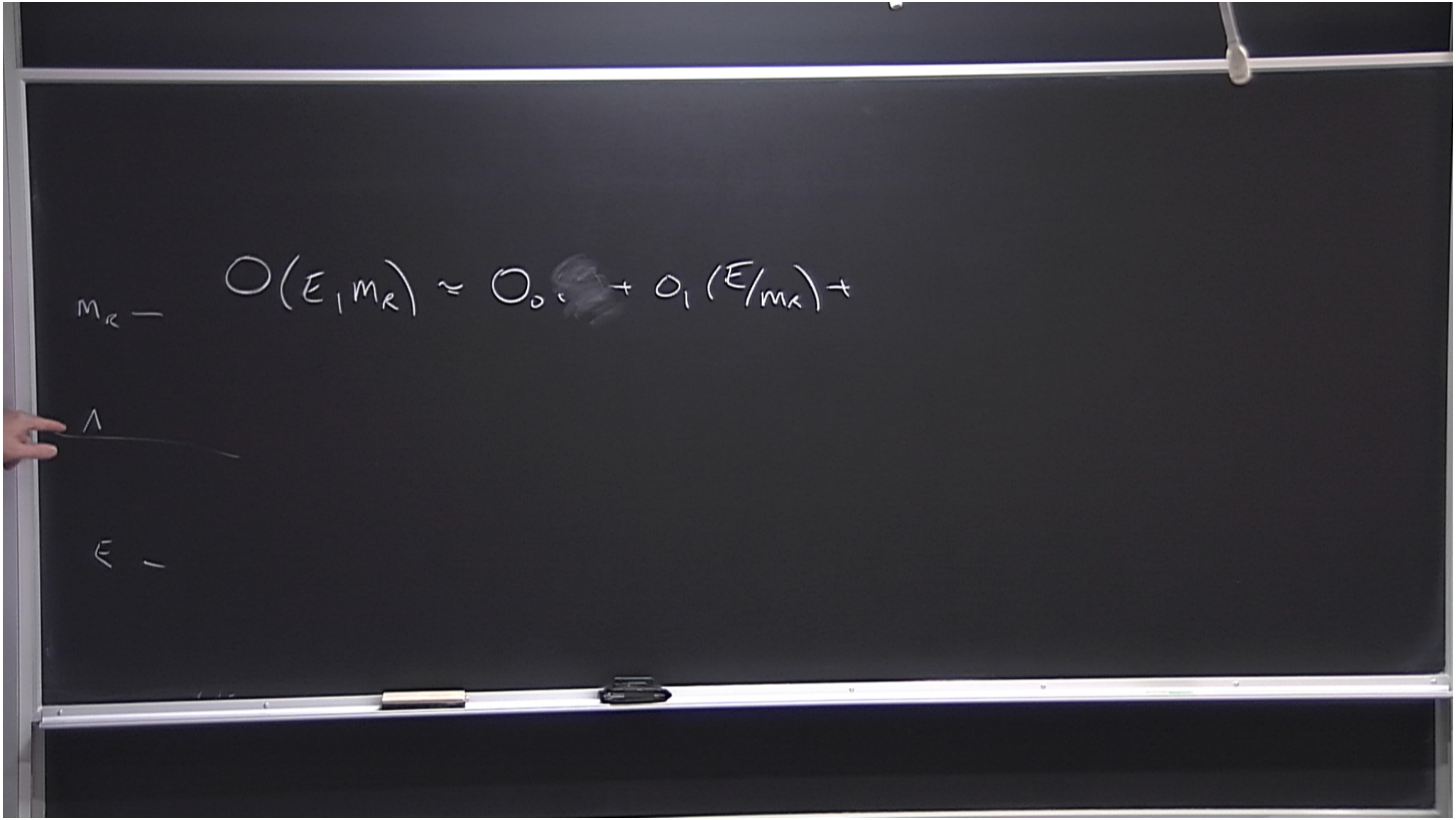
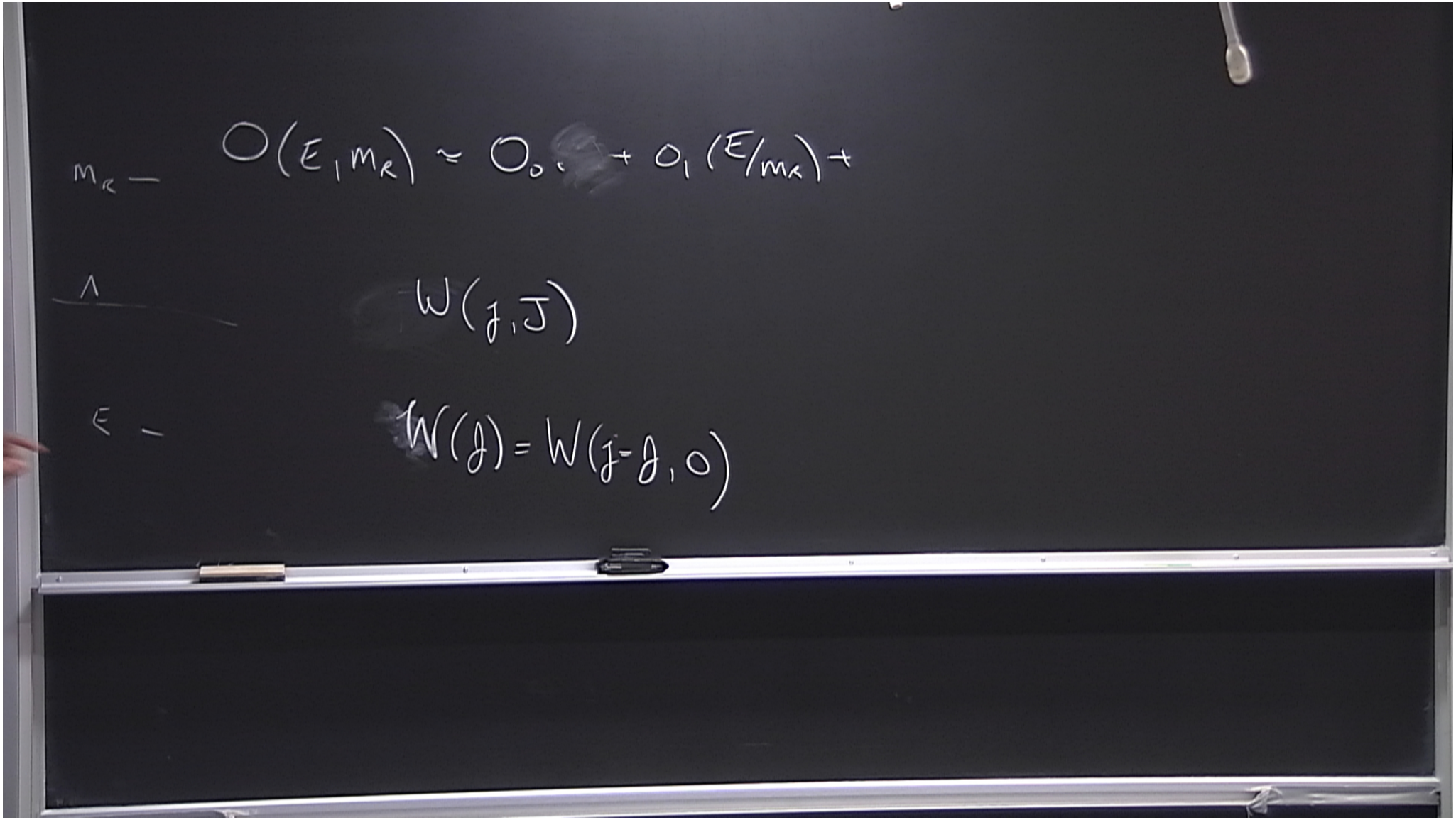
Title: Introduction to Effective Field Theories - Lecture 7

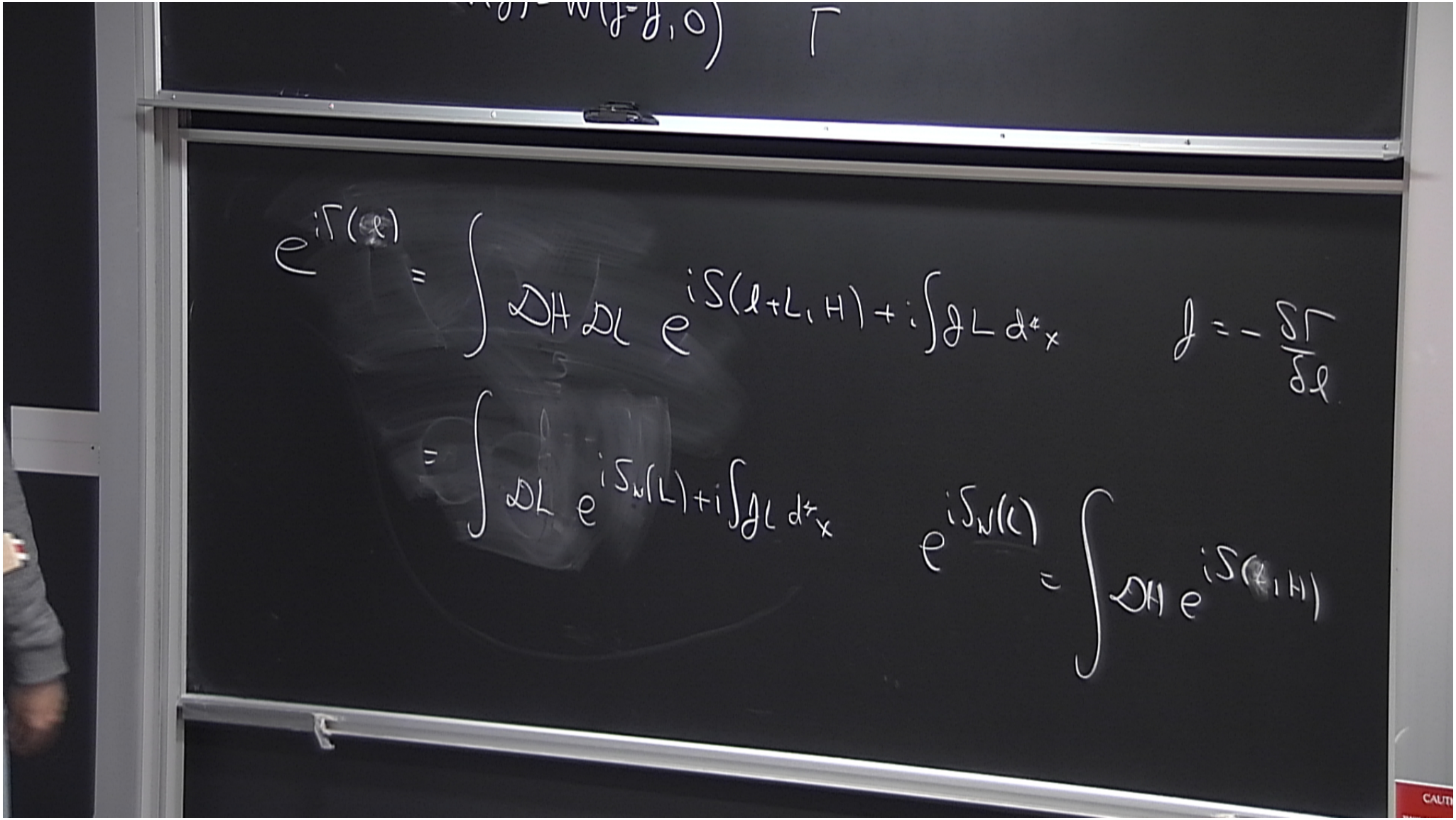
Date: Jan 31, 2014 09:00 AM

URL: http://pirsa.org/14010090

Abstract:


$O(E, m_R) \approx O_0 + O_1(E/m_R) +$
m_R
Λ
E

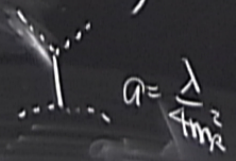

m_R —
$O(E, m_R) \approx O_0 \cdot$ + $O_1 (E/m_R)$ +
Λ
$W(J, J)$
ϵ —
$W(J) = W(J-J, 0)$


$W(J,0)$
Γ
$e^{i\Gamma(\ell)} = \int \mathcal{D}H\,\mathcal{D}L\; e^{iS(\ell+L,H)+i\int J L\, d^4x}$
$J = -\frac{\delta\Gamma}{\delta\ell}$
$= \int \mathcal{D}L\; e^{iS_W(L)+i\int J L\, d^4x}$
$e^{iS_W(L)} = \int \mathcal{D}H\, e^{iS(L,H)}$

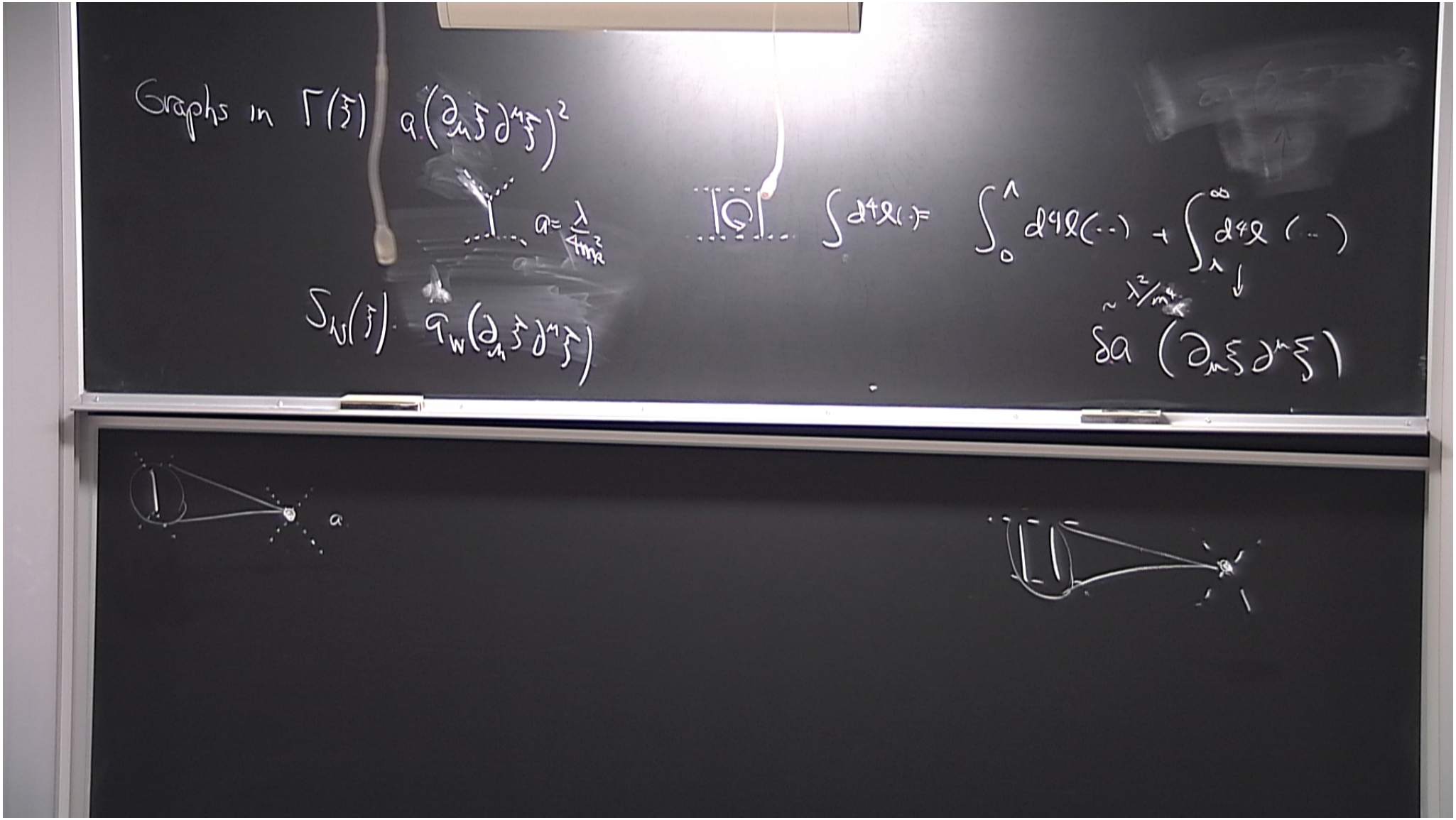
$$\mathcal{L} = -\frac{1}{2}\partial_\mu X \partial^\mu X - \frac{1}{2}\left(1+\frac{X}{\sqrt{2}v}\right)^2 \partial_\mu \xi \partial^\mu \xi - V(X)$$

$$V = \frac{\lambda}{4}\left(\sqrt{2}vX + \frac{X^2}{2}\right)^2$$

Graphs in $\Gamma(\xi)$: $a(\partial_\mu \xi \partial^\mu \xi)^2$



$S_W(\xi)$: $a_W(\partial_\mu \xi \partial^\mu \xi)$


Graphs in $\Gamma(\xi)$ $a(\partial_\mu\xi\partial^\mu\xi)^2$
$a=\frac{\lambda}{4\pi^2}$
$\int d^4x(\cdot) = \int_0^\Lambda d^4x(\cdot) + \int_\Lambda^\infty d^4x(\cdot)$
$\sim \lambda^2/m^4$
$S_N(\xi)$ $a_N(\partial_\mu\xi\partial^\mu\xi)$
$\delta a\ (\partial_\mu\xi\partial^\mu\xi)$
a

Graphs in $\Gamma(\xi)$ $a_\Gamma (\partial_\mu \xi \partial^\mu \xi)^2$

$a = \frac{\lambda}{4!}$ tree

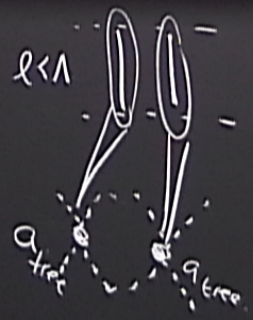
$\int d^4x (\cdot) = \int_0^\Lambda d^4p (\cdot) + \int_\Lambda^\infty d^4p (\cdot)$

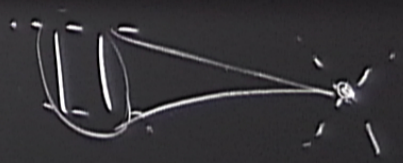
$\sim \lambda^2/m^4$

$S_M(\xi) \cdot a_M (\partial_\mu \xi \partial^\mu \xi)$

$\delta a \, (\partial_\mu \xi \partial^\mu \xi)$







$$W(J) = W(J-J,0) \qquad \Gamma$$

$$e^{i\Gamma(\ell)} = \int \mathcal{D}H\,\mathcal{D}L\; e^{iS(\ell+L,H)+i\int JL\,d^4x} \qquad J = -\frac{\delta\Gamma}{\delta\ell}$$

$$= \int \mathcal{D}L\; e^{iS_W(L)+i\int JL\,d^4x} \qquad e^{iS_W(L)} = \int \mathcal{D}H\, e^{iS(L,H)}$$

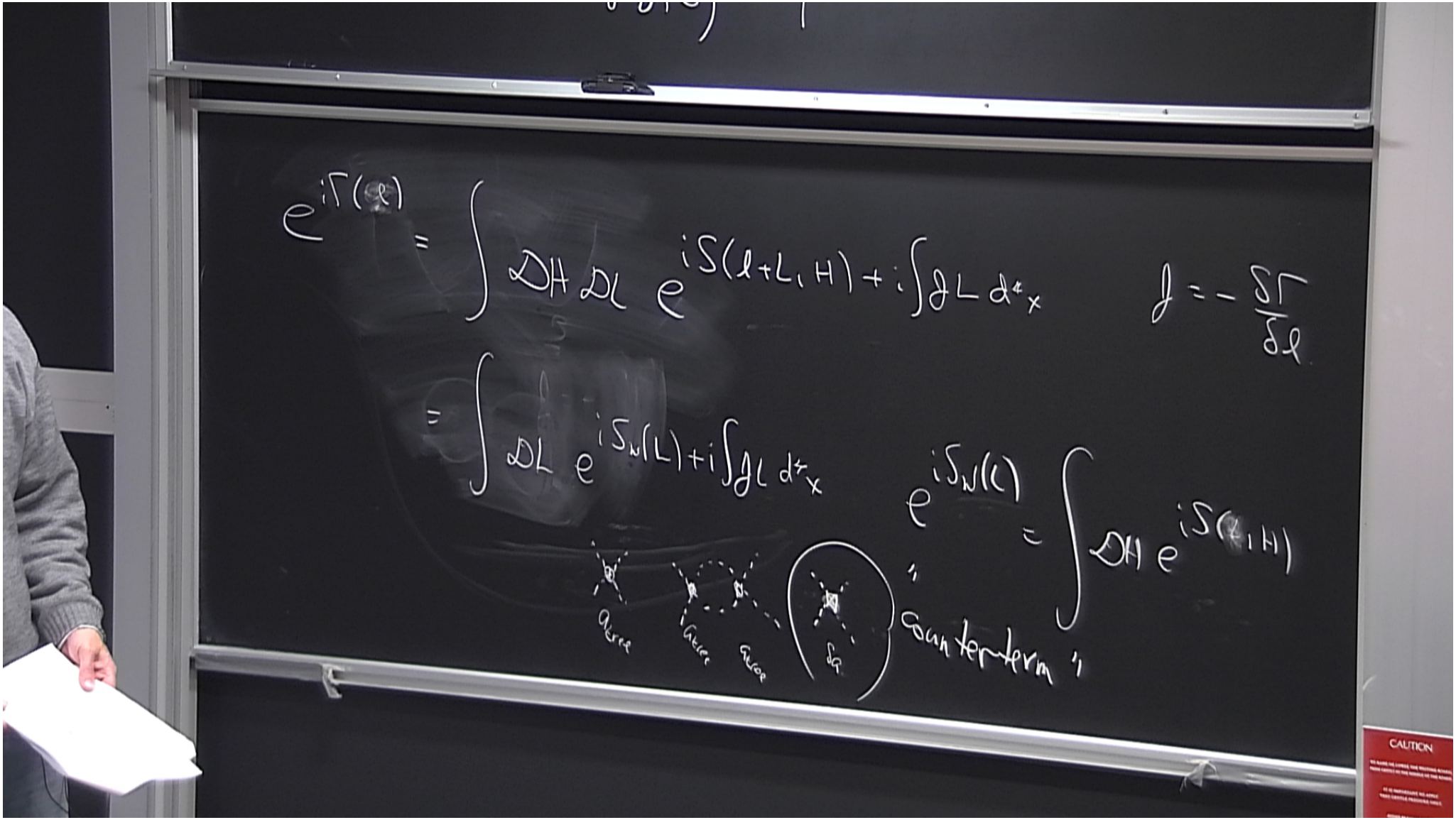




$$e^{i\Gamma(\ell)} = \int \mathcal{D}H\,\mathcal{D}L\; e^{iS(\ell+L,H)+i\int J L\,d^4x} \qquad J = -\frac{\delta\Gamma}{\delta\ell}$$

$$= \int \mathcal{D}L\; e^{iS_W(L)+i\int J L\,d^4x} \qquad e^{iS_W(L)} = \int \mathcal{D}H\; e^{iS(L,H)}$$



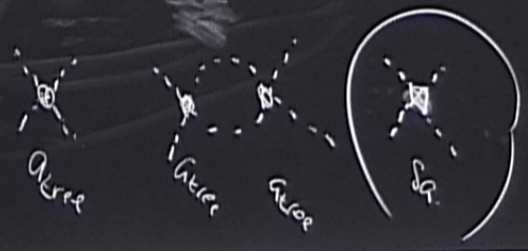

$e^{i\Gamma(J)} = \int \mathcal{D}H\,\mathcal{D}L\; e^{iS(J+L,H)+i\int JL\,d^4x}$
$J = -\frac{\delta\Gamma}{\delta J}$
$= \int \mathcal{D}L\; e^{iS_N(L)+i\int JL\,d^4x}$
$e^{iS_N(L)} = \int \mathcal{D}H\, e^{iS(L,H)}$
"counterterm"
CAUTION

$W(J) = W(J-J, 0)$ Γ

$$e^{i\Gamma(\ell)} = \int \mathcal{D}H\,\mathcal{D}L\; e^{iS(\ell+L,H)+i\int JL\,d^4x} \qquad J = -\frac{\delta\Gamma}{\delta\ell}$$

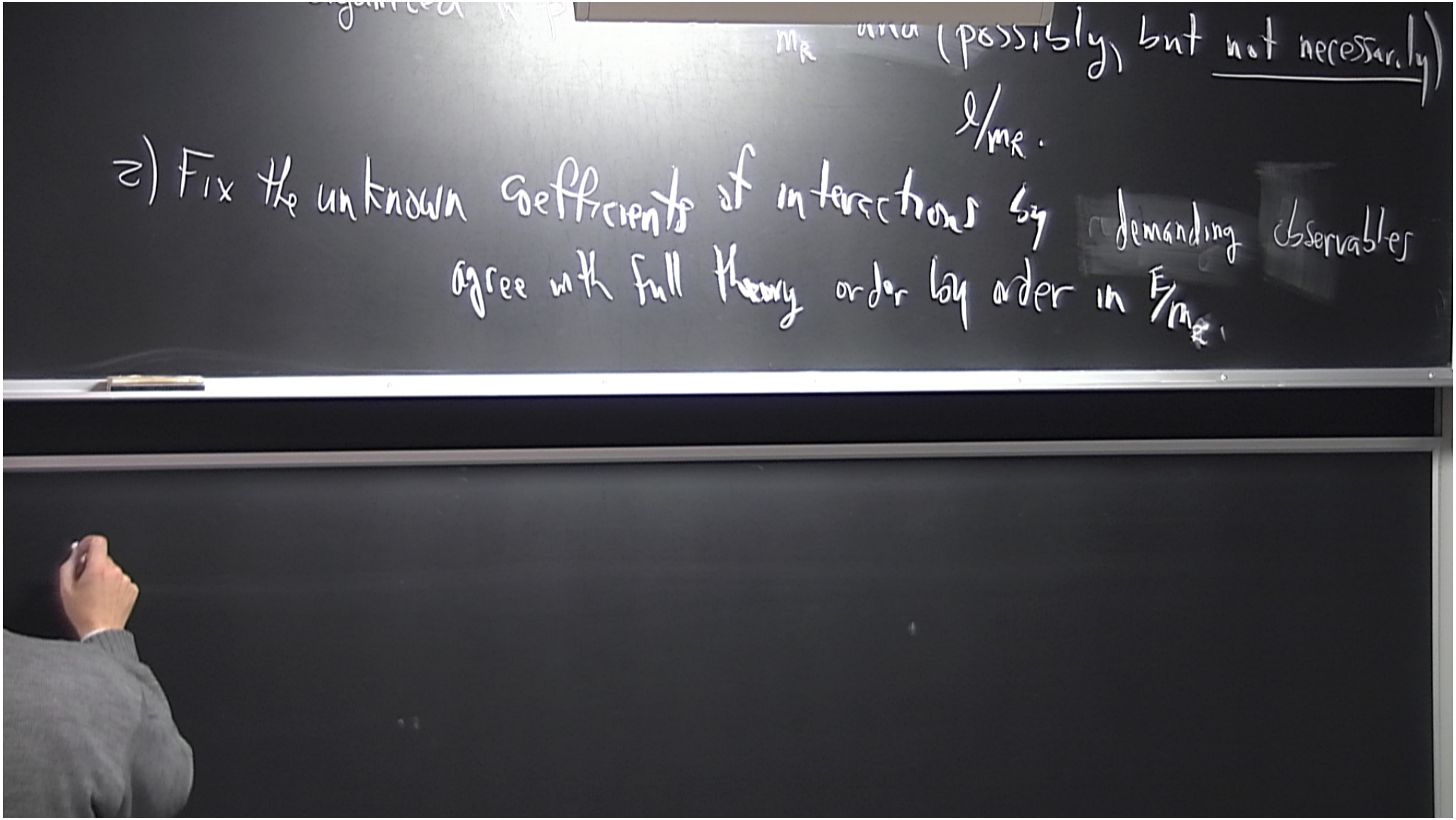
$$= \int \mathcal{D}L\; e^{iS_W(L)+i\int JL\,d^4x}$$

$g^2 \Delta 4$



$$e^{iS_W(L)} = \int \mathcal{D}H\; e^{iS(L,H)}$$

" counterterm "


m_R and (possibly, but not necessarily)
ℓ/m_R.
2) Fix the unknown coefficients of interactions by demanding observables
agree with full theory order by order in E/m_R.

(1) dimensional analysis of α^0

(2) "power count": track how $1/m_R$ appears in graphs.

Dimensional analysis: $(\hbar = 1,\ c = 1,\ k_B = 1)$

(unit choices)

e^{iS}

$$S = \int dt\, L = \int d^4x\, \mathcal{L}$$

$W(J) = W(J-J, 0)$ Γ

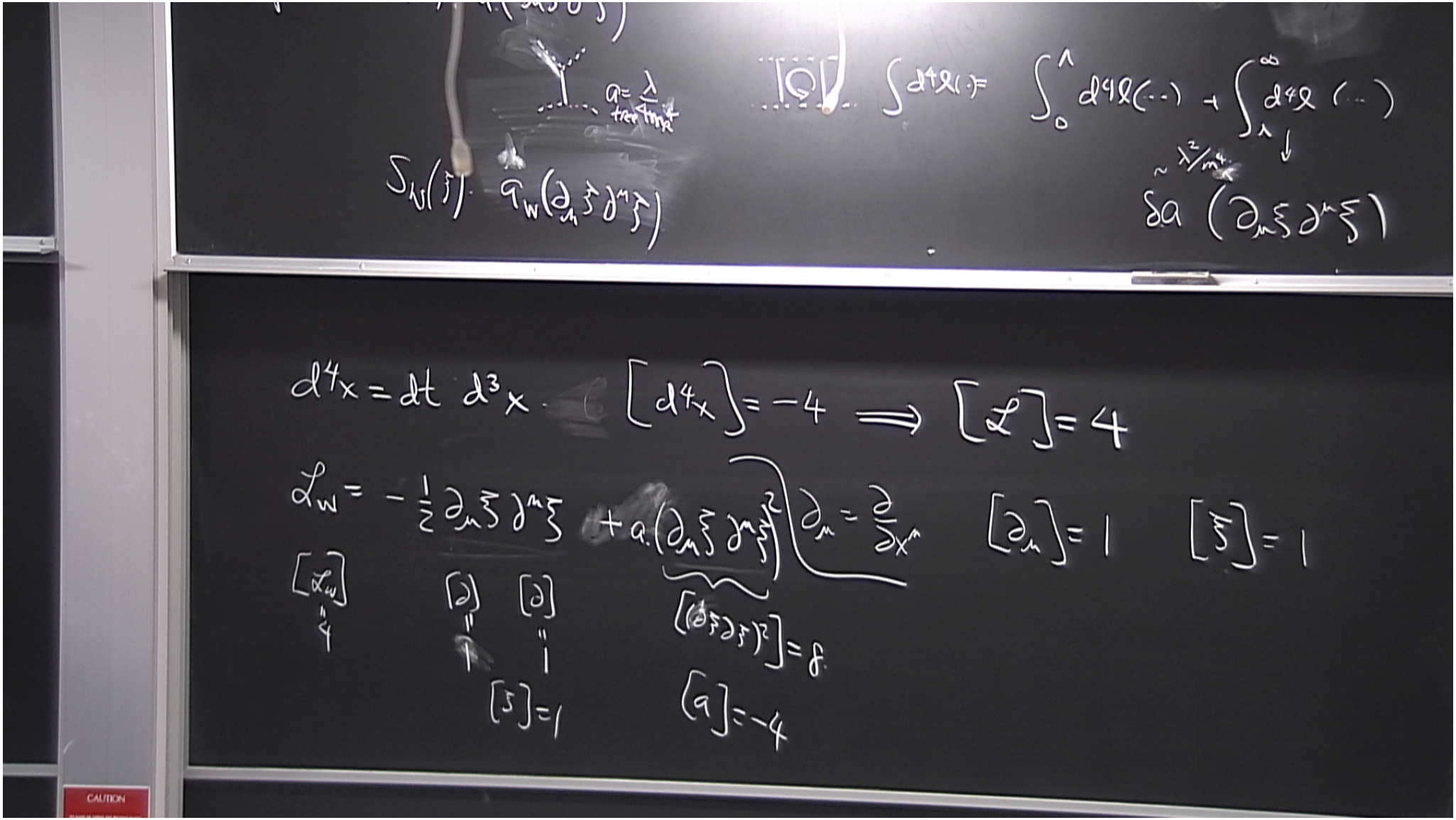
Dimensional analysis: $(\hbar = 1, c = 1, k_B = 1)$

(unit choices)

e^{iS} $S = \int dt\, L = \int d^4x\, \mathcal{L}$

write $[Q] = P \iff$ dimension of Q is E^P

eg: action $[S] = 0 \iff S$ is dimensionless (when $\hbar = 1$)


$a \sim \frac{\lambda}{4\pi}$ tree level
$\int d^4x(\cdots) = \int_0^{\Lambda} d^4x(\cdots) + \int_{\Lambda}^{\infty} d^4x(\cdots)$
$\sim \Lambda^2/m_W^4$
$S_W(\xi) \cdot q_W(\partial_m \xi \partial^m \xi)$
$\delta a\ (\partial_m \xi \partial^m \xi)$
$d^4x = dt\ d^3x$
$[d^4x] = -4 \Longrightarrow [\mathcal{L}] = 4$
$\mathcal{L}_W = -\frac{1}{2}\partial_m \xi \partial^m \xi + a(\partial_m \xi \partial^m \xi)^2$
$\partial_m = \frac{\partial}{\partial x^m}$
$[\partial_m] = 1$
$[\xi] = 1$
$[\mathcal{L}_W] = 4$
$[\partial] = 1$
$[\partial] = 1$
$[\xi] = 1$
$[(\partial_\xi \partial_\xi)^2] = 8$
$[a] = -4$
CAUTION

agree with full theory order by order in E/m_e.

(1) dimensional analysis of α^n

(2) "power count": track how $1/m_e$ appears in graphs.

$$\int dx\, e^{i\overbrace{\left[\frac{a}{2}x^2 - bx + cx^3\right]}^{S(x)}} x^n = \sqrt{\frac{1}{a}}\, e^{i S(x_c)}\, x_c^n \left[1 + O\left(\frac{1}{a}\right)\right]$$

$$\left.\frac{\partial S}{\partial x}\right|_{x_c} = 0 \qquad x_c = b/a$$

$$\int du\, e^{i\frac{a}{2}u^2}$$

$S_W(\xi) = a_W(\partial_\mu \xi \partial^\mu \xi)$ $\quad \delta a\ (\partial_\mu \xi \partial^\mu \xi)$

$d^4x = dt\ d^3x$ $\quad [d^4x] = -4 \Rightarrow [\mathcal{L}] = 4$

$\mathcal{L}_W = -\frac{1}{2}\partial_\mu \xi \partial^\mu \xi + a(\partial_\mu \xi \partial^\mu \xi)^2$ $\quad \partial_\mu = \frac{\partial}{\partial x^\mu}$ $\quad [\partial_\mu] = 1$ $\quad [\xi] = 1$

$[\mathcal{L}_W] = 4$ $\quad [\partial][\partial] = 1 + 1$ $\quad [\partial] = 1$ $\quad [\xi] = 1$

$[(\partial_\mu \xi \partial^\mu \xi)^2] = 8$ $\quad [a] = -4$

Fields + derivs have positive dimension so more complicated int'ns have coeffs with more neg dim.

QED

$$\mathcal{L} = -\bar{\psi}(\gamma^\mu D_\mu + m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\phi^* D^\mu\phi - V(\phi^*\phi)$$

$$-y\bar{\psi}\Gamma\psi\phi + c.c.$$

$$V = V_0 + a\phi^*\phi + b(\phi^*\phi)^2 + c(\phi^*\phi)^3 + \dots$$

$$D_\mu\psi = \partial_\mu\psi - ieA_\mu\psi \qquad D_\mu\phi = \partial_\mu\phi - ieQA_\mu\phi \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

counterterm

$[\mathcal{L}] = 4 \quad [\partial] = 1 \quad [eA_\mu] = 1 \quad [Q] = 0 \quad [V] = 4$

$-\partial_\mu \phi^* \partial^\mu \phi \to [\phi] = 1 \qquad [m] = 1 \qquad [V_0] = 4 \quad [a] = 2$

$-\bar{\psi} \gamma^\mu \partial_\mu \psi \to [\psi] = 3/2 \qquad [b] = 0 \quad [c] = -2$

$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} \partial_\mu A_\nu \partial^\mu A^\nu + \dots \quad [A_\mu] = 1 \to [e] = 0.$

agree with full theory order by order in E/m_e.

QED

$$\mathcal{L} = -\bar{\psi}(\gamma^\mu D_\mu + m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\phi^* D^\mu\phi - V(\phi^*\phi)$$

"yukawa" coupling $-y\,\bar{\psi}\Gamma\psi\phi + c.c$

$$V = V_0 + a\phi^*\phi + b(\phi^*\phi)^2 + c(\phi^*\phi)^3 + \dots$$

$$D_\mu\psi = \partial_\mu\psi - ieA_\mu\psi \qquad D_\mu\phi = \partial_\mu\phi - ieQA_\mu\phi \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

agree with full theory order by order in E/m_e.

QED

$$\mathcal{L} = -\bar{\psi}(\gamma^\mu D_\mu + m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\phi^* D^\mu\phi - V(\phi^*\phi)$$

"yukawa" coupling $-y\,\bar{\psi}\Gamma\psi\phi + z\,\bar{\psi}\psi\phi^*\phi\phi$

$$V = V_0 + a\phi^*\phi + b(\phi^*\phi)^2 + c(\phi^*\phi)^3 + \dots \qquad [z] = -2$$

$$D_\mu\psi = \partial_\mu\psi - ieA_\mu\psi \qquad D_\mu\phi = \partial_\mu\phi - ieQA_\mu\phi \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$[g] = 0$

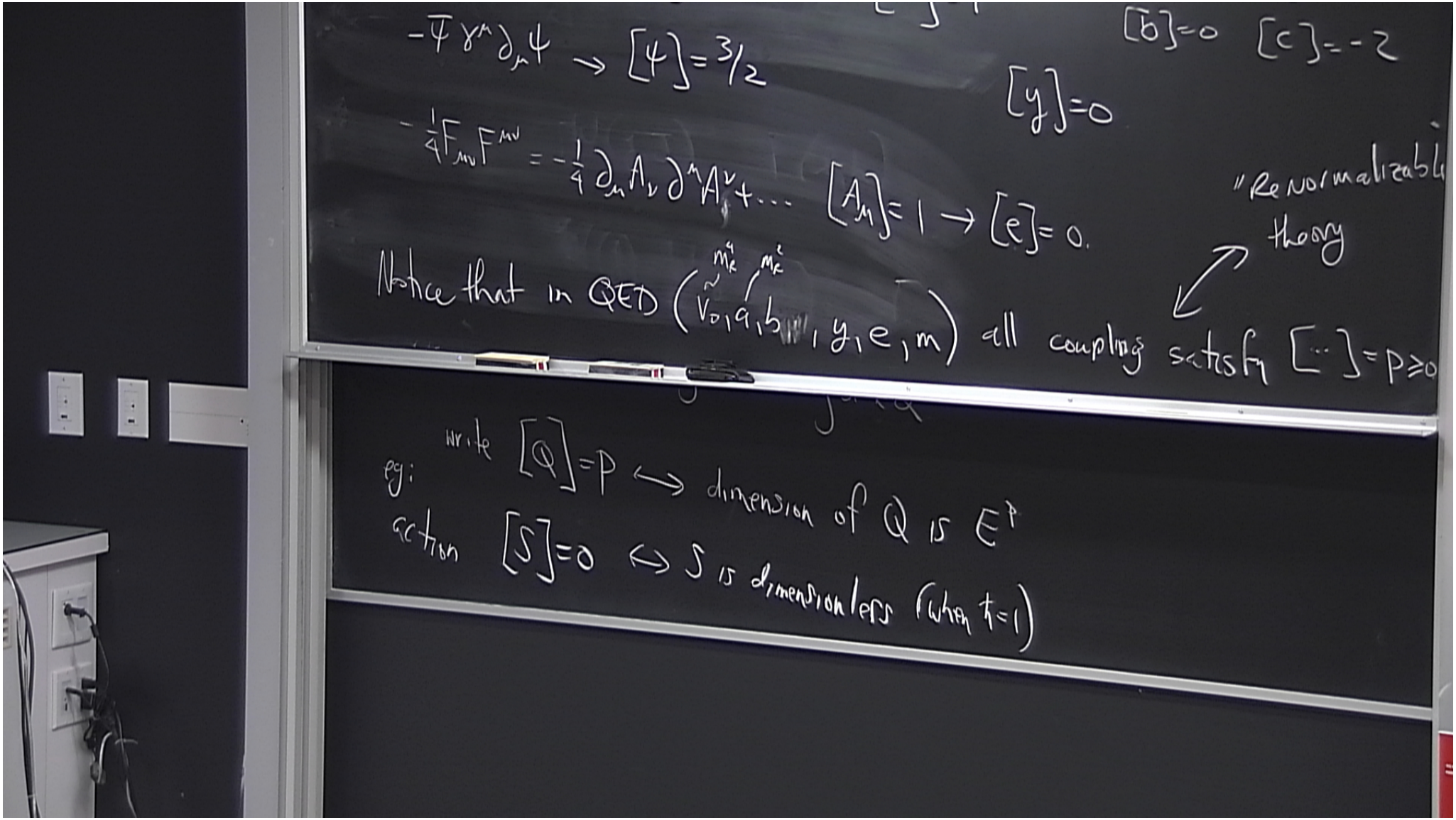
$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} \partial_\mu A_\nu \partial^\mu A^\nu + \dots \quad [A_\mu] = 1 \to [e] = 0.$

Notice that in QED (v, q, b, c, y, e, m) all couplings satisfy $[\dots] = p \geq 0$

eg: write $[Q] = p \iff$ dimension of Q is E^p

action $[S] = 0 \iff S$ is dimensionless (when $\hbar = 1$)

CAUTION


$-\bar{\psi}\gamma^\mu\partial_\mu\psi \to [\psi]=3/2$
$[b]=0$ $[c]=-2$
$[y]=0$
$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}\partial_\mu A_\nu \partial^\mu A^\nu + \dots$ $[A_\mu]=1 \to [e]=0.$
"Renormalizable theory"
Notice that in QED (V_0, q, b, W, y, e, m) all couplings satisfy $[\dots]=p\geq 0$
write $[Q]=p \leftrightarrow$ dimension of Q is E^p
eg: action $[S]=0 \leftrightarrow S$ is dimensionless (when $\hbar=1$)