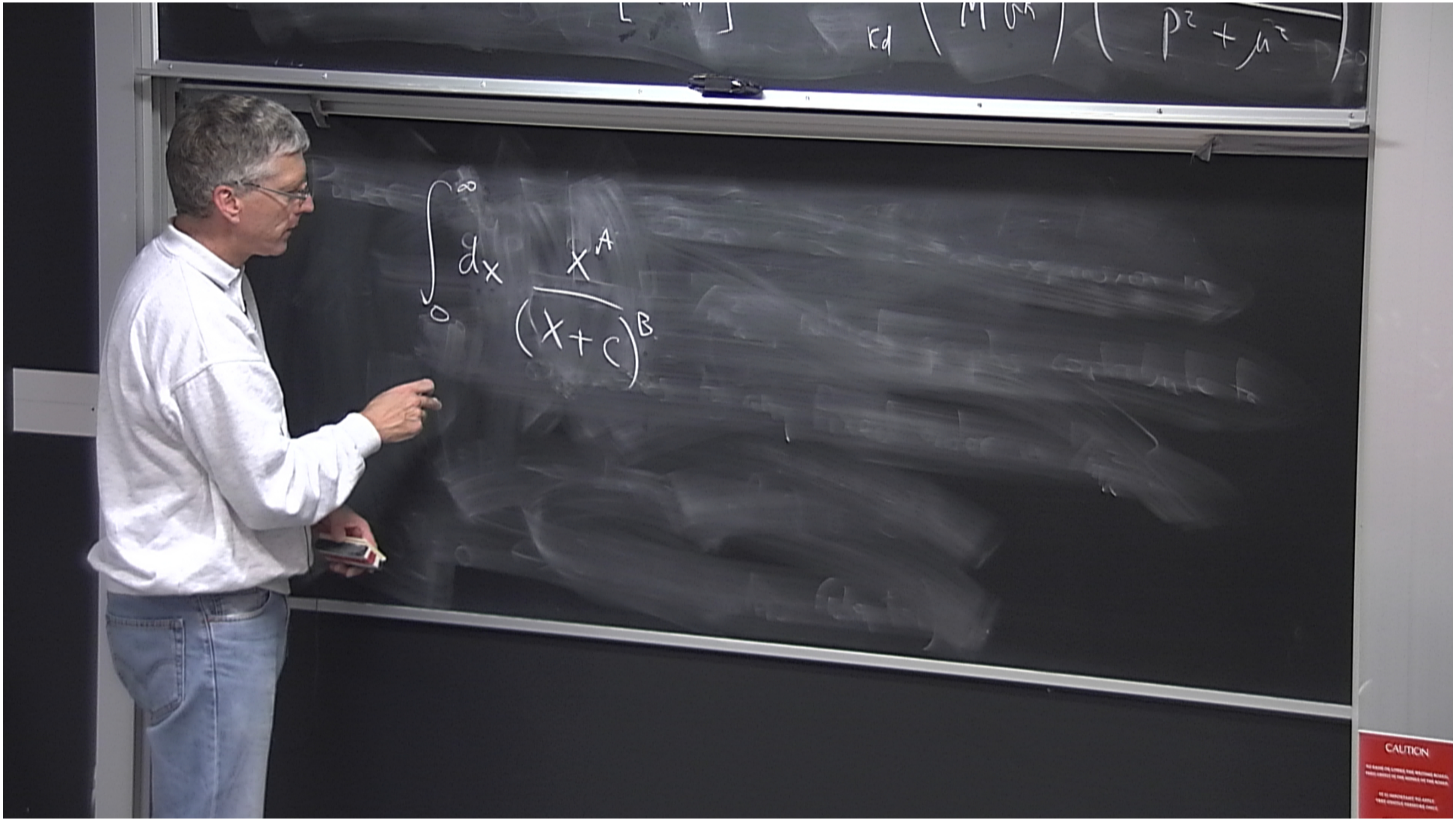

Simplify using 2 useful identities:
1) "Conservation of ends"
$2I+E = \sum_{d,k} V_{dk}\, k$
2) Definition of # of Loops
CAUTION

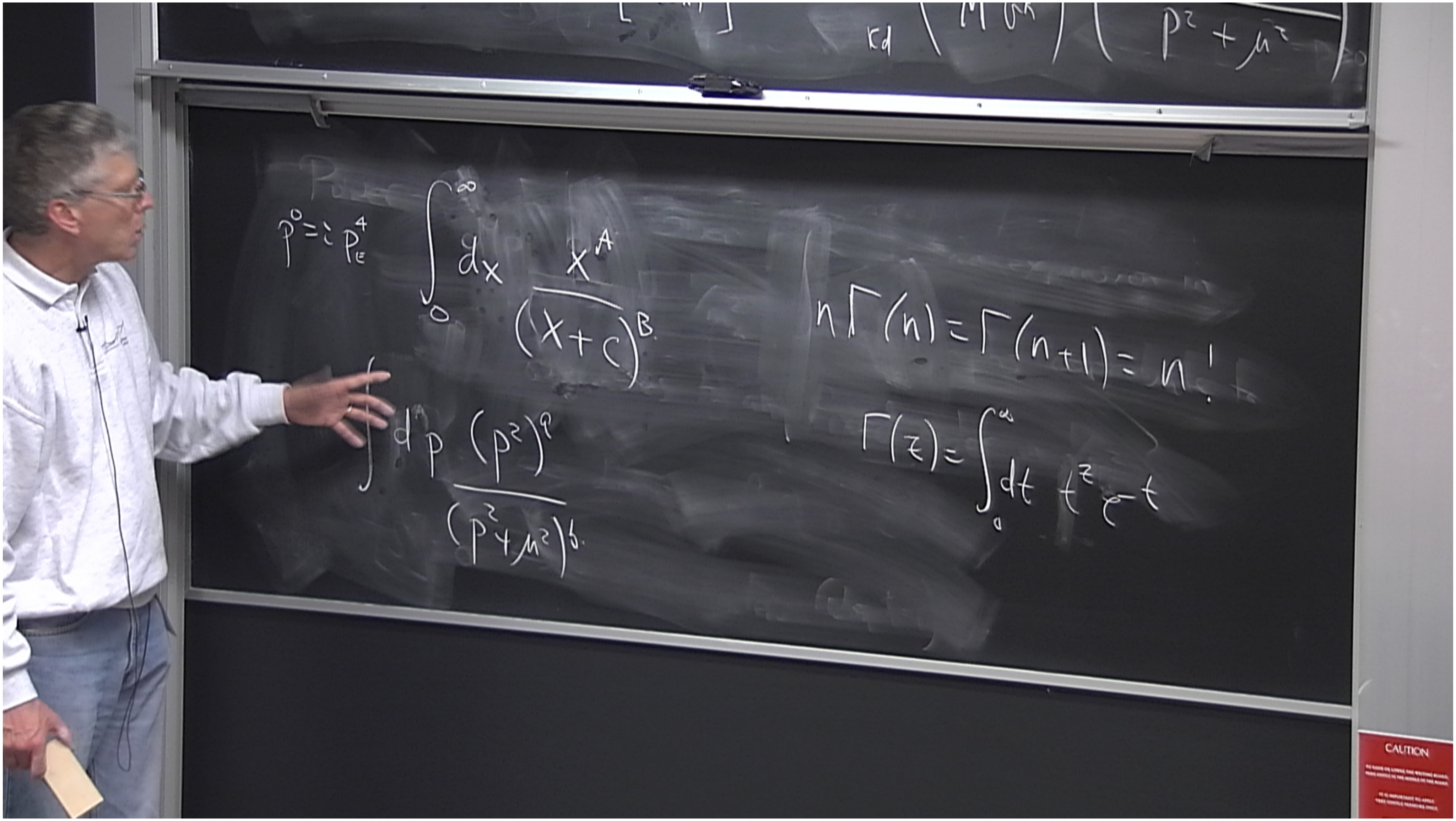

Euler #
χ = L − I + Σkd Vkd (= 1 for planar graph)
2 - 3 + 2
CAUTION

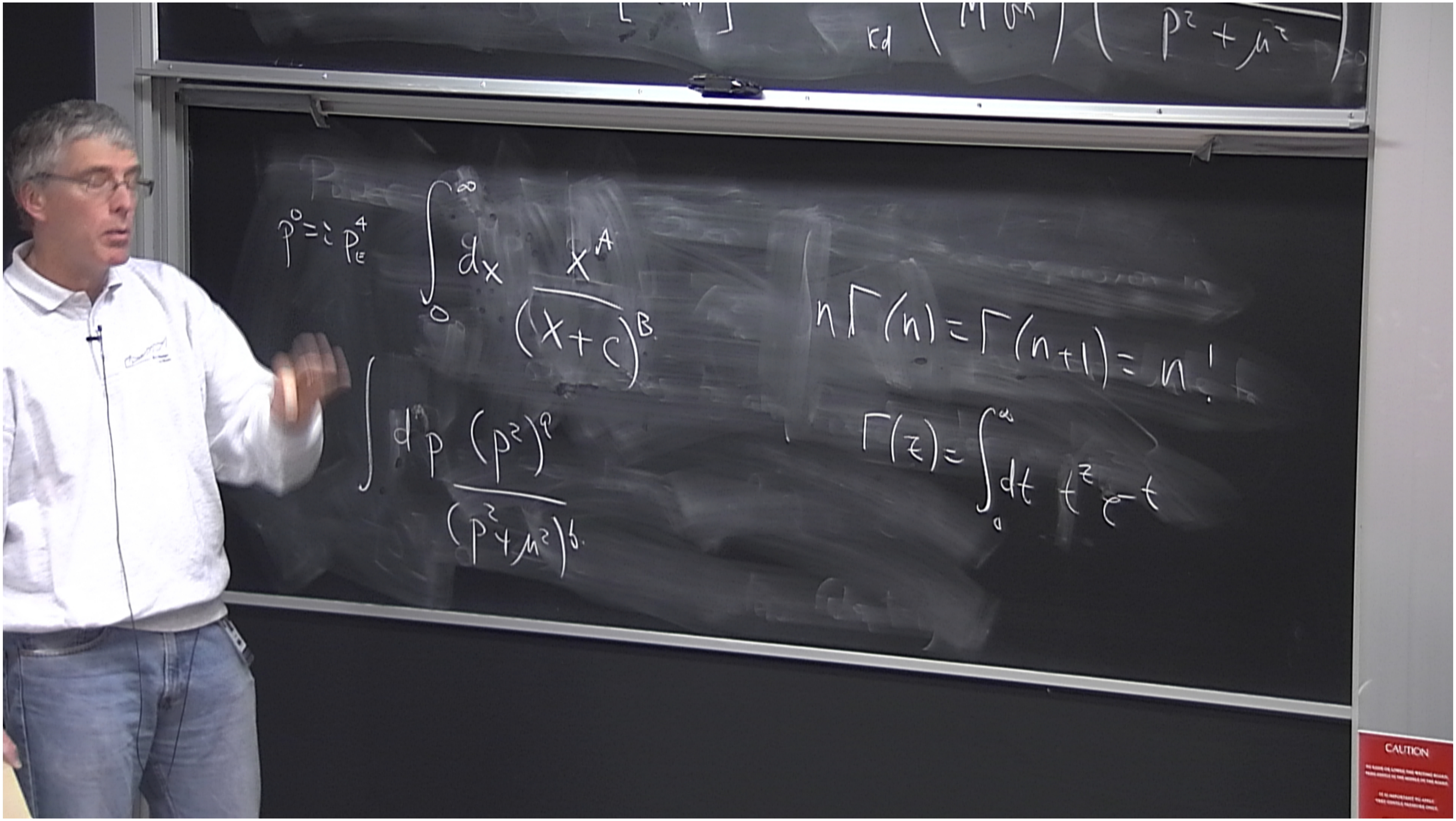

Euler #
χ = L − I + Σ_kd V_kd (= 1 for planar graph)
2 − 3 + 2
5 − 12 + 8 = 1
define for any
L = 1 + I − Σ_kd V_kd
L = 5
I = 12
Σ_kd V_kd = 8
L = 4
I = 6
ΣV = 4
χ = 4 − 6 + 4 = +2
CAUTION

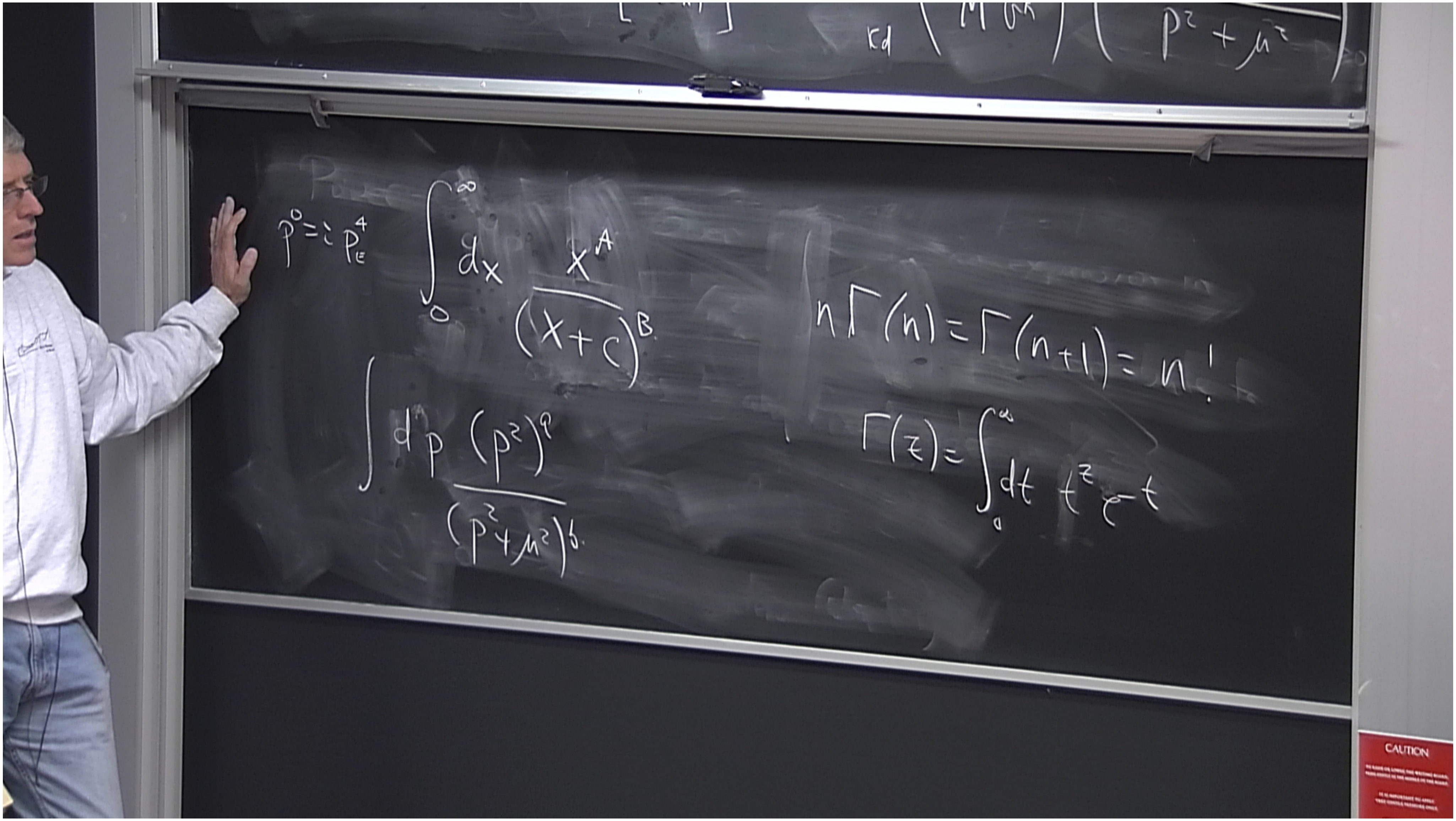

Evaluate integrals using
$\int \left[\frac{d^4p}{(2\pi)^4}\right]^L P^{\sum_{a<k} dV_{ak}}$

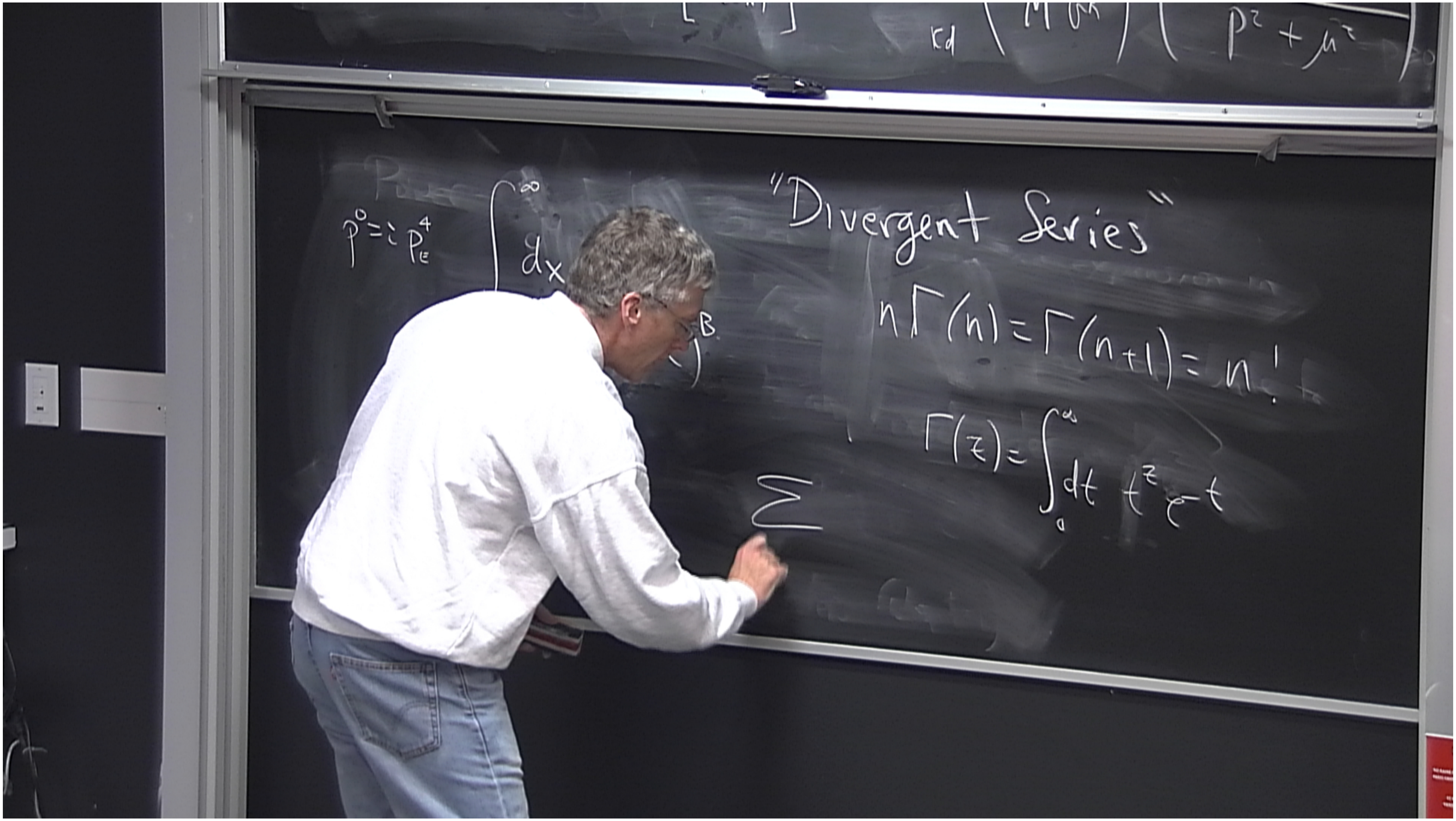

Evaluate integrals using
$\int^{\Lambda} \left[\frac{d^4p}{(2\pi)^4}\right]^L \frac{p^{\sum_{dk} dV_{dk}}}{(p^2+\mu^2)^I}$
$\simeq \left(\frac{1}{(2\pi)^4}\right)^L \times \Lambda^{4L+\sum_{dk} dV_{dk}}$
CAUTION

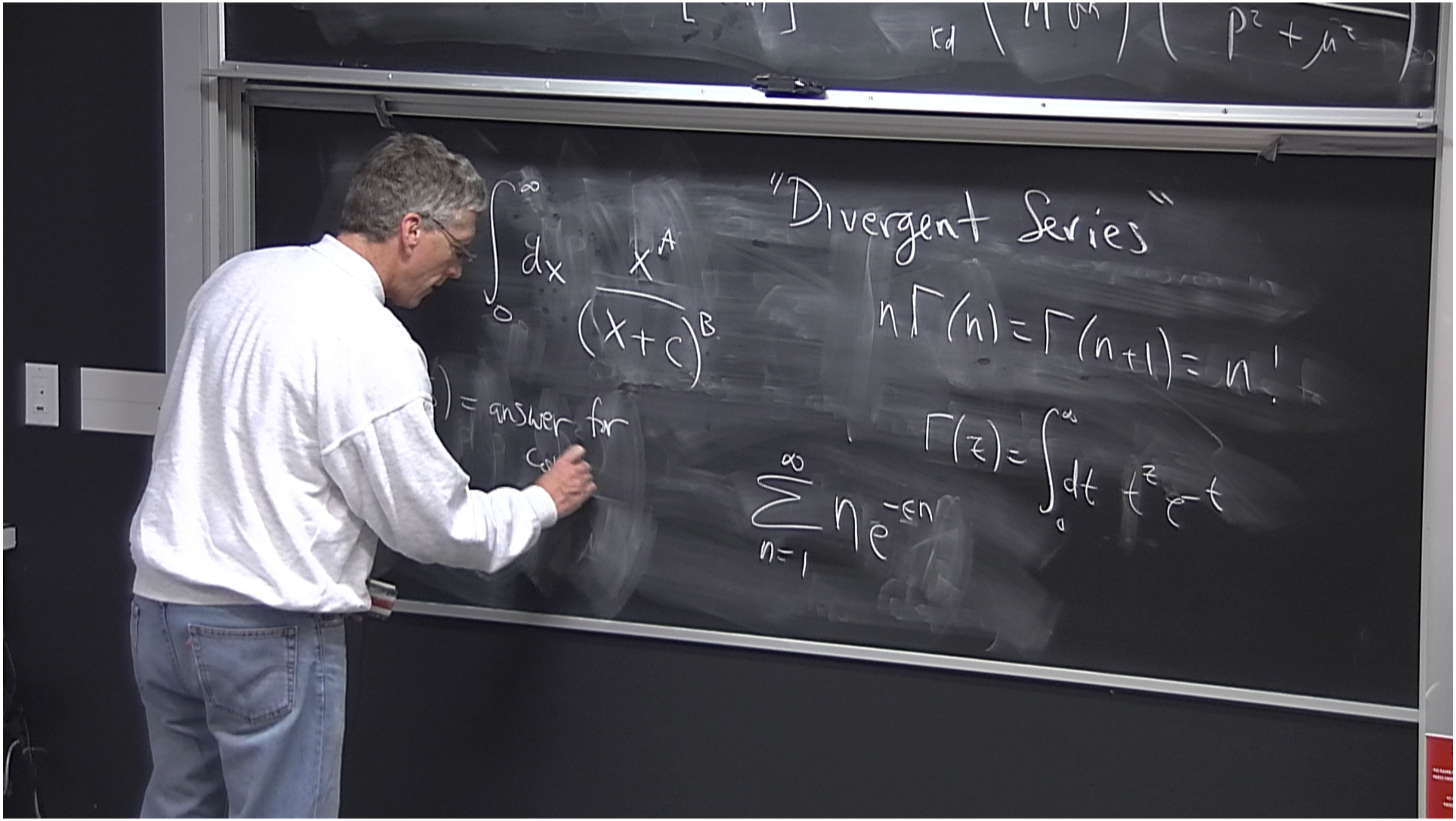

$\int_0^\infty dx \frac{x^A}{(x+c)^B}$
CAUTION


p^0 = i p_E^4
∫_0^∞ dx x^A / (x+c)^B
∫ d^4p (p^2)^a / (p^2+μ^2)^b
nΓ(n) = Γ(n+1) = n!
Γ(z) = ∫_0^∞ dt t^z e^-t
CAUTION

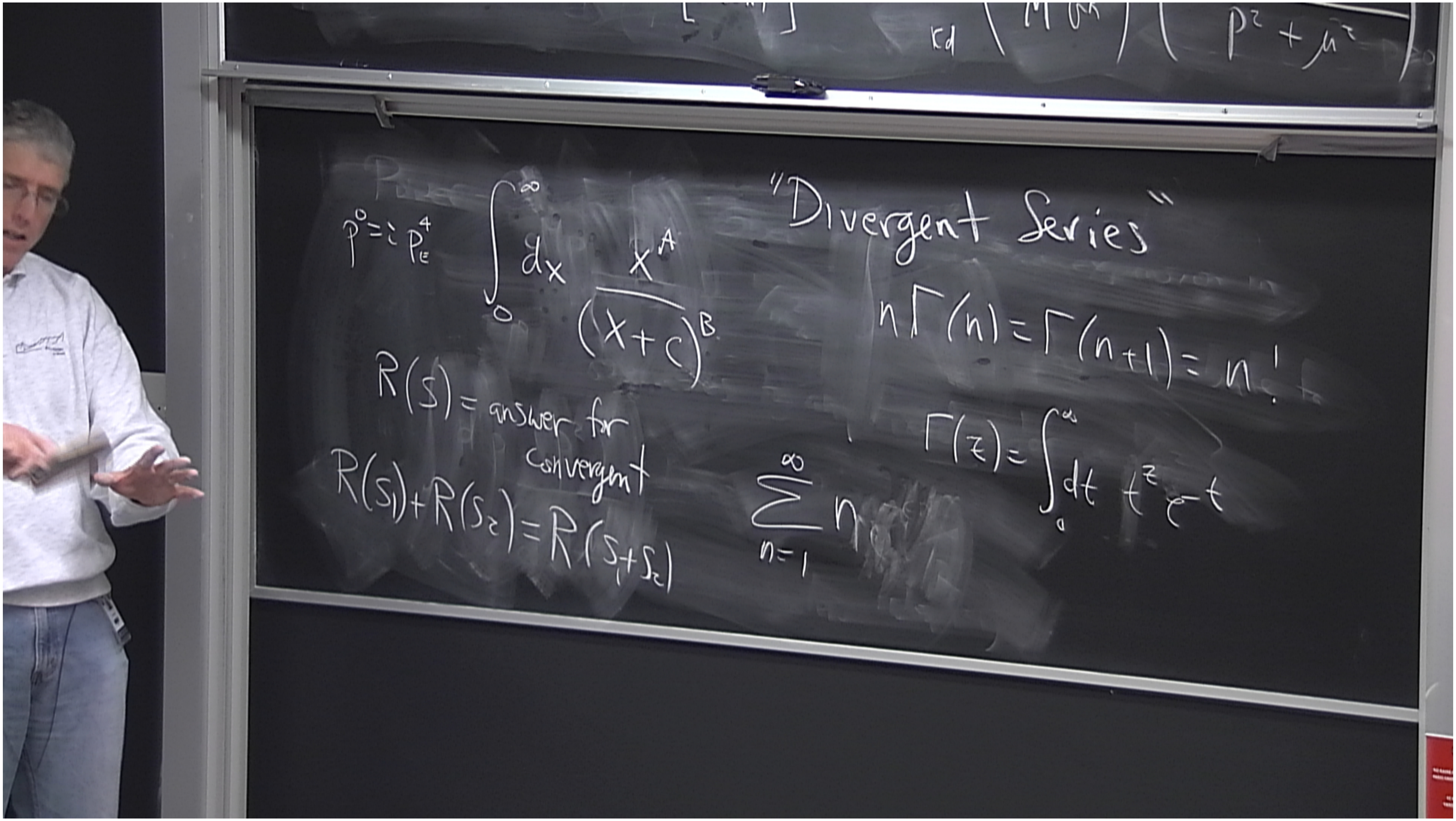

p⁰ = i p_E⁴
∫₀^∞ dx x^A / (x + c)^B
∫ d⁴p (p²)^a / (p² + μ²)^b
nΓ(n) = Γ(n+1) = n!
Γ(z) = ∫₀^∞ dt t^z e^(−t)
CAUTION


p^0 = i p_E^4
∫_0^∞ dx x^A / (x+c)^B
n Γ(n) = Γ(n+1) = n!
∫ d^4p (p^2)^a / (p^2+μ^2)^b
Γ(z) = ∫_0^∞ dt t^z e^{-t}
CAUTION


p° = i p_E^4
"Divergent Series"
nΓ(n) = Γ(n+1) = n!
Γ(z) = ∫₀^∞ dt t^z e^-t
Σ



"Divergent Series"
$n\Gamma(n) = \Gamma(n+1) = n!$
$\Gamma(z) = \int_0^\infty dt\, t^z e^{-t}$
$\sum_{n=1}^{\infty} n e^{-\epsilon n}$
$\int_0^\infty dx \frac{x^A}{(x+c)^B}$
) = answer for


"Divergent Series"
p° = i p_E^4
∫_0^∞ dx X^A / (X + C)^B
R(S) = answer for convergent
R(S_1) + R(S_2) = R(S_1 + S_2)
nΓ(n) = Γ(n+1) = n!
Γ(z) = ∫_0^∞ dt t^z e^-t
∑_{n=1}^∞ n