

Title: Introduction to Effective Field Theories - Lecture 5

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Abstract:

- Quantum gravity as an EFT

- EFT in condensed matter physics

$$\langle \Omega | T(\phi^i(x) \phi^j(y)) | \Omega \rangle = \int \mathcal{D}\phi e^{iS(\phi)} \phi^i(x) \phi^j(y)$$

$$= \int \mathcal{D}\phi e^{iS_0 + \Delta S} \phi^i(x) \phi^j(y) (1 + iS_{int} \dots)$$

$$S_0 = \frac{1}{2} \int d^4x \phi^i \Delta_{ij} \phi^j \quad S_{int} = \int d^4x \lambda \phi^4$$

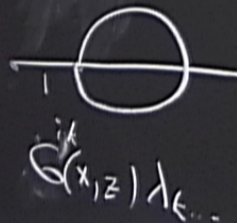


... in EFTs.
 - Quantum gravity as an EFT

- EFT in condensed matter physics

$$\langle \Omega | T(\phi^i(x) \phi^j(y)) | \Omega \rangle = \int \mathcal{D}\phi e^{iS(\phi)} \phi^i(x) \phi^j(y) \\ = \int \mathcal{D}\phi e^{iS_0 + \Delta S} \phi^i(x) \phi^j(y) (1 + iS_{int} \dots)$$

$$S_0 = \frac{1}{2} \int d^4x \phi^i \Delta_y \phi^j \quad S_{int} = \int d^4x \lambda_{ijkl} \phi^i \phi^j \phi^k \phi^l$$

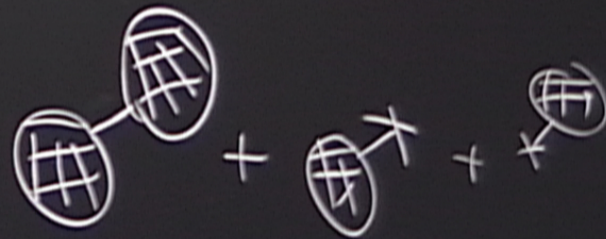


$$e^{iW(J)} = \int \mathcal{D}\phi \, e^{iS_0(\phi)} \left[1 + iS_{int} + \dots + i \int J_i \phi^i + \frac{i^2}{2} \int J_i \phi^i dx \int J_k \phi^k dy + \dots \right]$$

$$\star \int dx \, x - x J_1(x)$$

$$e^{i\Gamma(\phi)} \rightarrow \text{d.o.f. with } \bar{J}_i = -\frac{\delta \Gamma}{\delta \phi_i}$$

$$\star = - \text{ (circle with cross) }$$



$$\lambda \frac{d}{dx} x = x J_1(x)$$

$$e^{i\Gamma(\varphi)} \rightarrow \text{dots with } J_i = -\frac{\delta\Gamma}{\delta\varphi_i}$$

$$\rightarrow = - \text{ (diagram of a circle with a cross) }$$



$$f(x^i) \equiv f(R^i, x^i) + A \quad e^{i\Gamma(\varphi)} = \int d\varphi e^{iS(\varphi) + i \int J_i \varphi^i}$$

$$\frac{\partial f}{\partial x^i} = \frac{\partial f}{\partial y^j} R^j_i \quad J_i = -\frac{\delta\Gamma}{\delta\varphi^i}$$

Semiclassical approx: $S = \frac{1}{\lambda} S_0$ λ counts loops in the graphical expansion of $\int dx e^{iS(x)}$

$$S(\varphi+\phi) = S(\varphi) + \int d^4x \left(\frac{\delta S}{\delta \varphi^i} \right)_\varphi \phi^i + \frac{1}{2} \int d^4x \phi^i \Delta_i \phi^i + \dots$$

$$e^{i\Gamma(\varphi)} = \int \mathcal{D}\phi e^{iS(\varphi+\phi) + i\int J\phi} = e^{iS(\varphi)} \int \mathcal{D}\phi e^{\frac{i}{2} \int \phi^i \Delta_i \phi^i} [1 + \dots]$$

$\xrightarrow{\text{tree}}$
 $\downarrow$

$$= e^{iS(\varphi)} (\det \Delta)^{-1} [1 + \dots]$$

$$\Gamma = S - i p \ln \det \Delta + \dots$$

$$S(\varphi+\phi) = S(\varphi) + \int d^4x \left(\frac{\delta S}{\delta \phi^i} \right)_\varphi \phi^i + \frac{1}{2} \int d^4x \phi^i \Delta_i \phi^i + \dots$$

$$e^{i\Gamma(\varphi)} = \int \mathcal{D}\phi e^{iS(\varphi+\phi) + i\int J_i \phi^i} = e^{iS(\varphi)} \int \mathcal{D}\phi e^{\frac{i}{2} \int \phi^i \Delta_i \phi^i} [1 + \dots]$$

"tree level" "classical" $\rightarrow$
 "1-loop" $\rightarrow$
 $= e^{iS(\varphi)} (\det \Delta)^{-1/2} [1 + \dots]$

$$\Gamma = S - i \ln \det \Delta + \dots$$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \mathcal{L}_{int}$$

$$P_\Lambda^2 = \sum_{\substack{nm \\ E < \Lambda}} |n\rangle \langle n| m\rangle \langle m| = \sum_{E < \Lambda} |n\rangle \langle n|$$

$$Q_\Lambda = 1 - P_\Lambda \quad Q_\Lambda^2 = (1 - P_\Lambda)^2 = 1 - 2P_\Lambda + P_\Lambda^2 = 1 - P_\Lambda = Q_\Lambda$$

$$\text{if } |y\rangle \quad H|y\rangle = E|y\rangle \quad E < \Lambda \quad P_\Lambda |y\rangle = |y\rangle \\ Q_\Lambda |y\rangle = 0$$

$$|x\rangle \quad H|x\rangle = E|x\rangle \quad E > \Lambda \quad P_\Lambda |x\rangle = 0 \\ Q_\Lambda |x\rangle = |x\rangle$$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \mathcal{L}_{int}$$

$$P_\Lambda^2 = \sum_{\substack{n,m \\ E < \Lambda}} |n\rangle \langle n| m\rangle \langle m| = \sum_{\substack{n \\ E < \Lambda}} |n\rangle \langle n|$$

$$Q_\Lambda = 1 - P_\Lambda \quad Q_\Lambda^2 = (1 - P_\Lambda)^2 = 1 - 2P_\Lambda + P_\Lambda^2 = 1 - P_\Lambda = Q_\Lambda$$

$$\text{if } |\psi\rangle \quad H|\psi\rangle = E|\psi\rangle \quad E < \Lambda$$

$$P_\Lambda |\psi\rangle = |\psi\rangle$$

$$P_\Lambda H = H P_\Lambda$$

$$Q_\Lambda |\psi\rangle = 0$$

↓

$$|\chi\rangle \quad H|\chi\rangle = E|\chi\rangle \quad E > \Lambda$$

$$P_\Lambda |\chi\rangle = 0$$

$$Q_\Lambda |\chi\rangle = |\chi\rangle$$

$$P_\Lambda U(t, t_0) = U(t, t_0) P_\Lambda$$

High E

Low E

$$P_\Lambda = \sum_{\substack{n \\ E_n < \Lambda}} |\ln \chi_n|$$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \mathcal{L}_{int}$$

$H = H(k, l)$

$U = e^{-iH(t-t_0)}$

$= P_\Lambda$

$$P_\Lambda^2 = \sum_{\substack{nm \\ E_n < \Lambda}} |\ln \chi_n| |m\rangle \langle m| = \sum_{\substack{n \\ E_n < \Lambda}} |\ln \chi_n|$$

$$Q_\Lambda = 1 - P_\Lambda \quad Q_\Lambda^2 = (1 - P_\Lambda)^2 = 1 - 2P_\Lambda + P_\Lambda^2 = 1 - P_\Lambda = Q_\Lambda$$

$$\frac{\delta}{\delta q}(\Gamma - S)$$

$$\rightarrow \star = - \text{ (globe icon) }$$

Time evolution for low-energy states is given by.

$$\mathcal{U}(t, t_0) = P_\Lambda U(t, t_0) P_\Lambda \quad \mathcal{U}, U \text{ are equivalent if we only evolve states at low energy.}$$

$$\frac{\delta}{\delta q}(\Gamma - S)$$

$$\rightarrow \star = - \text{ (globe icon) }$$

Time evolution for low-energy states is given by.

$U(t, t_0) = P_\Lambda U(t, t_0) P_\Lambda$ U, U are equivalent if we only evolve states at low energy.

$U^\dagger U = 1 \rightarrow U^\dagger U = P_\Lambda$ so $U = e^{\underset{\substack{\uparrow \\ \text{define as Effective Hamiltonian}}}{-iH(t-t_0)}}$

$$+ = -$$

Time evolution for low-energy states, is given by

$U(t, t_0) = P_\Lambda U(t, t_0) P_\Lambda$ U, U are equivalent if we only evolve states at low energy.

$$U^\dagger U = 1 \rightarrow U^\dagger U = P_A \quad \text{So } U = e^{-iH_{\text{eff}}(t-t_0)}$$

$\uparrow$
define as Effective Hamiltonian.

$$\Gamma = S - i\pi \ln \det \Delta + \dots$$

Notice $U = e^{-iH(t-t_0)}$, $U = P_\Lambda e^{-iH(t-t_0)} = P_\Lambda U P_\Lambda$

$$P_\Lambda U P_\Lambda = P_\Lambda \left[1 - iH(t-t_0) + \frac{(-i)^2}{2} H^2 (t-t_0)^2 + \dots \right] P_\Lambda$$

$$= P_\Lambda - i P_\Lambda H P_\Lambda (t-t_0) + \frac{(-i)^2}{2} P_\Lambda H^2 P_\Lambda (t-t_0)^2 + \dots$$

$$H = P_\Lambda H P_\Lambda$$

$$\Gamma = S - i p \ln \det \Delta + \dots$$

Notice $U = e^{-iH(t-t_0)}$, $U = P_\Lambda e^{-iH(t-t_0)} = P_\Lambda U P_\Lambda$

$$P_\Lambda U P_\Lambda = P_\Lambda \left[1 - iH(t-t_0) + \frac{(-i)^2}{2} H^2 (t-t_0)^2 + \dots \right] P_\Lambda$$

$$= P_\Lambda - i P_\Lambda H P_\Lambda (t-t_0) + \frac{(-i)^2}{2} \underbrace{P_\Lambda H^2 P_\Lambda}_{\neq P_\Lambda H P_\Lambda H P_\Lambda} (t-t_0)^2 + \dots$$

$$\langle \beta | P_\Lambda H H P_\Lambda | \alpha \rangle \neq \sum_n \langle \beta | P_\Lambda H | n \rangle \langle n | H P_\Lambda | \alpha \rangle$$

$$\Gamma = S - i p \ln \det \Delta + \dots$$

Notice $U = e^{-iH(t-t_0)}$, $U = P_\Lambda e^{-iH(t-t_0)} = P_\Lambda U P_\Lambda$

$$P_\Lambda U P_\Lambda = P_\Lambda \left[1 - iH(t-t_0) + \frac{(-i)^2}{2} H^2 (t-t_0)^2 + \dots \right] P_\Lambda$$

$$= P_\Lambda - i P_\Lambda H P_\Lambda (t-t_0) + \frac{(-i)^2}{2} \underbrace{P_\Lambda H^2 P_\Lambda}_{\neq P_\Lambda H P_\Lambda H P_\Lambda} (t-t_0)^2 + \dots$$

$$\langle \beta | P_\Lambda H H P_\Lambda | \alpha \rangle \neq \sum_n \langle \beta | P_\Lambda H | n \rangle \langle n | H P_\Lambda | \alpha \rangle$$

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi_R \partial^\mu \phi_R - \frac{1}{2} m_R^2 \phi_R^2 - \frac{1}{2} \partial_\mu \phi_L \partial^\mu \phi_L + \mathcal{L}_{int}$$

$$P_\Lambda^2 = \sum_{nm} \ln \chi_n |n\rangle \langle m| = \sum_{E \leftarrow 1} \ln \chi_n$$

We wish to express $\mathcal{H}$ in terms of "light fields" only.

In the toy model a basis of fields was ϕ_L, ϕ_R

$$\phi_L(x) = \sum_n \left[a_n u_n(x) + \text{c.c.} \right]$$

$$\phi_R(x) = \sum_n \left[b_n v_n(x) + \text{c.c.} \right]$$

$$u_n(x) = e_n e^{ipx}$$

$$p_R^0 = E_R = \sqrt{\vec{p}^2 + m_R^2}$$

$$p_L^0 = E_L = \sqrt{\vec{p}^2}$$

$$p \cdot x = p_\mu x^\mu = \eta_{\mu\nu} p^\mu x^\nu = -E t + \vec{p} \cdot \vec{x}$$

$$\frac{\partial}{\partial q}(\Gamma - S)$$

$$\rightarrow * = - \oplus$$

Define $l_I^A = P_\Lambda \phi_I(x) P_\Lambda$

$$h_I(x) = Q_\Lambda \phi_I(x) Q_\Lambda$$

$$l_R(x) = P_\Lambda \phi_R(x) P_\Lambda$$

$$h_R(x) = Q_\Lambda \phi_R(x) Q_\Lambda$$

$$\frac{\delta}{\delta q}(\Gamma - S) \rightarrow * = - \text{ (globe symbol) }$$

Define $l_I^A = P_\Lambda \phi_I(x) P_\Lambda$ $h_I(x) = Q_\Lambda \phi_I(x) Q_\Lambda$

$$l_R(x) = P_\Lambda \phi_R(x) P_\Lambda$$

$$h_R(x) = Q_\Lambda \phi_R(x) Q_\Lambda$$

$$\Lambda \ll m_R = 0$$

$$\frac{\delta}{\delta g}(\Gamma - S)$$

$$\rightarrow = - \text{ (globe symbol) }$$

Define $h_I = P_\Lambda \phi_I(x) P_\Lambda$

$$h_R(x) = P_\Lambda \phi_R(x) P_\Lambda$$

$$= 0$$

$$\Lambda \ll m_R$$

$$h_I(x) = Q_\Lambda \phi_I(x) Q_\Lambda$$

$$h_R(x) = Q_\Lambda \phi_R(x) Q_\Lambda$$

Goal: construct $\tilde{H}(\phi, \psi)$ as function of h_I order by order in $\frac{1}{m_R}$.

Construct generating functional for correlations of $\ell_I(x)$.

$$\langle \Omega | T [\ell_I(x_1) \dots \ell_I(x_n)] | \Omega \rangle,$$

$$\mathcal{Z}(J) = \sum_{n=0}^{\infty} \frac{1}{n!} \langle \Omega | T [\ell_I(x_1) \dots \ell_I(x_n)] | \Omega \rangle J(x_1) \dots J(x_n)$$

construct generating functional for correlations of $\ell_I(x)$.

$$\langle \Omega | T [\ell_I(x_1) \dots \ell_I(x_n)] | \Omega \rangle$$

$$\mathcal{Z}(f) = \sum_{n=0}^{\infty} \frac{1}{n!} \langle \Omega | T [\ell_I(x_1) \dots \ell_I(x_n)] | \Omega \rangle f(x_1) \dots f(x_n)$$

$\mathcal{Z}(f)$ will have support only for f 's varying only over distances, times $\gtrsim 1/\Lambda \gg 1/m_F$

For toy model

$$e^{iW[j, J]} = \int D\phi_R D\phi_I e^{iS(\phi_R, \phi_I) + i \int j \phi_I dx + i \int J \phi_R dx}$$

slowly varying functions

$$\mathcal{Z} = e^{iW(j)} = e^{iW(j=l, J=0)}$$

$$\frac{\delta}{\delta g}(\Gamma - S) \Rightarrow * = - \oplus$$

Recall:

$$\Gamma(\varphi_I, \varphi_R) = W[J, \bar{J}] - \int d^4x \varphi_I J - \int d^4x \varphi_R \bar{J}$$

where

$$\frac{\delta W}{\delta J(x)} = \varphi_I(x)$$

$$\frac{\delta W}{\delta \bar{J}(x)} = \varphi_R(x)$$

m_R

$$\frac{\delta}{\delta g}(\Gamma - S) \Rightarrow * = - \oplus$$

Recall:

$$\Gamma(\varphi_I, \varphi_R) = W[J, \bar{J}] - \int d^4x \varphi_I J - \int d^4x \varphi_R \bar{J}$$

where

$$\frac{\delta W}{\delta J(x)} = \varphi_I(x)$$

$$\frac{\delta W}{\delta \bar{J}(x)} = \varphi_R(x)$$

$$\Rightarrow \frac{\delta \Gamma}{\delta \varphi_I(x)} + J(x) = 0 \quad \frac{\delta \Gamma}{\delta \varphi_R(x)} + \bar{J}(x) = 0$$

m_R

so $e^{i\Gamma(\varphi_I, \varphi_R)} = e^{iW(g, J) - i\int J\varphi_R - i\int g\varphi_I}$

$$= \int \mathcal{D}\phi_I \mathcal{D}\phi_R e^{iS(\varphi_I + \phi_I, \varphi_R + \phi_R) + i\int g\phi_I + i\int J\phi_R}$$

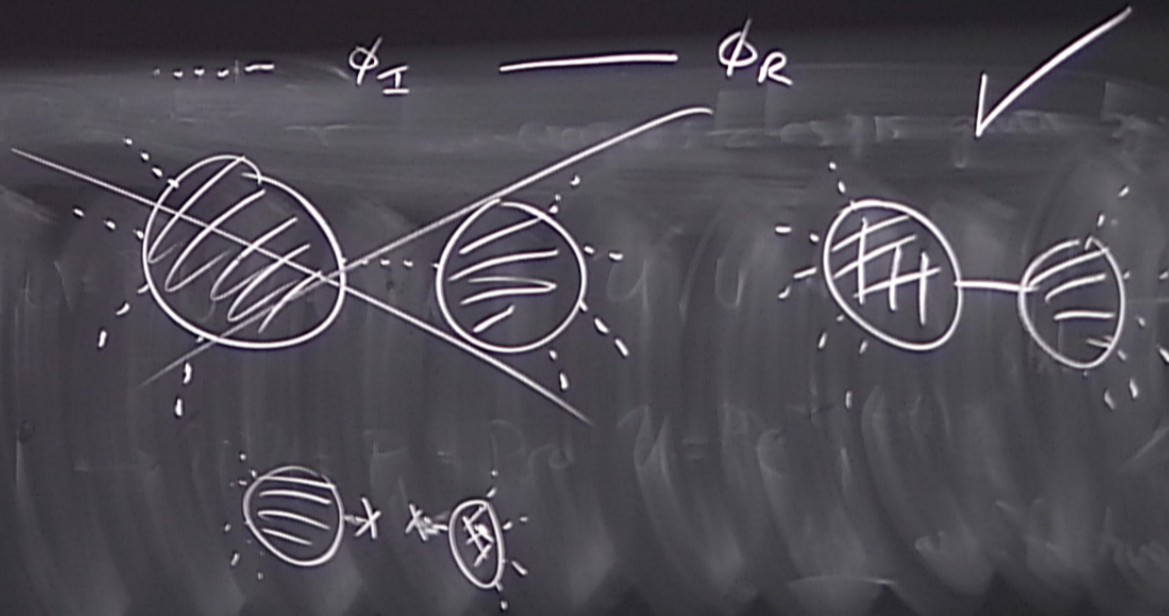
condition $J = -\frac{\delta\Gamma}{\delta\varphi_R}$, $g = -\frac{\delta\Gamma}{\delta\varphi_I} \Rightarrow$ keep only 1PI vacuum graphs

$$\text{so } e^{i\Gamma(\varphi_I, \varphi_R)} = e^{iW(g, J) - i\int J\varphi_R - i\int g\varphi_I}$$

$$= \int \mathcal{D}\phi_I \mathcal{D}\phi_R e^{iS(\varphi_I + \phi_I, \varphi_R + \phi_R) + i\int g\phi_I + i\int J\phi_R}$$

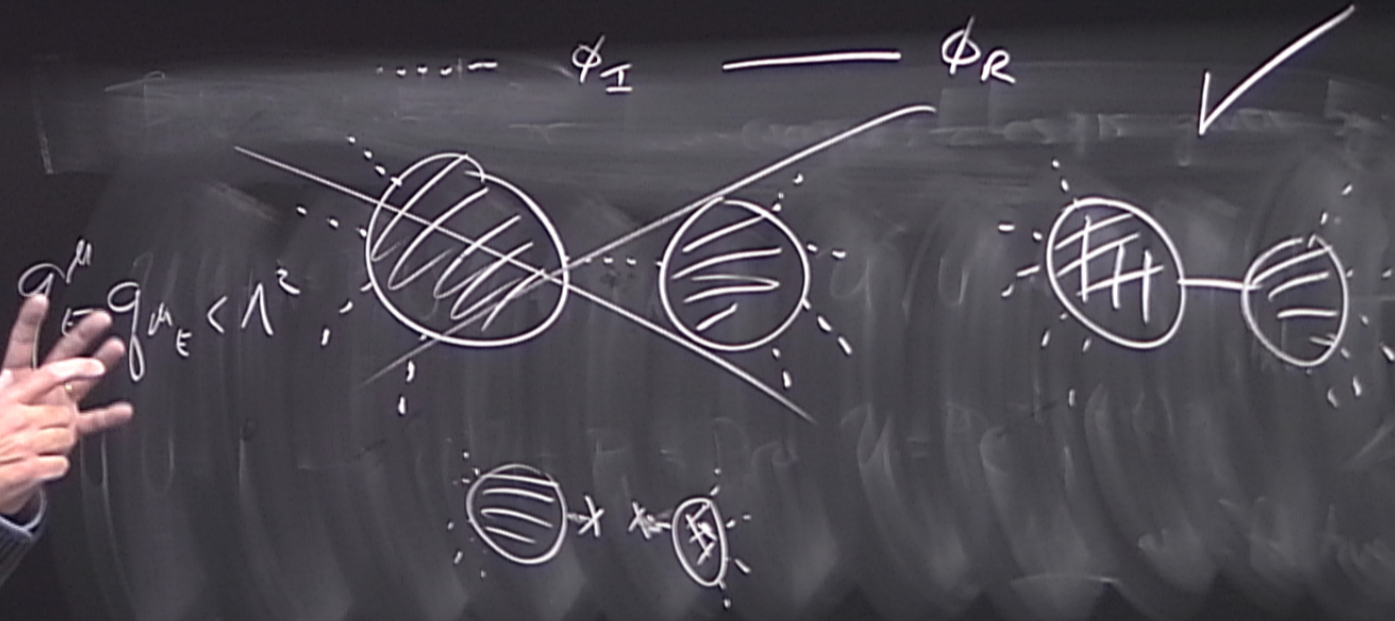
$$\text{condition } g = -\frac{\delta\Gamma}{\delta\phi_I}, J = -\frac{\delta\Gamma}{\delta\phi_R} \Rightarrow \text{keep only 1PI vacuum graphs}$$

$$e^{i\Gamma(\psi)} = \int \mathcal{D}\phi_R \mathcal{D}\phi_I \rho^{iS(\phi+\phi_I, \phi_R) + i\int \partial\phi_I d^4x} \rightarrow \text{all } \text{[LPI] graphs.}$$



$$e^{i\Gamma(\psi)} = \int \mathcal{D}\phi_R \mathcal{D}\phi_L \rho^{iS(\phi+\phi_L, \phi_R) + i\int \phi_L d^4x} \rightarrow \text{1LPI graphs.}$$

$\Gamma(\psi) = \text{sum of all}$



$$= e \int \mathcal{D}\phi_R \mathcal{D}\phi_I e^{iS(\phi_R, \phi_I)} \int \mathcal{D}\phi_I d^4x$$

$$\Gamma(\varphi) = W(j) - \int \varphi j d^4x$$

$\Gamma(\varphi)$ = sum of all

$$e^{i\Gamma(\varphi)} = \int \mathcal{D}\phi_R \mathcal{D}\phi_I e^{iS(\varphi + \phi_I, \phi_R) + i \int \phi_I j d^4x} \rightarrow \text{[LPT] graphs.}$$

$$e^{iS_w(\phi)} = \int \mathcal{D}\phi_R (\mathcal{D}\phi_I)^{p>1} e^{iS}$$

