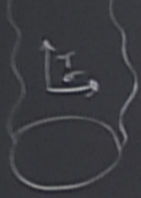


Title: 13/14 PSI - String Theory Review - Lecture 2

Date: Jan 28, 2014 10:15 AM

URL: <http://pirsa.org/14010071>

Abstract:



$$\tau = i t$$

$$\sigma \equiv \sigma + 2\pi L, \tau$$

$$s = \tau - i\sigma$$

$$\bar{s} = \tau + i\sigma$$

$$\partial_s \partial_{\bar{s}} X =$$

$$X(\sigma, \tau) = \mathcal{X} - \frac{2i}{L} p \tau$$

$$\tau = \tau + i\epsilon$$

$$\sigma \equiv \sigma + 2\pi L, \tau$$

$$s = \tau - i\sigma$$

$$\bar{s} = \tau + i\sigma$$

$$\partial_{\bar{s}} \partial_s X = 0$$

$$X(\sigma, \tau) = \mathcal{X} - \frac{2i}{L} p\tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i \frac{n}{L} (\sigma + i\tau)} \bar{a}_n$$

$$\sigma \equiv \sigma + 2\pi L, \tau$$

$$s = \tau - i\sigma$$

$$\bar{s} = \tau + i\sigma$$

$$\partial_s \partial_{\bar{s}} X$$

$$X(\sigma, \tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i \frac{n}{L} (\sigma + i\tau)} - \frac{i}{n} \bar{a}_n$$

$$[a_n, a_{-n}] = n$$

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$[x, p] = i$$

$$[X(\sigma, 0), i \partial_{\tau} X(\sigma', 0)] = \frac{2i}{L} \sum_n e^{i \frac{n}{L} (\sigma - \sigma')} = 4\pi i \delta(\sigma - \sigma')$$

$$\partial_s X = 0$$

$$-\frac{c}{m} \bar{a}_n e^{-i \frac{n}{L} (\sigma - i\tau)}$$

$$\mathcal{J}_a = \partial_a X$$

$$\nabla_a \mathcal{J}^a = \nabla' X = 0$$

$$i \partial_s X = \frac{p}{L} + \sum_{n \neq 0} \frac{a_n}{L} e^{-\frac{n}{L} s}$$

$$\partial_s \mathcal{J}_s +$$

$$\sigma \equiv \sigma + 2\pi L, \tau \quad S = \tau - i\sigma \quad \bar{S} = \tau + i\sigma \quad \partial_S \partial_{\bar{S}} X = 0$$

$$X(\sigma, \tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i \frac{n}{L} (\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-i \frac{n}{L} (\sigma + i\tau)}$$

$$[a_n, a_{-n}] = n$$

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$[x, p] = i$$

$$[X(\sigma, 0), i\partial_\tau X(\sigma', 0)] = \frac{2i}{L} \sum_n e^{i \frac{n}{L} (\sigma - \sigma')} = 4\pi i \delta(\sigma - \sigma')$$

$$a_n |p\rangle = 0 \quad n > 0$$

$$\hat{p}|p\rangle = p|p\rangle$$

$$x|p\rangle = i\partial_p |p\rangle$$

$$\equiv \sigma + 2\pi L, \tau \quad s = \tau - i\sigma \quad \bar{s} = \tau + i\sigma \quad \partial_s \partial_{\bar{s}} X = 0$$

$$X(\sigma, \tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i \frac{n}{L} (\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-i \frac{n}{L} (\sigma + i\tau)}$$

$$[a_n, a_{-n}] = n$$

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$[x, p] = i$$

$$[X(\sigma, 0), i\partial_\tau X(\sigma', 0)] = \frac{2i}{L} \sum_n e^{i \frac{n}{L} (\sigma - \sigma')} = 4\pi i \delta(\sigma - \sigma')$$

$$a_n |p\rangle = 0 \quad n > 0$$

$$\hat{p}|p\rangle = p|p\rangle$$

$$x|p\rangle = i\partial_p |p\rangle$$

$$|p\rangle,$$

$$a_1 a_{-1}^2 |p\rangle = a_{-1} a_1 a_{-1} |p\rangle + a_{-1} |p\rangle \\ = 2a_{-1} |p\rangle + a_{-1}^2 |p\rangle$$

$$= T + i\sigma \quad \partial_s \partial_{\bar{s}} X = 0 \\ e^{i\frac{m}{L}(\sigma + i\tau)} - \frac{c}{m} \bar{a}_n e^{-i\frac{m}{L}(\sigma - i\tau)}$$

$$[x, p] = i$$

$$= 4\pi i \delta(\sigma - \sigma')$$

$$|p\rangle = i\partial_p |p\rangle$$

$$|p\rangle, a_{-1}|p\rangle, a_{-2}|p\rangle, a_{-1}^2|p\rangle, \dots$$

$$T_0 = \partial_a X \quad \nabla_a J^a = \nabla' X = 0$$

$$i\partial_s X = \frac{p}{L} + \sum_{n \neq 0} \frac{a_n}{L} e^{-\frac{n}{L}s}$$

$$i\partial_{\bar{s}} X = \frac{p}{L} - \sum_{n \neq 0} \frac{\bar{a}_n}{L} e^{-\frac{n}{L}\bar{s}}$$



$$\sigma \equiv \sigma + 2\pi L, \tau$$

$$s = \tau - i\sigma$$

$$\bar{s} = \tau + i\sigma$$

$$X(\sigma, \tau) = x - \frac{2i}{L} p\tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i\frac{n}{L}(\sigma + i\tau)}$$

$$[a_n, a_{-n}] = n$$

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$[x, p] = i$$

$$[X(\sigma, 0), i\partial_\tau X(\sigma', 0)] = \frac{2i}{L} \sum_n e^{i\frac{n}{L}(\sigma - \sigma')} = 4\pi i \delta(\sigma - \sigma')$$

$$a_n |p\rangle = 0 \quad n > 0$$

$$\hat{p}|p\rangle = p|p\rangle$$

$$x|p\rangle = i\partial_p|p\rangle$$

$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2} \sum_n n e^{\frac{n}{L}(s'-s)}$$

$$\tau = c t$$

$$a_{-n}|0\rangle e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2} \sum_n n e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2}$$

$$\begin{aligned} a_1 a_{-1}^2 |P\rangle &= a_{-1} a_1 a_{-1} |P\rangle + a_{-1} |P\rangle \\ &= 2a_{-1} |P\rangle + \cancel{a_{-1}^2 |P\rangle} \end{aligned}$$

$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{R^2} = -\ln |s - s'|$$

$$\partial_{\bar{s}} \mathcal{J}_s + \partial_s \mathcal{J}_{\bar{s}} = 0 \quad \partial_{\bar{s}} [S(s) \mathcal{J}_s] = 0$$

$$\partial_{\bar{s}} \mathcal{J}_s = 0$$

$$\partial_s \mathcal{J}_{\bar{s}} = 0$$

$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^2} = -\ln|s-s'|$$

$$\partial_{\bar{s}} \mathcal{J}_s + \partial_s \mathcal{J}_{\bar{s}} = 0 \quad \partial_{\bar{s}} [S(s) \mathcal{J}_s] = 0$$

$$\partial_{\bar{s}} \mathcal{J}_s = 0$$

$$\partial_s \mathcal{J}_{\bar{s}} = 0$$

$h_{ab}, X^a$

$$\partial_{\bar{z}} T_{zz} = 0$$

$$T^{ab} = \frac{\delta S}{\delta h_{ab}} = 0$$

Diff  $\rightarrow$

$$\nabla_a T^{ab} = 0$$

$$\partial_{\bar{z}} T_{zz} + \partial_z \cancel{T_{z\bar{z}}} = 0$$

WEYL  $\rightarrow$

$$h_{ab} T^{ab} = 0$$

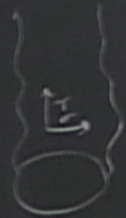
$$\equiv T_{z\bar{z}} = 0$$

$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{\frac{\pi n}{L} (s'-s)} = -\frac{1}{L^2}$$

$$T = \frac{1}{2} \lim_{s' \rightarrow s}$$

$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{\frac{\pi}{L}(s'-s)} = -\frac{1}{L^2}$$

$$T_{ss} = -\frac{1}{2} \lim_{s' \rightarrow s} \left[ \partial_s X(s) \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$



$$\tau = ct$$



$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{\frac{i}{L} n (s'-s)} = -\frac{1}{L^2}$$

$$T_{ss} = -\frac{1}{2} \lim_{s' \rightarrow s} \left[ \partial_s X(s) \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$



$$\tau = ct$$

$\tau = \tau E$

$$\langle X(s_1, \bar{s}_1) X(s_2, \bar{s}_2) \dots \rangle = \sum \pi G(s_1, \bar{s}_1, \bar{s}_2)$$

$$s \rightarrow s'(s)$$

$$X(s) \equiv X'(s'(s))$$

$$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$$

$$\dots \rightarrow = \sum \pi G(s_i, \bar{s}_i; s_j, \bar{s}_j)$$

$$X(s) \equiv X'(s'(s))$$

$$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$$

$$T_{ss} = \frac{\partial^2 s'}{\partial s^2} T_{s's'}$$

$$\dots \rightarrow = \sum \pi G(s_i, \bar{s}_i; s_j, \bar{s}_j)$$

$$T_{ss} = \left(\frac{\partial s'}{\partial s}\right)^2 T_{s's'} +$$

$$X(s) \equiv X'(s'(s))$$

$$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$$

$$\dots \rightarrow = \sum \pi G(s_i, \bar{s}_i; s_j, \bar{s}_j)$$

$$X(s) \equiv X'(s'(s))$$

$$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$$

$$T_{ss} = \left(\frac{\partial s'}{\partial s}\right)^2 \left[ T_{s's'} + \lim_{s' \rightarrow s} \frac{\partial s'}{\partial s} \frac{1}{s'} - \frac{1}{s} \right]$$

$$\dots \rightarrow = \sum \pi G(s_i, \bar{s}_i; s_j, \bar{s}_j)$$

$$X(s) \equiv X'(s'(s))$$

$$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$$

$$T_{ss} = \left(\frac{\partial s'}{\partial s}\right)^2 \left[ T_{s's'} + \lim_{s' \rightarrow s} \left[ \frac{\partial s'}{\partial s} \frac{\partial s''}{\partial s'} \frac{1}{(s-s')} - \frac{1}{(s-s')^2} \right] \right]$$

$$\dots > = \sum \pi G(s_i, \bar{s}_i; s_j, \bar{s}_j)$$

$$X(s) \equiv X'(s'(s))$$

$$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$$

$$T_{ss} = \frac{\partial^2}{\partial s^2} \left[ T_{s's'} + \lim_{s \rightarrow s'} \left[ \frac{\partial \bar{s}}{\partial s} \frac{\partial s'}{\partial s} \frac{1}{(s-s')} - \frac{1}{0-s'} \right] \right]$$

$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{i\omega_n s - i\omega_n s'} = -\frac{1}{L^2} \frac{(e^{i\omega_n s} - e^{i\omega_n s'})^2}{2i\omega_n}$$

$$T_{SS} = -\frac{1}{2} \lim_{s \rightarrow s'} \left[ \partial_s X(s) \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right] \quad L=1$$

$$\langle 0 | T_{SS} | 0 \rangle = \lim_{s \rightarrow s'} \left[ -\frac{1}{L^2} \frac{e^{i\omega_n(s+s')}}{(e^{i\omega_n s} - e^{i\omega_n s'})^2} + \frac{1}{(s-s')^2} \right] = -\frac{1}{24L^2}$$

$\tau = Lt$

$$\langle X(s_1, \bar{s}_1) X(s_2, \bar{s}_2) \dots \rangle = \int \prod \delta(s_i, \bar{s}_i; s_j, \bar{s}_j)$$

$$s_j \rightarrow s'_j(s)$$

$$X(s) = \dots$$

$$\partial_s X$$

$$T_{SS} = \left(\frac{\partial \bar{s}}{\partial s}\right)^2 T_{\bar{s}\bar{s}} + \lim_{s \rightarrow s'} \left[ \frac{\partial \bar{s}}{\partial s} \right] \dots$$

$$= \left(\frac{\partial \bar{s}}{\partial s}\right)^2 T_{\bar{s}\bar{s}} + \frac{1}{12} \left\{ \bar{s}, s \right\}$$



$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^1} = -\ln |s-s'|^2$$

FLAT

$$\text{SPACE} \lim_{s \rightarrow s'} \frac{1}{2} \left[ \langle \partial_s X(s) \partial_{s'} X(s') \rangle + \langle \partial_{\bar{s}} X(s) \partial_{\bar{s}'} X(s') \rangle \right] = T_{ss}$$

Shw

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n=1}^{\infty} a_n a_n! e^{-(n+1)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

[

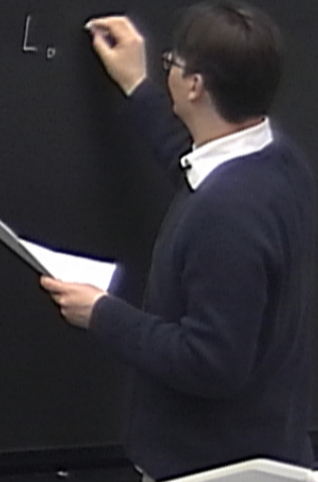


$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^1} = -\ln |s - s'|^2 \quad \text{FLAT} \quad \text{SPACE} \lim_{s \rightarrow s'} \left[ \frac{1}{2} \int_{\mathbb{R}^1} X(s) \partial_s X(s') + \langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle + \frac{1}{(s - s')^2} \right] = T_{ss}$$

Soln

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n=1}^{\infty} :a_n a_n: e^{-(n+1)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

[



$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^1} = -\ln |s - s'|^2 \quad \text{FLAT} \quad \text{SPACE} \lim_{s \rightarrow s'} \left[ \frac{1}{2} \langle \partial_s X(s) \partial_{s'} X(s') \rangle + \frac{1}{(s - s')^2} \langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle \right] = T_{ss}$$

Soln

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n,m} :a_n a_m : e^{-(m+n)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$L_0 |p\rangle = \frac{p^2}{2}$$

$$L_0 - \frac{1}{24} = E$$

$$[L_n, a_m] = m a_{n+m} \quad [L_n, \partial_s X(s)] = \partial_n (e^{-ns} \partial_s X)$$

$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^t} = -\ln |s-s'|^2 \quad \text{FLAT} \quad \text{SPACE} \lim_{s \rightarrow s'} \left[ \frac{1}{2} \langle \partial_s X(s) \partial_{s'} X(s') \rangle + \frac{1}{(s-s')^2} \right] = T_{ss}$$

Soln

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n,m} :a_n a_m: e^{-(m+n)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$L_0 |p\rangle = \frac{p^2}{2}$$

$$L_0 - \frac{1}{24} = E$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X] = \partial_n (e^{-ns} \partial_s X)$$

$$[L_n, X(s, \bar{s})] = -X$$

$$[L_n, \partial_s X(s) \partial_{s'} X(s')] = \partial_s (e^{-ns} \partial_{s'} X(s'))$$

$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^t} = -\ln |s-s'|^2 \quad \text{FLAT} \quad \text{SPACE} \lim_{s \rightarrow s'} \left[ \frac{1}{2} \langle \partial_s X(s) \partial_{s'} X(s') \rangle + \frac{1}{(s-s')^2} \right] = T_{ss}$$

Soln

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n,m} :a_n a_m: e^{-(m+n)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$[L_n, a_m] = m a_{n+m}$$

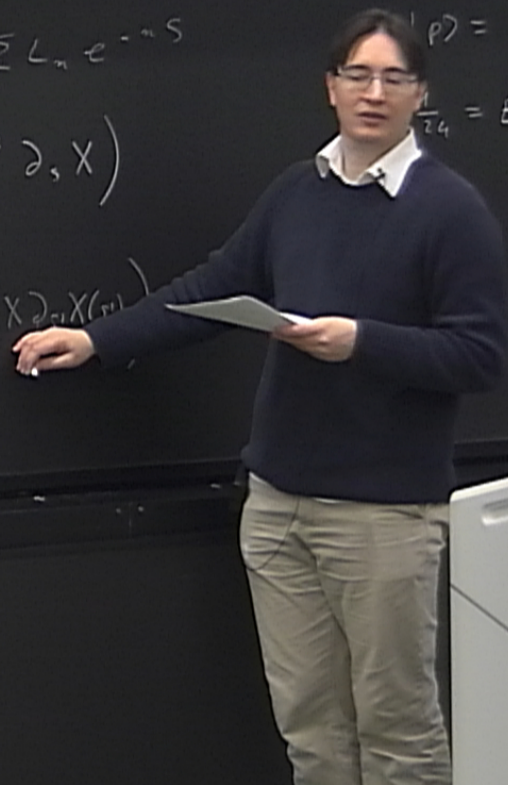
$$[L_n, \partial_s X(s)] = \partial_n (e^{ns} \partial_s X)$$

$$[L_n, X(s) \bar{X}(s')] = e^{ns} \partial_s X$$

$$[L_n, \partial_s X \partial_{s'} X(s')] = i \partial_s (e^{ns} \partial_s X \partial_{s'} X(s')) + i \partial_{s'} (e^{ns'} \partial_s X \partial_{s'} X(s'))$$

$$[p] = \frac{p^2}{2}$$

$$\frac{1}{24} = E$$



$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^1} = -\ln |s-s'|^2 \quad \text{FLAT} \quad \text{SPACE} \lim_{s \rightarrow s'} \left[ \frac{1}{2} \langle \partial_s X(s) \partial_{s'} X(s') \rangle + \frac{1}{(s-s')^2} \langle \partial_s X(s) \partial_{s'} X(s') | 0 \rangle \right] = T_{ss}$$

Soln

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n,m} :a_n a_m: e^{-(m+n)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_n (e^{ns} \partial_s X)$$

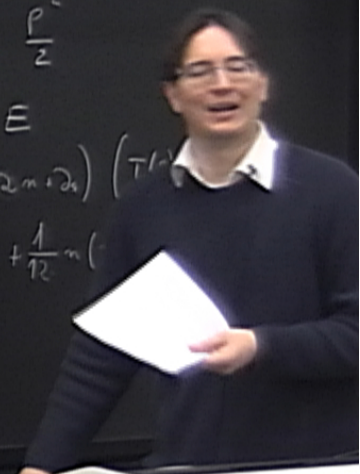
$$[L_n, X(s) \bar{X}(s')] = e^{ns} \partial_s X$$

$$[L_n, \partial_s X(s) \partial_{s'} X(s')] = i \partial_s (e^{ns} \partial_s X(s) \partial_{s'} X(s')) + i \partial_{s'} (e^{ns'} \partial_s X(s) \partial_{s'} X(s'))$$

$$L_0 |p\rangle = \frac{p^2}{2}$$

$$L_0 - \frac{1}{24} = E$$

$$[L_n, T(s)] = e^{-s} (2n+24) (T(s) + \frac{1}{12} n)$$



$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^2} = -\ln|s-s'|^2 \quad \text{FLAT} \quad \text{SPACE} \lim_{s' \rightarrow s} \left[ \frac{1}{2} \langle \partial_s X(s) \partial_{s'} X(s') \rangle + \frac{1}{(s-s')^2} \langle \partial_s X(s) \partial_{s'} X(s') | 0 \rangle \right] = T_{ss}$$

show

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n \neq 0} :a_{-n} a_n: e^{-in s} = -\frac{1}{24} + \sum L_n e^{-in s}$$

$$L_0 |p\rangle = \frac{p^2}{2}$$

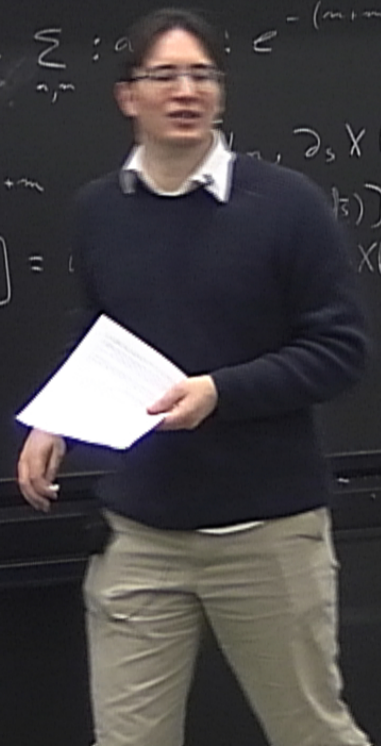
$$L_0 - \frac{1}{24} = E$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_n (e^{-ns} \partial_s X)$$

$$[L_n, T(s)] = e^{-ns} (2n \partial_s) \left( T(s) + \frac{1}{24} \right) + \frac{1}{12} n(n^2-1) e^{-ns}$$

$$[L_n, \partial_s X(s) \partial_{s'} X(s')] = \partial_n \left( e^{-ns} \partial_s X(s) \partial_{s'} X(s') \right) + i \partial_{s'} \left( e^{-ns'} \partial_s X(s) \partial_{s'} X(s') \right)$$



$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^2} = -\ln|s-s'|^2 \quad \text{FLAT} \quad \text{SPACE} \lim_{s \rightarrow s'} \left[ \frac{1}{2} [X(s) \partial_s X(s') + \langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle] + \frac{1}{(s-s')^2} \right] = T_{ss}$$

Sho:

$$T(s) = -\frac{1}{24} + \frac{1}{2} \sum_{n,m} :a_n a_m: e^{-(m+n)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$L_0 |p\rangle = \frac{p^2}{2}$$

$$L_0 - \frac{1}{24} = E$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_n (e^{-ns} \partial_s X)$$

$$[L_n, T(s)] = e^{-ns} (2n \partial_s) \left( T(s) + \frac{1}{24} \right) + \frac{1}{12} n(n-1) e^{-ns}$$

$$[L_n, X(s) \bar{X}(s')] = e^{-ns} \partial_s X$$

$$[L_n, \partial_s X(s) \partial_{s'} X(s')] = i \partial_s (e^{-ns} \partial_s X(s) \partial_{s'} X(s')) + i \partial_{s'} (e^{-ns'} \partial_s X \partial_{s'} X(s'))$$



$$[L_m, L_m] = (m-m) L_m = 0$$

$$\langle X(s, \bar{s}) X(s', \bar{s}), \mathbb{R} \rangle$$

$-s'^2$

FLAT

$$\text{SPACE} \lim_{s \rightarrow s'} \left[ \frac{1}{2} \langle \partial_s X(s) \partial_{s'} X(s') | 0 \rangle + \frac{1}{(s-s')^2} \right] = T_{ss}$$

$$s_s = -\frac{1}{2} \partial_s X \partial_s X$$

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{1}{12} m(m^2-1) \delta_{m+n,0}$$

$$\langle X(s, \bar{s})$$

$$|s-\bar{s}|^2$$

FLAT

$$\text{SPACE} \lim_{s \rightarrow \bar{s}} \left[ \frac{1}{2} \langle \partial_s X(s) \partial_{\bar{s}} X(\bar{s}) \rangle + \frac{1}{(s-\bar{s})^2} \right] = T_{s\bar{s}}$$

$$s_s = -\frac{1}{2} \partial_s X \partial_s X$$

$$E = L_m + 1$$

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{1}{12} m(m^2-1) \delta_{m+n,0}$$

$$\langle X(s, \bar{s}) X(s', \bar{s}') \rangle_{\mathbb{R}^1} = -\ln |s-s'|^2$$

FLAT

$$\text{SPACE} \lim_{s \rightarrow s'} \frac{1}{2} \partial_s X \partial_{s'} X + \frac{1}{(s-s')^2} \dots$$

$$\dots] = T_{ss}$$