Title: Introduction to Effective Field Theories - Lecture 6

Date: Jan 24, 2014 02:30 PM

URL: http://pirsa.org/14010093

Abstract:

h, l

$$e^{i\Gamma(l)} = \int \mathcal{D}L\,\mathcal{D}H\; e^{iS(l+L,H) + i\int L J\, d^4x}$$

$$W(J) = W[J; J=0] \qquad \text{so:} \qquad \Gamma(l) = \Gamma(l, h(l))$$

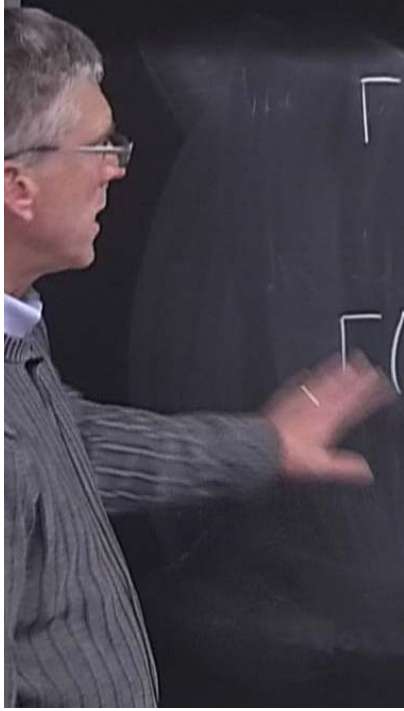
$$\boxed{\frac{\delta\Gamma}{\delta h}} + J = \frac{\delta\Gamma}{\delta l} + J = 0$$

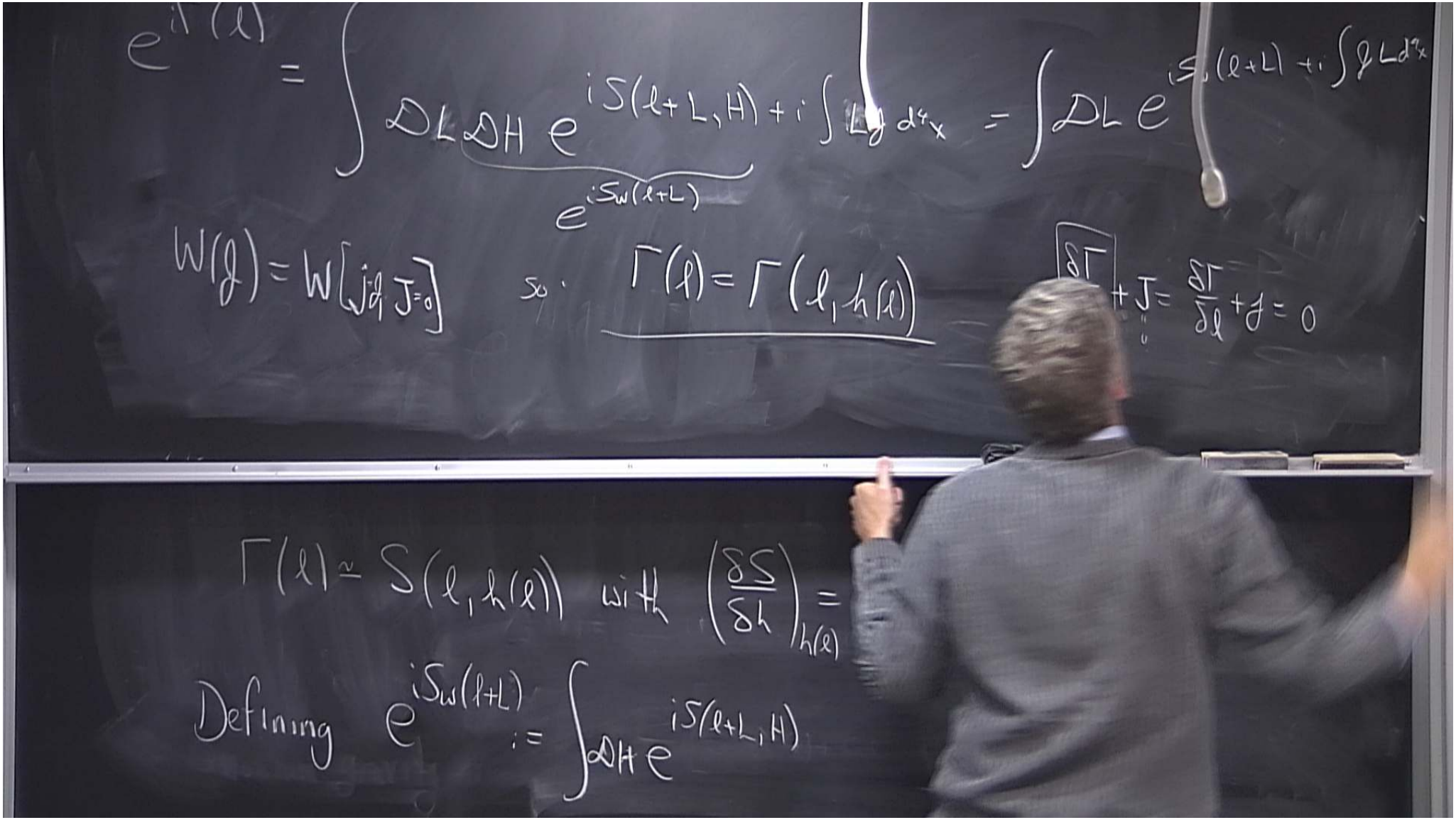
$W(J) = W[J; J=0]$ so: $\Gamma(\phi) = \Gamma(\phi, h(\phi))$

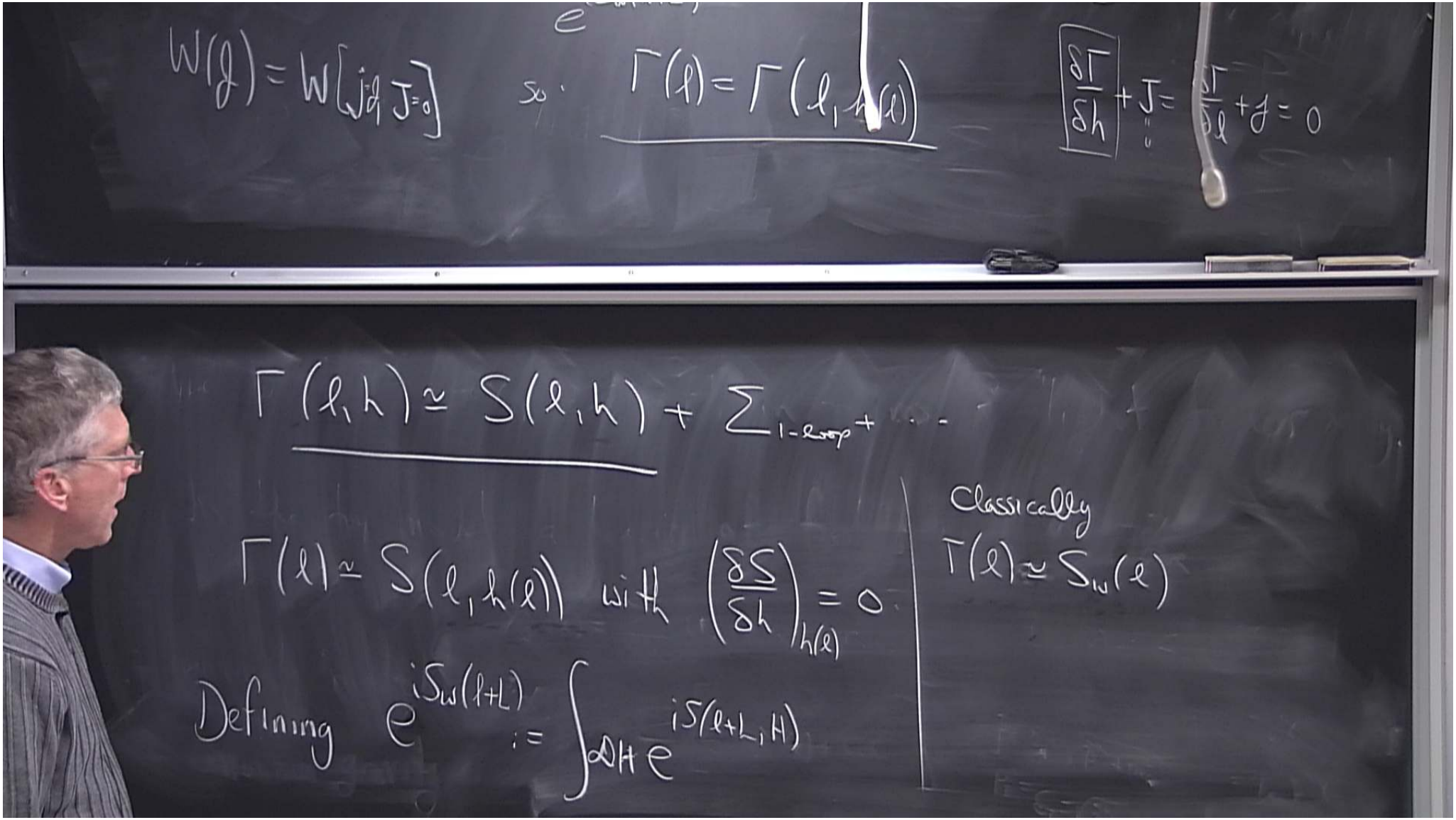
$\boxed{\frac{\delta\Gamma}{\delta h}} + J = \frac{\delta\Gamma}{\delta\phi} + J = 0$

$\Gamma(\phi, h) \simeq S(\phi, h) + \Sigma_{1\text{-loop}} + \cdots$

$\Gamma(\phi) \simeq S(\phi, h(\phi))$ with $\left(\frac{\delta S}{\delta h}\right)_{h(\phi)} = 0$




$e^{i\Gamma(\ell)} = \int \mathcal{D}L\,\mathcal{D}H\, e^{iS(\ell+L,H) + i\int L J\, d^4x} = \int \mathcal{D}L\, e^{iS_W(\ell+L) + i\int J L\, d^4x}$
$e^{iS_W(\ell+L)}$
$W(J) = W[j=J, J=0]$
so: $\Gamma(\ell) = \Gamma(\ell, h(\ell))$
$\boxed{\delta\Gamma} + J = \frac{\delta\Gamma}{\delta\ell} + J = 0$
$\Gamma(\ell) \simeq S(\ell, h(\ell))$ with $\left(\frac{\delta S}{\delta h}\right)_{h(\ell)} =$
Defining $e^{iS_W(\ell+L)} := \int \mathcal{D}H\, e^{iS(\ell+L,H)}$


$W(J) = W[J; J=0]$
so: $\Gamma(\phi) = \Gamma(\phi, h(\phi))$
$\frac{\delta\Gamma}{\delta h} + J = \frac{\delta\Gamma}{\delta\phi} + J = 0$
$\Gamma(\phi, h) \simeq S(\phi, h) + \Sigma_{1\text{-loop}} + \cdots$
$\Gamma(\phi) \simeq S(\phi, h(\phi))$ with $\left(\frac{\delta S}{\delta h}\right)_{h(\phi)} = 0$
Defining $e^{iS_W(\phi+L)} := \int \mathcal{D}H\, e^{iS(\phi+L, H)}$
Classically
$\Gamma(\phi) \simeq S_W(\phi)$

Example: $\phi = \left(v + \frac{\chi}{\sqrt{2}}\right) e^{i\xi/\sqrt{2}v}$

$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$ $\qquad V = \frac{\lambda}{4}\left(\phi^* \phi - v^2\right)^2 = \frac{\lambda}{4}\left(\sqrt{2}v\chi + \frac{\chi^2}{2}\right)^2$

$= -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{1}{2}\left(1 + \frac{\chi}{\sqrt{2}v}\right)^2 \partial_\mu \xi \partial^\mu \xi - V[\chi]$

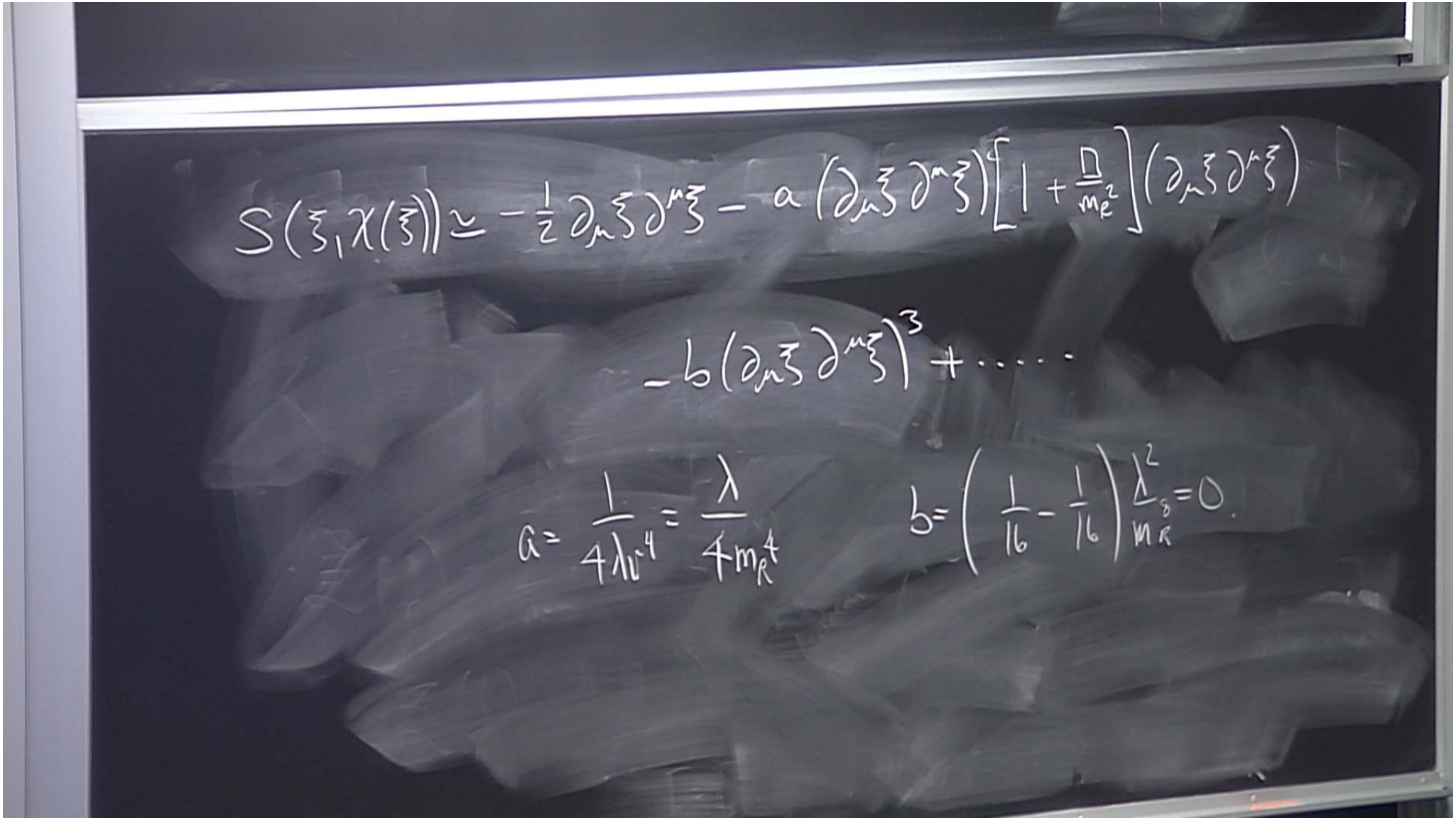
$$\partial\chi - \Box\chi - \left(1+\frac{\chi}{\sqrt{2}v}\right)\frac{1}{\sqrt{2}v}\partial_\mu\xi\partial^\mu\xi - \frac{\lambda}{2}\left(\sqrt{2}v\chi+\frac{\chi^2}{2}\right)\left(\sqrt{2}v+\chi\right) = 0$$

$$\left(-\Box + m_R^2 + \frac{1}{2v^2}\partial_\mu\xi\,\partial^\mu\xi\right)\chi = -\frac{1}{\sqrt{2}v}\partial_\mu\xi\,\partial^\mu\xi - \frac{3\lambda v}{2\sqrt{2}}\chi^2 - \frac{\lambda}{4}\chi^3$$

$$\left(m_R^2 - \Box + \frac{1}{2v^2}\partial_\mu\xi\,\partial^\mu\xi\right)^{-1} = \frac{1}{m_R^2}\left[1 + \frac{\Box}{m_R^2} - \frac{1}{2v^2 m_R^2}\partial_\mu\xi\,\partial^\mu\xi + \dots\right]$$

$$\chi = -\frac{1}{\sqrt{2}v m_R^2}\partial_\mu\xi\,\partial^\mu\xi + O\left(\frac{1}{m_R^3}\right)$$

$$\mathcal{L}\left(\xi,\chi(\xi)\right) = +\frac{1}{2}\chi\Box\chi - \frac{1}{2}\left(\partial_\mu\xi\,\partial^\mu\xi\right) - \frac{\chi}{\sqrt{2}v}\left(\partial_\mu\xi\,\partial^\mu\xi\right) - \frac{\chi^2}{4v^2}\left(\partial_\mu\xi\,\partial^\mu\xi\right) - V(\chi)$$


$S(\xi, \chi(\xi)) \simeq -\frac{1}{2}\partial_\mu\xi\partial^\mu\xi - a(\partial_\mu\xi\partial^\mu\xi)\left[1 + \frac{\Box}{m_R^2}\right](\partial_\mu\xi\partial^\mu\xi)$
$- b(\partial_\mu\xi\partial^\mu\xi)^3 + \cdots\cdots$
$a = \frac{1}{4\lambda v^4} = \frac{\lambda}{4m_R^4}$
$b = \left(\frac{1}{16} - \frac{1}{16}\right)\frac{\lambda^2}{m_R^6} = 0.$

$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi) \qquad V = \frac{\lambda}{4}\left(\phi^* \phi - v^2\right)^2 = \frac{\lambda}{4}\left(\sqrt{2}v\chi + \frac{\chi^2}{2}\right)^2$$

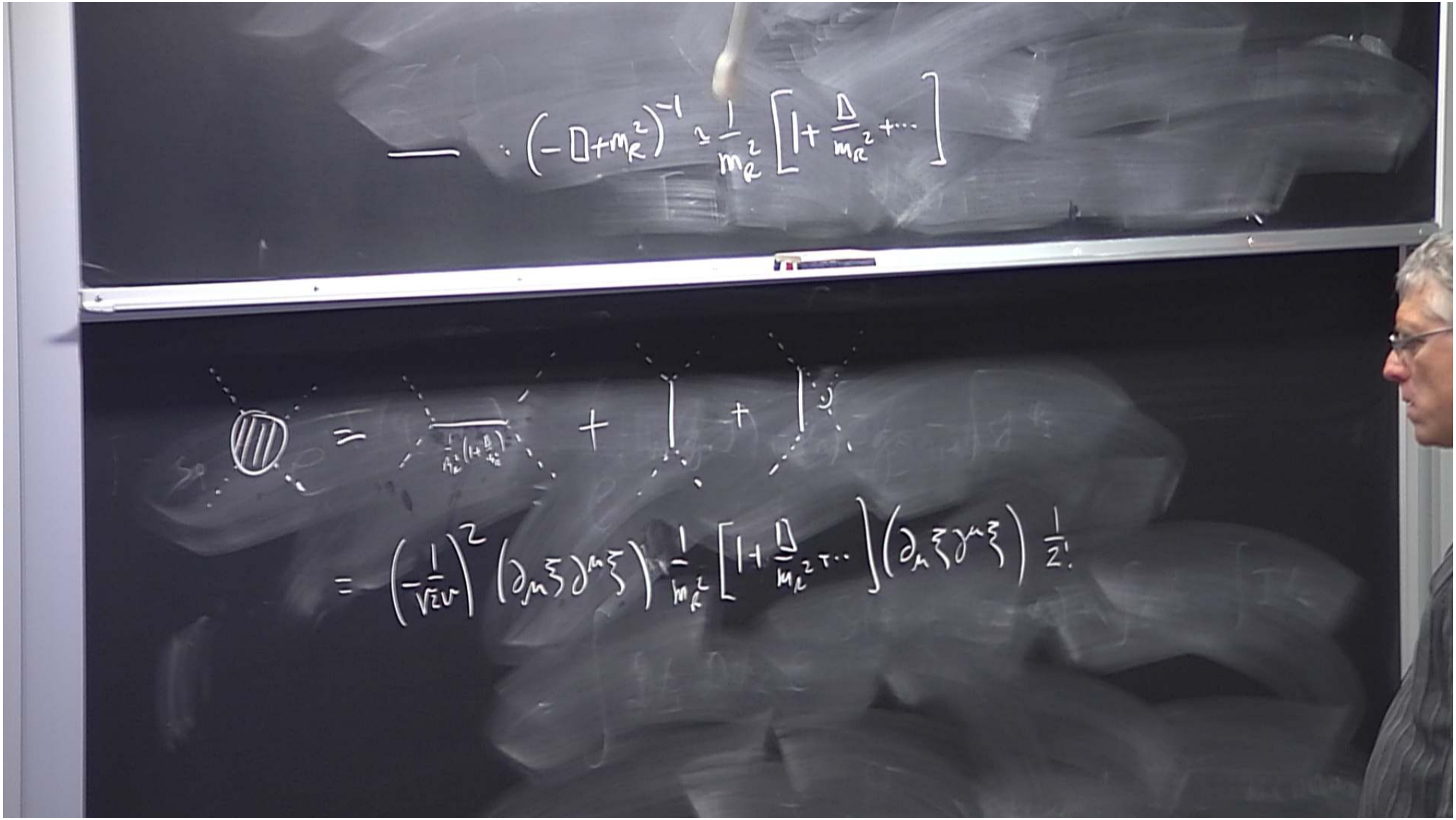
$$= -\frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{1}{2}\left(1 + \frac{\chi}{\sqrt{2}v}\right)^2 \partial_\mu \xi \partial^\mu \xi - V[\chi] \qquad = \frac{\lambda}{2} v^2 \chi^2 + \cdots \quad m_\chi^2 = \lambda v^2$$

$$\frac{\delta S}{\delta \chi} = 0 = +\Box\chi - \left(1 + \frac{\chi}{\sqrt{2}v}\right)\frac{1}{\sqrt{2}v}\partial_\mu \xi \partial^\mu \xi - \frac{\lambda}{2}\left(\sqrt{2}v\chi + \frac{\chi^2}{2}\right)\left(\sqrt{2}v + \chi\right) = 0$$

$$\leftrightarrow -\frac{\chi}{\sqrt{2}v}\partial_\mu \xi \partial^\mu \xi$$

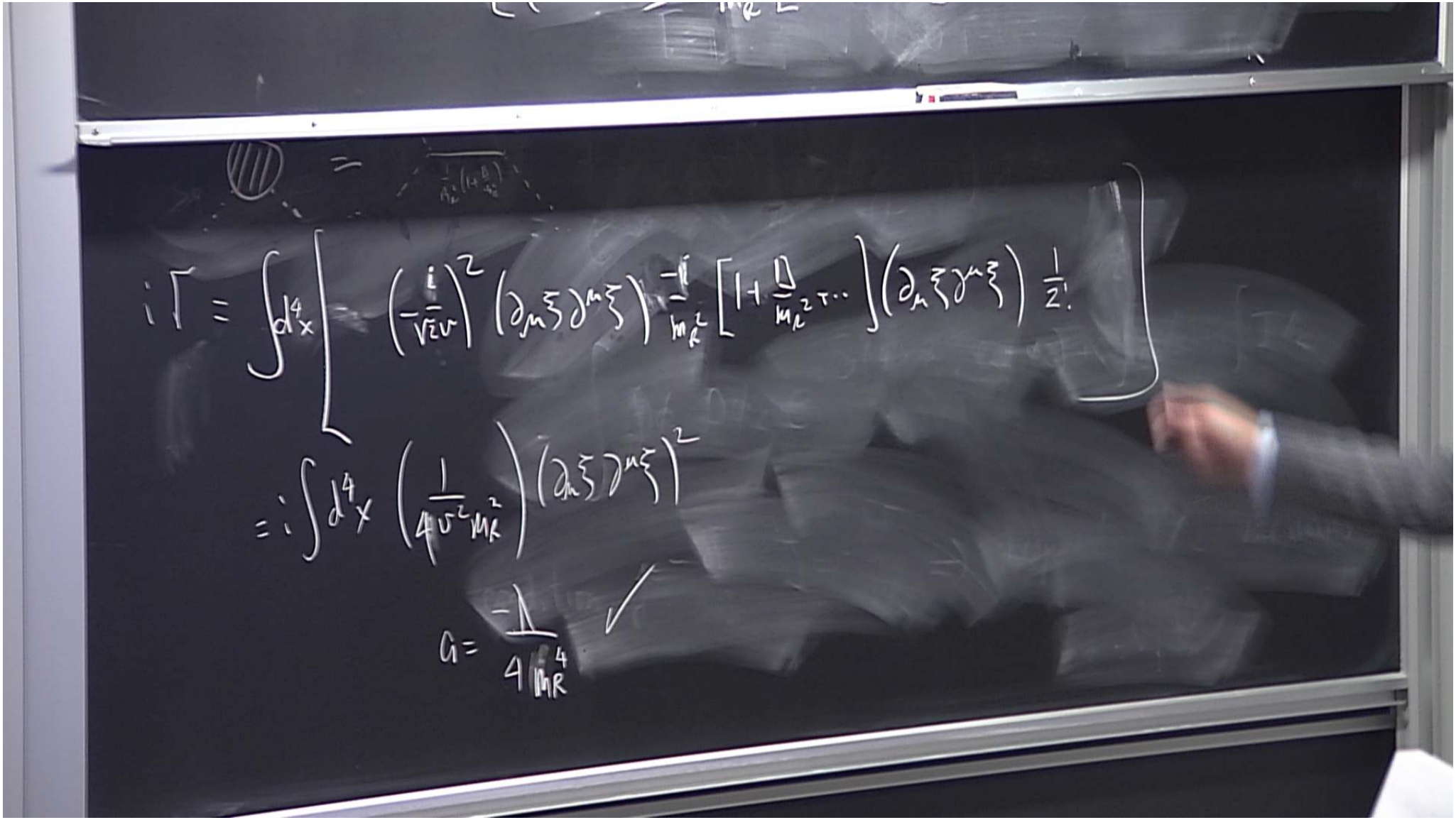
$$\left(-\Box + m_\chi^2 + \frac{\lambda}{2m_\chi^2}\partial_\mu \xi \partial^\mu \xi\right)\chi = -\frac{1}{\sqrt{2}v}\partial_\mu \xi \partial^\mu \xi - \frac{3\lambda v}{2\sqrt{2}}\chi^2 - \frac{\lambda}{4}\chi^3$$

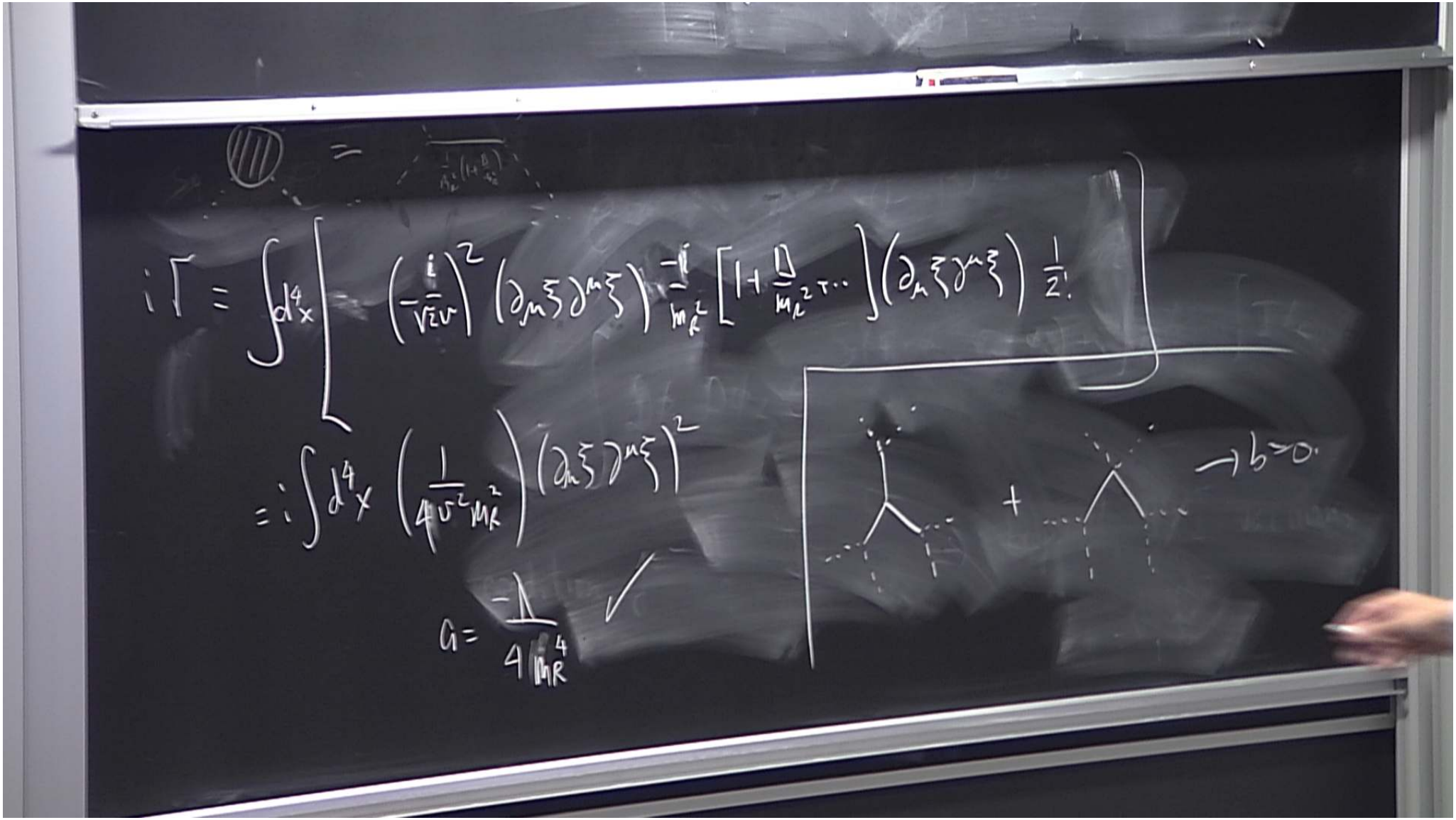
$$\left(m_\chi^2 - \Box + \frac{\lambda}{2v^2}\partial_\mu \xi \partial^\mu \xi\right)^{-1} = \frac{1}{m_\chi^2}\left[1 + \frac{\Box}{m_\chi^2} - \frac{1}{2v^2 m_\chi^2}\partial_\mu \xi \partial^\mu \xi + \cdots\right]$$


$\sim (-\Box + m_R^2)^{-1} \simeq \frac{1}{m_R^2}\left[1 + \frac{\Box}{m_R^2} + \cdots\right]$
$= \left(-\frac{1}{\sqrt{2}v}\right)^2 (\partial_\mu \xi \partial^\mu \xi) \frac{1}{m_R^2}\left[1 + \frac{\Box}{m_R^2} + \cdots\right](\partial_\mu \xi \partial^\mu \xi)\frac{1}{2!}$

$$\text{—} \cdot \left(-\Box + m_R^2\right)^{-1} \simeq \frac{1}{m_R^2}\left[1 + \frac{\Box}{m_R^2} + \cdots\right]$$

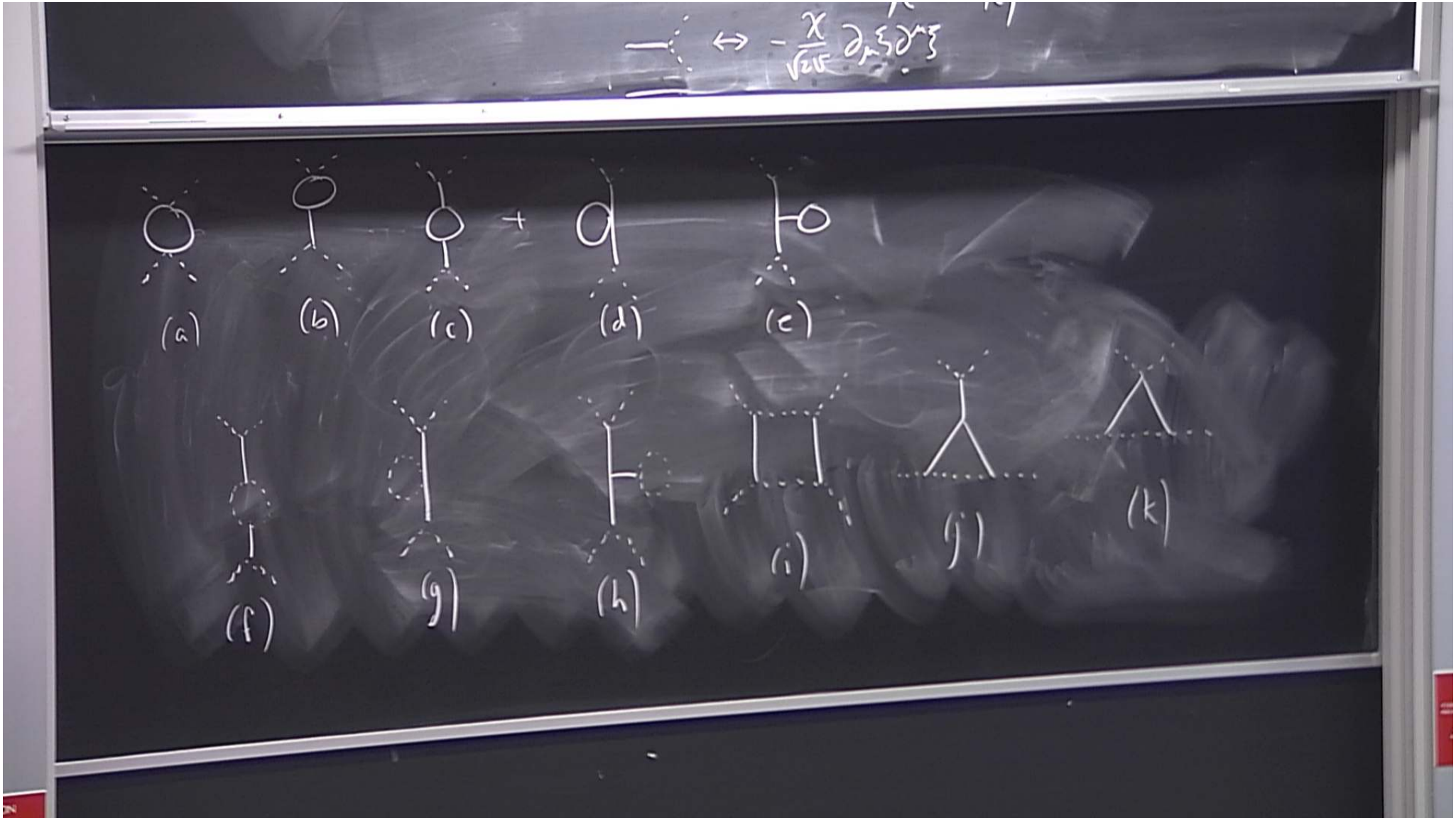
$$= \left(-\frac{1}{\sqrt{2}v}\right)^2 \left(\partial_\mu \xi \partial^\mu \xi\right) \frac{1}{m_R^2}\left[1 + \frac{\Box}{m_R^2} + \cdots\right]\left(\partial_\mu \xi \partial^\mu \xi\right) \frac{1}{2!} \times 2$$

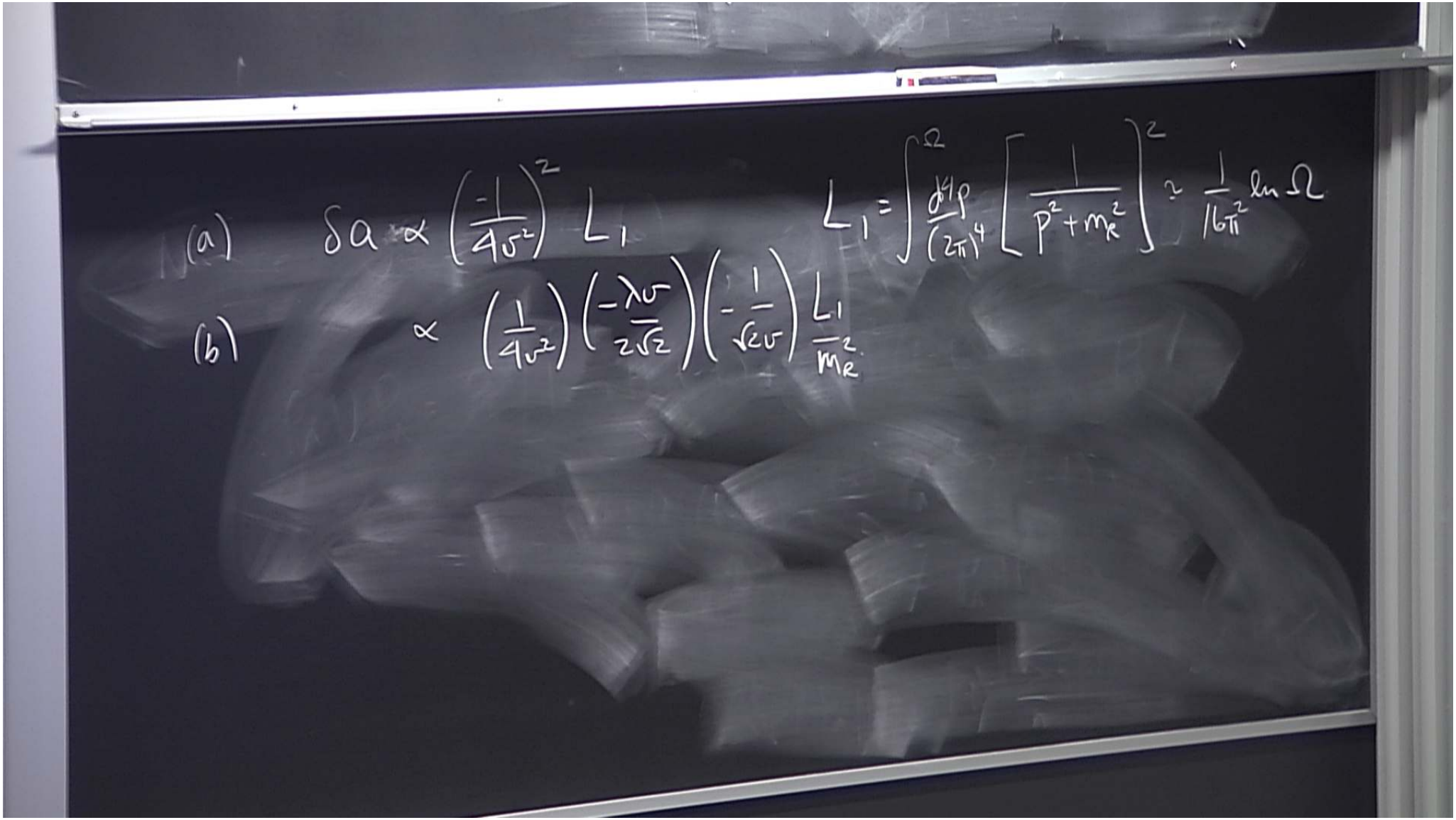

$i\Gamma = \int d^4x \left[\left(\frac{i}{\sqrt{2}v}\right)^2 (\partial_\mu \xi \partial^\mu \xi) \frac{-i}{m_R^2} \left[1 + \frac{\Box}{m_R^2} + \cdots\right] (\partial_\mu \xi \partial^\mu \xi) \frac{1}{2!} \right]$
$= i \int d^4x \left(\frac{1}{4v^2 m_R^2}\right) (\partial_\mu \xi \partial^\mu \xi)^2$
$G = \frac{-\lambda}{4 m_R^4}$

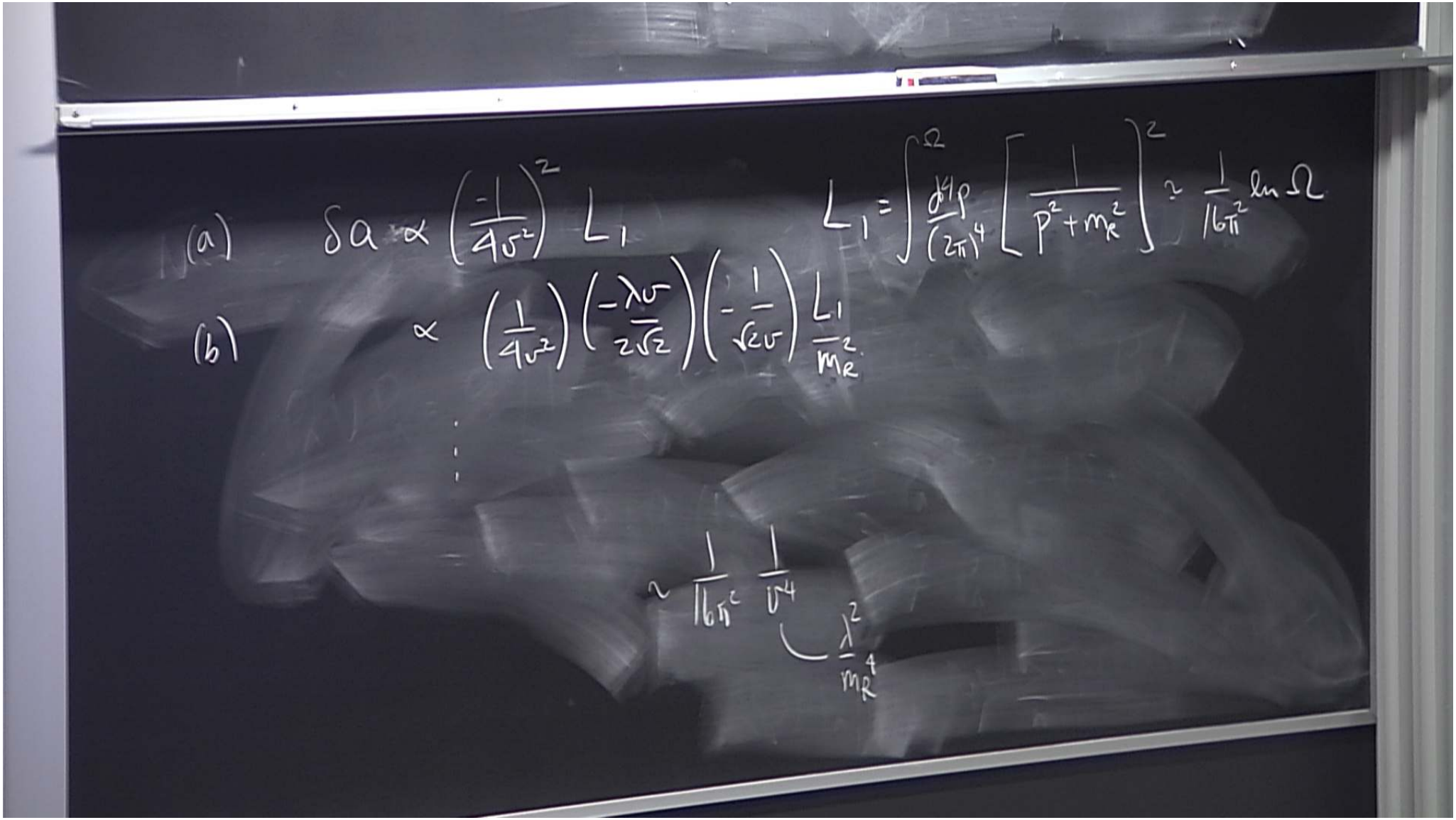

$i\Gamma = \int d^4x \left[-\left(\frac{i}{\sqrt{2}v}\right)^2 (\partial_\mu \xi \partial^\mu \xi) \frac{-i}{m_R^2} \left[1 + \frac{\Box}{m_R^2} + \cdots\right] (\partial_\mu \xi \partial^\mu \xi) \frac{1}{2!} \right.$
$= i \int d^4x \left(\frac{1}{4v^2 m_R^2}\right) (\partial_\mu \xi \partial^\mu \xi)^2$
$G = \frac{-1}{4 m_R^4}$
$\rightarrow b > 0.$

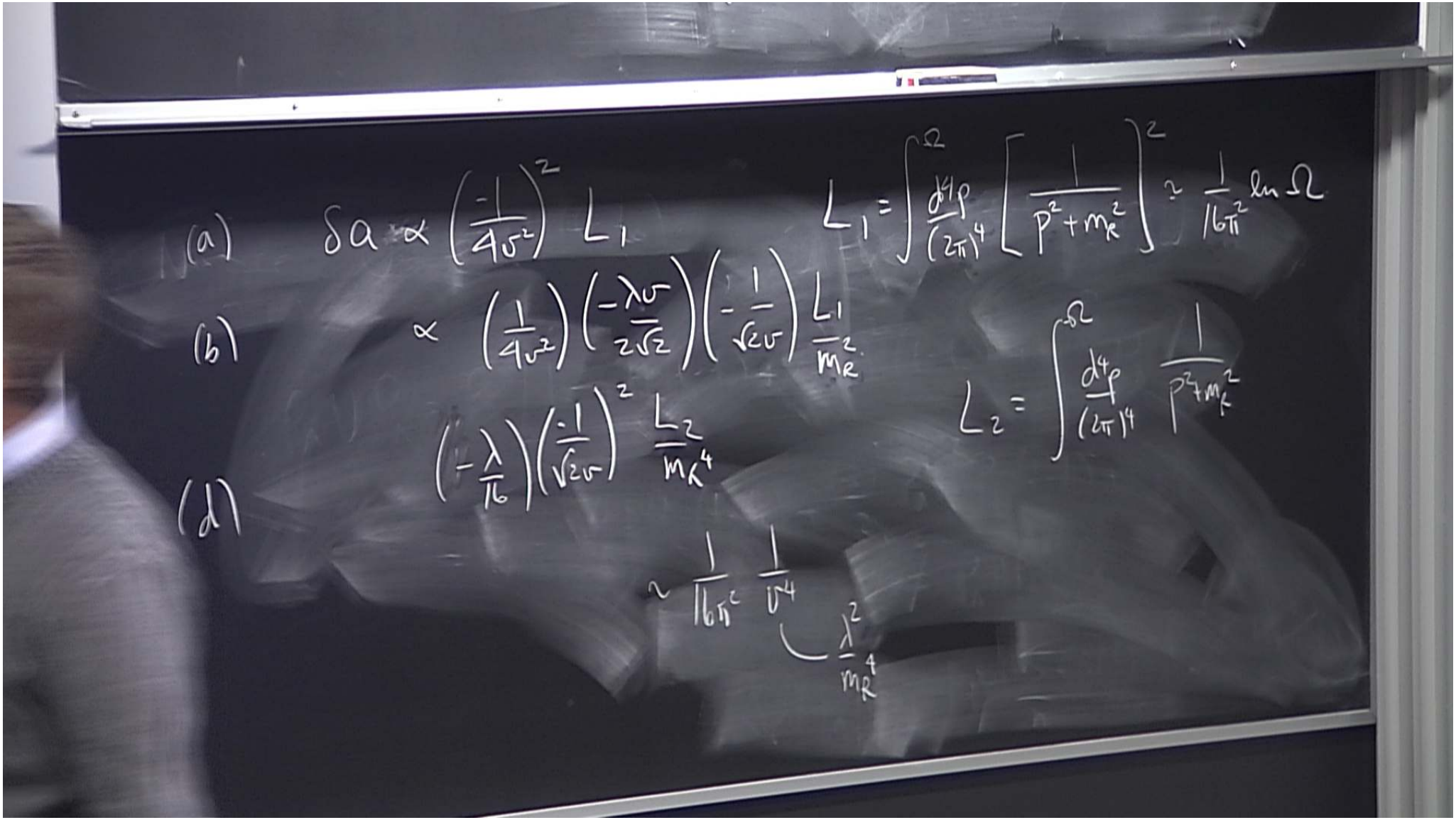
To see a difference between $T(\ell)$ and $S_W(\ell)$ must go beyond classical approx:

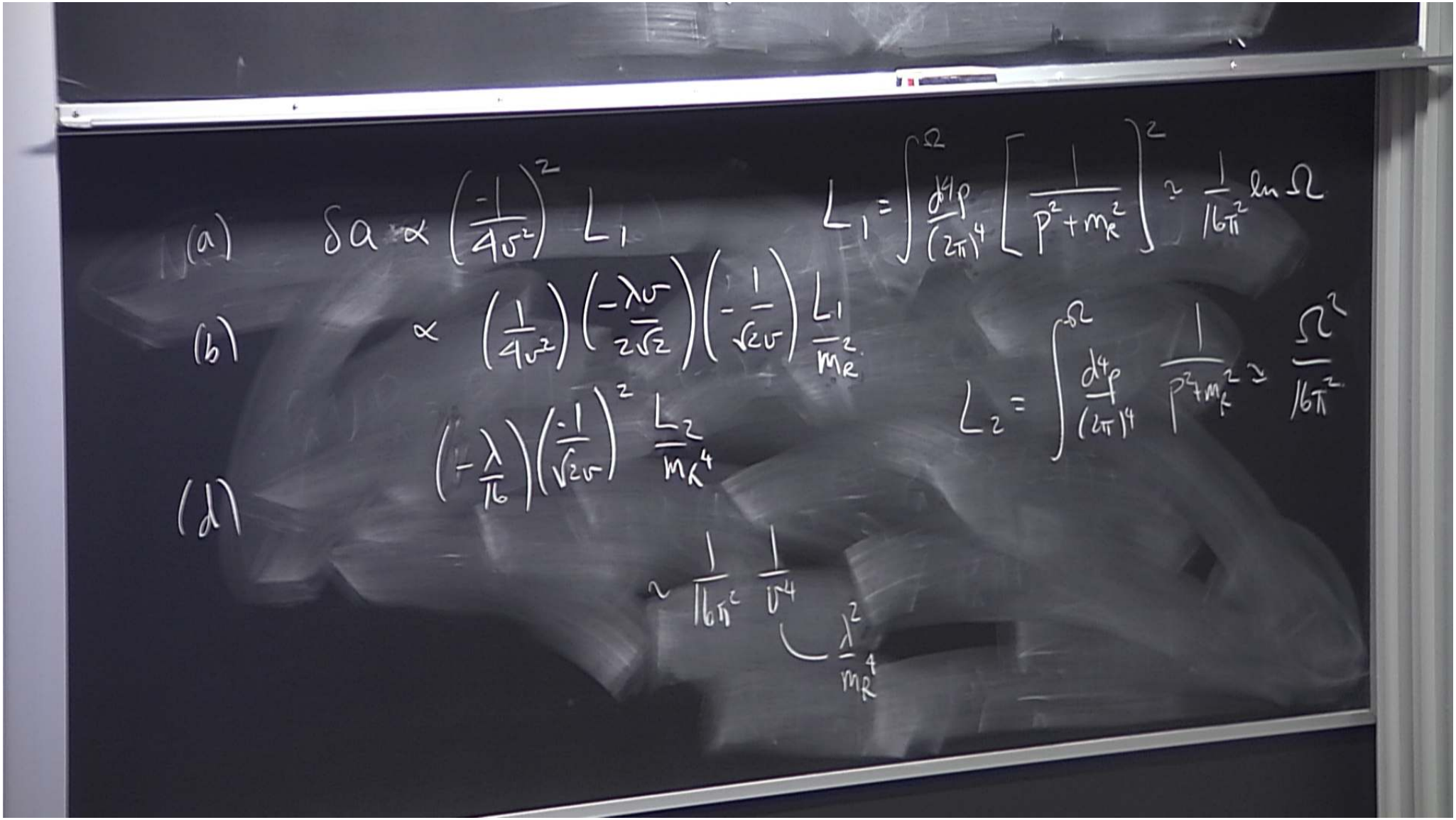
Compute 1-loop correction to $a(\partial_\mu \xi \partial^\mu \xi)^2$

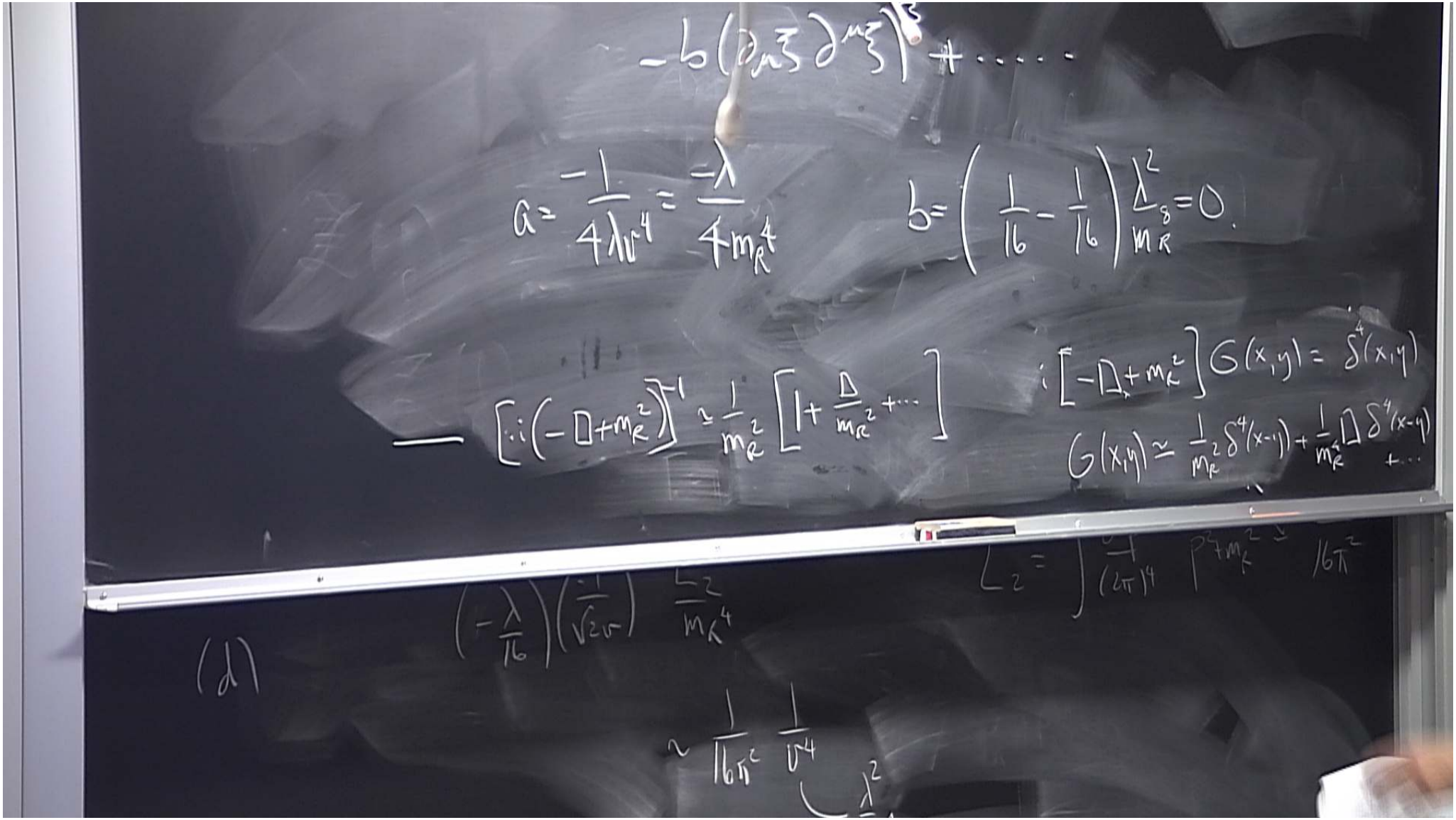

$\leftrightarrow -\frac{\chi}{\sqrt{2}\epsilon}\partial_\mu\xi\partial^\mu\xi$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)

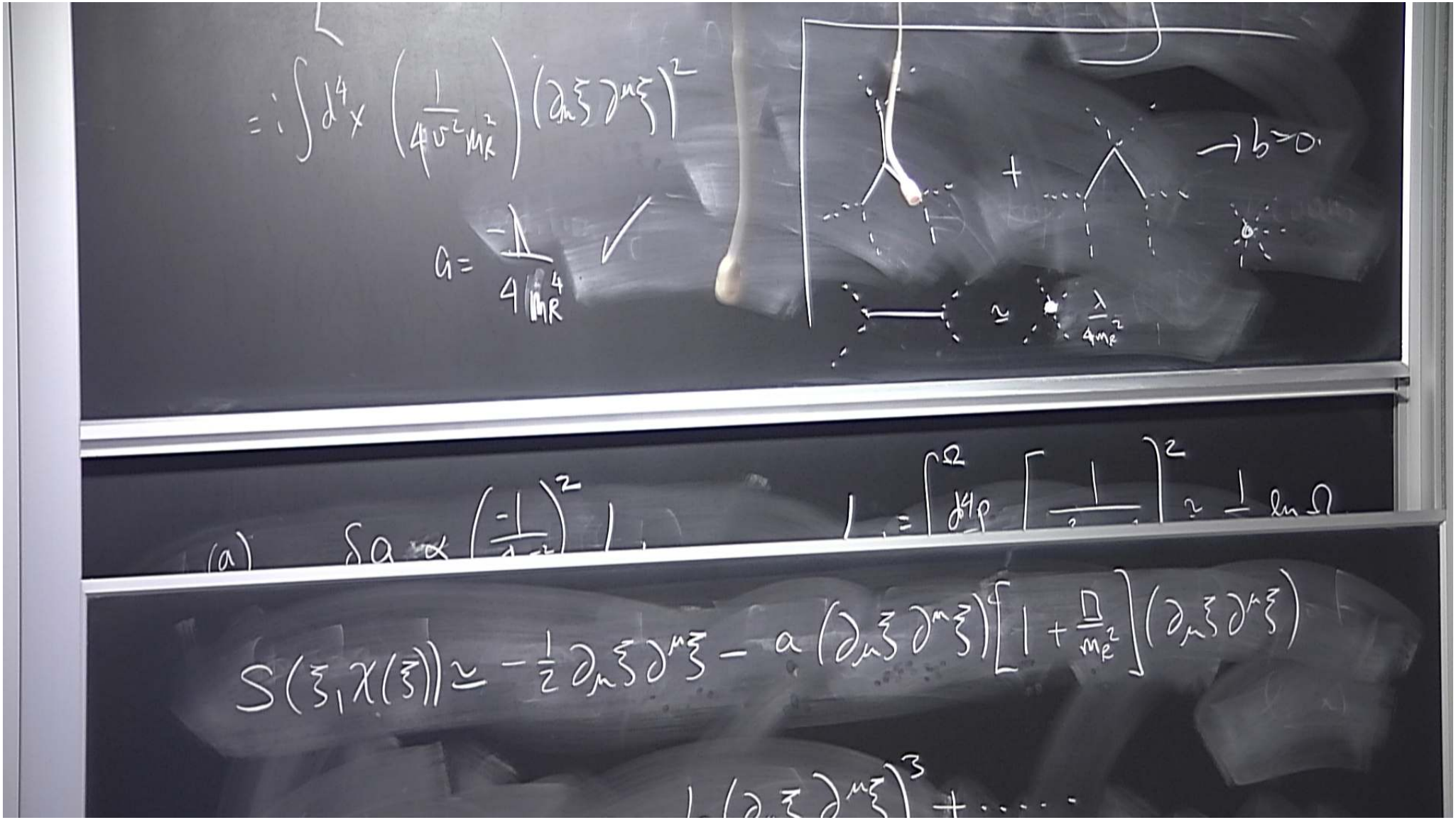

(a) $\delta a \propto \left(\frac{-1}{4v^2}\right)^2 L_1$
$L_1 = \int^{\Omega} \frac{d^4p}{(2\pi)^4} \left[\frac{1}{p^2+m_R^2}\right]^2 \simeq \frac{1}{16\pi^2} \ln \Omega$
(b) $\propto \left(\frac{1}{4v^2}\right)\left(\frac{-\lambda v}{2\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}v}\right)\frac{L_1}{m_R^2}$

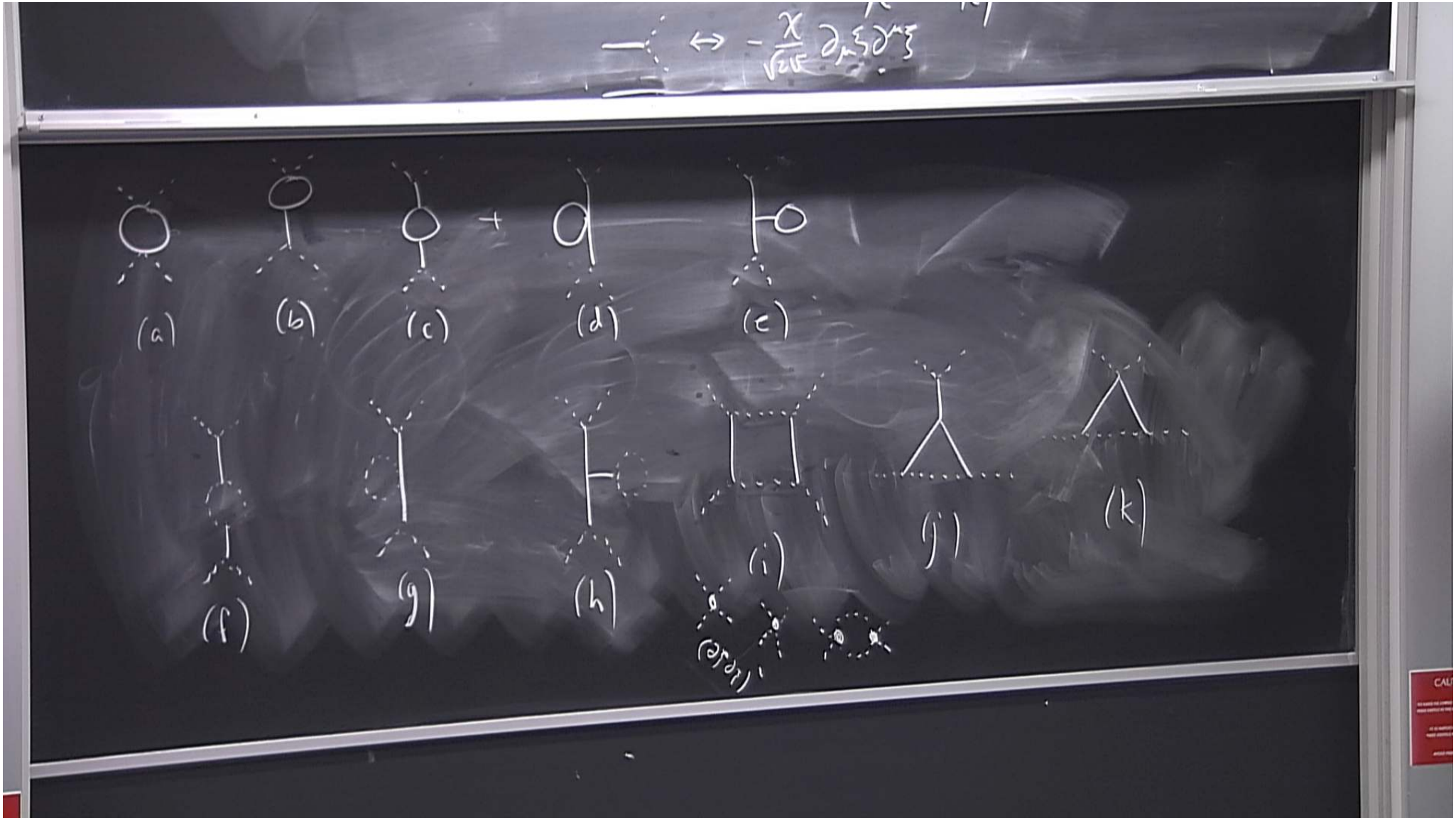

(a) $\delta a \propto \left(\frac{-1}{4v^2}\right)^2 L_1$
$L_1 = \int^{\Omega} \frac{d^4p}{(2\pi)^4} \left[\frac{1}{p^2+m_R^2}\right]^2 \simeq \frac{1}{16\pi^2} \ln \Omega$
(b) $\propto \left(\frac{1}{4v^2}\right)\left(\frac{-\lambda v}{2\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}v}\right)\frac{L_1}{m_R^2}$
$\sim \frac{1}{16\pi^2}\frac{1}{v^4}$
$\sim \frac{\lambda^2}{m_R^4}$

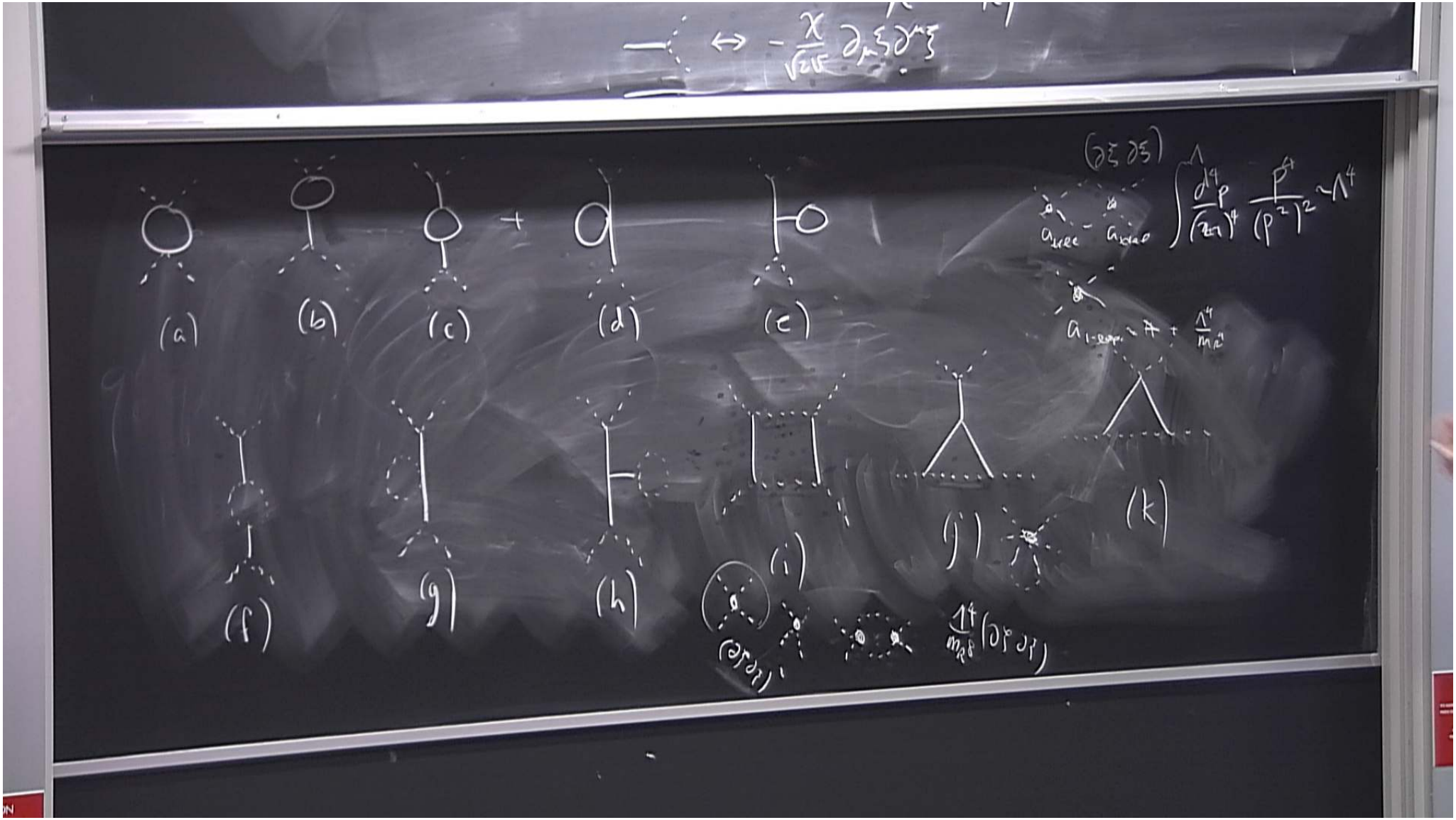

(a) $\delta a \propto \left(\frac{-1}{4v^2}\right)^2 L_1$
$L_1 = \int^{\Omega} \frac{d^4p}{(2\pi)^4} \left[\frac{1}{p^2+m_R^2}\right]^2 \simeq \frac{1}{16\pi^2} \ln \Omega$
(b) $\propto \left(\frac{1}{4v^2}\right)\left(\frac{-\lambda v}{2\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}v}\right) \frac{L_1}{m_R^2}$
$L_2 = \int^{\Omega} \frac{d^4p}{(2\pi)^4} \frac{1}{p^2+m_R^2}$
(d) $\left(\frac{-\lambda}{16}\right)\left(\frac{-1}{\sqrt{2}v}\right)^2 \frac{L_2}{m_R^4}$
$\sim \frac{1}{16\pi^2} \frac{1}{v^4}$
$\frac{\lambda^2}{m_R^4}$

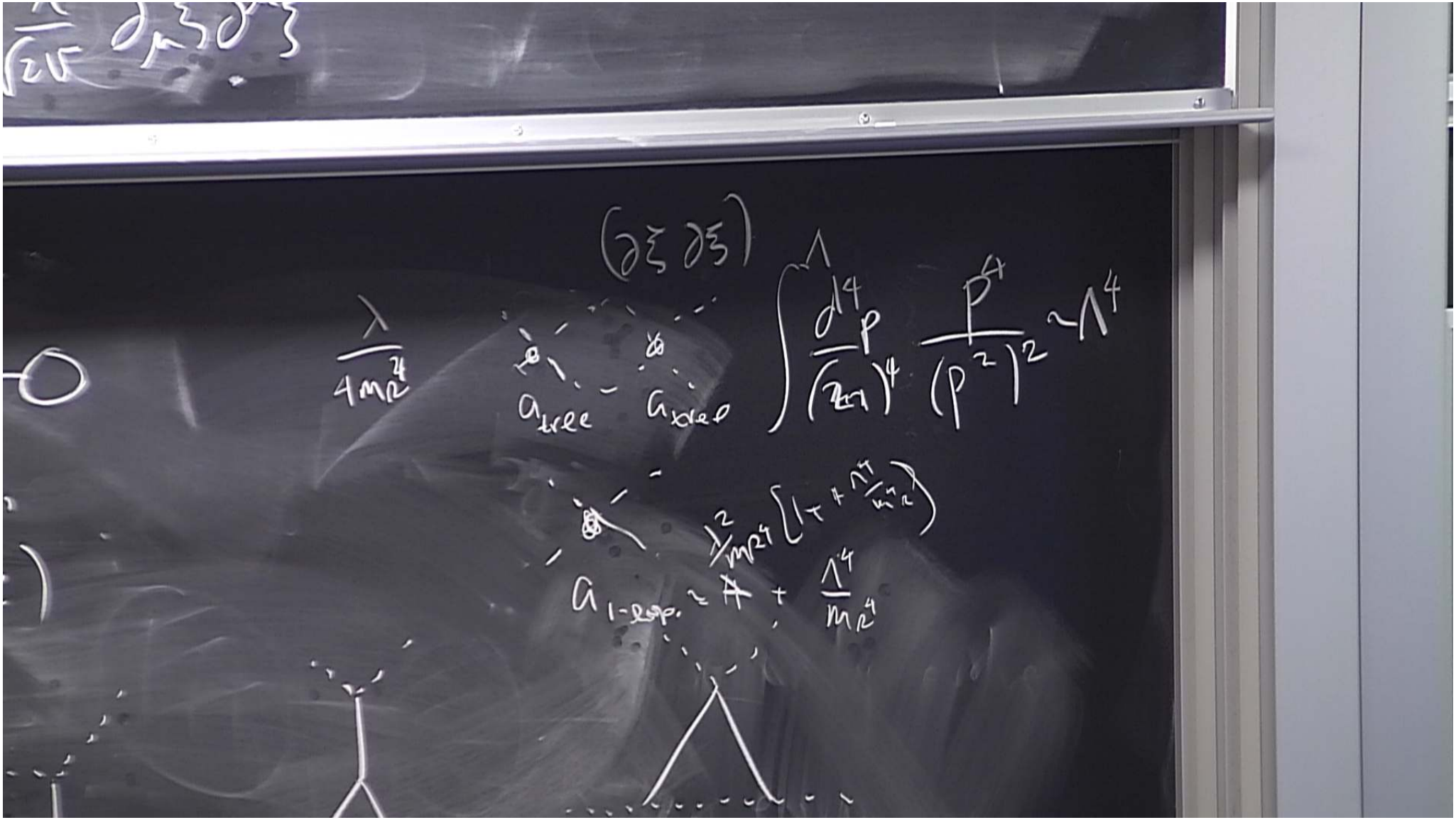

(a) $\delta a \propto \left(\frac{-1}{4v^2}\right)^2 L_1$
$L_1 = \int^{\Omega} \frac{d^4p}{(2\pi)^4} \left[\frac{1}{p^2+m_R^2}\right]^2 \simeq \frac{1}{16\pi^2} \ln \Omega$
(b) $\propto \left(\frac{1}{4v^2}\right)\left(\frac{-\lambda v}{2\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}v}\right) \frac{L_1}{m_R^2}$
$L_2 = \int^{\Omega} \frac{d^4p}{(2\pi)^4} \frac{1}{p^2+m_R^2} \simeq \frac{\Omega^2}{16\pi^2}$
(d) $\left(-\frac{\lambda}{16}\right)\left(\frac{-1}{\sqrt{2}v}\right)^2 \frac{L_2}{m_R^4}$
$\sim \frac{1}{16\pi^2} \frac{1}{v^4}$
$\frac{\lambda^2}{m_R^4}$


$-b(\partial_\mu \xi \partial^\mu \xi)^2 + \cdots$
$a = \frac{-1}{4\lambda v^4} = \frac{-\lambda}{4 m_R^4}$
$b = \left(\frac{1}{16} - \frac{1}{16}\right)\frac{\lambda^2}{m_R^8} = 0$
$i\left[i(-\Box + m_R^2)\right]^{-1} = \frac{1}{m_R^2}\left[1 + \frac{\Box}{m_R^2} + \cdots\right]$
$i\left[-\Box_x + m_R^2\right] G(x,y) = \delta^4(x,y)$
$G(x,y) \simeq \frac{1}{m_R^2}\delta^4(x-y) + \frac{1}{m_R^4}\Box\delta^4(x-y) + \cdots$
(d)
$\left(-\frac{\lambda}{16}\right)\left(\frac{1}{\sqrt{2}v}\right)\frac{L_2}{m_R^4}$
$L_2 = \int \frac{1}{(2\pi)^4} \frac{1}{p^2 + m_R^2} \frac{1}{16\pi^2}$
$\sim \frac{1}{16\pi^2}\frac{1}{v^4}\lambda^2$


$= i\int d^4x \left(\frac{1}{4v^2 m_R^2}\right)(\partial_\mu\xi\,\partial^\mu\xi)^2$
$a = \frac{-1}{4 m_R^4}$
$\to b > 0$
$\simeq \frac{\lambda}{4m_R^2}$
(a) $\delta a \propto \left(\frac{-1}{4m_R^2}\right)^2 I$
$I = \int^\Omega d^4p \left[\frac{1}{p^2}\right]^2 \simeq \frac{1}{2}\ln\Omega$
$S(\xi, \chi(\xi)) \simeq -\frac{1}{2}\partial_\mu\xi\,\partial^\mu\xi - a(\partial_\mu\xi\,\partial^\mu\xi)\left[1 + \frac{\Box}{m_R^2}\right](\partial_\mu\xi\,\partial^\mu\xi)$
$+ b(\partial_\mu\xi\,\partial^\mu\xi)^3 + \ldots$


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)


$\frac{\lambda}{4m_R^4}$
a_{tree}
a_{tree}
$\int \frac{d^4p}{(2\pi)^4} \frac{p^4}{(p^2)^2} \sim \Lambda^4$
$a_{1\text{-loop}} \simeq$
$+ \frac{\Lambda^4}{m_R^4}$