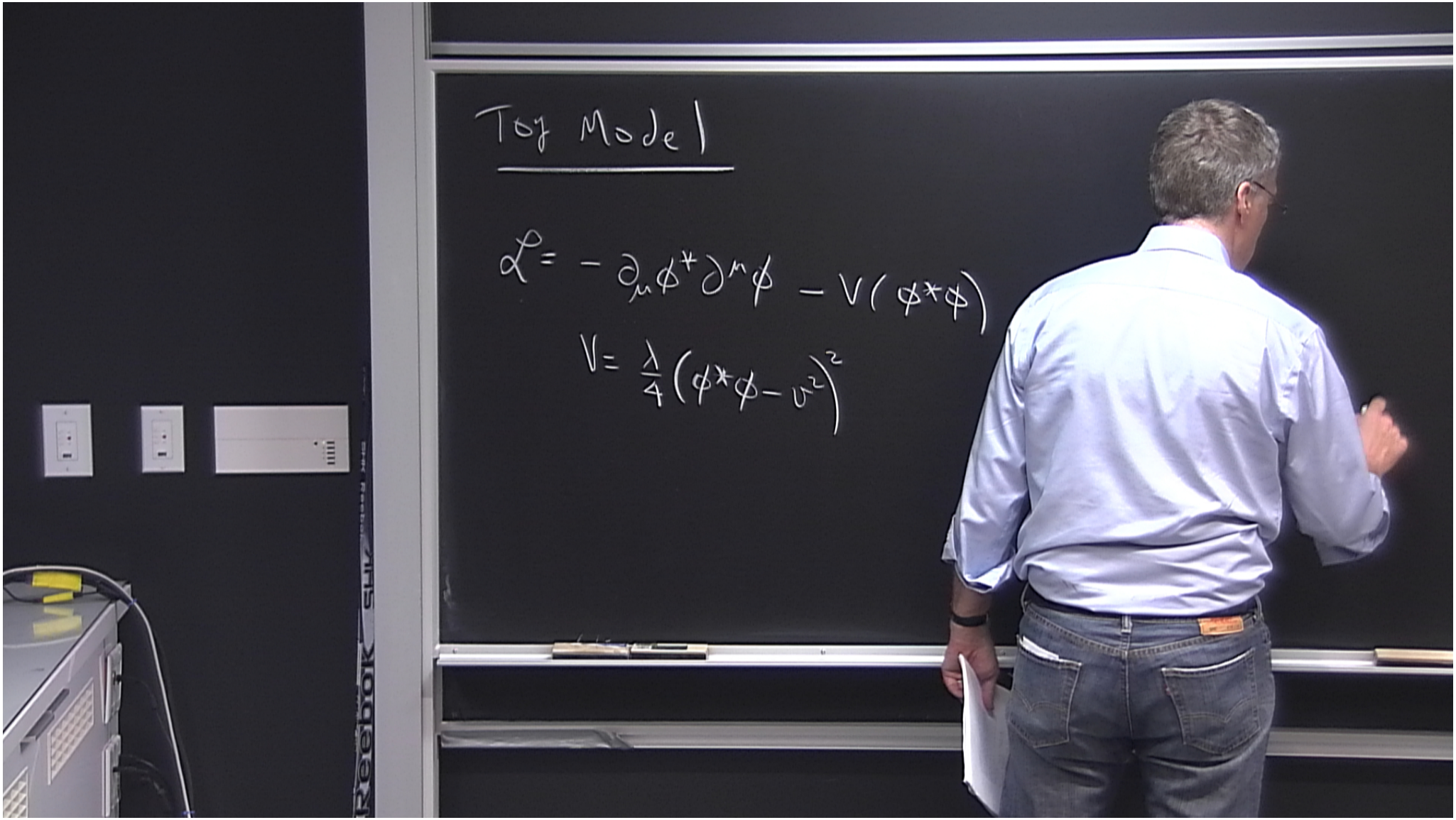
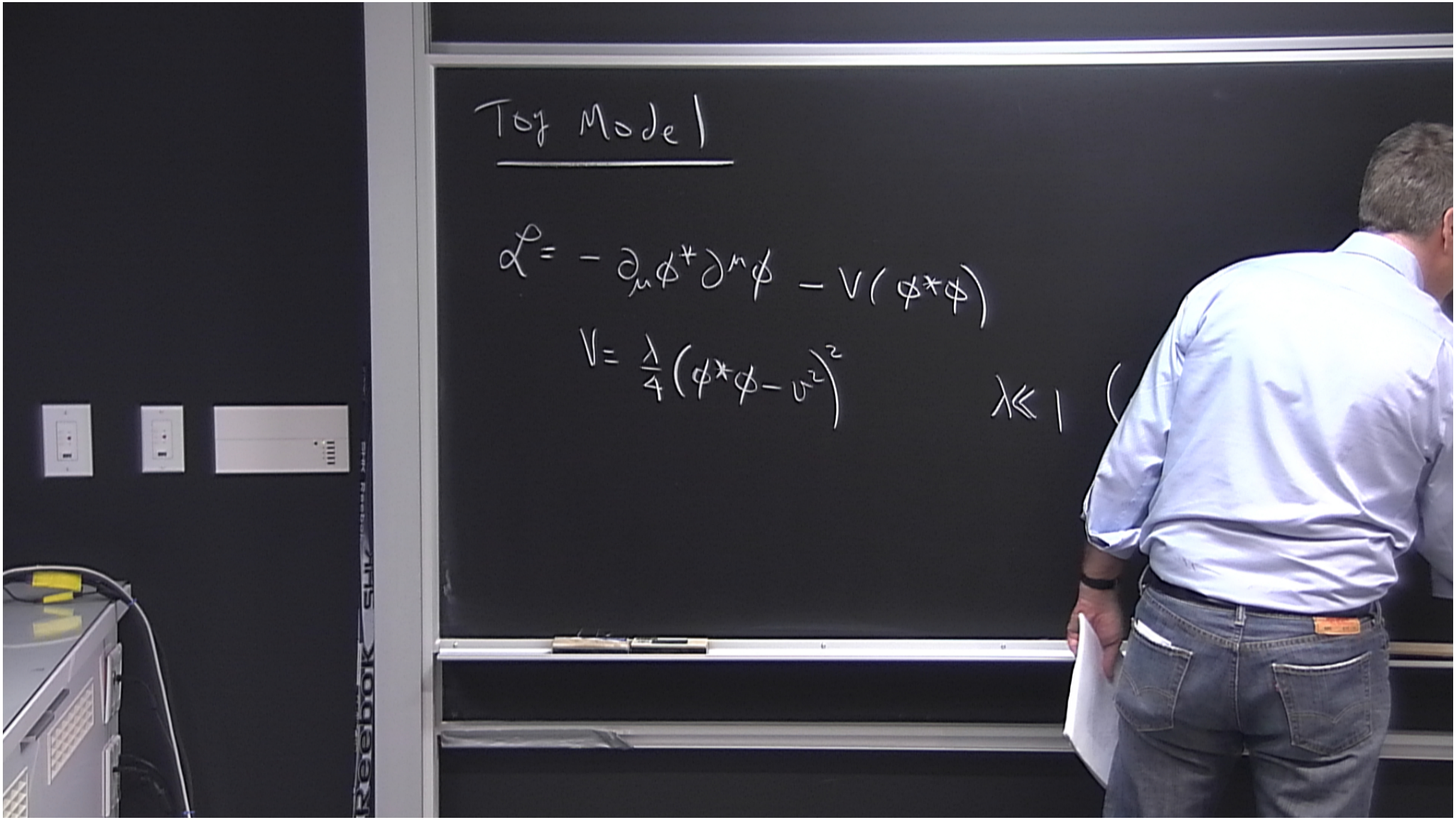
Title: Introduction to Effective Field Theories - Lecture 3

Date: Jan 17, 2014 09:00 AM

URL: http://pirsa.org/14010088

Abstract:


Toy Model
$\mathcal{L} = -\partial_\mu \phi^* \partial^\mu \phi - V(\phi^* \phi)$
$V = \frac{\lambda}{4}(\phi^* \phi - v^2)^2$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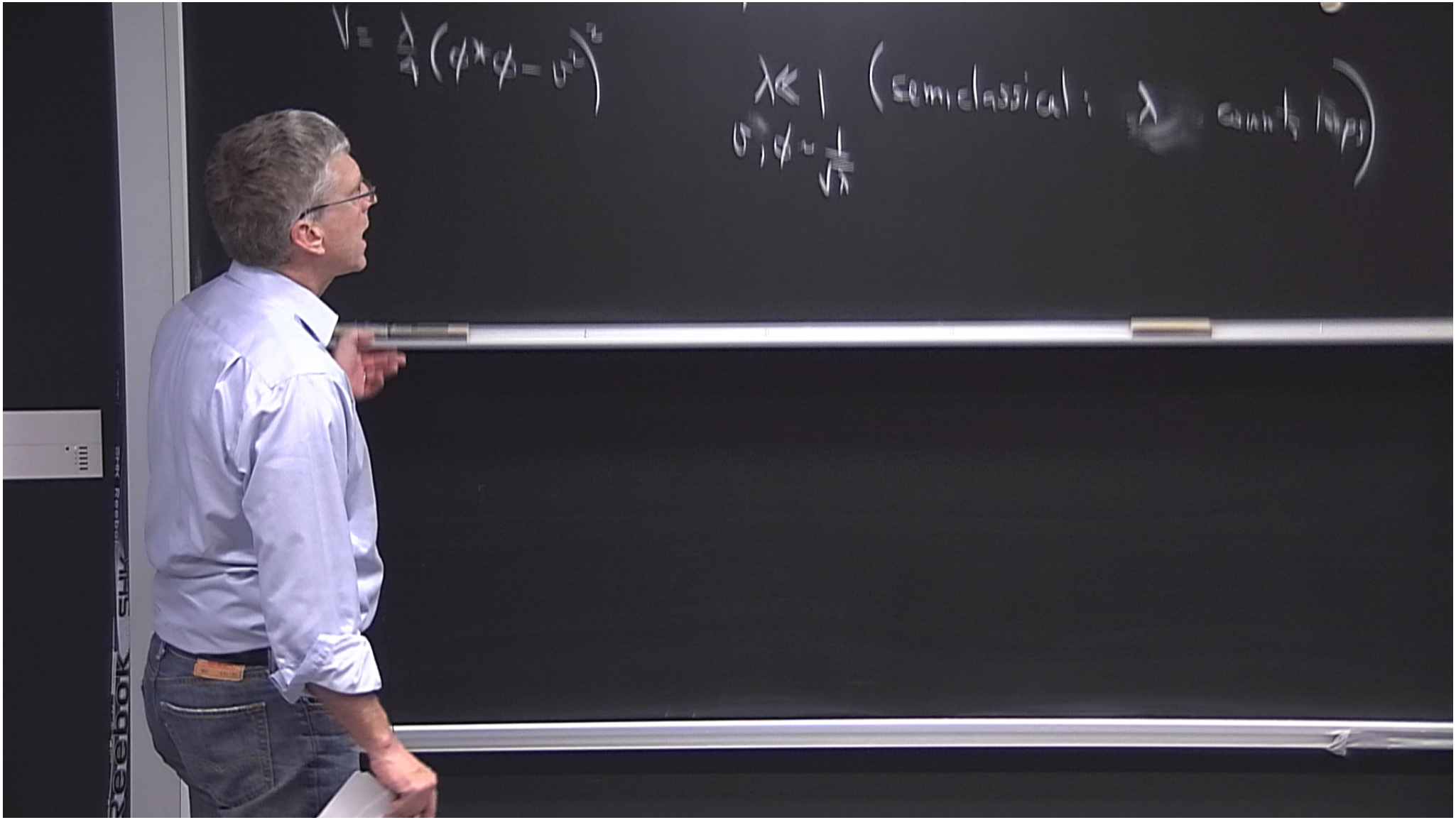
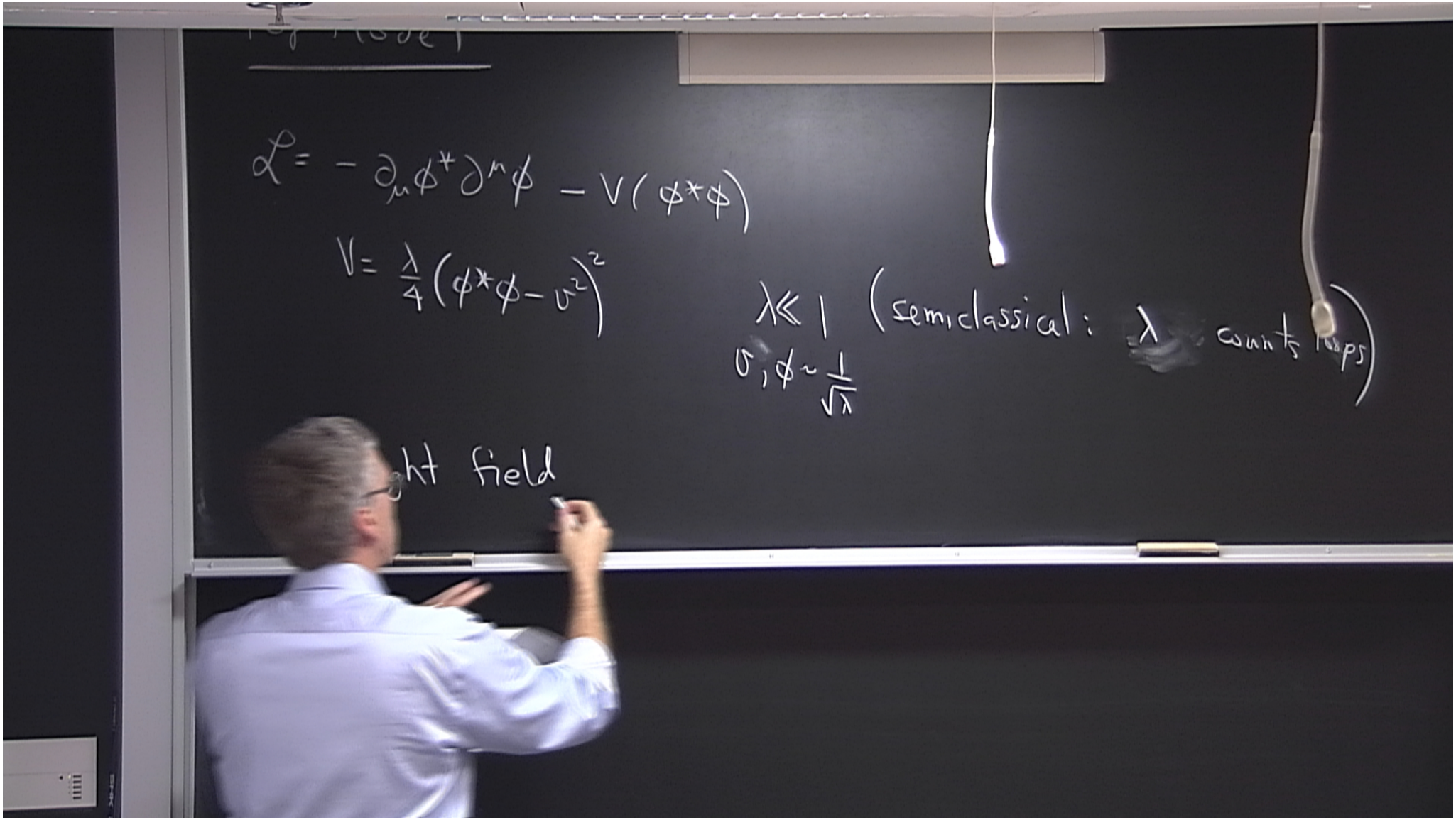
$\lambda \ll 1$


$V = \frac{\lambda}{4}(\phi^*\phi - v^2)^2$
$\lambda \ll 1$
(semiclassical: λ
$\sigma, \phi \sim \frac{1}{\sqrt{\lambda}}$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$V = \frac{\lambda}{4}(\phi^* \phi - v^2)^2$
$\lambda \ll 1$ (semiclassical: λ counts loops)
$v, \phi \sim \frac{1}{\sqrt{\lambda}}$
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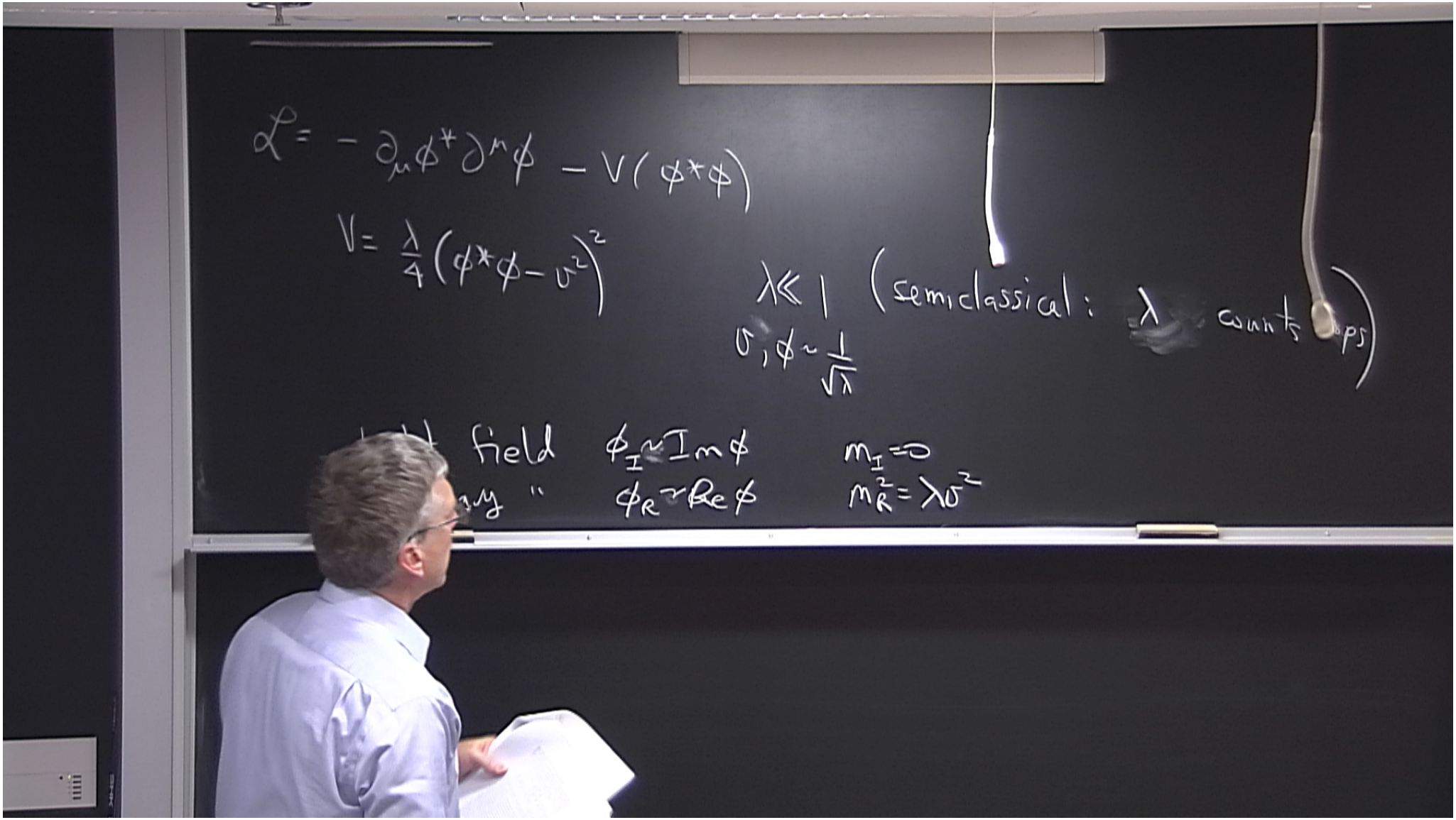
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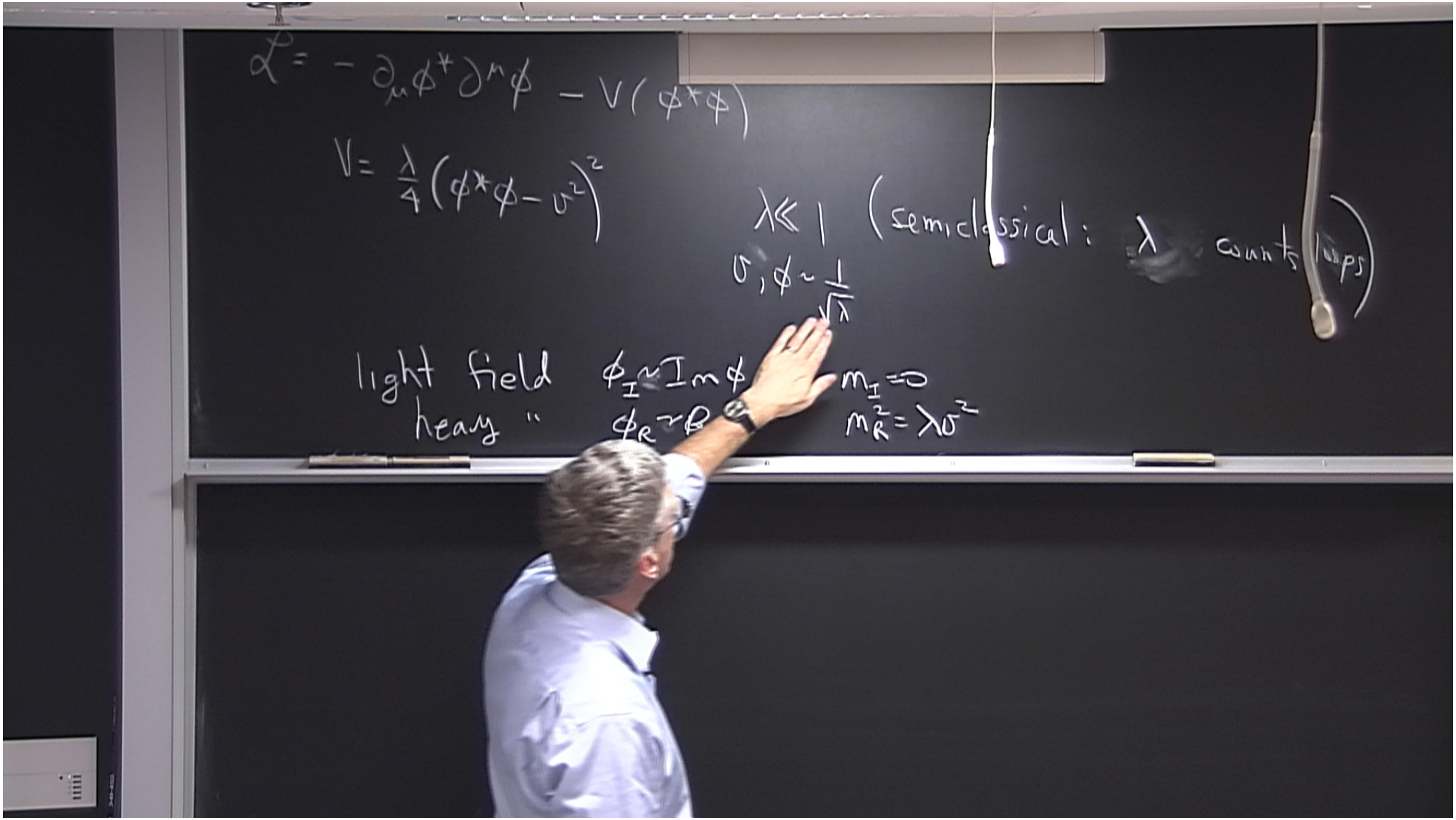
field $\phi_I \sim \mathrm{Im}\,\phi$ $m_I = 0$
$\phi_R \sim \mathrm{Re}\,\phi$ $m_R^2 = \lambda v^2$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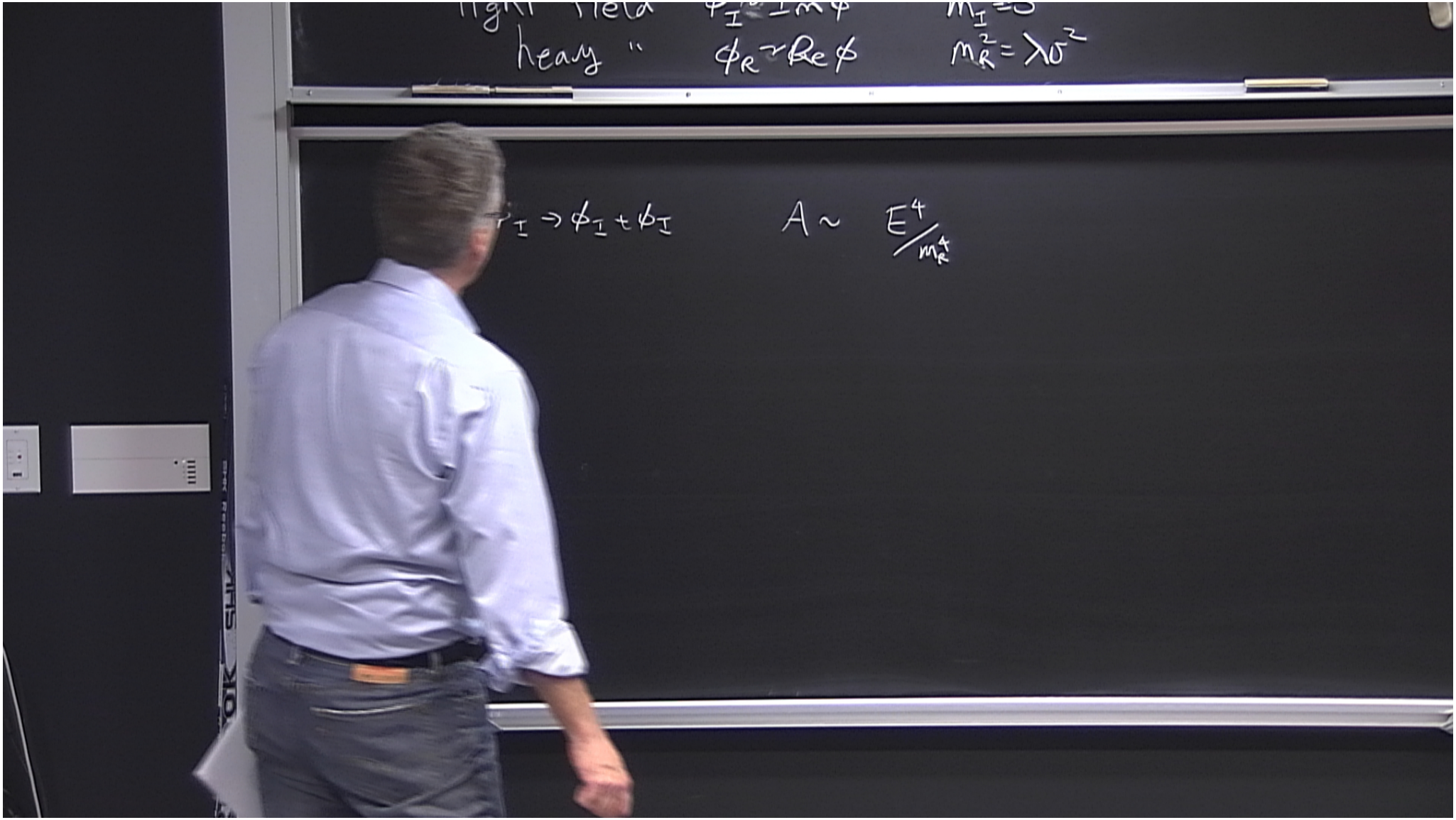
light field $\phi_I \sim \mathrm{Im}\,\phi$ $m_I = 0$
heavy " $\phi_R \sim R$ $m_R^2 = \lambda v^2$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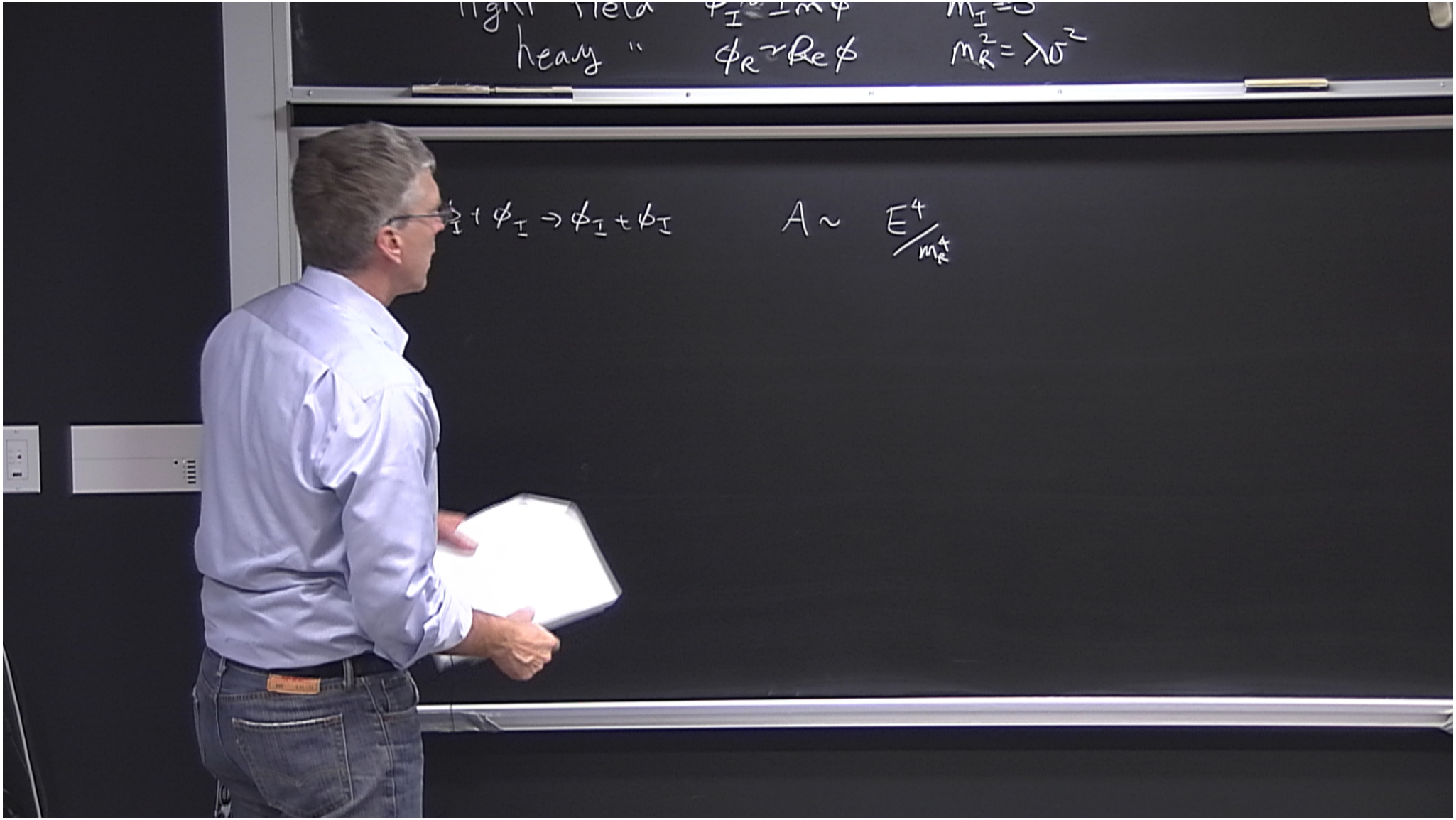
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\phi_I \to \phi_I + \phi_I$
$A \sim E^4 / m_R^4$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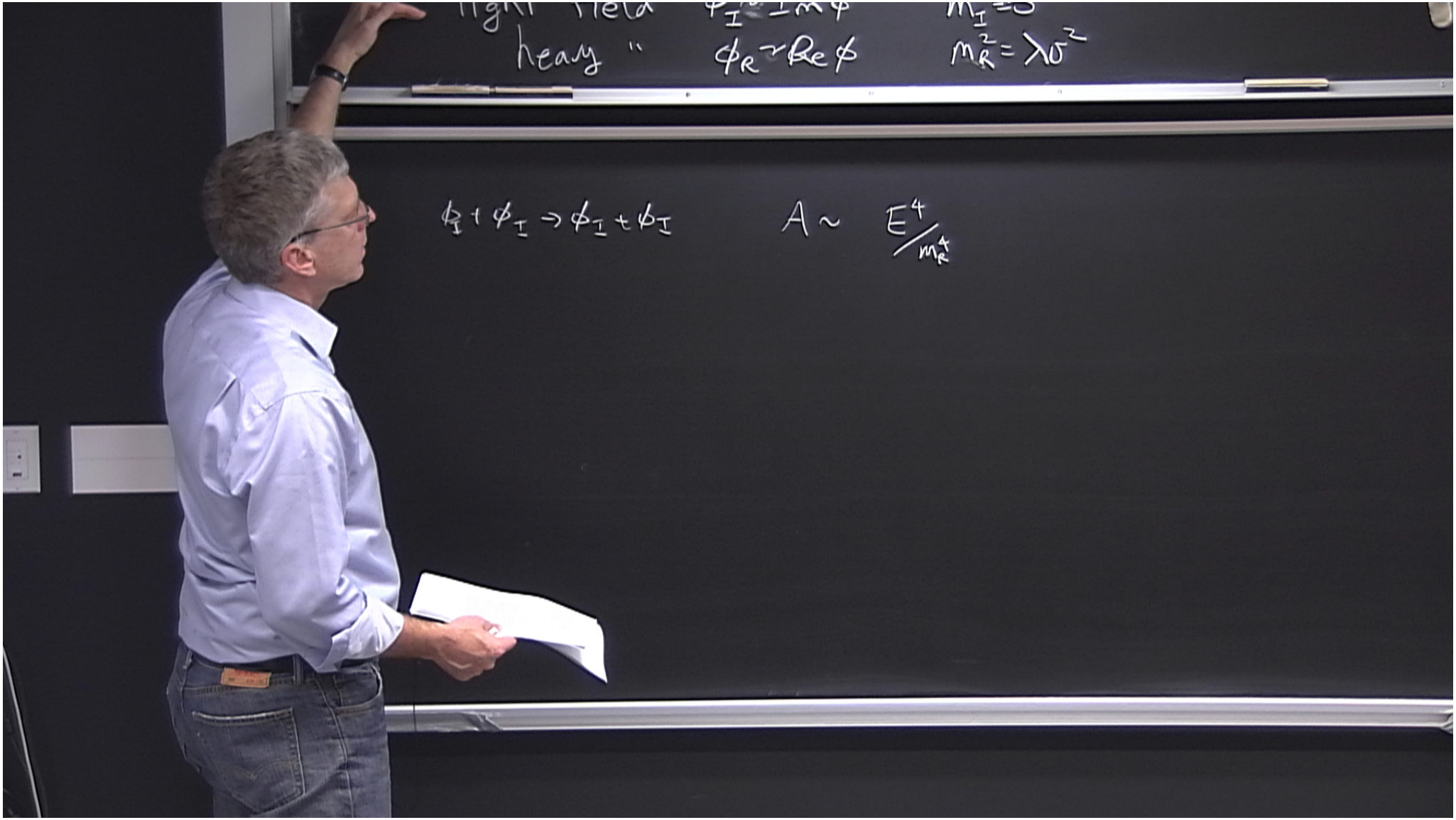
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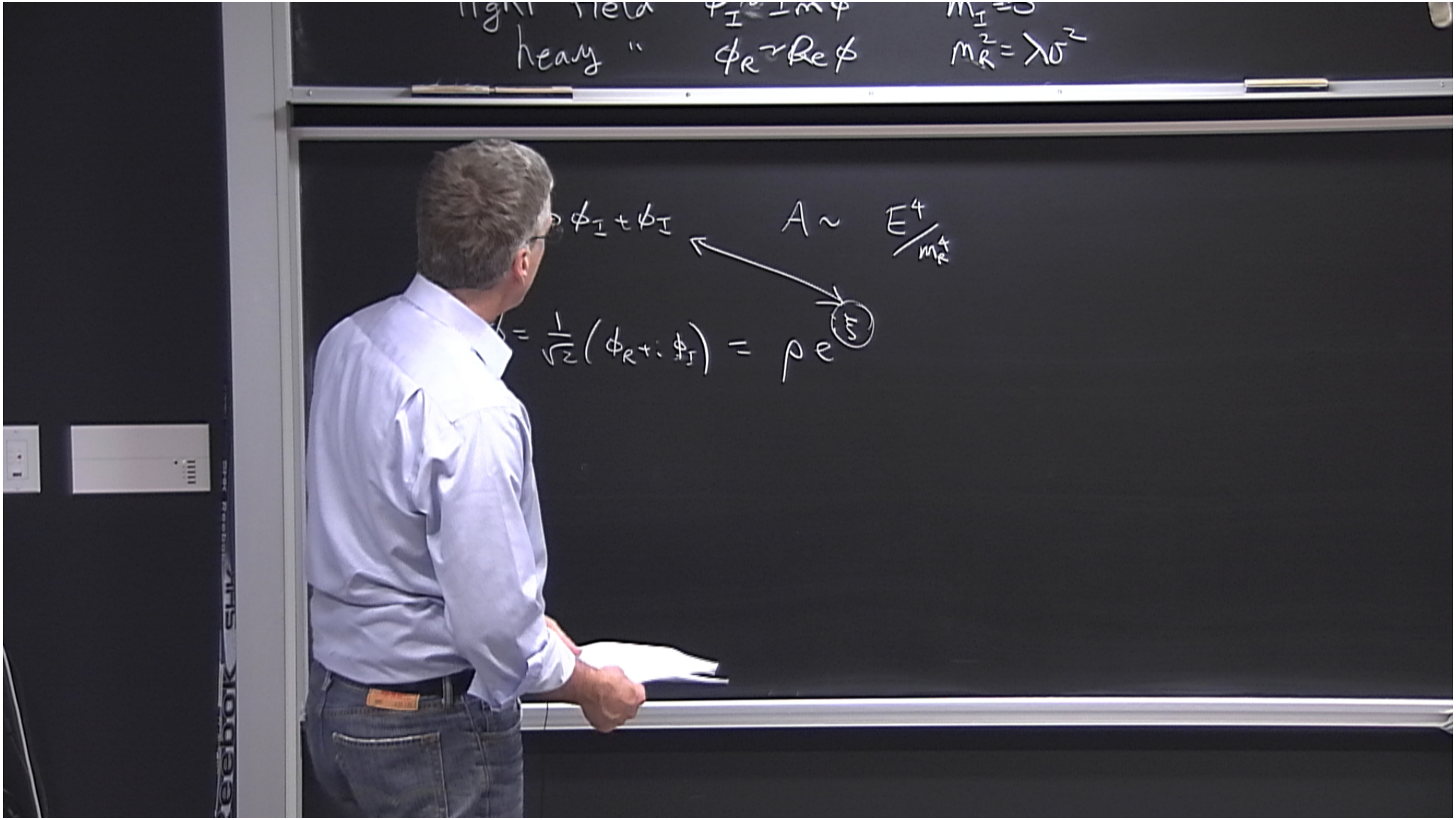
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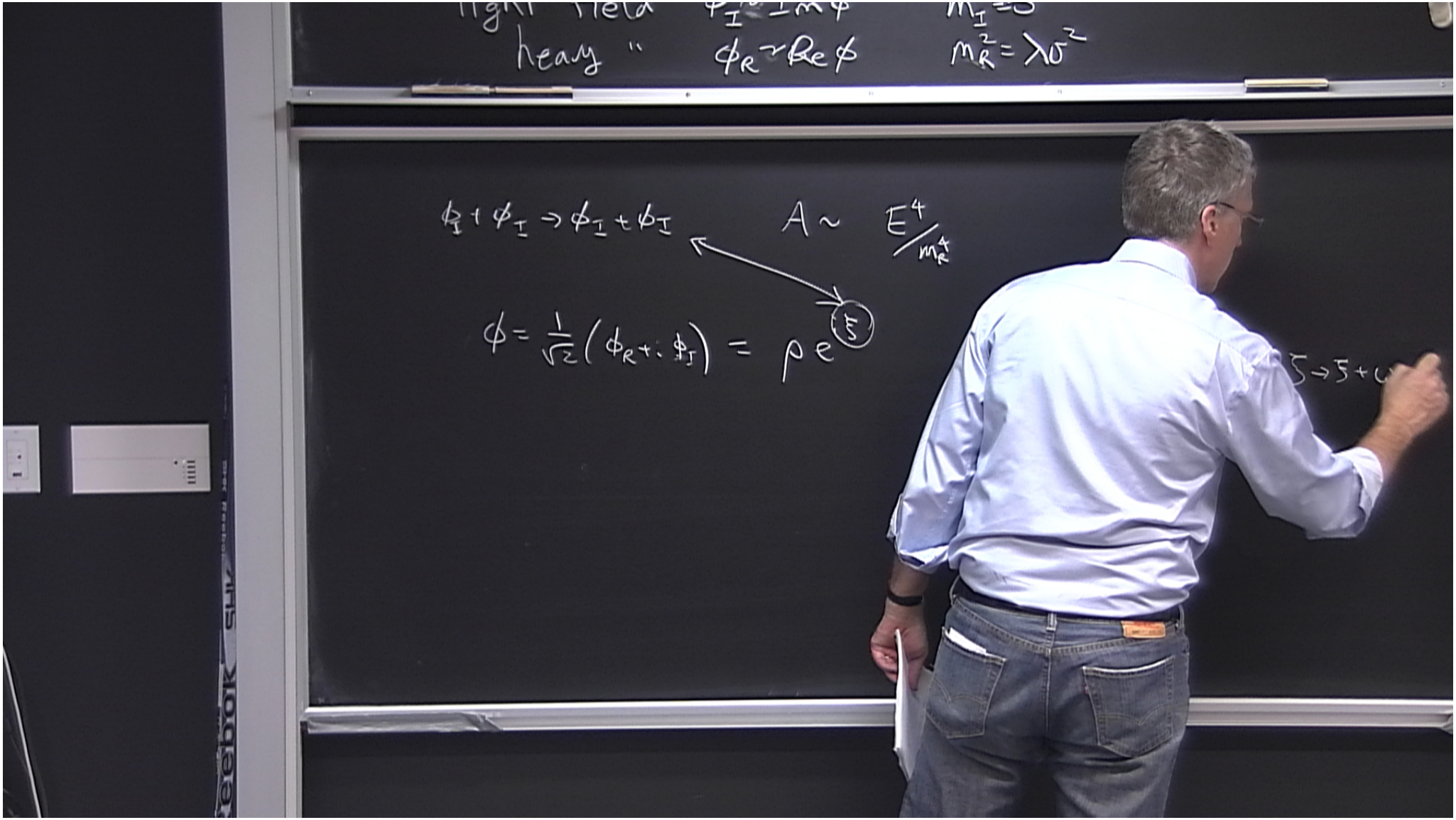
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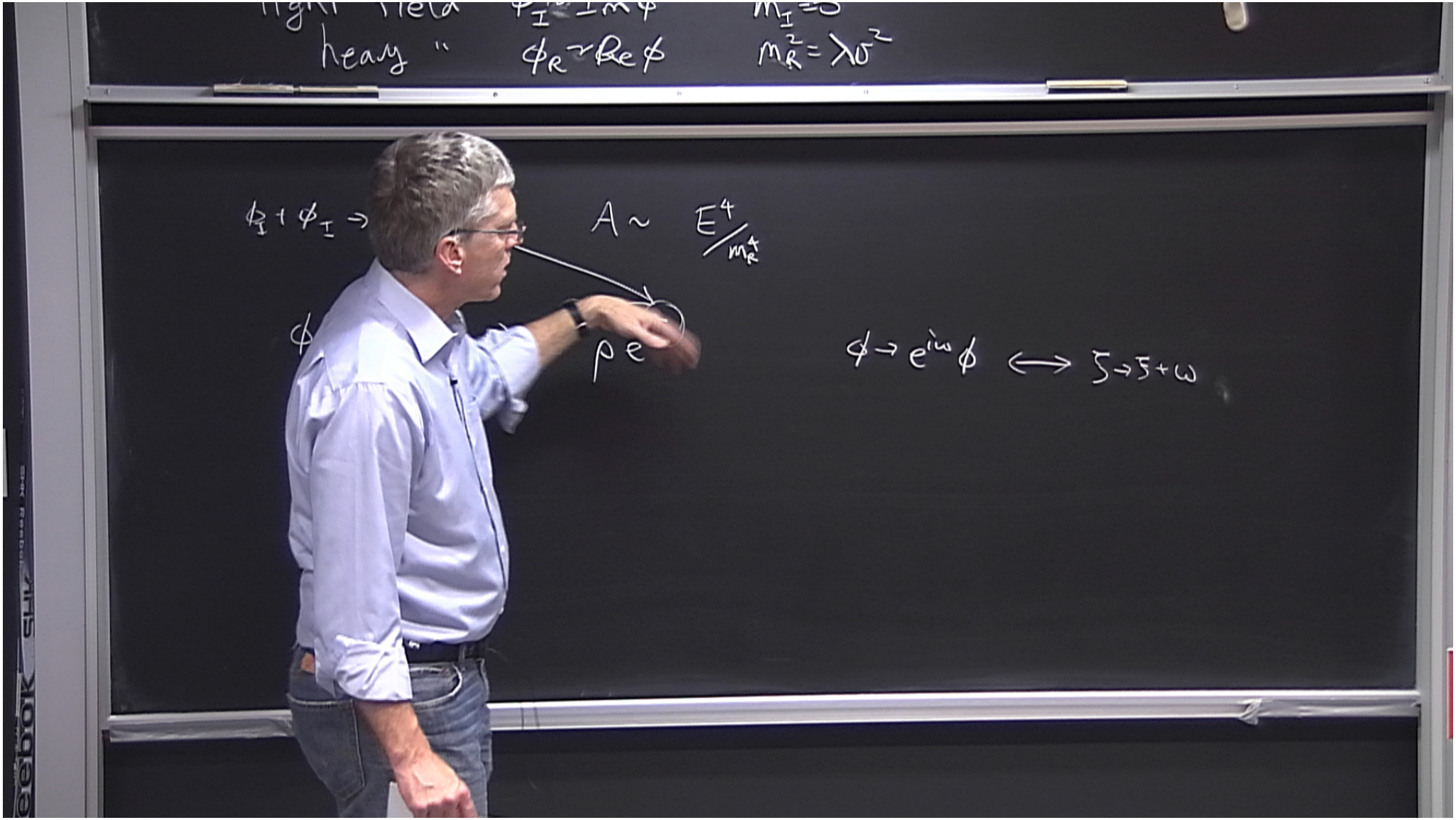
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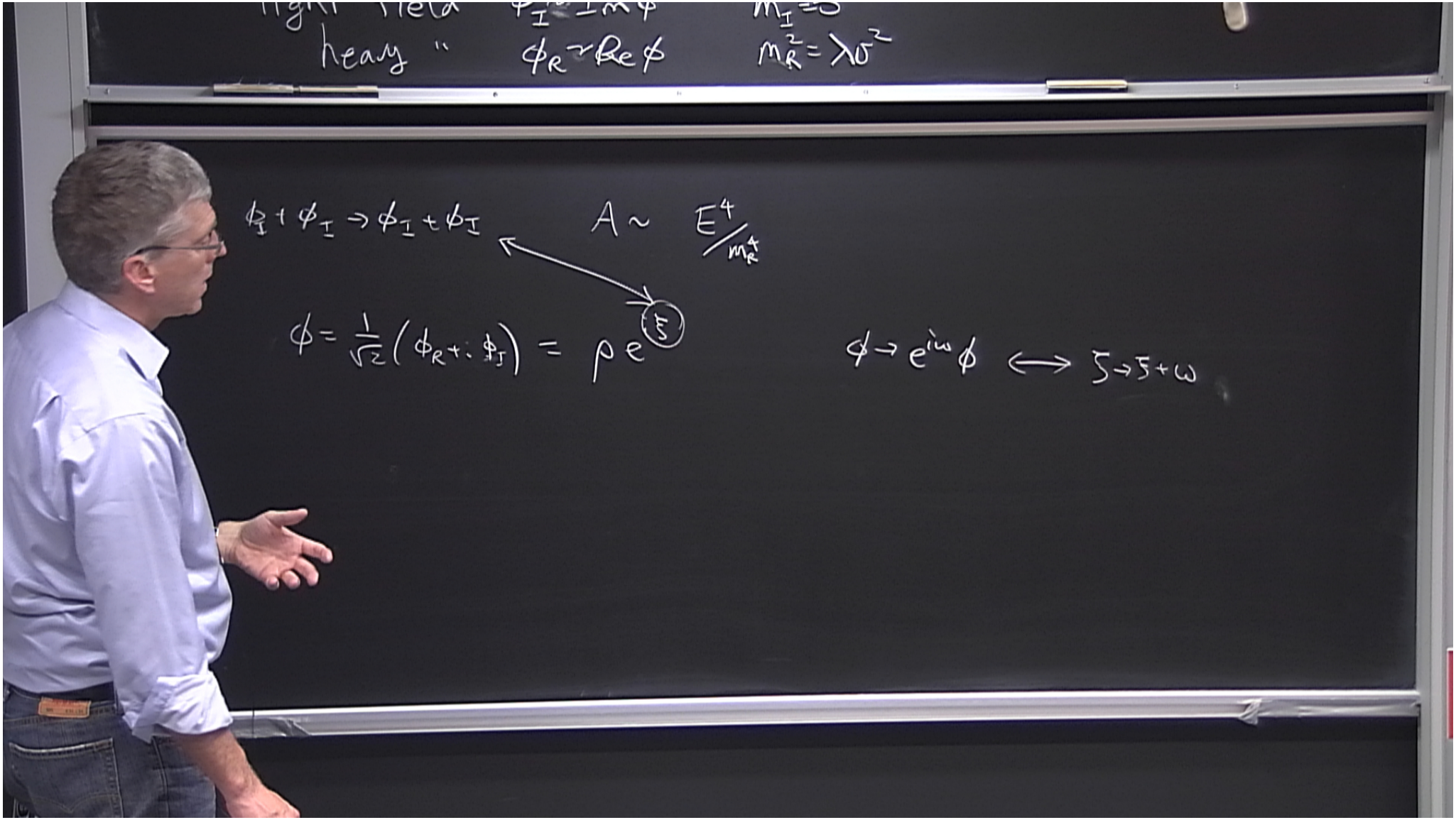
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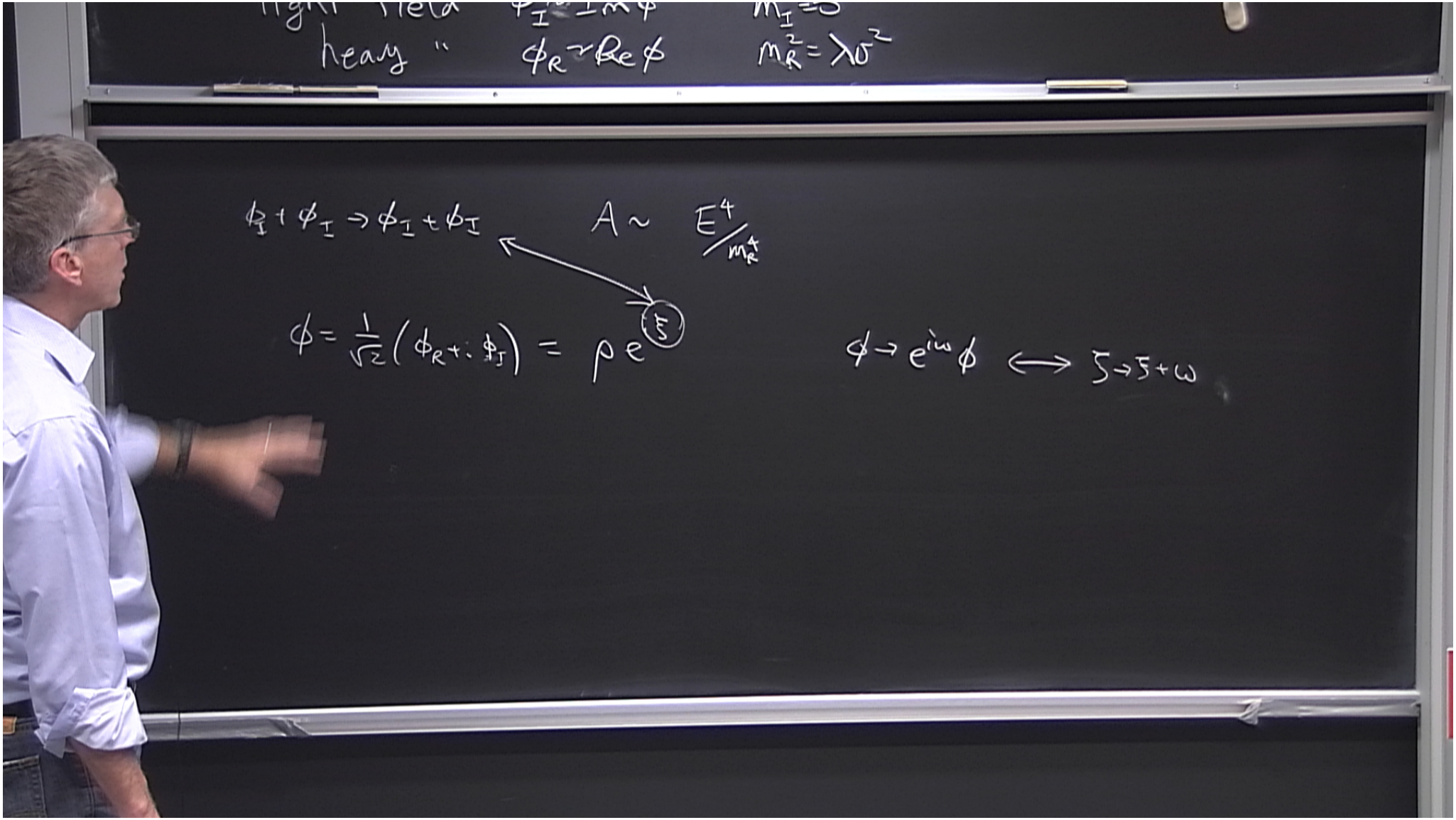
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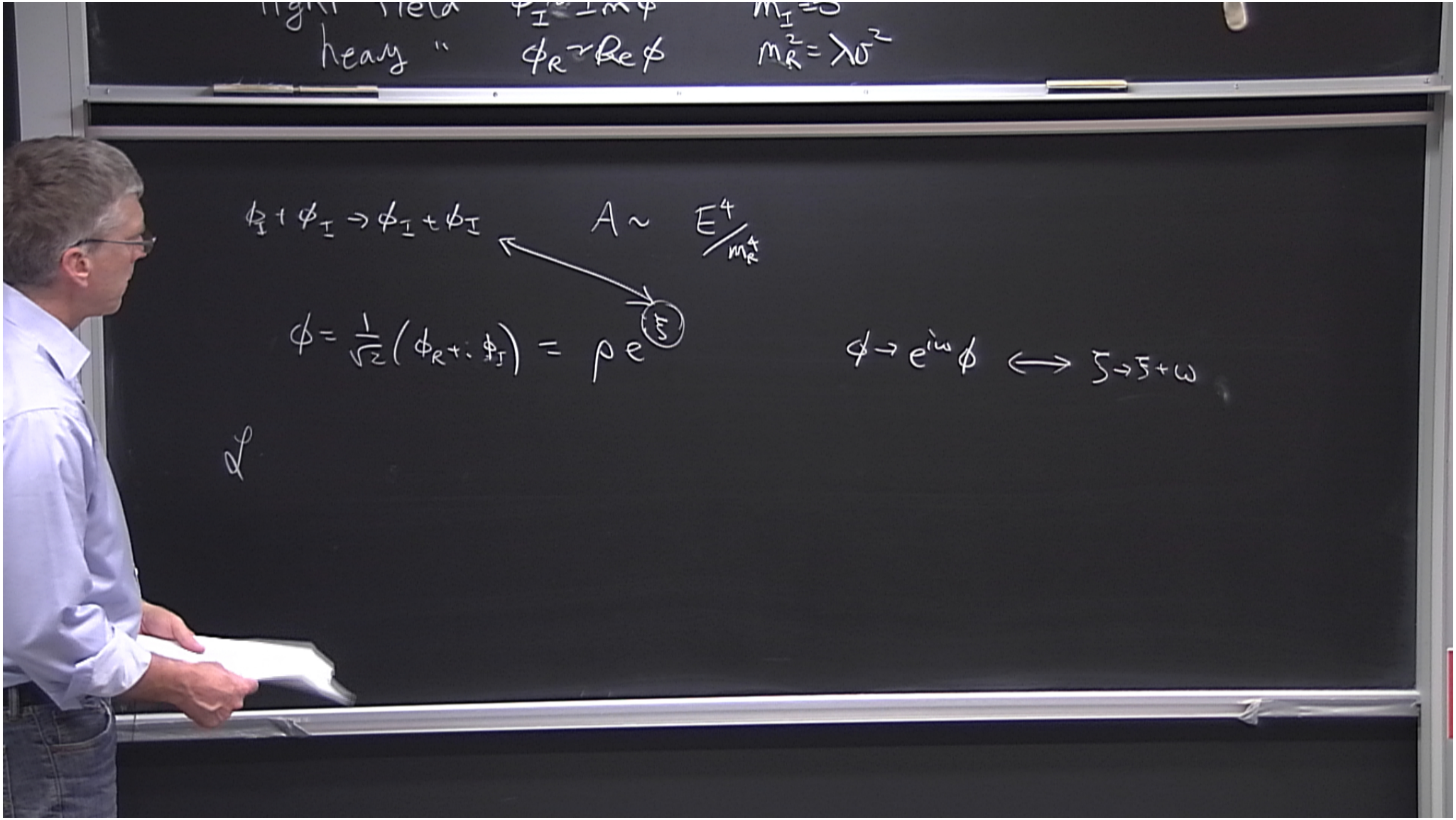
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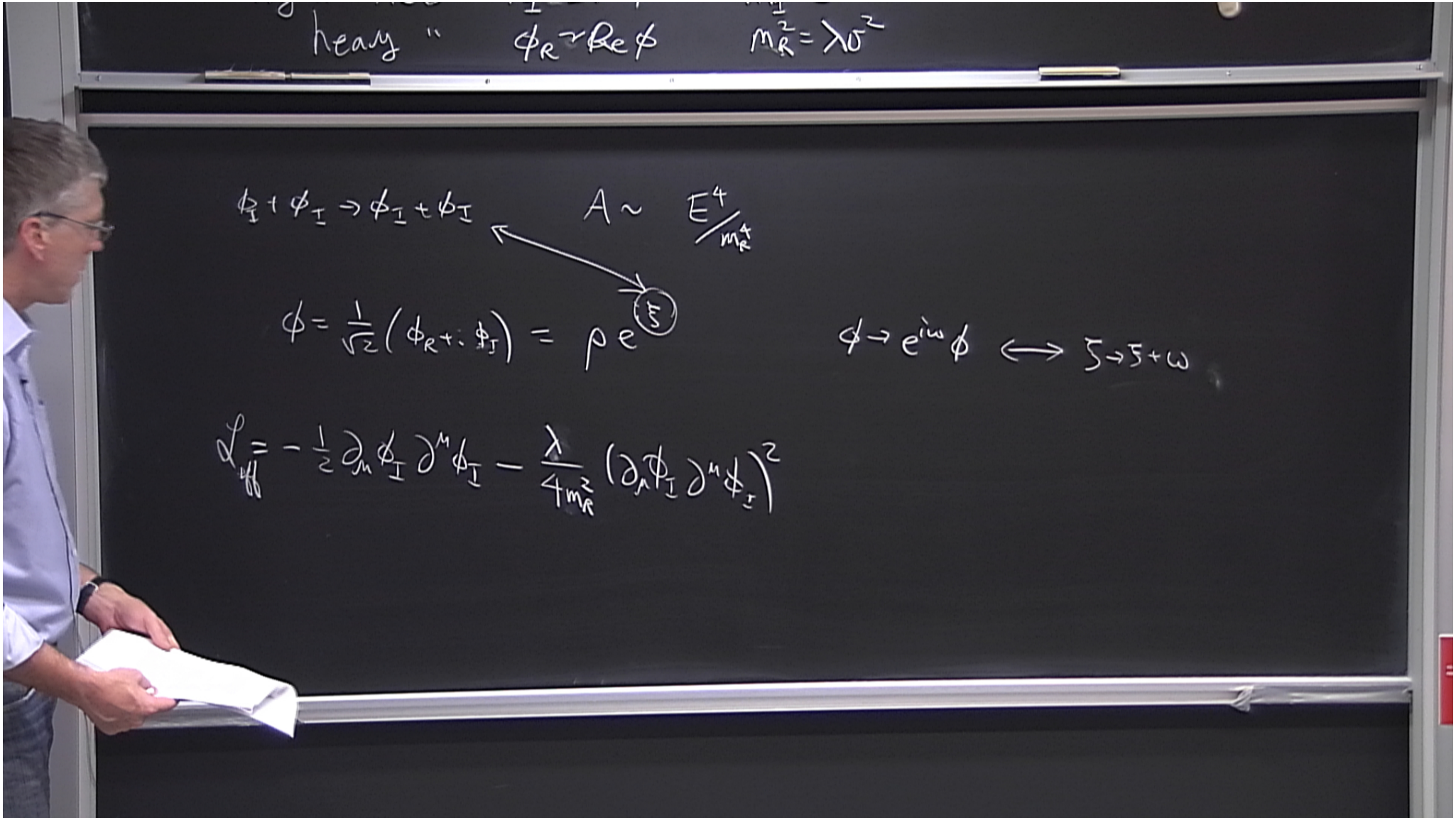
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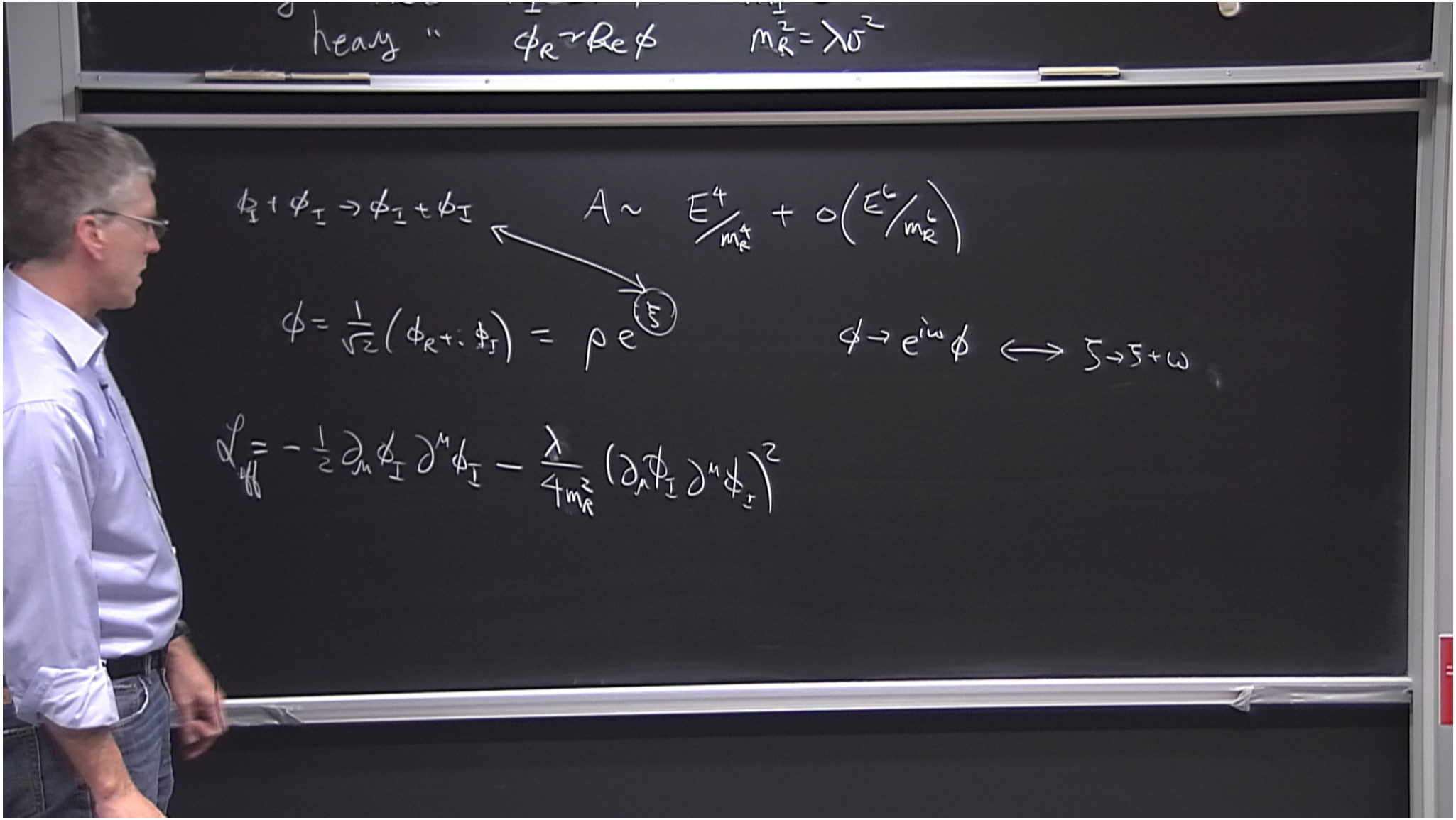
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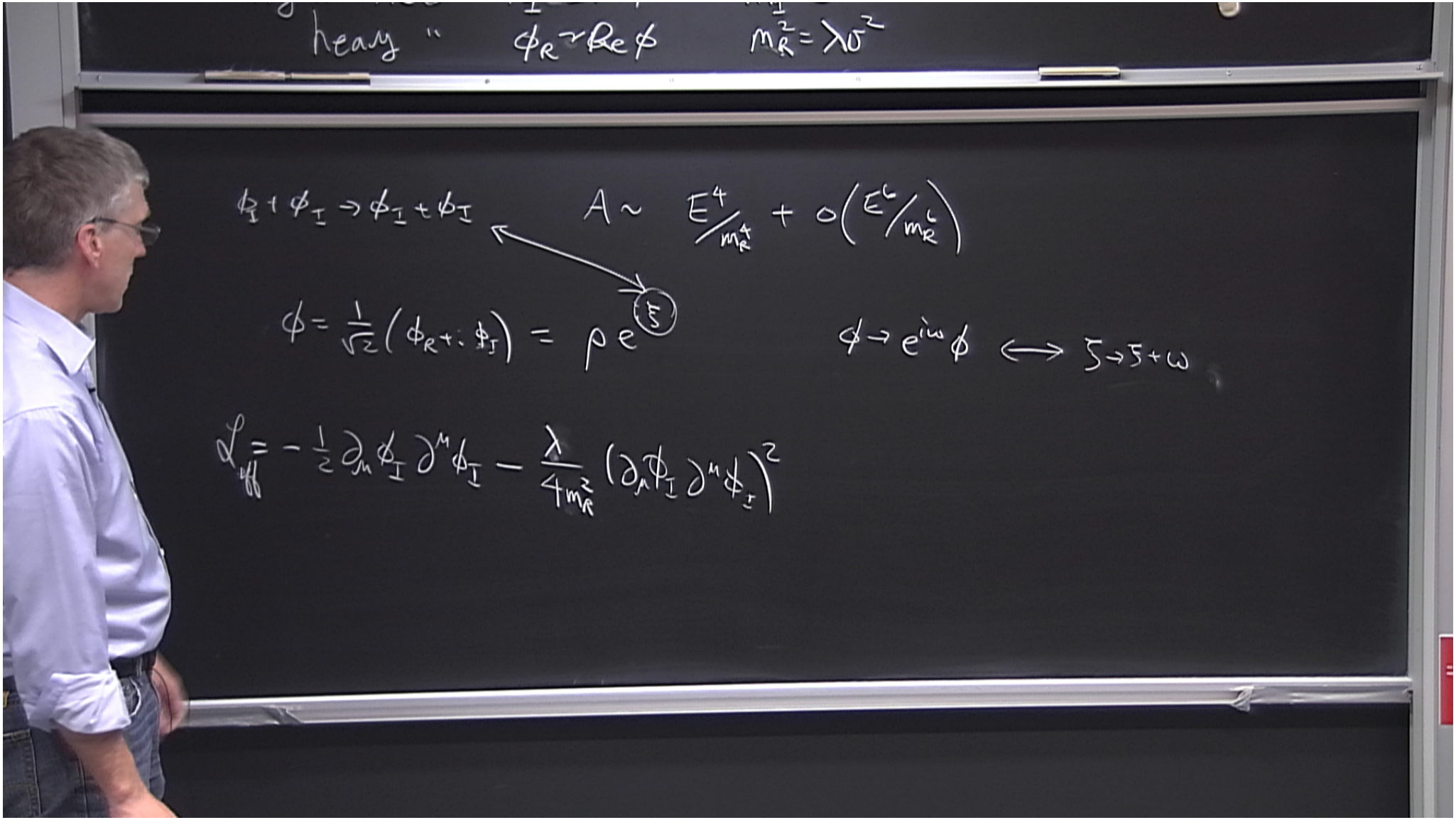
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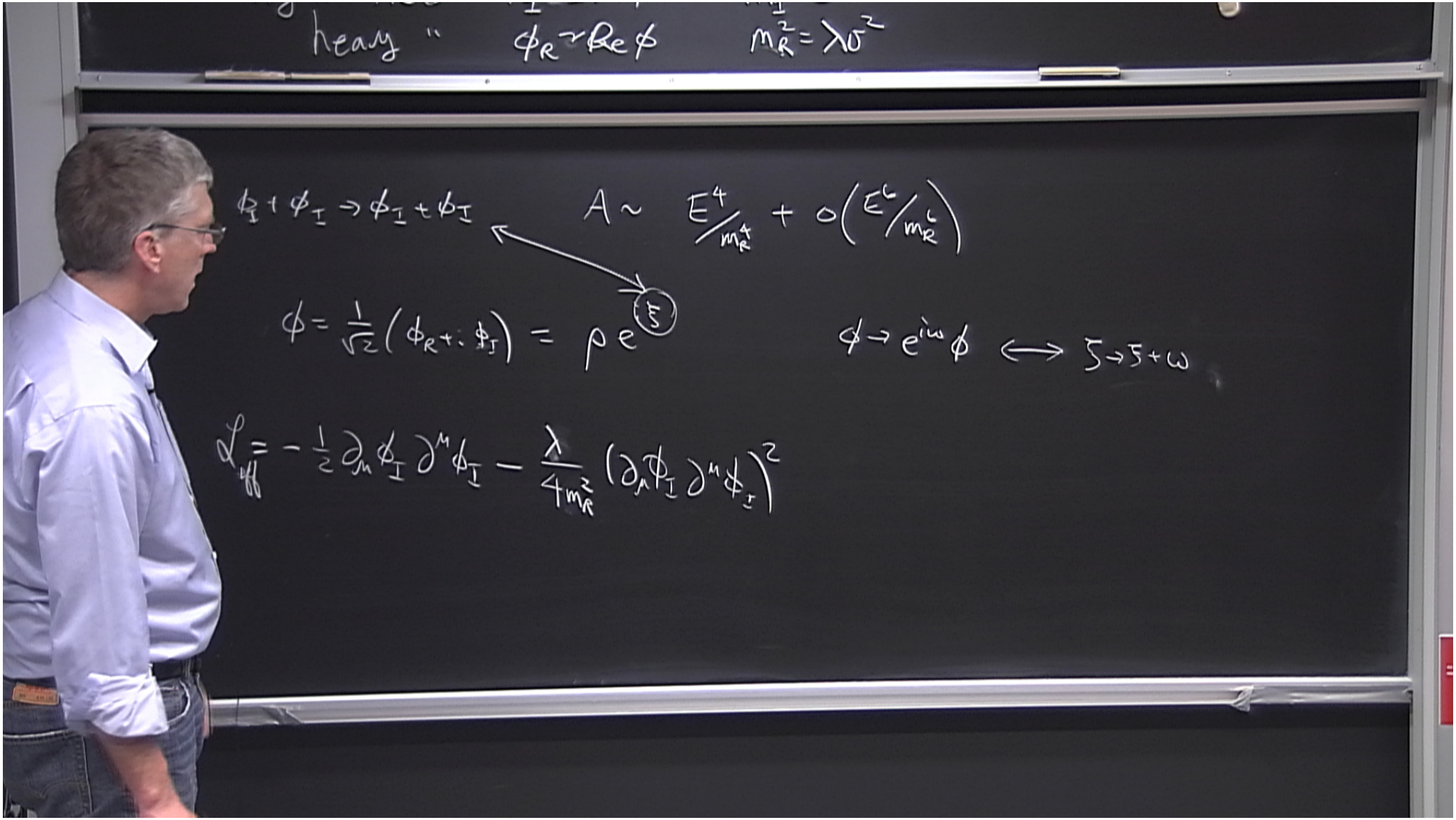
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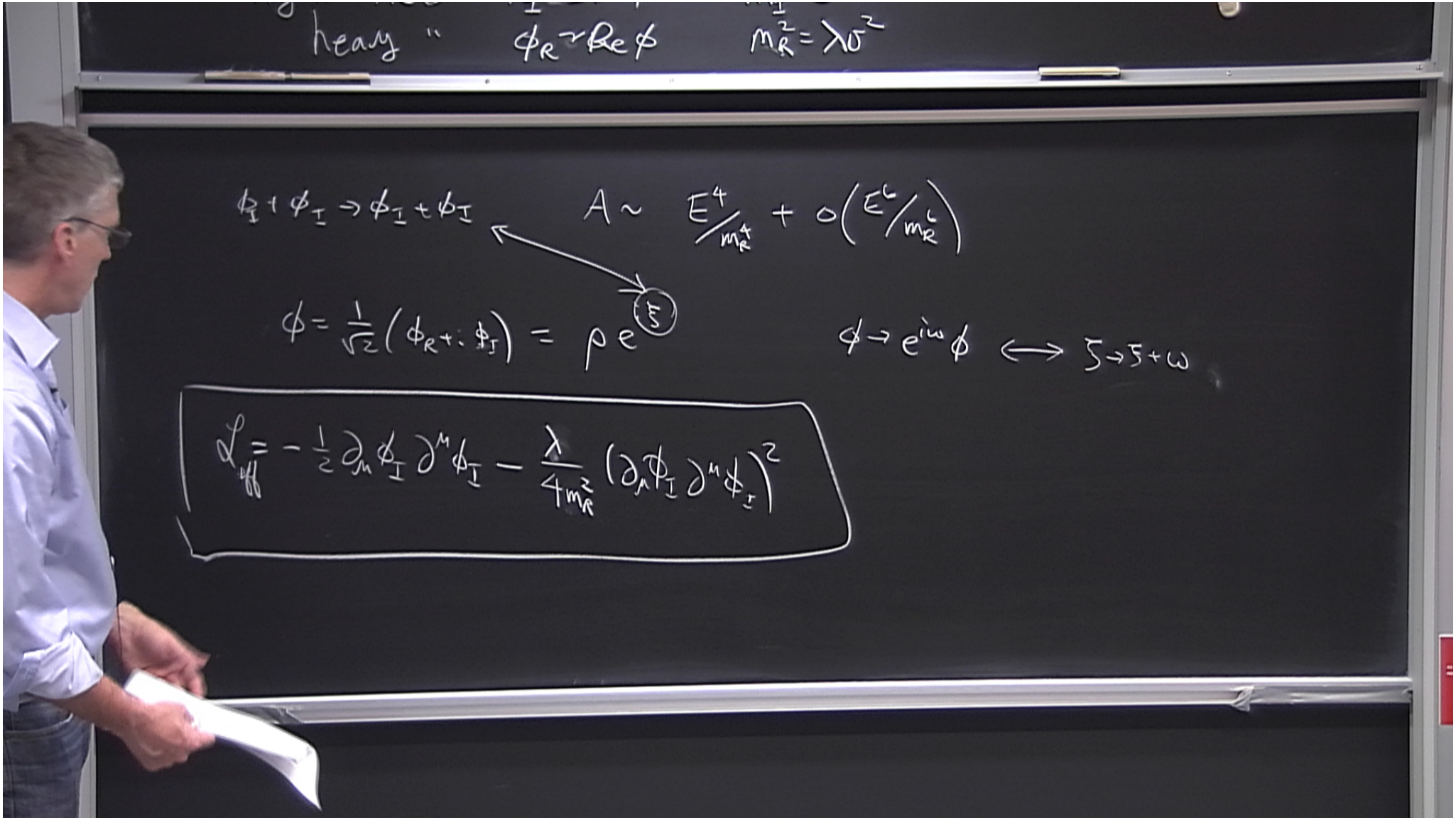
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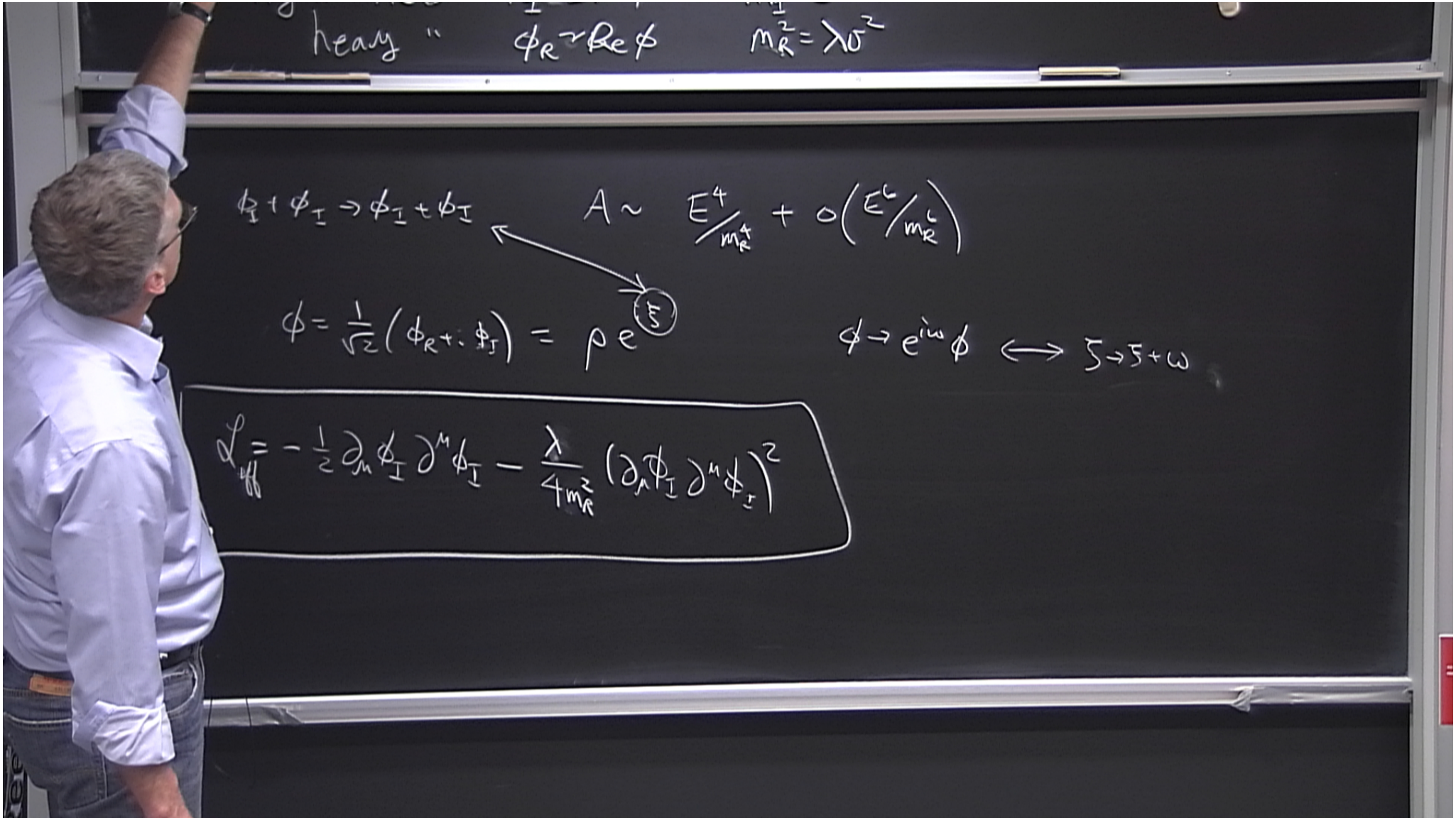
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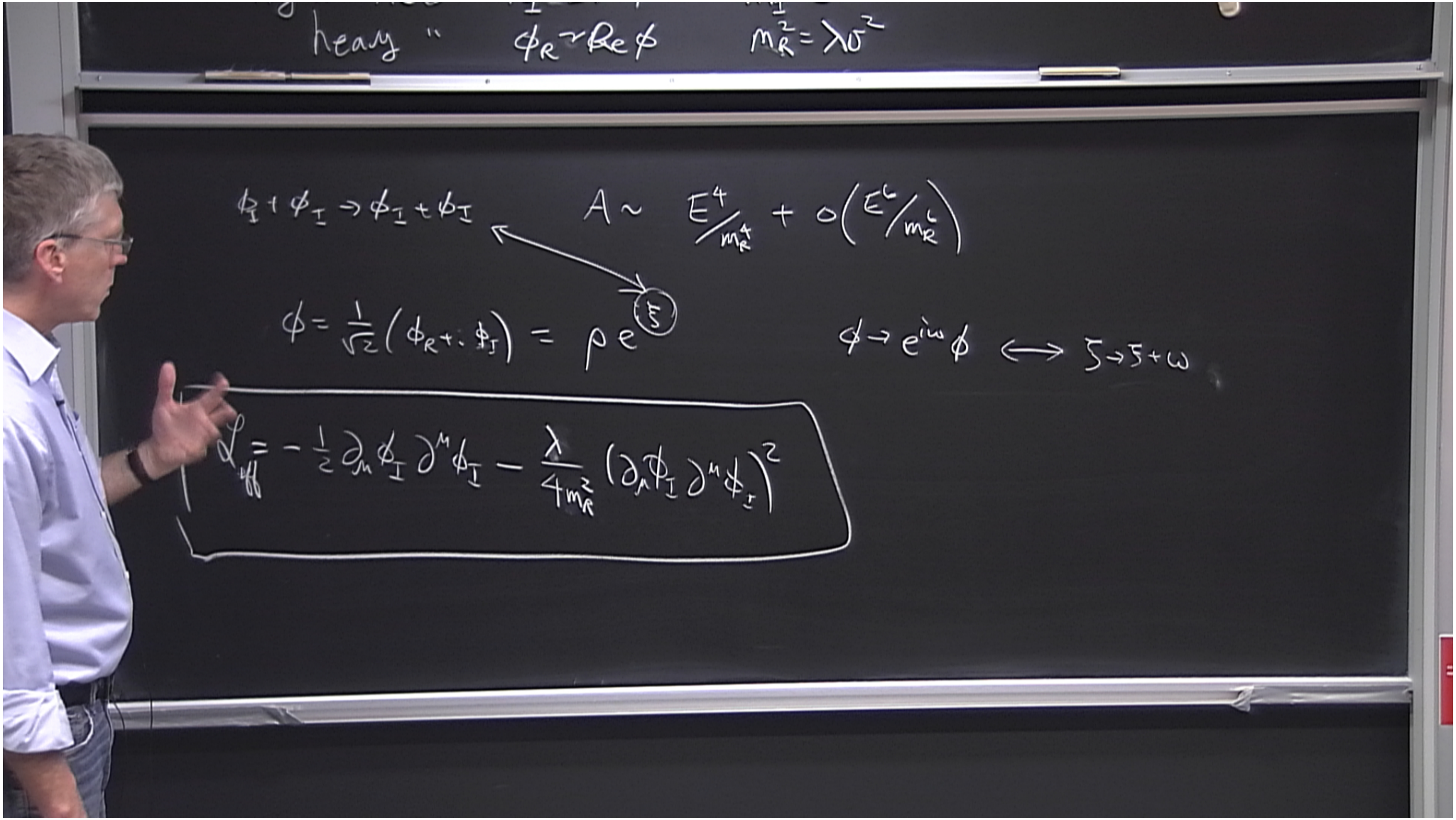
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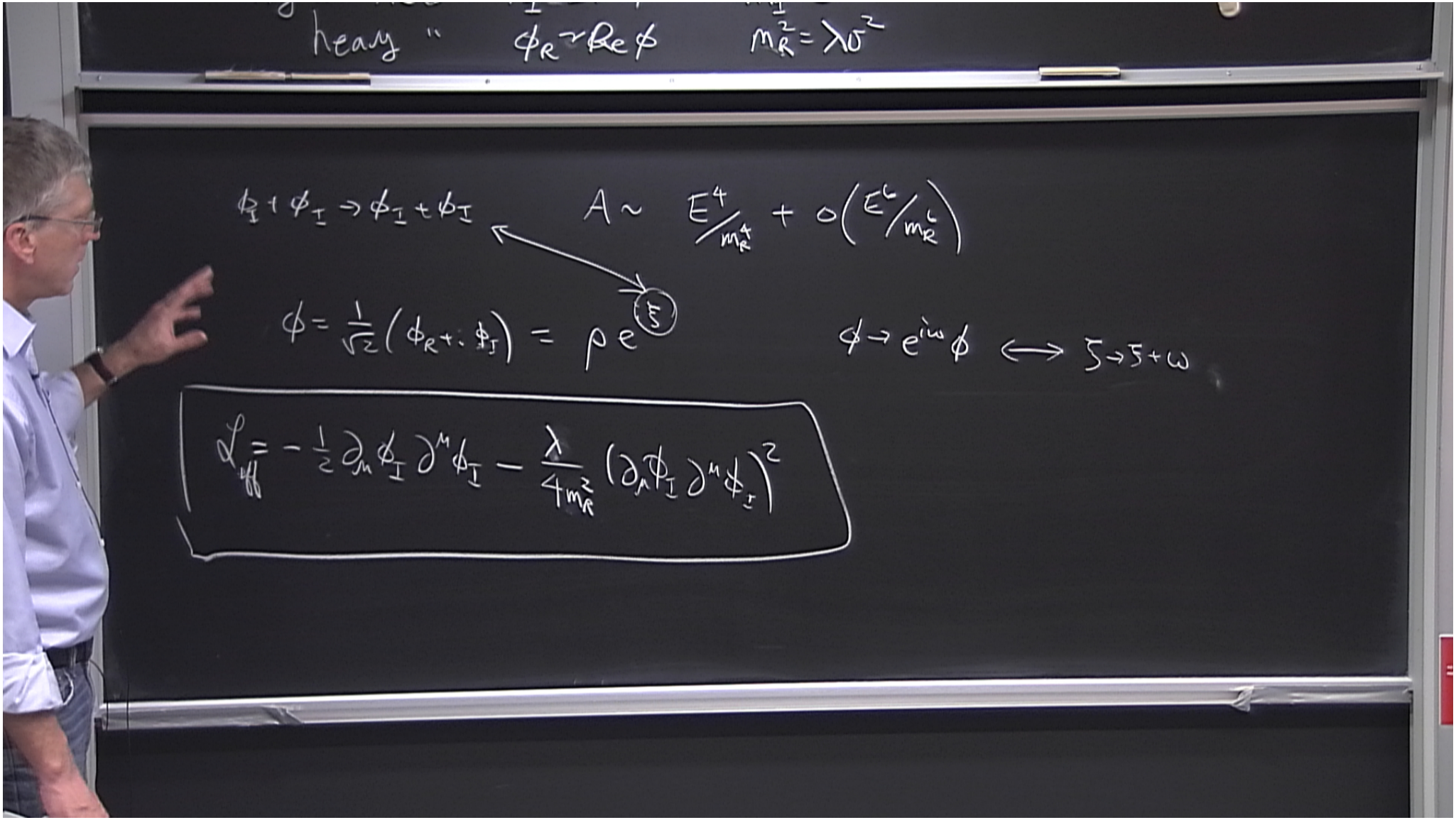
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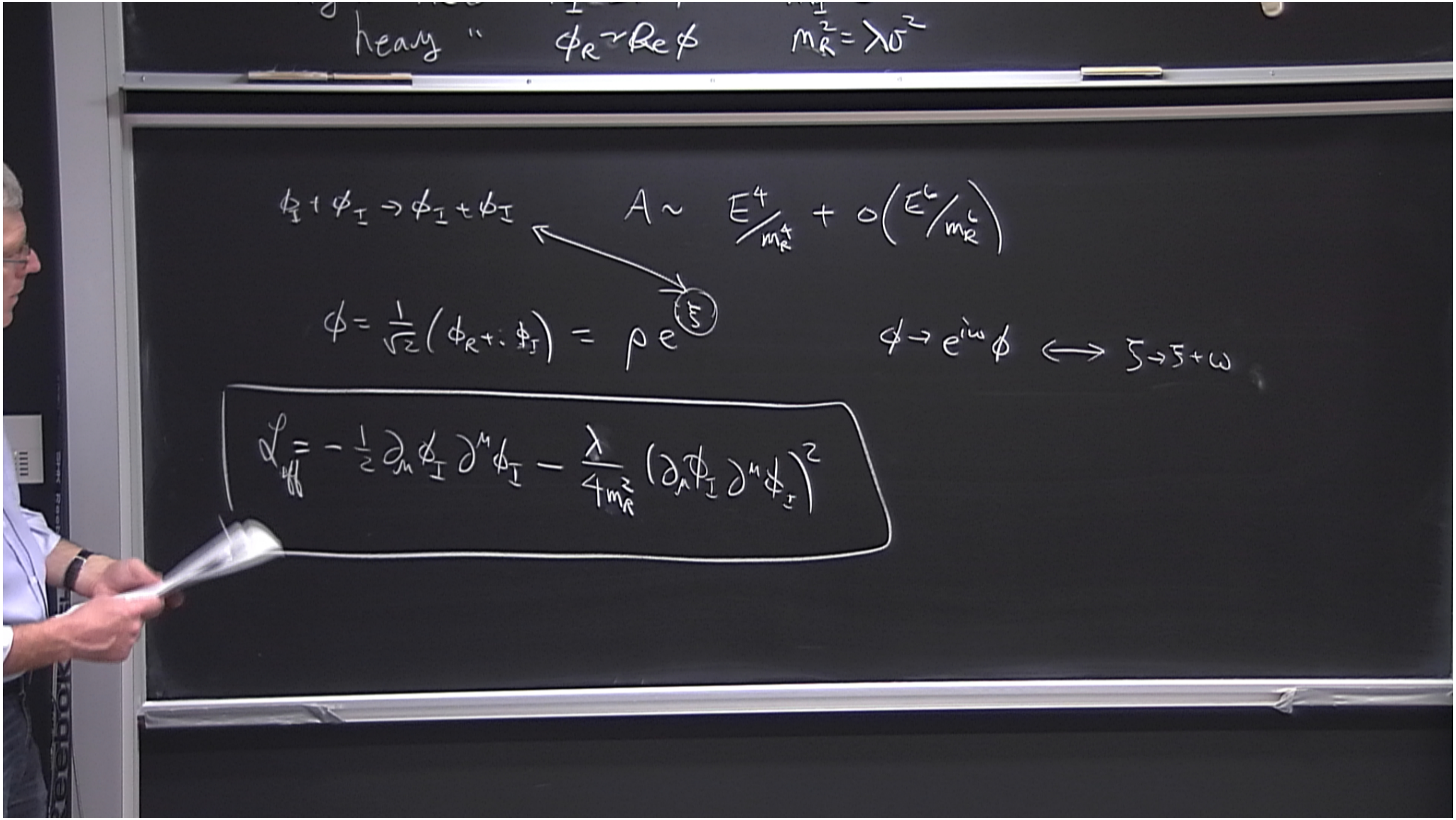
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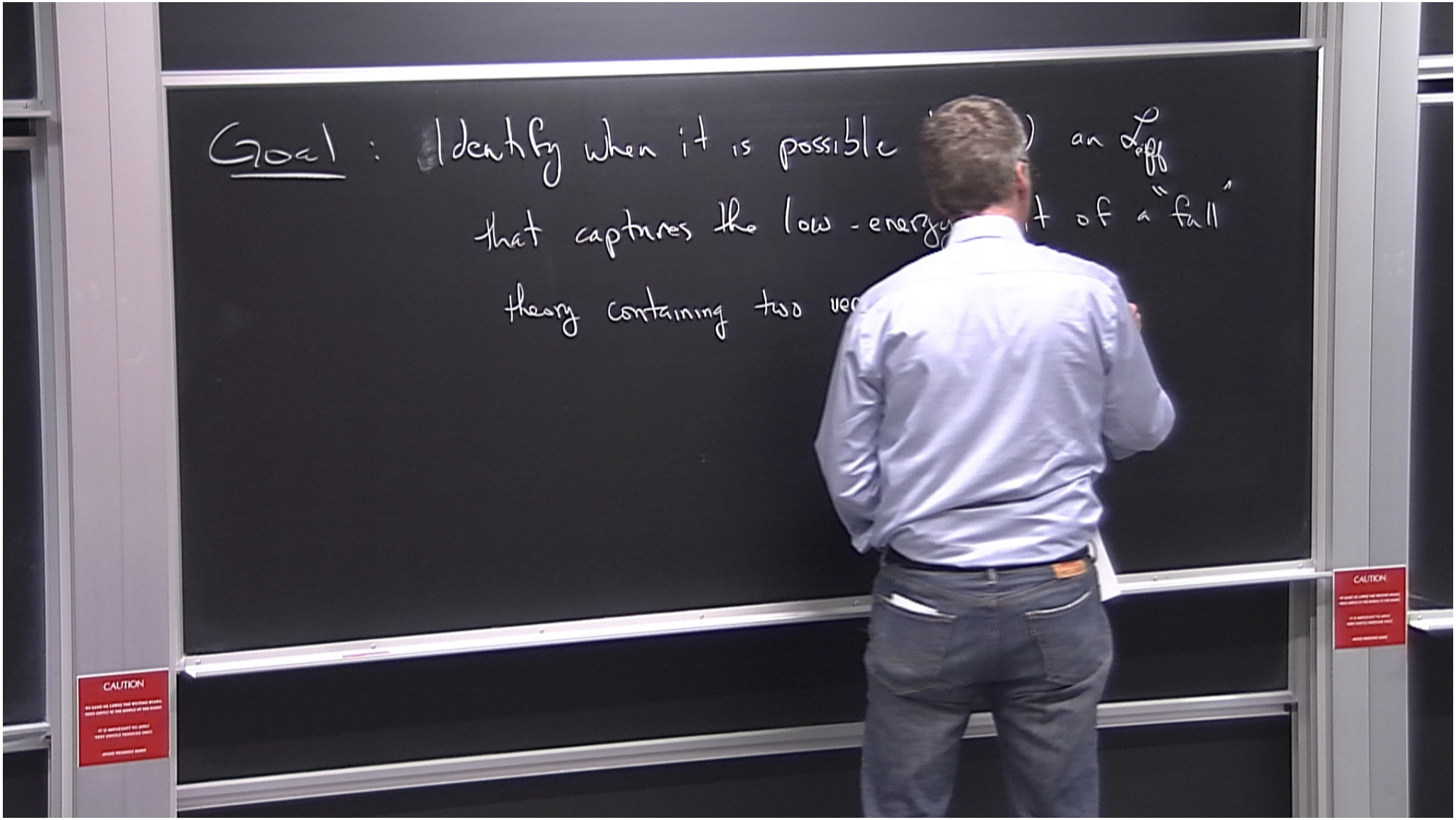
Goal : Identify when it is possible
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CAUTION
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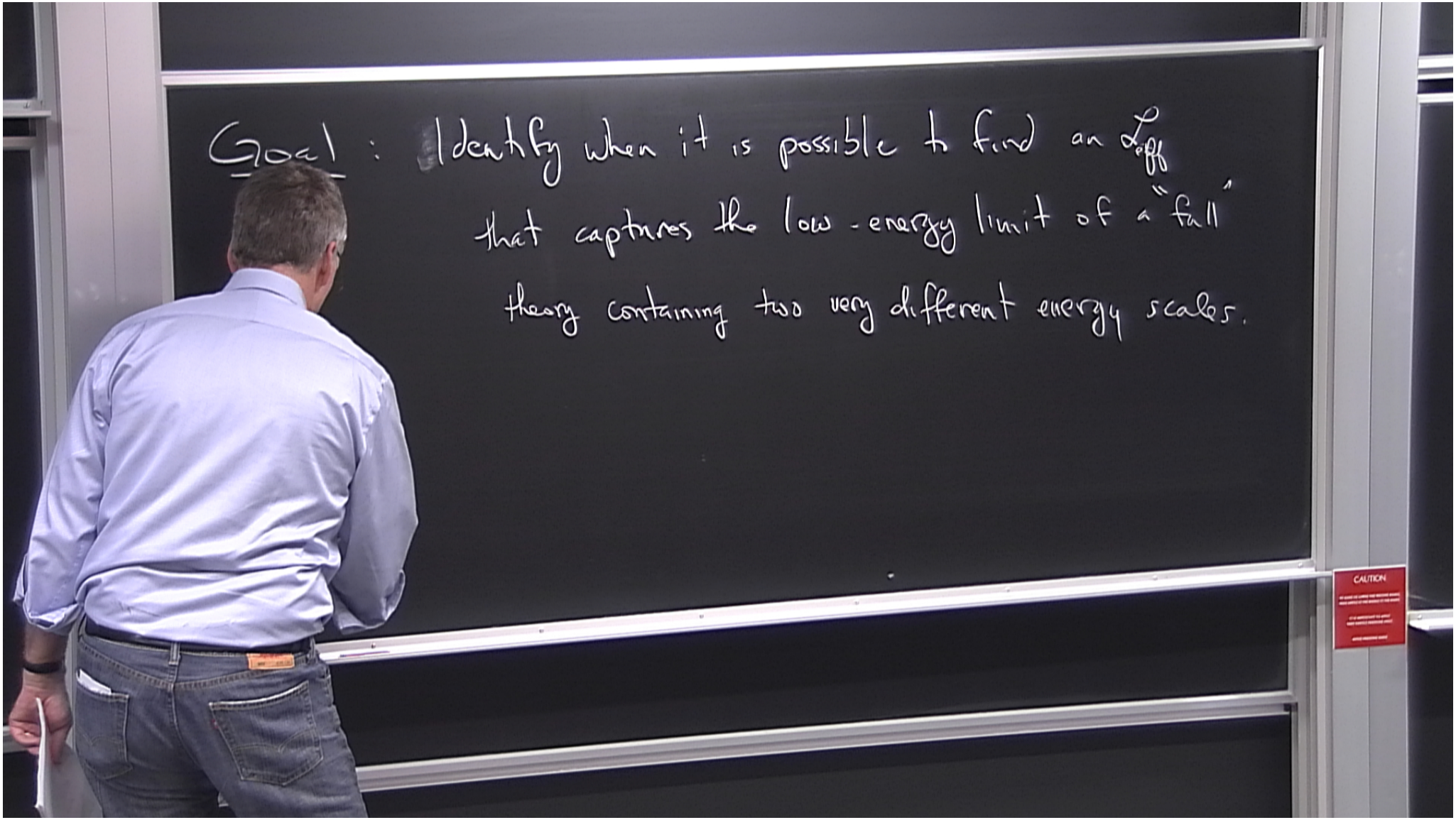
Goal: Identify when it is possible to find an $\mathcal{L}_{eff}$
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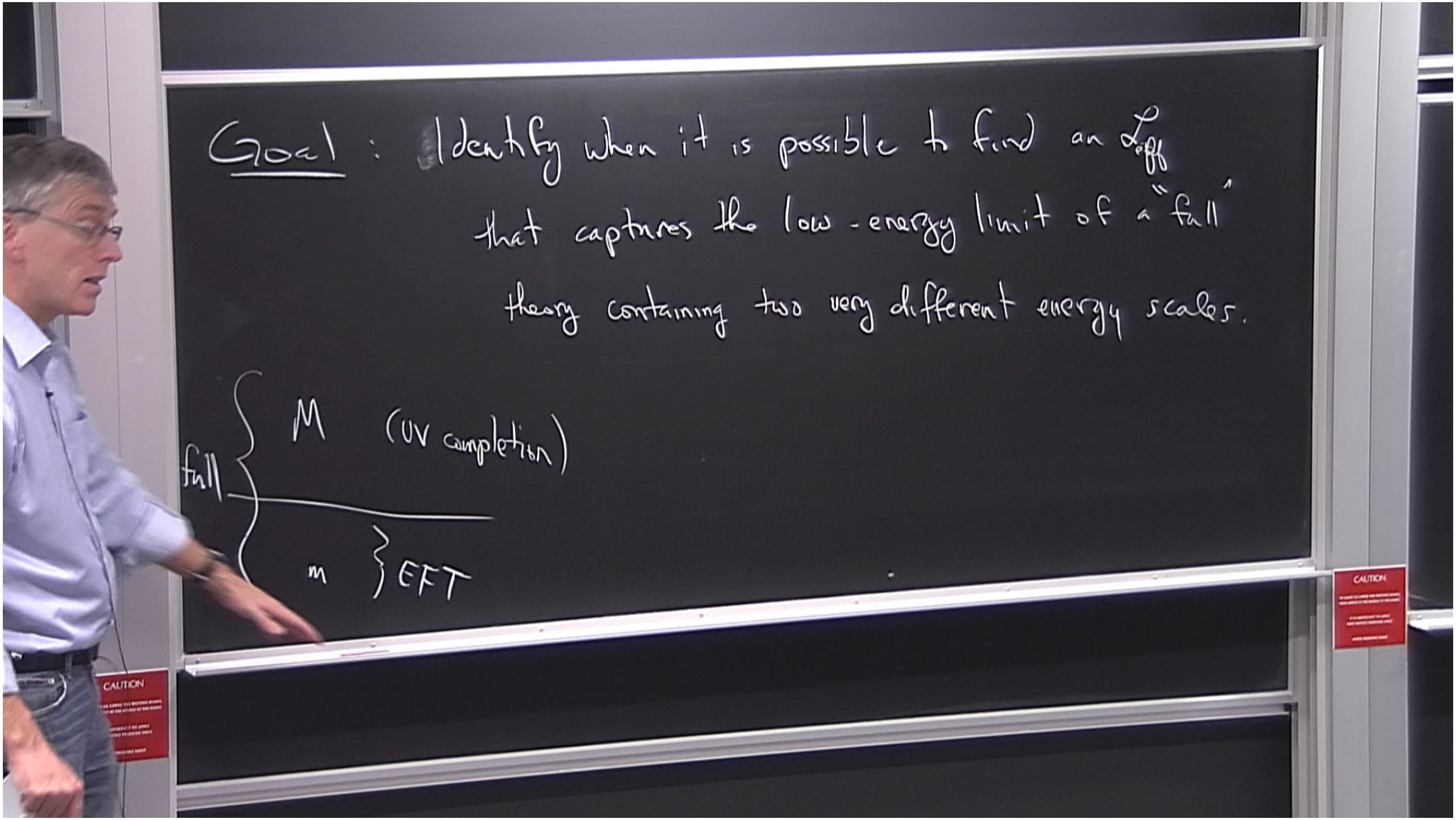
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M (UV completion)
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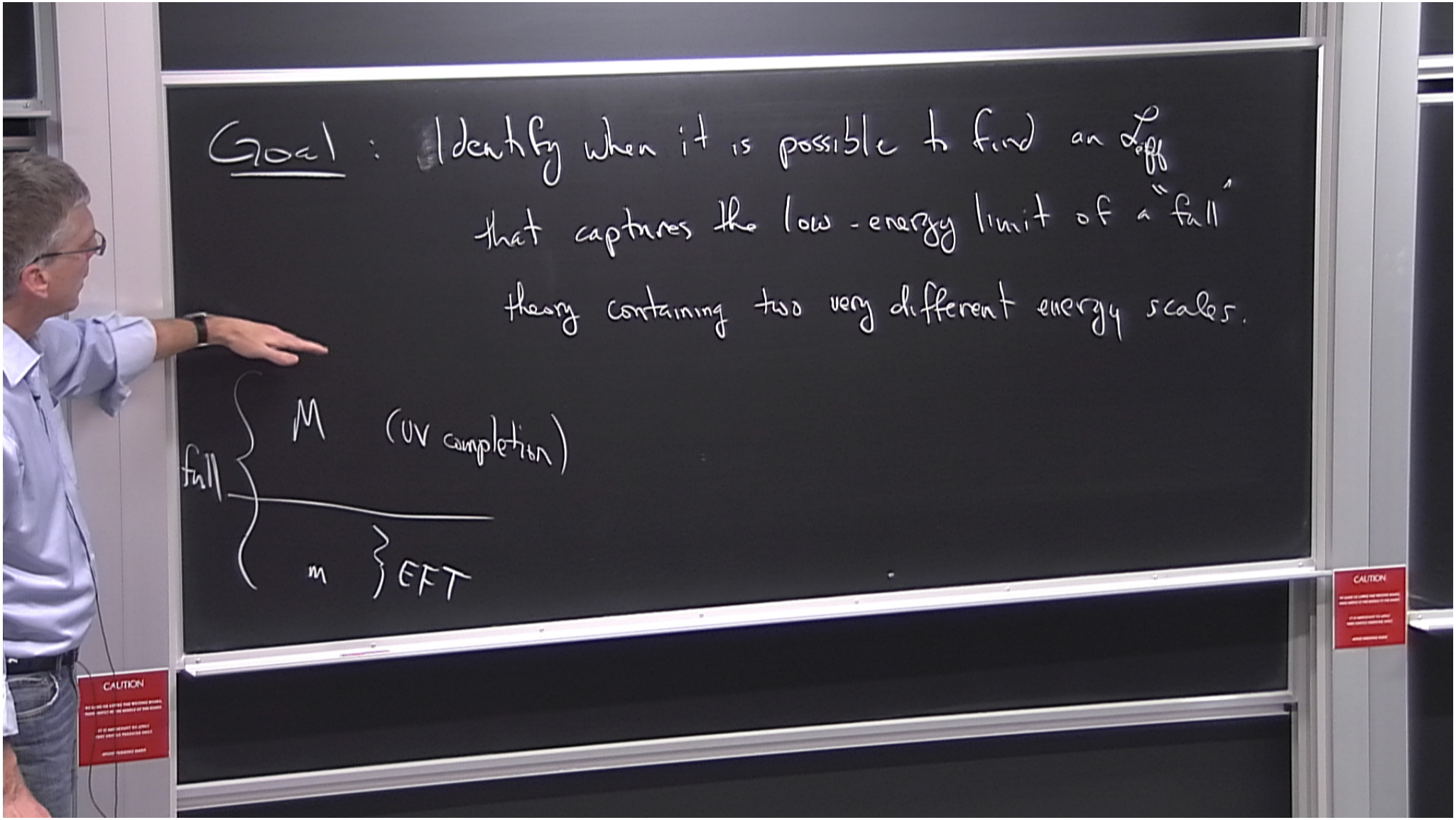
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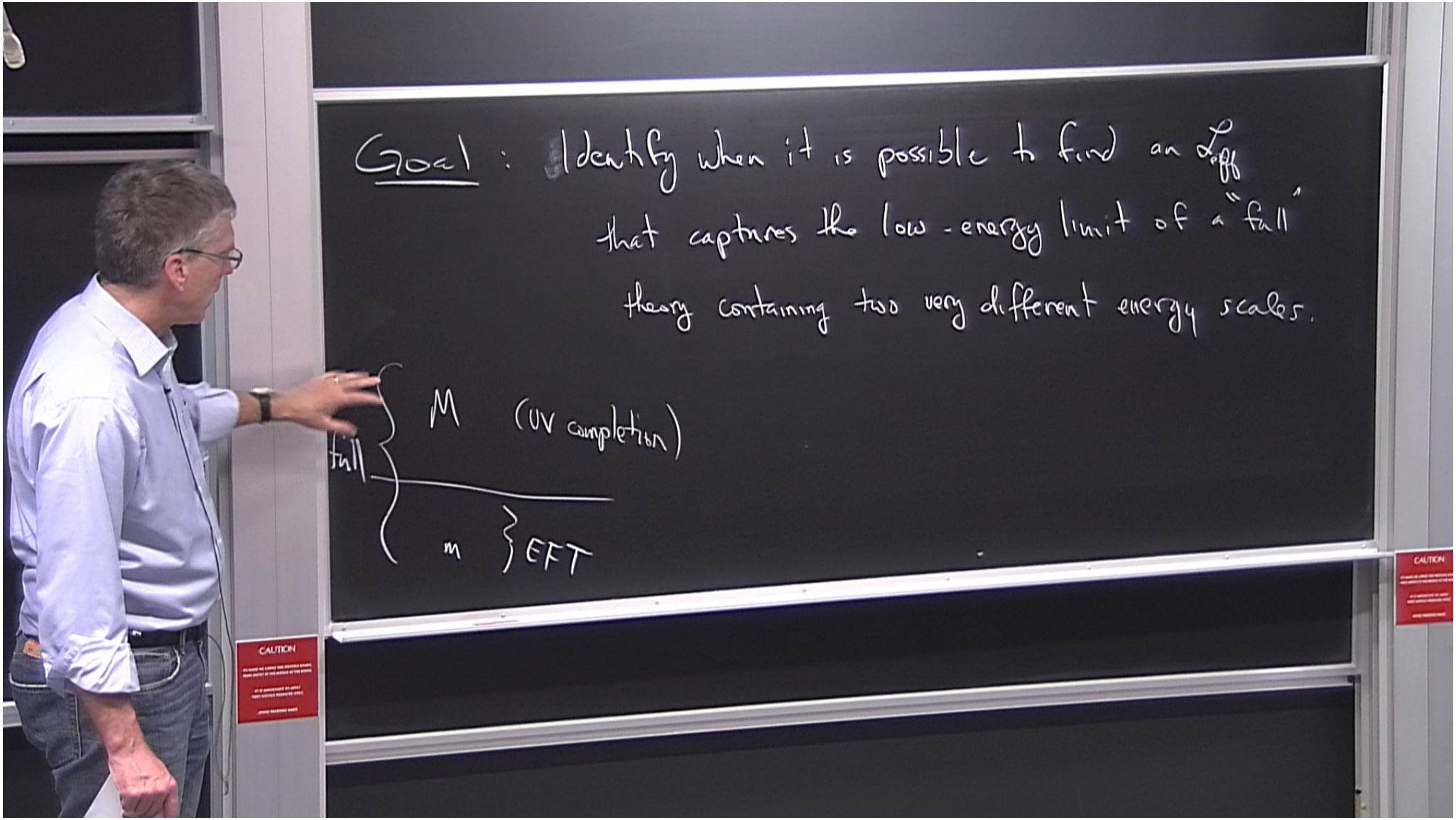
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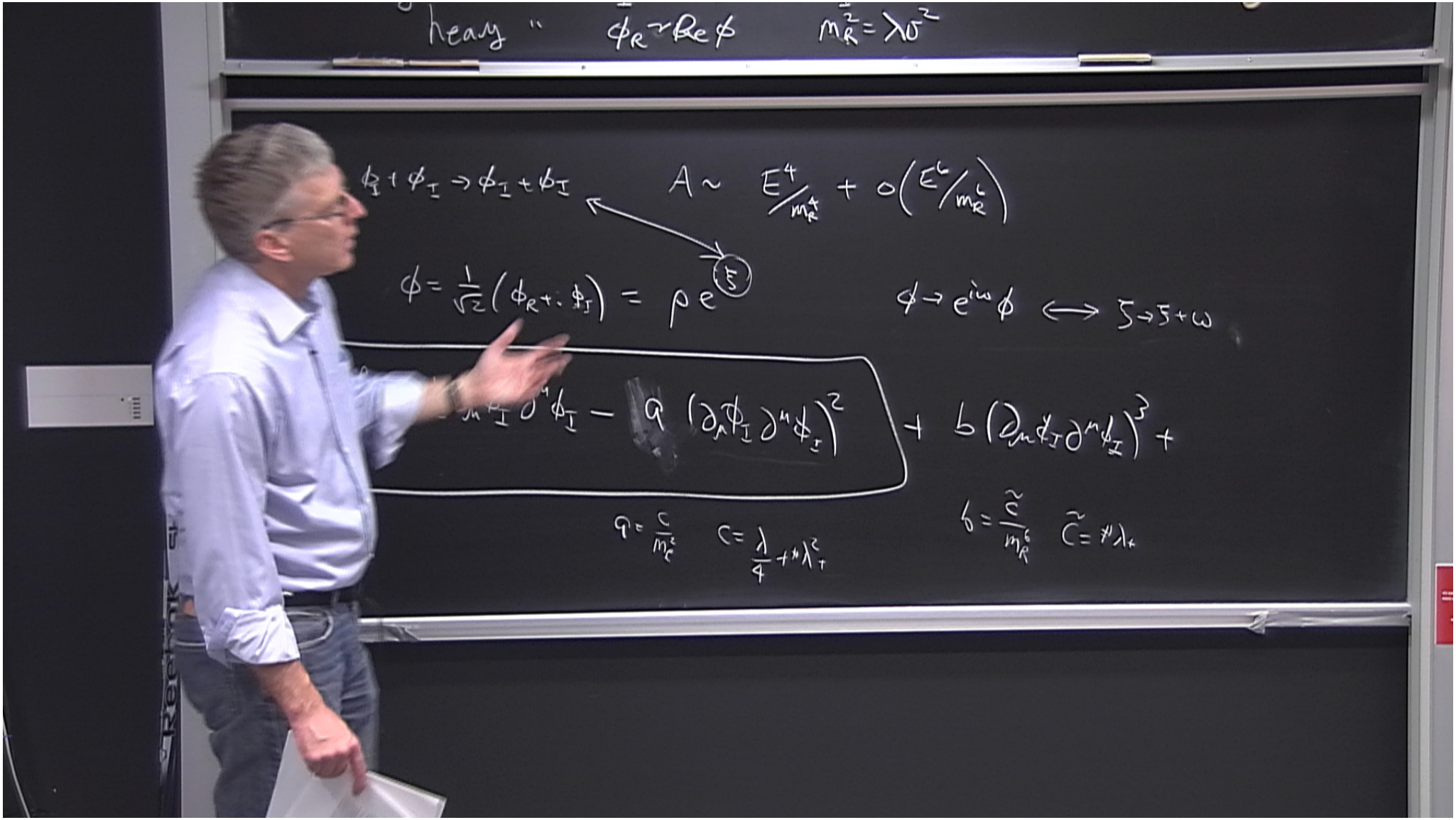
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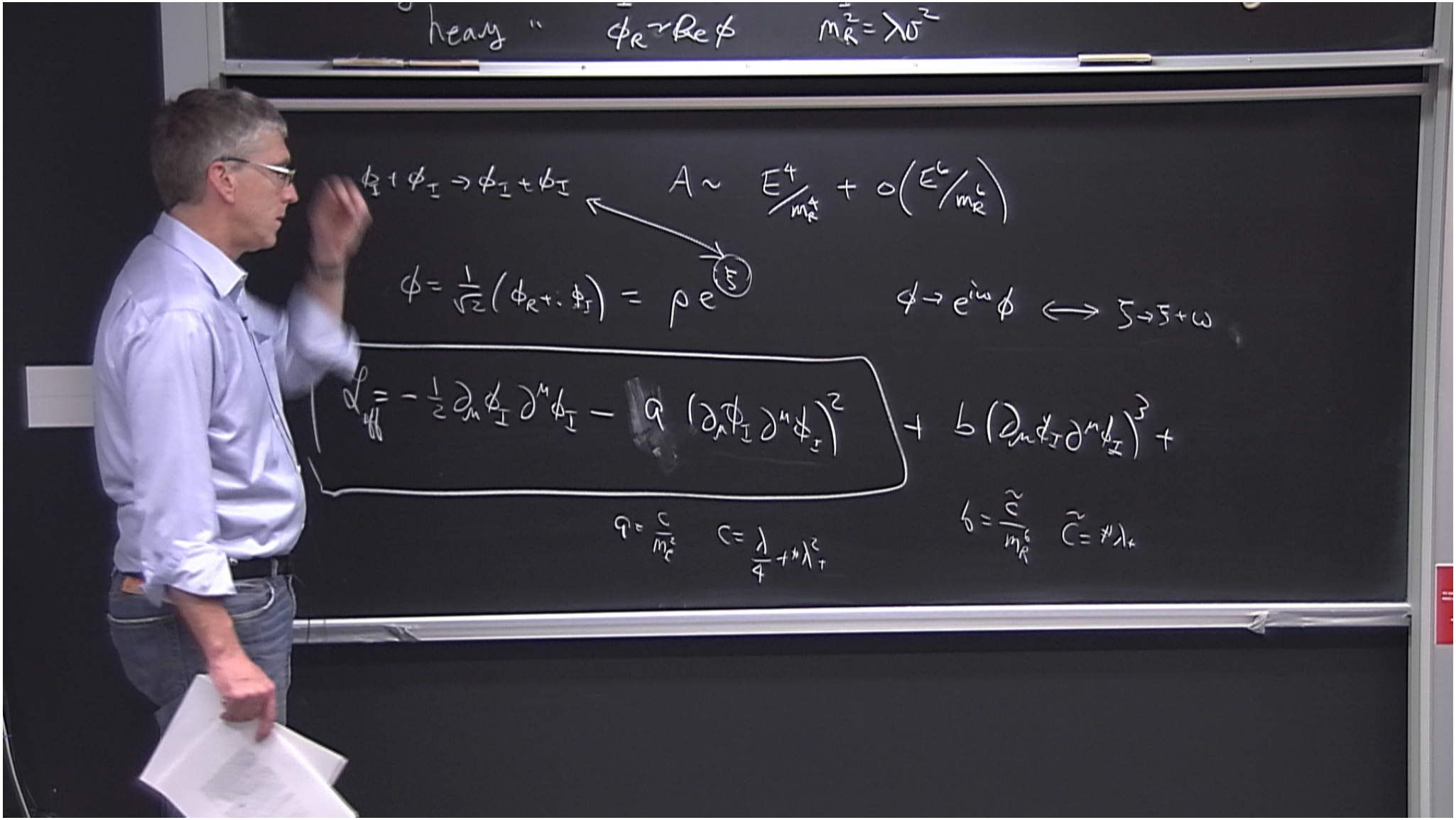
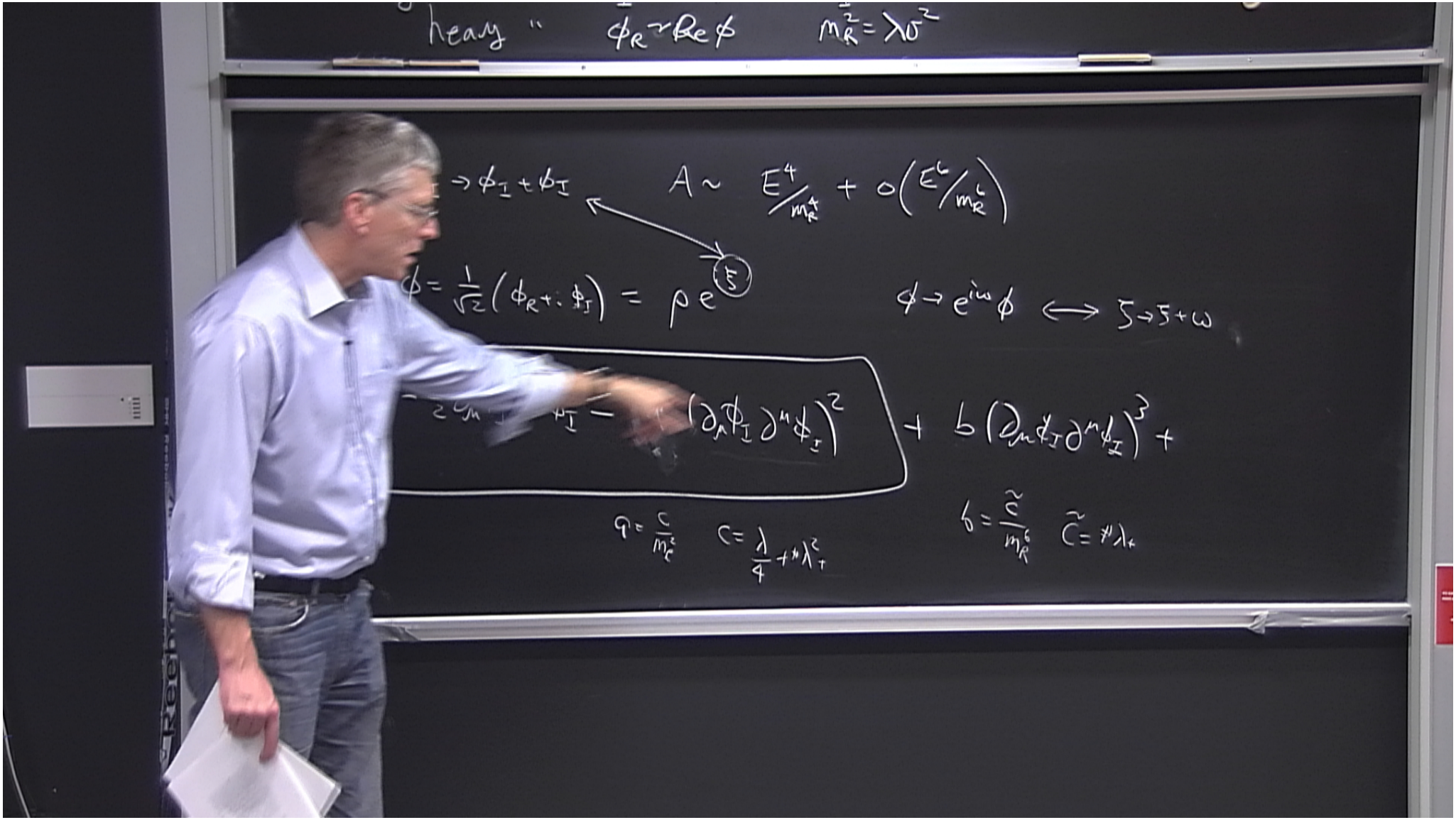
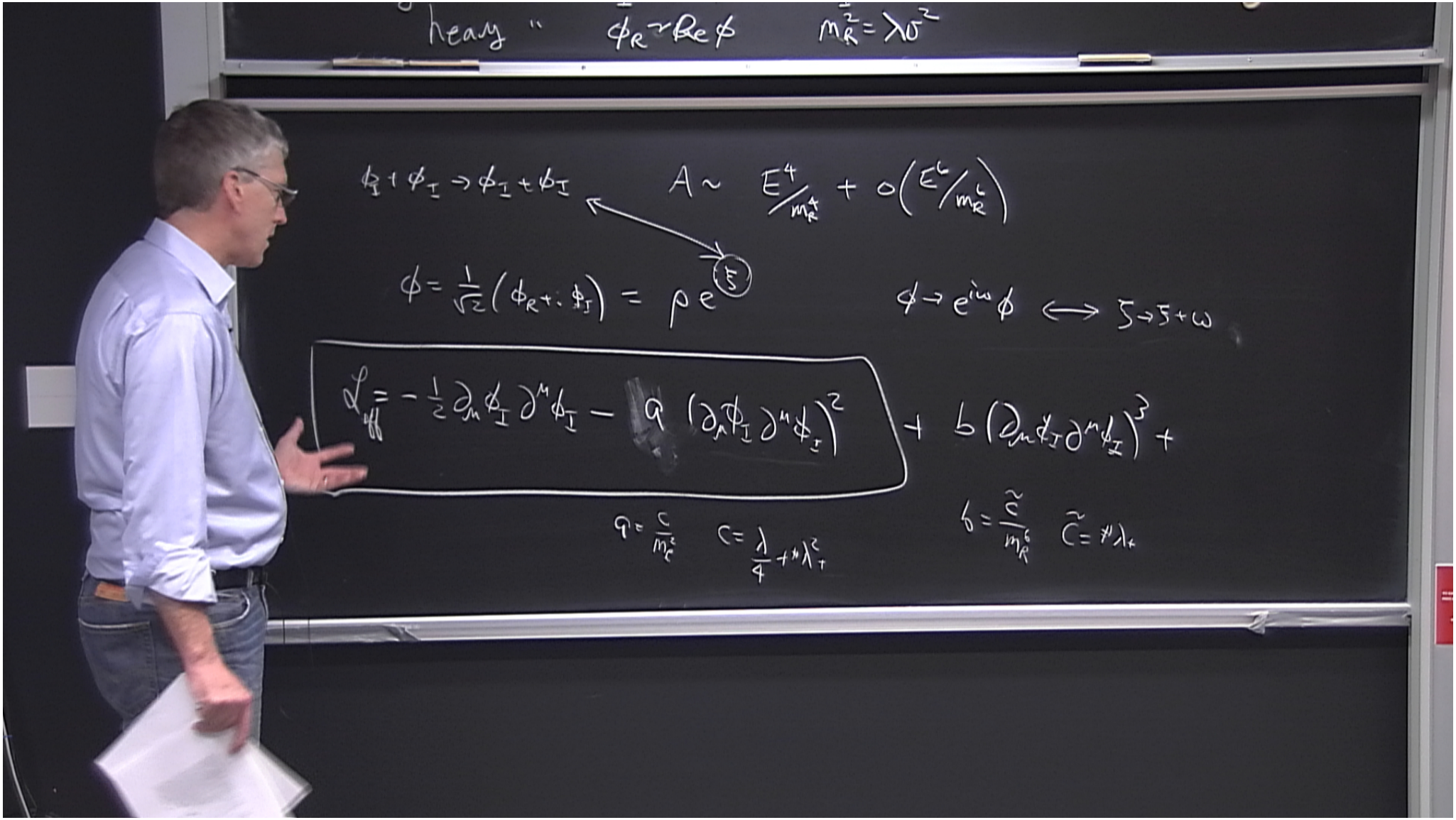
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$- a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \#\lambda^2 + \ldots$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \#\lambda + \ldots$


heavy "
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + O(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \longleftrightarrow \xi \to \xi + \omega$
$\mathcal{L}_{eff} = -\frac{1}{2}\partial_\mu\phi_I\partial^\mu\phi_I - a(\partial_\mu\phi_I\partial^\mu\phi_I)^2 + b(\partial_\mu\phi_I\partial^\mu\phi_I)^3 +$
$a = \frac{c}{m_R^2}$
$c = \frac{\lambda}{4} + \#\lambda_+^2$
$b = \frac{\tilde{c}}{m_R^6}$
$\tilde{c} = \#\lambda_+$


heavy "
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \longleftrightarrow \xi \to \xi + \omega$
$(\partial_\mu \phi_I \partial^\mu \phi_I)^2$
$+ b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$
$c = \frac{\lambda}{4} + \#\lambda_+^2$
$b = \frac{\tilde{c}}{m_R^6}$
$\tilde{c} = \#\lambda_+$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$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \longleftrightarrow \xi \to \xi + \omega$
$\mathcal{L}_{eff} = -\frac{1}{2}\partial_\mu\phi_I\partial^\mu\phi_I - a(\partial_\mu\phi_I\partial^\mu\phi_I)^2 + b(\partial_\mu\phi_I\partial^\mu\phi_I)^3 +$
$a = \frac{c}{m_R^2}$
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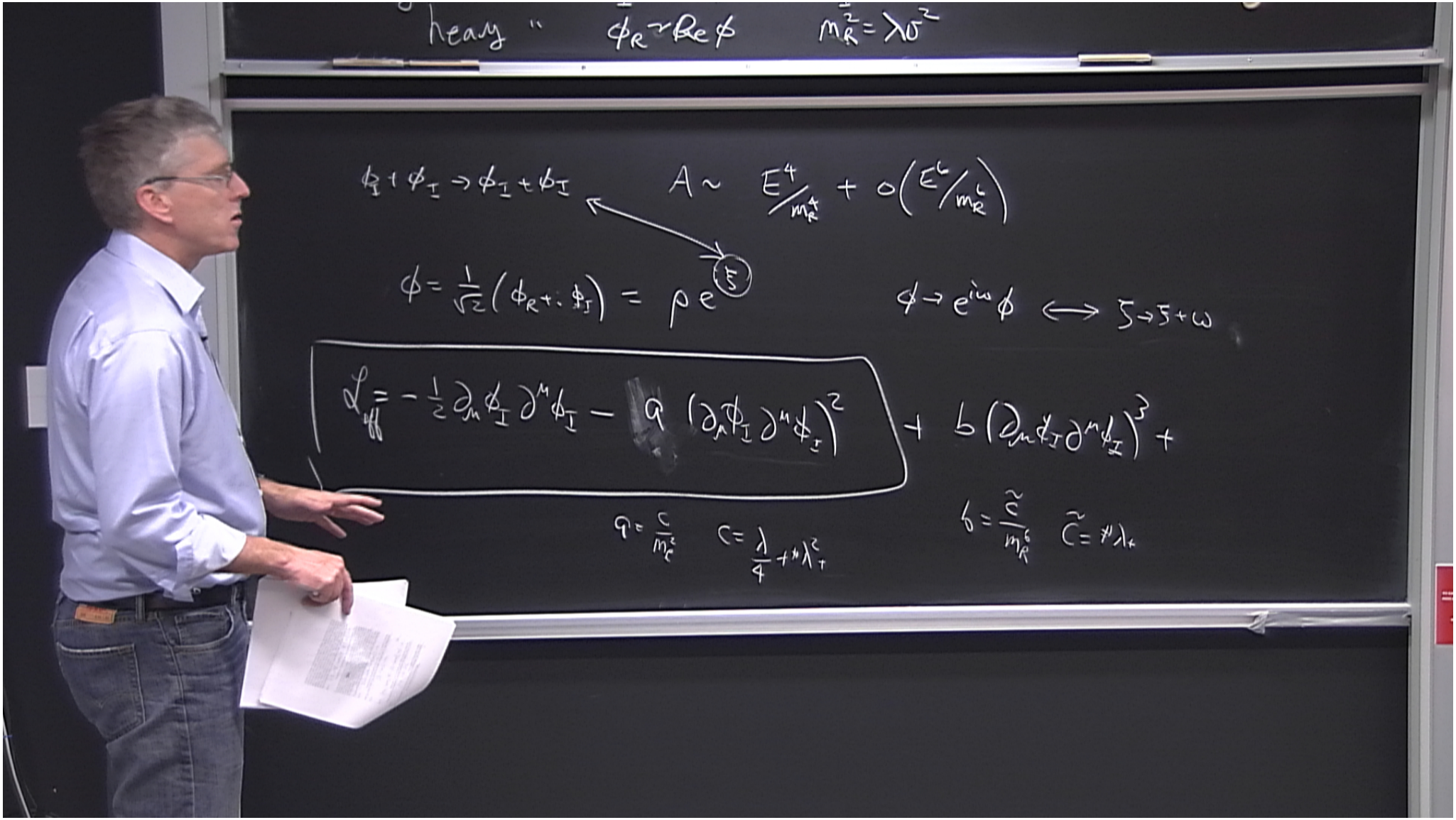
$b = \frac{\tilde{c}}{m_R^6}$
$\tilde{c} = \#\lambda_+$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$\phi_R \sim \mathrm{Re}\,\phi$
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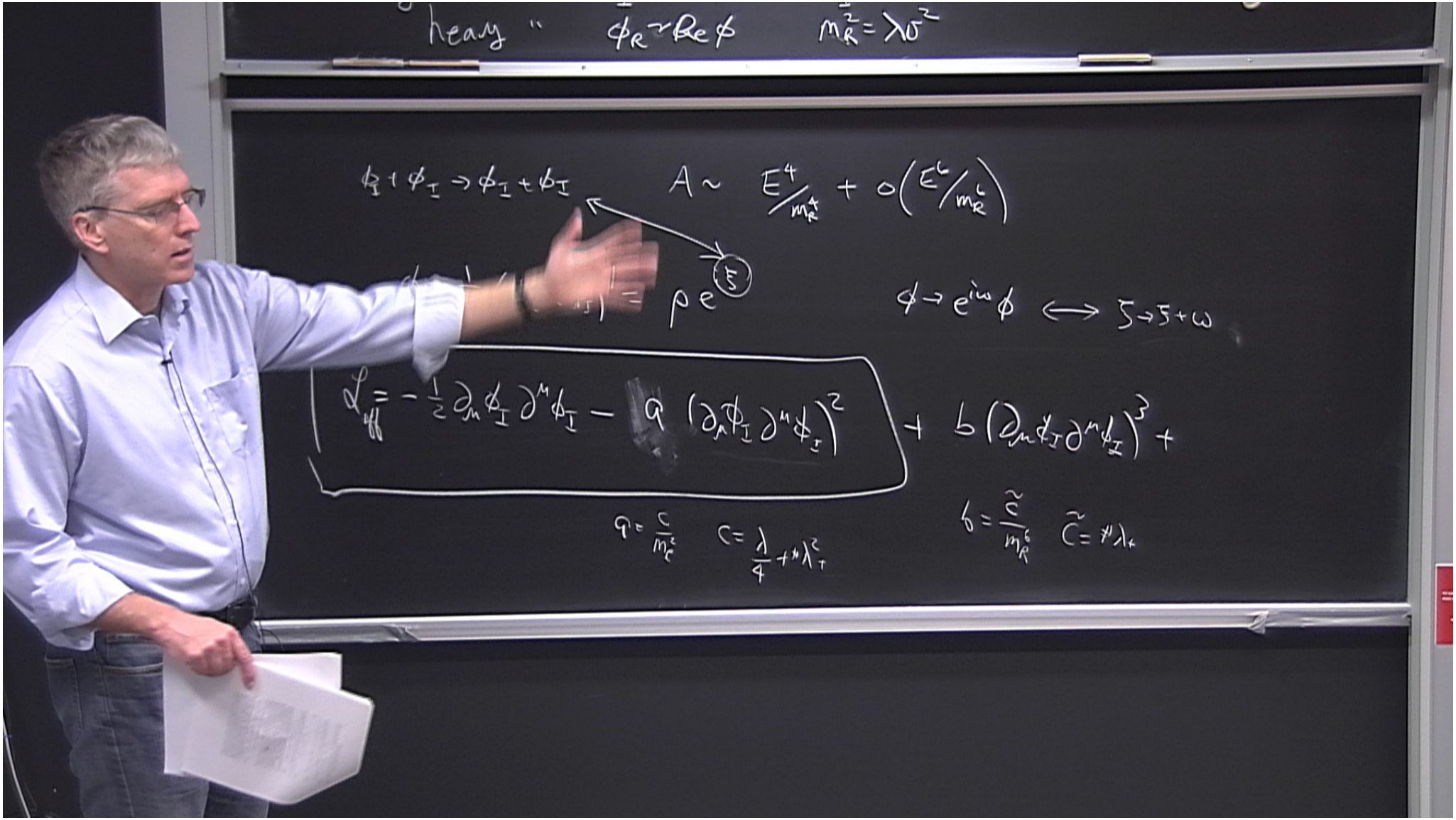
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \ldots\lambda^2 + \ldots$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \ldots\lambda + \ldots$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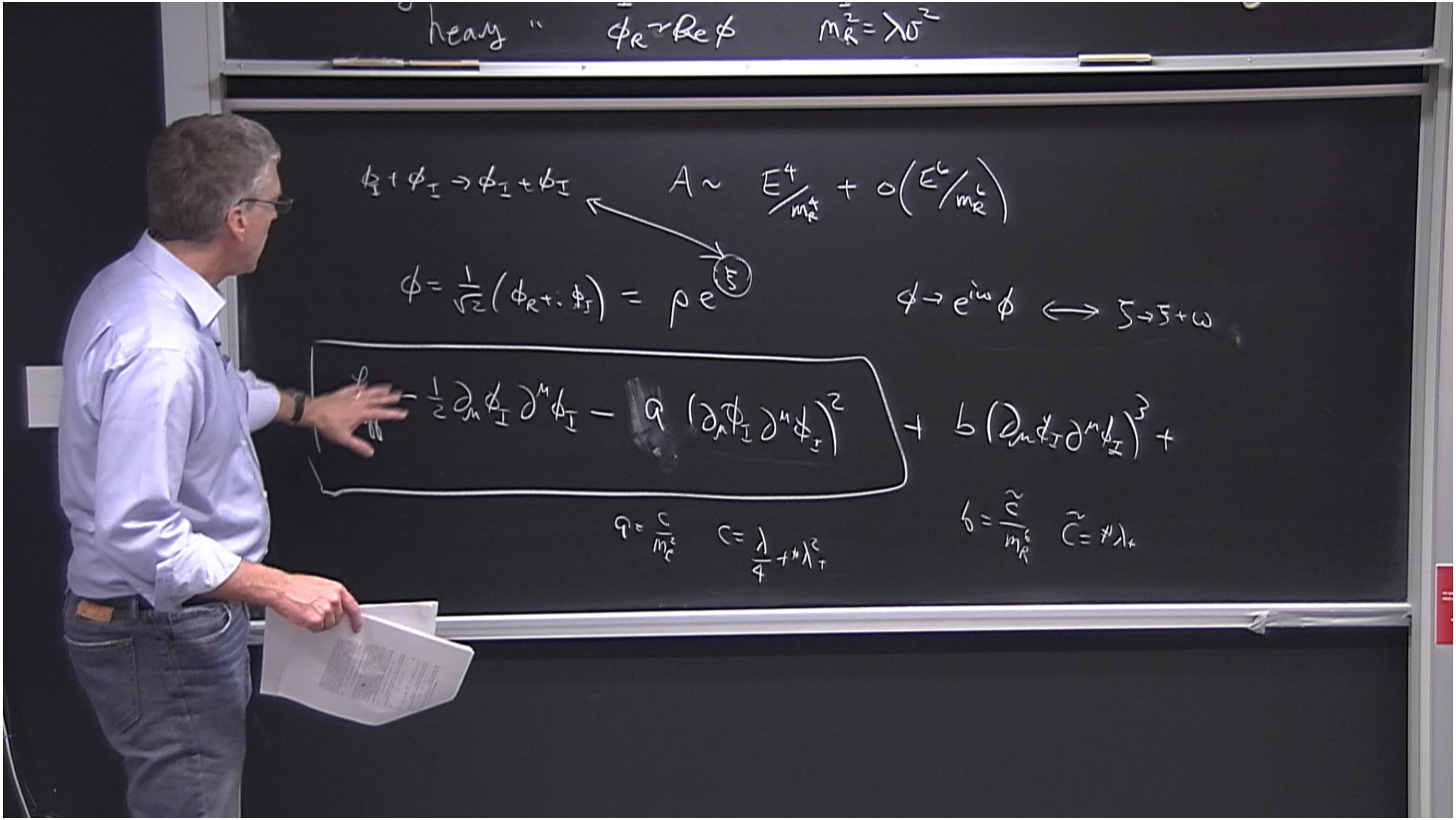
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
$\rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$
$c = \frac{\lambda}{4} + \ldots \lambda_+^2$
$b = \frac{\tilde{c}}{m_R^6}$
$\tilde{c} = \ldots \lambda_+$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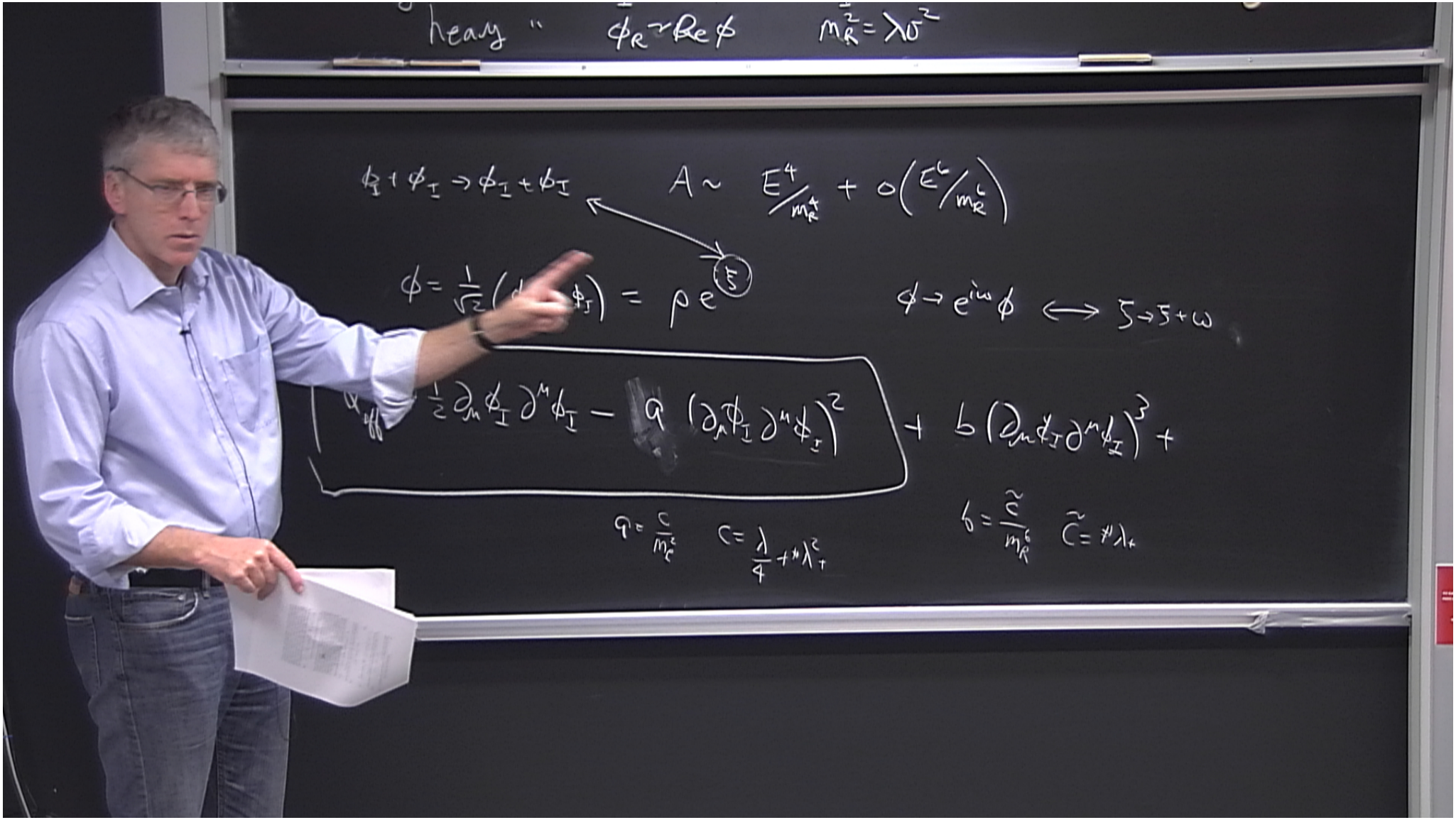
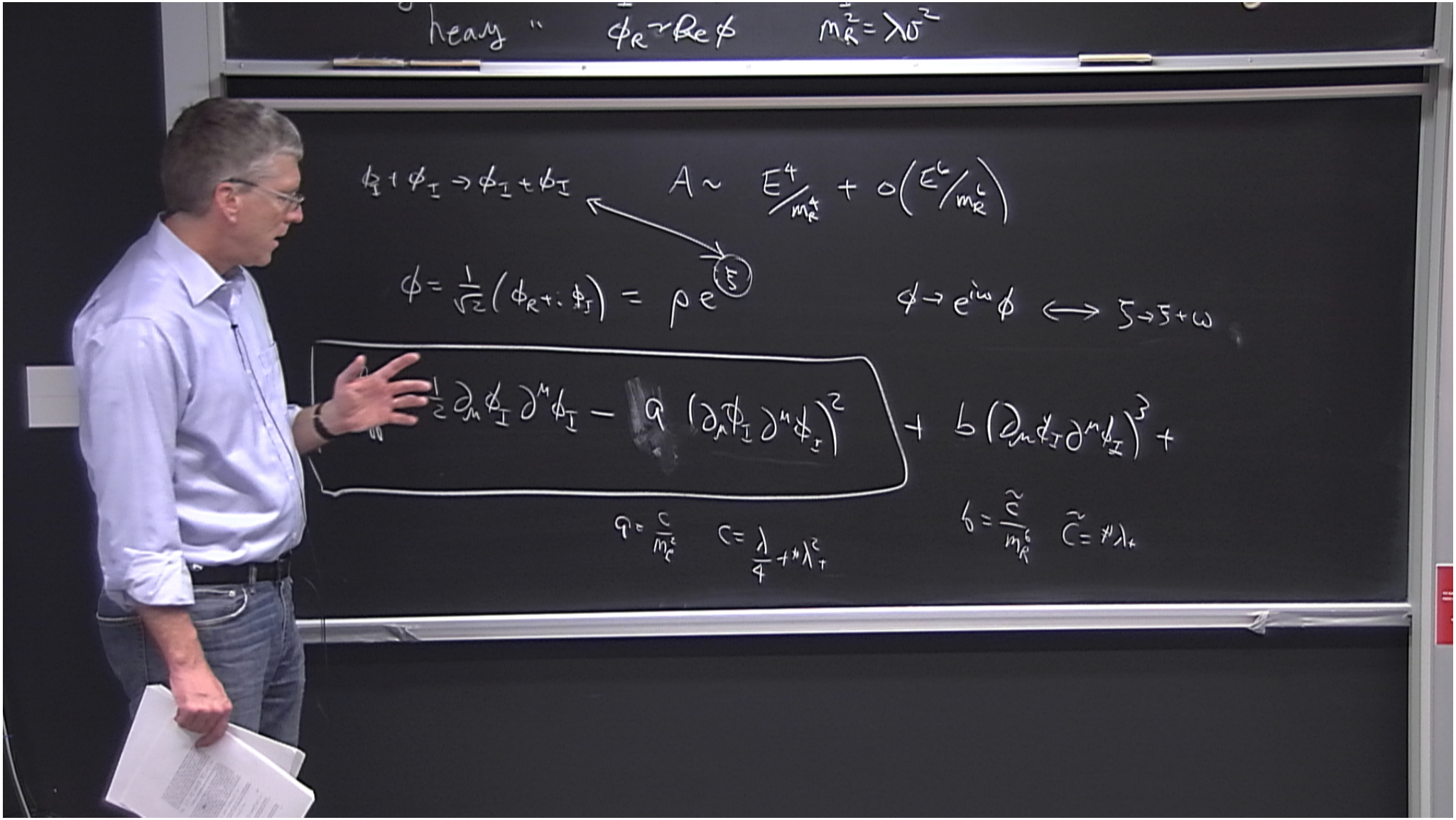
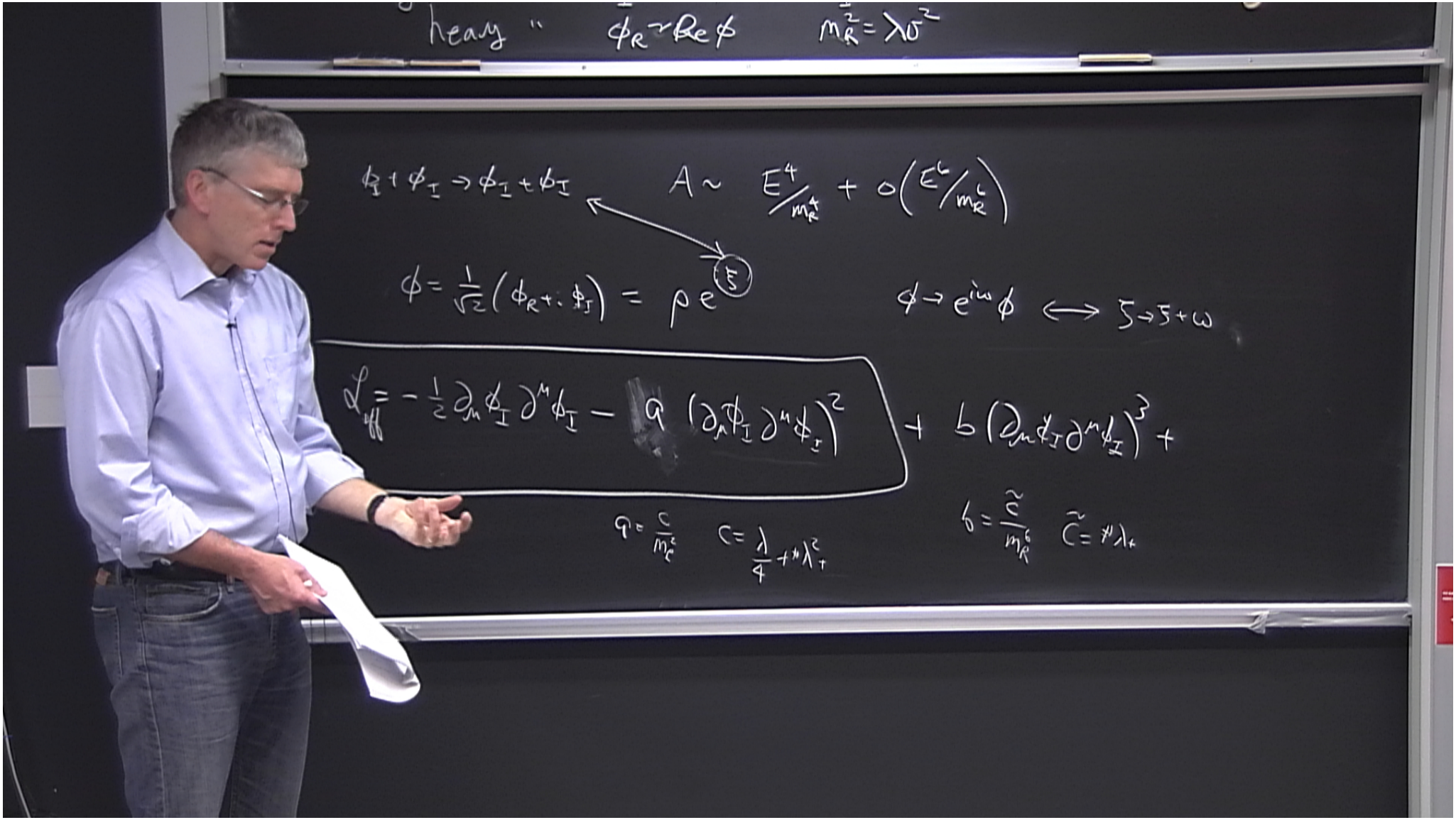
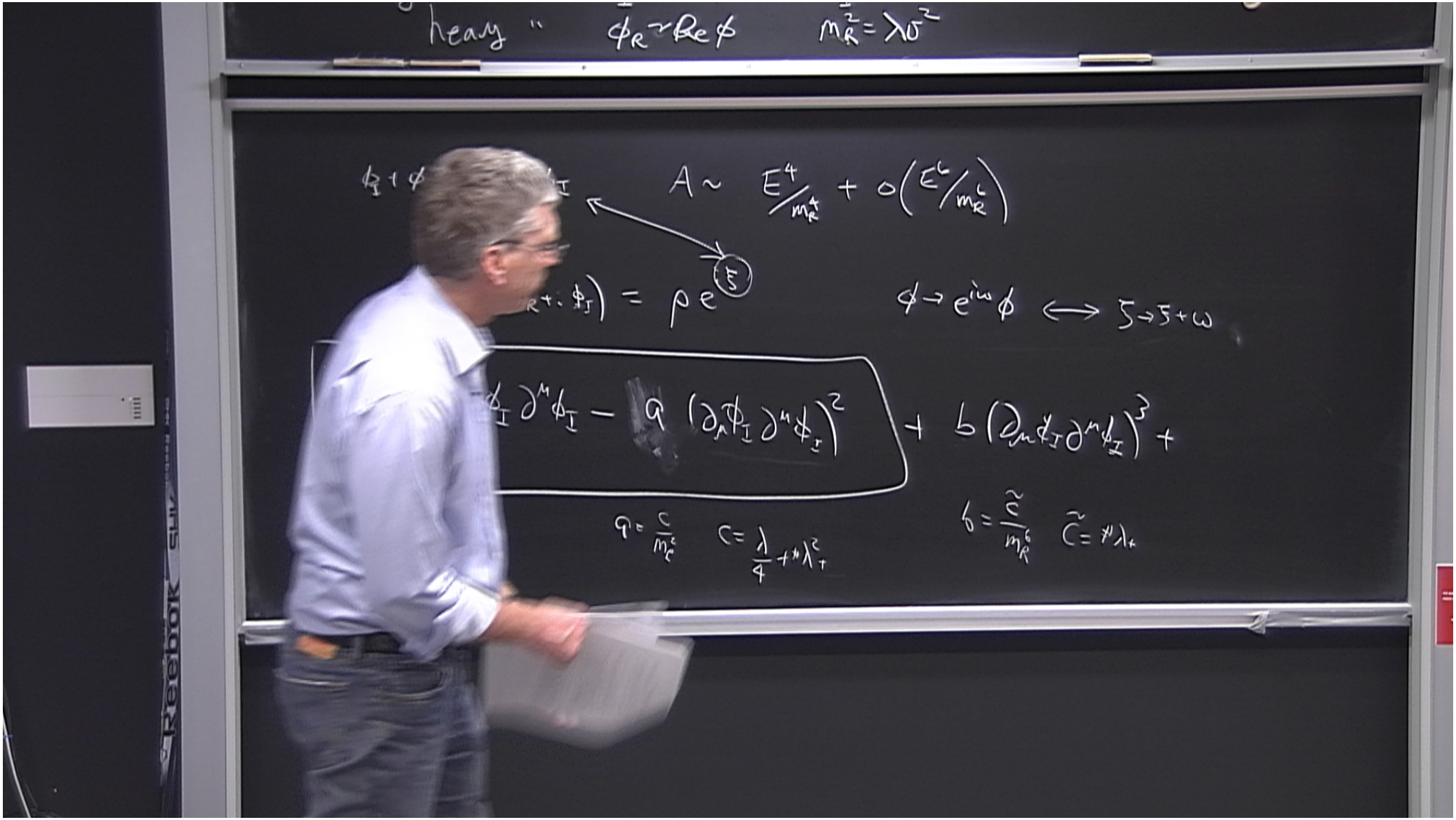
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + O(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \longleftrightarrow \xi \to \xi + \omega$
$\mathcal{L}_{eff} = -\frac{1}{2}\partial_\mu\phi_I\partial^\mu\phi_I - a(\partial_\mu\phi_I\partial^\mu\phi_I)^2 + b(\partial_\mu\phi_I\partial^\mu\phi_I)^3 +$
$a = \frac{c}{m_R^2}$
$c = \frac{\lambda}{4} + \#\lambda_+^2$
$b = \frac{\tilde{c}}{m_R^6}$
$\tilde{c} = \#\lambda_+$

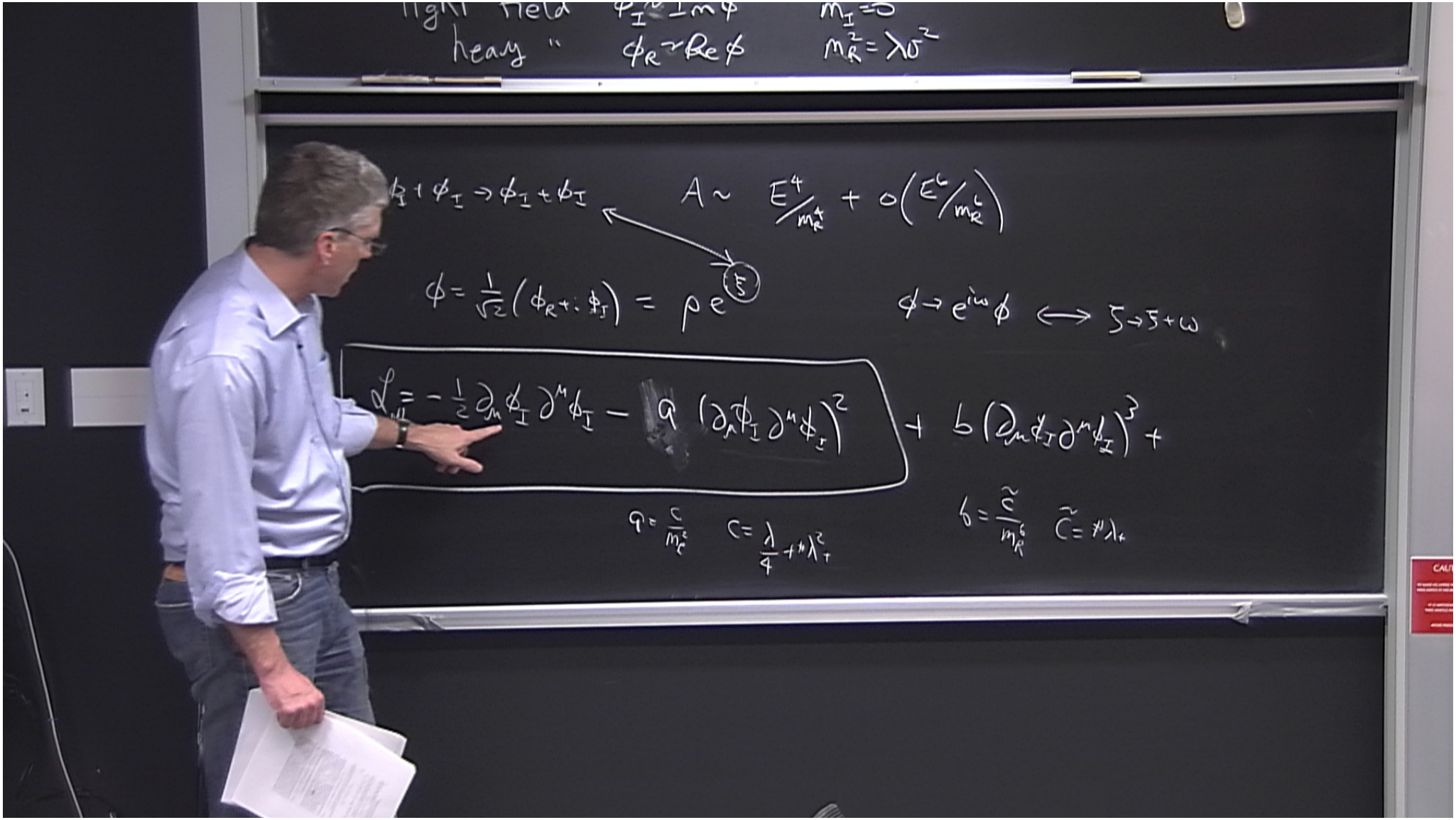

heavy
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + O(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\ldots) = \rho e^{i\xi/v}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\mathrm{eff}} = \frac{1}{2}\partial_\mu\phi_I\partial^\mu\phi_I - a(\partial_\mu\phi_I\partial^\mu\phi_I)^2 + b(\partial_\mu\phi_I\partial^\mu\phi_I)^3 + \ldots$
$a = \frac{c}{m_R^4}$ $c = \frac{\lambda}{4} + \ldots\lambda^2 + \ldots$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \ldots\lambda + \ldots$

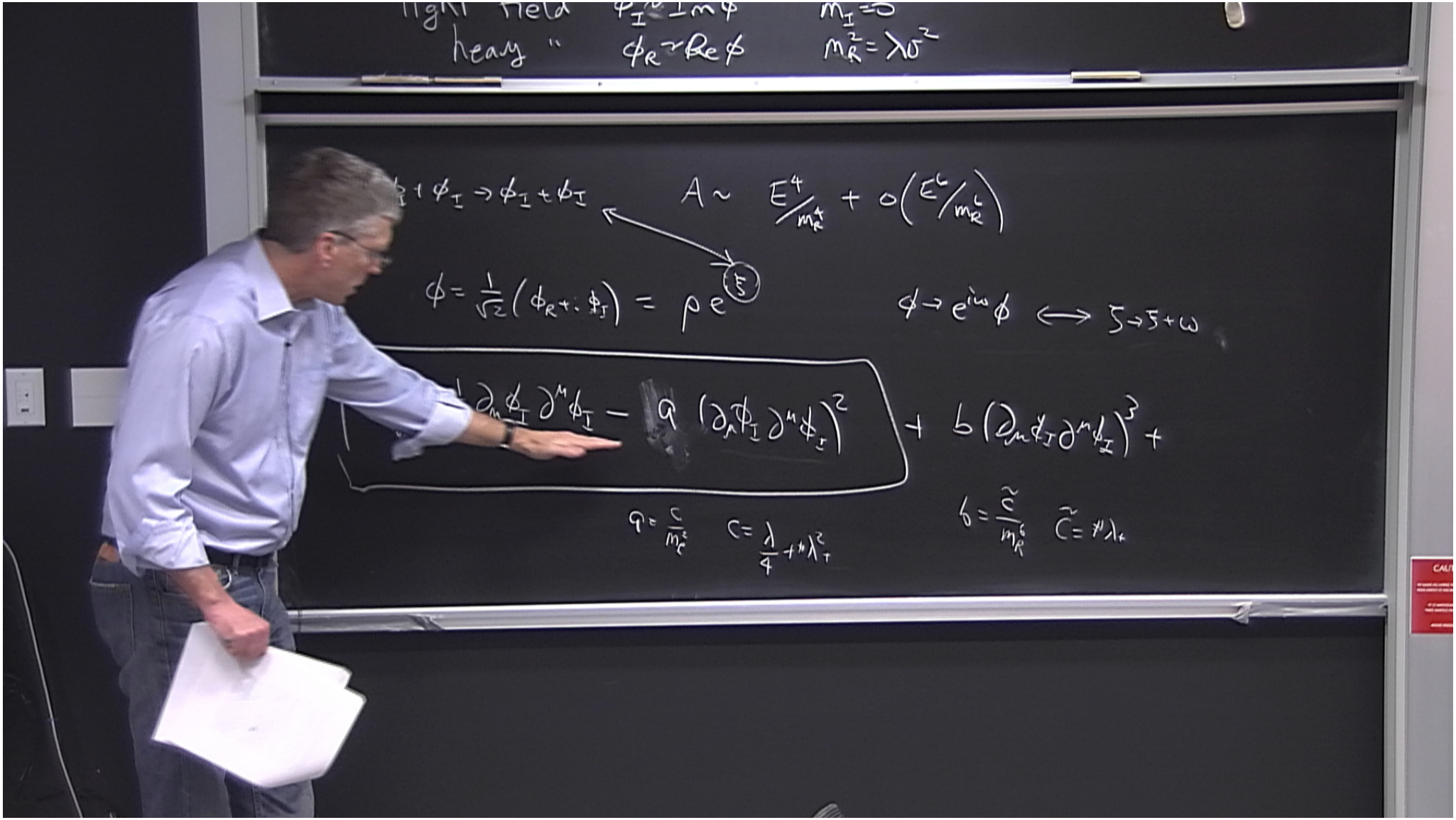

heavy
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \ldots\lambda^2 +$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \ldots\lambda +$


heavy
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + O(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi/v}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2}\partial_\mu\phi_I\,\partial^\mu\phi_I - a(\partial_\mu\phi_I\,\partial^\mu\phi_I)^2 + b(\partial_\mu\phi_I\,\partial^\mu\phi_I)^3 +$
$a = \frac{c}{m_R^4}$ $c = \frac{\lambda}{4} + \cdots\lambda^2 +$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \cdots\lambda +$


heavy "
$\phi_R \sim \mathrm{Re}\,\phi$
$m_R^2 = \lambda v^2$
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
$\phi \to e^{i\omega}\phi \longleftrightarrow \xi \to \xi + \omega$
$\phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$
$c = \frac{\lambda}{4} + \#\lambda_+^2$
$b = \frac{\tilde{c}}{m_R^6}$
$\tilde{c} = \#\lambda_+$


different energy scales
M (UV completion)
EFT


light field
heavy "
$\phi_I \sim \mathrm{Im}\,\phi$
$\phi_R \sim \mathrm{Re}\,\phi$
$m_I = 0$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\lambda \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \ldots \lambda_T^2$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \ldots \lambda_T$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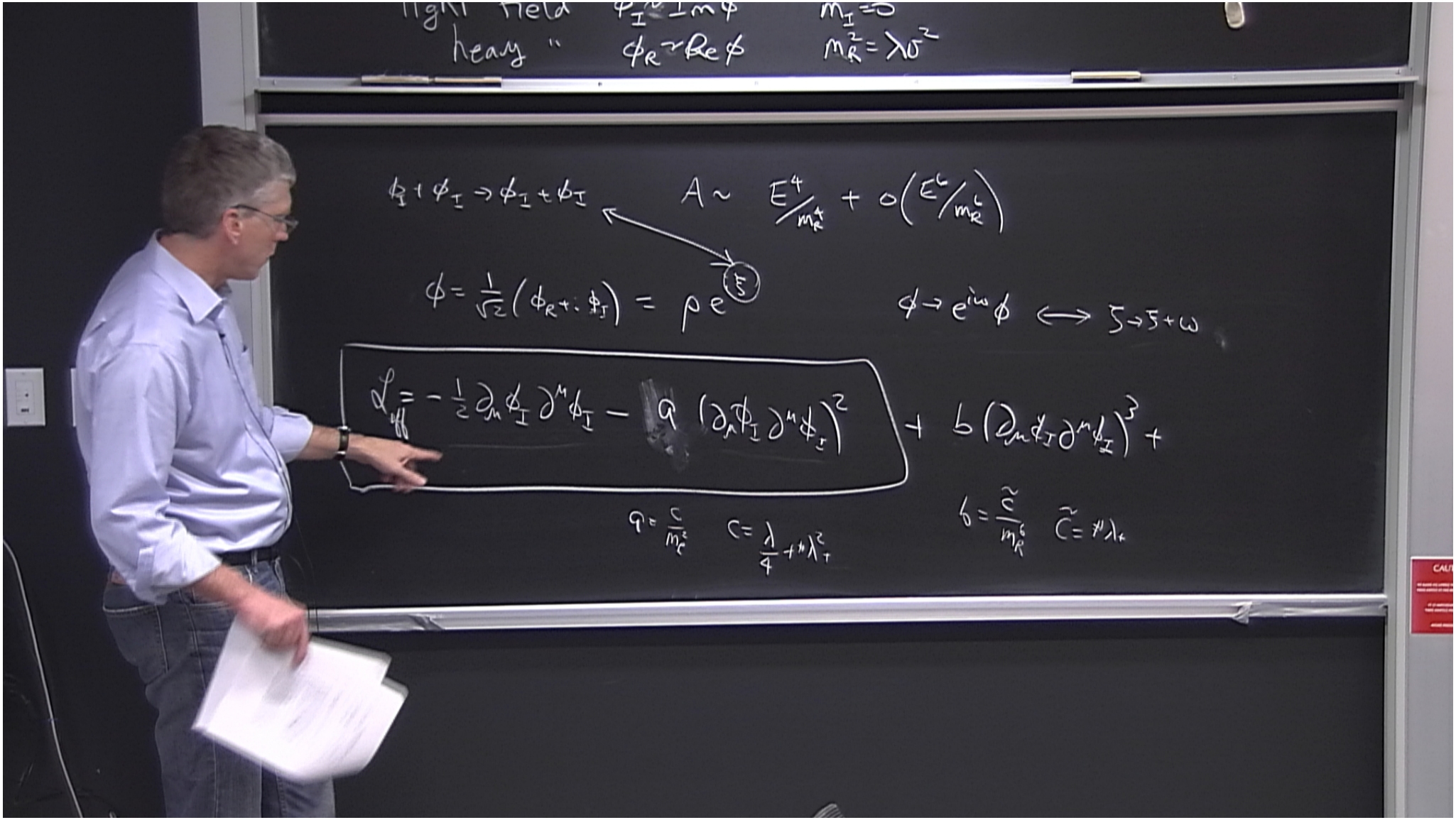
$\phi_I \sim \mathrm{Im}\,\phi$
$\phi_R \sim \mathrm{Re}\,\phi$
$m_I = 0$
$m_R^2 = \lambda v^2$
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \ldots \lambda_\tau^2$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \ldots \lambda_t$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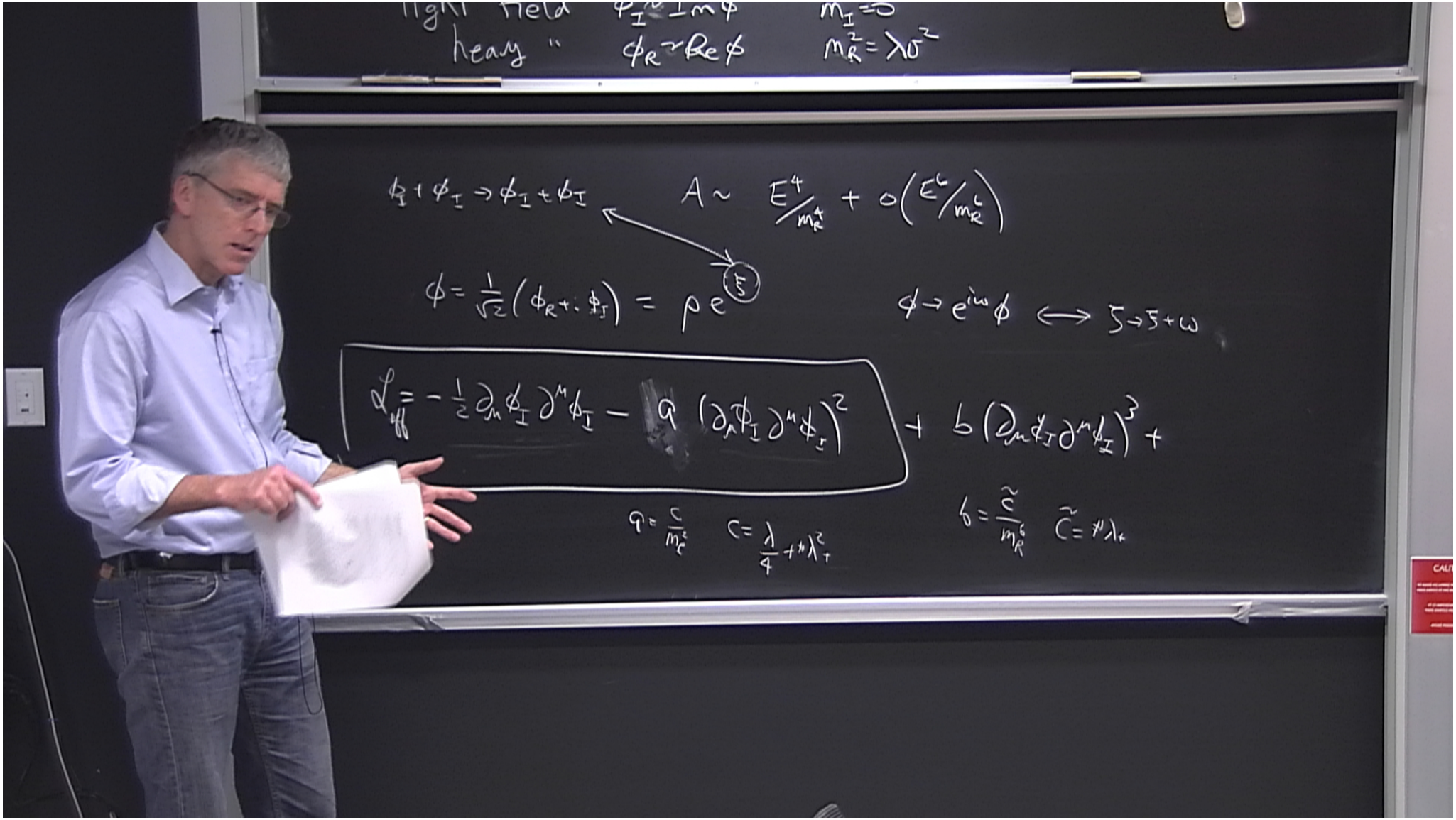
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = c/m_R^2 \quad c = \frac{\lambda}{4} + \lambda_T^2$
$b = \tilde{c}/m_R^6 \quad \tilde{c} = \lambda_T$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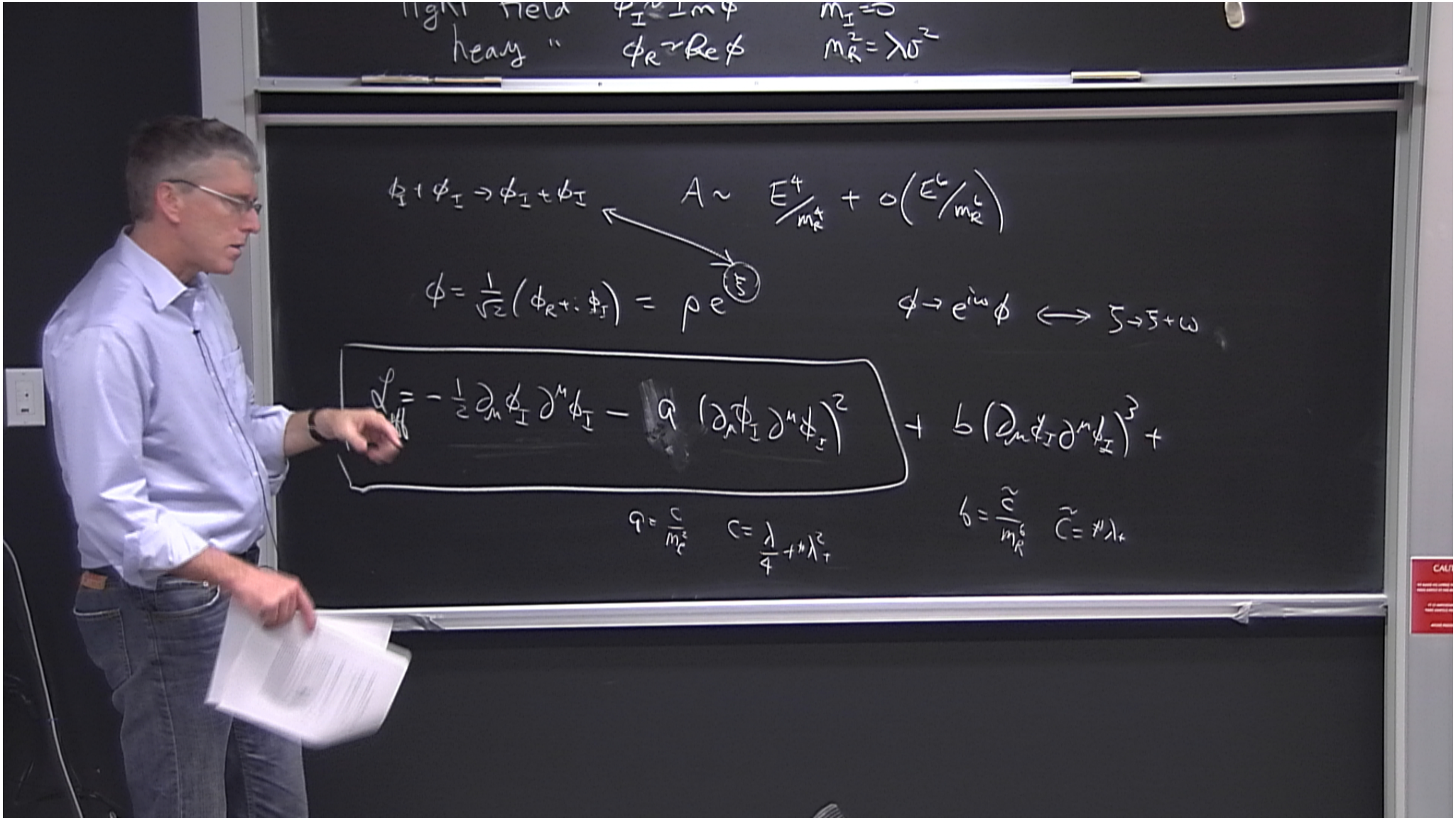
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = c/m_R^2 \quad c = \frac{\lambda}{4} + \text{"}\lambda_T^2$
$b = \tilde{c}/m_R^6 \quad \tilde{c} = \text{"}\lambda_T$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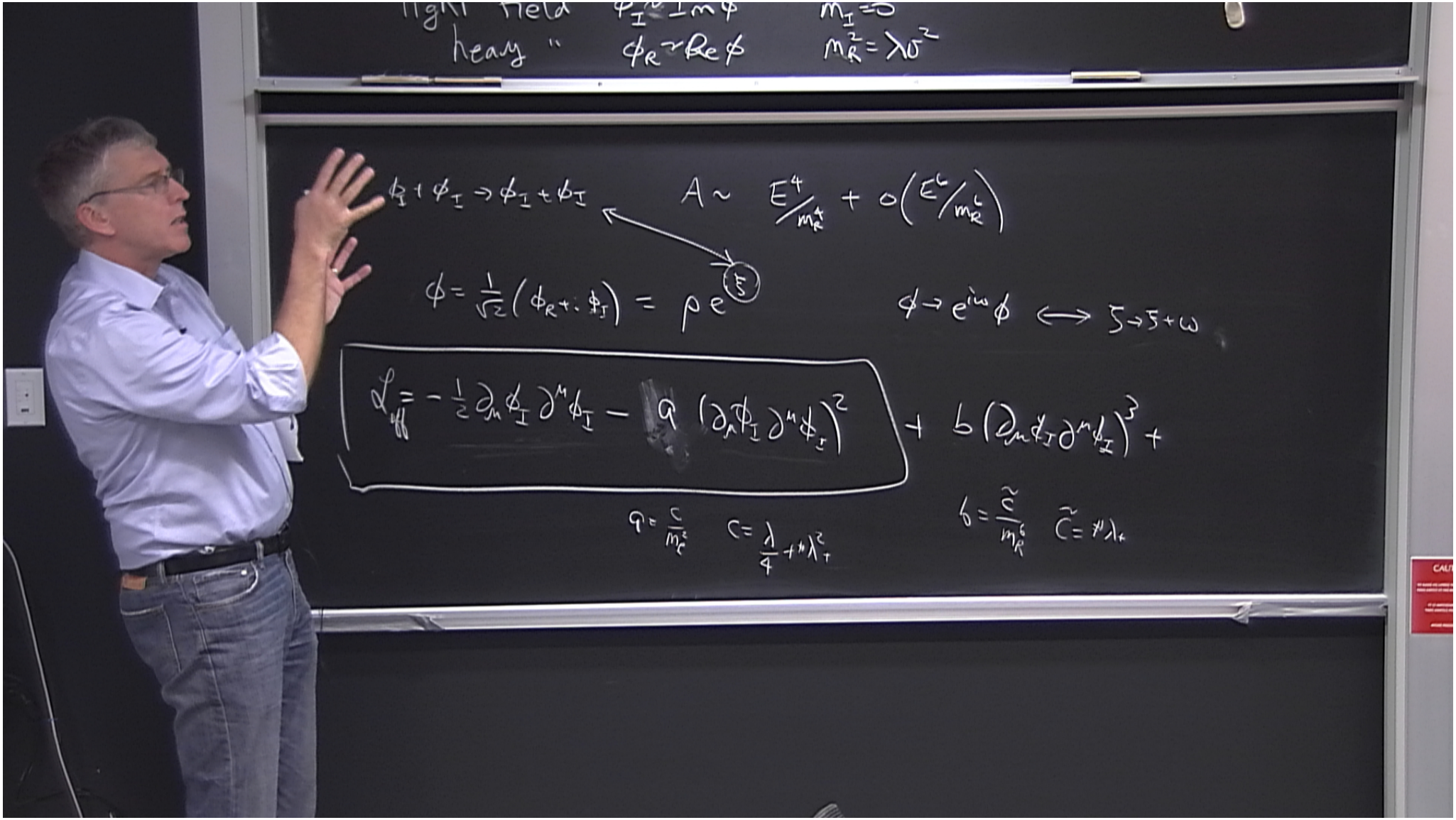
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_\mu\phi_I\,\partial^\mu\phi_I - a(\partial_\mu\phi_I\,\partial^\mu\phi_I)^2 + b(\partial_\mu\phi_I\,\partial^\mu\phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \#\lambda_T^2$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \#\lambda_T$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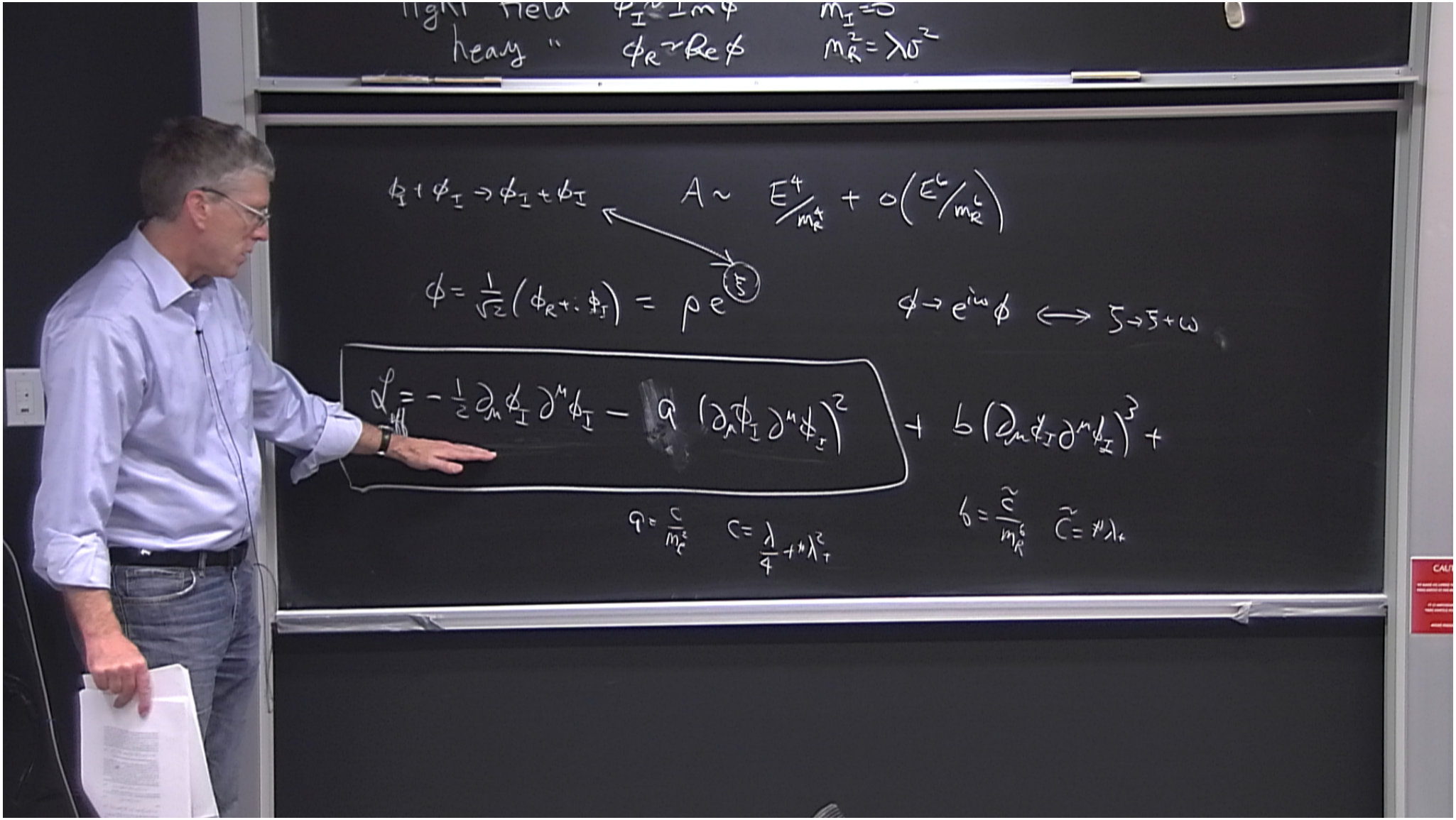
$\phi_I + \phi_I \to \phi_I + \phi_I$
$A \sim E^4/m_R^4 + o(E^6/m_R^6)$
E
$\phi = \frac{1}{\sqrt{2}}(\phi_R + i\phi_I) = \rho e^{i\xi}$
$\phi \to e^{i\omega}\phi \iff \xi \to \xi + \omega$
$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_\mu\phi_I\,\partial^\mu\phi_I - a(\partial_\mu\phi_I\,\partial^\mu\phi_I)^2 + b(\partial_\mu\phi_I\,\partial^\mu\phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \frac{1}{4}\lambda_T^2$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \frac{1}{8}\lambda_T^3$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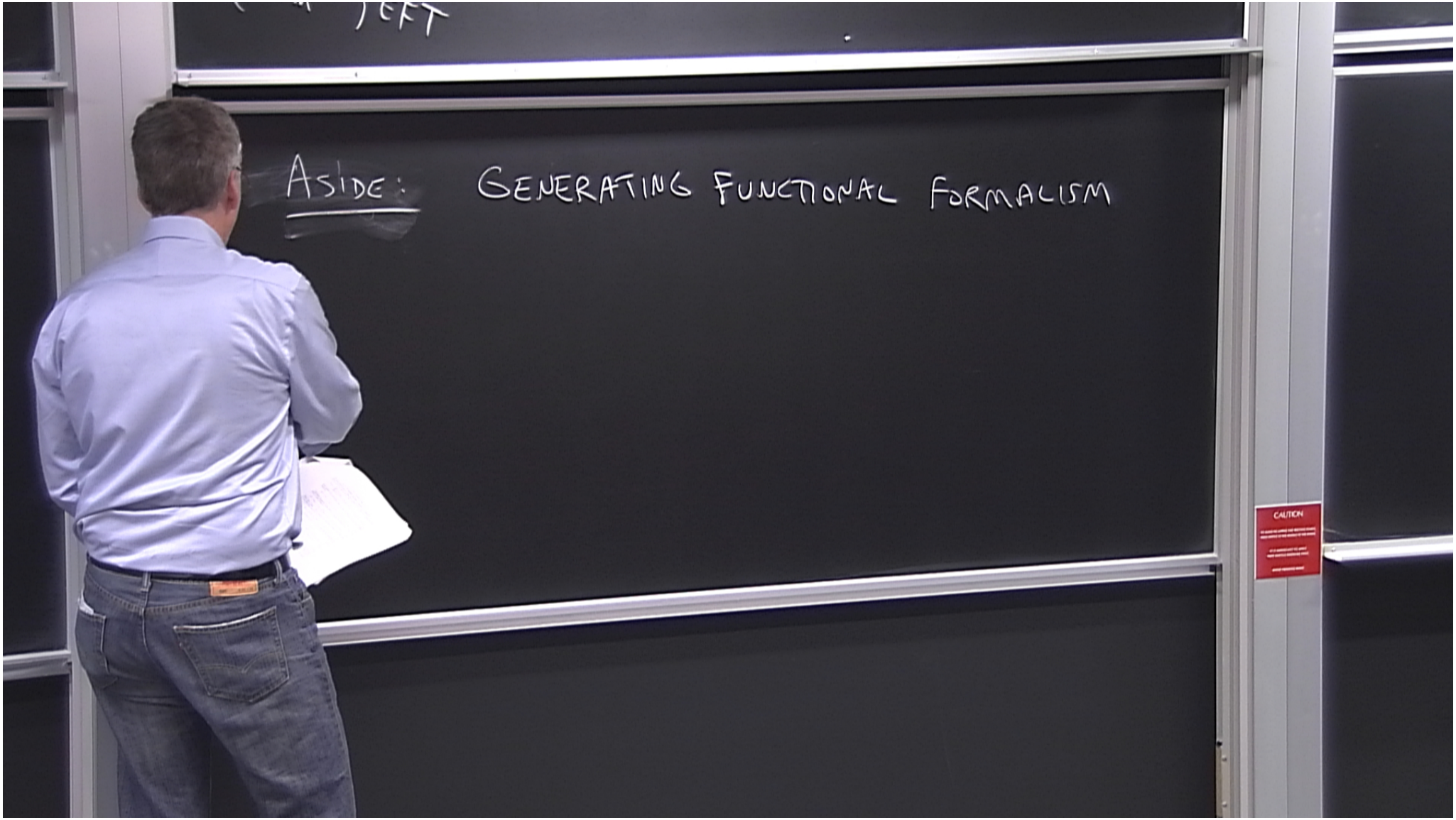
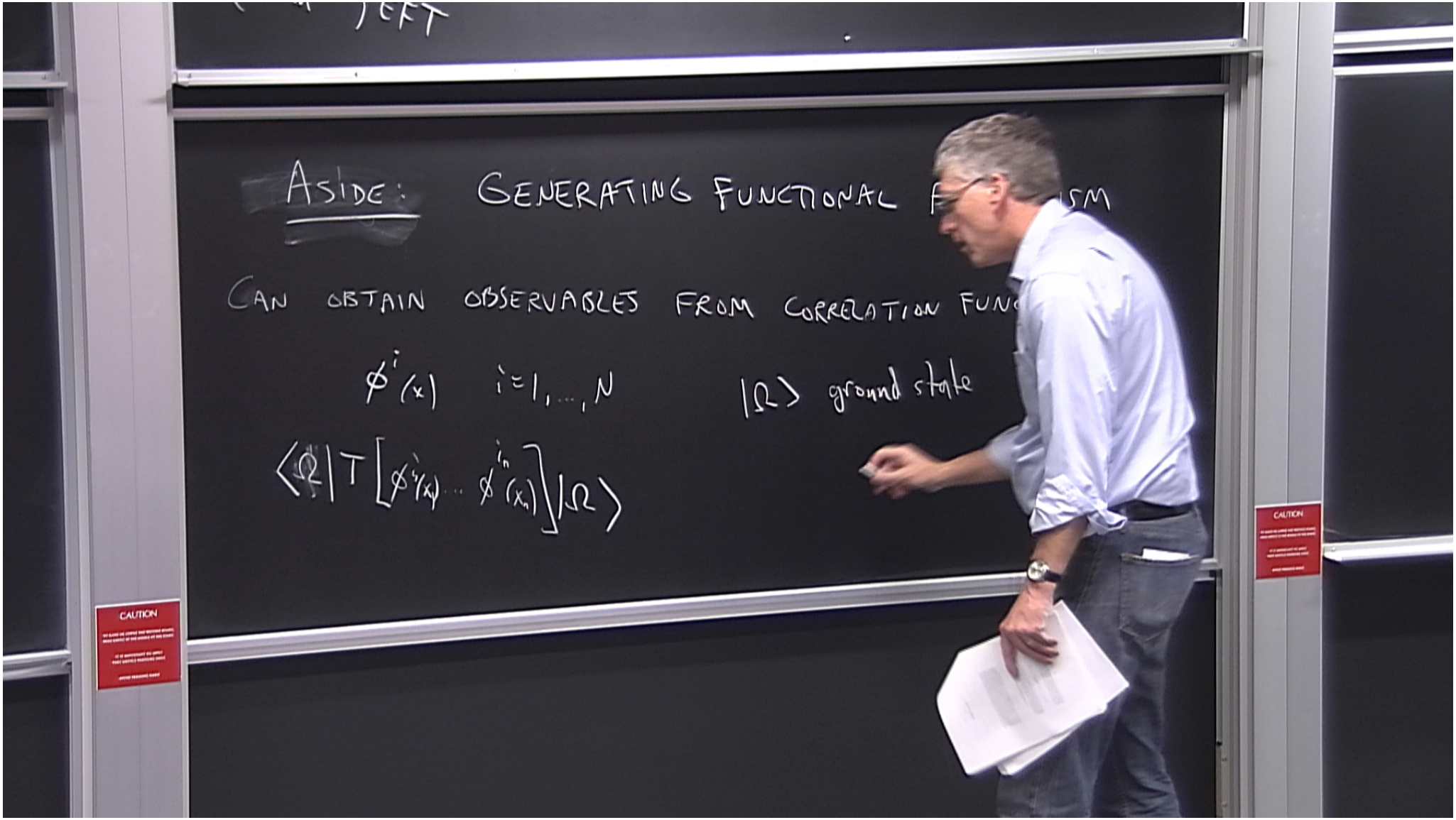
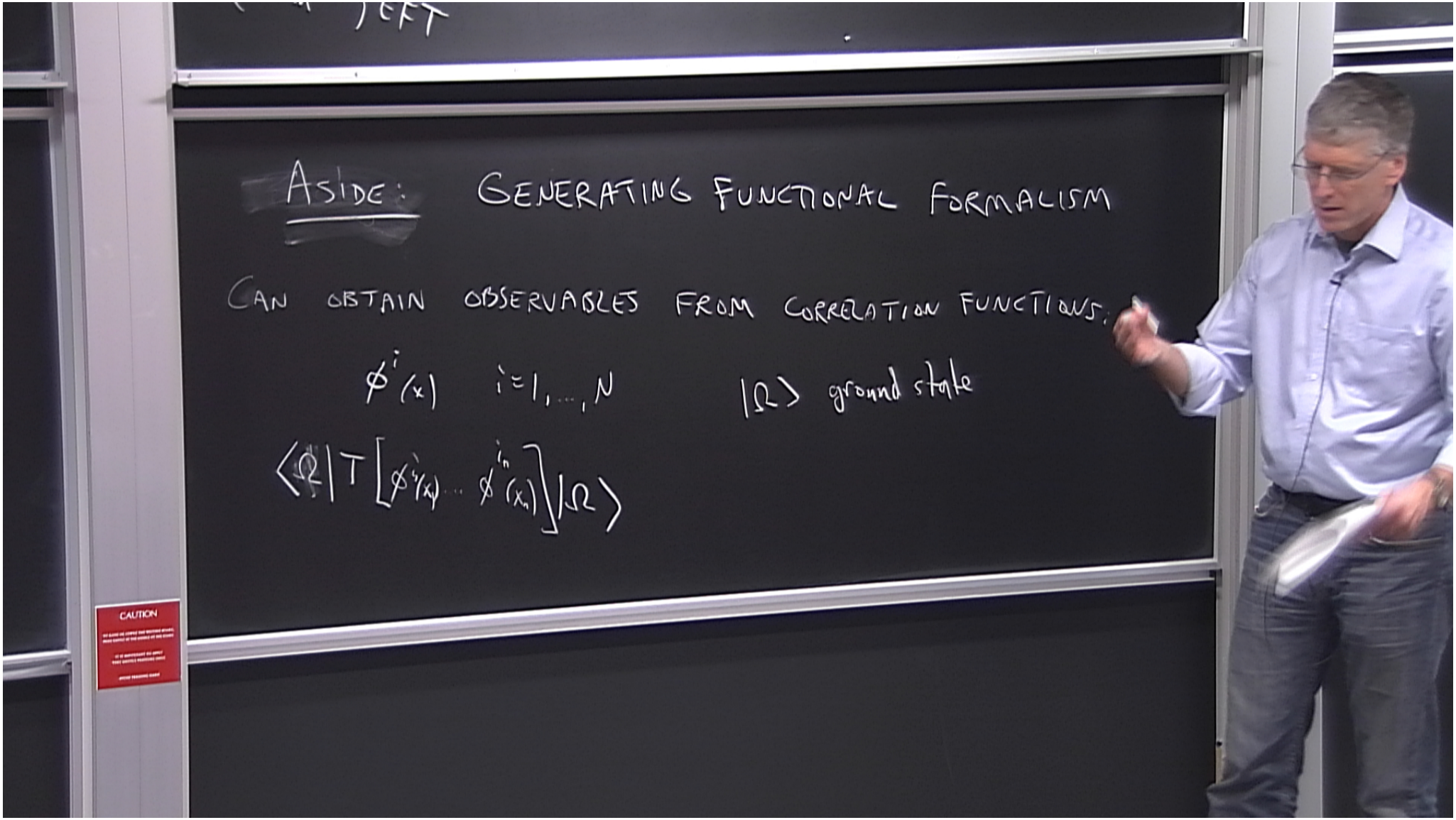
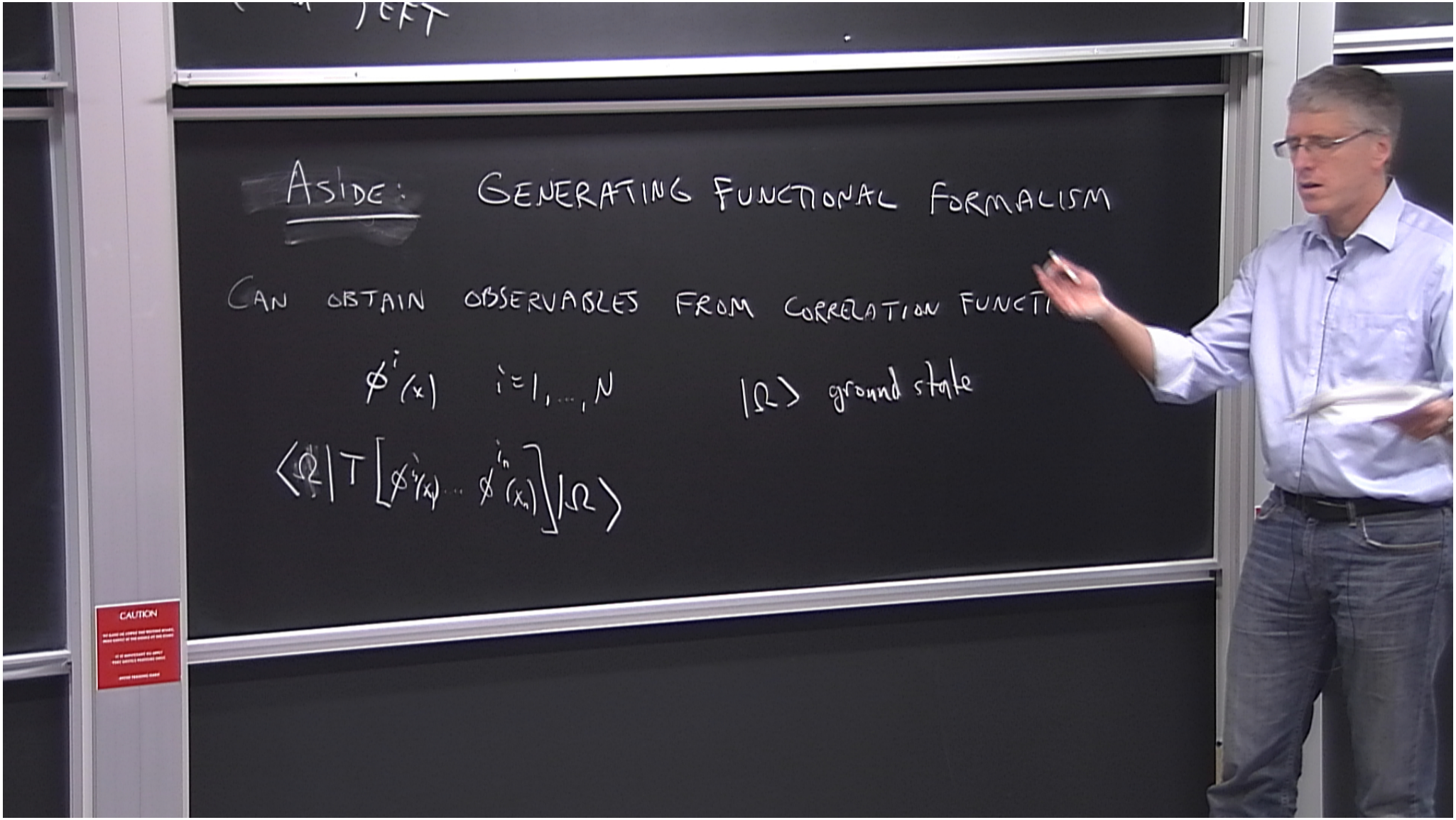
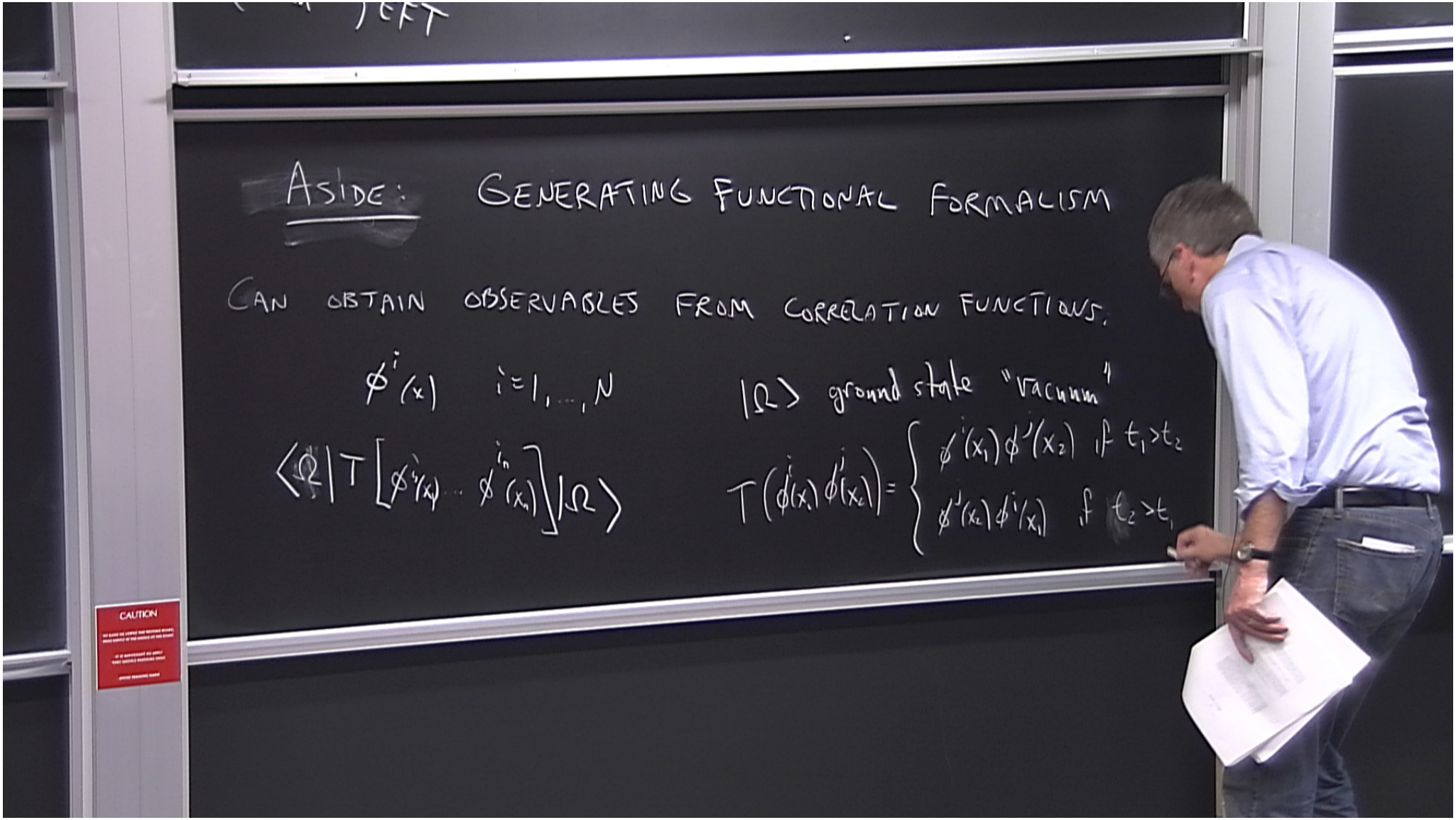
$\mathcal{L}_{\text{eff}} = -\frac{1}{2}\partial_\mu \phi_I \partial^\mu \phi_I - a(\partial_\mu \phi_I \partial^\mu \phi_I)^2 + b(\partial_\mu \phi_I \partial^\mu \phi_I)^3 +$
$a = \frac{c}{m_R^2}$ $c = \frac{\lambda}{4} + \#\lambda_T^2$
$b = \frac{\tilde{c}}{m_R^6}$ $\tilde{c} = \#\lambda_T$


ASIDE: GENERATING FUNCTIONAL FORMALISM
CAUTION


Aside: Generating Functional
Can obtain observables from correlation
$\phi^i(x)$ $i = 1, \ldots, N$
$|\Omega\rangle$ ground state
$\langle\Omega|T[\phi^{i_1}(x_1) \ldots \phi^{i_n}(x_n)]|\Omega\rangle$
CAUTION
CAUTION


Aside: Generating Functional Formalism
Can obtain observables from correlation functions.
$\phi^i(x)$ $i = 1, \ldots, N$
$|\Omega\rangle$ ground state
$\langle\Omega|T[\phi^{i_1}(x_1)\ldots\phi^{i_n}(x_n)]|\Omega\rangle$
CAUTION


Aside: Generating Functional Formalism
Can obtain observables from correlation functi
$\phi^i(x)$ $i = 1, \ldots, N$
$|\Omega\rangle$ ground state
$\langle\Omega|T[\phi^{i_1}(x_1)\ldots\phi^{i_n}(x_n)]|\Omega\rangle$
CAUTION


Aside: Generating Functional Formalism
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$\phi^i(x)$ $i = 1, \ldots, N$
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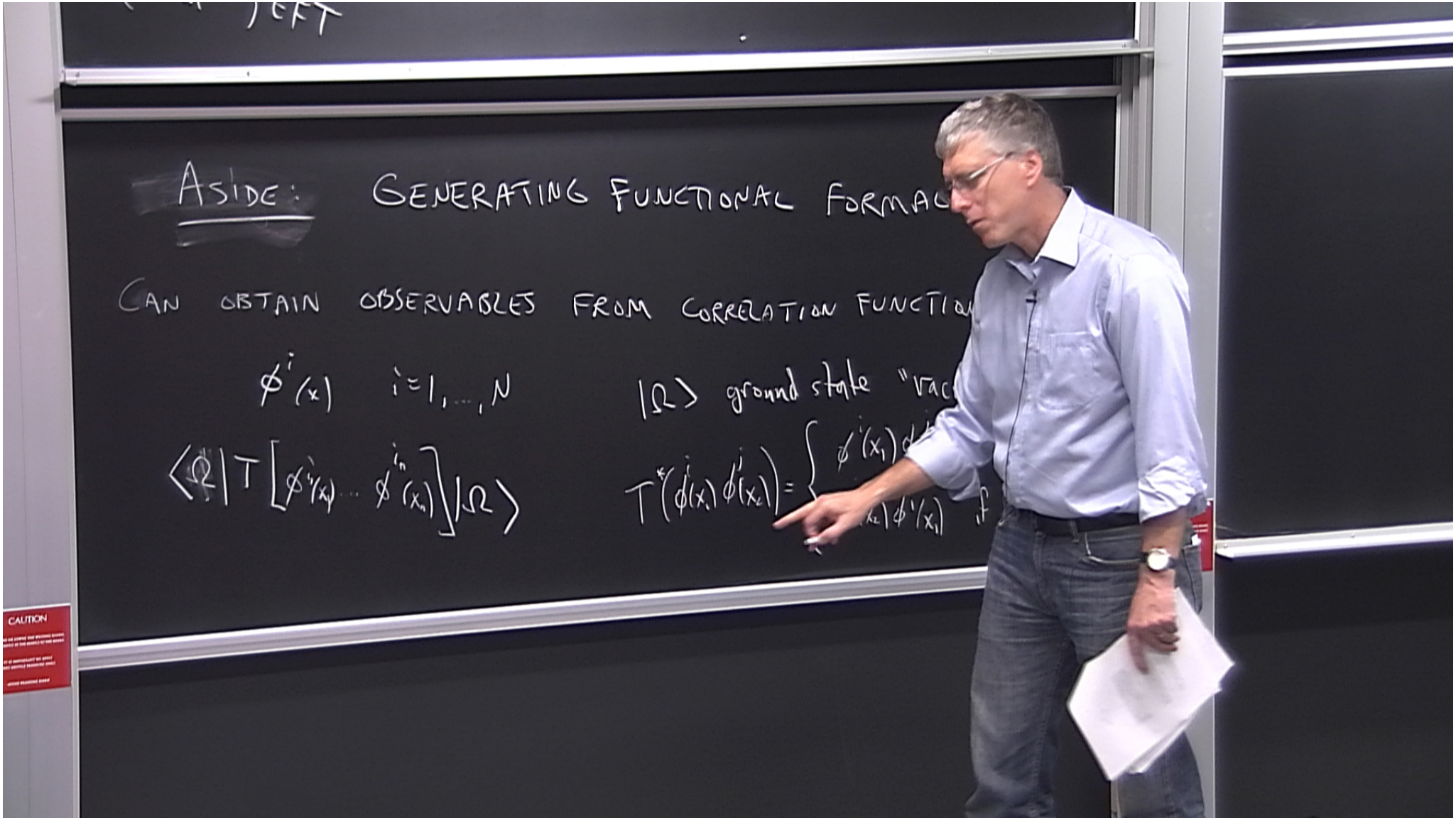
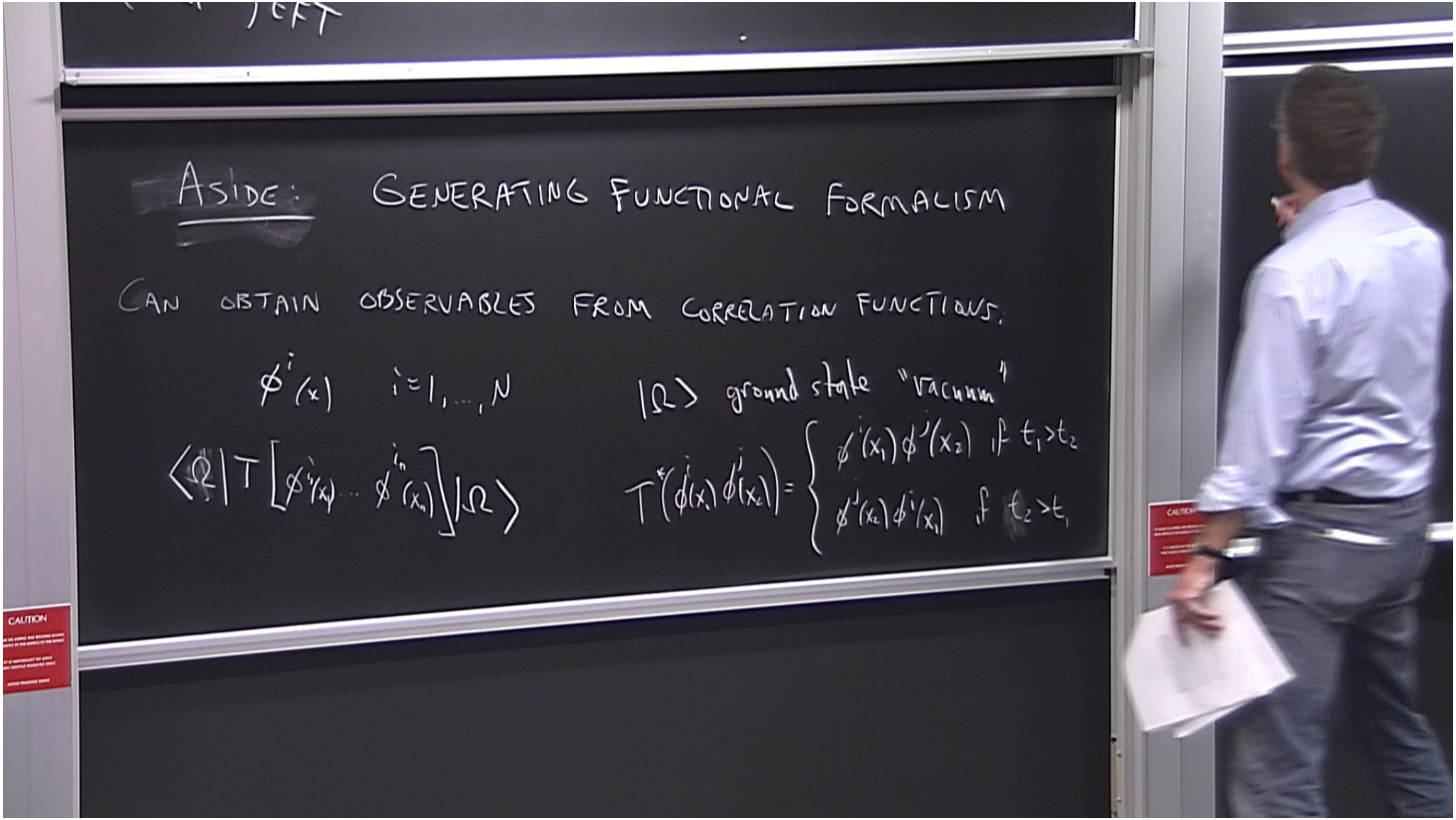
$\langle\Omega|T[\phi^{i_1}(x_1)\ldots\phi^{i_n}(x_n)]|\Omega\rangle$
$T(\phi^i(x_1)\phi^j(x_2)) = \begin{cases} \phi^i(x_1)\phi^j(x_2) & \text{if } t_1 > t_2 \\ \phi^j(x_2)\phi^i(x_1) & \text{if } t_2 > t_1 \end{cases}$
CAUTION


Aside: Generating Functional Formac
Can obtain observables from correlation functio
$\phi^i(x)$ $i=1,\dots,N$
$|\Omega\rangle$ ground state "vac
$\langle\Omega|T[\phi^{i_1}(x_1)\dots\phi^{i_n}(x_n)]|\Omega\rangle$
$T^*(\phi(x_1)\phi(x_2))=\{$
CAUTION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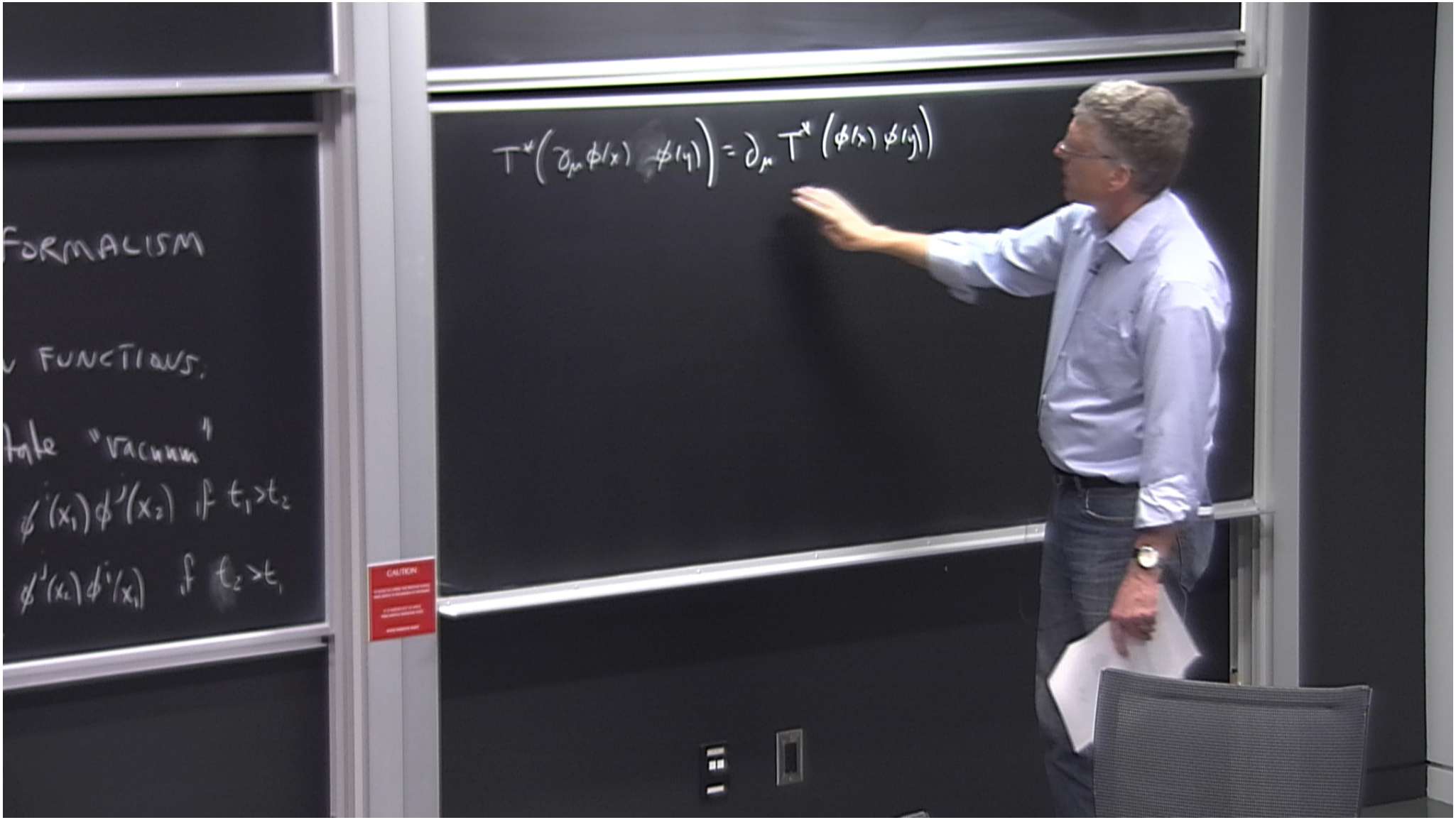
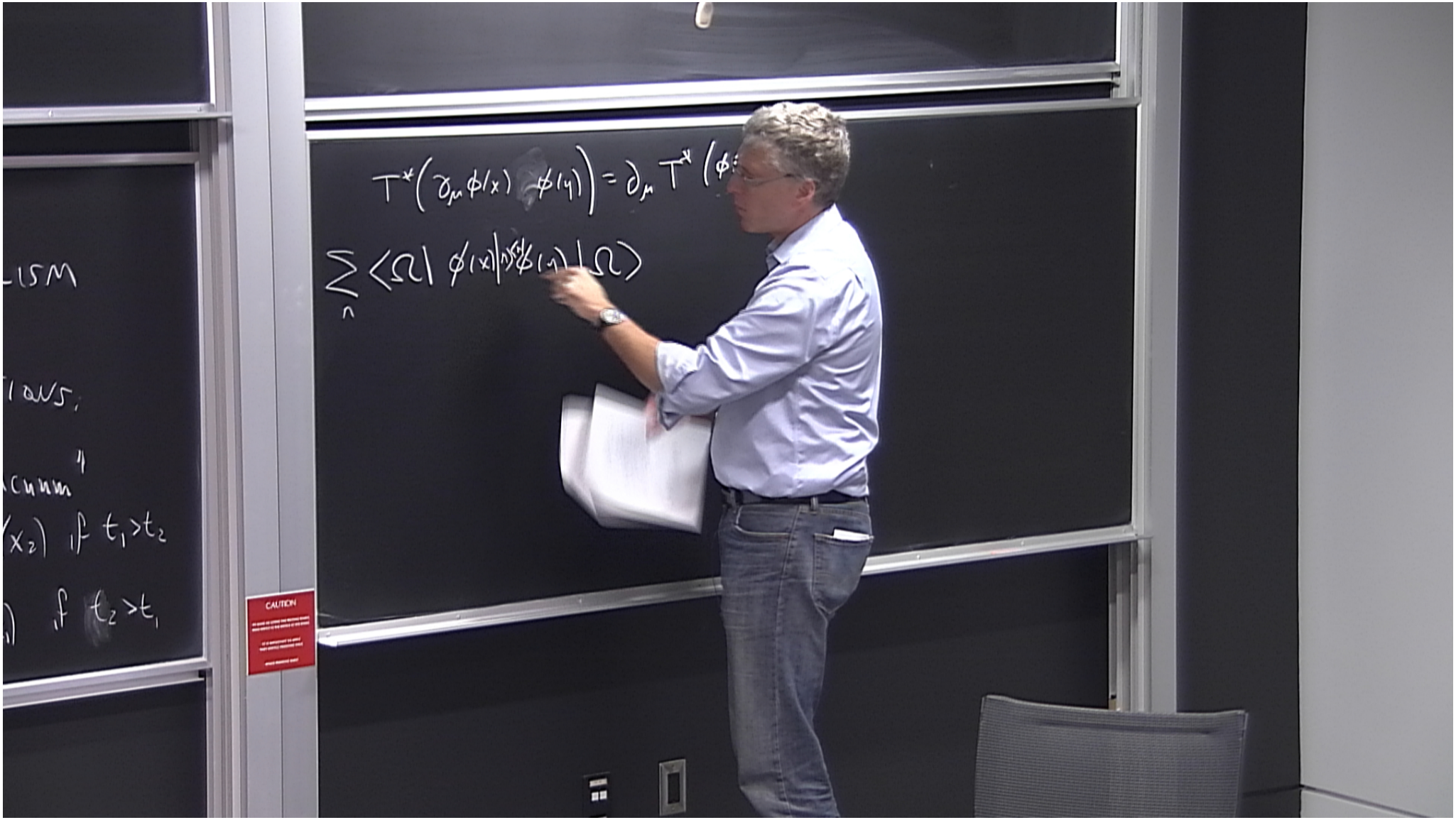
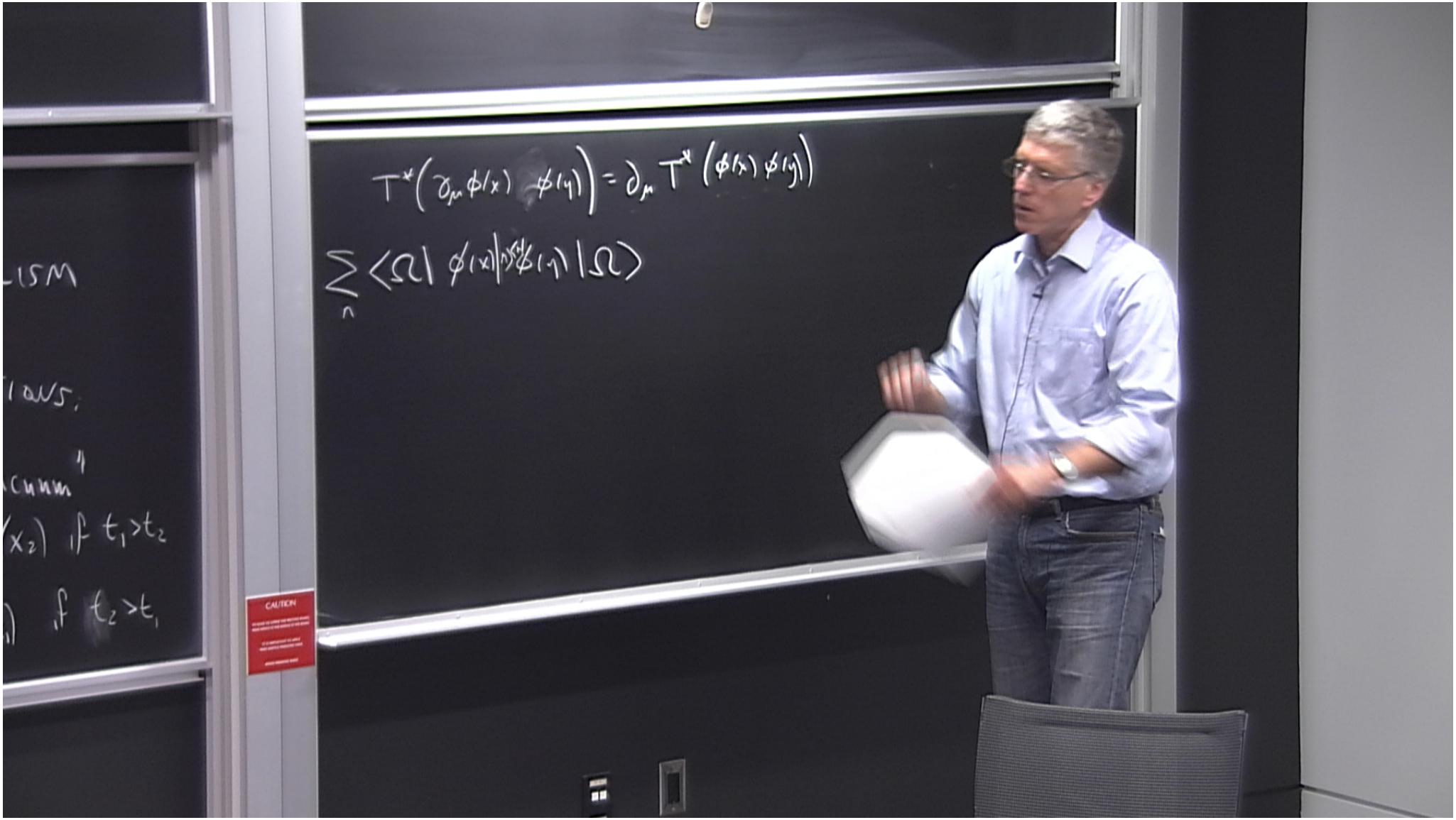

$T^{\nu}\left(\partial_{\mu}\phi(x)\,\phi(y)\right) = \partial_{\mu}\,T^{\nu}\left(\phi(x)\,\phi(y)\right)$
FORMALISM
FUNCTIONS:
"vacuum"
$\phi^{i}(x_1)\phi^{j}(x_2)$ if $t_1 > t_2$
$\phi^{j}(x_2)\phi^{i}(x_1)$ if $t_2 > t_1$
CAUTION


$T^*(\partial_\mu \phi(x)\ \phi(y)) = \partial_\mu T^*(\phi$
$\sum_n \langle\Omega|\phi(x)|n\rangle\langle n|\phi(y)|\Omega\rangle$
CAUTION


$T^*(\partial_\mu \phi(x)\ \phi(y)) = \partial_\mu T^*(\phi(x)\phi(y))$
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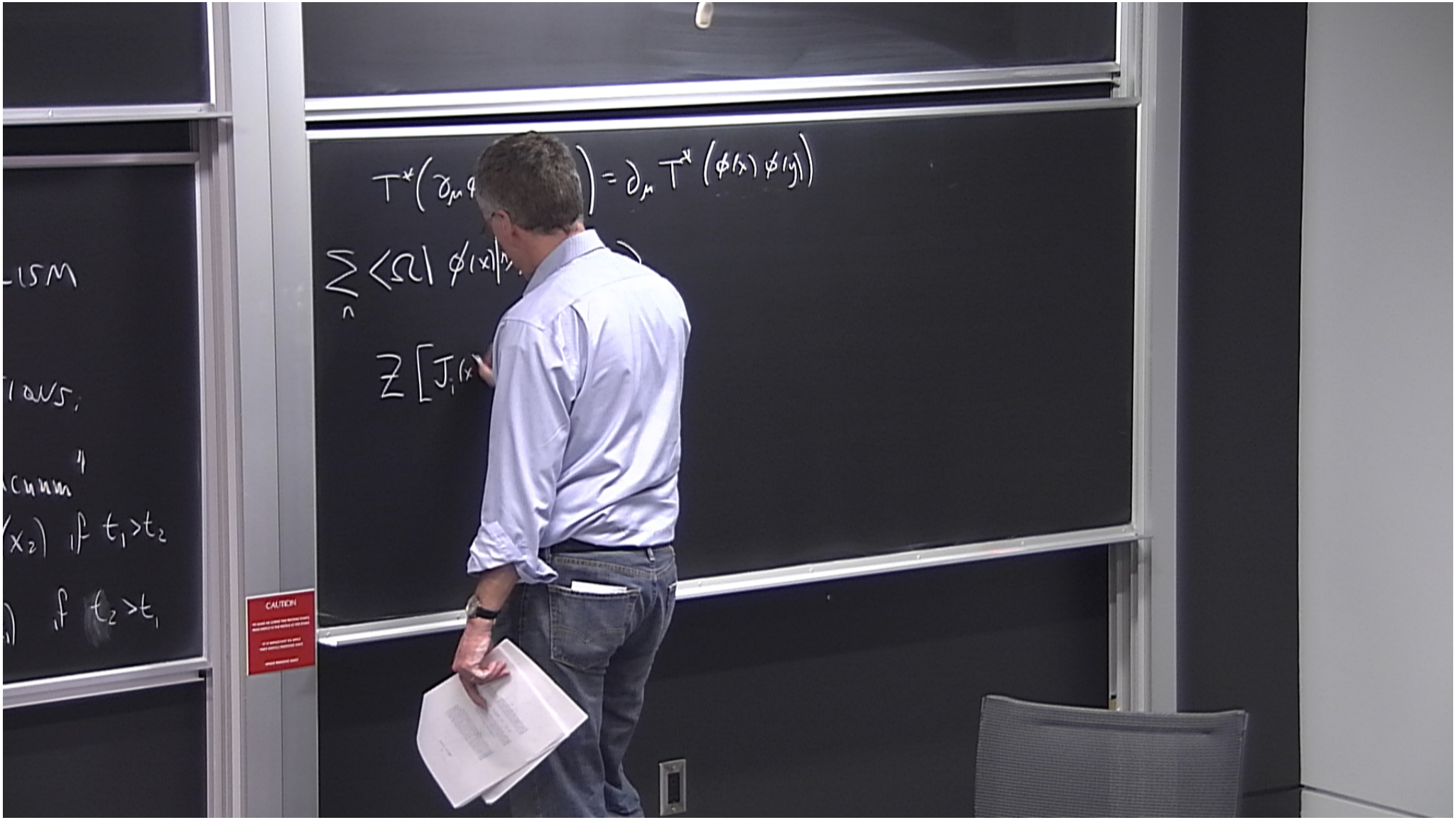
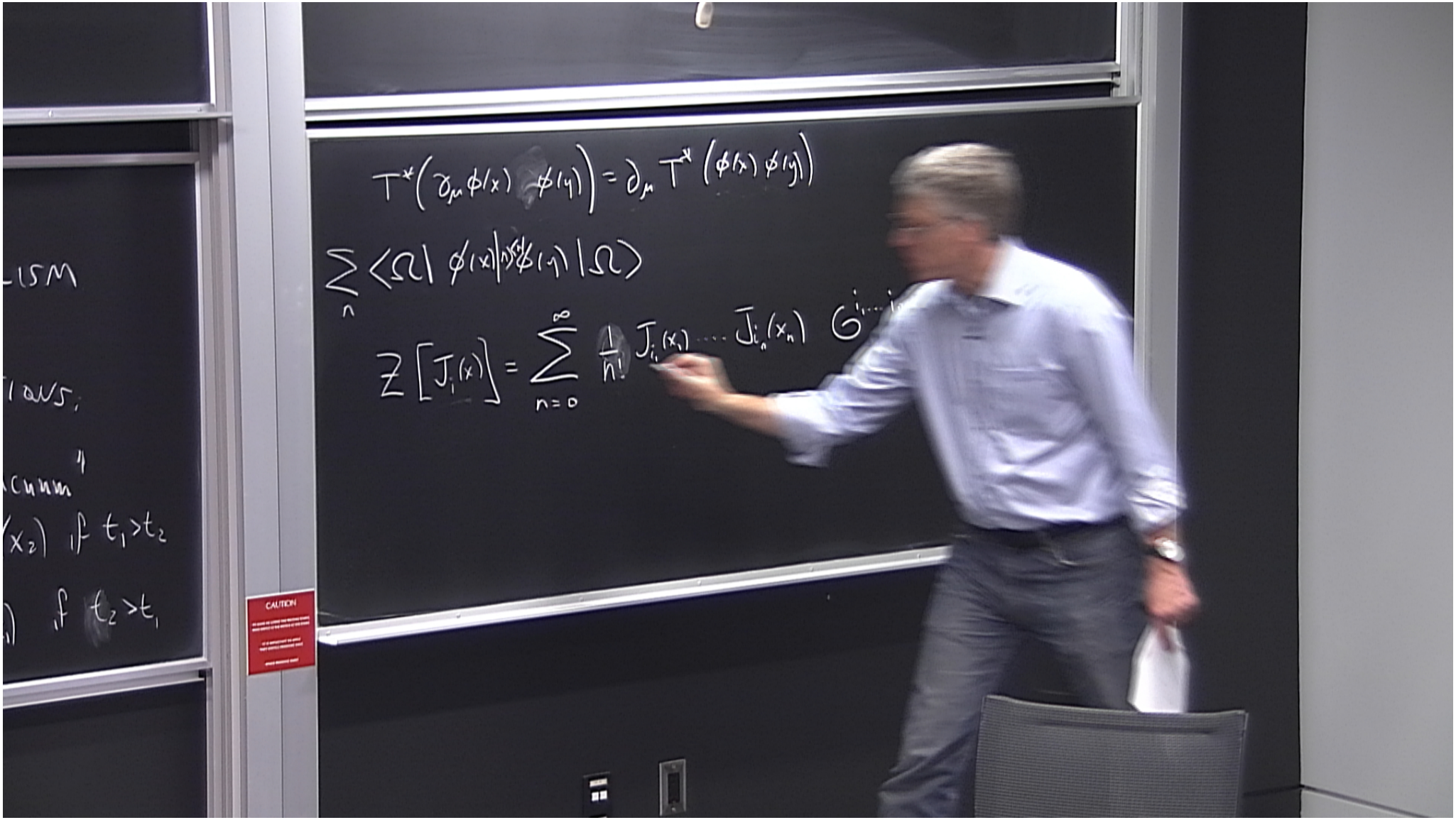
CAUTION


$T^*(\partial_\mu \phi \ldots) = \partial_\mu T^*(\phi(x)\phi(y))$
$\sum_n \langle \Omega | \phi(x) | n \rangle \ldots$
$Z[J_i(x$
(x_2) if $t_1 > t_2$
if $t_2 > t_1$
CAUTION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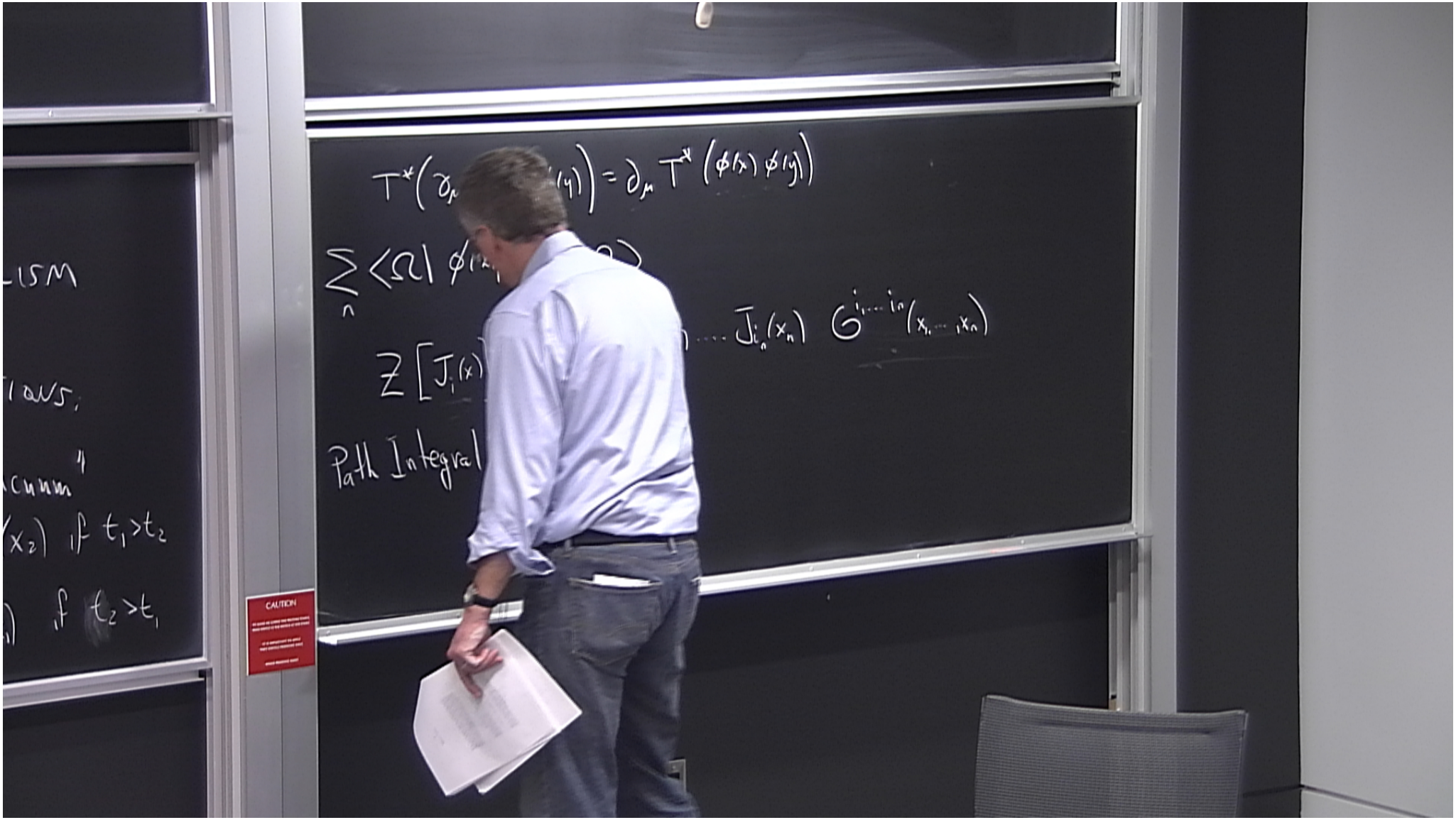
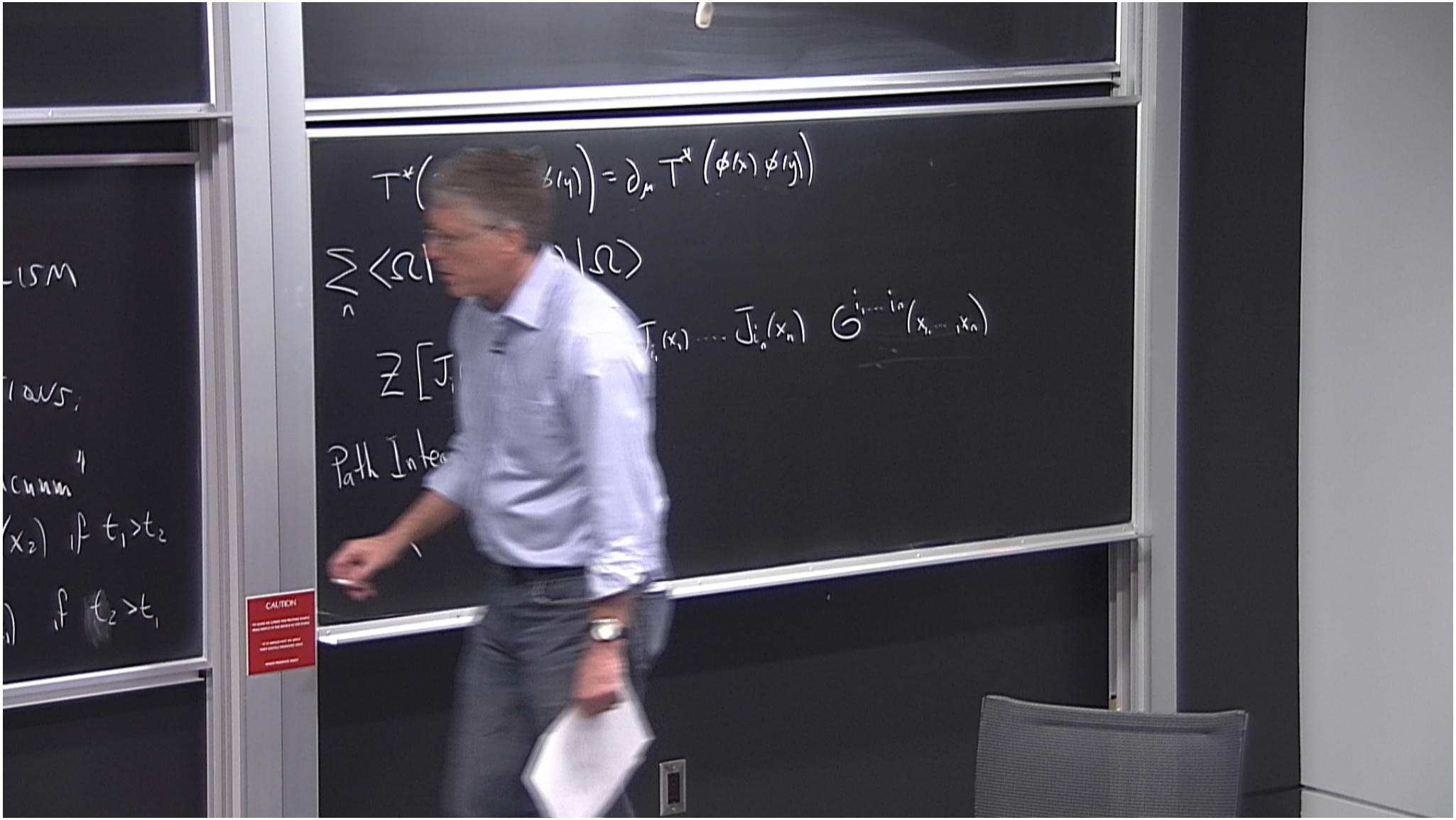
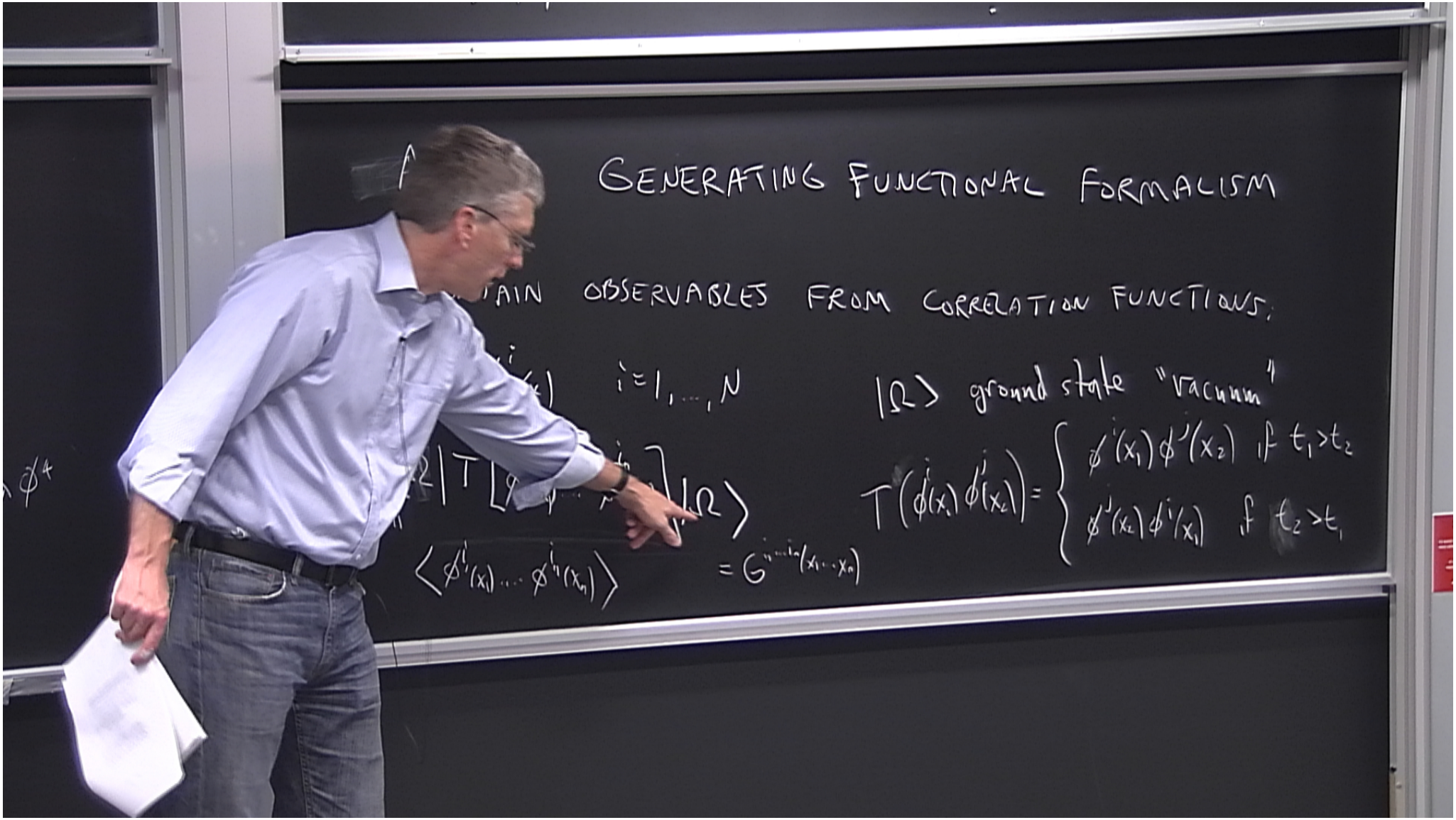
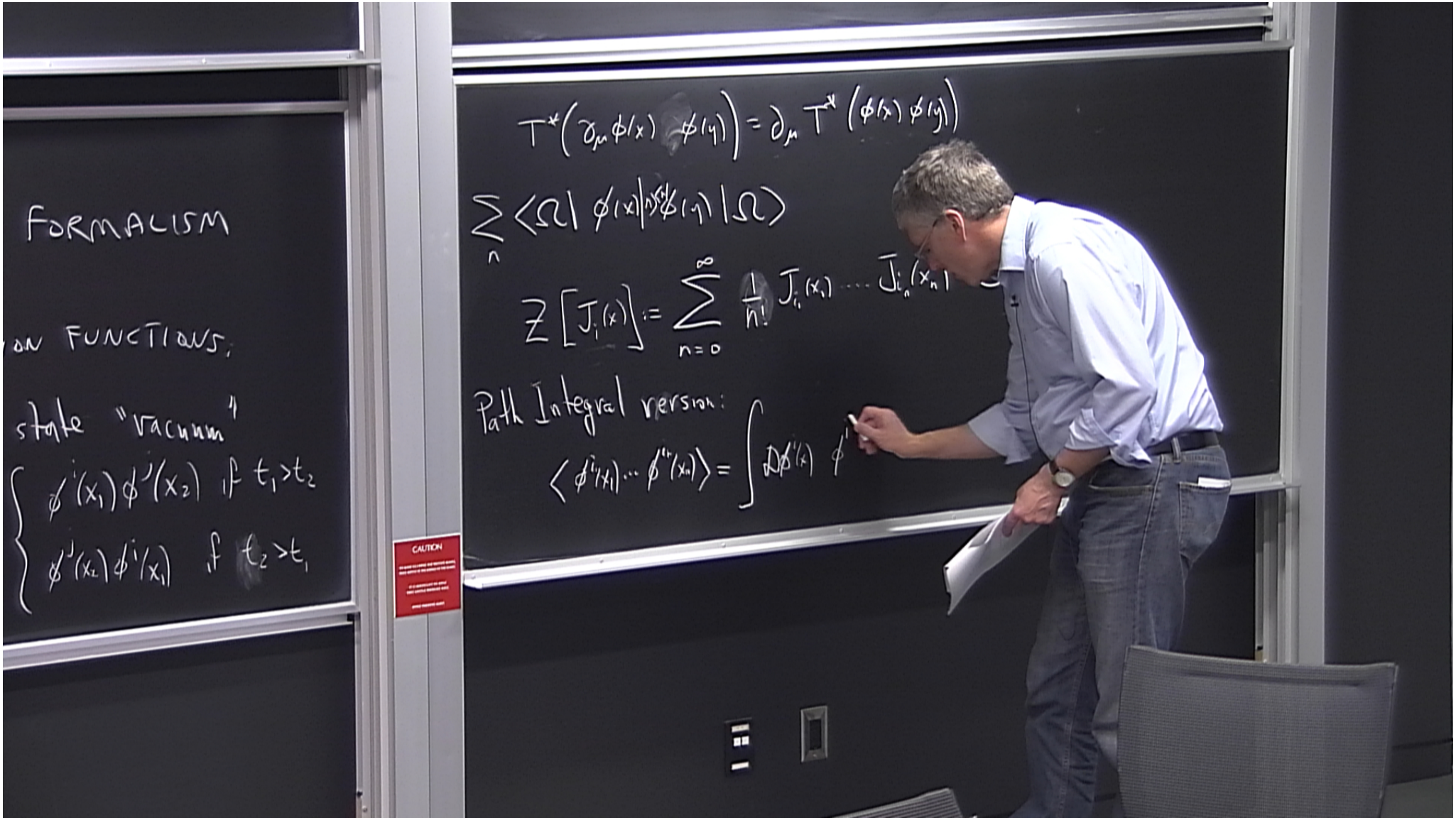
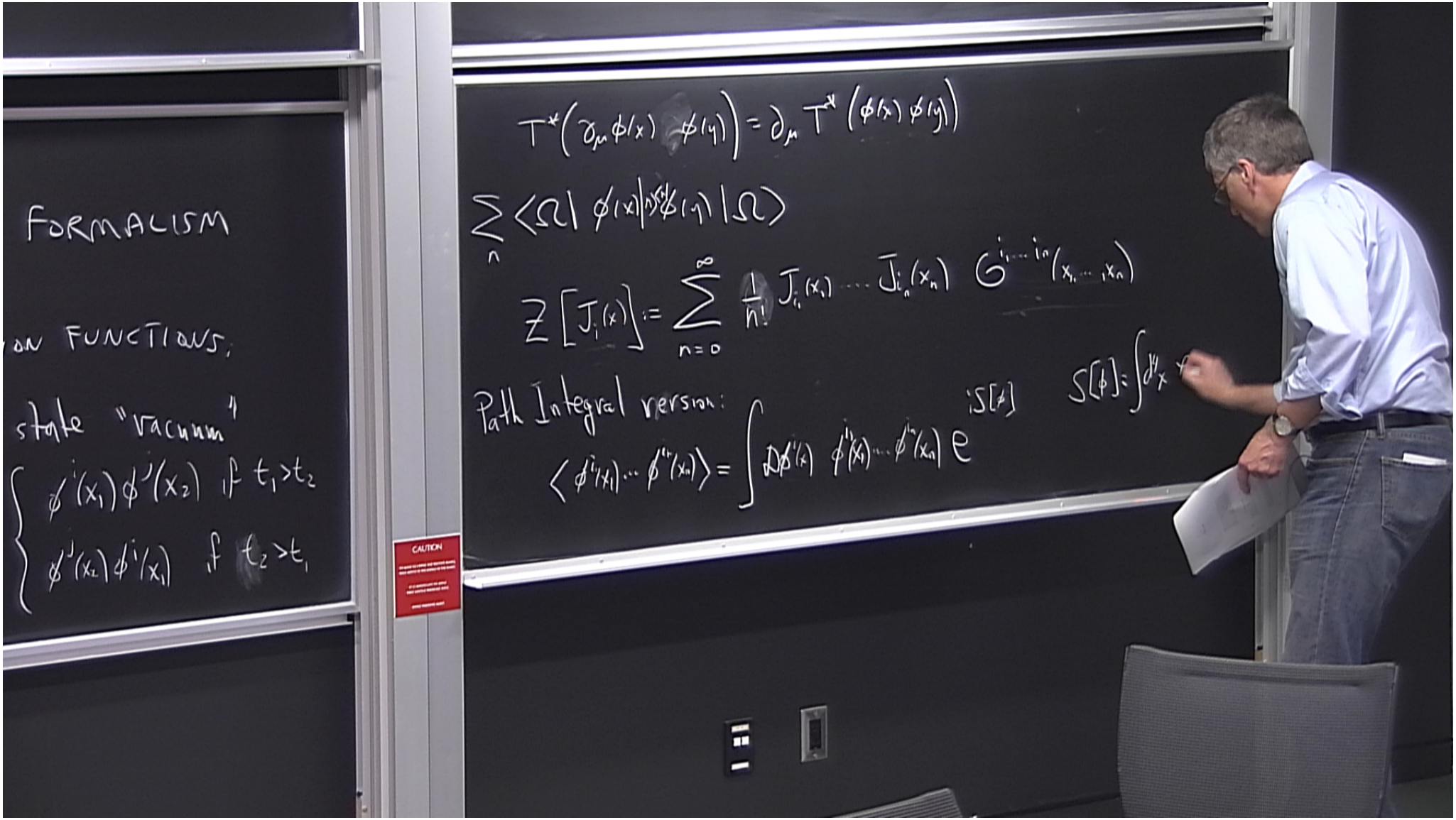
$Z[J_i(x)] = \sum_{n=0}^{\infty} \frac{1}{n!} J_{i_1}(x_1) \cdots J_{i_n}(x_n)$
CAUTION

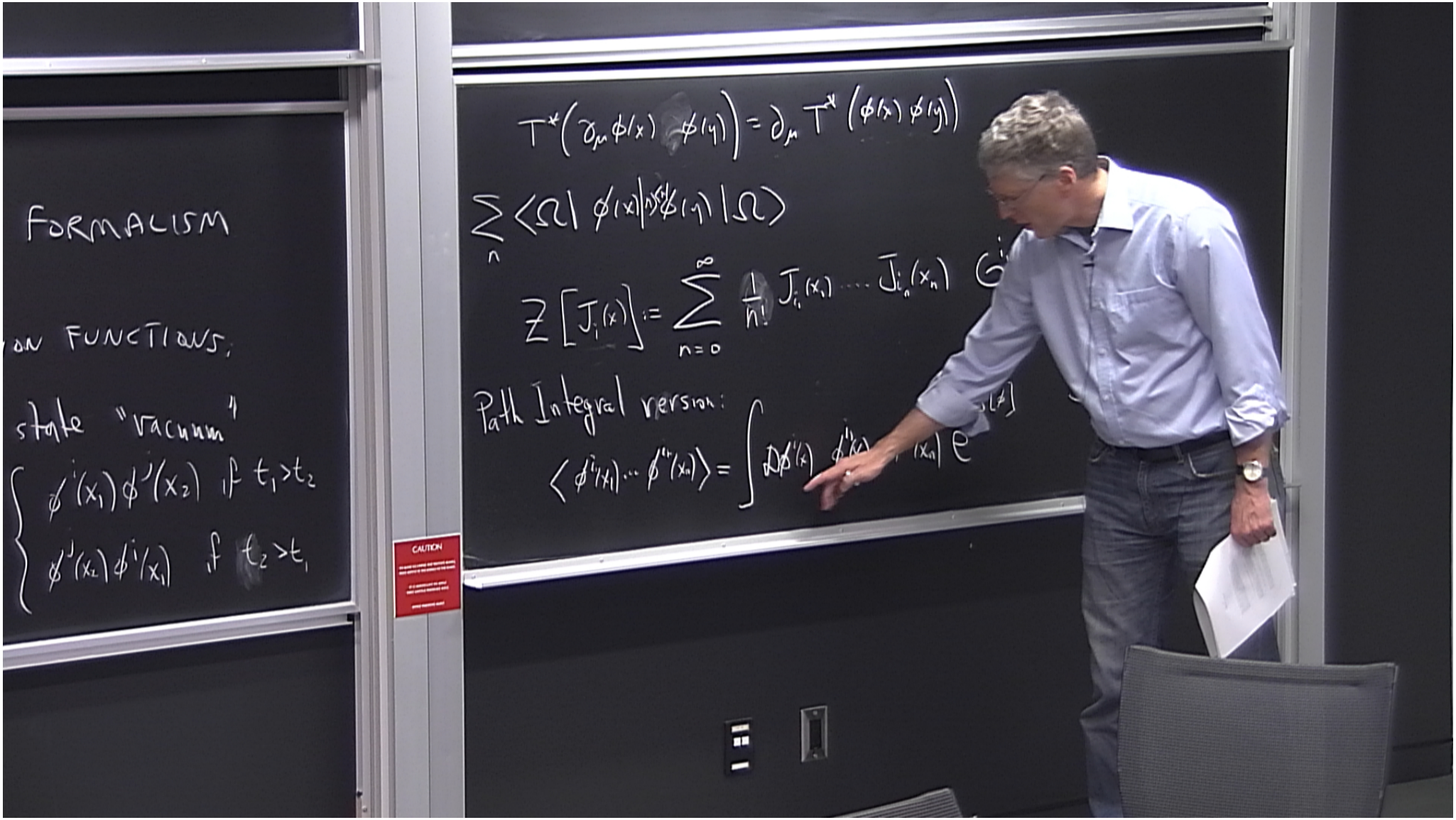

Path Integral
Z [J_i(x)
CAUTION


CAUTION


GENERATING FUNCTIONAL FORMALISM
OBSERVABLES FROM CORRELATION FUNCTIONS:
$i=1,\dots,N$
$|\Omega\rangle$ ground state "vacuum"
$T(\phi^i(x_1)\phi^j(x_2)) = \begin{cases} \phi^i(x_1)\phi^j(x_2) & \text{if } t_1 > t_2 \\ \phi^j(x_2)\phi^i(x_1) & \text{if } t_2 > t_1 \end{cases}$
$\langle \phi^{i_1}(x_1)\cdots\phi^{i_n}(x_n)\rangle = G^{i_1\cdots i_n}(x_1\cdots x_n)$


FORMALISM
ON FUNCTIONS:
state "vacuum"
Path Integral version:
CAUTION


FORMALISM
ON FUNCTIONS:
state "vacuum"
$T^*(\partial_\mu \phi(x)\ \phi(y)) = \partial_\mu T^*(\phi(x)\phi(y))$
$\sum_n \langle\Omega|\phi(x)|n\rangle\langle n|\phi(y)|\Omega\rangle$
$Z[J_i(x)] := \sum_{n=0}^{\infty} \frac{1}{n!} J_{i_1}(x_1) \cdots J_{i_n}(x_n)\ G^{i_1 \cdots i_n}(x_1 \cdots x_n)$
Path Integral version:
$\langle \phi^{i_1}(x_1) \cdots \phi^{i_n}(x_n) \rangle = \int \mathcal{D}\phi^i(x)\ \phi^{i_1}(x_1) \cdots \phi^{i_n}(x_n)\ e^{iS[\phi]}$
$S[\phi] = \int d^4x$
CAUTION


FORMALISM
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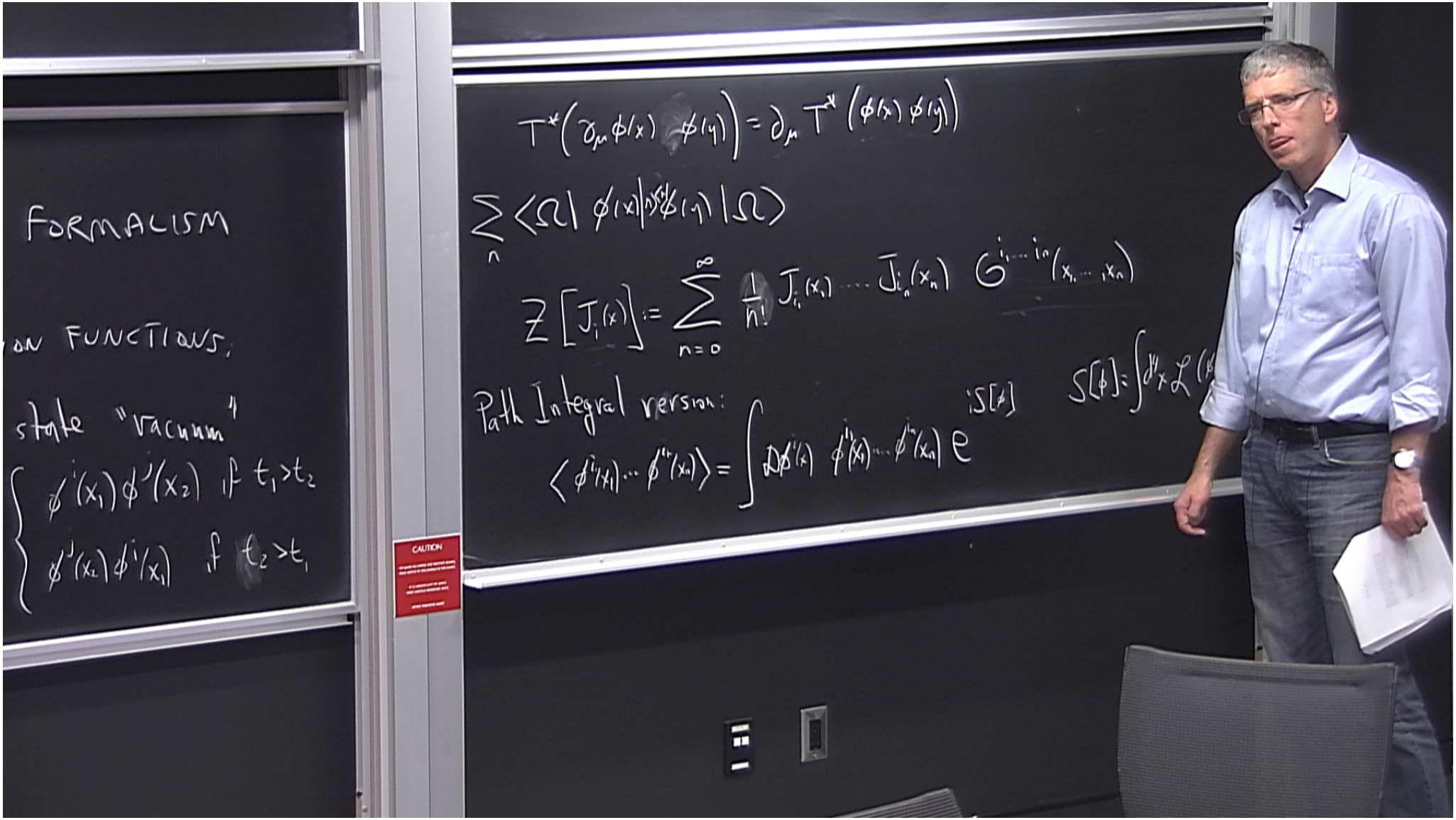
Path Integral version:
CAUTION


FORMALISM
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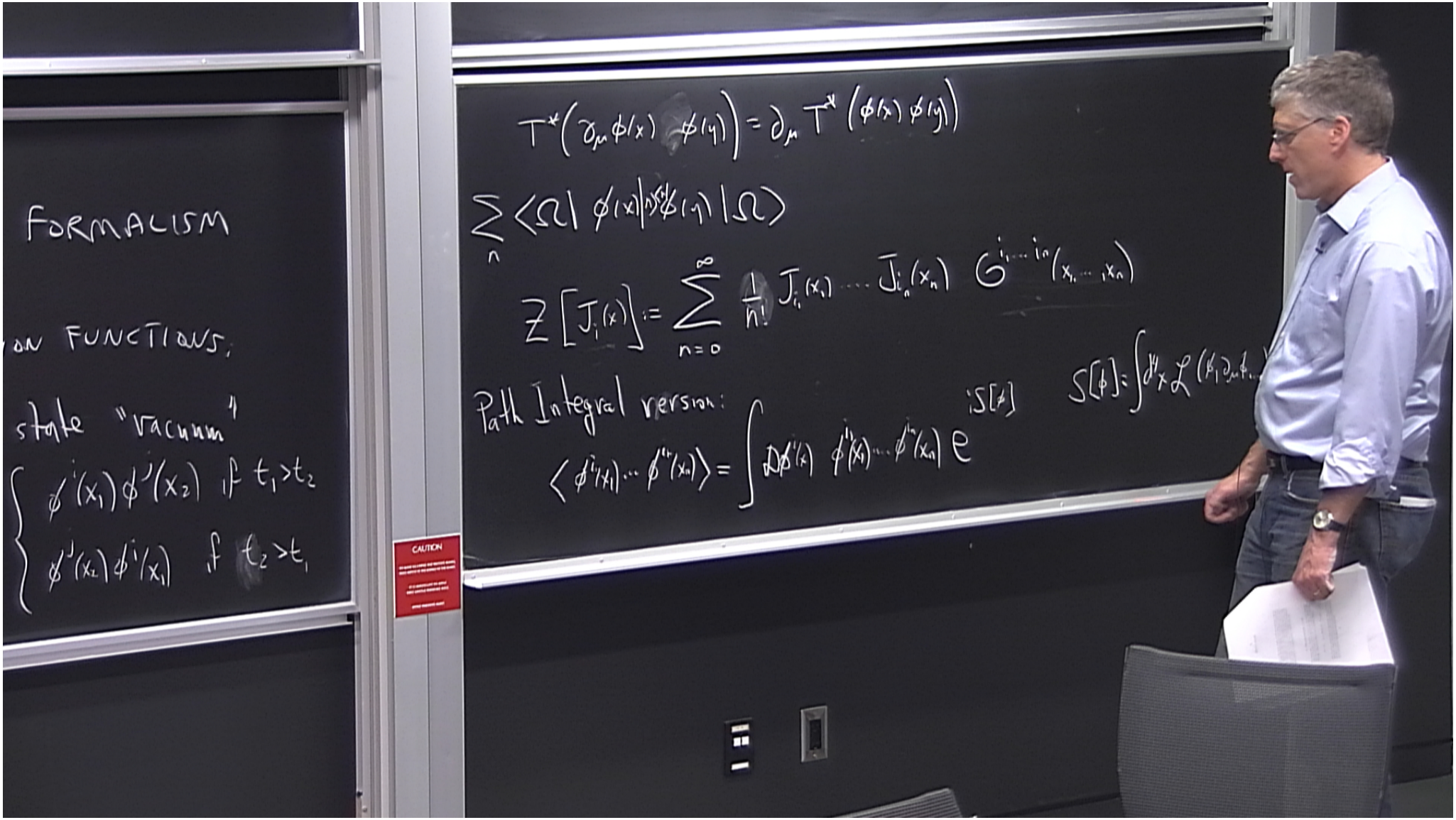
Path Integral version:
CAUTION


FORMALISM
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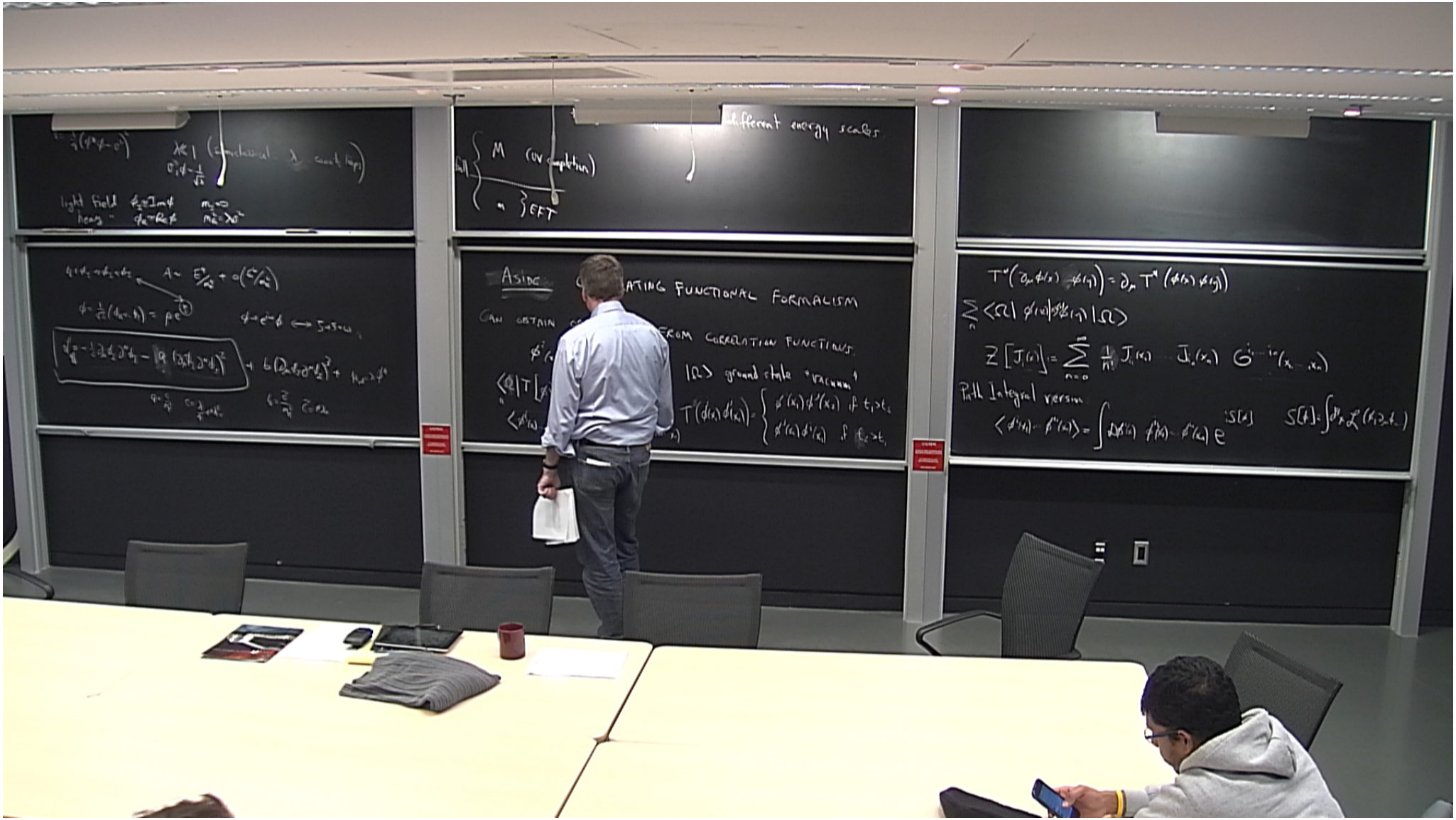
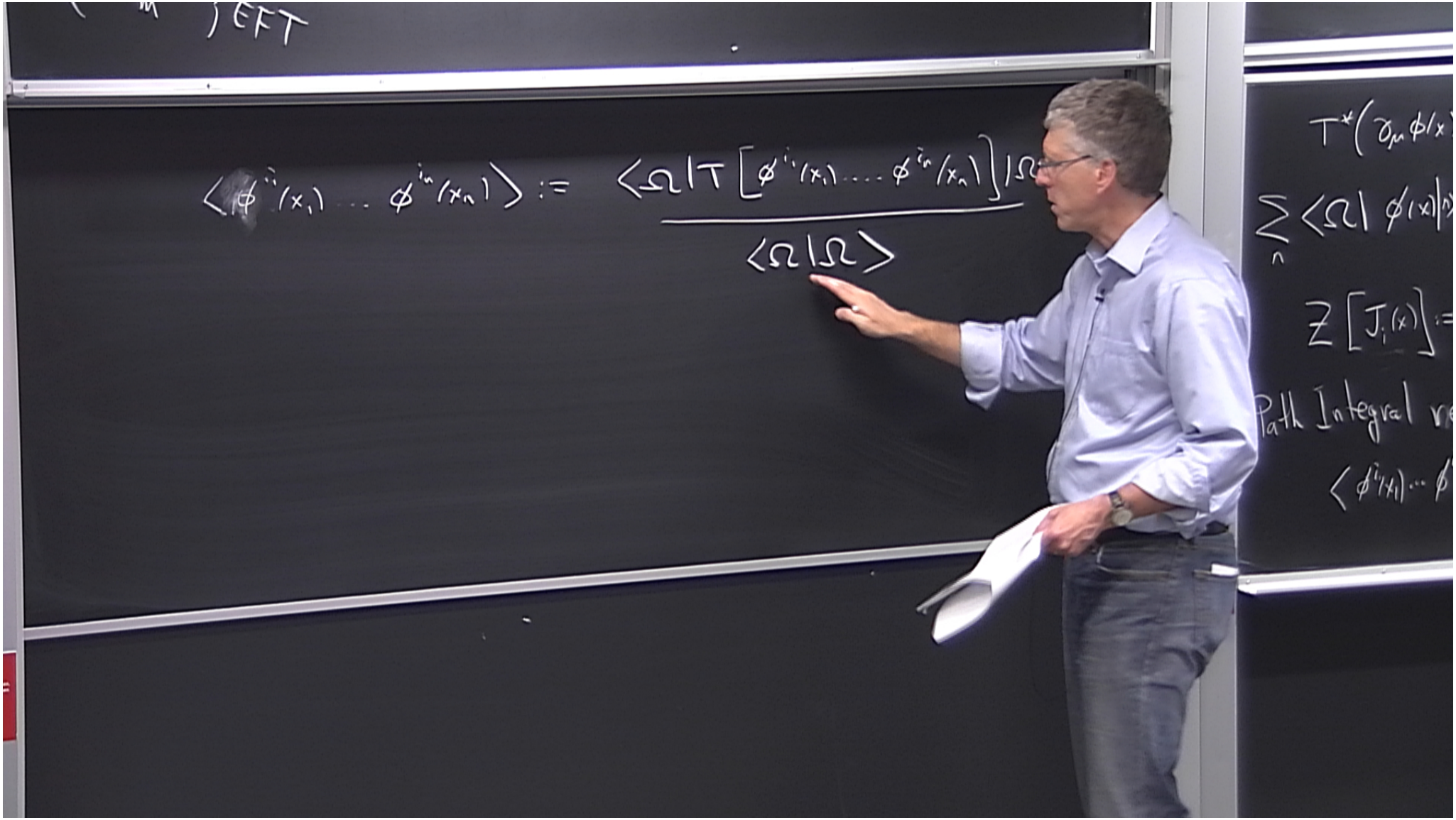
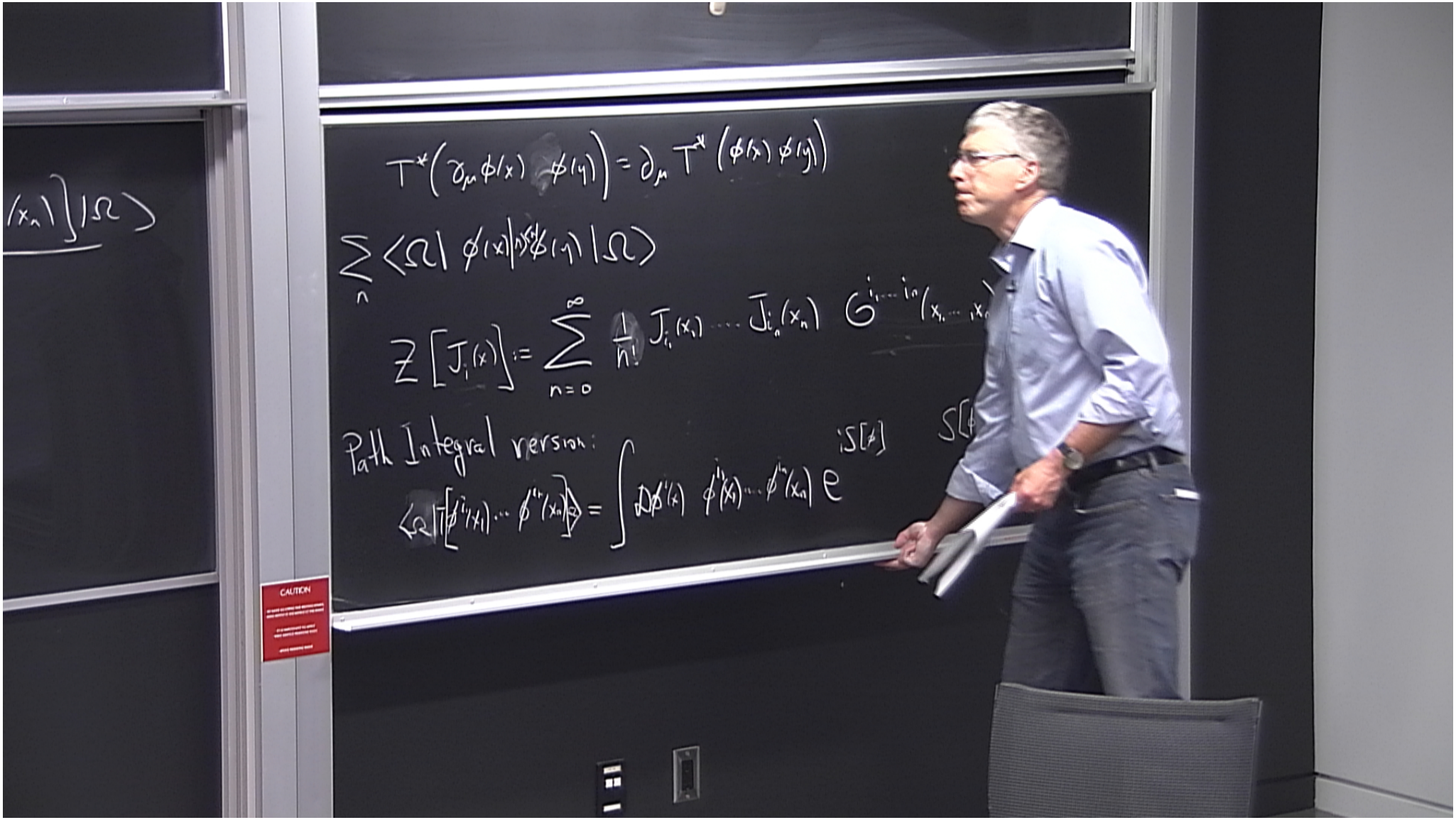
state "vacuum"
Path Integral version:
CAUTION


different energy scales
M (UV completion)
EFT
light field
heavy
Aside
FUNCTIONAL FORMALISM
CAN OBTAIN
FROM CORRELATION FUNCTIONS
|Ω⟩ ground state "vacuum"
Path Integral version


$\langle \phi^{i_1}(x_1) \dots \phi^{i_n}(x_n) \rangle := \frac{\langle \Omega | T[\phi^{i_1}(x_1) \dots \phi^{i_n}(x_n)] | \Omega \rangle}{\langle \Omega | \Omega \rangle}$
$T^*(\partial_\mu \phi(x)$
$\sum_n \langle \Omega | \phi(x) |$
$Z[J_i(x)] :=$
Path Integral re
$\langle \phi^{i_1}(x_1) \dots$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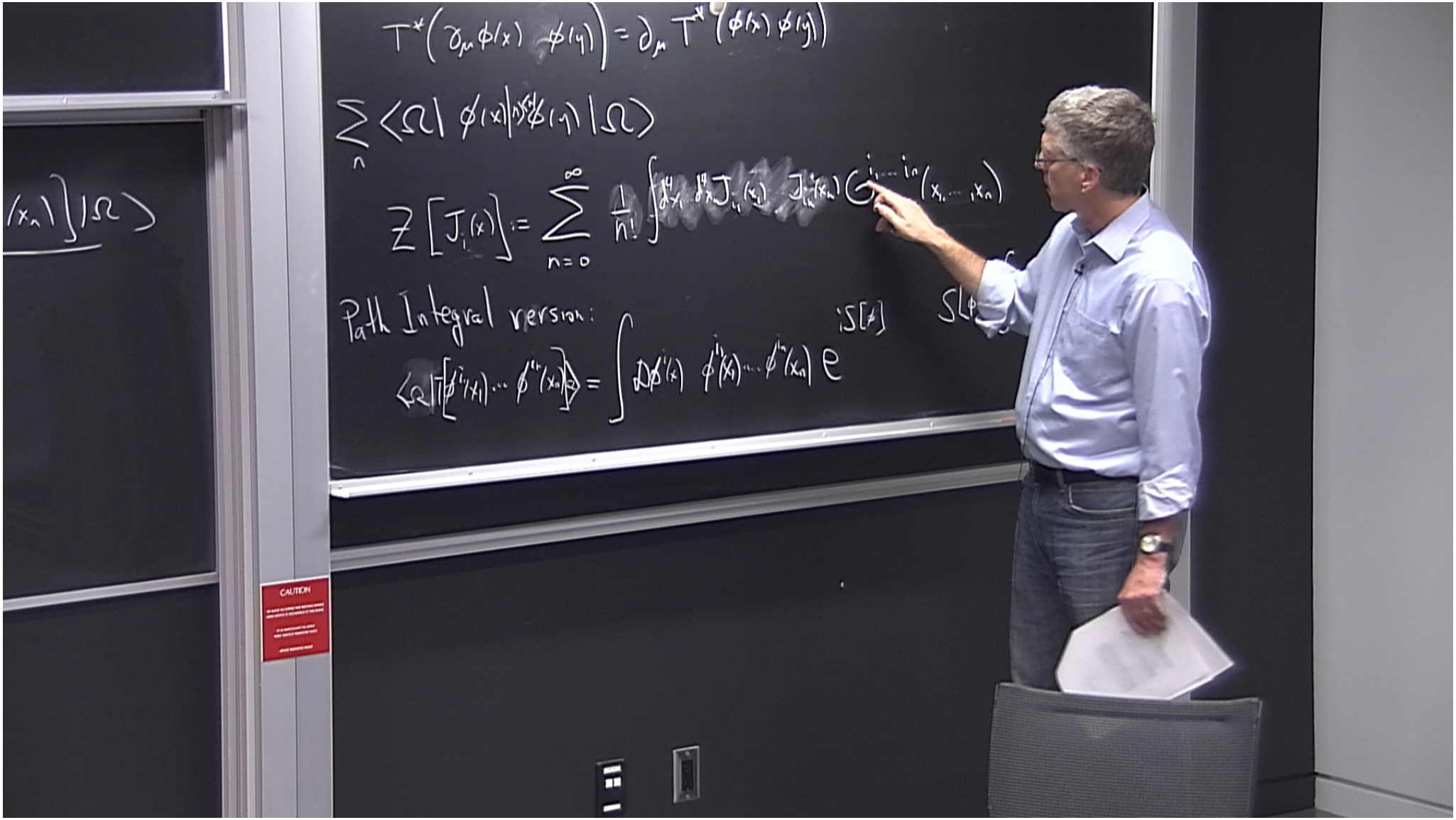
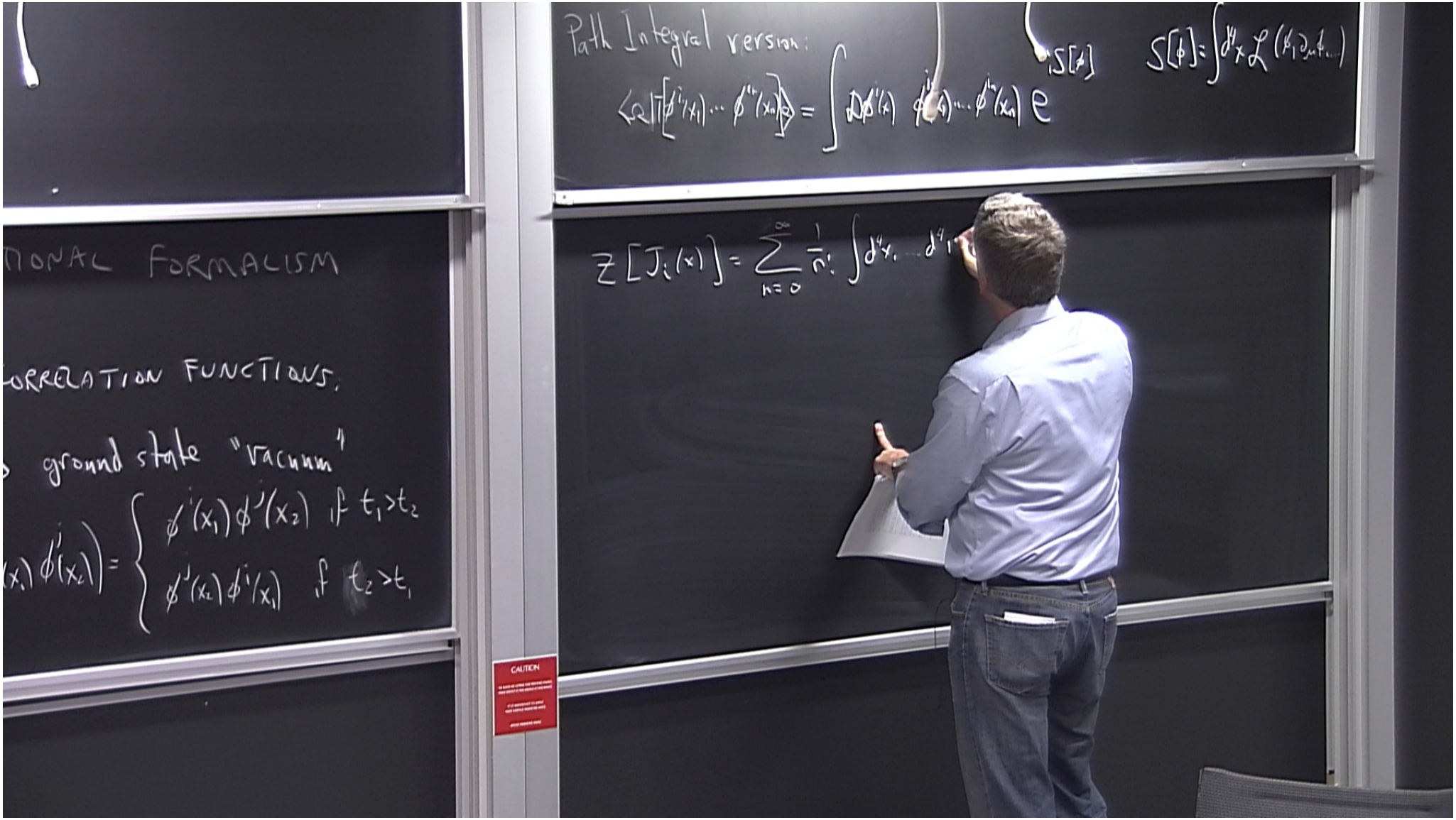
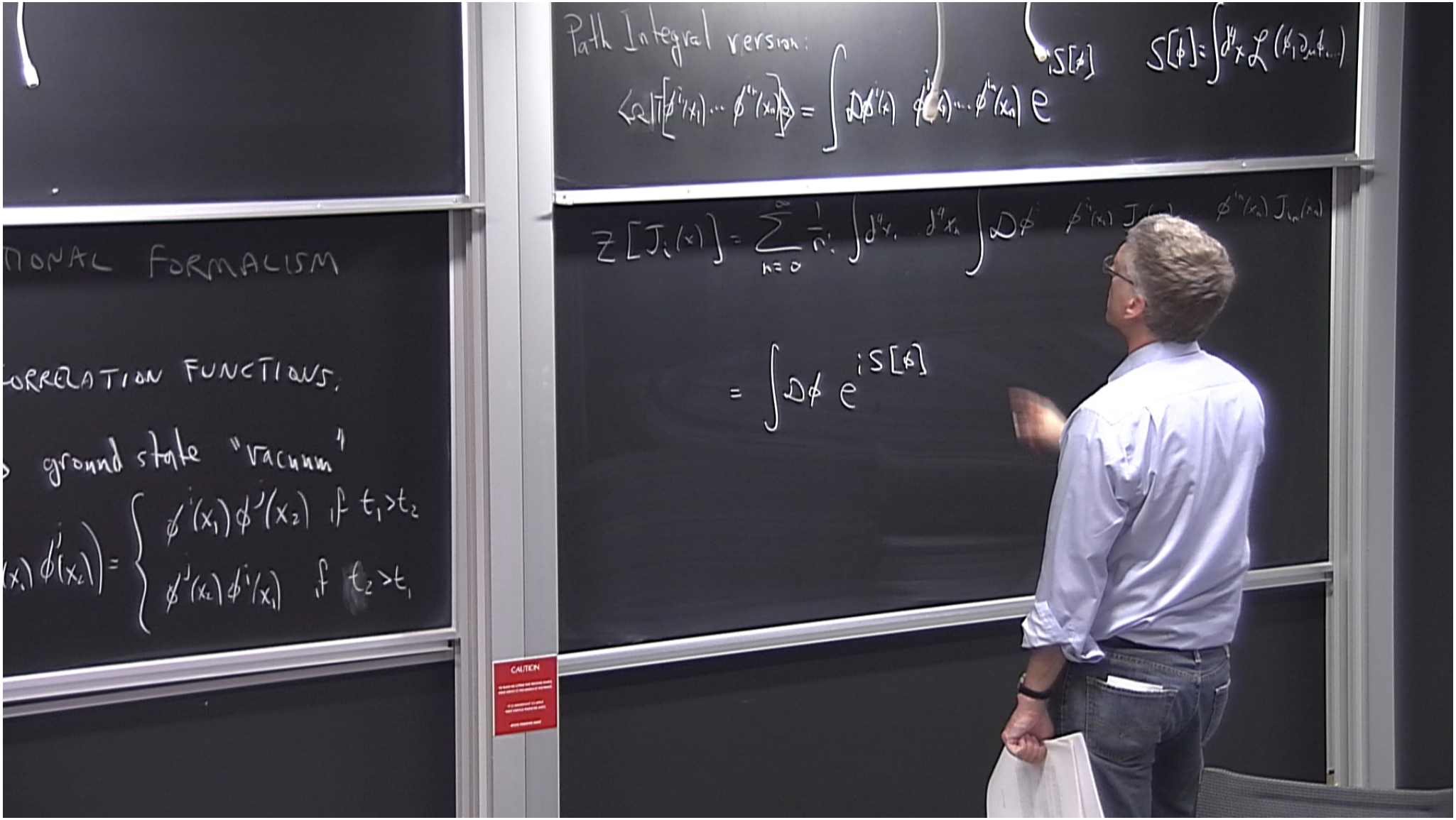
Path Integral version:
$\int \mathcal{D}\phi^i(x)\ \phi^{i_1}(x_1)\cdots\phi^{i_n}(x_n)\, e^{iS[\phi]}$
CAUTION


Path Integral version:
CAUTION


Path Integral version:
TIONAL FORMALISM
CORRELATION FUNCTIONS:
ground state "vacuum"
CAUTION


Path Integral version:
$S[\phi] = \int d^4x \, \mathcal{L}(\phi, \partial_\mu \phi, \ldots)$
$Z[J_i(x)] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n \int \mathcal{D}\phi \; \phi^{i_1}(x_1) J_{i_1}(x_1) \cdots \phi^{i_n}(x_n) J_{i_n}(x_n)$
$= \int \mathcal{D}\phi \, e^{iS[\phi]}$
TIONAL FORMALISM
CORRELATION FUNCTIONS:
ground state "vacuum"
$\phi^i(x_1)\phi^j(x_2) = \begin{cases} \phi^i(x_1)\phi^j(x_2) & \text{if } t_1 > t_2 \\ \phi^j(x_2)\phi^i(x_1) & \text{if } t_2 > t_1 \end{cases}$
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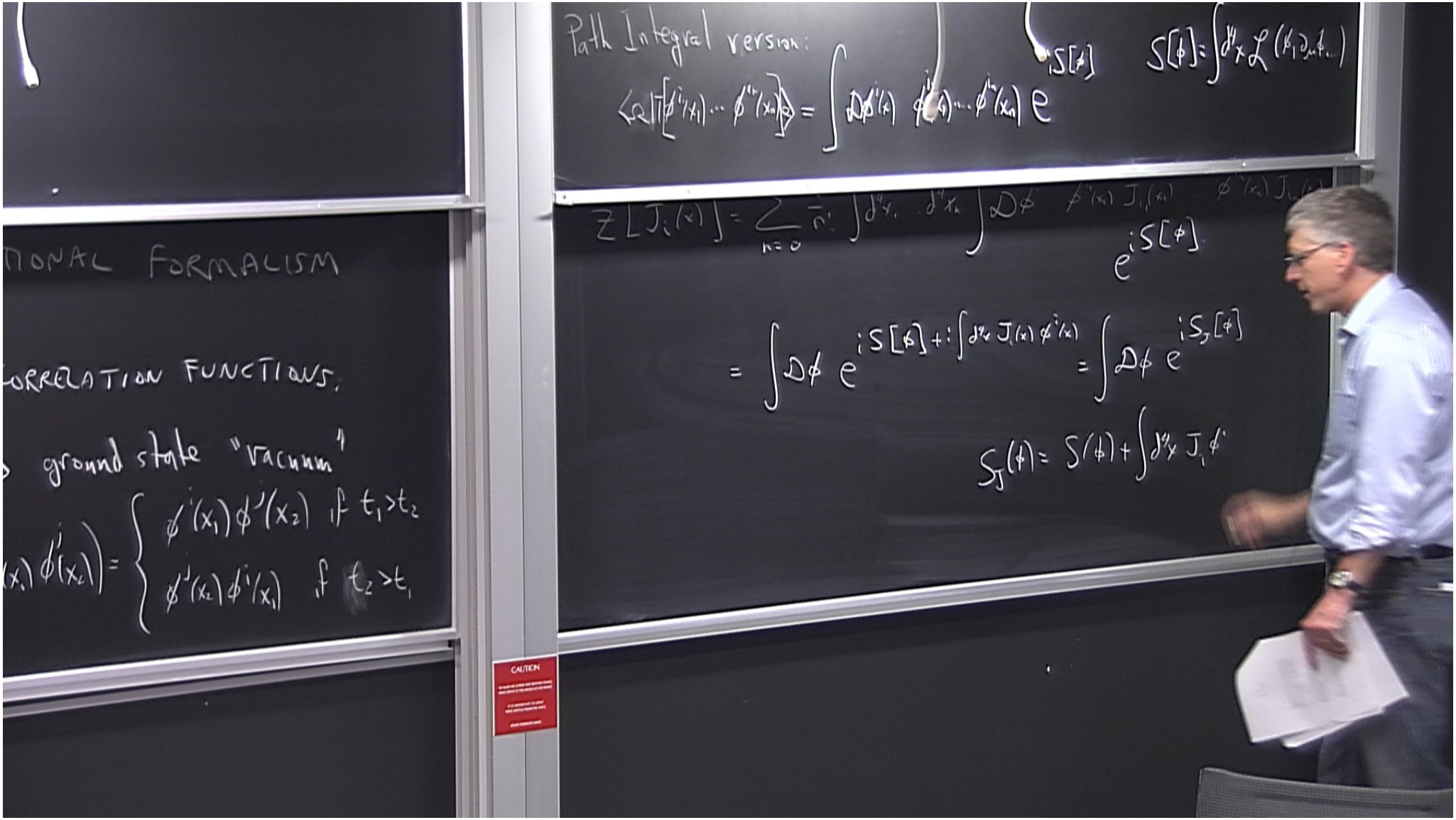
Path Integral version:
TIONAL FORMALISM
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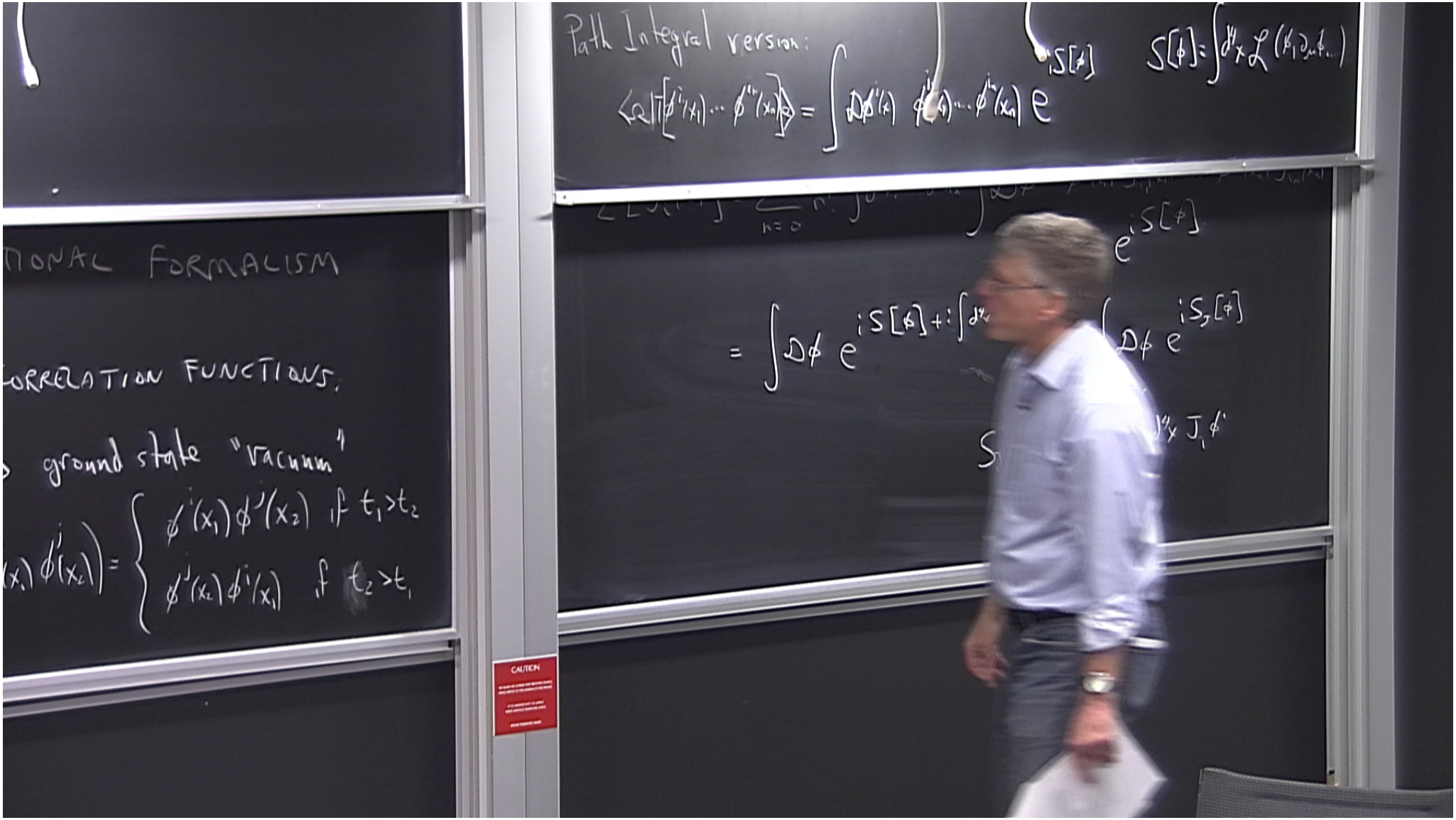
ground state "vacuum"
CAUTION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TIONAL FORMALISM
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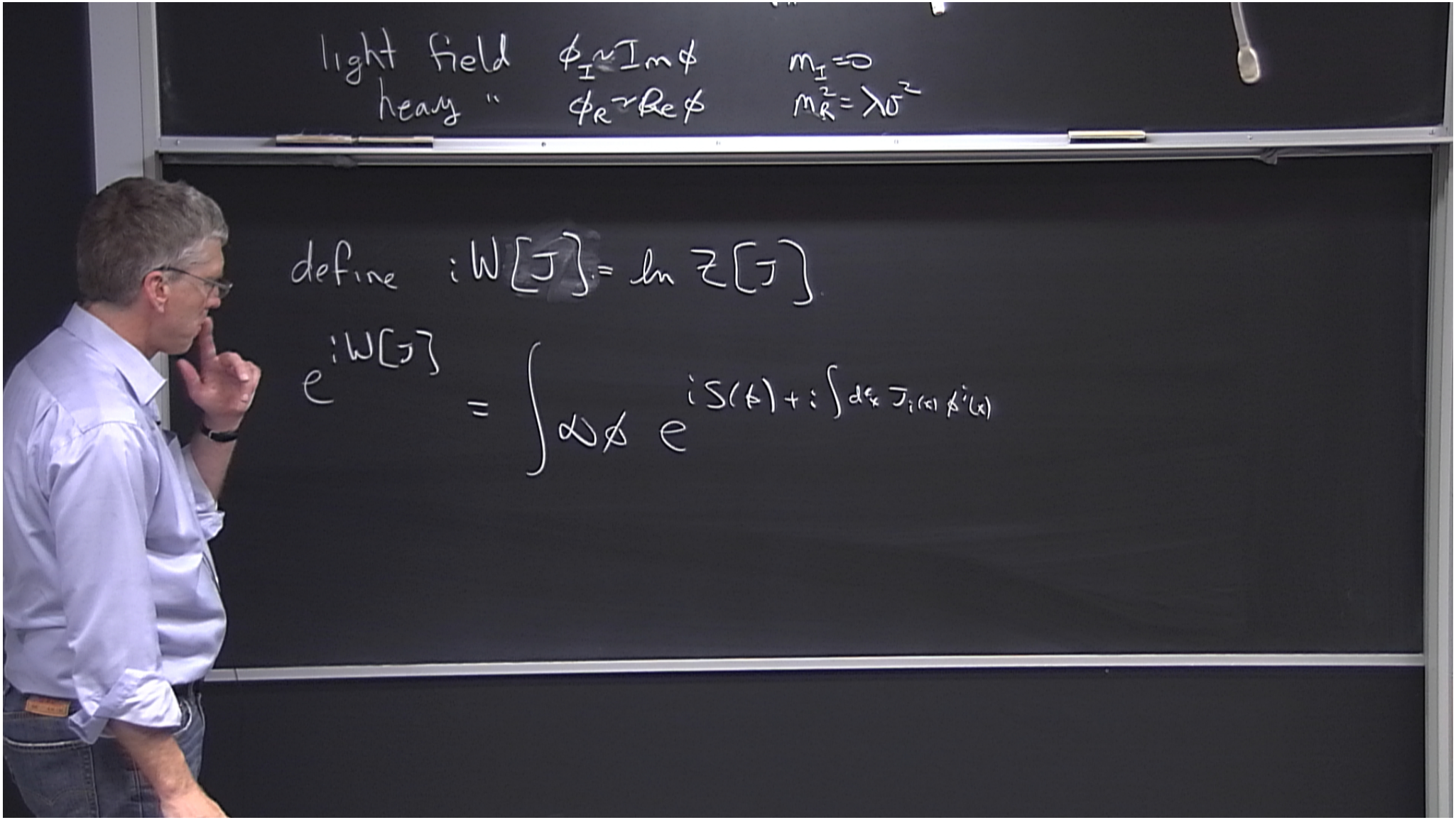
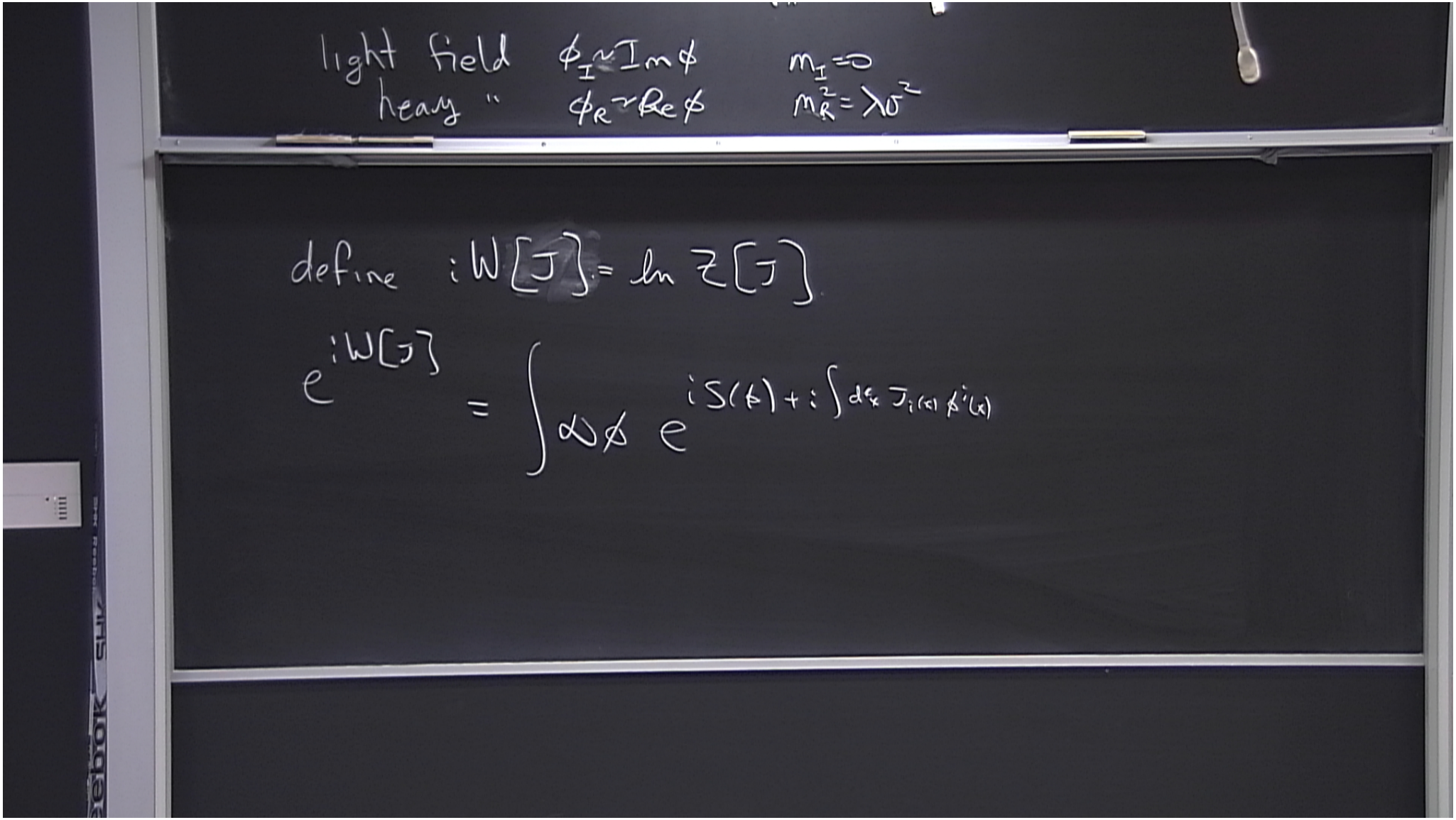

light field $\phi_I \sim \mathrm{Im}\,\phi$ $m_I = 0$
heavy " $\phi_R \sim \mathrm{Re}\,\phi$ $m_R^2 = \lambda v^2$
define $iW[J] = \ln Z[J]$
$e^{iW[J]} = \int \mathcal{D}\phi\, e^{iS(\phi) + i\int d^4x\, J_i(x)\phi^i(x)}$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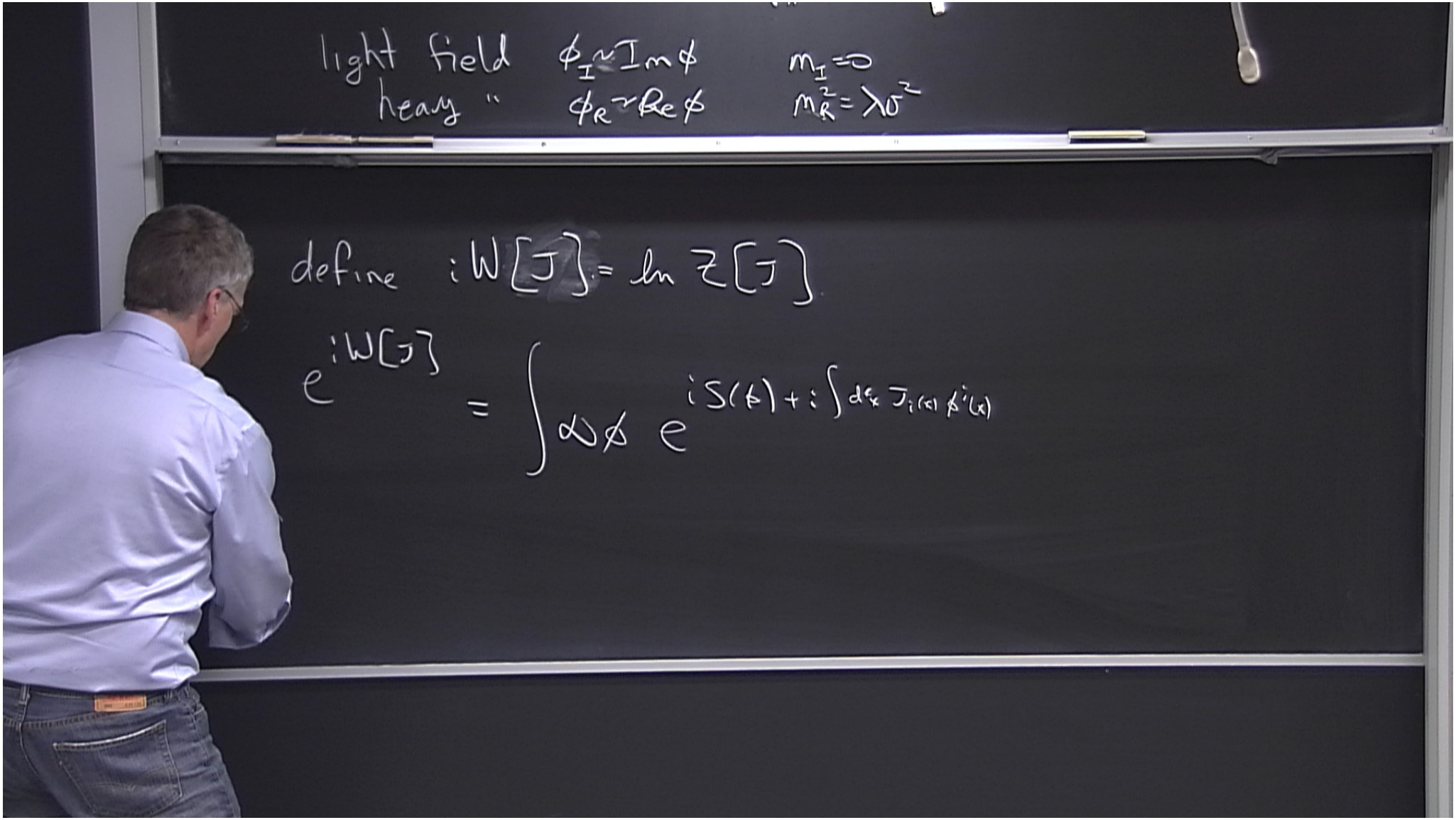
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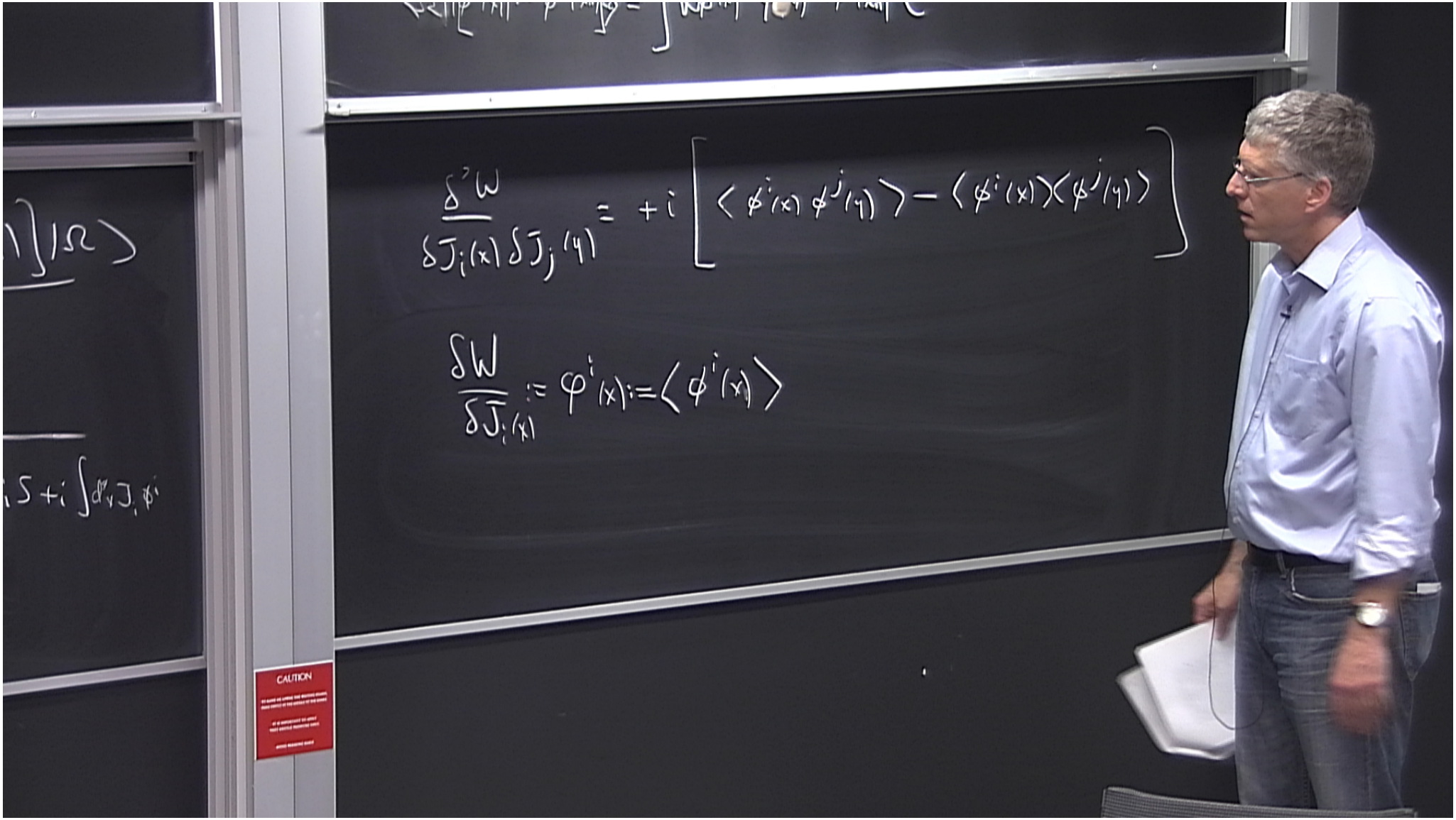
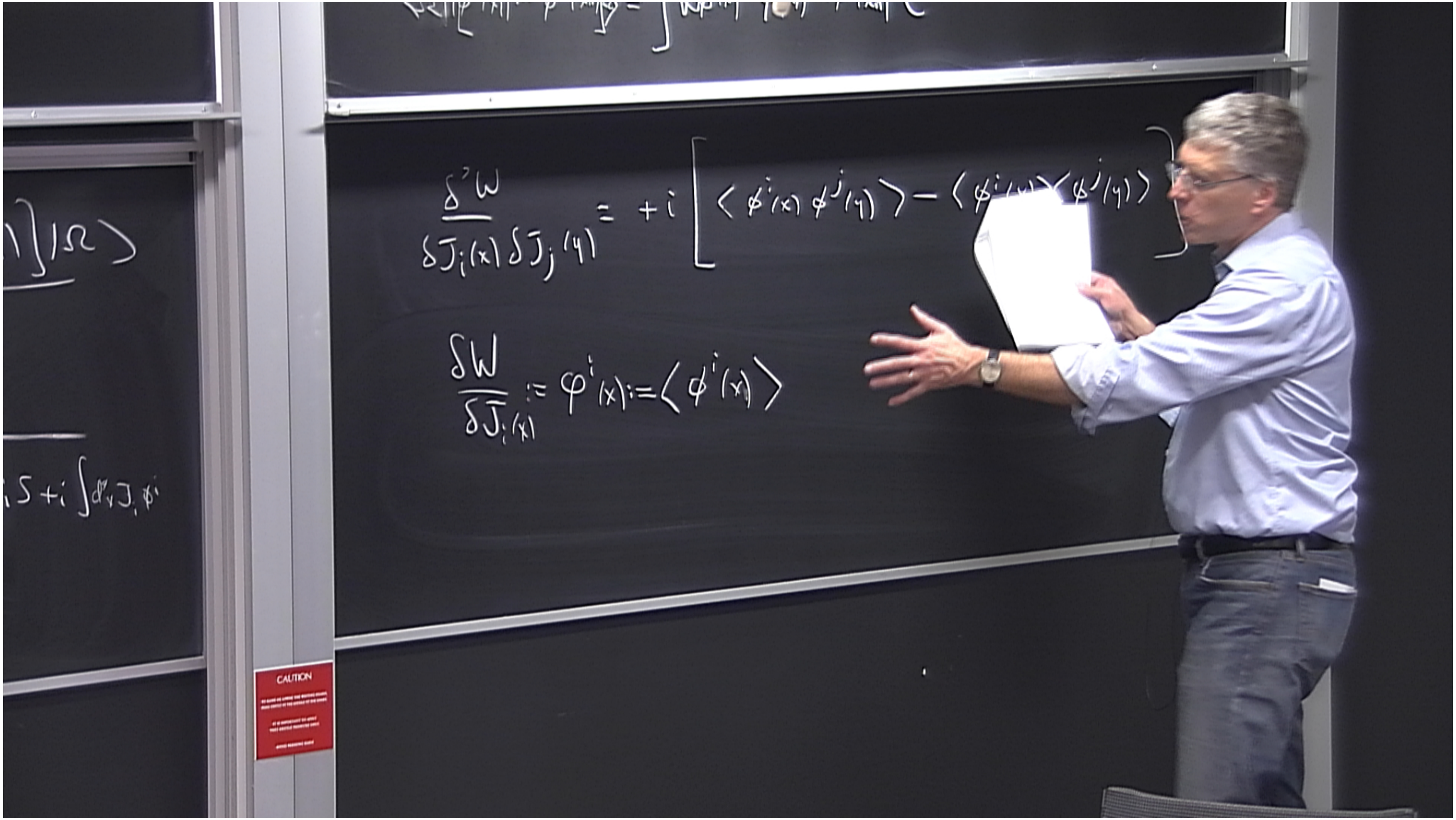

$\frac{\delta^2 W}{\delta J_i(x)\delta J_j(y)} = +i\left[\langle \phi^i(x)\phi^j(y)\rangle - \langle \phi^i(x)\rangle\langle \phi^j(y)\rangle\right]$
$\frac{\delta W}{\delta J_i(x)} := \varphi^i(x) := \langle \phi^i(x)\rangle$
CAUTION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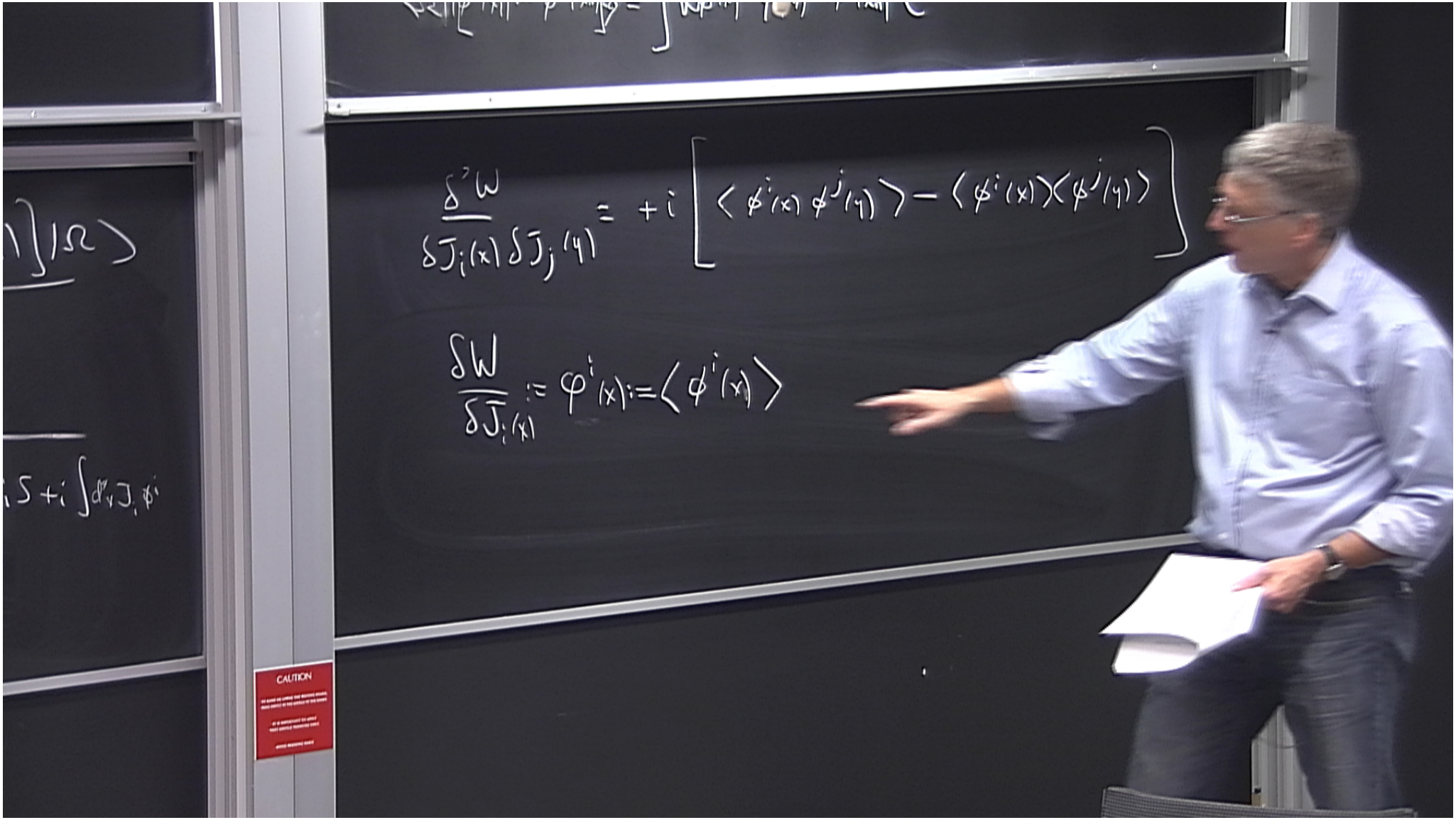
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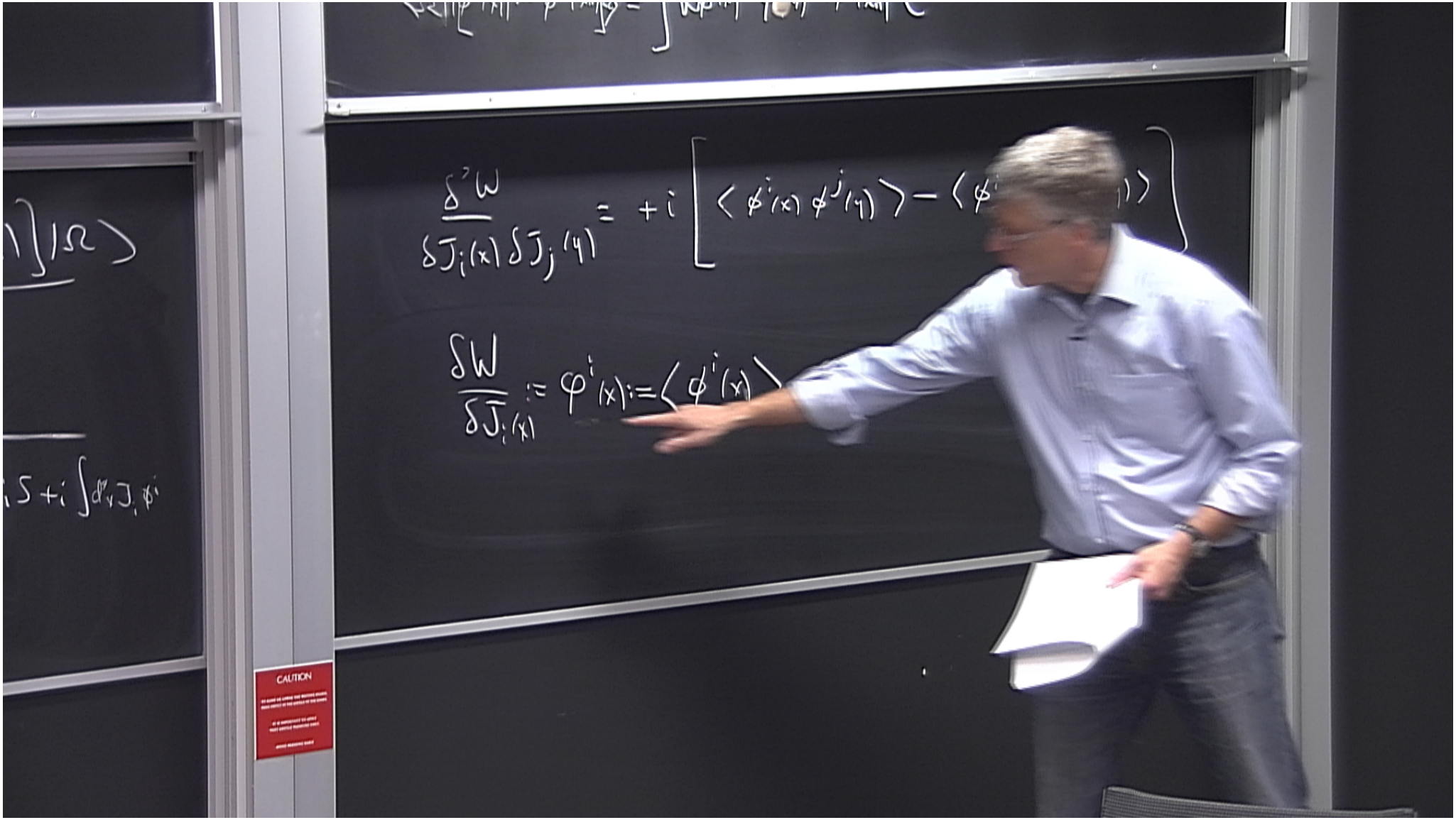
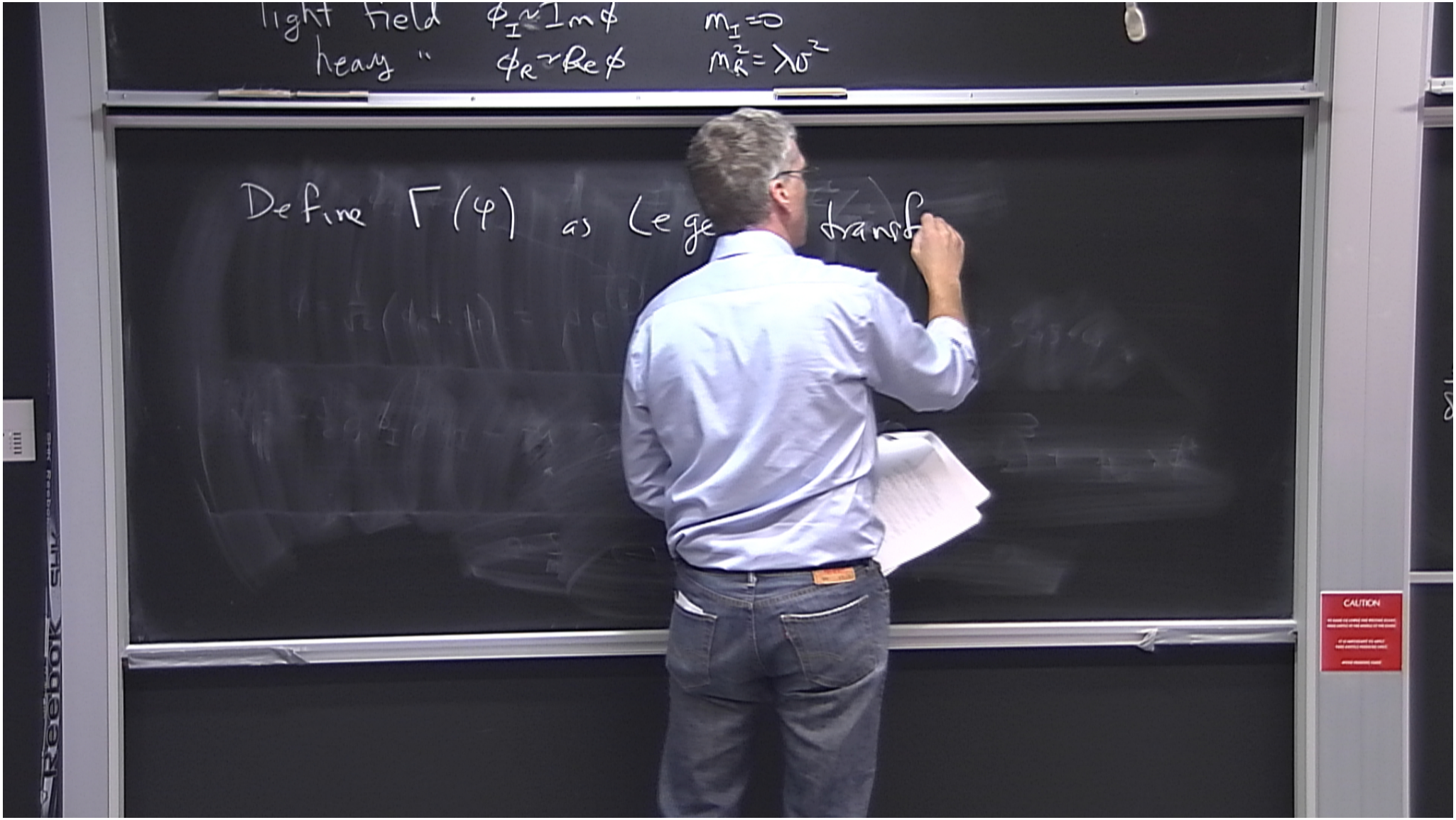

$\frac{\delta^2 W}{\delta J_i(x)\delta J_j(y)} = +i\left[\langle \phi^i(x)\phi^j(y)\rangle - \langle \phi^i \quad \rangle\right]$
$\frac{\delta W}{\delta J_i(x)} := \varphi^i(x) := \langle \phi^i(x) \rangle$
CAUTION


light field $\phi_I \sim \mathrm{Im}\,\phi$ $m_I = 0$
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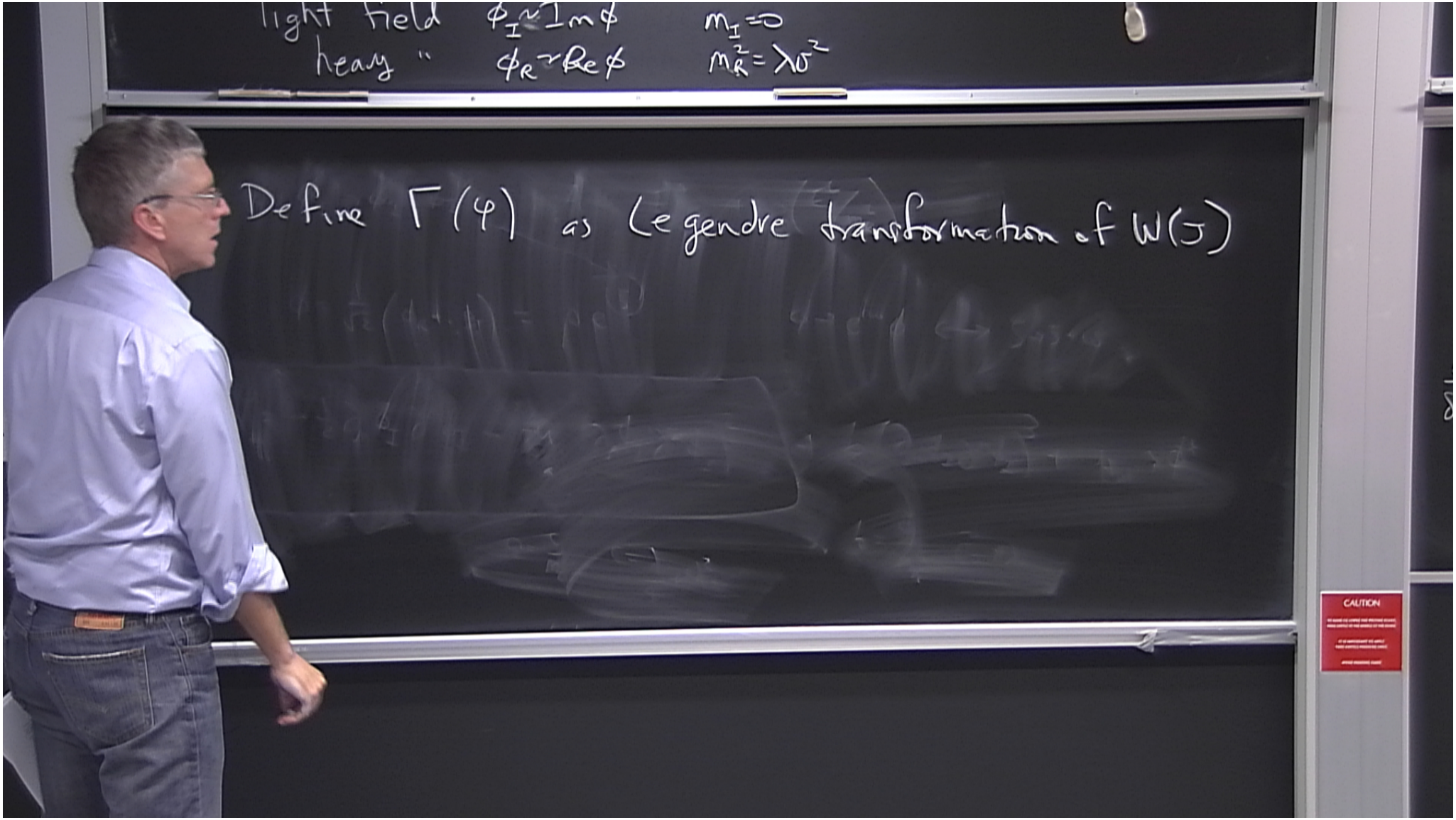
Define $\Gamma(\varphi)$ as Lege transf
CAUTION


light field $\phi_I \sim \mathrm{Im}\,\phi$ $m_I = 0$
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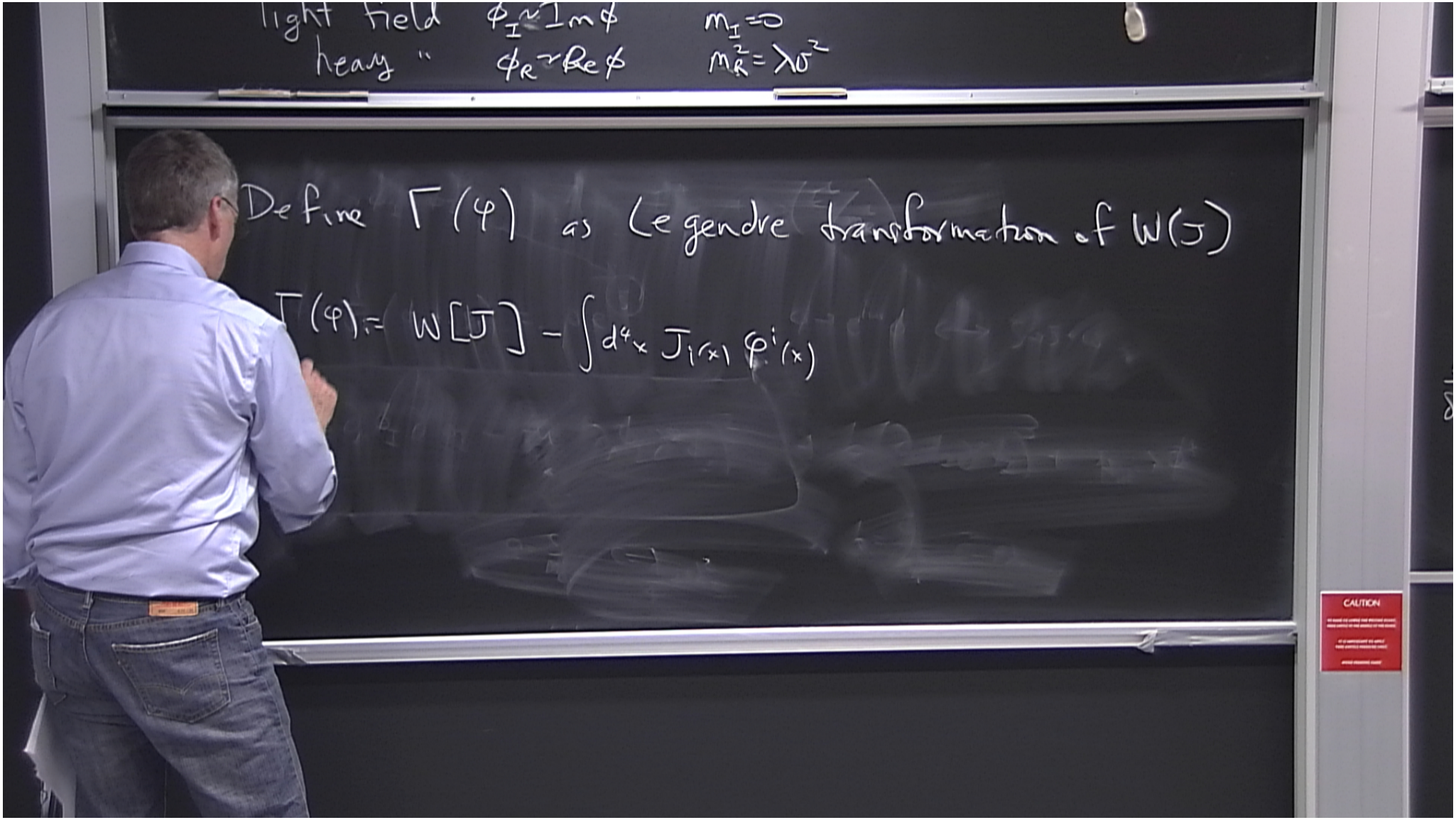
Define $\Gamma(\varphi)$ as Legendre transformation of $W(J)$
CAUTION


light field $\phi_I \sim \mathrm{Im}\,\phi$ $m_I = 0$
heavy " $\phi_R \sim \mathrm{Re}\,\phi$ $m_R^2 = \lambda v^2$
Define $\Gamma(\varphi)$ as Legendre transformation of $W(J)$
$\Gamma(\varphi) = W[J] - \int d^4x\, J_i(x)\, \varphi^i(x)$
CAUTION

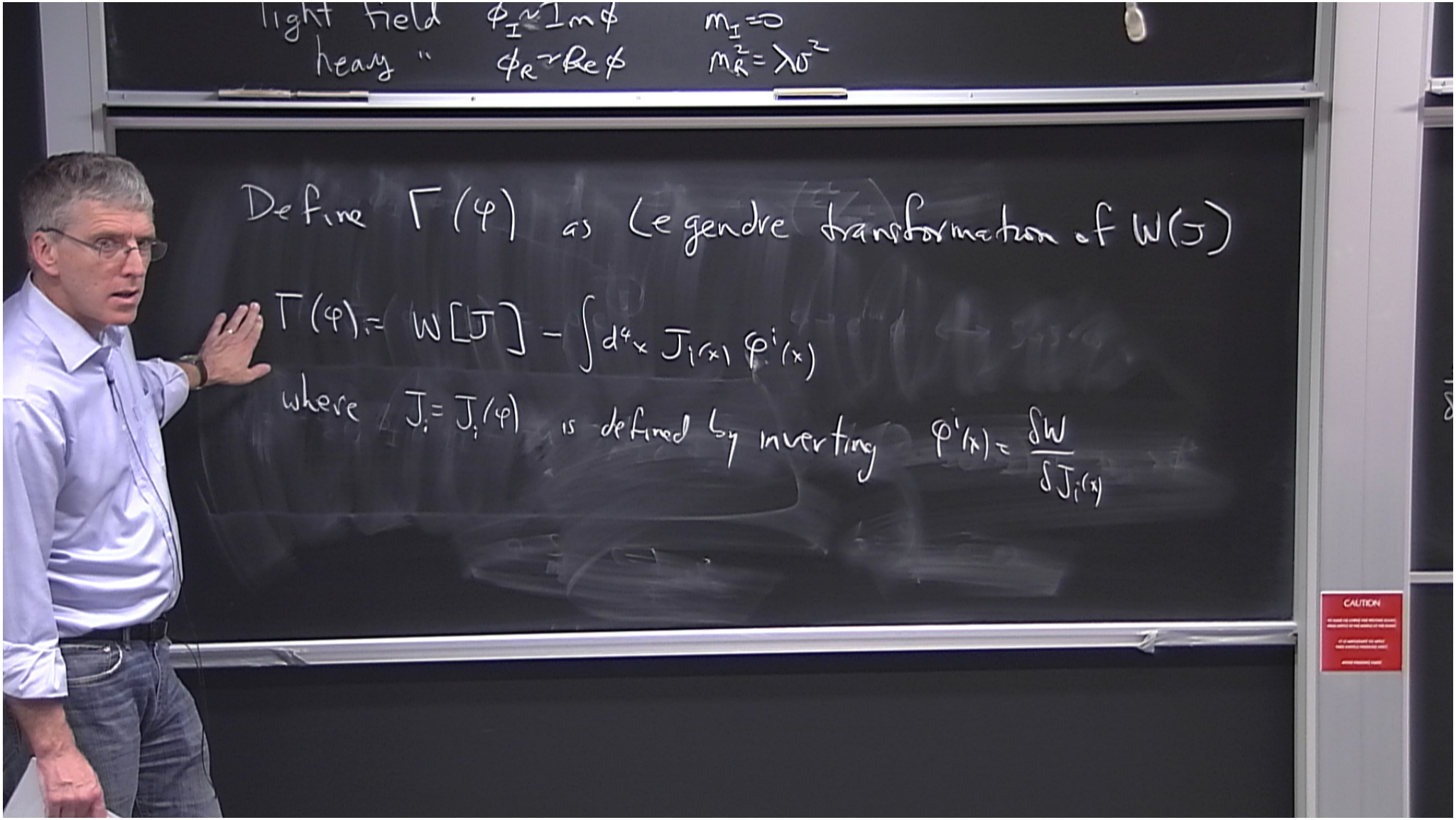
light field $\phi_I \sim \mathrm{Im}\,\phi$ $\quad m_I = 0$

heavy " $\phi_R \sim \mathrm{Re}\,\phi$ $\quad m_R^2 = \lambda v^2$

Define $\Gamma(\varphi)$ as Legendre transformation of $W(J)$

$$\Gamma(\varphi) := W[J] - \int d^4x\, J_i(x)\, \varphi^i(x)$$

where $J_i = J_i(\varphi)$ is defined by inverting $\varphi^i(x) = \dfrac{\delta W}{\delta J_i(x)}$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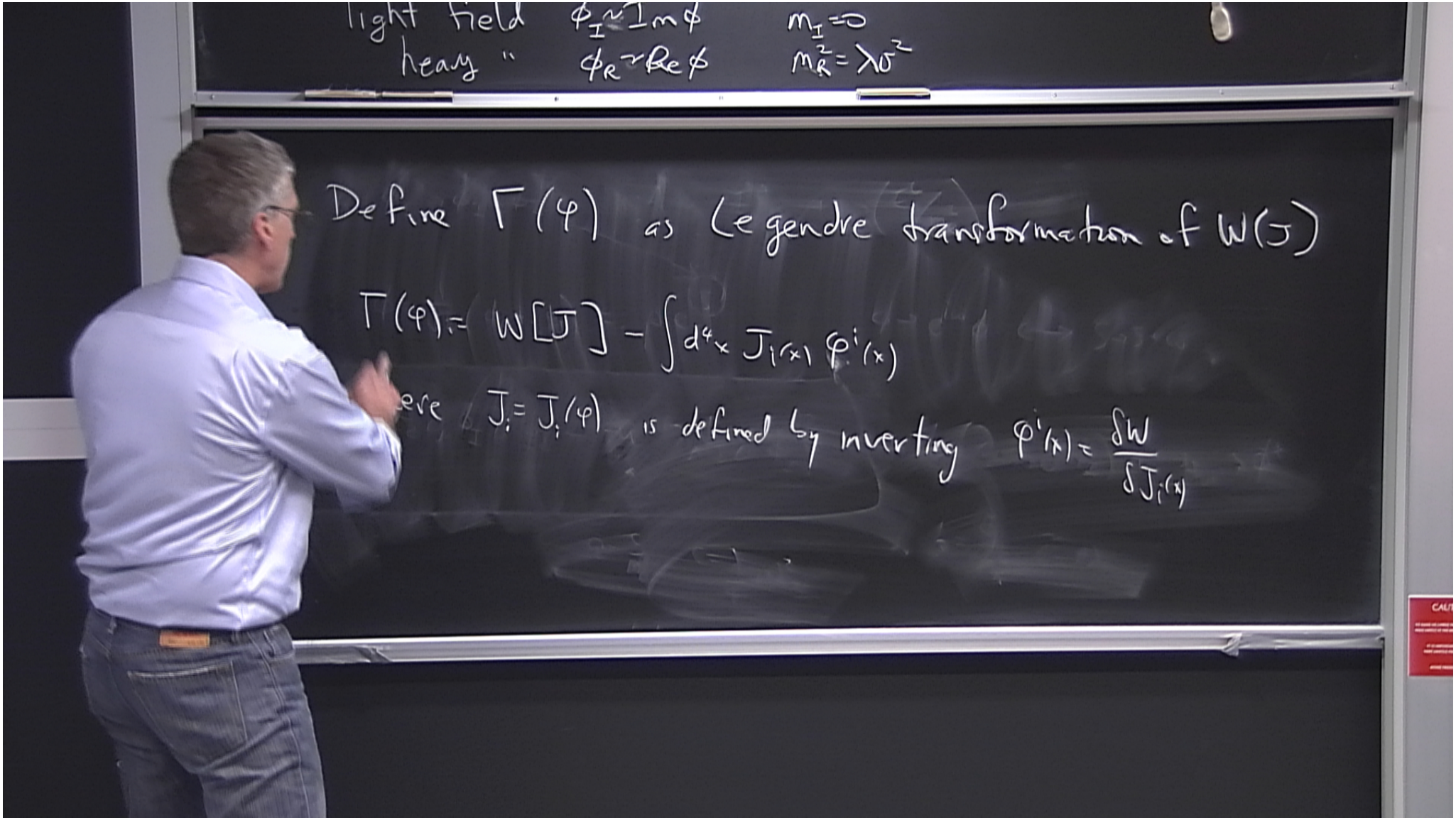
CAUTION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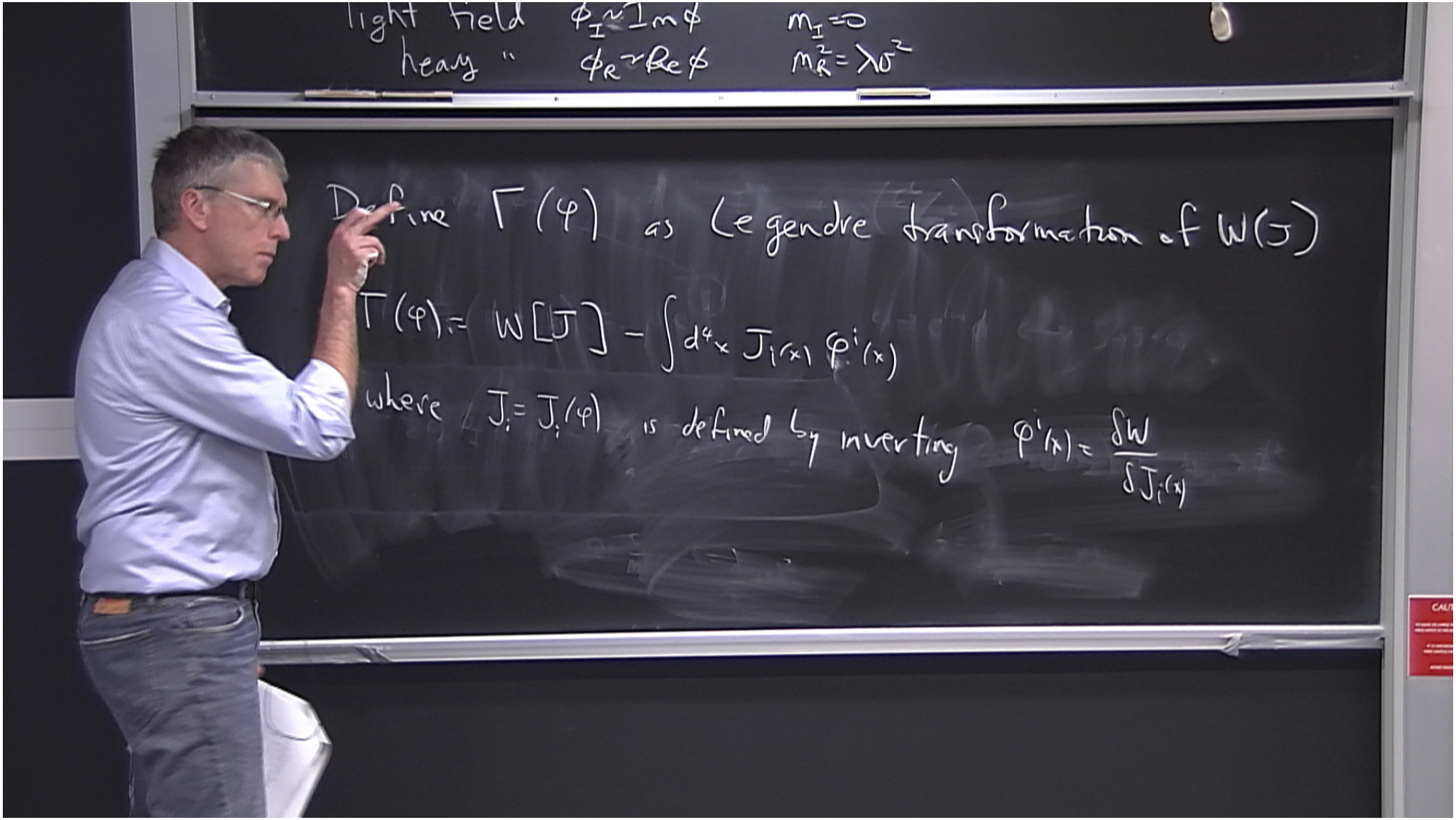
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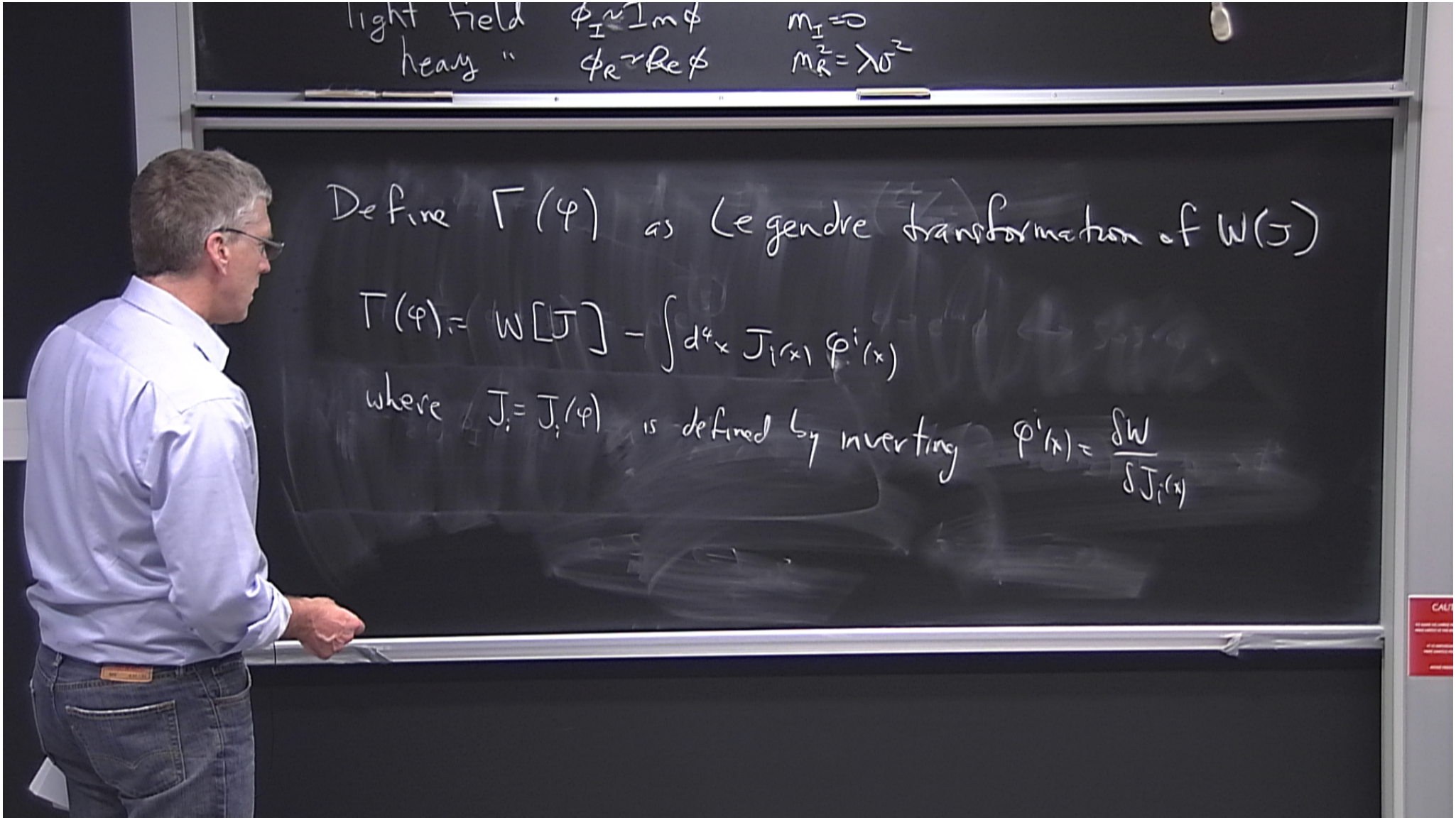
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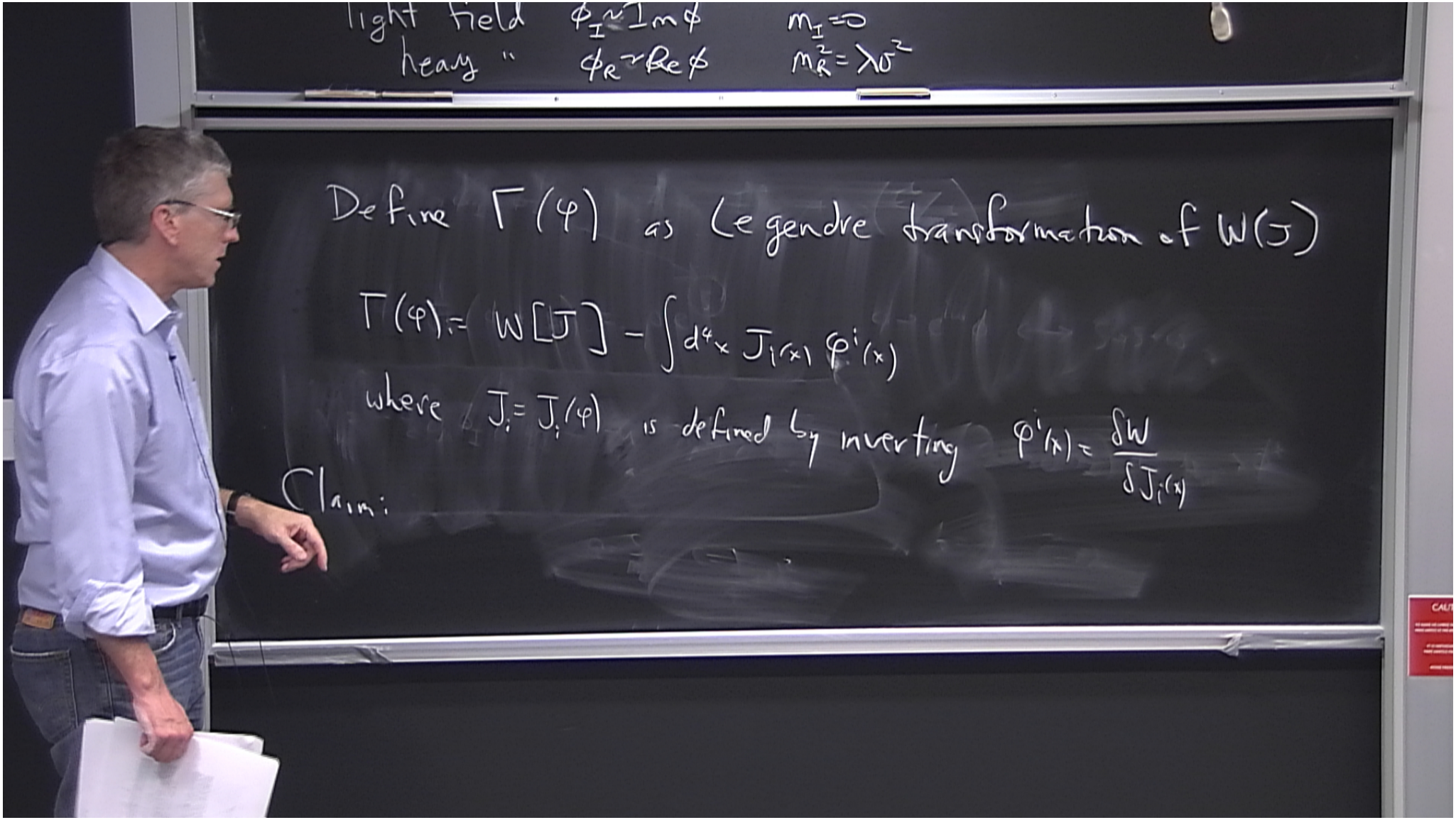
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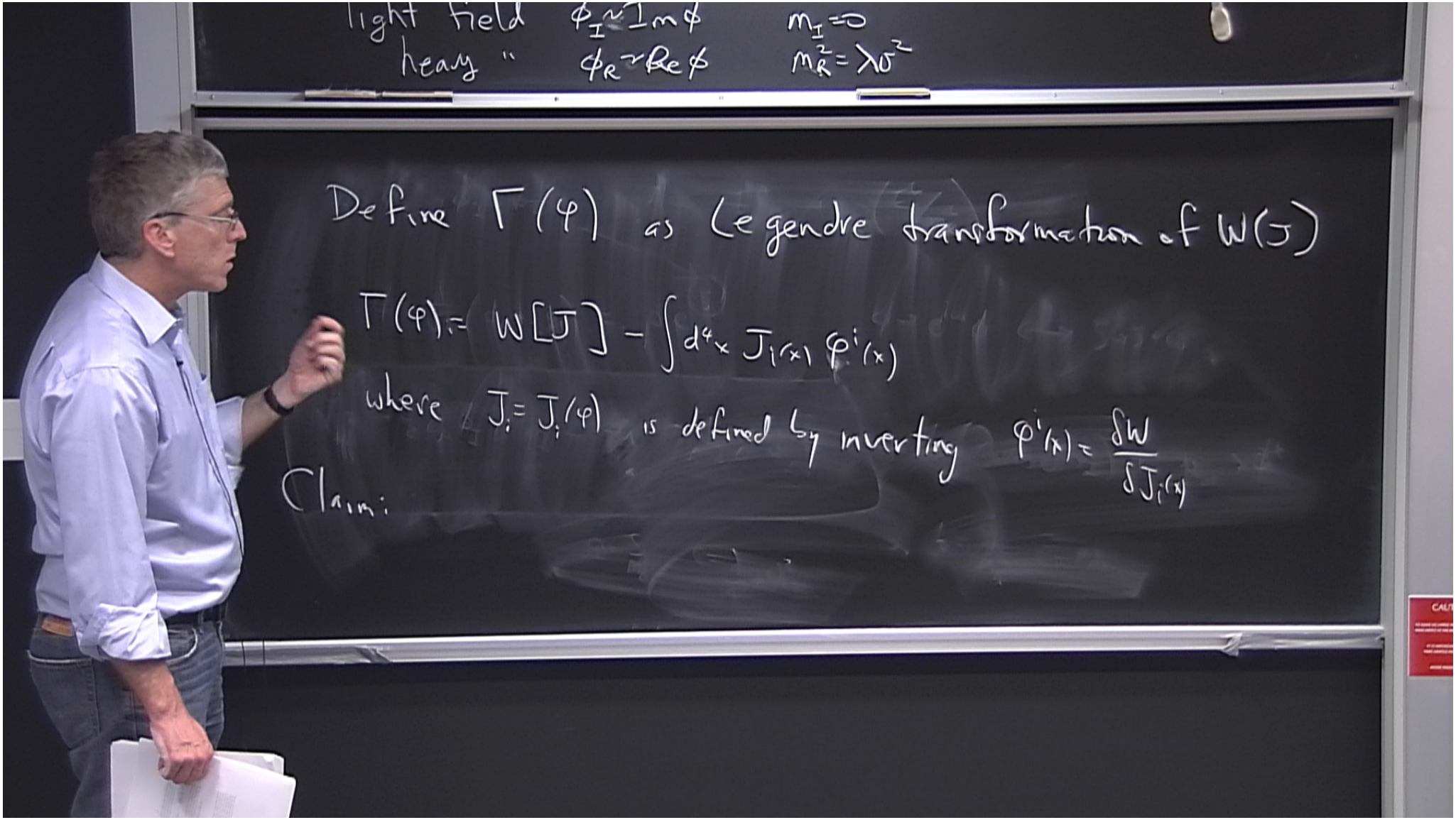
Claim: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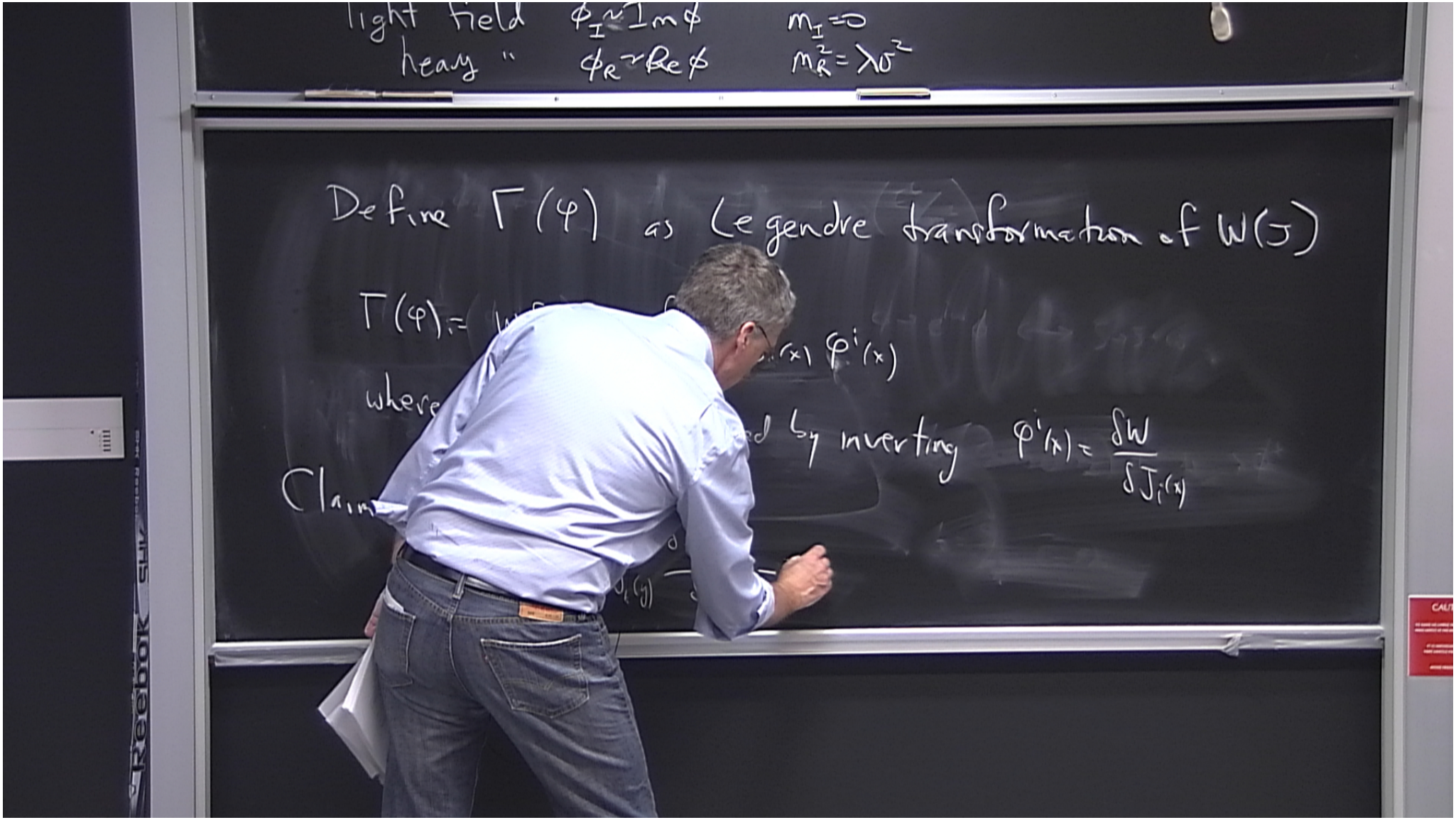
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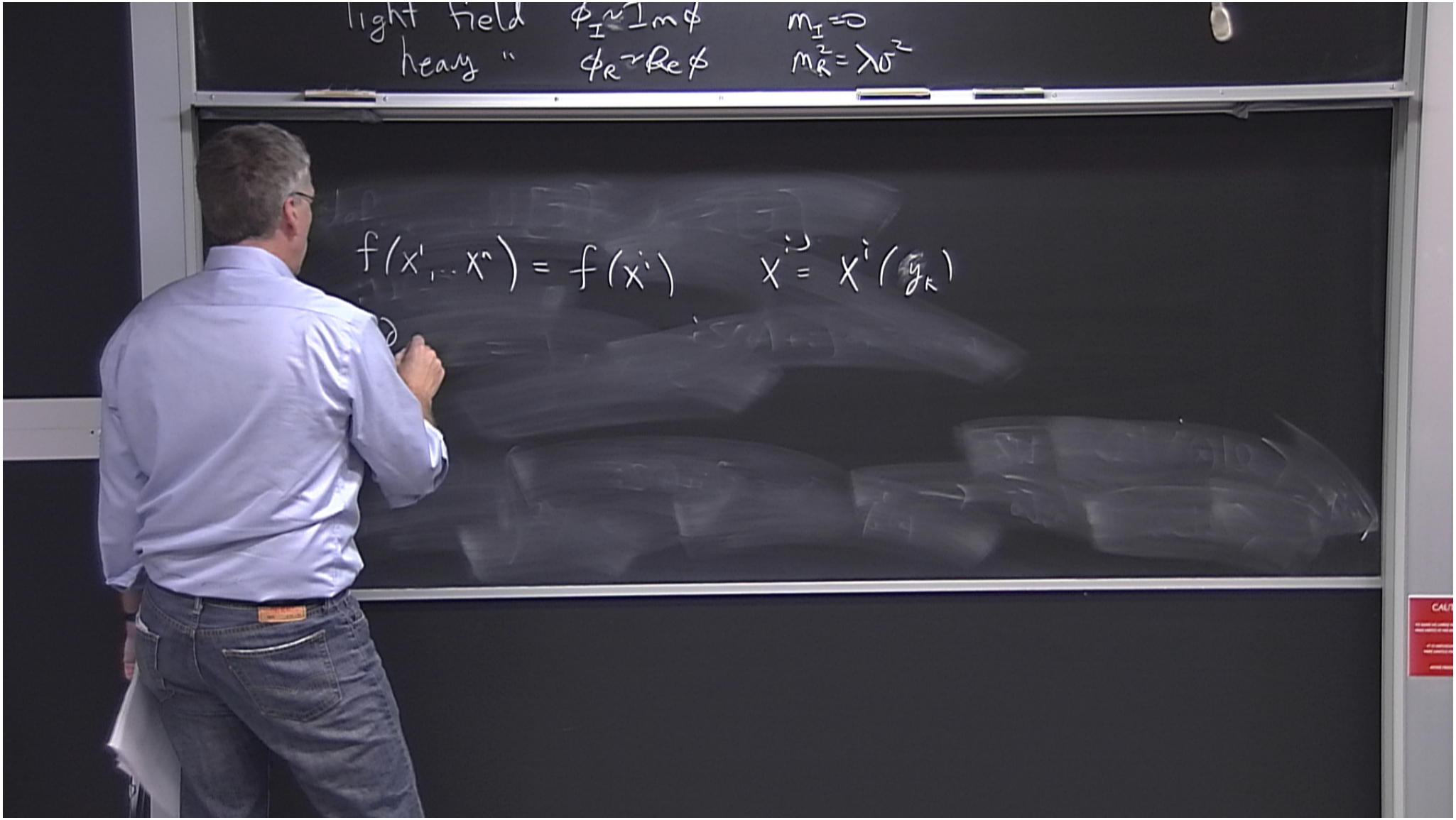
$\Gamma(\varphi) :=$
where
Claim
$\varphi^i(x)$
by inverting $\varphi^i(x) = \frac{\delta W}{\delta J_i(x)}$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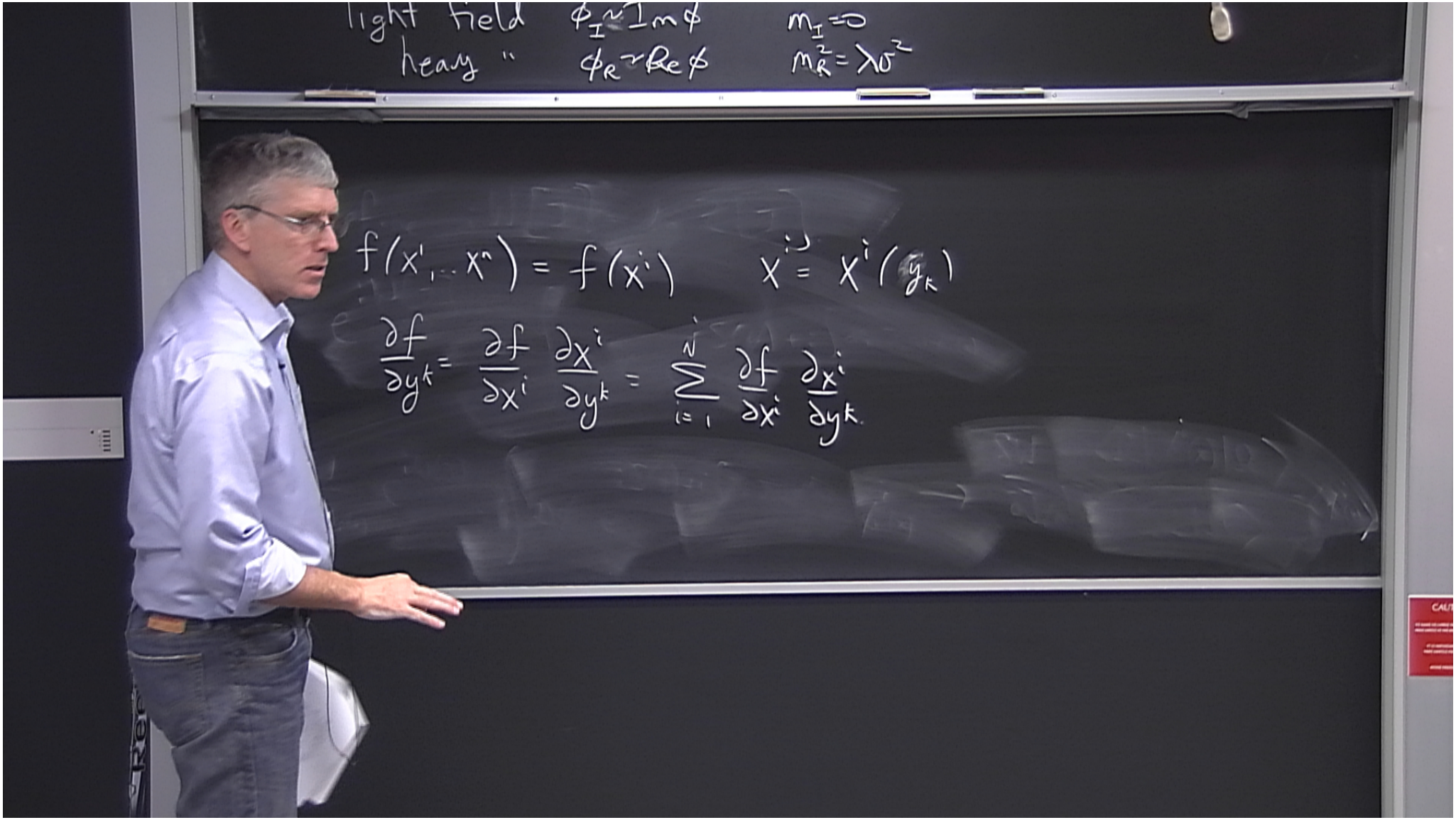
$f(x^1, \ldots x^n) = f(x^i)$ $x^{i;j} = x^i(y_k)$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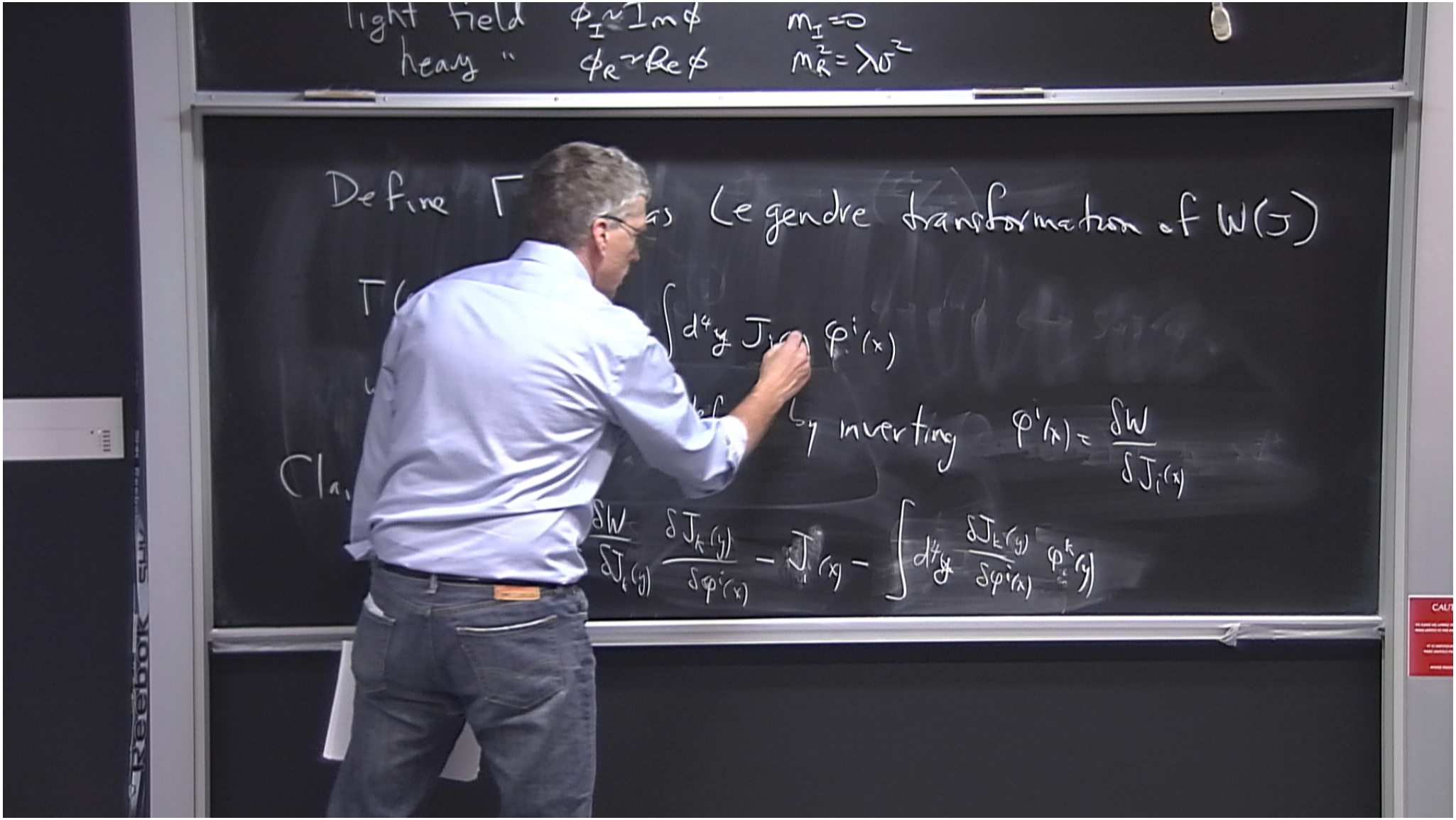
$f(x^1, \ldots x^n) = f(x^i)$ $x^{i'} = x^i(y_k)$
$\frac{\partial f}{\partial y^k} = \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial y^k} = \sum_{i=1}^{N} \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial y^k}$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Define Γ as Legendre transformation of $W(J)$
$\int d^4y\, J_i(y)\, \varphi^i(x)$
by inverting $\varphi^i(x) = \frac{\delta W}{\delta J_i(x)}$
$\frac{\delta W}{\delta J_k(y)} \frac{\delta J_k(y)}{\delta \varphi^i(x)} = J_i(x) - \int d^4y \frac{\delta J_k(y)}{\delta \varphi^i(x)} \varphi^k(y)$

light field $\phi_I \sim \mathrm{Im}\,\phi$ $\quad m_I = 0$

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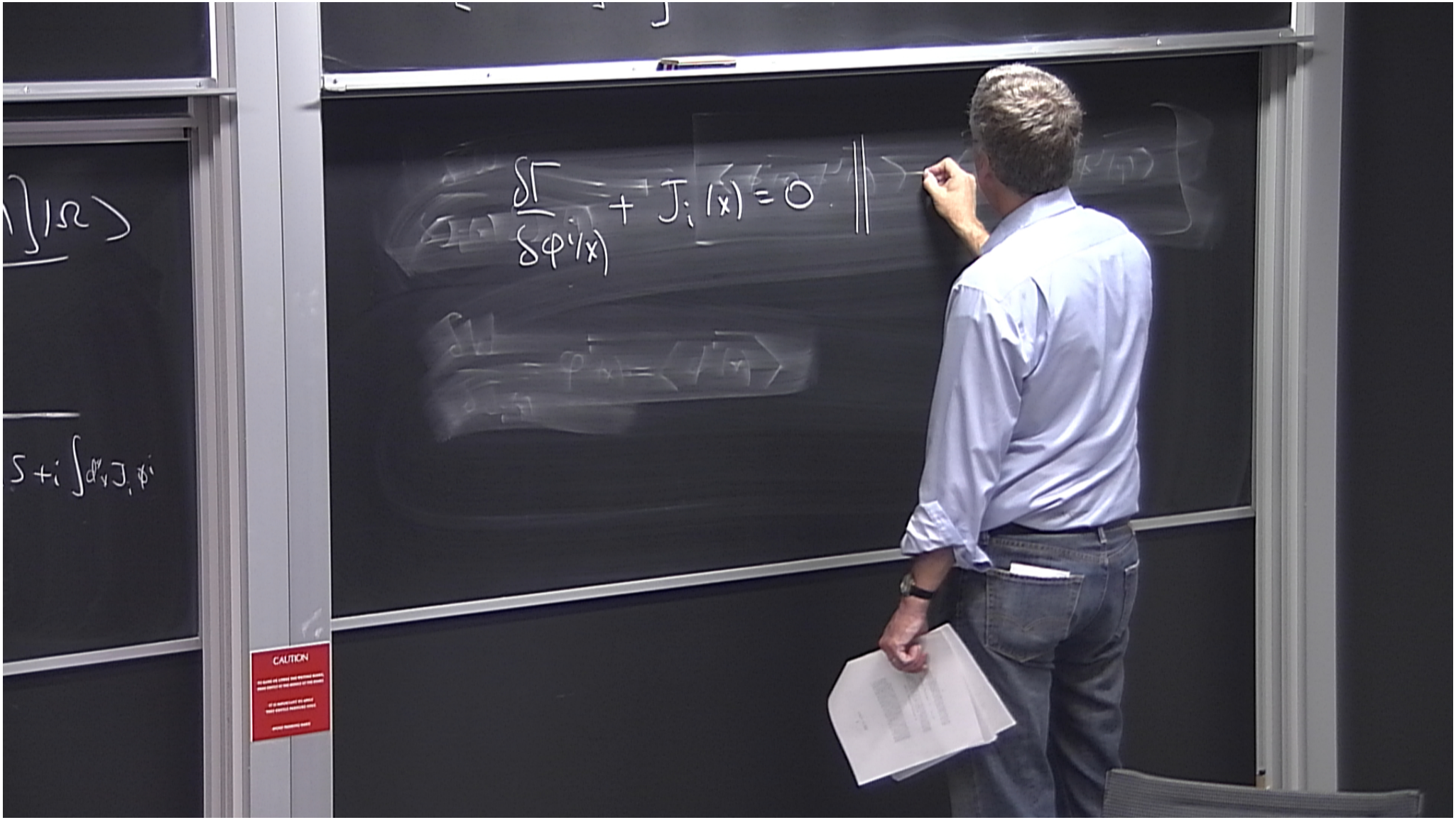
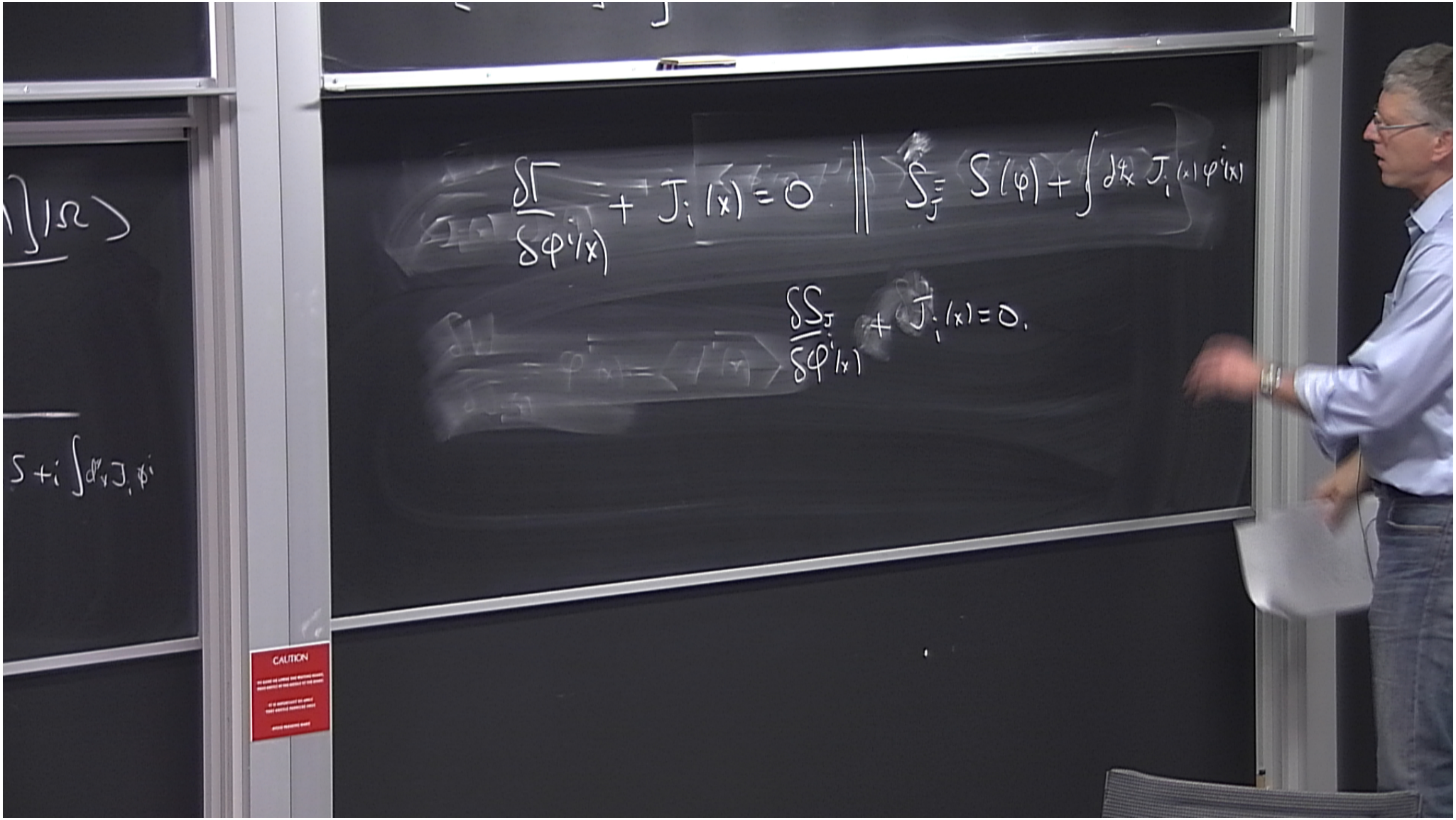
Define $\Gamma(\varphi)$ as Legendre transformation of $W(J)$

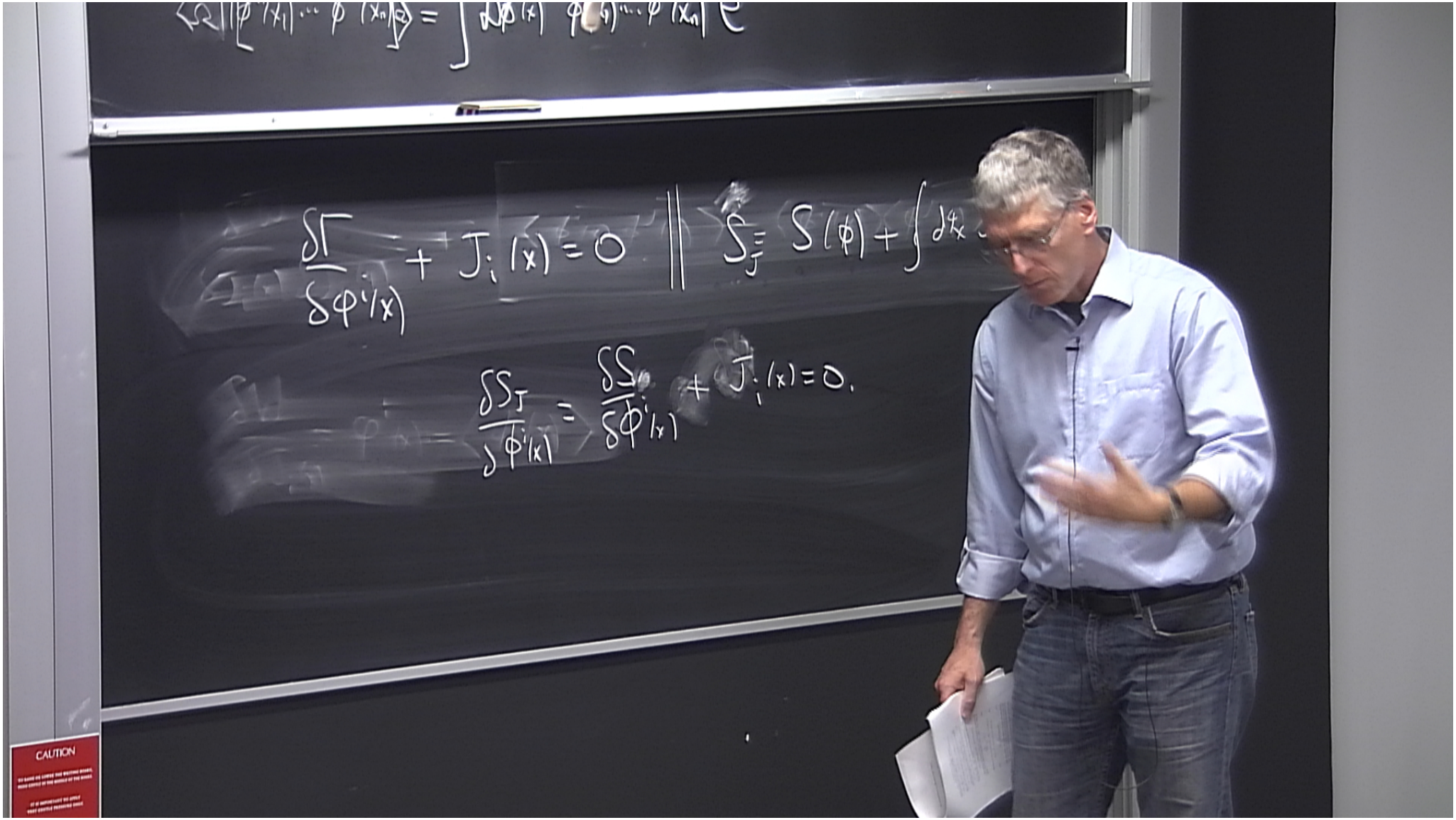
$$\Gamma(\varphi) := W[J] - \int d^4y\, J_i(y)\, \varphi^i(y)$$

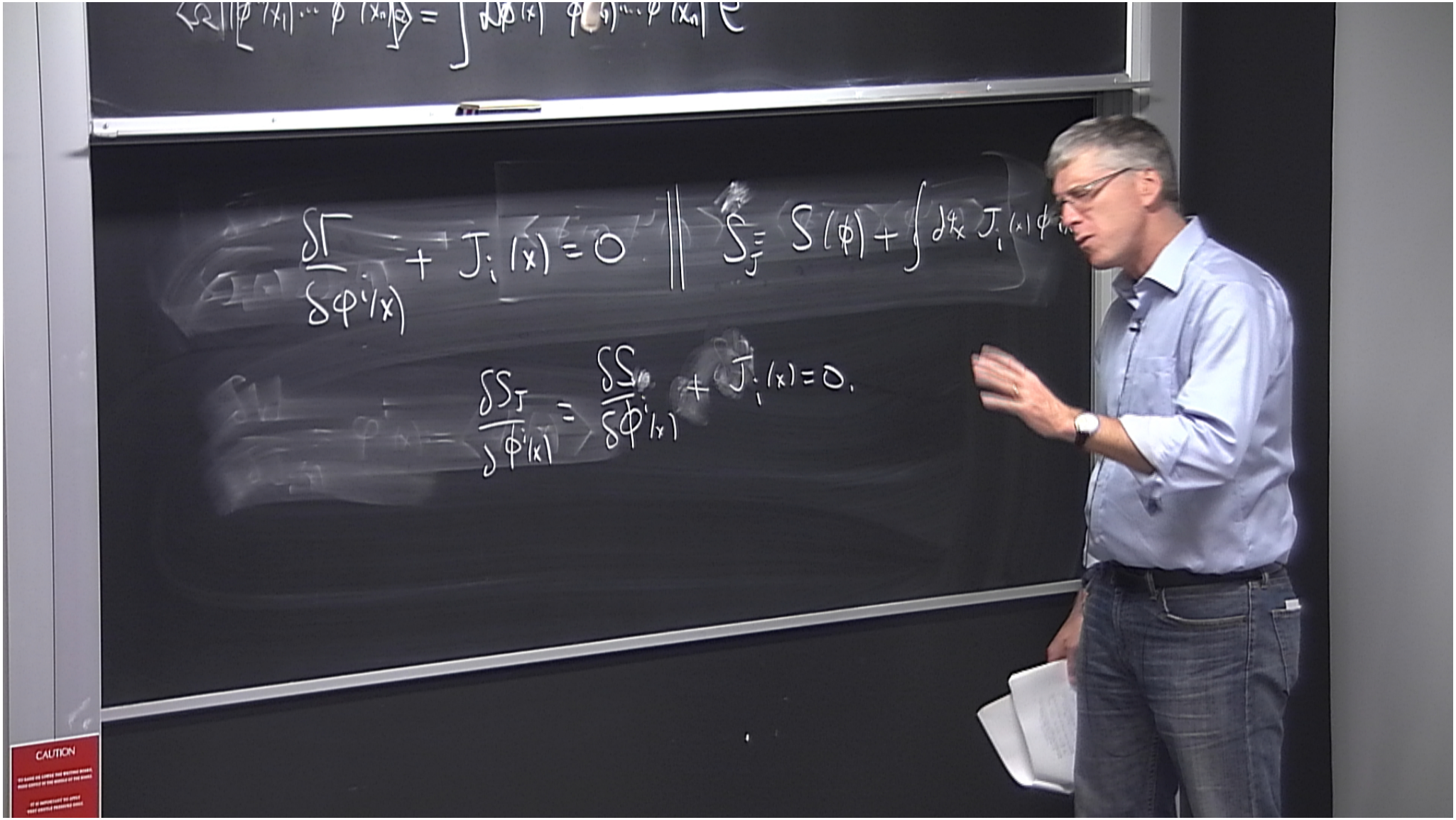
where $J_i = J_i(\varphi)$ is defined by inverting $\varphi^i(x) = \frac{\delta W}{\delta J_i(x)}$

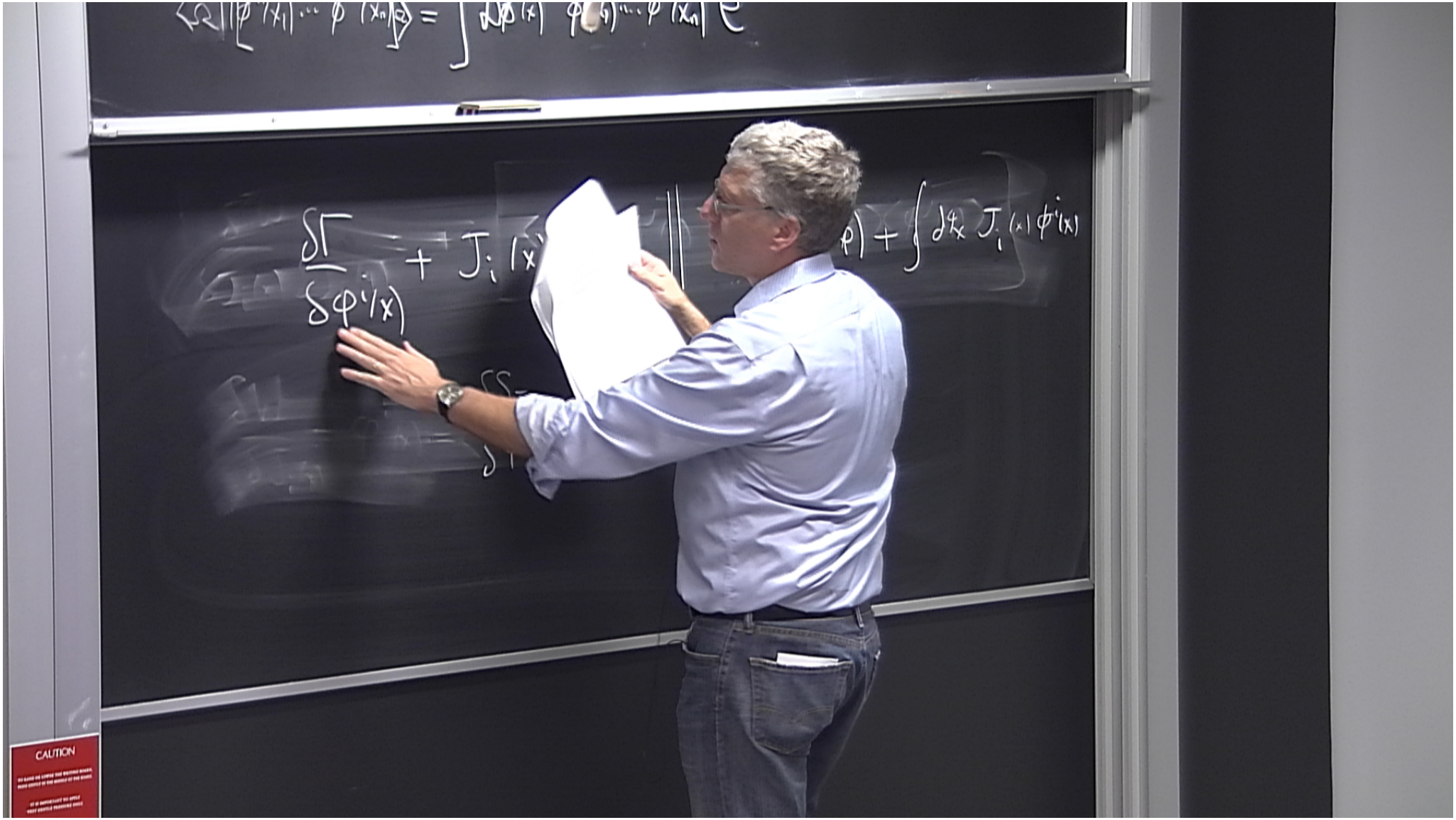
Claim: $\frac{\delta \Gamma}{\delta \varphi^i(x)} = \int d^4y \boxed{\frac{\delta W}{\delta J_k(y)}}^{\varphi^k(y)} \frac{\delta J_k(y)}{\delta \varphi^i(x)} - J_i(x) - \int d^4y \frac{\delta J_k(y)}{\delta \varphi^i(x)} \varphi^k(y)$


$\frac{\delta\Gamma}{\delta\varphi(x)} + J_i(x) = 0$
CAUTION


$\frac{\delta\Gamma}{\delta\varphi^i(x)} + J_i(x) = 0$
$S_J = S(\varphi) + \int dx\, J_i(x)\varphi^i(x)$
$\frac{\delta S_J}{\delta\varphi^i(x)} + J_i(x) = 0.$
CAUTION


$\frac{\delta\Gamma}{\delta\phi_i(x)} + J_i(x) = 0$
$\frac{\delta S_J}{\delta\phi_i(x)} = \frac{\delta S}{\delta\phi_i(x)} + J_i(x) = 0.$
CAUTION


$\frac{\delta\Gamma}{\delta\phi_i(x)} + J_i(x) = 0$
$S_J = S(\phi) + \int dx\, J_i(x)\phi$
$\frac{\delta S_J}{\delta\phi_i(x)} = \frac{\delta S}{\delta\phi_i(x)} + J_i(x) = 0.$
CAUTION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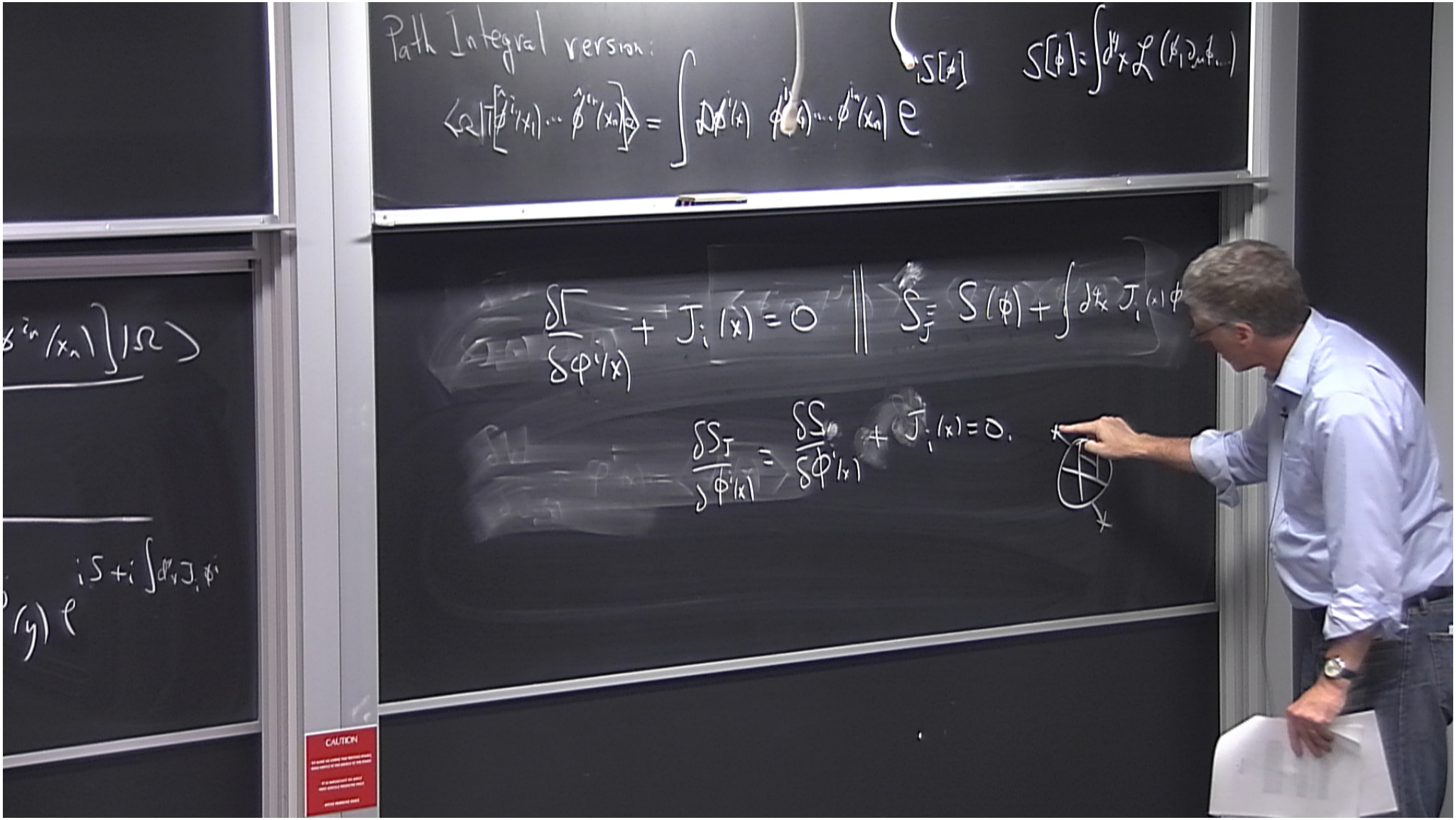
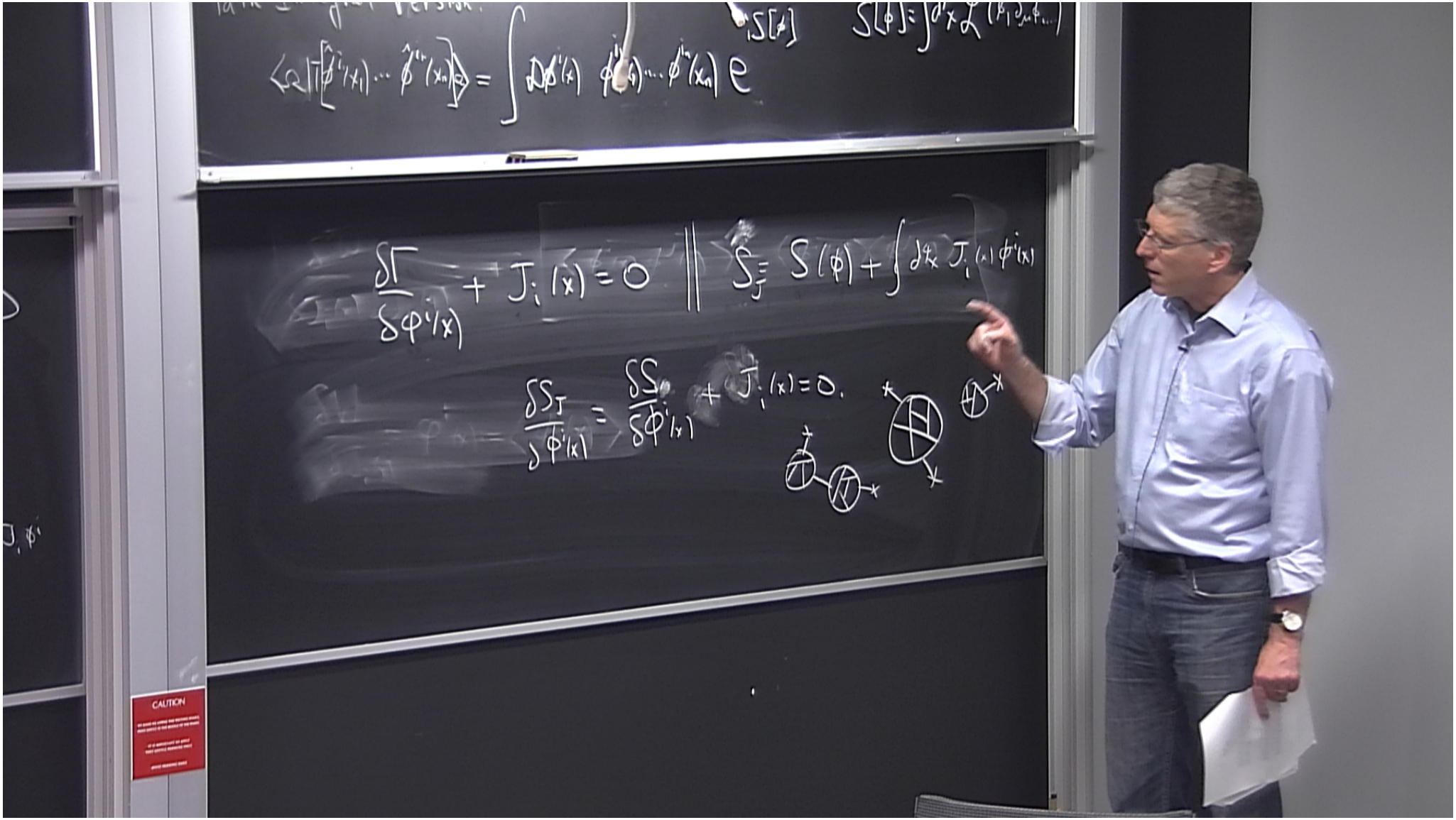

Path Integral version:
$S[\phi] = \int d^4x \mathcal{L}(\phi_1, \partial_\mu \phi, \ldots)$
$\frac{\delta \Gamma}{\delta \phi^i(x)} + J_i(x) = 0$
$S_J = S(\phi) + \int d^4x J_i(x) \phi$
$\frac{\delta S_J}{\delta \phi^i(x)} = \frac{\delta S}{\delta \phi^i(x)} + J_i(x) = 0.$
CAUTION


$\frac{\delta\Gamma}{\delta\phi^i(x)} + J_i(x) = 0$
$S_J = S(\phi) + \int dx\, J_i(x)\phi^i(x)$
$\frac{\delta S_J}{\delta\phi^i(x)} = \frac{\delta S}{\delta\phi^i(x)} + J_i(x) = 0.$
CAUTION