

Title: Wilson Lines in Higher Spin Gravity

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Abstract: In this talk I will review the interpretation of Wilson line operators in the context of higher spin gravity in 2+1 dim and holography. I will show how a Wilson line encapsulates the thermodynamics of black holes. Furthermore it provides an elegant description of massive particles. This opens a new window of observables which will allow us to probe the true geometrical nature of higher spin gravity.

Wilson lines in Higher Spin Gravity

w M Ammon & N Iqbal
1306.4338

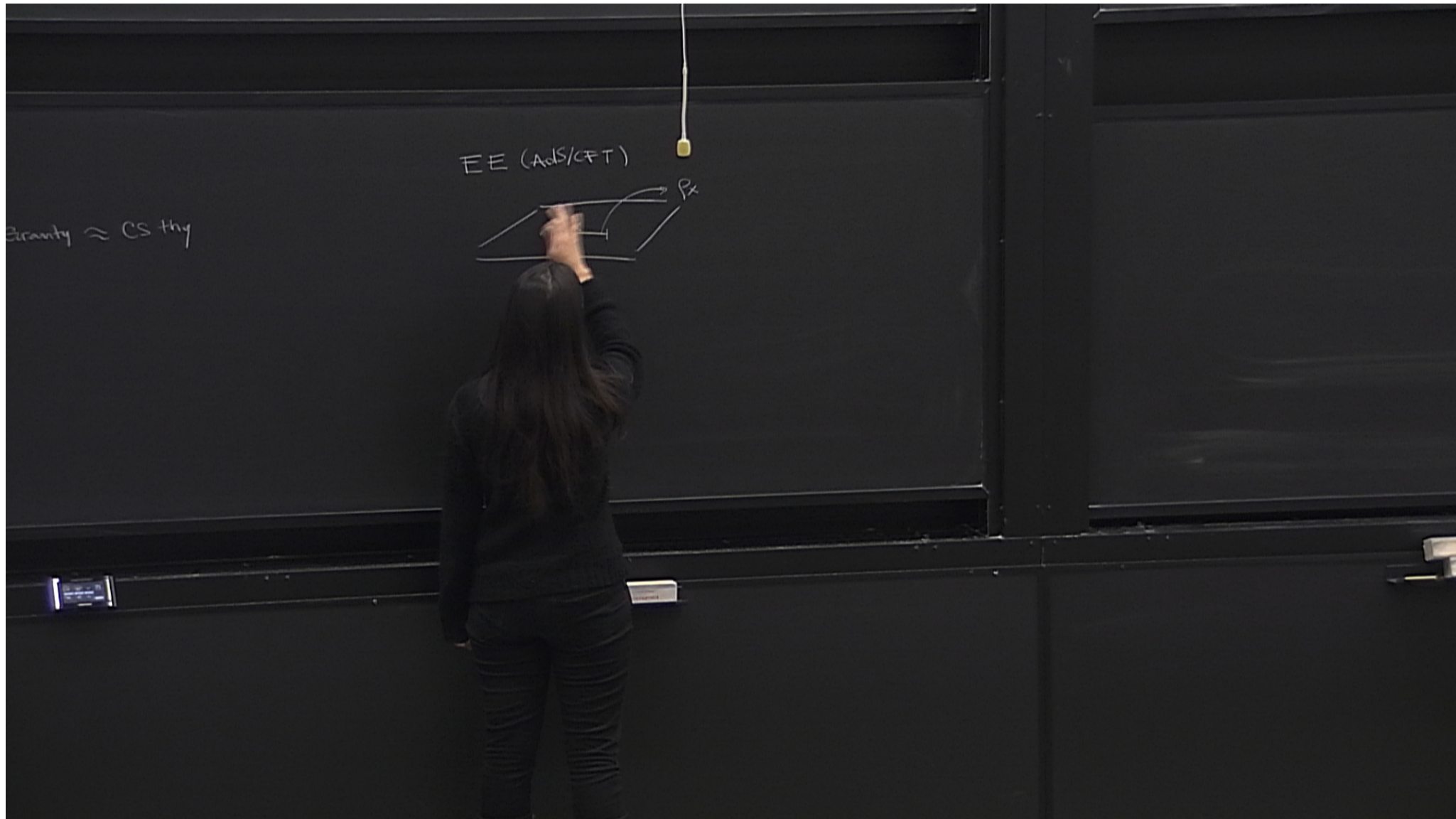
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AdS/CFT

Higher-Spin Grav (Vasiliev's thy)

$(Z+1)$ -D Gravity $\approx$ CS thy



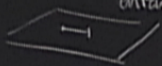
Wilson lines in Higher Spin Gravity

by M. Ammon & N. Iqbal
1306.4338

AdS_d/CFT_d

→ geometry knows about
thermo

→ geometry knows about
entanglement entropy



Higher Spin Grav (Vasiliev's thry)

test geom nature "strange" thys

Gauge
group

diff

matter content

$s=2,3,4,\dots,\infty$

$(2+1)$ -D Gravity $\approx$ CS thry

- 3D grav is a gauge th_y
- topological nature of 3D grav
- easy to construct HS th_y

⇒ How does $SL(2, \mathbb{R})$ CS th_y know about geo?

$$SL(2, \mathbb{R}) \Rightarrow SL(N, \mathbb{R}) \supset SL(2, \mathbb{R})$$

$$s = 2, 3, \dots, N$$

$$\Rightarrow hs[\lambda]$$

$$s = 2, \dots, \infty$$

⇒ Vasiliev th_y: $hs[\lambda] + \text{scalars}$

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$$s = 2, 3, \dots, N$$

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$$s = 2, \dots, \infty$$

$$\Rightarrow \text{Vasiliev thry: } hs[\lambda] + sc$$

$\Rightarrow$ How does $SL(2, \mathbb{R})$ CS thry know about geo?

$$W_\mu(C) = \text{Tr}_R \left(P \exp \int_C A \right)$$

$\downarrow$ $\rightarrow$ curve
 rep of G

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$$W_\mu(C) = \text{Tr}_R \left(\mathcal{P} \exp \int_C A \right)$$

$\downarrow$ curve
 $\downarrow$ rep of G

→ if C is closed, $W_\mu(C)$ is gauge inv.

→ if C is open, only depends on end-points

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- topological nature of 3D grav
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$\Rightarrow$ How does $SL(2, \mathbb{R})$ CS thry know about good ?

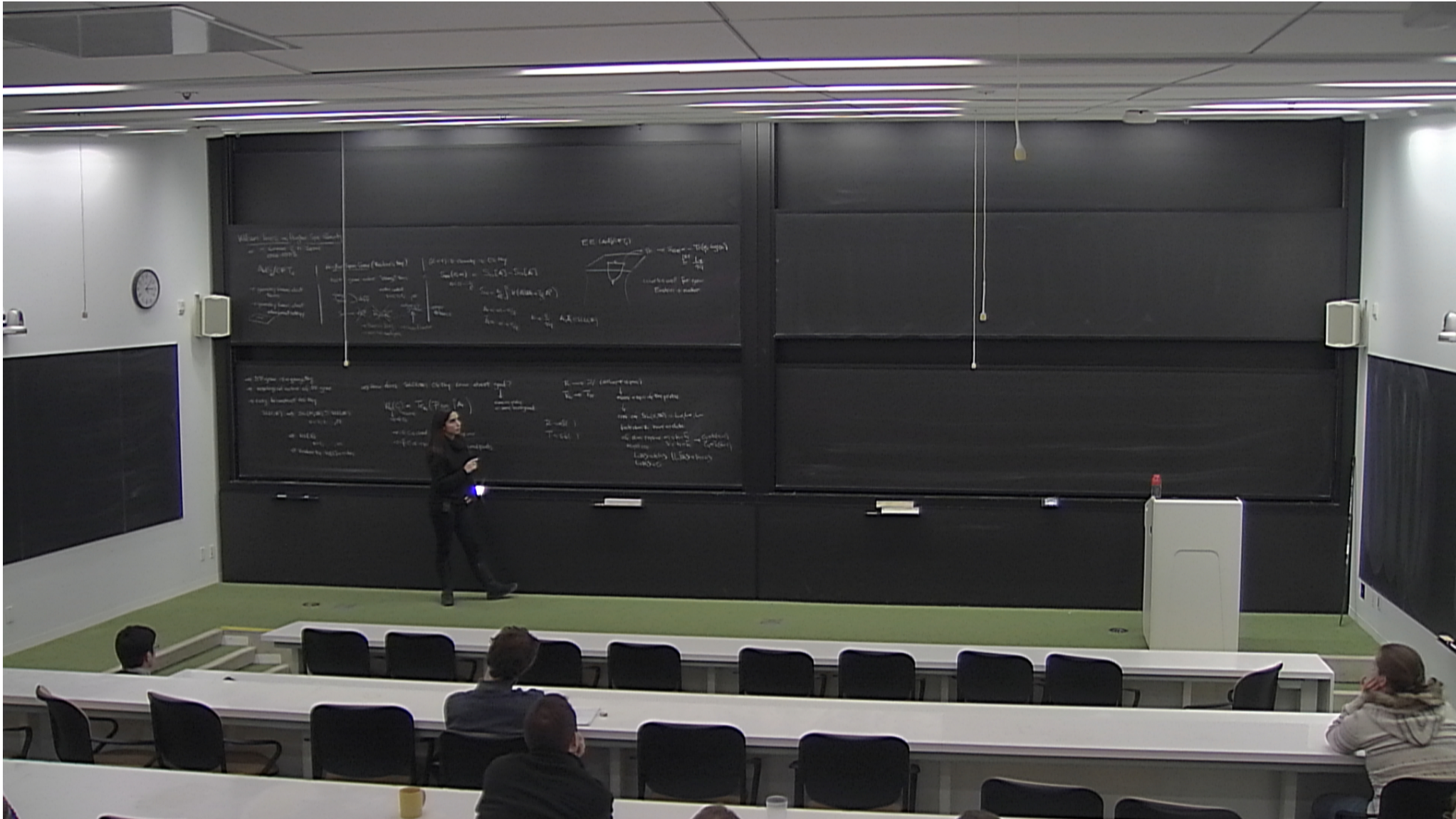
$$W_\mu(C) = \text{Tr}_R \left(P \exp \int_C A \right)$$

$\downarrow$ curve
 $\downarrow$ rep of G

$\downarrow$
maximal probe
on some background

C is closed, $W_\mu(C)$ is gauge inv.

if C is open, only depends on end-points



CS they know about good?

$$(P \exp \int_c A)$$

maximize probe
on some background

$K_2(c)$ is gauge inv.

only depends on order

$$\mathcal{R} \rightarrow \mathcal{H} \text{ (Hilbert space)}$$

$$\overline{T}_{\mathcal{R}} \rightarrow \overline{T}_{\mathcal{H}}$$

mass + spin of the probe

$$\text{reps of } SL(2, \mathbb{R}) = L_0, L_{\pm}, \bar{L}_{\pm}$$

finite dim $\mathcal{R}$ have no data

$$\text{inf dim reps} \rightarrow \begin{matrix} m = h + \bar{h} \\ S = h - \bar{h} \end{matrix} \rightarrow \begin{matrix} C_2 = h(h-1) \\ \bar{C}_2 = \bar{h}(\bar{h}-1) \end{matrix}$$

height rep

$$\begin{aligned} L_0 |h\rangle &= h |h\rangle & (L_{-})^n |h\rangle &= |h+n\rangle \\ L_{+} |h\rangle &= 0 \end{aligned}$$

$$\mathcal{R} \sim \mathcal{MS}()$$

$$\mathcal{T} \sim \mathcal{SS}()$$

$W_F(c)$ in the highest weight rep

$$W_F(c) = \int_{\text{Tr} \neq 1} dU e^{\underbrace{-S_0(U) + S(U, A)}_{\text{local } SL(2, \mathbb{R}) \text{ symm}}}$$

Aux probe $U \in SL(2, \mathbb{R})$

$$\begin{aligned} G_2 &= h(h-1) \\ \bar{G}_2 &= \bar{h}(\bar{h}-1) \end{aligned}$$

$|h+n\rangle$

$W_P(c)$ in the highest weight rep

$$W_P(c) = \int \underbrace{DU}_{\substack{\text{Aux probe} \\ U \in \text{SL}(2, \mathbb{R}) \\ \downarrow \\ \text{conj. mom } P \in \mathfrak{sl}(2, \mathbb{R})}} e^{\underbrace{-S_0(U) + S(U, A)}_{\substack{\text{local } \text{SL}(2, \mathbb{R}) \text{ symm}}}}$$

$$S_0(U, P) = \int_c ds \text{Tr}(P U^{-1} \dot{U}) + \lambda (\text{Tr}(P^2) - c)$$

EOM

$$\frac{d}{ds} \left(U^{-1} \dot{U} + U^{-1} A U - \bar{A} \right) + [\bar{A}, U^{-1} \dot{U} + U^{-1} A U] = 0$$

EOM

$$\frac{d}{ds} \left(U^{-1} \dot{U} + U^{-1} A U - \bar{A} \right) + [\bar{A}, U^{-1} \dot{U} + U^{-1} A U] = 0$$

$$A = \Delta_\mu \frac{dx^\mu}{ds}$$

EOM

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$$) = \mathbb{I} \rightarrow \text{eqn } \frac{dx^{\mu}}{ds} \text{ geod eqn}$$

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$$A = \Delta_\mu \frac{dx^\mu}{ds}$$

$$U = \mathbb{1} \rightarrow \text{eqn } \frac{dx^\mu}{ds} \text{ geod eqn}, \quad S = \int ds \sqrt{g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds}}$$

EOM

$$\frac{d}{ds} \left(U^{-1} \dot{U} + U^{-1} A U - \bar{A} \right) + [\bar{A}, U^{-1} \dot{U} + U^{-1} A U] = 0$$

$$A = A_\mu \frac{dx^\mu}{ds}$$

$$U = \mathbb{1} \rightarrow \text{eqn } \frac{dx^\mu}{ds} \text{ geod eqn}, \quad S = \int ds \sqrt{\text{Tr}(A - \bar{A})^2}$$

EOM

$$\frac{d}{ds} \left(U^{-1} \dot{U} + U^{-1} A U - \bar{A} \right) + [\bar{A}, U^{-1} \dot{U} + U^{-1} A U] = 0$$

$$A = \Delta_\mu \frac{dx^\mu}{ds}$$

$$U = \mathbb{1} \rightarrow \text{eqn } \frac{dx^\mu}{ds} \text{ geod eqn}, \quad S = \int ds \sqrt{\text{Tr}(A - \bar{A})^2} = \text{proper distance}$$

$\rightarrow W_2(C)$

$$S = S_0 + S(U, A) = \int ds \text{Tr}(U \dot{U}) + \lambda(U, A)$$

EOM

$$\frac{d}{ds} (U^{-1} \dot{U} + U^{-1} A U - \bar{A}) + [\bar{A}, U^{-1} \dot{U} + U^{-1} A U] = 0$$

$$A = \Delta_f \frac{dx^\mu}{ds}$$

$$U = \mathbb{1} \rightarrow \text{eqn } \frac{dx^\mu}{ds} \text{ geod eqn}, \quad S = \int ds \sqrt{\text{Tr}(A - \bar{A})^2} = \text{proper distance}$$

$\rightarrow W_2(C)$ is indep of the path



→ Penrose Diagram → causal structure
→ Horizon ambiguous

$$A = \omega + e/2 \quad K = \frac{1}{4G} \quad A, A \in SL(2, \mathbb{R})$$

Conclusions/Results

- 1) Show $W_p(C)$ for $SL(2, \mathbb{R})$ correct obj to describe massive
- 2) Test $W_p(C)$ proper distance. $SL(3, \mathbb{R})$ → $U=1$ is inconsistent, no eqn for the path
→ C is

→ Penrose Diagram → causal structure
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$$A = \omega + e/2$$

$$k = \frac{1}{4G}$$

$$A, A \in SL(2, \mathbb{R})$$

Conclusions/Results

1) Show $W_h(C)$ for $SL(2, \mathbb{R})$ correct obj to describe massive

2) Test proper distance $SL(3, \mathbb{R})$ → $U=1$ is inconsistent, no eqn for the path
→ C is closed reproduces entropy of HS Bth
→ C is open relation to EE

left

→ Penrose Diagram → Causal structure
→ Horizon ambiguous

$$A = \omega + e/2$$

$$K = \frac{1}{4G}$$

$$A, A \in SL(2, \mathbb{R})$$

Conclusions/Results

1) Show $W_P(C)$ for $SL(2, \mathbb{R})$ correct obj to describe massive

2) Test $W_P(C)$ proper distance $SL(3, \mathbb{R})$ → $U=1$ is inconsistent, no eqn for the path
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⇒ How to define obs in gravity that are non-local