Title: 13/14 PSI - Condensed Matter Review - Lecture 3

Date: Jan 29, 2014 09:00 AM

URL: http://pirsa.org/14010077

Abstract:

$$H = -h\sum_i^N s_i - J\sum_i^N s_i s_{i+1}$$

$$\boxed{\begin{aligned} \lambda &= \beta h \\ K &= \beta J \end{aligned}}$$

$$\langle \hat\sigma_i^z \hat\sigma_j^z \rangle = \frac{1}{Z}\,\mathrm{Tr}\left[e^{-H}\hat\sigma_i^z \hat\sigma_j^z\right] =$$

$$\langle i | \hat\sigma_i^z | i \rangle = s_i$$

$$Z = \mathrm{Tr}\, e^{-\beta H} = \mathrm{Tr}\left(T^N\right)$$

$$\begin{cases} T_1 = e^{\lambda \hat\sigma^z} \\ T_2 = e^{K}\left[\mathbb{1} + e^{-2K}\hat\sigma^x\right] \end{cases}$$

$$T = \begin{pmatrix} e^{\lambda+K} & e^{-K} \\ e^{-K} & e^{-\lambda+K} \end{pmatrix} = \underbrace{\begin{pmatrix} e^{\lambda} & \\ & e^{-\lambda} \end{pmatrix}}_{\equiv\, T_1}\underbrace{\begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix}}_{\equiv\, T_2}$$

$$H = -h\sum_i^N s_i - J\sum_i^N s_i s_{i+1}$$

$$\boxed{\begin{aligned}\lambda &= \beta h \\ K &= \beta J\end{aligned}}$$

$$\langle \hat\sigma_i^z \hat\sigma_j^z \rangle = \frac{1}{Z}\,\mathrm{Tr}\left[e^{-H}\hat\sigma_i^z\hat\sigma_j^z\right] =$$

$$\langle i | \hat\sigma_i^z | i \rangle = s_i$$

$$Z = \mathrm{Tr}\, e^{-\beta H} = \mathrm{Tr}\left(T^N\right) = \lambda_1^N + \lambda_2^N$$

$$\begin{cases} T_1 = e^{\lambda\hat\sigma^z} \\ T_2 = e^{K}\left[\mathbb{1} + e^{-2K}\hat\sigma^x\right] \end{cases}$$

$$T = \begin{pmatrix} e^{\lambda+K} & e^{-K} \\ e^{-K} & e^{-\lambda+K} \end{pmatrix} = \underbrace{\begin{pmatrix} e^{\lambda} & \\ & e^{-\lambda} \end{pmatrix}}_{T_1}\underbrace{\begin{pmatrix} e^{K} & e^{-K} \\ e^{-K} & e^{K} \end{pmatrix}}_{T_2}$$

$$\downarrow$$

$$\lambda_{1,2} = e^{K}\left[\cosh\lambda \pm \sqrt{\sinh^2\lambda + e^{-4K}}\right]$$

$$-H\ \hat\sigma^z_i \hat\sigma^z_j] = \frac{1}{\lambda_1^N+\lambda_2^N}\mathrm{Tr}[\ \hat\sigma^z_j T_2^N\ \hat\sigma^z_i\ ] = \frac{1}{\lambda_1^N+\lambda_2^N}\mathrm{Tr}\left[T_2^{N-i}\hat\sigma^z_j T_2^{i-j}\hat\sigma^z_i T_2^j\right]$$

$$h=0 \Rightarrow \lambda=0$$

$$T_2^N = T_2^{N-i}T_2^{i-j}T_2^j$$

$$\begin{pmatrix}\lambda_1^{N-i} & \\ & \lambda_2^{N-i}\end{pmatrix}$$

$$T_2|\pm\rangle = \lambda_{1,2}|\pm\rangle$$

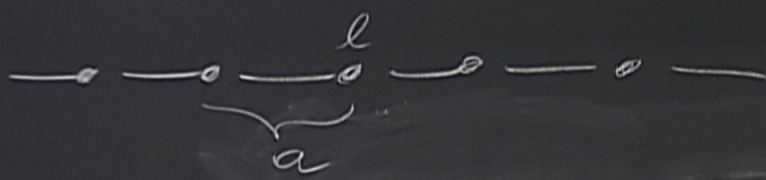
$$\hat\sigma^z = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$= \frac{\lambda_1^{N-i+j}\lambda_2^{i-j} + \lambda_2^{N-i+j}\lambda_1^{i-j}}{\lambda_1^N+\lambda_2^N} \xrightarrow[N\to\infty]{TDL}$$

$$\hat\sigma^x]$$

$$C_{ij} = \langle\hat\sigma^z_i\hat\sigma^z_j\rangle = (\tanh K)^{i-j}$$

12



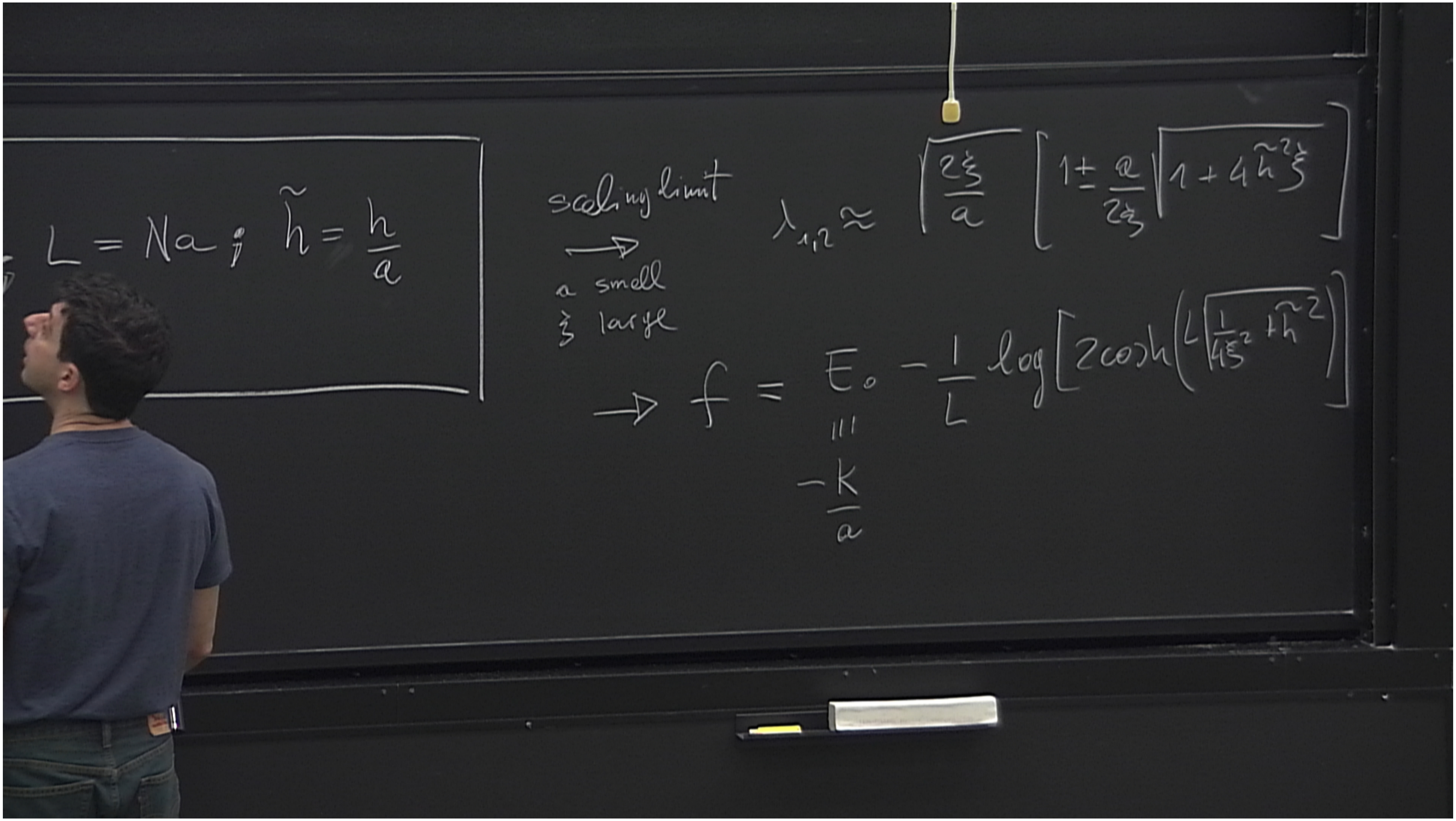
$$\tau \equiv a\ell$$

$$\xi^{-1} = \frac{1}{a} \log \coth K$$

$$\boxed{C(\tau) = e^{-|\tau|/\xi}}$$

$$\boxed{\frac{a}{\xi} \simeq 2 e^{-2K} \; ; \; L = Na \; ; \; \tilde{h} = \frac{h}{a}}$$

for large K (large β)
ξ does diverge


L = Na ; h̃ = h/a
scaling limit
a small
ξ large
λ_{1,2} ≈ √(2ξ/a) [1 ± (a/2ξ) √(1 + 4h̃²ξ²)]
f = E_0 − (1/L) log[2cosh(L √(1/4ξ² + h̃²))]
E_0 = −K/a

$$\boxed{L = Na \; ; \; \tilde{h} = \frac{h}{a}}$$

$\xrightarrow{\text{scaling limit}}$

a small

ξ large

$$\lambda_{1,2} \approx \sqrt{\frac{2\xi}{a}} \left[1 \pm \frac{a}{2\xi} \sqrt{1 + 4\tilde{h}^2 \xi^2} \right]$$

$$\rightarrow f = E_0 - \frac{1}{L} \log\left[2\cosh\left(L \sqrt{\frac{1}{4\xi^2} + \tilde{h}^2} \right) \right]$$

$$E_0 \equiv -\frac{K}{a}$$

$$\rightarrow C(\tau) = \frac{e^{-|\tau|/\xi} + e^{-(L-|\tau|)/\xi}}{1 + e^{-L/\xi}}$$

$$\begin{cases} f = \Phi_f\left(\frac{L}{\xi}, L\tilde{h}\right) \\ C(\tau) = \Phi_\sigma\left(\frac{L}{\xi}, \tau/\xi\right) \end{cases}$$

Universality: Φ_f, Φ_σ are universal functions

$$e^{-2K} \simeq \frac{a}{2\xi}$$

$$E_0 \simeq -\frac{K}{a}$$

$$\lambda = a\tilde{h}$$

Mapping <u>Classical - Quantum</u>

$$T \simeq \underbrace{e^{a\tilde{h}\hat{\sigma}^z}}_{T_1} \underbrace{e^{a\left(-E_0 + \frac{1}{2\xi}\hat{\sigma}^x\right)}}_{T_2}$$

$$e^{a\hat{O}_1} e^{a\hat{O}_2} = e^{a(\hat{O}_1 + \hat{O}_2)} + \mathcal{O}(a^2)$$

$$e^{-2K} \simeq \frac{a}{2\xi}$$

$$E_0 \simeq -K \sim a$$

$$\lambda = a h$$

Mapping Classical-Quantum

$$T \simeq \underbrace{e^{a\tilde{h}\hat{\sigma}^z}}_{T_1}\,\underbrace{e^{a\left(-E_0 + \frac{1}{2\xi}\hat{\sigma}^x\right)}}_{T_2} \approx e^{a\left[\tilde{h}\hat{\sigma}^z - E_0 + \frac{1}{2\xi}\hat{\sigma}^x\right]}$$

$$e^{aO_1}e^{aO_2} = e^{a(O_1+O_2)} + \mathcal{O}(a^2)$$

$$e^{-2K} \approx \frac{a}{2\xi}$$

$$E_0 \approx -\frac{K}{a}$$

$$\lambda = a\tilde{h}$$

Mapping Classical-Quantum

$$T \simeq \underbrace{e^{a\tilde{h}\hat{\sigma}^z}}_{T_1} \underbrace{e^{a\left(-E_0 + \frac{1}{2\xi}\hat{\sigma}^x\right)}}_{T_2} \approx e^{a\left[\tilde{h}\hat{\sigma}^z - E_0 + \frac{1}{2\xi}\hat{\sigma}^x\right]}$$

$$\xi^{-1} \equiv \Delta$$

$$H_Q = \tilde{h}\hat{\sigma}^z - E_0 + \frac{\Delta}{2}\hat{\sigma}^x$$

$$e^{-2K} \approx \frac{a}{2\xi}$$

$$E_0 \approx -K \sim a$$

$$\lambda = a\tilde{h}$$

Mapping Classical - Quantum

$$T \approx \underbrace{e^{a\tilde{h}\hat{\sigma}^z}}_{T_1} \underbrace{e^{a\left(-E_0 + \frac{1}{2\xi}\hat{\sigma}^x\right)}}_{T_2} \approx e^{-a\left[-\tilde{h}\hat{\sigma}^z + E_0 - \frac{1}{2\xi}\hat{\sigma}^x\right]}$$

$$\xi^{-1} \equiv \Delta$$

$$H_Q = -\tilde{h}\hat{\sigma}^z + E_0 - \frac{\Delta}{2}\hat{\sigma}^x$$

$$T_{s_i s_{i+1}}$$

$$a \sim \tau$$

$$T = e^{-\tau H_Q}$$

Mapping Classical - Quantum

$$e^{-2K} \approx \frac{a}{2\xi}$$

$$E_0 \approx -K \sim a$$

$$\lambda = a\tilde{h}$$

$$T \simeq \underbrace{e^{a\tilde{h}\hat{\sigma}^z}}_{T_1}\,\underbrace{e^{a\left(-E_0 + \frac{1}{2\xi}\hat{\sigma}^x\right)}}_{T_2} \approx e^{-a\left[-\tilde{h}\hat{\sigma}^z + E_0 - \frac{1}{2\xi}\hat{\sigma}^x\right]}$$

$$\xi^{-1} \equiv \Delta$$

$$T_{s_i s_{i+1}}$$

$$H_Q = -\tilde{h}\hat{\sigma}^z + E_0 - \frac{\Delta}{2}\hat{\sigma}^x$$

$$a \sim \tau$$

$$T = e^{-\tau H_Q}$$

$$Z = \mathrm{Tr}\, T^N \approx \mathrm{Tr}\, e^{-H_Q L} = \mathrm{Tr}\, e^{-\frac{H_Q}{T}}$$

$$\boxed{\frac{1}{T} = L}$$

$$f = -T\log Z = E_0 - T\log\left[2\cosh\sqrt{\left(\frac{\Delta}{2}\right)^2+\tilde{h}^2}\,/T\right]$$

$$E_{1,2} = E_0 \pm \sqrt{\tilde{h}^2 + \left(\frac{\Delta}{2}\right)^2}$$

$$f = -T \log Z = E_0 - T \log \left[2 \cosh \left(\sqrt{\left(\frac{\Delta}{2}\right)^2 + \tilde{h}^2} \,/\, T \right) \right]$$

$$E_{1,2} = E_0 \pm \sqrt{\tilde{h}^2 + \left(\frac{\Delta}{2}\right)^2}$$

1D classical Stat-mech $\longleftrightarrow$ 0-D Quantum Im. time evol.

$/T]$

$$H = -Jg \sum_i^M \hat{\sigma}_i^x - J \sum_i^M \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z \qquad L_{space} \equiv Ma$$

a

$$\mathcal{U}_a = e^{-aH} = T_1 T_2 + \mathcal{O}(a^2)$$

$$\begin{cases} T_1 = \exp\left[Jga \sum_i \hat{\sigma}_i^x \right] \\ T_2 = \exp\left[Ja \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z \right] \end{cases} \qquad Z = e^{-\frac{H}{T}}$$

$$\frac{1}{T} \equiv L_\tau = Na$$

$$H = -Jg\sum_i^M \hat{\sigma}_i^x - J\sum_i^M \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z \qquad L_{space} \equiv Ma$$

$$a \qquad \mathcal{U}_a = e^{-aH} = T_1 T_2 + \mathcal{O}(a^2)$$

$$\begin{cases} T_1 = \exp\left[Jga\sum_i \hat{\sigma}_i^x\right] \\ T_2 - \exp\left[Ja\sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z\right] \end{cases}$$

$$\mathcal{Z} = e^{-\frac{H}{T}} \qquad \frac{1}{T} \equiv L_\tau$$

$$= \mathrm{Tr}\left(e^{-aH}\right)^N \qquad = Na$$

$$Z = \mathrm{Tr}\,(T_1 T_2)^N$$

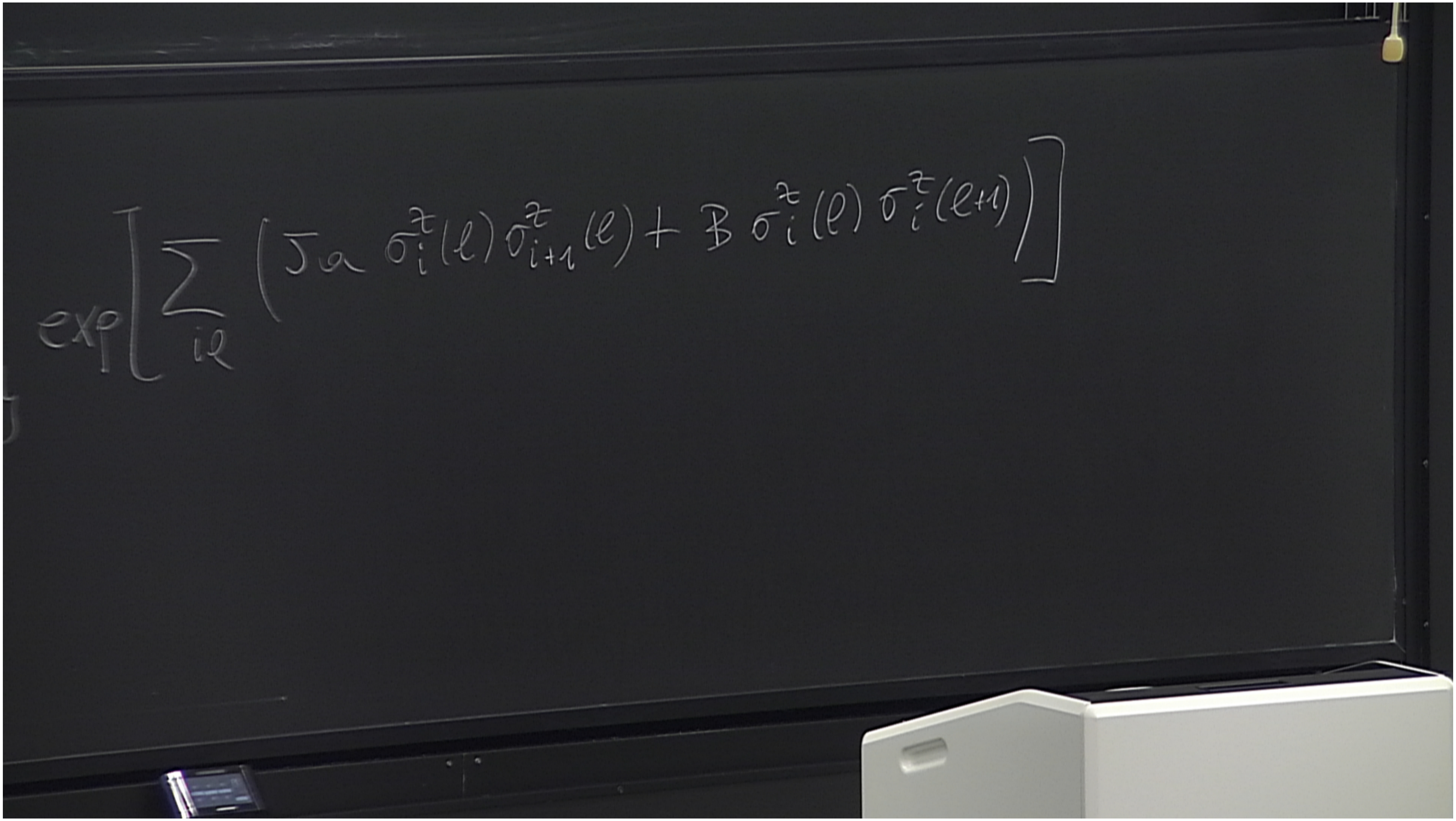
$$T_2 |\{\sigma_i^z\}\rangle = e^{-Ja\sum_i \sigma_i^z \sigma_{i+1}^z} |\{\sigma_i^z\}\rangle$$

$$Z = \sum_{\{\sigma_i^z(l)\}} \prod_{l=1}^{N} \langle\{\sigma_i^z(l)\}|T_1 T_2|\{\sigma_i^z(l+1)\}\rangle = \sum_{\{\sigma_i^z(l)\}} \exp\Big[\sum_{il} Ja\,\sigma_i^z$$

$$\langle\{\sigma_i^z(l)\}|T_1|\{\sigma_i^z(l+1)\}\rangle = A e^{B\,\sigma_i^z(l)\sigma_i^z(l+1)}$$

$$A = \frac{1}{2}\cosh(2Jga)$$

$$e^{-2B} = \tanh(Jga)$$


$\exp\left[\sum_{i\ell}\left(J a\, \sigma_i^z(\ell)\, \sigma_{i+1}^z(\ell) + B\, \sigma_i^z(\ell)\, \sigma_i^z(\ell+1)\right)\right]$

$$\exp\Big[\underbrace{\sum_{i\ell}\Big(J a\,\sigma_i^z(\ell)\,\sigma_{i+1}^z(\ell)+B\,\sigma_i^z(\ell)\,\sigma_i^z(\ell+1)\Big)}_{-H_{2D}}\Big]$$