

Title: Introduction to Effective Field Theories - Lecture 2

Date: Jan 10, 2014 02:30 PM

URL: <http://pirsa.org/14010091>

Abstract:

"EFT"

Physica A96 (1979) 327

H. Georgi [www.people.fas.harvard.edu/~hgeorgi/review.pdf](http://www.people.fas.harvard.edu/~hgeorgi/review.pdf)

A. Manohar "EFT's" [hep-ph/9606222](http://hep-ph/9606222)

D.B. Kaplan [nucl-th/0510023](http://nucl-th/0510023)



D.B. Kaplan  $\text{nucl-th}/0510023$

A. Pich "EFT"  $\text{hep-ph}/9806303$

CB "Intro to EFT"  $\text{hep-th}/0701053$

Polchinski: "EFT + the Fermi Surface"  $\text{hep-th}/9210046$



D.B. Kaplan  $\text{nucl-th}/0510023$

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CB "Intro to EFT"  $\text{hep-th}/0701053$

Polchinski: "EFT + the Fermi Surface"  $\text{hep-th}/9210046$

I. Rothstein "TASI Lectures on EFT"  $\text{hep-ph}/0308266$

Donoghue "Intro to EFT description of gravity"  $\text{gr-qc}/9512024$

CB "QG in Everyday Life, GR as an EFT"  $\text{gr-qc}/0311082$



$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$$

$$\phi \rightarrow \phi e^{i\omega}$$

$\omega \text{ const}$

$$\hat{\phi} = \hat{\phi}_R + i \hat{\phi}_I = v + \hat{\psi}$$

$$V = \frac{\lambda}{4} (\phi^* \phi - v^2)^2$$

$$\lambda \gg 1$$

$$v \approx \frac{\mu}{\sqrt{\lambda}}$$



$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$$

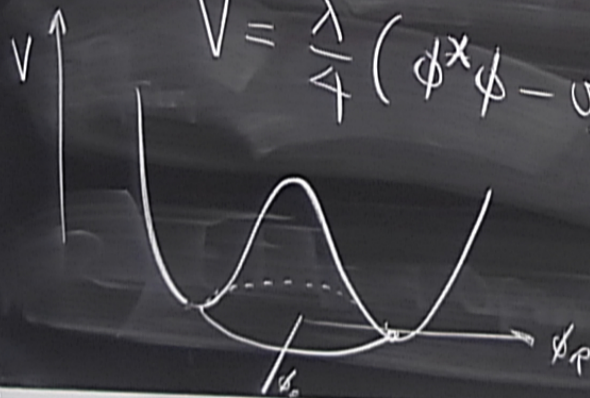
$$\phi \rightarrow \phi e^{i\omega}$$

$\omega \text{ const}$

$$\hat{\phi} = \hat{\phi}_R + i \hat{\phi}_I = v + \hat{\psi}$$

$$V = \frac{\lambda}{4} (\phi^* \phi - v^2)^2$$

$$\lambda \ll 1 \quad v \approx \frac{\mu}{\sqrt{\lambda}}$$



CAUTION  
DO NOT TOUCH THE BOARD OR THE BOARDER.  
IF YOU REMOVED THE BOARDER,  
THE BOARDER WILL BE DAMAGED.  
DO NOT TOUCH THE BOARDER.



$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$$

$$\phi \rightarrow \phi e^{i\omega t}$$

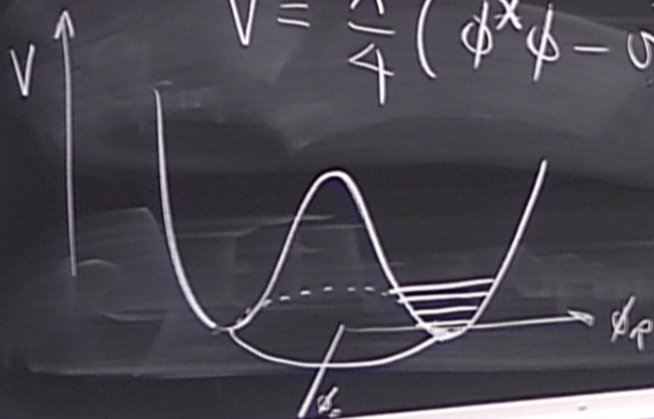
$\omega \text{ const}$

$$\hat{\phi} = \frac{\hat{\phi}_R + i \hat{\phi}_I}{\sqrt{2}} = v + \hat{\psi} = v + \frac{\psi_R + i \psi_I}{\sqrt{2}}$$

$$V = \frac{\lambda}{4} (\phi^* \phi - v^2)^2$$

$$\lambda \ll 1$$

$$v \simeq \frac{\mu}{\sqrt{\lambda}}$$





$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_0 = -\frac{1}{2} \partial_\mu \psi_R \partial^\mu \psi_R - \frac{1}{2} \partial_\mu \psi_I \partial^\mu \psi_I - \frac{\lambda v^2}{2} \psi_R^2$$

$$\mathcal{L}_{\text{int}} = -\frac{\lambda v}{2\sqrt{2}} \psi_R (\psi_R^2 + \psi_I^2) - \frac{\lambda}{16} (\psi_R^2 + \psi_I^2)^2$$



$$H = \sum_k \left[ a_k^\dagger a_k \epsilon_k + b_k^\dagger b_k \right]$$

$$\psi_R = E_k = \sqrt{k^2 + \lambda \sigma^2}$$

$$m_R^2 = \lambda \sigma^2 \quad \text{massive}$$

$\psi_I$

$$\epsilon_k = \sqrt{k^2 + 0} = |k|$$

massless.



A. Manshary, EFTs, hep-th/0503032

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$$\frac{\vec{p}}{\dots} \quad \begin{matrix} \psi_R \\ \psi_L \end{matrix}$$

$$\frac{-i}{p^2 + m^2 - i\epsilon}$$

$$p^2 = p_\mu p^\mu = \eta_{\mu\nu} p^\mu p^\nu = -E^2 + \vec{p}^2$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$$

$$p^\mu = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$$



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$$\overrightarrow{p} \quad \psi_R$$

$$\overrightarrow{p} \quad \psi_I$$

$$\frac{-i}{p^2 + m^2 - i\epsilon}$$

$$\frac{-i}{p^2 - i\epsilon}$$

$$p^2 = p_\mu p^\mu = \eta_{\mu\nu} p^\mu p^\nu = -E^2 + \vec{p}^2$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & \ddots \end{pmatrix}$$

$$p^\mu = \begin{pmatrix} E \\ \vec{p} \end{pmatrix}$$

$$\psi_R^3$$



$$\rightarrow \left( -\frac{\lambda \gamma}{2\sqrt{2}} \right)$$

$$\psi_R \psi_I^2$$



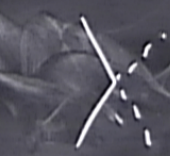
$$\psi_R^4$$



$$\psi_I^4$$



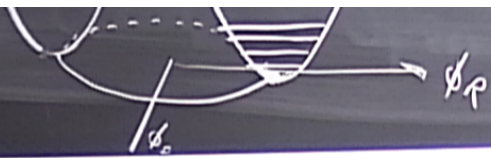
$$\left( -\frac{\lambda}{16} \right)$$



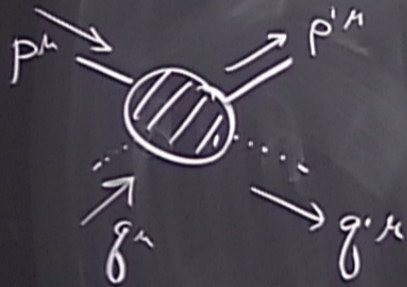
$$\psi_R^2 \psi_I^2$$

$$\left( -\frac{\lambda}{8} \right)$$





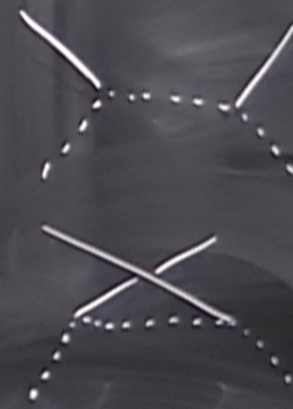
$$\psi_I(q) + \psi_R(p) \rightarrow \psi_I(q') + \psi_R(p')$$



+



+





$$A \sim \int \mathcal{D}\phi e^{iS_0 + iS_{\text{int}}} = \int \mathcal{D}\phi e^{iS_0} \left[ 1 + iS_{\text{int}} + \frac{1}{2} S_{\text{int}}^2 + \frac{i}{3!} S_{\text{int}}^3 + \dots \right]$$



Donoghue "Intro to EFT description of gravity" gr-qc/9512024  
(\*) CB "QG in Everyday Life, GR as an EFT" gr-qc/0311082

$$A(\psi_R(p) + \psi_I(q) \rightarrow \psi_R(p') + \psi_I(q')) = (2\pi)^4 \delta^4(p+q-p'-q') A.$$

$$A \propto \left[ i \left( -\frac{\lambda}{8} \right) \right]$$



Donoghue "Intro to EFT description of gravity" gr-qc/9512124

$$A(\psi_R(p) + \psi_I(q) \rightarrow \psi_R(p') + \psi_I(q')) = (2\pi)^4 \delta^4(p+q-p'-q') A.$$

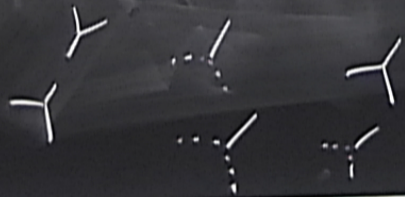
$$A \propto \left[ i \left( -\frac{\lambda}{8} \right) 4 \right]$$



$$H = \sum_k \left[ a_k^\dagger a_k E_k + b_k^\dagger b_k \epsilon_k + \text{const} \right]$$

$$A \sim \int \mathcal{D}\phi e^{iS_0 + iS_{\text{int}}} = \int \mathcal{D}\phi e^{iS_0} \left[ 1 + iS_{\text{int}} + \frac{i^2}{2} S_{\text{int}}^2 + \frac{i^3}{6} S_{\text{int}}^3 + \dots \right]$$

$$S_{\text{int}}^2 = \left( \psi_R^3 + \psi_R \psi_I^2 \right)^2$$





$$A \propto \left[ i \left( -\frac{\lambda}{8} \right) 4 + \frac{i^2}{2} \left( \frac{-\lambda 5}{2\sqrt{2}} \right)^2 \frac{-i(2 \times 2 \times 3 \times 2)}{(q-q')^2 + M_R^2 - i\epsilon} \right]$$

$$+ \frac{i^2}{2} \left( \frac{-\lambda 5}{2\sqrt{2}} \right)^2 \frac{-i( )}{(p+q)^2 - i\epsilon}$$

$$\psi_R^4 \quad \times \quad ; \quad \psi_I^4 \quad \times \quad \left( -\frac{\lambda}{16} \right) \quad \times \quad \psi_R^2 \psi_I^2 \quad \left( -\frac{\lambda}{8} \right)$$

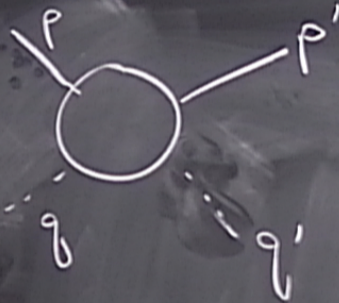


$$\begin{aligned}
 & (8) \quad \left[ \frac{i^2 \left( \frac{-15}{2\sqrt{2}} \right)^2 - i(2 \times 2 \times 2)}{(p+q)^2 - i\epsilon} + \frac{i^2 \left( \frac{-15}{2\sqrt{2}} \right)^2 - i(2+2+2)}{(p-q)^2 - i\epsilon} \right]
 \end{aligned}$$



$$\begin{aligned}
 A \propto & \left\{ -\frac{i\lambda}{2} + \frac{i\lambda^2 u^2}{2} \left[ \frac{3}{(q-q')^2 + m_R^2} + \frac{1}{(p+q)^2} + \frac{1}{(p-q')^2} \right] \right\} \\
 = & +\frac{i}{2} \left\{ \lambda + \underbrace{\lambda^2 u^2}_{\substack{O(1) \\ \text{BCS } u \sim \frac{h}{\sqrt{A}}}} \left[ \frac{3}{(q-q')^2 + m_R^2} + \frac{1}{(p+q)^2} + \frac{1}{(p-q')^2} \right] \right\}
 \end{aligned}$$

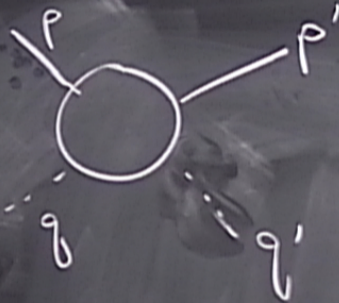




$$p^2 = m_R^2 = p'^2 \quad E_p = \sqrt{\vec{p}^2 + m_R^2}$$

$$q^2 = q'^2 = 0 \quad E_q = |\vec{q}|$$





$$p^2 = -m_R^2 = p'^2 \quad E_p = \sqrt{\vec{p}^2 + m_R^2}$$

$$q^2 = q'^2 = 0 \quad E_q = |\vec{q}|$$

$$(q - q')^2 = -2q \cdot q'$$

$$(p + q)^2 = p^2 + 2p \cdot q + q^2 = -m_R^2 + 2p \cdot q$$

$$(p - q')^2 = p^2 - 2p \cdot q' + q'^2 = -m_R^2 - 2p \cdot q'$$



$$\psi(q) + \psi(p) \rightarrow \psi_T(q') + \psi_R(p')$$

$$A \approx -\frac{i\lambda}{2} + \frac{i\lambda^2 \omega^2}{2m_R^2} \left[ \frac{3}{1 - 2q \cdot q' / m_R^2} - \frac{1}{1 - 2p \cdot q / m_R^2} - \frac{1}{1 + 2p \cdot q' / m_R^2} \right]$$

$$\left( \frac{q \cdot q'}{m_R^2} \right), \frac{p \cdot q'}{m_R^2}, \frac{p \cdot q}{m_R^2} \ll 1$$

$$A \rightarrow -\frac{i\lambda}{2} + \frac{i\lambda^2 \omega^2}{2m_R^2} = 0$$



$$\psi_T(q) + \psi_e(p) \rightarrow \psi_T(q') + \psi_e(p')$$

$$A \approx -\frac{i\lambda}{2} + \frac{i\lambda^2}{2m_R^2} \left[ \frac{3}{1-2q \cdot q'/m_R^2} - \frac{1}{1-2p \cdot q/m_R^2} - \frac{1}{1+2p \cdot q'/m_R^2} \right]$$

$$\left(\frac{q \cdot q'}{m_R^2}\right), \frac{p \cdot q'}{m_R^2}, \frac{p \cdot q}{m_R^2} \ll 1 \quad A \rightarrow -\frac{i\lambda}{2} + \frac{i\lambda^2}{2m_R^2} = 0$$

$$A \approx 2i\lambda \left( \frac{q \cdot q'}{m_R^2} \right) + \mathcal{O}\left(\frac{1}{m_R^4}\right)$$



$$H = \sum_k \left[ a_k^\dagger a_k \epsilon_k + b_k^\dagger b_k \epsilon_k \right]$$

$$A(\psi_I(p) + \psi_I(q) \rightarrow \psi_I(p') + \psi_I(q'))$$



$$H = \sum_k \left[ \alpha_k^* \alpha_k \epsilon_k + \beta_k \right]$$

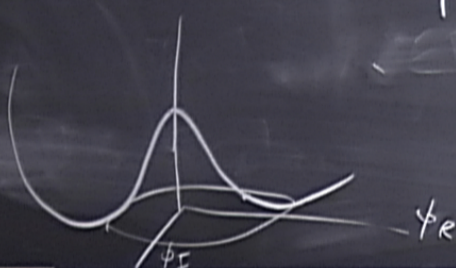


$$A(\psi_I(p) + \psi_I(q) \rightarrow \psi_I(p') + \psi_I(q')) \propto \left[ -\frac{3i\lambda}{2} + \frac{i(\lambda v)^2}{2m_R^2} \left[ \frac{1}{1 + 2p \cdot q / m_R^2} + \frac{1}{1 - 2q \cdot q' / m_R^2} + \frac{1}{1 - 2p \cdot q' / m_R^2} \right] \right]$$



$$A = \lambda \left( \frac{E}{m_R} \right)^4 + \underbrace{\lambda^2 \left( \frac{E}{m_R} \right)^4}_{1\text{-loop}} + \lambda^3 (\#)^2$$



$$+ \frac{i^2}{2} \left( \frac{-\lambda v}{2\sqrt{2}} \right)^2 \frac{-i(2+2+2)}{(p+q)^2 - i\epsilon} + \frac{i^2}{2} \left( \frac{-\lambda v}{2\sqrt{2}} \right)^2 \frac{-i(2+2+2)}{(p-q)^2 - i\epsilon}$$


A Better way to analyse this system.

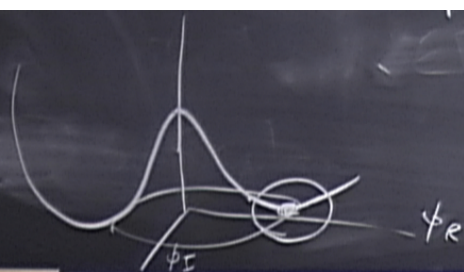
$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda}{4} (\phi^* \phi - v^2)^2$$

$$\phi \rightarrow \phi e^{i\omega}$$

$$\phi = \frac{\phi_R + i\phi_I}{\sqrt{2}}$$

$$\begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix} \rightarrow \begin{pmatrix} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix}$$





A Better way to analyse this system.

$$\mathcal{L} = -\partial_\mu \phi^\dagger \partial^\mu \phi - \frac{\lambda}{4} (\phi^\dagger \phi - v^2)^2$$

$$\phi \rightarrow \phi e^{i\omega}$$

$$\phi = \frac{\phi_R + i\phi_I}{\sqrt{2}}$$

$$\psi_R, \psi_I$$

$$\psi_R = \phi_R - \frac{v}{\sqrt{2}}$$

$$\begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix} \rightarrow \begin{pmatrix} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix}$$

"linear realization"



A Better way to analyse this system.

$$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - \frac{\lambda}{4} (\phi^* \phi - v^2)^2$$

$$\boxed{\phi \rightarrow \phi e^{i\omega}}$$

$$\phi = \frac{\phi_R + i\phi_I}{\sqrt{2}}$$

$$\begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix} \rightarrow \begin{pmatrix} \cos \omega & \sin \omega \\ -\sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} \phi_R \\ \phi_I \end{pmatrix}$$

$$\psi_R, \psi_I$$

$$\psi_R = \phi_R - \frac{v}{\sqrt{2}}$$

"linear realization"



$$\phi = \left( v + \frac{p}{\sqrt{2}} \right) e^{i\tilde{\zeta}}$$

$$\tilde{\zeta} \rightarrow \tilde{\zeta} + \omega$$

$$\rho \rightarrow \rho$$

$$\phi^* \phi = \left( v + \frac{p}{\sqrt{2}} \right)^2$$

$$\phi^* \phi - v^2 = \sqrt{2} \rho v + \frac{\rho^2}{2}$$

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} \partial_\mu \rho \partial^\mu \rho - \left( v + \frac{p}{\sqrt{2}} \right)^2 \partial_\mu \tilde{\zeta} \partial^\mu \tilde{\zeta} - \frac{\lambda}{4} \left( \sqrt{2} \rho v + \frac{\rho^2}{2} \right)^2 \\ &= \mathcal{L}_0 + \mathcal{L}_{int} \end{aligned}$$

$$\mathcal{L}_0 = -\frac{1}{2} \partial_\mu \rho \partial^\mu \rho - v^2 \partial_\mu \tilde{\zeta} \partial^\mu \tilde{\zeta} - \frac{\lambda v^2}{2} \rho^2$$



$$\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

$$\phi = \left( v + \frac{\rho}{\sqrt{2}} \right) e^{i\tilde{\zeta}}$$

$$\tilde{\zeta} \rightarrow \tilde{\zeta} + \omega$$

$$\rho \rightarrow \rho$$

$$\phi^* \phi = \left( v + \frac{\rho}{\sqrt{2}} \right)^2$$

$$\phi^* \phi - v^2 = \sqrt{2} \rho v + \frac{\rho^2}{2}$$

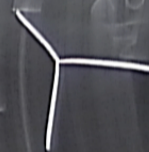
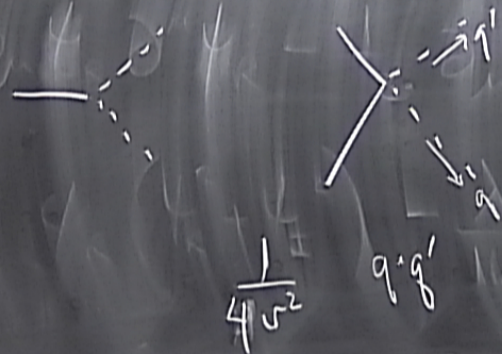
$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} \partial_\mu \rho \partial^\mu \rho - \left( v + \frac{\rho}{\sqrt{2}} \right)^2 \partial_\mu \tilde{\zeta} \partial^\mu \tilde{\zeta} - \frac{\lambda}{4} \left( \sqrt{2} \rho v + \frac{\rho^2}{2} \right)^2 \\ &= \mathcal{L}_0 + \mathcal{L}_{int} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_0 &= -\frac{1}{2} \partial_\mu \rho \partial^\mu \rho - v^2 \partial_\mu \tilde{\zeta} \partial^\mu \tilde{\zeta} - \frac{\lambda v^2}{2} \rho^2 \\ &\quad - \frac{1}{2} \partial_\mu \tilde{\zeta} \partial^\mu \tilde{\zeta} \end{aligned}$$

$$\tilde{\zeta} = \frac{\zeta}{v\sqrt{2}}$$



$$\mathcal{L}_{\text{int}} = -\frac{1}{2} \left( \sqrt{2} \frac{p}{g} + \frac{p^2}{2v^2} \right) \underline{\partial_\mu \phi \partial^\mu \phi} - \frac{1}{4} \left( \sqrt{2} p^3 + \frac{p^4}{4} \right)$$



$\sim E^2$



$\sim E^4$



$$\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2$$

$$-\frac{1}{2} \partial_\mu \psi \partial^\mu \psi$$

$$A = \lambda \left( \frac{c}{m_R} \right) + \underbrace{\lambda \left( \frac{c}{m_R} \right)}_{1\text{-loop}} + \lambda (\#)$$

Claim: <sup>same</sup>  $A(\psi_I \psi_I \rightarrow \psi_I \psi_I)$  comes from

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \psi_I \partial^\mu \psi_I + \frac{\lambda}{4m_R^4} (\partial_\mu \psi_I \partial^\mu \psi_I)^2$$