

Title: Introduction to Effective Field Theories - Lecture 6

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URL: <http://pirsa.org/14010093>

Abstract:

$h, l$

$$e^{i\Gamma(l)} = \int \mathcal{D}L \mathcal{D}H e^{iS(l+L, H) + i \int L g d^4x}$$

$$W(j) = W[j, J=0]$$

so

$$\Gamma(l) = \Gamma(l, h(l))$$

$$\left[ \frac{\delta \Gamma}{\delta h} \right]_{J=0} + J = \frac{\delta \Gamma}{\delta l} + j = 0$$



$$W(l) = W[l, J=0]$$

$$\text{so } \Gamma(l) = \Gamma(l, h(l))$$

$$\left[ \frac{\delta \Gamma}{\delta h} \right] + J = \left[ \frac{\delta \Gamma}{\delta l} \right] + J = 0$$

$$\Gamma(l, h) \approx S(l, h) + \sum_{1\text{-loop}} + \dots$$

$$\Gamma(l) \approx S(l, h(l)) \text{ with } \left( \frac{\delta S}{\delta h} \right)_{h(l)} = 0$$



$$e^{i\Gamma(l)} = \int \mathcal{D}L \mathcal{D}H e^{iS(l+L, H) + i \int L g dx} = \int \mathcal{D}L e^{iS(l+L) + i \int L g dx}$$

$$W(g) = W[j, J=0] \quad \text{so} \quad \Gamma(l) = \Gamma(l, h(l))$$

$$\left[ \frac{\delta \Gamma}{\delta l} \right] + J = \frac{\delta \Gamma}{\delta l} + J = 0$$

$$\Gamma(l) \approx S(l, h(l)) \quad \text{with} \quad \left( \frac{\delta S}{\delta h} \right)_{h(l)} =$$

$$\text{Defining } e^{iS_W(l+L)} = \int \mathcal{D}H e^{iS(l+L, H)}$$



$$W(j) = W[j, j=0]$$

$$\Gamma(l) = \Gamma(l, h(l))$$

$$\left[ \frac{\delta \Gamma}{\delta h} \right] + j = \left[ \frac{\delta \Gamma}{\delta h} \right] + j = 0$$

$$\Gamma(l, h) \approx S(l, h) + \sum_{1-loop} + \dots$$

$$\Gamma(l) \approx S(l, h(l)) \text{ with } \left( \frac{\delta S}{\delta h} \right)_{h(l)} = 0$$

$$\text{Defining } e^{iS_W(l+L)} = \int dH e^{iS(l+L, H)}$$

Classically

$$\Gamma(l) \approx S_W(l)$$



Example:  $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right) e^{i\zeta/\sqrt{2}v}$

$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi) \quad V = \frac{\lambda}{4} (\phi^* \phi - v^2)^2 = \frac{\lambda}{4} \left( \sqrt{2} v \chi + \frac{\chi^2}{2} \right)^2$$

$$= -\frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} \left( 1 + \frac{\chi}{\sqrt{2}v} \right)^2 \partial_\mu \zeta \partial^\mu \zeta - V[\chi]$$



$$\partial \chi = 0 \Rightarrow \square \chi - \left(1 + \frac{\lambda}{\sqrt{2}v}\right) \frac{1}{\sqrt{2}v} \partial_\mu \xi \partial^\mu \xi - \frac{\lambda}{2} \left(\sqrt{2}v \chi + \frac{\chi^2}{2}\right) \left(\sqrt{2}v + \chi\right) = 0$$

$$\left(-\square + m_R^2 + \frac{\lambda}{2m_R^2} \partial_\mu \xi \partial^\mu \xi\right) \chi = -\frac{1}{\sqrt{2}v} \partial_\mu \xi \partial^\mu \xi - \frac{3\lambda v}{2\sqrt{2}} \chi^2 - \frac{\lambda}{4} \chi^3$$

$$\left(m_R^2 - \square + \frac{\lambda}{2v^2} \partial_\mu \xi \partial^\mu \xi\right)^{-1} = \frac{1}{m_R^2} \left[1 + \frac{\square}{m_R^2} - \frac{1}{2v^2 m_R^2} \partial_\mu \xi \partial^\mu \xi + \dots\right]$$

$$\chi \simeq -\frac{1}{\sqrt{2}v m_R^2} \partial_\mu \xi \partial^\mu \xi + o\left(\frac{1}{m_R^2}\right)$$

$$\mathcal{L}(\xi, \chi(\xi)) = +\frac{1}{2} \chi \square \chi - \frac{1}{2} (\partial_\mu \xi \partial^\mu \xi) - \frac{\lambda}{\sqrt{2}v} (\partial_\mu \xi \partial^\mu \xi) - \frac{\lambda^2}{4v^2} (\partial_\mu \xi \partial^\mu \xi) - V(\chi)$$



$$S(\xi, \chi(\xi)) \simeq -\frac{1}{2} \partial_\mu \xi \partial^\mu \xi - a (\partial_\mu \xi \partial^\mu \xi) \left[ 1 + \frac{D}{m_P^2} \right] (\partial_\mu \xi \partial^\mu \xi)$$

$$- b (\partial_\mu \xi \partial^\mu \xi)^3 + \dots$$

$$a = \frac{1}{4\lambda v^4} = \frac{\lambda}{4m_R^4}$$

$$b = \left( \frac{1}{16} - \frac{1}{16} \right) \frac{\lambda^2}{m_R^2} = 0.$$



$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi) \quad V = \frac{\lambda}{4} (\phi^* \phi - v^2)^2 = \frac{\lambda}{4} \left( \sqrt{2} v \chi + \frac{\chi^2}{2} \right)^2$$

$$= -\frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{2} \left( 1 + \frac{\chi}{\sqrt{2} v} \right)^2 \partial_\mu \xi \partial^\mu \xi - V[\chi] = \frac{1}{2} v^2 \chi^2 + \dots \quad m_\chi^2 = \lambda v^2$$

$$\frac{\delta S}{\delta \chi} = 0 = +\square \chi - \left( 1 + \frac{\chi}{\sqrt{2} v} \right) \frac{1}{\sqrt{2} v} \partial_\mu \xi \partial^\mu \xi - \frac{\lambda}{2} \left( \sqrt{2} v \chi + \frac{\chi^2}{2} \right) \left( \sqrt{2} v + \chi \right) = 0$$

$$\Leftrightarrow -\frac{\chi}{\sqrt{2} v} \partial_\mu \xi \partial^\mu \xi$$

$$\left(-\square + m_R^2 + \frac{1}{2M_K^2} \partial_\mu \tilde{\zeta} \partial^\mu \tilde{\zeta}\right) \chi = -\frac{1}{\sqrt{2}v} \partial_\mu \tilde{\zeta} \partial^\mu \tilde{\zeta} - \frac{5\lambda v}{2\sqrt{2}} \chi^2 - \frac{\lambda}{4} \chi^3$$

$$(m_R^2 - \square + \frac{1}{2v^2} \partial_\mu \xi \partial^\mu \xi)^{-1} = \frac{1}{m_R^2} \left[ 1 + \frac{\square}{m_R^2} - \frac{1}{2v^2 m_R^2} \partial_\mu \xi \partial^\mu \xi + \dots \right]$$



$$- \cdot (-\square + m_R^2)^{-1} \approx \frac{1}{m_R^2} \left[ 1 + \frac{\Delta}{m_R^2} + \dots \right]$$

$$\text{diagram} = \frac{1}{A_R^2 (1 + \frac{\Delta}{m_R^2})} + \text{diagram} + \text{diagram}$$

$$= \left( \frac{1}{\sqrt{2}v} \right)^2 (\partial_\mu \xi \partial^\mu \xi) \frac{1}{m_R^2} \left[ 1 + \frac{\Delta}{m_R^2} + \dots \right] (\partial_\mu \xi \partial^\mu \xi) \frac{1}{2!}$$



$$= (-\square + m_R^2)^{-1} \approx \frac{1}{m_R^2} \left[ 1 + \frac{\Delta}{m_R^2} + \dots \right]$$



$$= \left( -\frac{1}{\sqrt{2}v} \right)^2 (\partial_\mu \xi \partial^\mu \xi) \frac{1}{m_R^2} \left[ 1 + \frac{\Delta}{m_R^2} + \dots \right] (\partial_\mu \xi \partial^\mu \xi) \frac{1}{2!} \times 2$$



$$i\Gamma = \int d^4x \left[ \left( \frac{i}{\sqrt{2}v} \right)^2 (\partial_\mu \xi \partial^\mu \xi) - \frac{i}{m_R^2} \left[ 1 + \frac{1}{m_R^2} \tau \dots \right] (\partial_\mu \xi \partial^\mu \xi) \right] \frac{1}{2!}$$

$$= i \int d^4x \left( \frac{1}{4v^2 m_R^2} \right) (\partial_\mu \xi \partial^\mu \xi)^2$$

$$G = \frac{-1}{4 m_R^4} \quad \checkmark$$



$$i\Gamma = \int d^4x \left[ \left( \frac{i}{\sqrt{2}v} \right)^2 (\partial_\mu \xi \partial^\mu \xi) - \frac{i}{m_R^2} \left[ 1 + \frac{1}{m_R^2} \tau \dots \right] (\partial_\mu \xi \partial^\mu \xi) \frac{1}{2!} \right]$$

$$= i \int d^4x \left( \frac{1}{4v^2 m_R^2} \right) (\partial_\mu \xi \partial^\mu \xi)^2$$

$$G = \frac{-1}{4 m_R^4}$$





To see a difference between  $T(l)$  and  $S_W(l)$   
must go beyond classical approx:

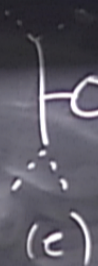
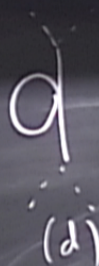
Compute 1-loop correction to  $a(\partial_\mu \xi \partial^\mu \bar{\xi})^2$



$$- \leftrightarrow - \frac{\chi}{\sqrt{2V}} \partial_\mu \xi \partial^\mu \xi$$



+





$$(a) \quad \delta a \propto \left(\frac{-1}{4v^2}\right)^2 L_1$$

$$(b) \quad \propto \left(\frac{1}{4v^2}\right) \left(\frac{-\lambda v}{2\sqrt{2}}\right) \left(\frac{-1}{\sqrt{2}v}\right) \frac{L_1}{m_R^2}$$

$$L_1 = \int^{\Omega} \frac{d^4 p}{(2\pi)^4} \left[ \frac{1}{p^2 + m_R^2} \right]^2 \approx \frac{1}{16\pi^2} \ln \Omega$$



$$(a) \quad \delta a \propto \left( \frac{-1}{4v^2} \right)^2 L_1$$

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$$(b) \quad \propto \left( \frac{1}{4v^2} \right) \left( \frac{-\lambda v}{2\sqrt{2}} \right) \left( \frac{-1}{\sqrt{2}v} \right) \frac{L_1}{m_R^2}$$

$$\sim \frac{1}{16\pi^2} \frac{1}{v^4} \left( \frac{\lambda^2}{m_R^4} \right)$$



$$(a) \quad \delta a \propto \left(\frac{-1}{4v^2}\right)^2 L_1$$

$$L_1 = \int^{\Omega} \frac{d^4 p}{(2\pi)^4} \left[ \frac{1}{p^2 + m_R^2} \right]^2 \sim \frac{1}{16\pi^2} \ln \Omega$$

$$(b) \quad \propto \left(\frac{1}{4v^2}\right) \left(\frac{-\lambda v}{2\sqrt{2}}\right) \left(\frac{-1}{\sqrt{2}v}\right) L_1 \frac{1}{m_R^2}$$

$$L_2 = \int^{\Omega} \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_R^2}$$

$$(d) \quad \left(\frac{-\lambda}{16}\right) \left(\frac{-1}{\sqrt{2}v}\right)^2 L_2 \frac{1}{m_R^4}$$

$$\sim \frac{1}{16\pi^2} \frac{1}{v^4} \left( \frac{\lambda^2}{m_R^4} \right)$$



$$(a) \quad \delta a \propto \left(\frac{-1}{4v^2}\right)^2 L_1$$

$$L_1 = \int \frac{d^4 p}{(2\pi)^4} \left[ \frac{1}{p^2 + m_R^2} \right]^2 \approx \frac{1}{16\pi^2} \ln \Omega$$

$$(b) \quad \propto \left(\frac{1}{4v^2}\right) \left(\frac{-\lambda v}{2\sqrt{2}}\right) \left(\frac{-1}{\sqrt{2}v}\right) \frac{L_1}{m_R^2}$$

$$L_2 = \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + m_R^2} \approx \frac{\Omega^2}{16\pi^2}$$

$$(d) \quad \left(\frac{-\lambda}{16}\right) \left(\frac{-1}{\sqrt{2}v}\right)^2 \frac{L_2}{m_R^4}$$

$$\sim \frac{1}{16\pi^2} \frac{1}{v^4} \left( \frac{\lambda^2}{m_R^4} \right)$$



$$-b(\partial_{\mu} \xi \partial^{\mu} \xi) + \dots$$

$$a = \frac{-1}{4\lambda v^4} = \frac{-\lambda}{4m_R^4}$$

$$b = \left( \frac{1}{16} - \frac{1}{16} \right) \frac{\lambda^2}{m_R^2} = 0$$

$$[i(-\square + m_R^2)]^{-1} \approx \frac{1}{m_R^2} \left[ 1 + \frac{\square}{m_R^2} + \dots \right]$$

$$[-\square + m_R^2] G(x, y) = \delta^4(x, y)$$

$$G(x, y) \approx \frac{1}{m_R^2} \delta^4(x, y) + \frac{1}{m_R^4} \square \delta^4(x, y) + \dots$$

(d)

$$\left( -\frac{\lambda}{16} \right) \left( \frac{-1}{\sqrt{2}v} \right) \frac{\lambda^2}{m_R^4}$$

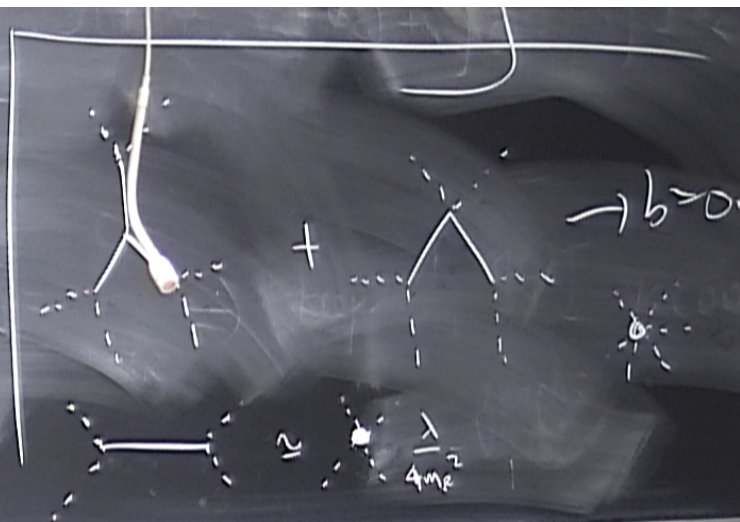
$$\sim \frac{1}{16v^2} \frac{1}{v^4} \lambda^2$$

$$L_2 = \int \frac{1}{(2\pi)^4} p^4 m_R^2 \approx 16\pi^2$$



$$= i \int d^4x \left( \frac{1}{4v^2 m_R^2} \right) (\partial_\mu \xi \partial^\mu \xi)^2$$

$$a = \frac{-1}{4m_R^4} \quad \checkmark$$



$$(a) \quad \delta a \propto \left( \frac{-1}{4m_R^4} \right)^2$$

$$1 = \int d^4p \left[ \frac{1}{p^2} \right]^2 \approx \frac{1}{2} \ln \Omega$$

$$S(\xi, \chi(\xi)) \simeq -\frac{1}{2} \partial_\mu \xi \partial^\mu \xi - a (\partial_\mu \xi \partial^\mu \xi) \left[ 1 + \frac{D}{m_R^2} \right] (\partial_\mu \xi \partial^\mu \xi)$$

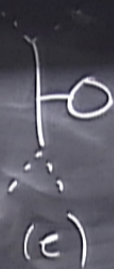
$$+ \frac{1}{6} (\partial_\mu \xi \partial^\mu \xi)^3 + \dots$$



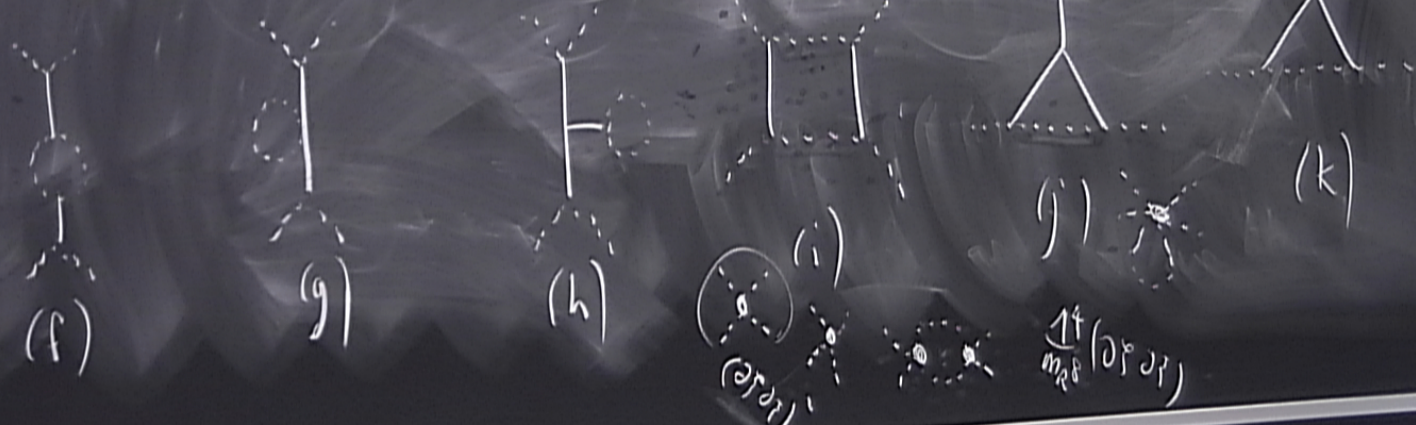
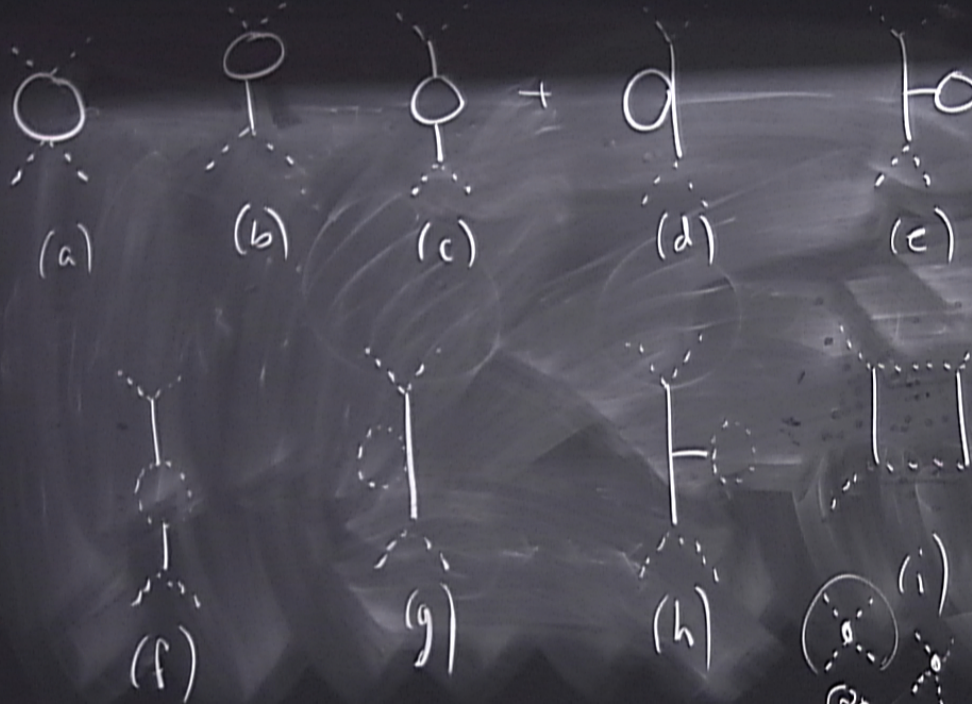
$$- \leftrightarrow - \frac{\chi}{\sqrt{2}i} \partial_\mu \xi \partial^\mu \xi$$



+







$$\text{---} \leftrightarrow -\frac{\chi}{\sqrt{2}iF} \partial_\mu \xi \partial^\mu \xi$$

$$a_{tree} - a_{loop} \int \frac{d^4 p}{(2\pi)^4} \frac{p^4}{(p^2)^2} \sim \Lambda^4$$

$$a_{1-loop} \sim \frac{1}{m^2} + \frac{\Lambda^4}{m^2}$$

$$\frac{1}{m^2} (\partial_\mu \partial_\nu)$$



$$\frac{1}{\sqrt{2}} \delta_{\mu\nu} \delta^{\mu\nu}$$

$$(\partial_\mu \partial_\mu)$$

$$\frac{\lambda}{4m^2}$$

$$a_{tree}$$

$$a_{loop}$$

$$\int \frac{d^4 p}{(2\pi)^4} \frac{p^4}{(p^2)^2} \sim \Lambda^4$$

$$a_{1-loop} \sim \frac{1}{4} + \frac{1}{m^2} \left( 1 + \frac{\lambda}{4m^2} \right)$$

