Title: The McV black hole and the scalar theories that support it

Date: Jan 21, 2014 11:00 AM

URL: http://pirsa.org/14010082

Abstract: Systems in which the local gravitational attraction is coupled to the expansion of the Universe have been studied since the early stages of General Relativity as the pioneering works of McVittie show. In this talk I start reviewing the McVittie black hole solution and its variable mass generalization from a classical fluid approach to understand its properties. I then move to a field theoretical analysis to investigate the scalar theories that support such black holes.

The McVittie black hole and the scalar theories that support it

E.Abdalla, N.Afshordi, MF, D.C.Guariento, E.Papantonopoulos arXiv:1312.3682

D.C.Guariento, MF, A.M. DaSilva, E.Abdalla Phys RevD.86.124020

A.M. DaSilva, MF, D.C.Guariento, Phys RevD.87.064030

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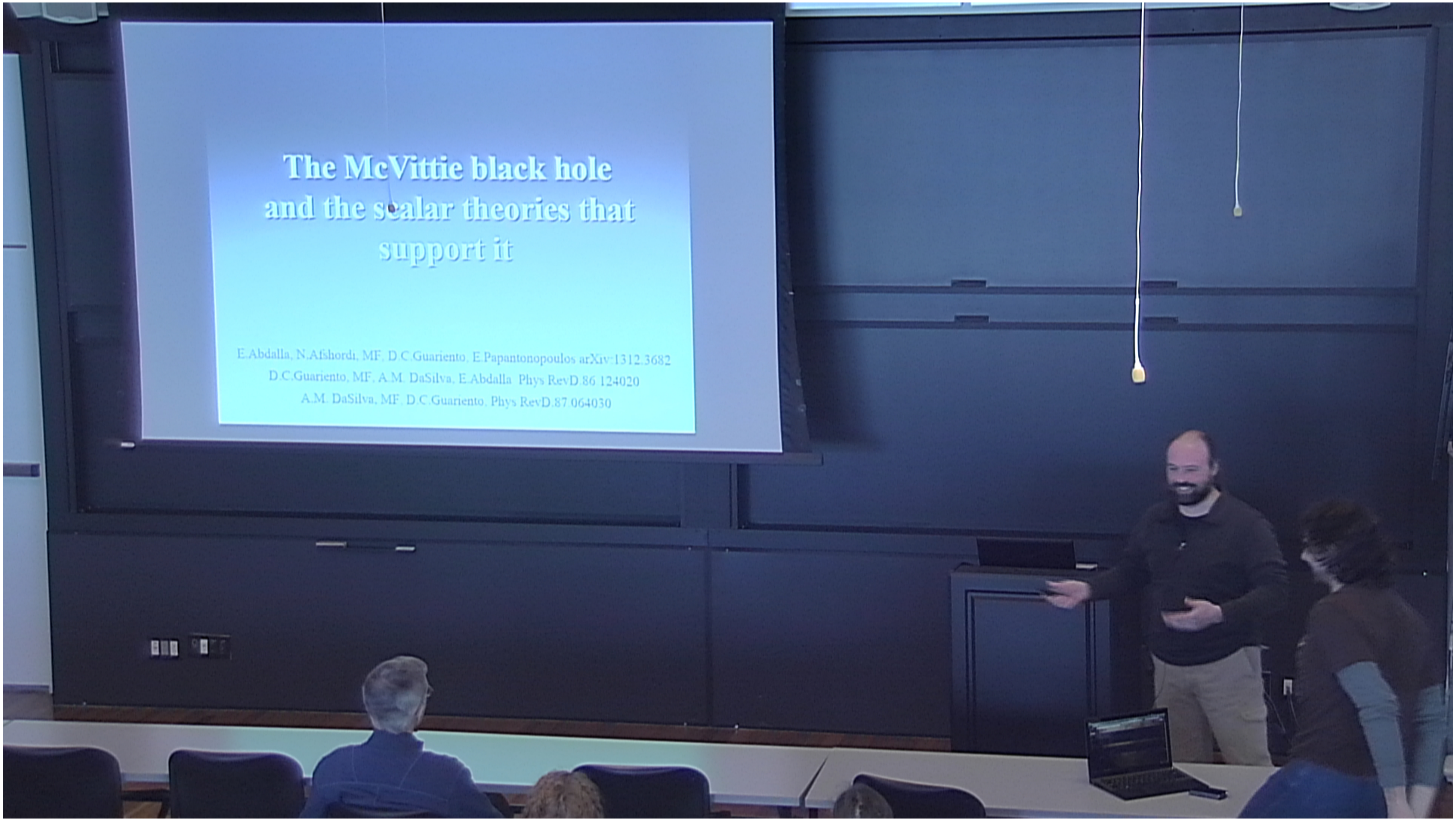
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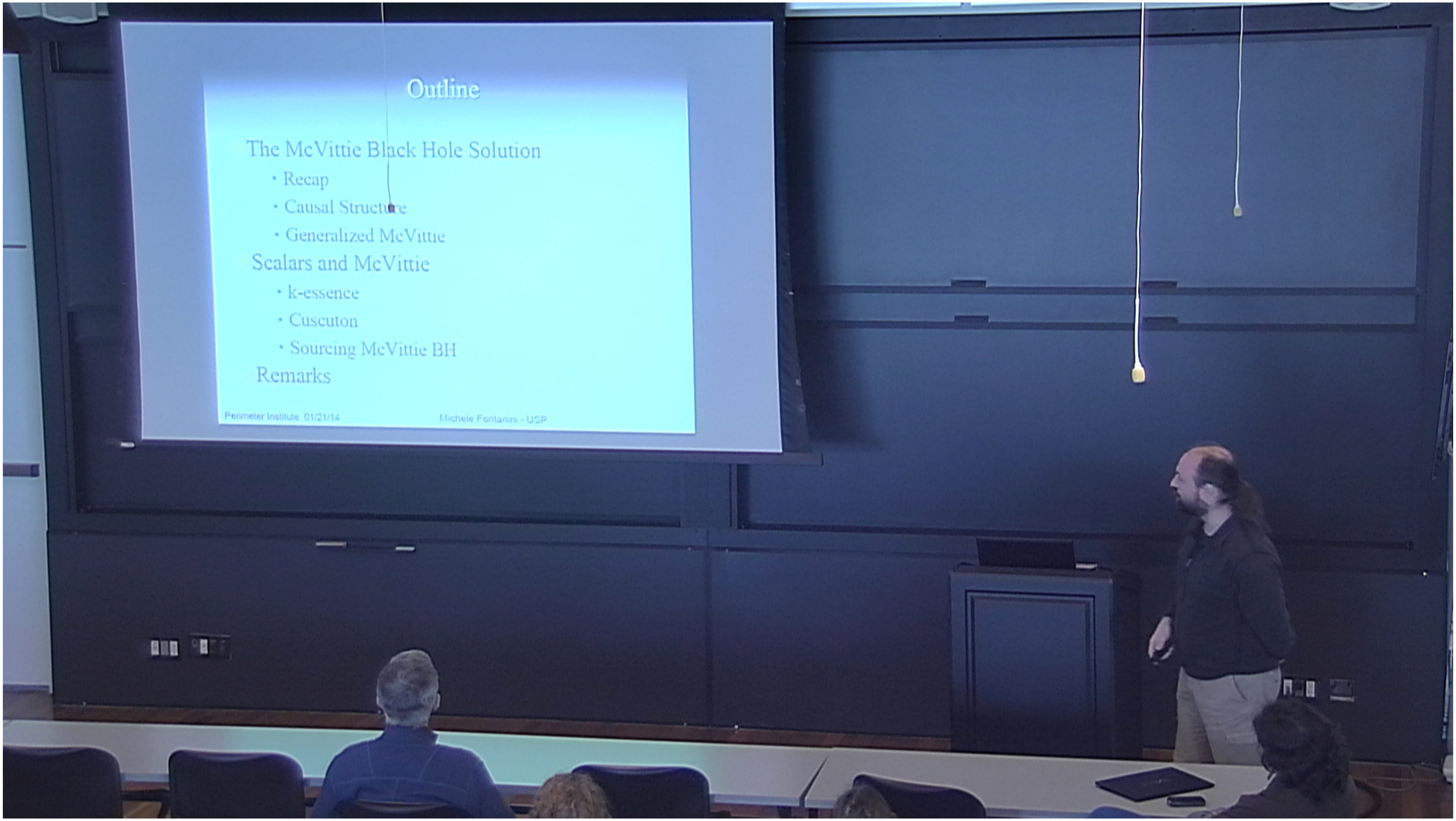
Outline
The McVittie Black Hole Solution
Recap
Causal Structure
Generalized McVittie
Scalars and McVittie
k-essence
Cuscuton
Sourcing McVittie BH
Remarks
Perimeter Institute 01/21/14
Michele Fontanini - USP

Outline

- The McVittie Black Hole Solution
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  - Causal Structure
  - Generalized McVittie
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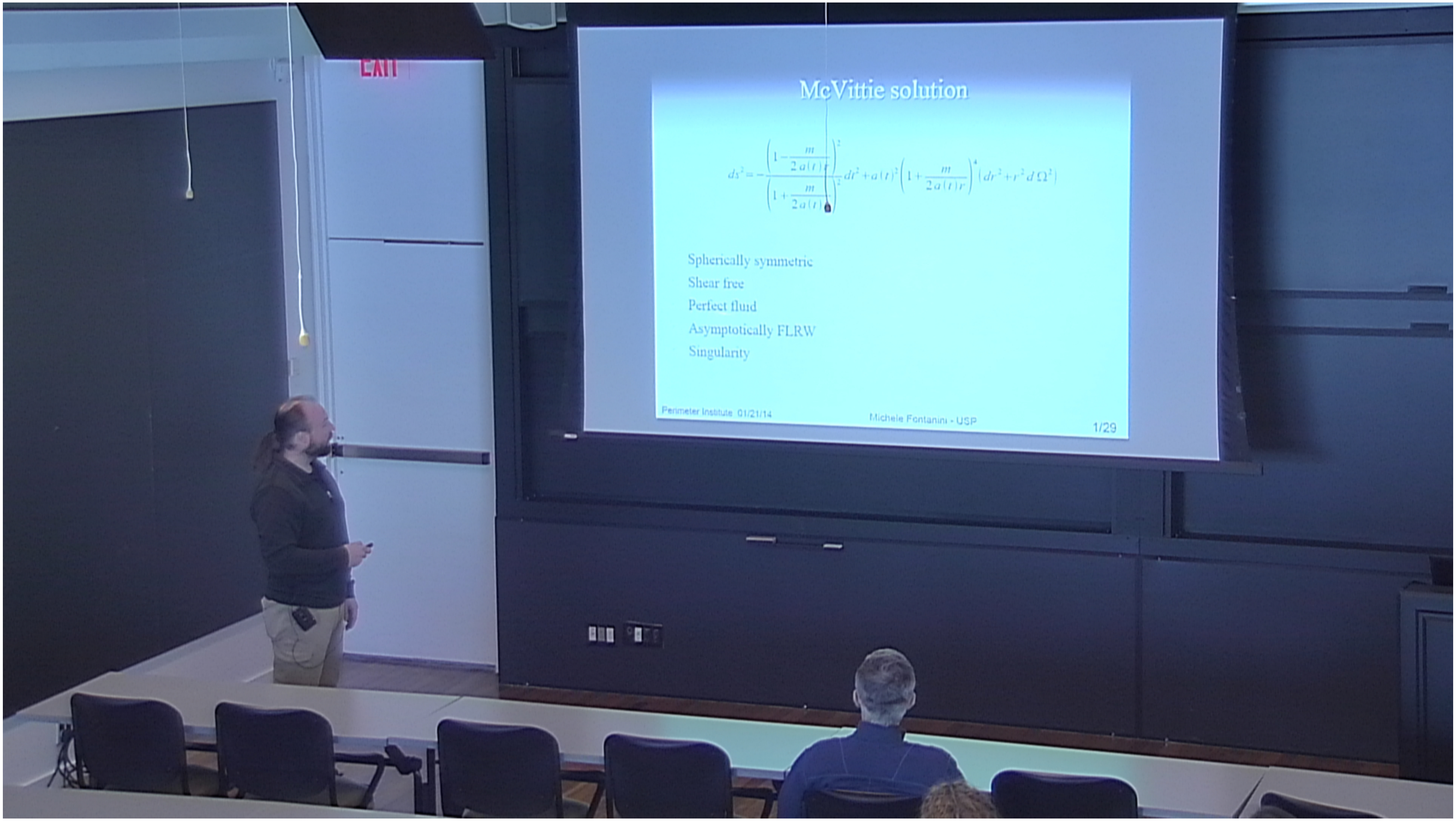
McVittie solution

$$ds^2 = -\frac{\left(1 - \frac{m}{2a(t)r}\right)^2}{\left(1 + \frac{m}{2a(t)r}\right)^2} dt^2 + a(t)^2 \left(1 + \frac{m}{2a(t)r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

McVittie solution

$$ds^2 = -\frac{\left(1 - \frac{m}{2a(t)r}\right)^2}{\left(1 + \frac{m}{2a(t)r}\right)^2} dt^2 + a(t)^2 \left(1 + \frac{m}{2a(t)r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

- Spherically symmetric
- Shear free
- Perfect fluid
- Asymptotically FLRW
- Singularity


McVittie solution
Spherically symmetric
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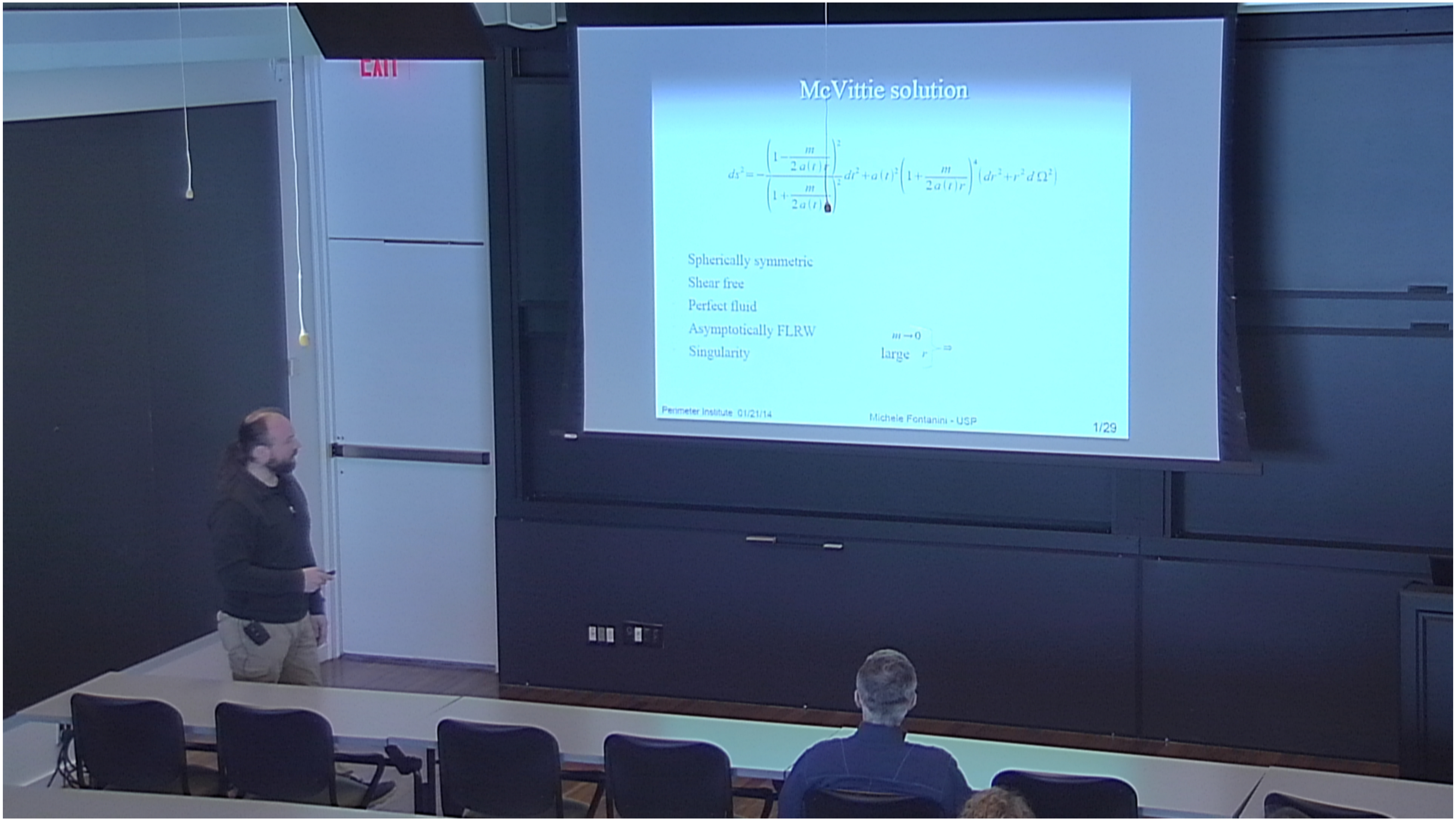
EXIT


EXIT
McVittie solution
$ds^2 = -\frac{\left(1-\frac{m}{2a(t)r}\right)^2}{\left(1+\frac{m}{2a(t)r}\right)^2}dt^2 + a(t)^2\left(1+\frac{m}{2a(t)r}\right)^4\left(dr^2+r^2d\Omega^2\right)$
Spherically symmetric
Shear free
Perfect fluid
Asymptotically FLRW
Singularity
$m \to 0$
large r
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McVittie solution

$$ds^2 = -\frac{\left(1-\frac{m}{2a(t)r}\right)^2}{\left(1+\frac{m}{2a(t)r}\right)^2}dt^2 + a(t)^2\left(1+\frac{m}{2a(t)r}\right)^4\left(dr^2+r^2 d\Omega^2\right)$$

- Spherically symmetric
- Shear free
- Perfect fluid
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- Singularity

$$\left.\begin{array}{r} m \to 0 \\ \text{large} \quad r \end{array}\right\} \Rightarrow$$

McVittie solution

$$ds^2 = -dt^2 + a(t)^2 \left(dr^2 + r^2 d\Omega^2 \right)$$

- Spherically symmetric
- Shear free
- Perfect fluid
- Asymptotically FLRW
- Singularity

$$\left. \begin{array}{r} m \to 0 \\ \text{large} \quad r \end{array} \right\} \Rightarrow \quad \text{FLRW}$$

McVittie solution – canonical coordinates

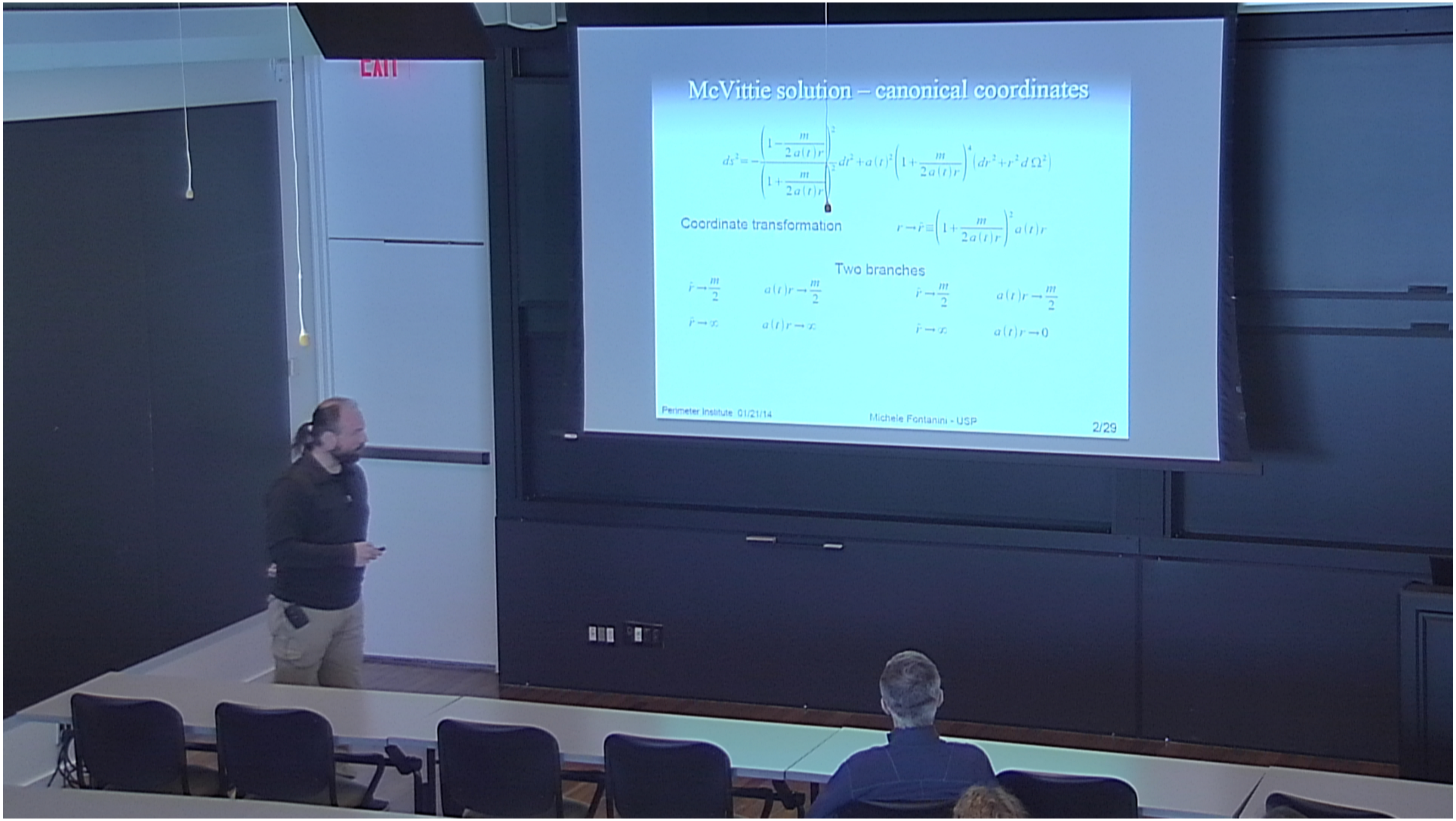
$$ds^2 = -\frac{\left(1 - \frac{m}{2a(t)r}\right)^2}{\left(1 + \frac{m}{2a(t)r}\right)^2} dt^2 + a(t)^2 \left(1 + \frac{m}{2a(t)r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right)$$

Coordinate transformation $\quad r \to \hat{r} \equiv \left(1 + \frac{m}{2a(t)r}\right)^2 a(t) r$

Two branches

$\hat{r} \to \frac{m}{2} \qquad a(t) r \to \frac{m}{2} \qquad\qquad \hat{r} \to \frac{m}{2} \qquad a(t) r \to \frac{m}{2}$

$\hat{r} \to \infty \qquad a(t) r \to \infty \qquad\qquad \hat{r} \to \infty \qquad a(t) r \to 0$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$ds^2 = -\frac{\left(1-\frac{m}{2a(t)r}\right)^2}{\left(1+\frac{m}{2a(t)r}\right)^2}dt^2 + a(t)^2\left(1+\frac{m}{2a(t)r}\right)^4\left(dr^2+r^2d\Omega^2\right)$
Coordinate transformation
$r \rightarrow \hat{r} \equiv \left(1+\frac{m}{2a(t)r}\right)^2 a(t)r$
Two branches
$\hat{r} \rightarrow \frac{m}{2}$ $a(t)r \rightarrow \frac{m}{2}$
$\hat{r} \rightarrow \infty$ $a(t)r \rightarrow \infty$
$\hat{r} \rightarrow \frac{m}{2}$ $a(t)r \rightarrow \frac{m}{2}$
$\hat{r} \rightarrow \infty$ $a(t)r \rightarrow 0$
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McVittie solution – canonical coordinates

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Two branches

$\hat{r} \to \frac{m}{2}$	$a(t)r \to \frac{m}{2}$	$\hat{r} \to \frac{m}{2}$	$a(t)r \to \frac{m}{2}$
$\hat{r} \to \infty$	$a(t)r \to \infty$	$\hat{r} \to \infty$	$a(t)r \to 0$
physical		Terminates on spacelike curvature singularities on both past and future	

McVittie solution – canonical coordinates

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Coordinate transformation $\quad r \to \hat{r} \equiv \left(1+\frac{m}{2a(t)r}\right)^2 a(t)\, r$

physical branch

$\hat{r} \to \frac{m}{2} \qquad a(t)r \to \frac{m}{2} \qquad \hat{r} \to \infty \qquad a(t)r \to \infty$

$$ds^2 = -\left(R^2 - r^2 H^2\right) dt^2 - 2r\frac{H}{R}\, dt\, dr + \frac{dr^2}{R^2} + r^2 d\Omega^2$$

$R[t,r] = \sqrt{1-\frac{2\mathrm{m}}{r}}$

McVittie solution – canonical coordinates

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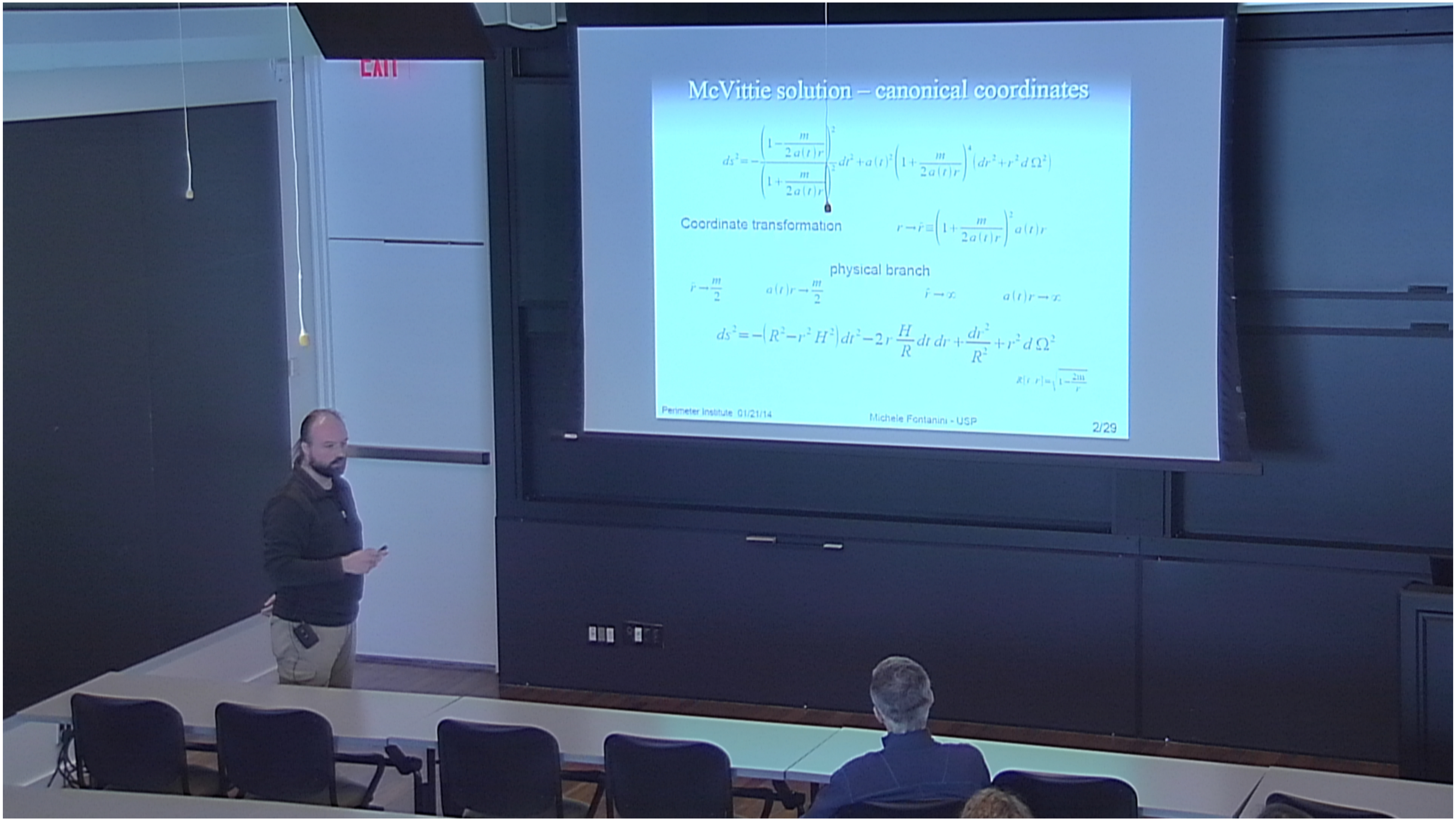

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$ds^2=-\left(R^2-r^2H^2\right)dt^2-2r\frac{H}{R}dt\,dr+\frac{dr^2}{R^2}+r^2d\Omega^2$
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McVittie – causal structure

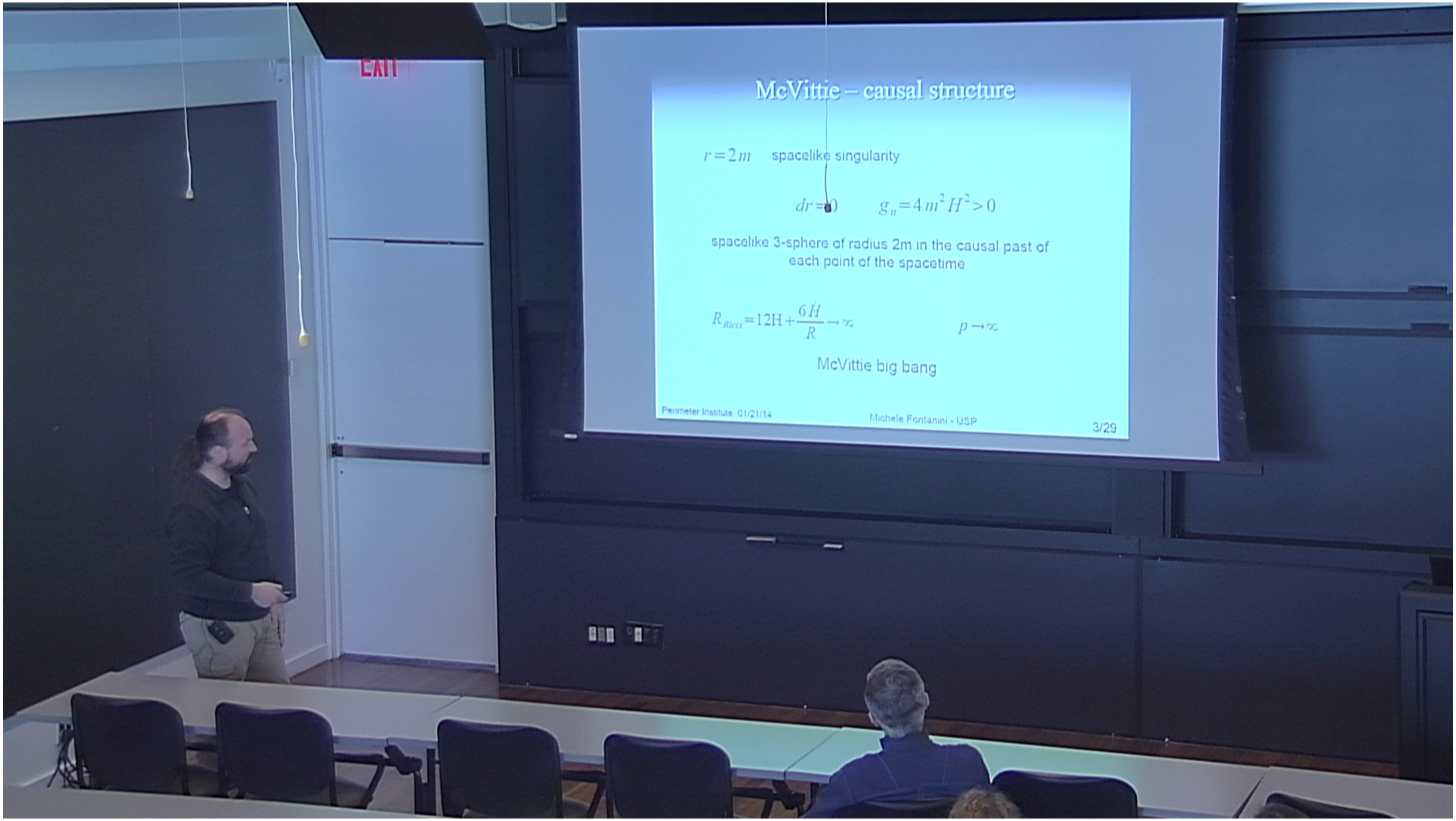
- $r = 2m$ spacelike singularity

$$dr = 0 \qquad g_{tt} = 4m^2 H^2 > 0$$

spacelike 3-sphere of radius 2m in the causal past of each point of the spacetime

$$R_{Ricci} = 12\mathrm{H} + \frac{6\dot{H}}{R} \rightarrow \infty \qquad p \rightarrow \infty$$

McVittie big bang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McVittie – causal structure
r=2m spacelike singularity
dr=0 g_tt=4m^2H^2>0
spacelike 3-sphere of radius 2m in the causal past of each point of the spacetime
R_Ricci=12H+6Ḣ/R→∞
p→∞
McVittie big bang
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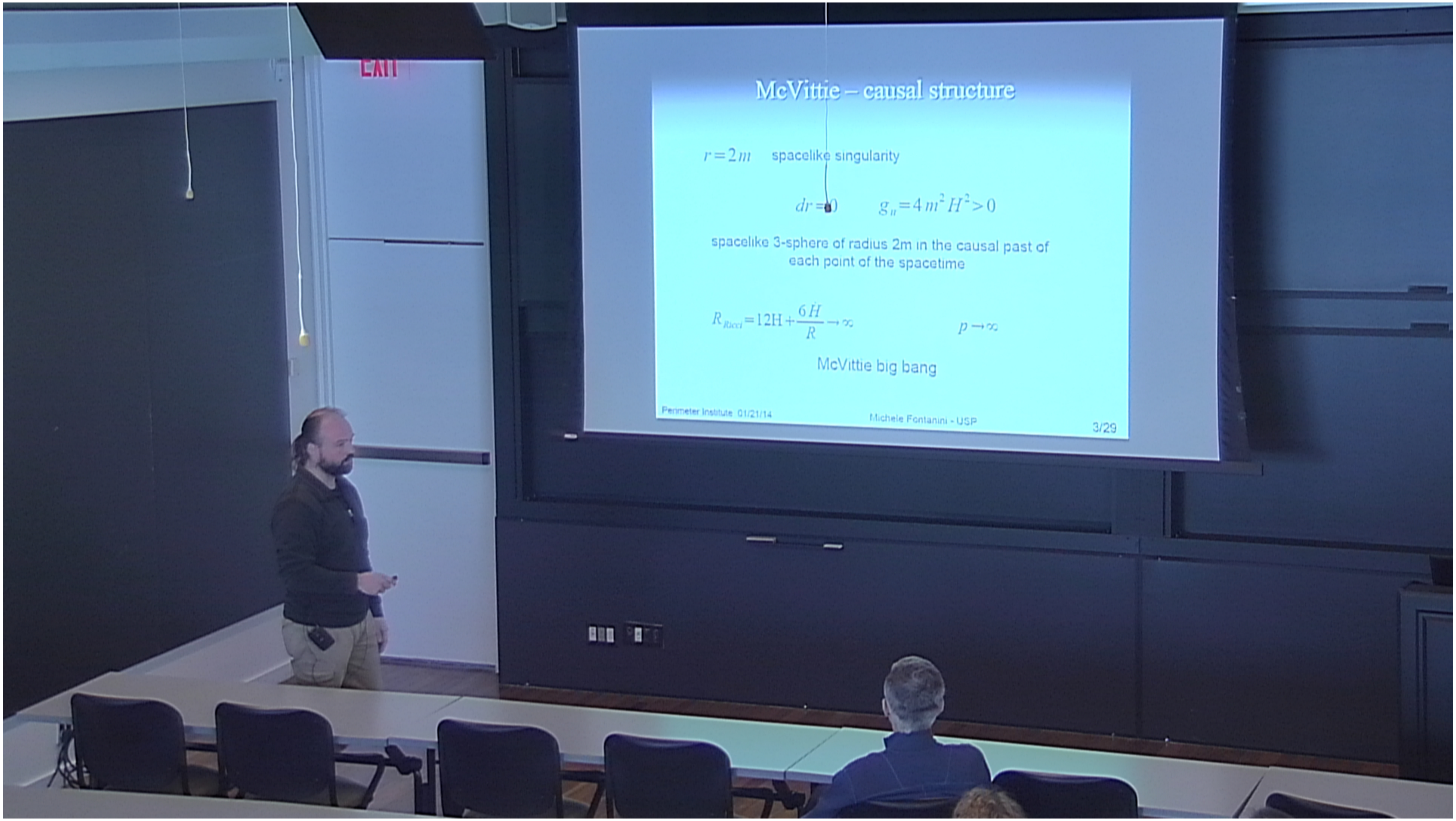
McVittie – causal structure
$r=2m$ spacelike singularity
$dr=0$ $g_{tt}=4m^2H^2>0$
spacelike 3-sphere of radius 2m in the causal past of each point of the spacetime
$R_{Ricci}=12H+\frac{6\dot{H}}{R}\rightarrow\infty$ $p\rightarrow\infty$
McVittie big bang
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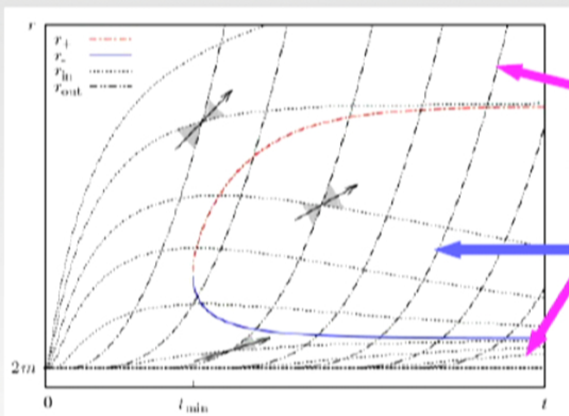
McVittie – causal structure

- apparent horizons $\dot{r} = R(r)\left[r\,H[t] \pm R(r)\right]$

- 2 solutions "outer" $r_+(t)$ and "inner" $r_-(t)$ apparent horizons branch off at the bifurcation 2-sphere $\left(t_{0,}\, r_\pm(t_0)\right)$



antitrapping region

normal region (r is spacelike)

McVittie – causal structure

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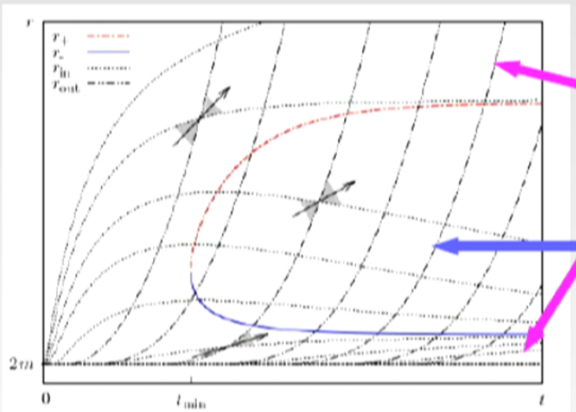

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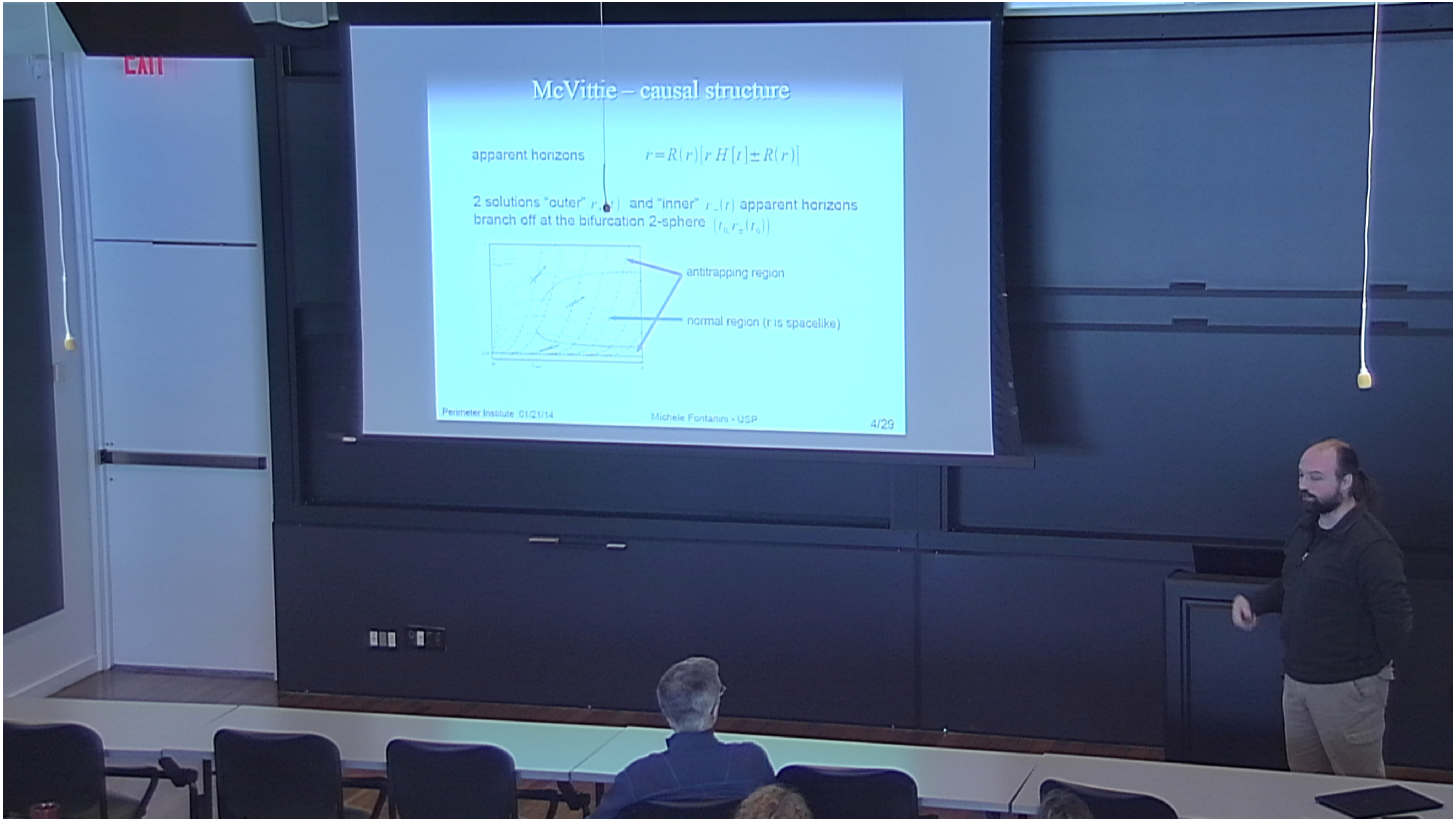

McVittie – causal structure
apparent horizons
2 solutions "outer" and "inner" apparent horizons
branch off at the bifurcation 2-sphere
antitrapping region
normal region (r is spacelike)
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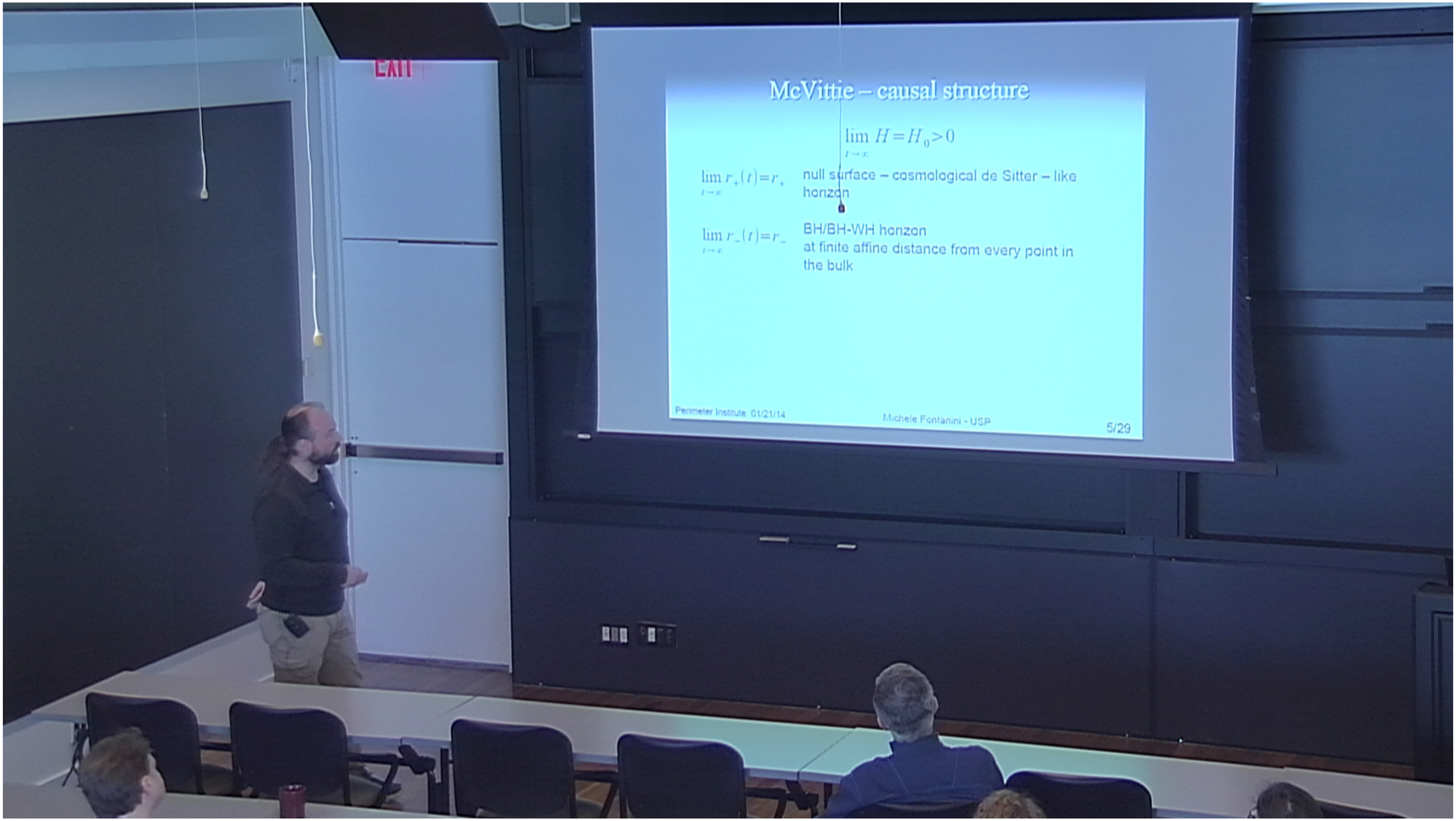
McVittie – causal structure
$\lim_{t\to\infty} H = H_0 > 0$
$\lim_{t\to\infty} r_+(t) = r_+$
null surface – cosmological de Sitter – like horizon
$\lim_{t\to\infty} r_-(t) = r_-$
BH/BH-WH horizon
at finite affine distance from every point in the bulk
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EXIT

McVittie – causal structure

$$\lim_{t\to\infty} H = H_0 > 0$$

$\lim_{t\to\infty} r_+(t) = r_+$ null surface – cosmological de Sitter – like horizon

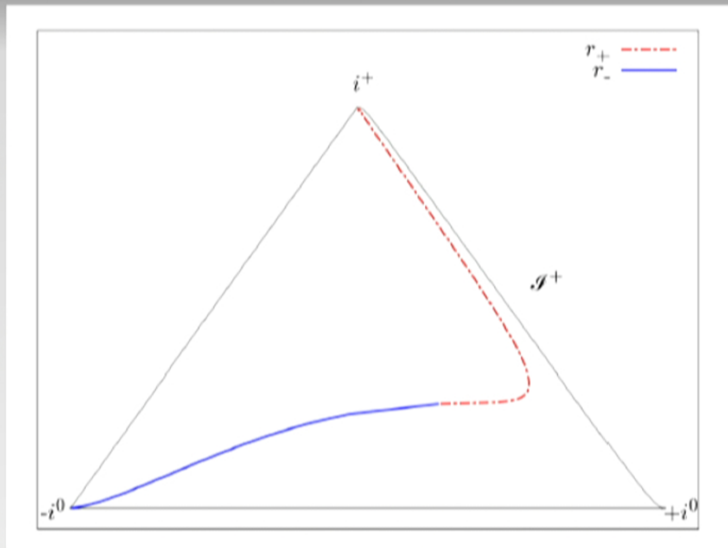
$\lim_{t\to\infty} r_-(t) = r_-$ BH/BH-WH horizon
at finite affine distance from every point in the bulk

$$H_0 = 0$$

r_+ null FLRW infinity

r_- null singularity

McVittie – causal structure



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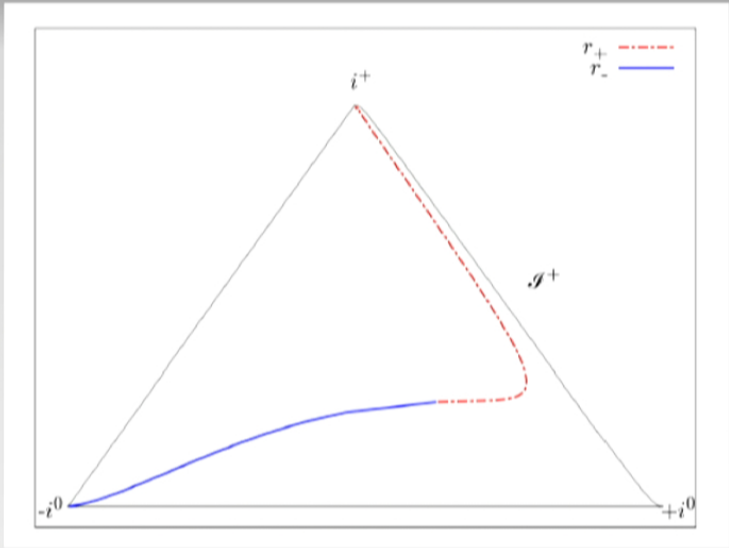

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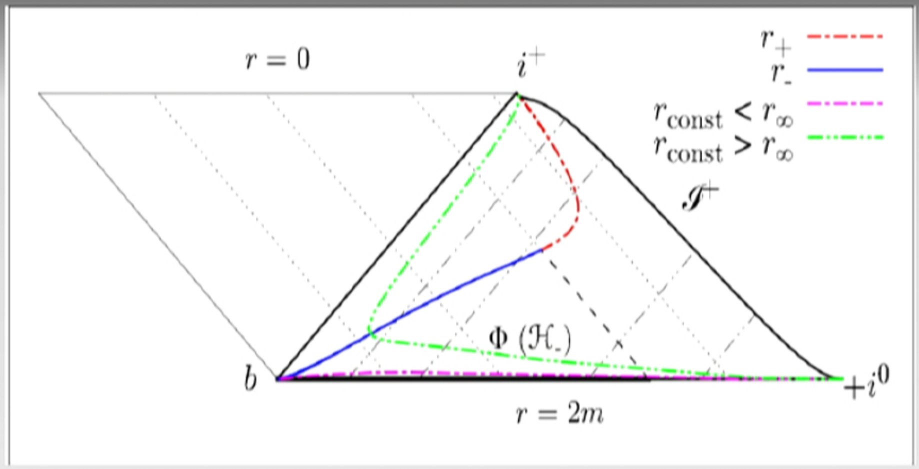
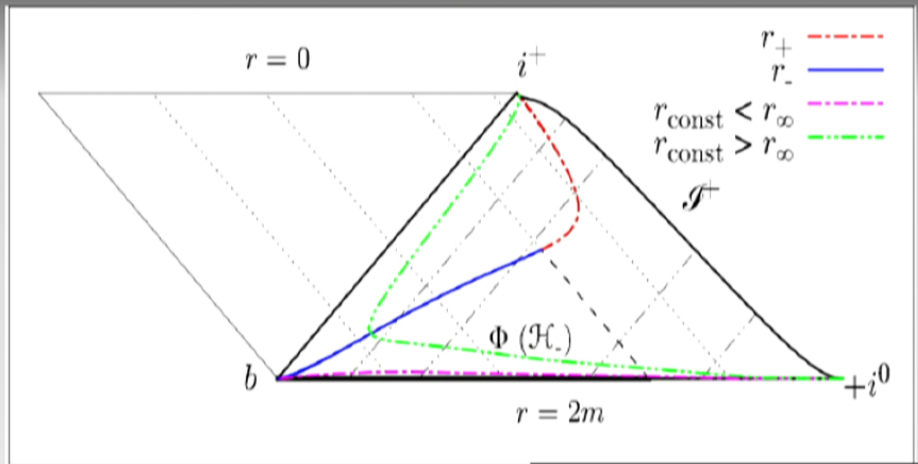
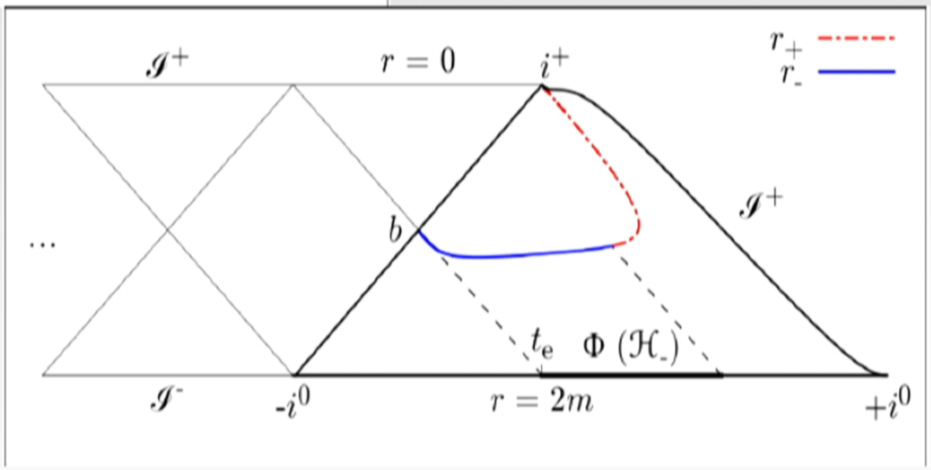


McVittie – causal structure





McVittie – causal structure

BH or BH/WH couple?
Do all ingoing geodesics* enter the normal region?
Is the back-projection of the inner horizon compact on the big bang surface?

McVittie – causal structure

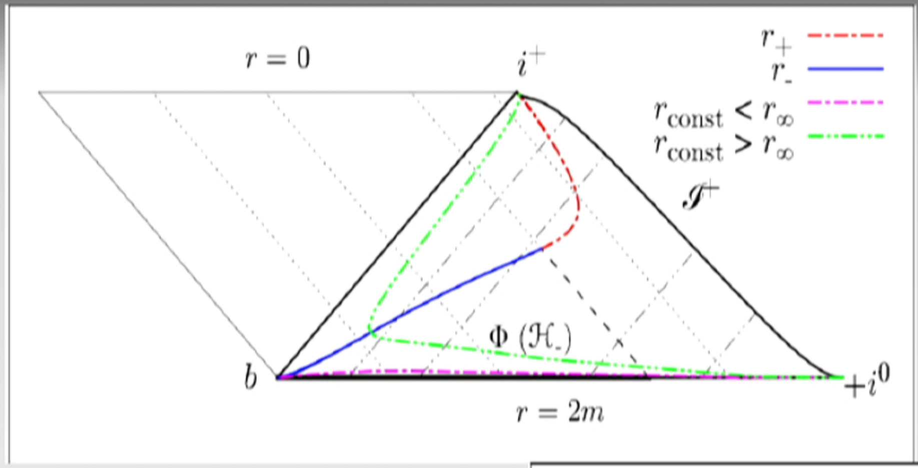
BH or BH/WH couple?
Do all ingoing geodesics* enter the normal region?
Is the back-projection of the inner horizon compact on the big bang surface?

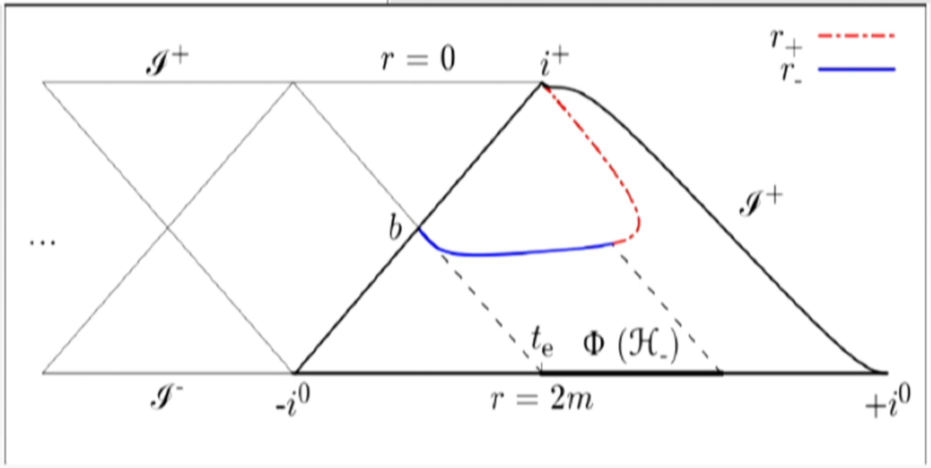
$$\Delta H(t) = H(t) - H_0$$

Determines the answer!

In almost every case it is possible to follow a light geodesic close to the inner apparent horizon to find whether it crosses to the normal region or not

McVittie – causal structure





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McVittie – causal structure

ΛCDM case: dust + cosmological constant $\quad H(t) = H_0 \coth\left(\frac{3}{2} H_0 t\right)$

a) $\eta < 0$ image bounded — BH-WH
b) $\eta > 0$ image unbounded — BH
c) $\eta = 0$ inconclusive — any

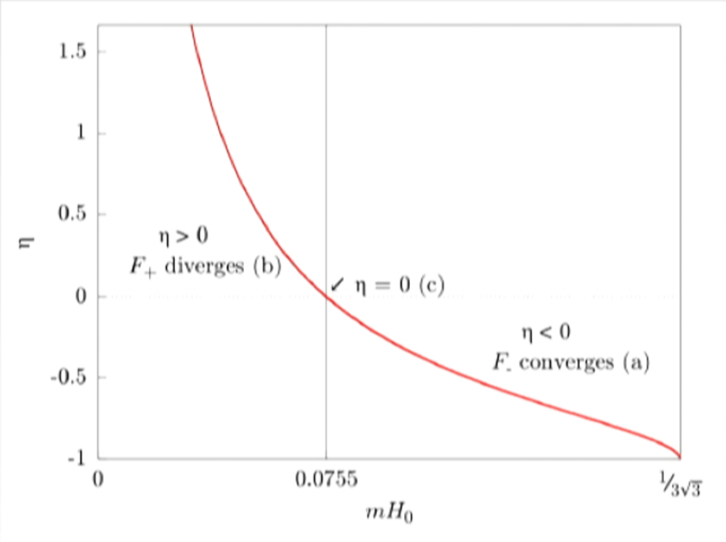
$$\eta \equiv \frac{B}{3H_0} - 1$$

$$B \equiv R(r_-)(R'(r_-) - H_0)$$

η depends only on

$$m H_0 \equiv \lambda \in \left(0, \frac{1}{3\sqrt{3}}\right)$$

For non-extremal BH



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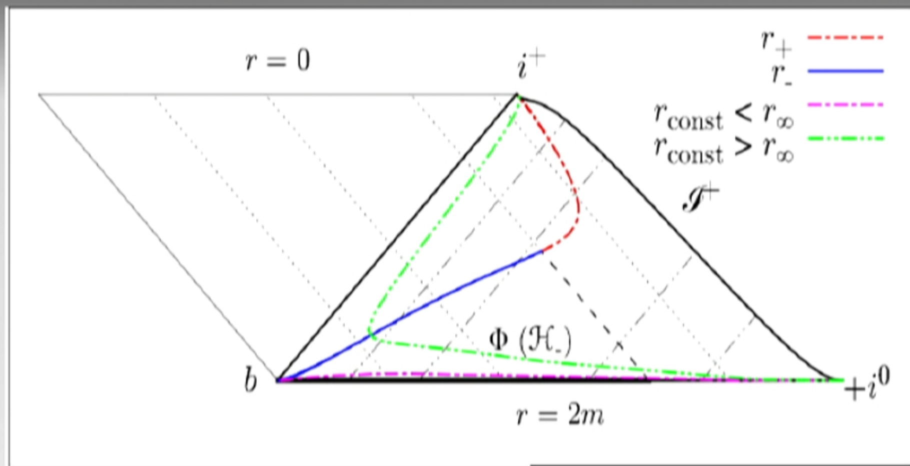
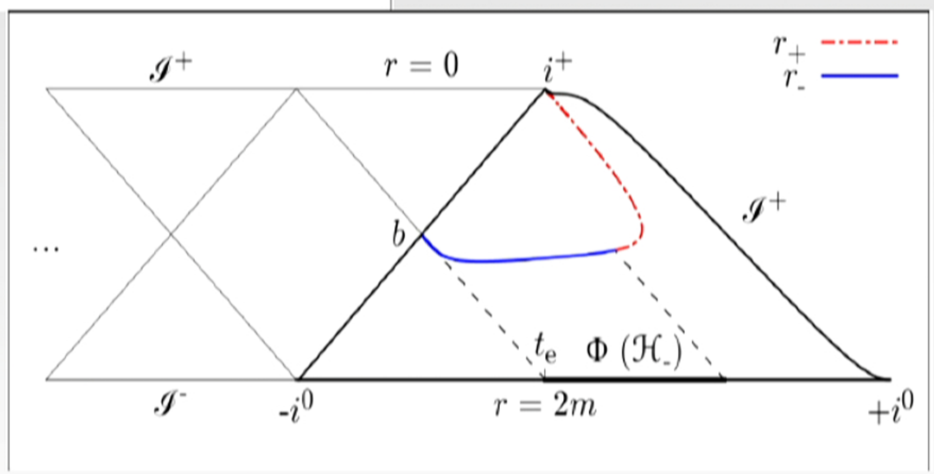
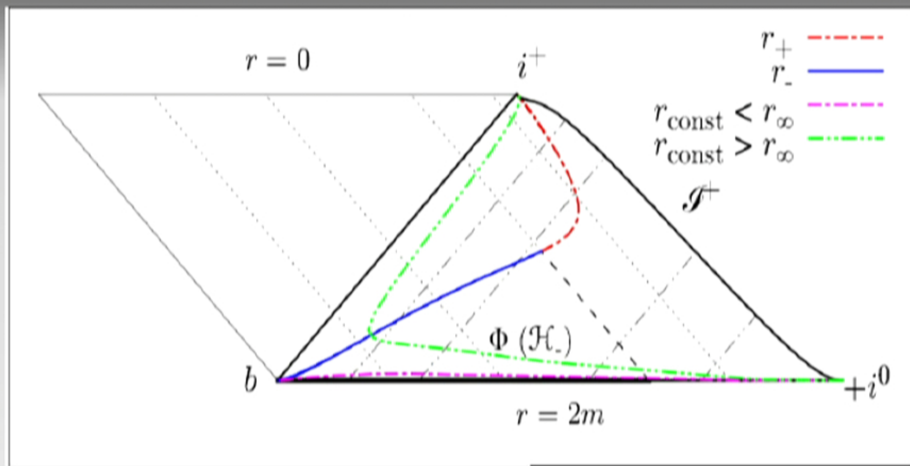
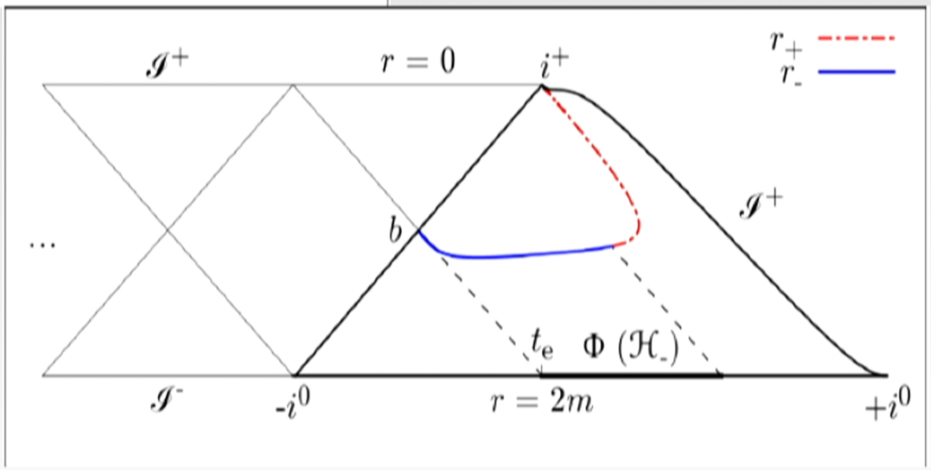
McVittie – causal structure





McVittie – causal structure





Generalized McV

$$ds^2 = -\left(R^2 - r^2 H^2\right) dt^2 - 2r\frac{H}{R} dt\, dr + \frac{dr^2}{R^2} + r^2 d\Omega^2$$

$$R[t,r] = \sqrt{1 - \frac{2\mathrm{m}(t)}{r}}$$

Generalized McV

$$ds^2 = -\left(R^2 - r^2 H^2\right) dt^2 - 2\, r\, \frac{H}{R}\, dt\, dr + \frac{dr^2}{R^2} + r^2\, d\Omega^2$$

$$R[t,r] = \sqrt{1 - \frac{2\mathrm{m}(t)}{r}}$$

$$m \rightarrow m(t)$$

Generalized McV

$$ds^2 = -\left(R^2 - r^2 H^2\right) dt^2 - 2r\frac{H}{R} dt\, dr + \frac{dr^2}{R^2} + r^2 d\Omega^2$$

$$R[t,r] = \sqrt{1 - \frac{2\mathrm{m}(t)}{r}}$$

$$m \to m(t) \quad \Rightarrow \quad H \to H - M\left(1 - \frac{1}{R}\right)$$

$$H = \frac{\dot{a}}{a}, \quad M = \frac{\dot{m}}{m}$$

$$ds^2 = -\left(R^2 - r^2\left(H - M + \frac{M}{R}\right)^2\right) dt^2 - 2\frac{r}{R}\left(H - M + \frac{M}{R}\right) dt\, dr + \frac{dr^2}{R^2} + r^2 d\Omega^2$$

GMcV – construction

- Multiple perfect fluids (→ phantom fluid)
- Imperfect fluid

GMcV – construction

- Multiple perfect fluids ($\to$ phantom fluid)
- Imperfect fluid

$$G^{t}{}_{r} \propto \dot{m} \Rightarrow$$

Non comoving fluids
+
Ricci isotropy

$$G^{r}{}_{r} = G^{\theta}{}_{\theta}$$

$\to$ phantom fluid + fine tuned cancellation

GMcV – imperfect fluid

$$T^{\mu\nu}=(\rho+p)u^{\mu}u^{\nu}+p\,g^{\mu\nu}-\zeta h^{\mu\nu}u^{\gamma}{}_{;\gamma}-\chi 2h^{\gamma(\mu}u^{\nu)}q_{\gamma}$$

Einstein's equations

Eckart – Landau-Lifshitz model

$$q_{\mu}=\partial_{\mu}T+T\,u_{\mu;\gamma}u^{\gamma}$$

$$T=\left[T_{\infty}(t)+\frac{M}{4\pi\chi}\frac{\ln(R)}{R}\right]$$

GMcV – imperfect fluid

$$T^{\mu\nu}=(\rho+p)u^{\mu}u^{\nu}+p\,g^{\mu\nu}-\zeta h^{\mu\nu}u^{\gamma}{}_{;\gamma}-\chi 2h^{\gamma(\mu}u^{\nu)}q_{\gamma}$$

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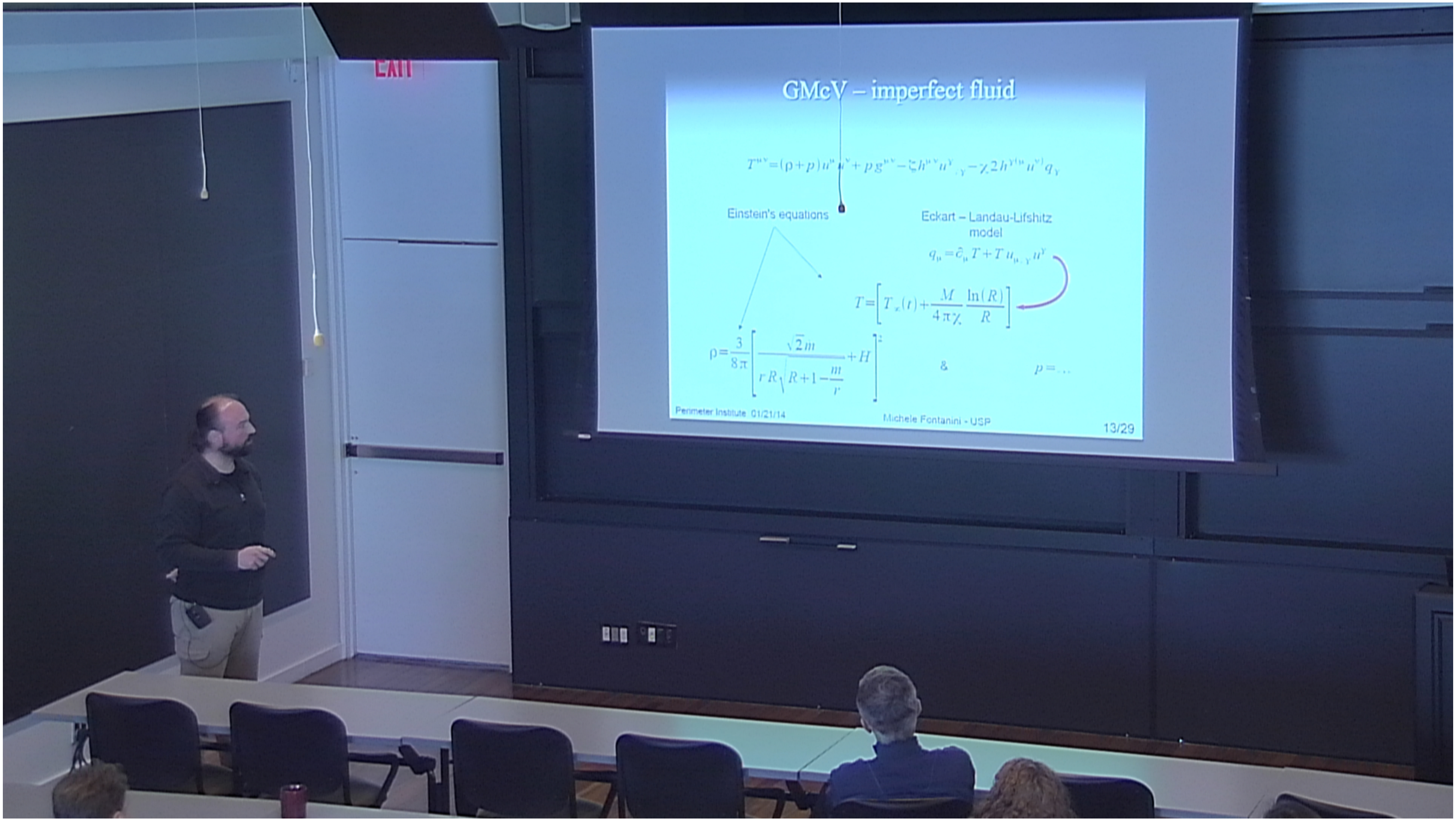
$$q_{\mu}=\partial_{\mu}T+T\,u_{\mu;\gamma}u^{\gamma}$$

$$T=\left[T_{\infty}(t)+\frac{M}{4\pi\chi}\frac{\ln(R)}{R}\right]$$

$$\rho=\frac{3}{8\pi}\left[\frac{\sqrt{2}\,\dot{m}}{rR\sqrt{R+1-\frac{m}{r}}}+H\right]^{2}$$

&

$$p=\ldots$$


GMcV – imperfect fluid
Einstein's equations
Eckart – Landau-Lifshitz model
&
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EXIT

GMcV – imperfect fluid

$$T^{\mu\nu} = (\rho+p)u^{\mu}u^{\nu} + p\,g^{\mu\nu} - \zeta h^{\mu\nu}u^{\gamma}{}_{;\gamma} - \chi 2 h^{\gamma(\mu}u^{\nu)}q_{\gamma}$$

Einstein's equations

Eckart – Landau-Lifshitz model

$$q_{\mu} = \partial_{\mu}T + T\,u_{\mu;\gamma}u^{\gamma}$$

$$T = \left[T_{\infty}(t) + \frac{M}{4\pi\chi}\frac{\ln(R)}{R}\right]$$

$$\rho = \frac{3}{8\pi}\left[\frac{\sqrt{2}\,\dot{m}}{r R\sqrt{R+1-\frac{m}{r}}} + H\right]^{2}$$

&

$$p = \dots$$

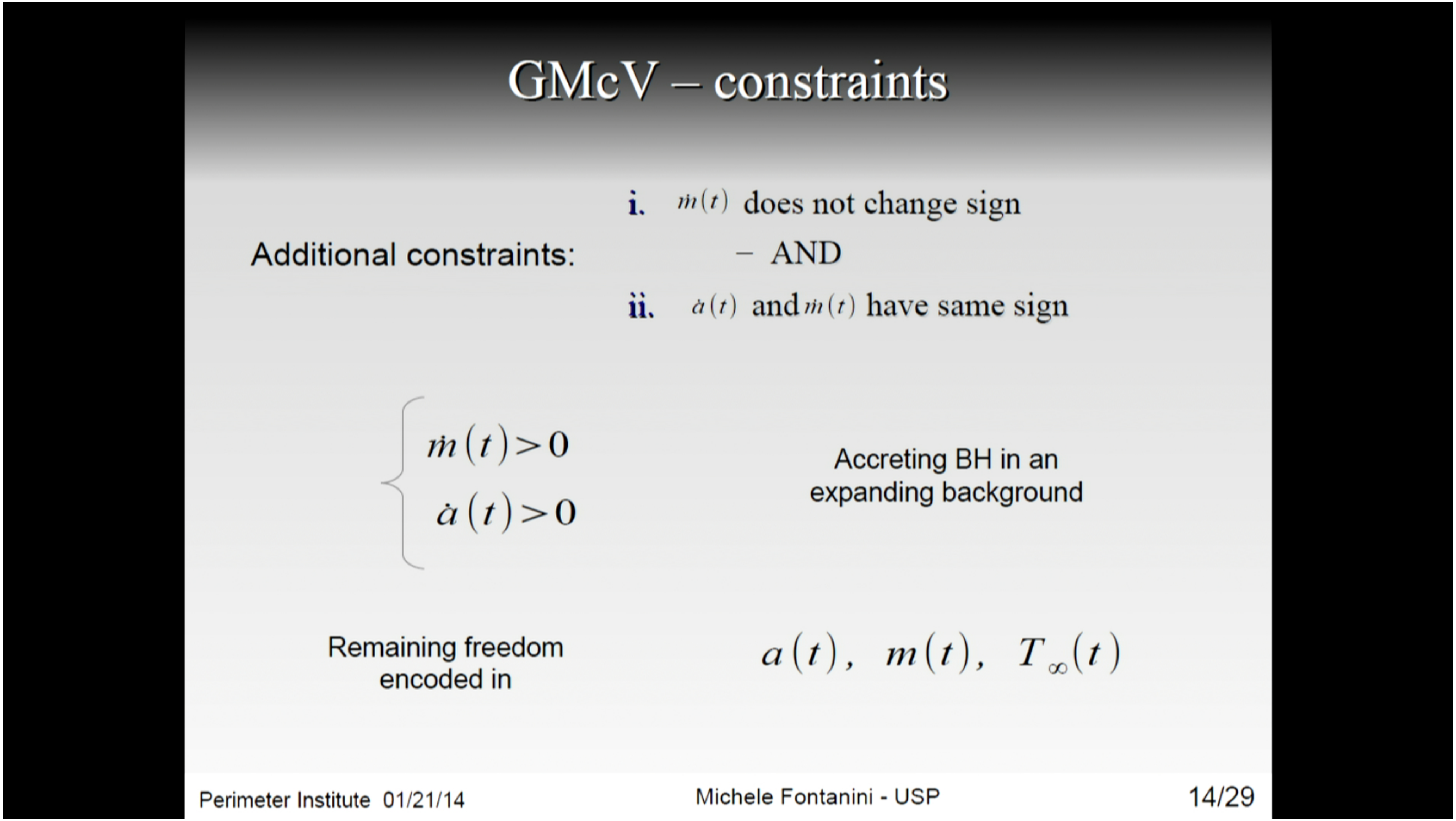
GMcV – constraints

Additional constraints:

i. $\dot{m}(t)$ does not change sign

– AND

ii. $\dot{a}(t)$ and $\dot{m}(t)$ have same sign


GMcV – constraints
Additional constraints:
i. $\dot{m}(t)$ does not change sign
– AND
ii. $\dot{a}(t)$ and $\dot{m}(t)$ have same sign
$\begin{cases} \dot{m}(t) > 0 \\ \dot{a}(t) > 0 \end{cases}$
Accreting BH in an expanding background
Remaining freedom encoded in
$a(t), \quad m(t), \quad T_\infty(t)$
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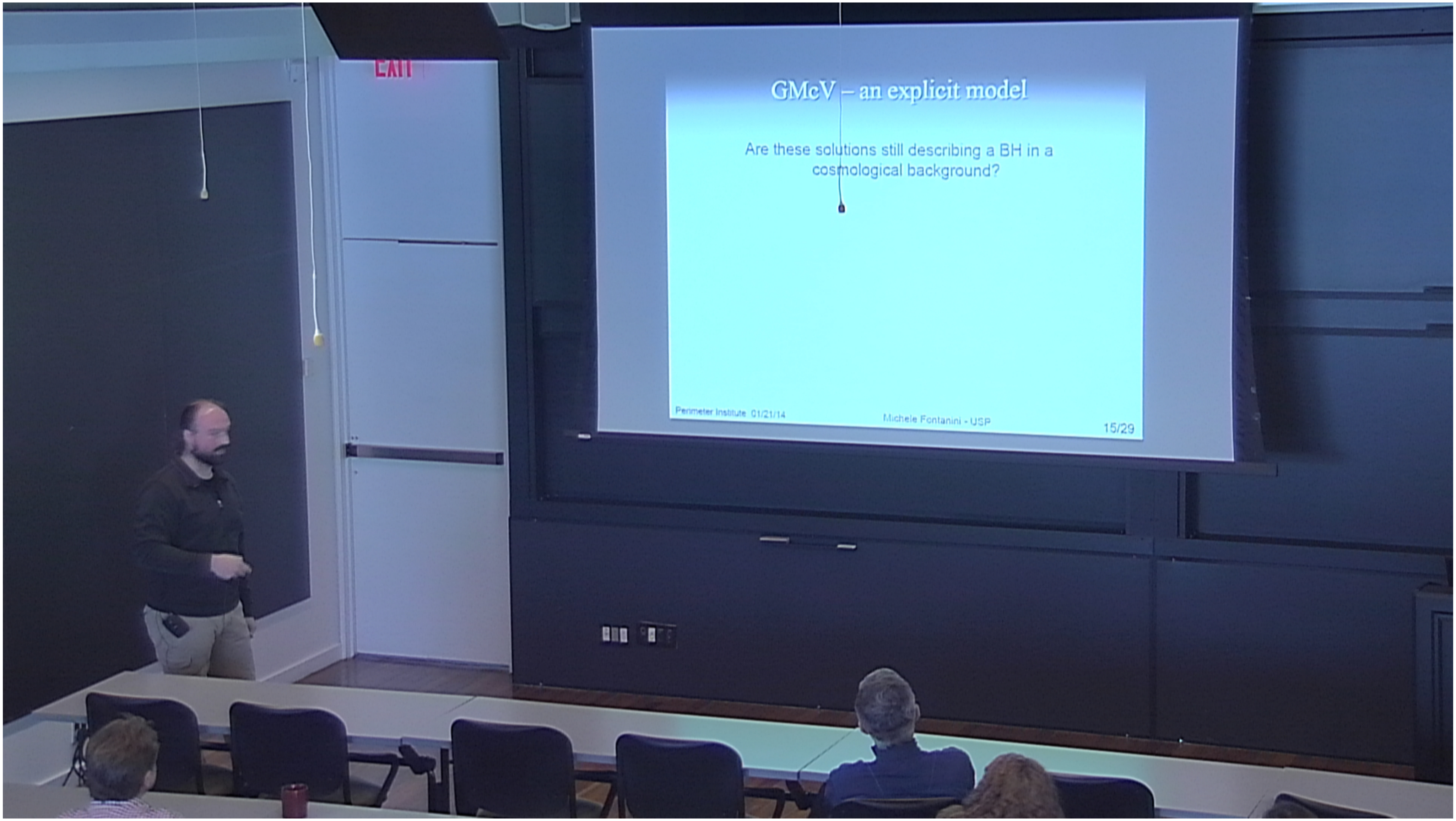
GMcV – an explicit model
Are these solutions still describing a BH in a cosmological background?
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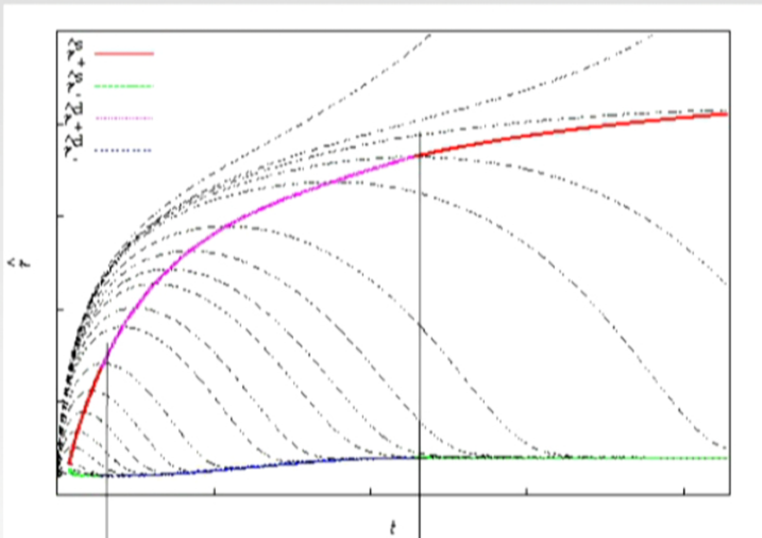
GMcV – an explicit model

Are these solutions still describing a BH in a cosmological background?

GMcV – an explicit model

Are these solutions still describing a BH in a cosmological background?



$m = m_1$ $m = m(t)$ $m = m_2$

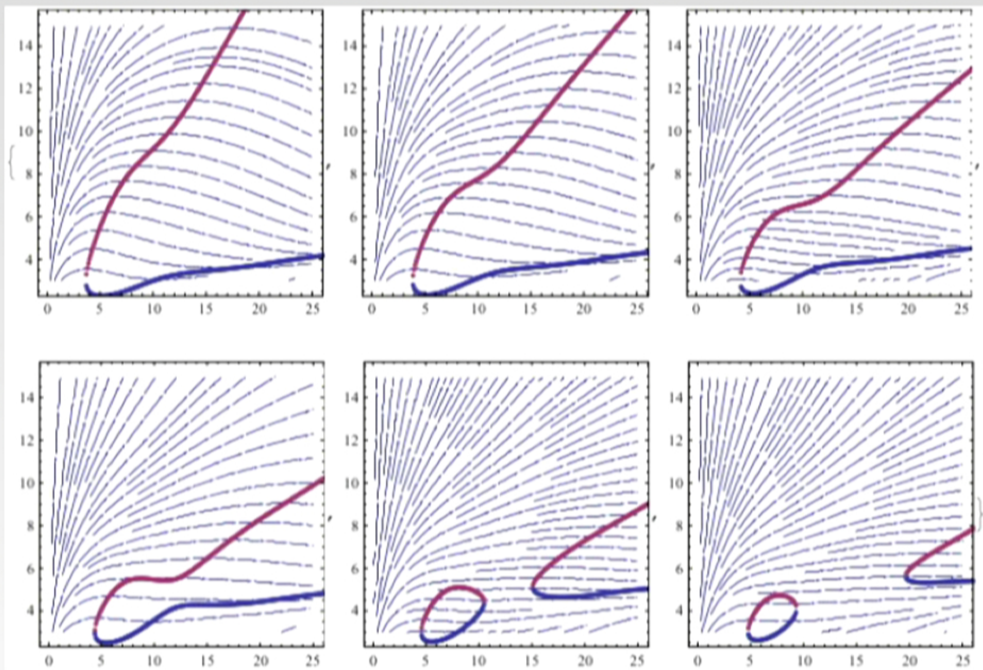
Patching McV and GMcV

$$m(t) \sim t - \sin(t)$$

smooth connection
and smooth derivatives
→
continuous pressure and
energy density

GMcV – pushing the boundaries

Even models smoothly patched to McV can produce unexpected results and need to be investigated



Outline

- The McVittie Black Hole Solution
  - Recap
  - Causal Structure
  - Generalized McVittie
- Scalars and McVittie
  - k-essence
  - Cuscuton
  - Sourcing McVittie BH
- Remarks

k-essence

$$S_\phi = \int d^4x \sqrt{-g}\, L(X,\phi) \qquad X = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

- Introduced to solve the c.c. Problem
- Tracking solutions
- Issues with causality due to superluminal propagation of perturbations

k-essence

Stress-Energy tensor $$T_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = L_{,X} \nabla_\mu \phi \nabla_\nu \phi + g_{\mu\nu} L$$

An equivalent hydrodynamical description exists

Defining velocity $$u_\mu \equiv -\frac{\nabla_\mu \phi}{\sqrt{2X}}$$

Hortogonal projector $$h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

$T_{\mu\nu}$ can be recast as $$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu}$$

$$\rho \equiv 2X L_{,X} - L$$

$$p \equiv L$$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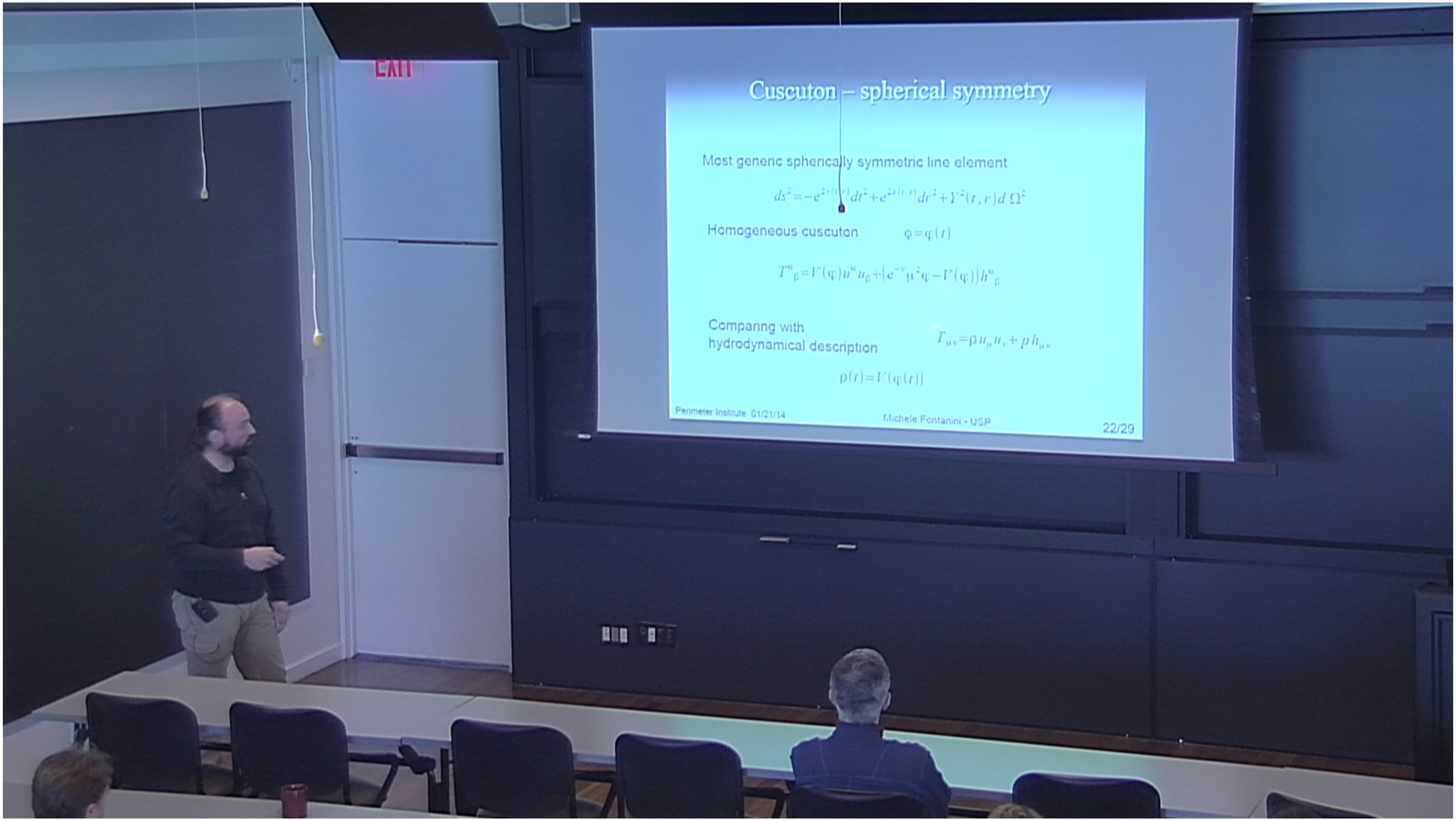
Cuscuton – spherical symmetry
Most generic spherically symmetric line element
Homogeneous cuscuton
Comparing with
hydrodynamical description
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Cuscuton – spherical symmetry

Field equations

$$\left(\frac{2\dot{Y}}{Y}+\dot{\lambda}\right)e^{-\nu}+\frac{1}{\mu^2}V_{,\varphi}=0$$

Bianchi identities

$$\left(\frac{2\dot{Y}}{Y}+\dot{\lambda}\right)e^{-\nu}=-\frac{\dot{\rho}}{\mu^2\dot{\varphi}}\equiv 3H(t)$$

Just a label for now

$$\frac{V_{,\varphi}}{\mu^2}=3H$$

Field equation in FLRW

Cuscuton sourcing McVittie

Homogeneous cuscuton ~ homogeneous density fluid

McVittie requires homogeneous energy density

Can McVittie be a solution for the system $S_{HE}+S_{Cuscuton}$?

Einstein equations

$$\begin{cases} V(\varphi)=\dfrac{3}{8\pi}H^2 \\ 2\dot{H}\dfrac{2ar+m}{2ar-m}+3H^2=8\pi\left(V-\dfrac{2ar+m}{2ar-m}\mu^2\dot{\varphi}\right) \end{cases}$$

Field equations

$$\frac{V_{,\varphi}}{\mu^2}-3\frac{\dot{a}}{a}=0$$

Cuscuton sourcing McVittie

$$\begin{cases} \dot{H} = -4\pi\mu^2\dot{\varphi} \\ \left(\dfrac{dV}{d\varphi}\right)^2 - 24\pi\mu^4 V = 0 \\ V(\varphi) = \dfrac{3}{8\pi}H^2 \end{cases} \Longrightarrow V(\varphi) = -6\pi\mu^4(\varphi + C)^2$$

Homogeneous Cuscuton with quadratic potential admits/sources McVittie black hole

k-essence sourcing McVittie

t-t component Einstein eqns

$$2X L_{,X} - L = \frac{3}{8\pi} H^2$$

no r dependence

take r derivative

$$4 m a \dot{\varphi}^2 \frac{(2ar+m)}{(2ar-m)^3} \left(2 X L_{,XX} + L_{,X}\right) = 0$$

$$L(X, \varphi) = A(\varphi) + B(\varphi)\sqrt{X}$$

Homogeneous Cuscuton as unique k-essence source for McVittie

Spoiler

How about generalized McVittie BH?

k-essence does not seem to work

Horndeski fields do

(stay tuned)

Conclusions

- McVittie as non-vacuum BH solution.
- Possibility of obtaining a BH/WH couple.
- BH in expanding background with non-constant mass parameter generated by an imperfect fluid.
- McVittie from fields: Cuscuton as unique k-essence model supporting it.