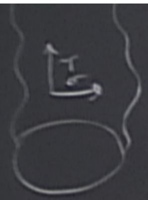
Title: 13/14 PSI - String Theory Review - Lecture 2

Date: Jan 28, 2014 10:15 AM

URL: http://pirsa.org/14010071

Abstract:

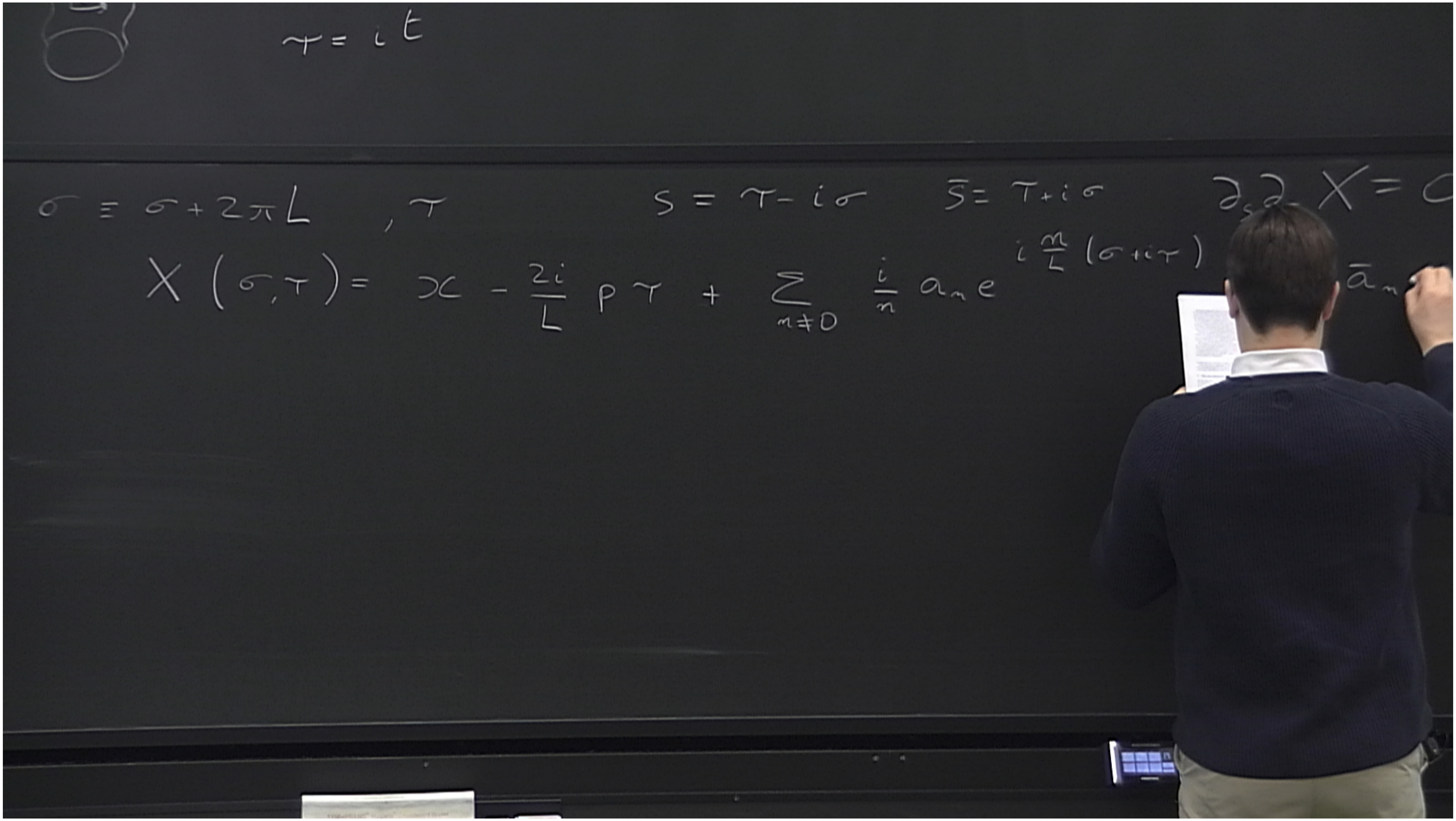


$$\tau = i t$$

$$\sigma \equiv \sigma + 2\pi L \quad , \tau$$

$$s = \tau - i\sigma \qquad \bar{s} = \tau + i\sigma \qquad \partial_s \partial_{\bar{s}} X =$$

$$X(\sigma, \tau) = x - \frac{2i}{L} p \tau$$



$\tau = it$
$\sigma \equiv \sigma + 2\pi L \quad , \tau$
$s = \tau - i\sigma \quad \bar{s} = \tau + i\sigma \quad \partial_s \partial_{\bar{s}} X = 0$
$X(\sigma,\tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i \frac{n}{L}(\sigma + i\tau)}$
$\bar{a}_n$

$$\sigma \equiv \sigma + 2\pi L \,, \quad \tau \qquad s = \tau - i\sigma \qquad \bar{s} = \tau + i\sigma \qquad \partial_s \partial_{\bar{s}} X$$

$$X(\sigma,\tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i \frac{n}{L}(\sigma + i\tau)} - \frac{i}{n} \bar{a}$$

$$[a_n, a_{-n}] = n \qquad [\bar{a}_n, \bar{a}_{-n}] = n \qquad [x, p] = i$$

$$[X(\sigma,0), i\partial_\tau X(\sigma',0)] = \frac{2i}{L} \sum_n e^{i\frac{n}{L}(\sigma - \sigma')} = 4\pi i \, \delta(\sigma - \sigma')$$

$$\partial_{\bar z} X = 0$$

$$J_a = \partial_a X \qquad \nabla_a J^a = \nabla^2 X = 0$$

$$e^{-i\frac{n}{L}(\sigma - i\tau)}$$

$$i\partial_s X = \frac{p}{L} + \sum_{n\neq 0} \frac{a_n}{L} e^{-\frac{n}{L} s}$$

$$- \frac{i}{n}\, \bar a_n\, e$$

$$\partial_{\bar z} J_s +$$

$\sigma \equiv \sigma + 2\pi L \quad , \tau \qquad s = \tau - i\sigma \qquad \bar{s} = \tau + i\sigma \qquad \partial_s \partial_{\bar{s}} X = 0$

$$X(\sigma,\tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i\frac{n}{L}(\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-i}$$

$[a_n, a_{-n}] = n \qquad [\bar{a}_n, \bar{a}_{-n}] = n \qquad [x, p] = i$

$$[X(\sigma,0), i\partial_\tau X(\sigma',0)] = \frac{2i}{L} \sum_n e^{i\frac{n}{L}(\sigma - \sigma')} = 4\pi i\, \delta(\sigma - \sigma')$$

$a_n |p\rangle = 0 \quad n > 0 \qquad \hat{p}|p\rangle = p|p\rangle \qquad x|p\rangle = i\partial_p |p\rangle$

$\equiv \sigma + 2\pi L \quad , \tau$ $\qquad s = \tau - i\sigma \qquad \bar{s} = \tau + i\sigma \qquad \partial_s \partial_{\bar{s}} X = 0$

$$X(\sigma,\tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \frac{i}{n} a_n e^{i \frac{n}{L}(\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-i \frac{n}{L}}$$

$[a_n, a_{-n}] = n \qquad [\bar{a}_n, \bar{a}_{-n}] = n \qquad [x, p] = i$

$$[X(\sigma,0), i\partial_\tau X(\sigma',0)] = \frac{2i}{L} \sum_n e^{i \frac{n}{L}(\sigma - \sigma')} = 4\pi i \, \delta(\sigma - \sigma')$$

$a_n |p\rangle = 0 \quad n > 0 \qquad \hat{p}|p\rangle = p|p\rangle \qquad x|p\rangle = i\partial_p |p\rangle \qquad |p\rangle ,$

$$a_1 a_{-1}^2 |p\rangle = a_{-1} a_1 a_{-1} |p\rangle + a_{-1}|p\rangle$$
$$= 2a_{-1}|p\rangle + \cancel{a_{-1}^2 a_1 |p\rangle}$$

$$J_a = \partial_a X \qquad \nabla_a J^a = \nabla^2 X = 0$$

$$\bar{s} = \tau + i\sigma \qquad \partial_s \partial_{\bar s} X = 0$$

$$i\partial_s X = \frac{p}{L} + \sum_{n\neq 0} \frac{a_n}{L} e^{-\frac{n}{L}s}$$

$$i\partial_{\bar s} X = \frac{p}{L} - \sum_{n\neq 0} \frac{\bar a_n}{L} e^{-\frac{n}{L}\bar s}$$

$$e^{i\frac{n}{L}(\sigma + i\tau)} \qquad - \frac{i}{n}\bar a_n e^{-i\frac{n}{L}(\sigma - i\tau)}$$

$$[x, p] = i$$

$$= 4\pi i\, \delta(\sigma - \sigma')$$

$$|p\rangle = i\partial_p |p\rangle$$

$$|p\rangle,\ a_{-1}|p\rangle,\ a_{-2}|p\rangle,\ a_{-1}^2|p\rangle \ldots\ldots$$

$$\sigma \equiv \sigma + 2\pi L \quad , \tau \qquad s = \tau - i\sigma \qquad \bar{s} = \tau + i\sigma \qquad \partial$$

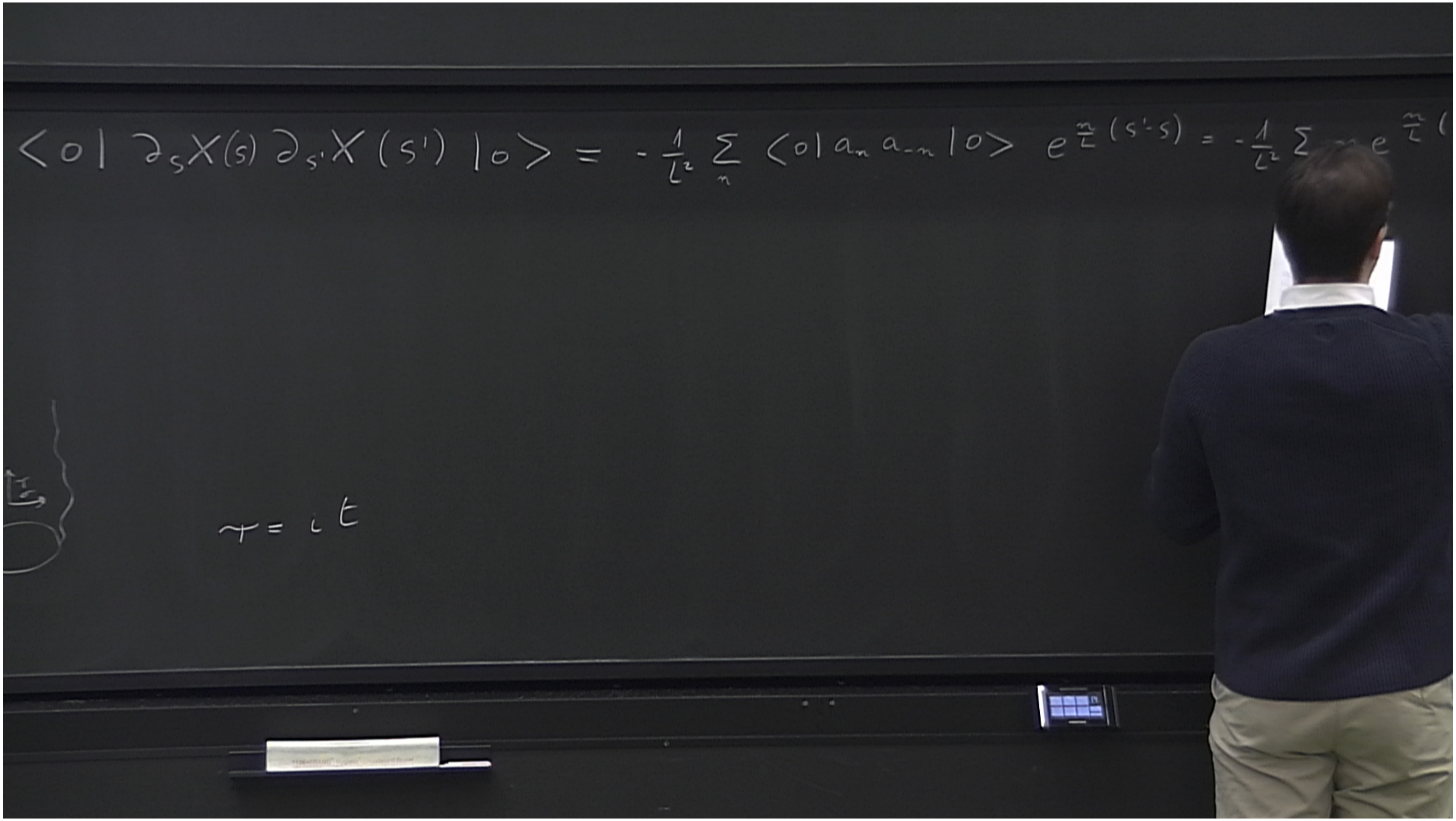
$$X(\sigma,\tau) = x - \frac{2i}{L} p\tau + \sum_{n\neq 0} \frac{i}{n} a_n e^{i\frac{n}{L}(\sigma + i\tau)}$$

$$[x,p] = i$$

$$[a_n, a_{-n}] = n \qquad [\bar{a}_n, \bar{a}_{-n}] = n$$

$$[X(\sigma,0), i\partial_\tau X(\sigma',0)] = \frac{2i}{L} \sum_n e^{i\frac{n}{L}(\sigma-\sigma')} = 4\pi i\, \delta(\sigma-\sigma')$$

$$a_n|p\rangle = 0 \quad n>0 \qquad \hat{p}|p\rangle = p|p\rangle \qquad x|p\rangle = i\partial_p|p\rangle$$


$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2} \sum n e^{\frac{n}{L}(}$
$\tau = i t$

$$a_{-n}|0\rangle \; e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2}\sum_n n\, e^{\frac{n}{L}(s'-s)} = -\frac{1}{L^2}\frac{e^{\frac{1}{L}(s+s')}}{\left(e^{\frac{s}{L}}-e^{\frac{s'}{L}}\right)^2}$$

$$\begin{aligned} a_1\, a_{-1}^2|p\rangle &= a_{-1}a_1a_{-1}|p\rangle + a_{-1}|p\rangle \\ &= 2a_{-1}|p\rangle + \cancel{a_{-1}^2 a_1|p\rangle} \end{aligned}$$

$$\langle X(\zeta,\bar\zeta)\, X(\zeta',\bar\zeta') \rangle_{\mathbb{R}^2} = -\ln|\zeta-\zeta'|$$

$$\partial_{\bar\zeta} J_\zeta + \partial_\zeta J_{\bar\zeta} = 0 \qquad \partial_{\bar\zeta}[f(\zeta)\, J_\zeta] = 0$$

$$\partial_{\bar\zeta} J_\zeta = 0$$

$$\partial_\zeta J_{\bar\zeta} = 0$$

$$\langle X(\zeta,\bar\zeta)\, X(\zeta',\bar\zeta') \rangle_{\mathbb{R}^2} = -\ln|\zeta-\zeta'|$$

$$\partial_{\bar\zeta} J_\zeta + \partial_\zeta J_{\bar\zeta} = 0 \qquad \partial_{\bar\zeta}[f(\zeta)\, J_\zeta] = 0$$

$$\partial_{\bar\zeta} J_\zeta = 0$$

$$\partial_\zeta J_{\bar\zeta} = 0$$

h_{ab} , X^μ

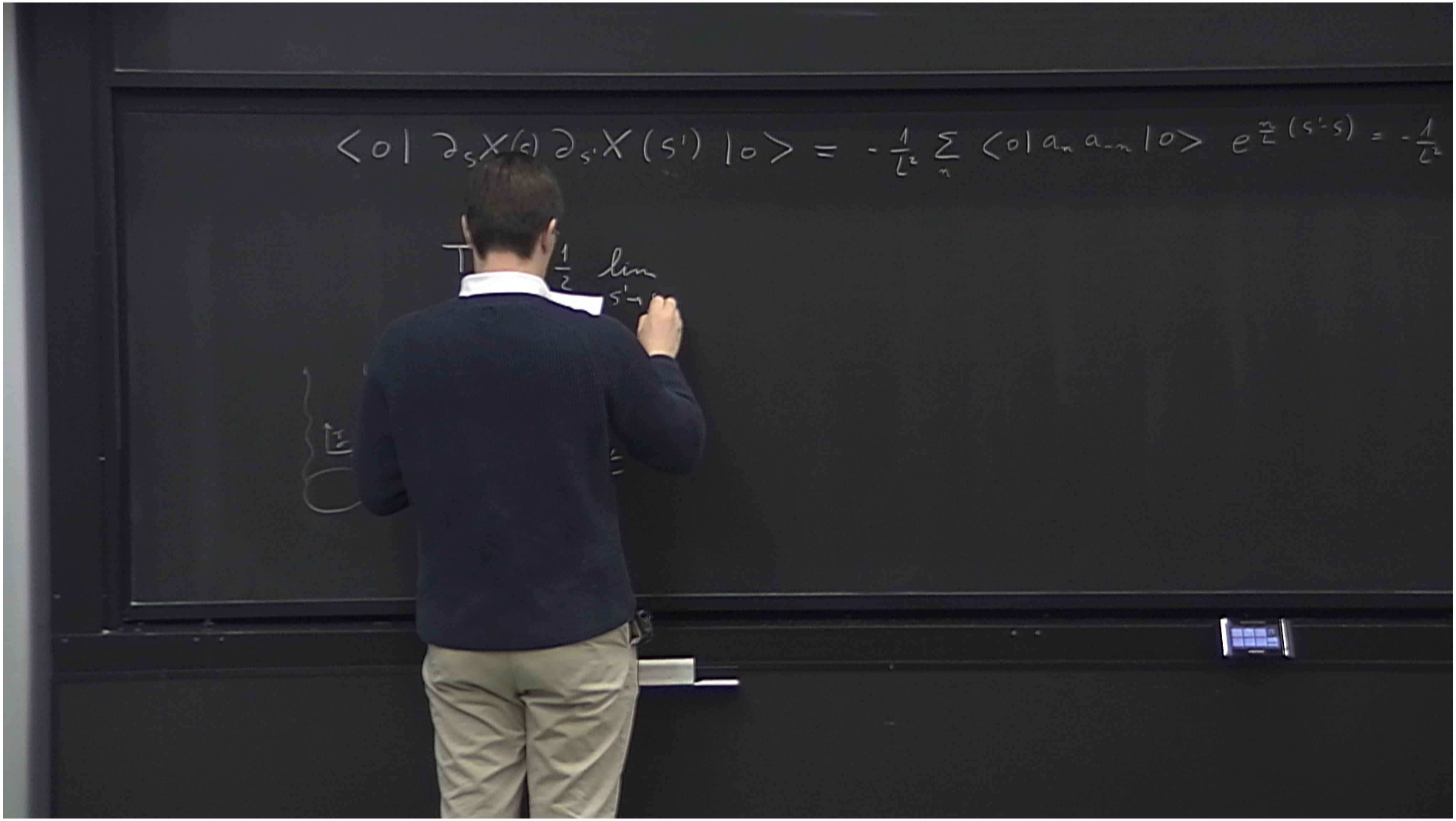
$\partial_{\bar z} T_{zz} = 0$

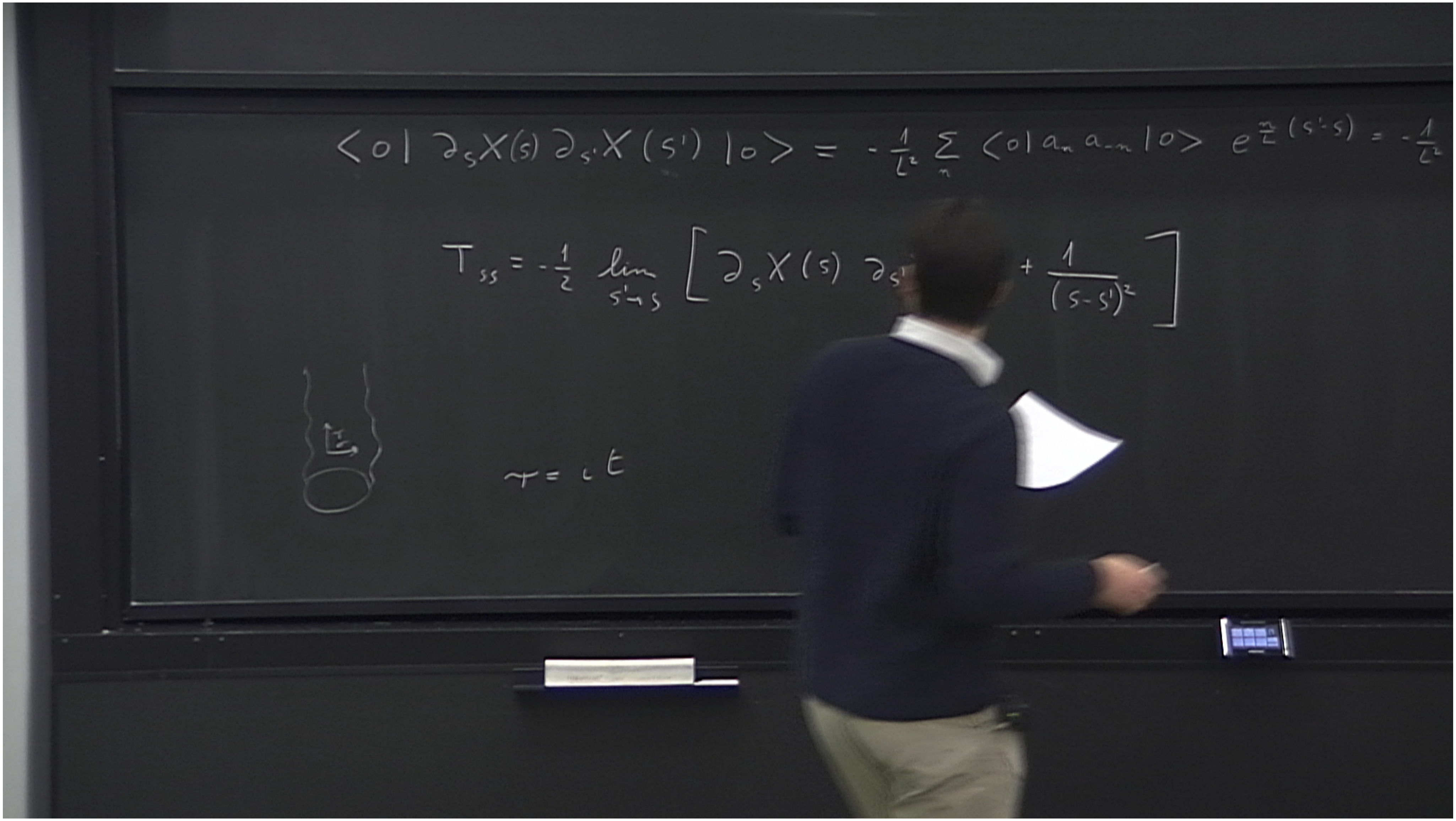
$$T^{ab} = \frac{\delta S}{\delta h_{ab}} = 0$$

$$\partial_{\bar z} T_{zz} + \partial_z \cancel{T}_{\bar z z} = 0$$

Diff $\Rightarrow$ $\nabla_a T^{ab} = 0$

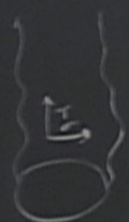
WEYL $\rightarrow$ $h_{ab} T^{ab} = 0 \quad \equiv \quad T_{z \bar z} = 0$


$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{l^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{\frac{n}{l}(s'-s)} = -\frac{1}{l^2}$

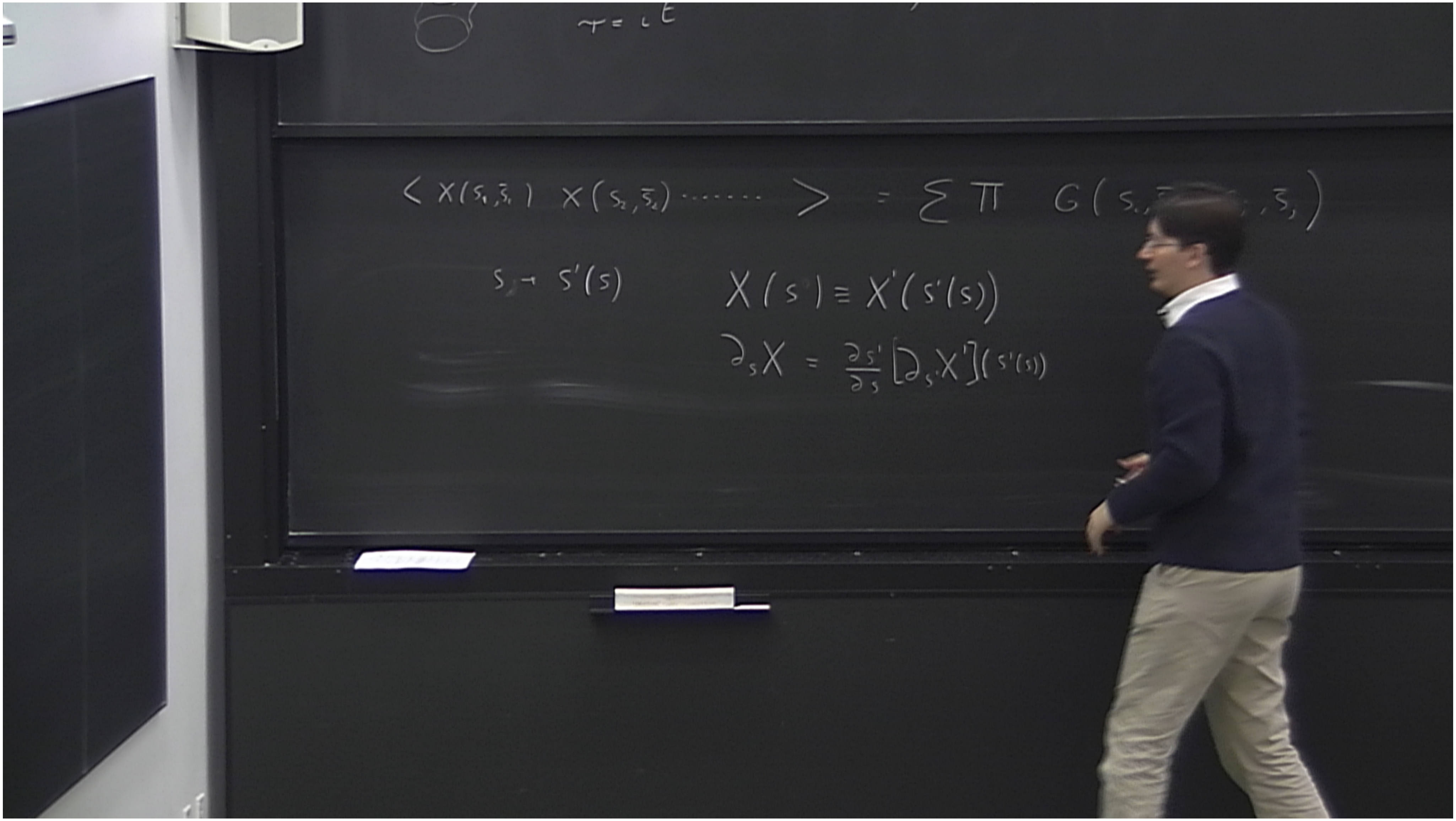

$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{\ell^2} \sum_n \langle 0 | a_n a_{-n} | 0 \rangle e^{\frac{n}{\ell}(s'-s)} = -\frac{1}{\ell^2}$
$T_{ss} = -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \partial_{s'} \quad + \frac{1}{(s-s')^2} \right]$
$\tau = i t$

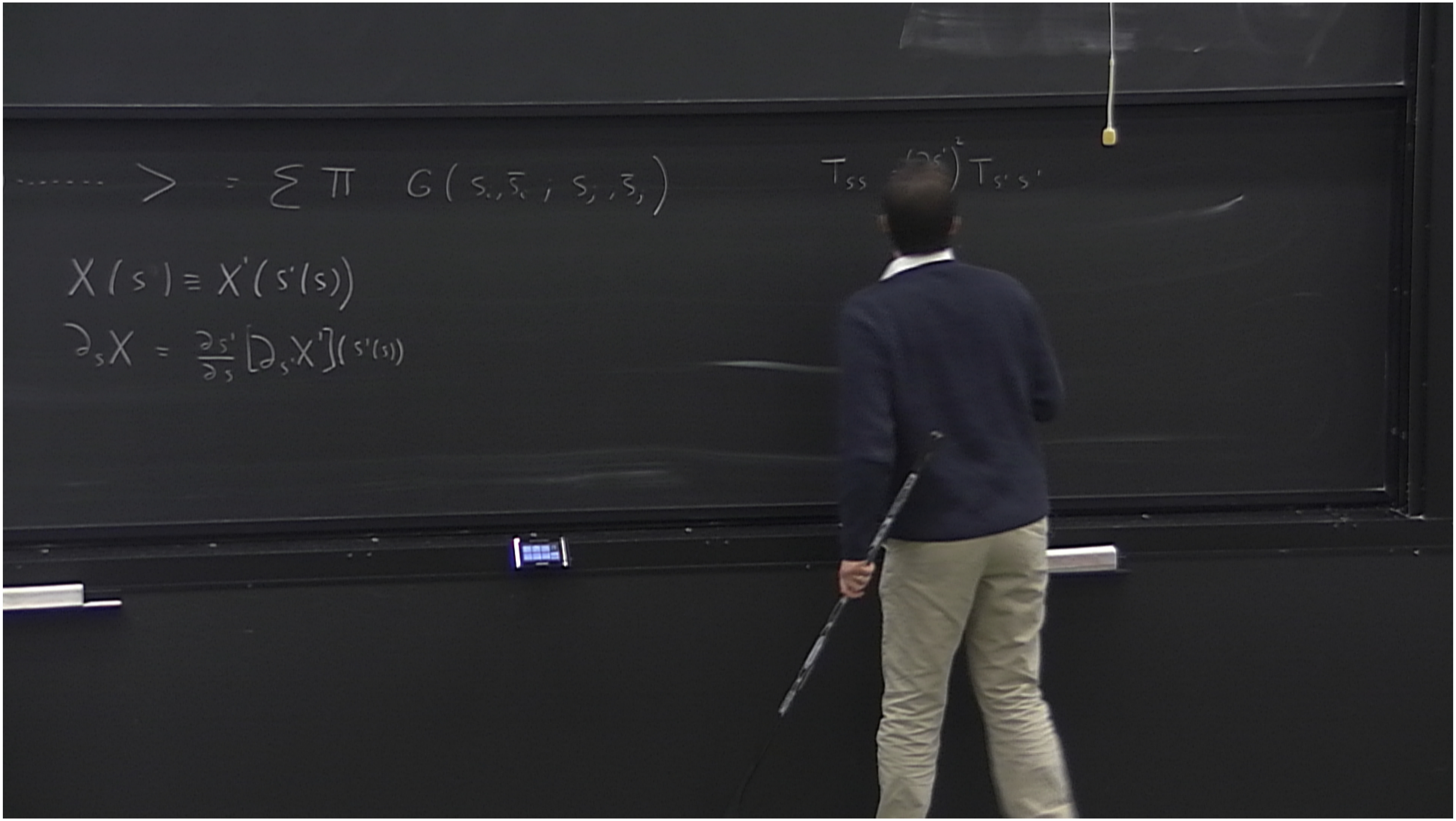
$$\langle 0|\,\partial_s X(s)\,\partial_{s'} X(s')\,|0\rangle = -\frac{1}{\ell^2}\sum_n \langle 0|\,a_n a_{-n}\,|0\rangle\, e^{\frac{n}{\ell}(s'-s)} = -\frac{1}{\ell^2}$$

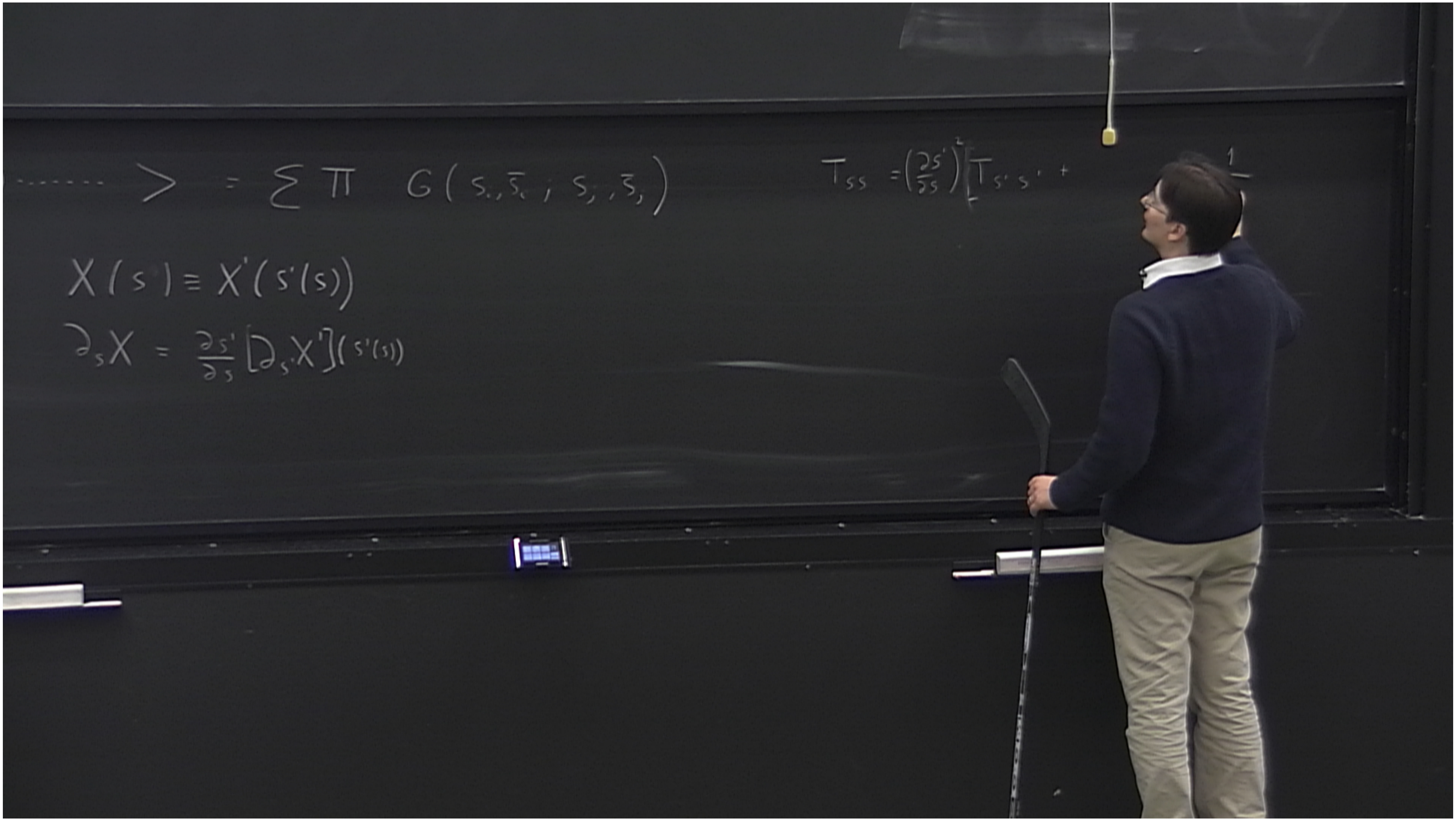
$$T_{ss} = -\frac{1}{2}\lim_{s'\to s}\left[\partial_s X(s)\,\partial_{s'} X(s') + \frac{1}{(s-s')^2}\right]$$

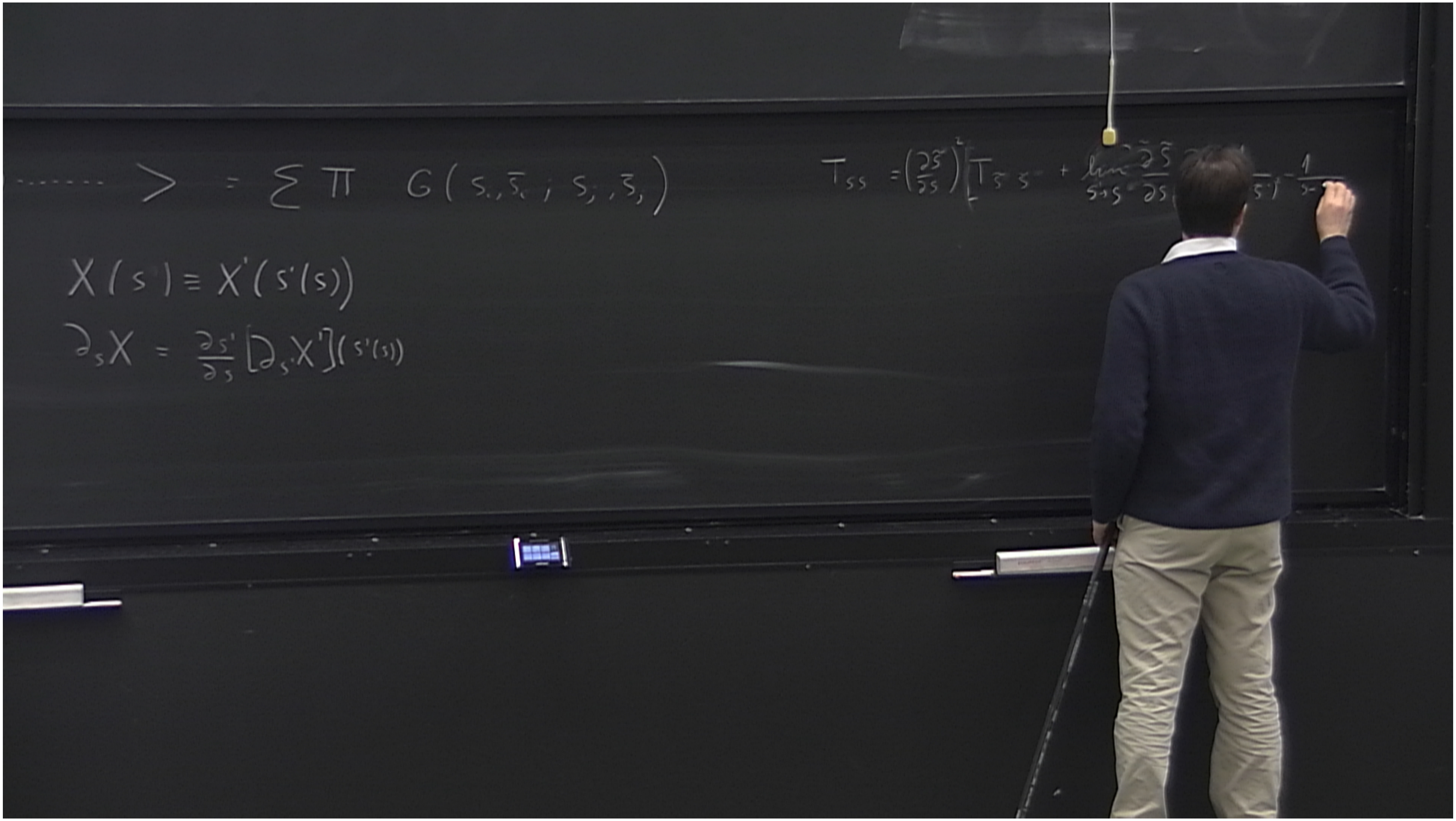


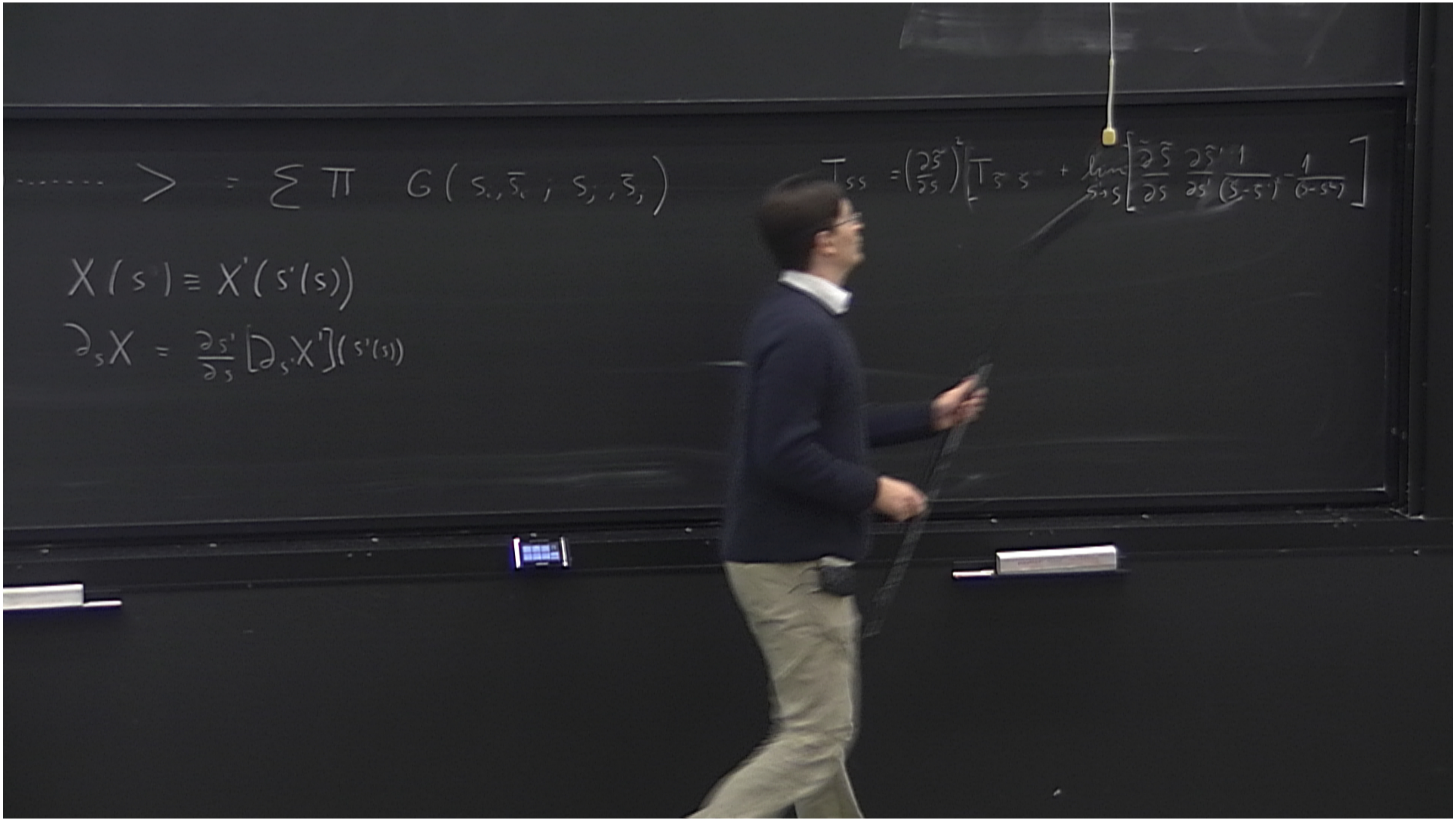
$$\tau = i t$$

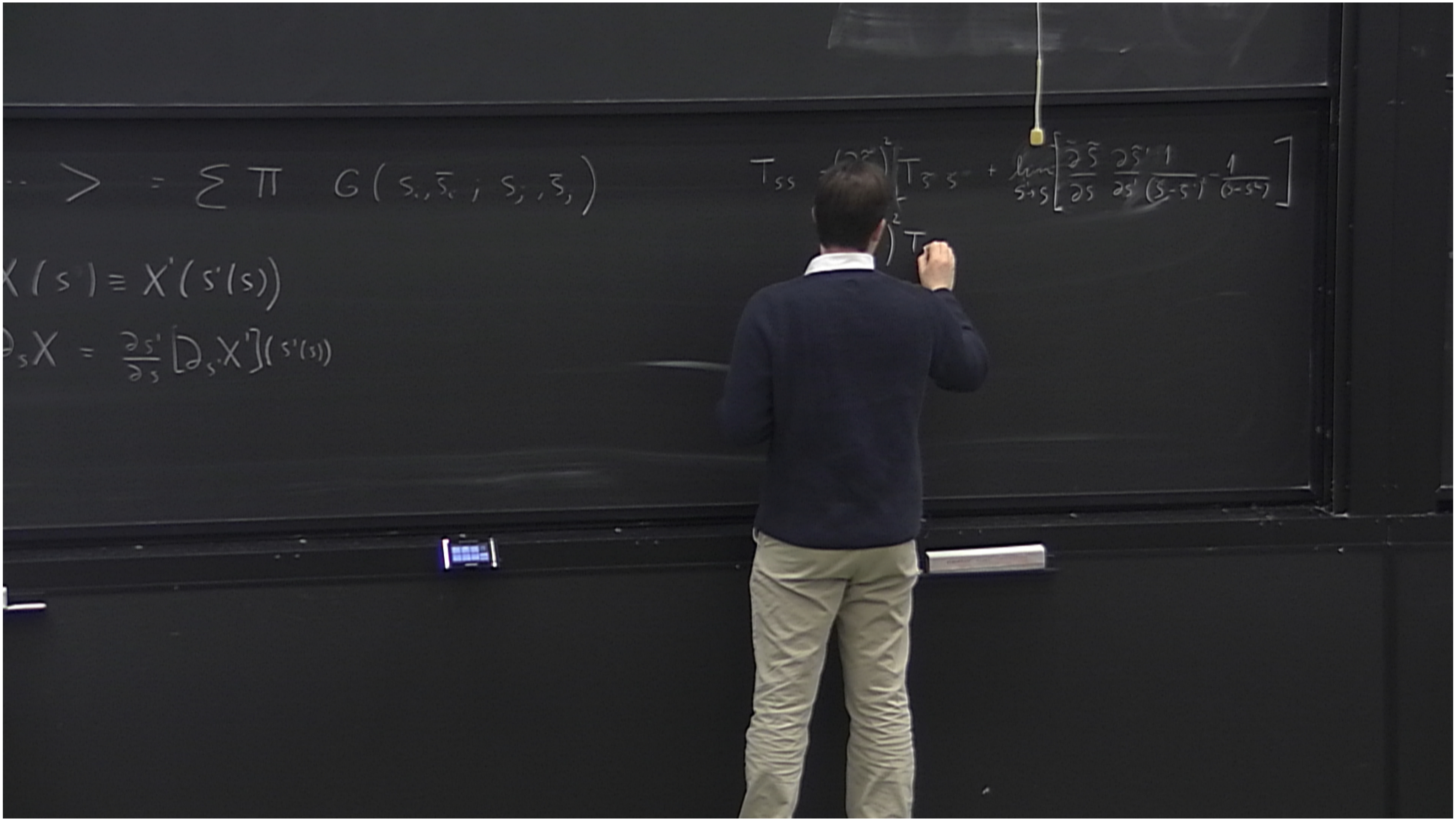

τ = i t
< X(s1,s̄1) X(s2,s̄2) ······· > = Σ Π G(s ... ,s̄)
s → s'(s)
X(s) ≡ X'(s'(s))
∂s X = (∂s'/∂s) [∂s' X'](s'(s))

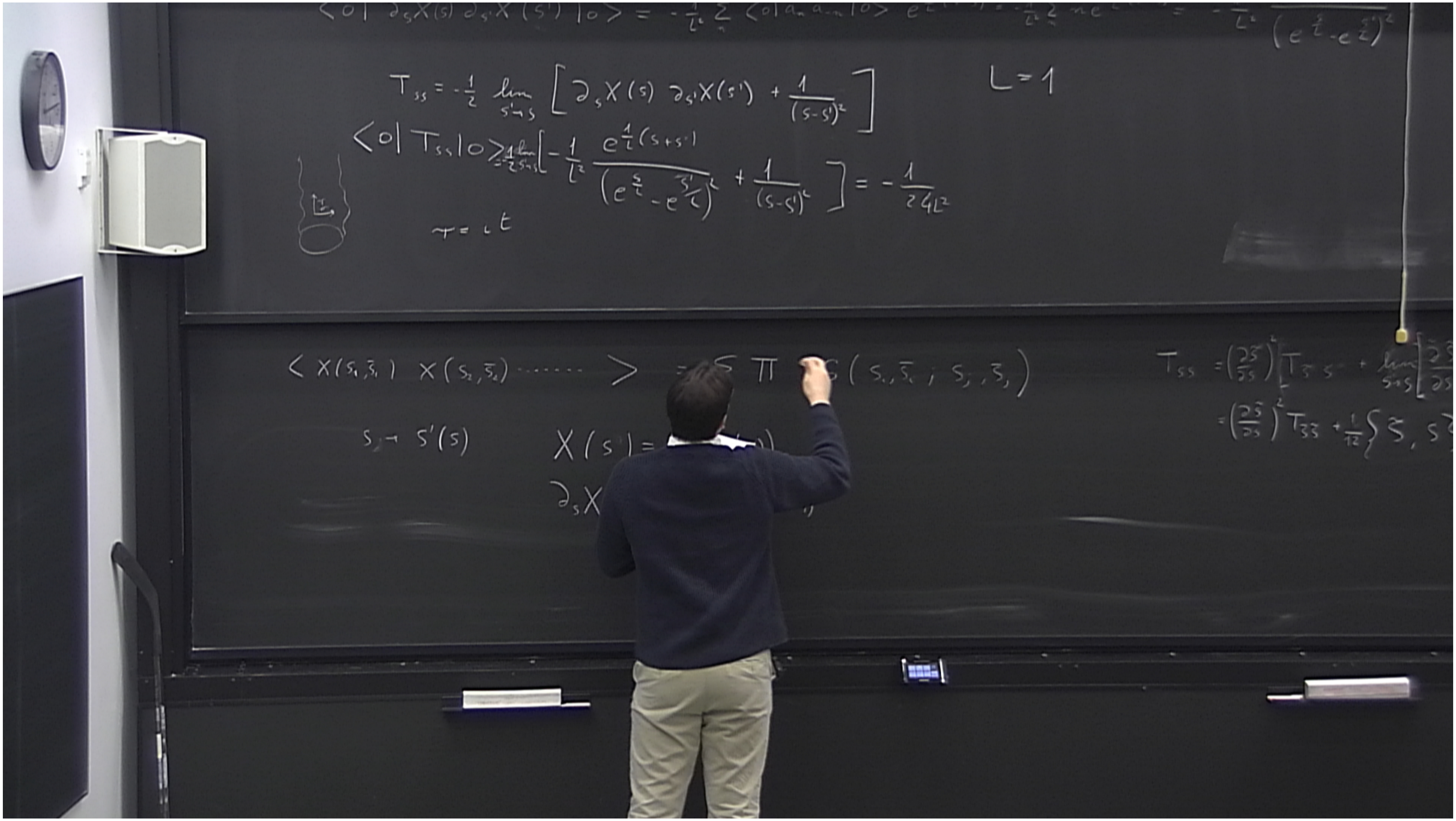

$= \sum \prod G(s_i, \bar{s}_i ; s_j, \bar{s}_j)$
$X(s) \equiv X'(s'(s))$
$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$
T_{ss}
$T_{s's'}$

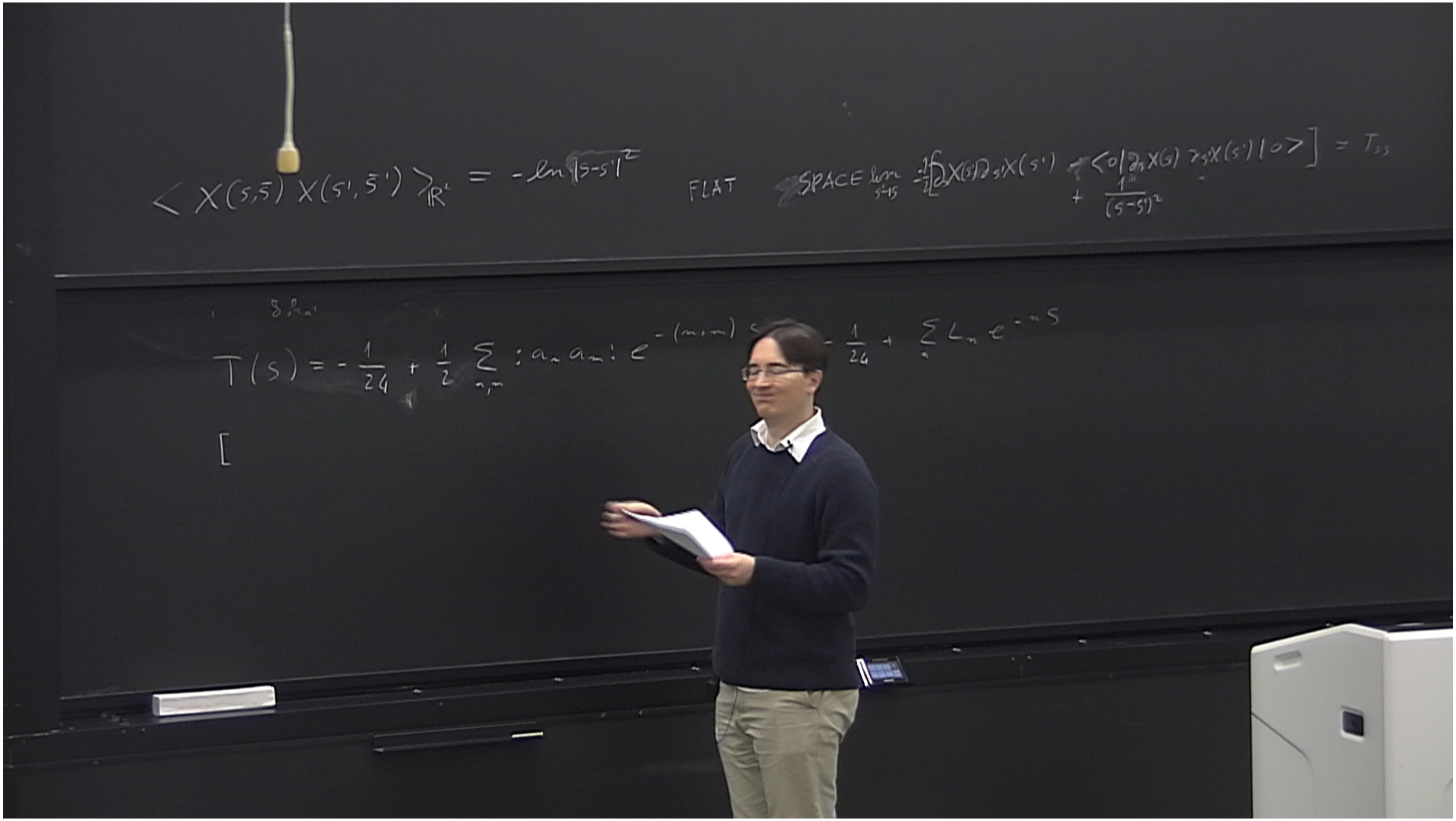

$\cdots\cdots > = \sum \prod G(s_i, \bar{s}_i ; s_j, \bar{s}_j)$
$T_{ss} = \left(\frac{\partial s'}{\partial s}\right)^2 \left[T_{s's'} + \right.$
$X(s) \equiv X'(s'(s))$
$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$


$X(s) \equiv X'(s'(s))$
$\partial_s X = \frac{\partial s'}{\partial s} [\partial_{s'} X'](s'(s))$


X (s) ≡ X' (s'(s))
∂sX = ∂s'/∂s [∂s'X'] (s'(s))


X(s) ≡ X'(s'(s))
∂_s X = ∂s'/∂s [∂_{s'} X'](s'(s))


L = 1
τ = i t



$< X(s,\bar{s}) X(s',\bar{s}') >_{\mathbb{R}^2} = -\ln|s-s'|^2$
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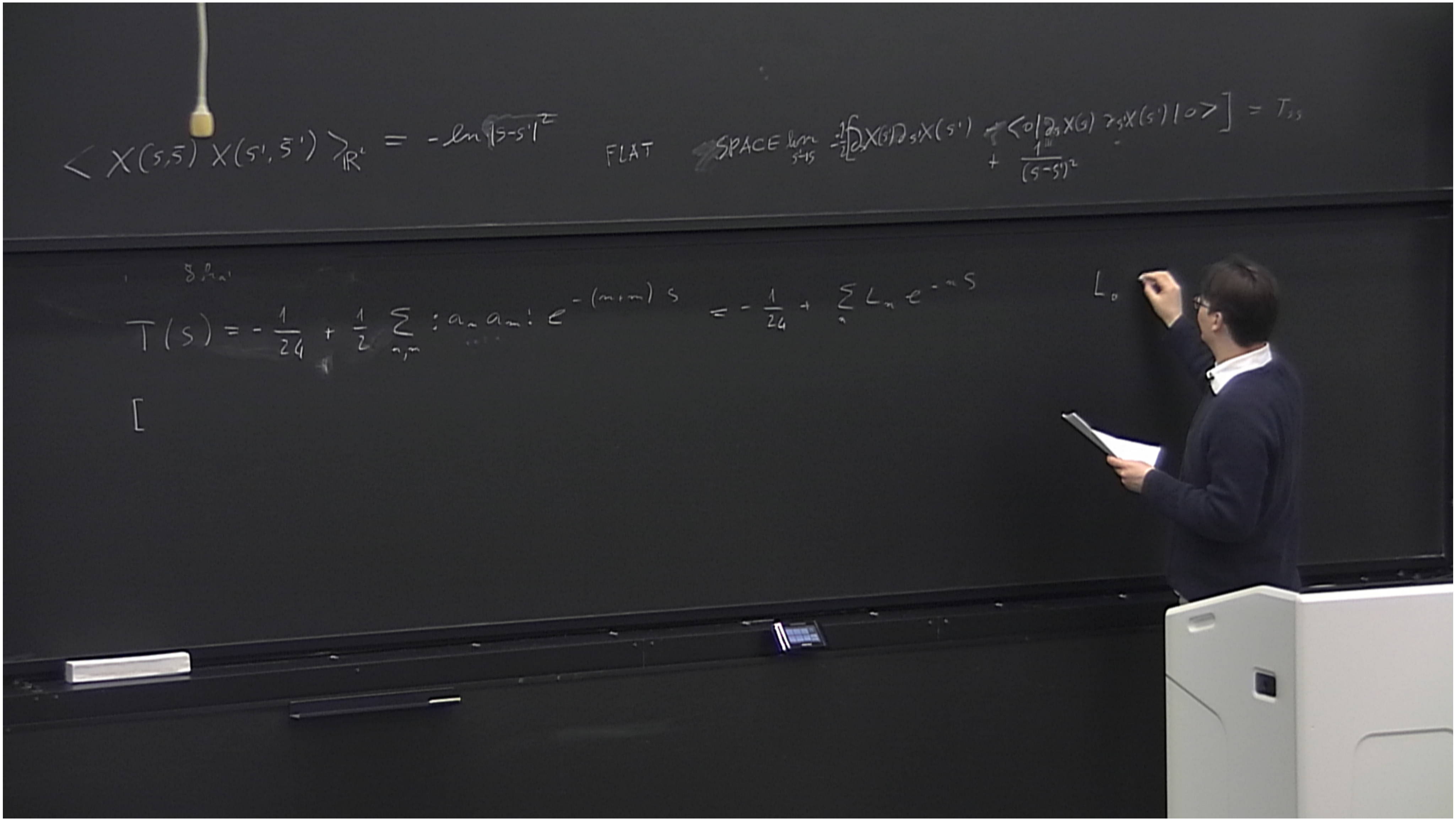
$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{n,m} :a_n a_m: e^{-(n+m)s}$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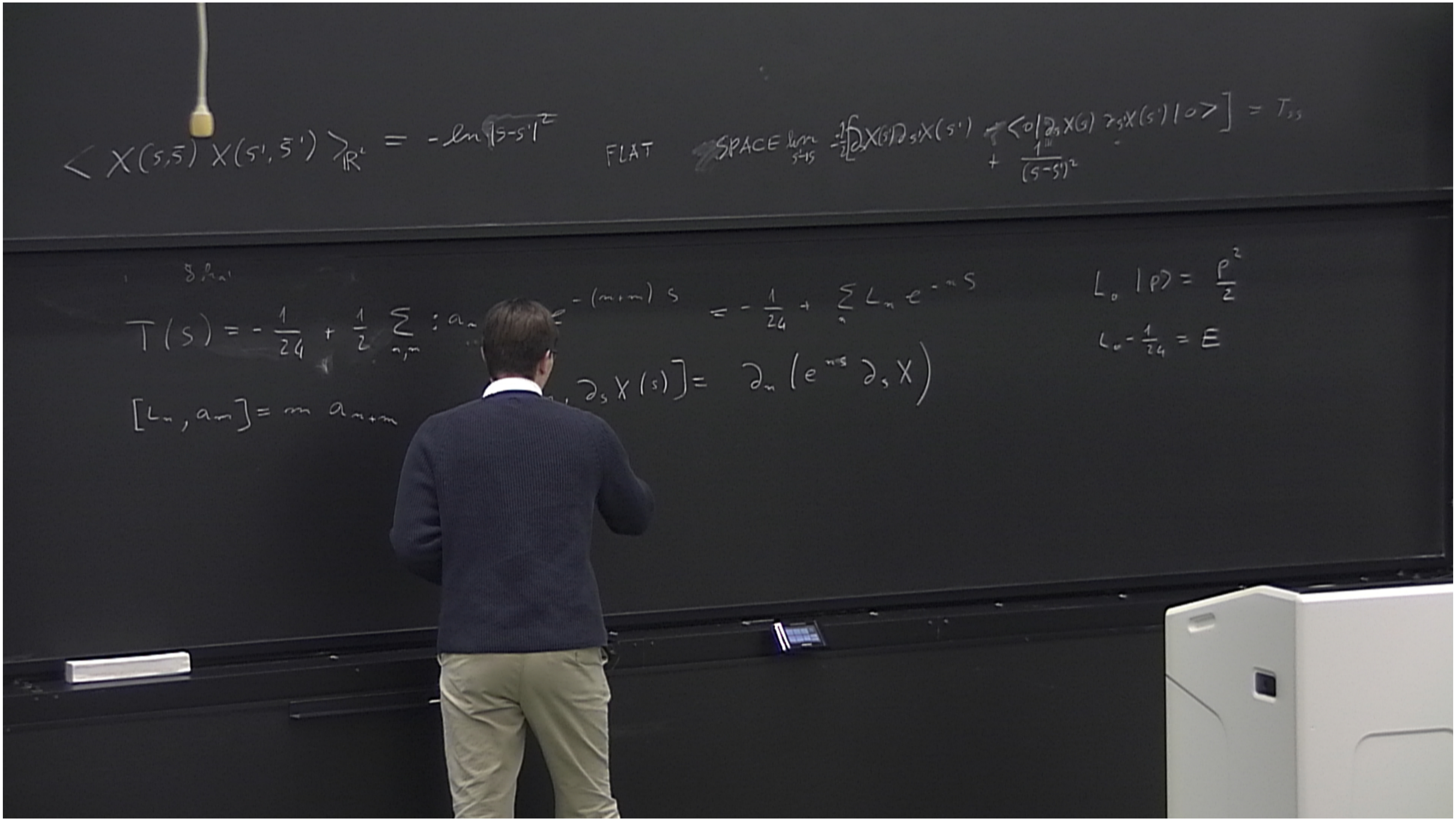
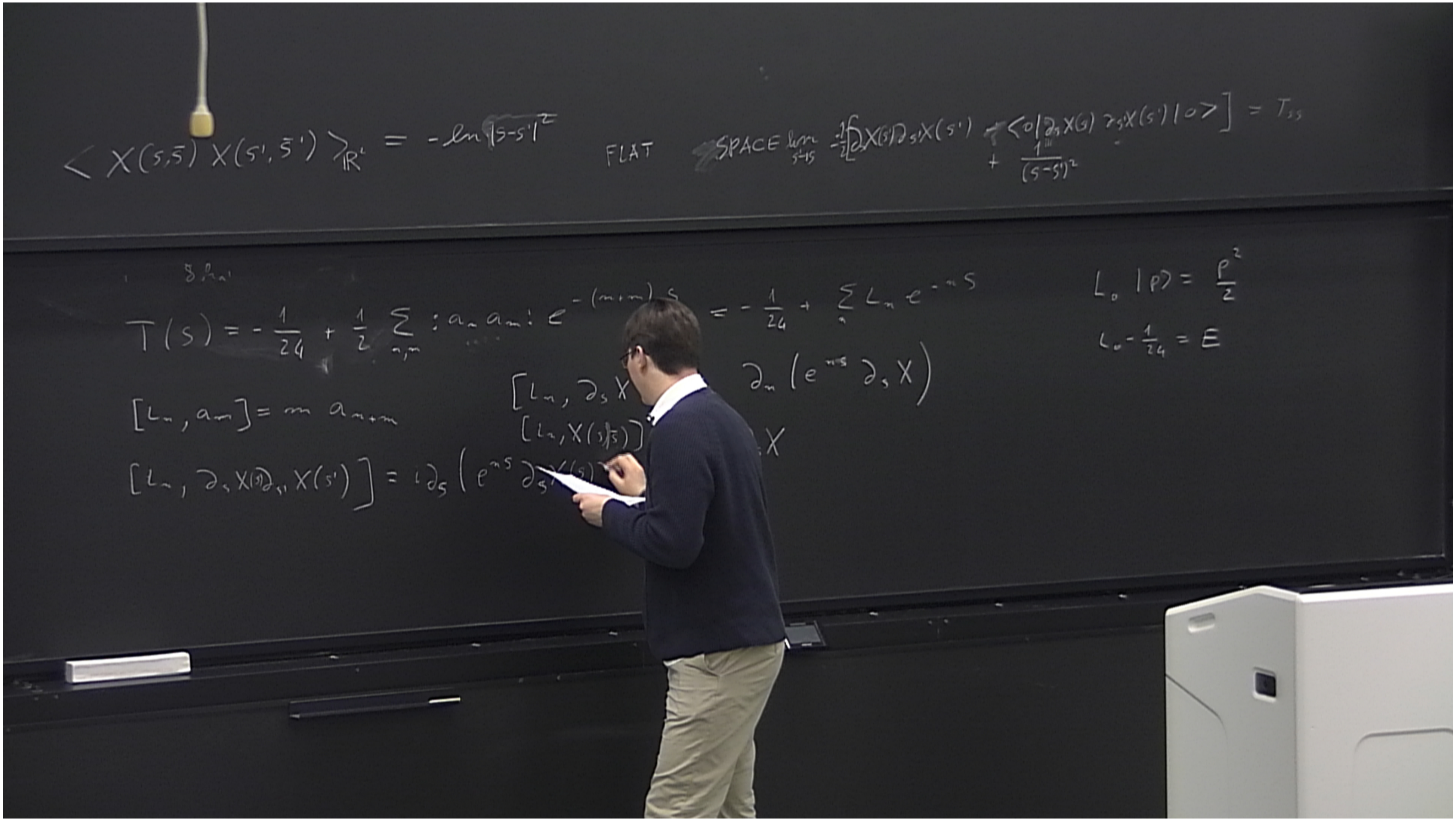

< X(s,s̄) X(s',s̄') >_{R^2} = - ln |s-s'|^2
FLAT SPACE
L_0 |p> = p^2/2
L_0 - 1/24 = E
T(s) = - 1/24 + 1/2 Σ_{n,m} : a_n
= - 1/24 + Σ_n L_n e^{-ns}
[L_n , a_m] = m a_{n+m}
∂_s X(s)] = ∂_n (e^{ns} ∂_s X)



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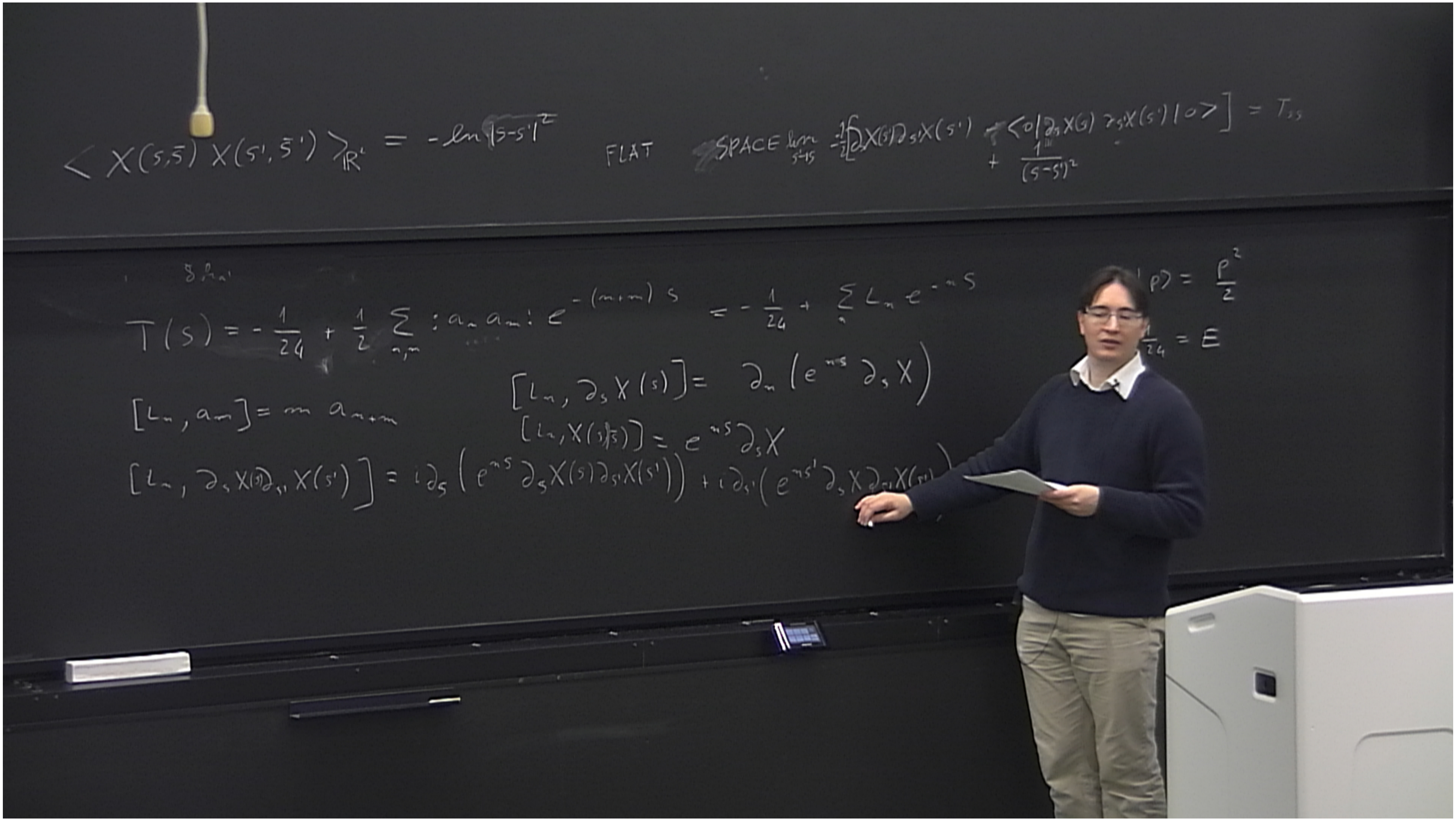
T(s) = - 1/24 + 1/2 Σ :a_n a_m: e^{-(n+m)s} = - 1/24 + Σ L_n e^{-ns}
L_0 |p> = p^2/2
L_0 - 1/24 = E
[L_n, a_m] = m a_{n+m}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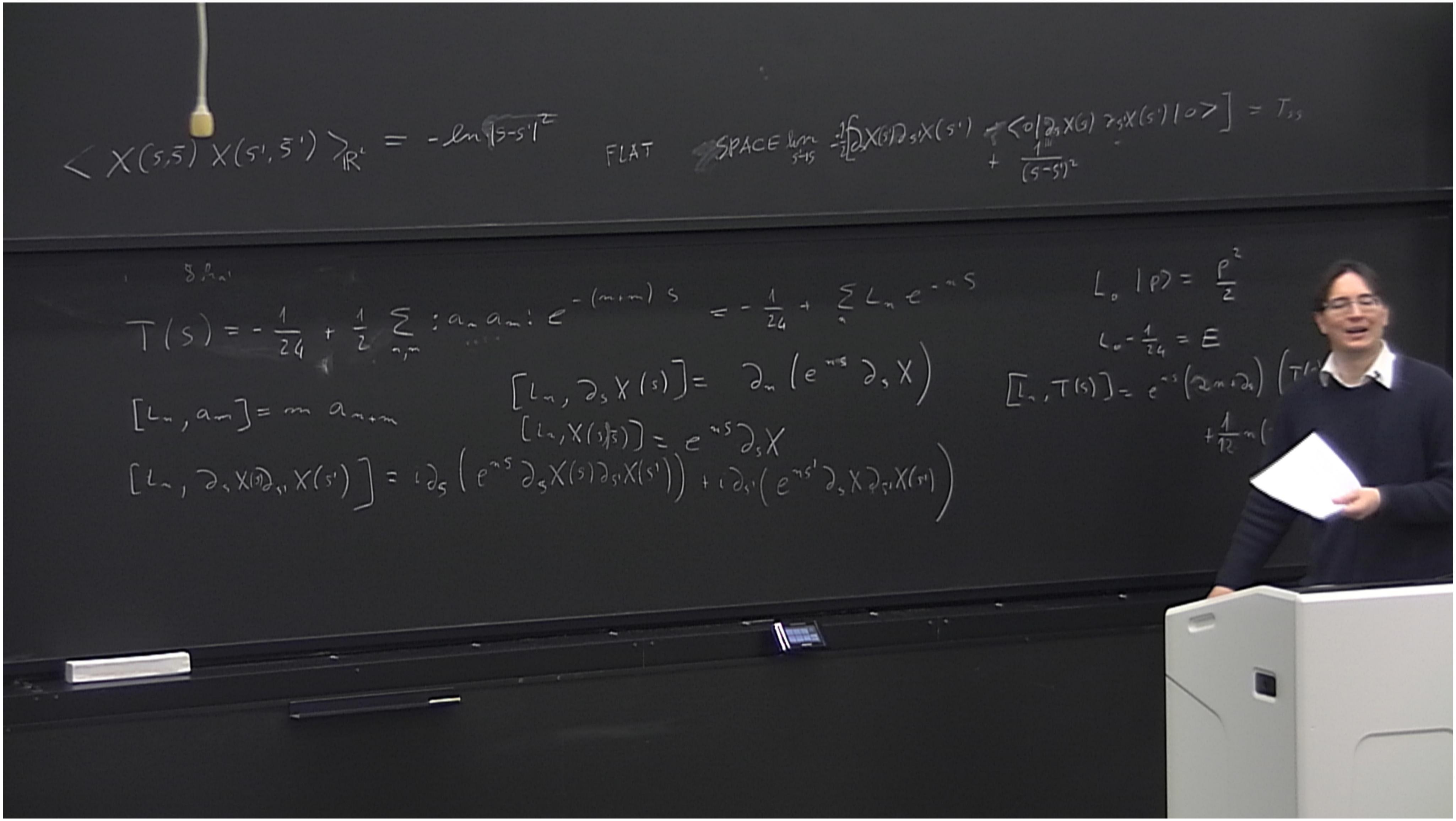
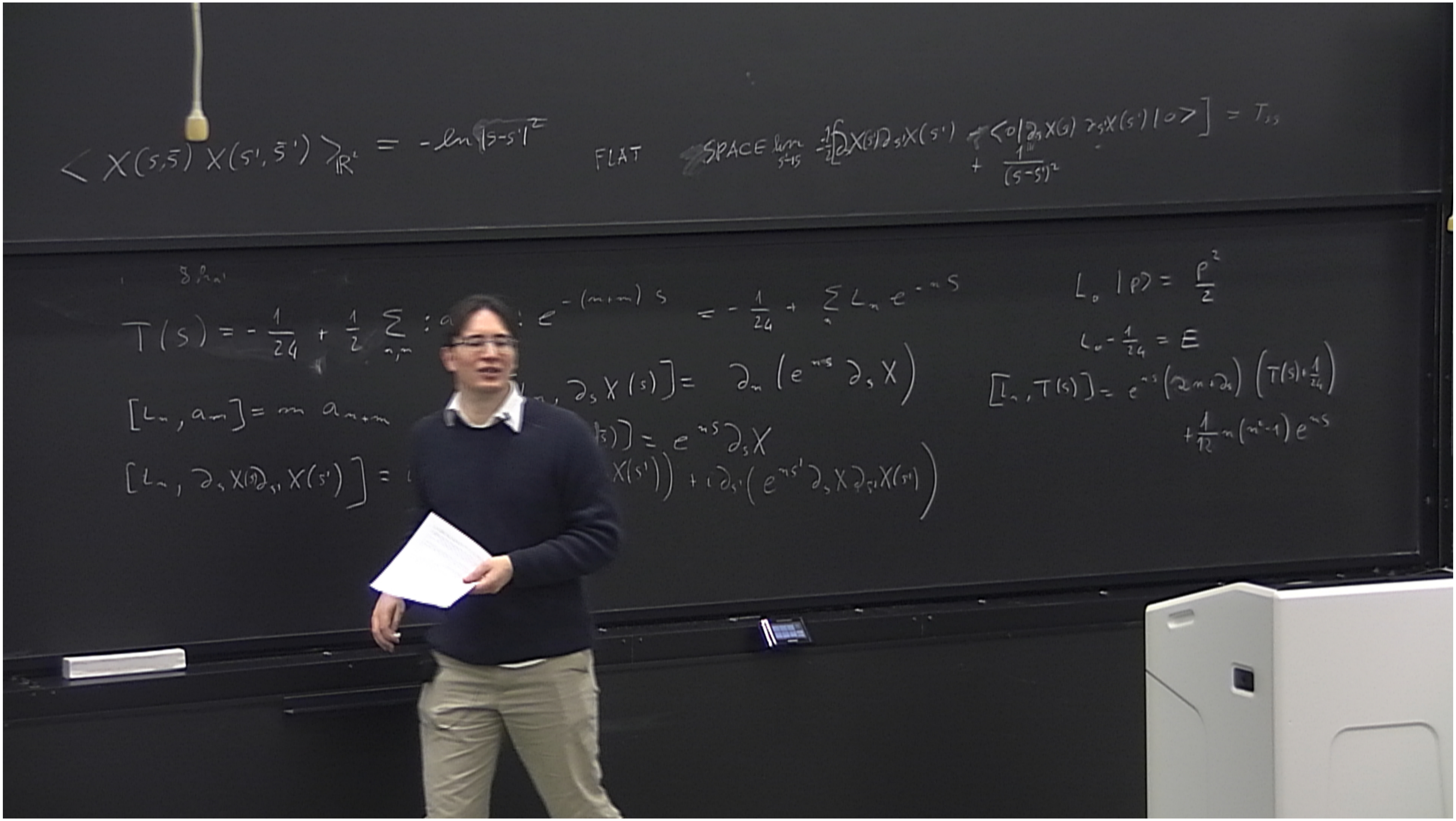

$< X(s,\bar{s}) X(s',\bar{s}') >_{\mathbb{R}^2} = -\ln|s-s'|^2$
FLAT SPACE
$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{n,m} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$
$L_0 |p\rangle = \frac{p^2}{2}$
$L_0 - \frac{1}{24} = E$
$[L_n, a_m] = m\, a_{n+m}$
$[L_n, \partial_s X(s)] = \partial_n (e^{ns} \partial_s X)$
$[L_n, X(s,\bar{s})] = e^{ns}\partial_s X$
$[L_n, \partial_s X(s) \partial_{s'} X(s')] = i\partial_s (e^{ns} \partial_s X(s) \partial_{s'} X(s')) + i\partial_{s'} (e^{ns'} \partial_s X \partial_{s'} X(s'))$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$$\langle X(s,\bar{s})\, X(s',\bar{s}') \rangle_{\mathbb{R}^2} = -\ln|s-s'|^2$$

FLAT SPACE $\lim_{s\to s'} -\frac{1}{2}\left[\partial_s X(s)\partial_{s'} X(s') - \langle 0|\partial_s X(s)\, \partial_{s'} X(s')|0\rangle\right] = T_{ss}$

$+ \frac{1}{(s-s')^2}$

$\delta_{n,n'}$

$$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{n,m} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$L_0|p\rangle = \frac{p^2}{2}$$

$$L_0 - \frac{1}{24} = E$$

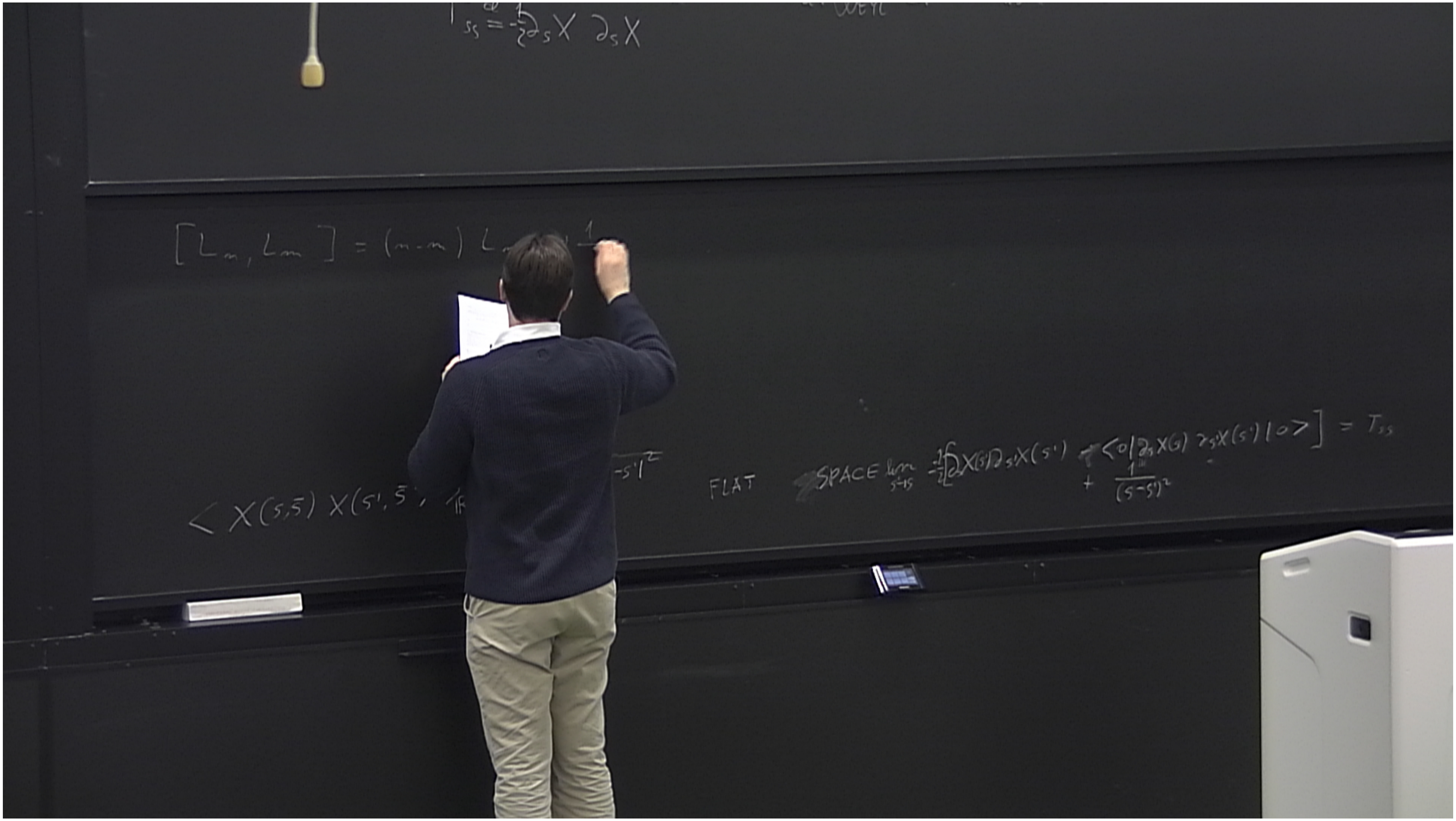
$$[L_n, a_m] = m\, a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_s\left(e^{ns}\partial_s X\right)$$

$$[L_n, X(s,\bar{s})] = e^{ns}\partial_s X$$

$$[L_n, T(s)] = e^{ns}(2n+\partial_s)\left(T(s)+\frac{1}{24}\right) + \frac{1}{12}n(n^2-1)e^{ns}$$

$$[L_n, \partial_s X(s)\partial_{s'} X(s')] = i\partial_s\left(e^{ns}\partial_s X(s)\partial_{s'}X(s')\right) + i\partial_{s'}\left(e^{ns'}\partial_s X\partial_{s'}X(s')\right)$$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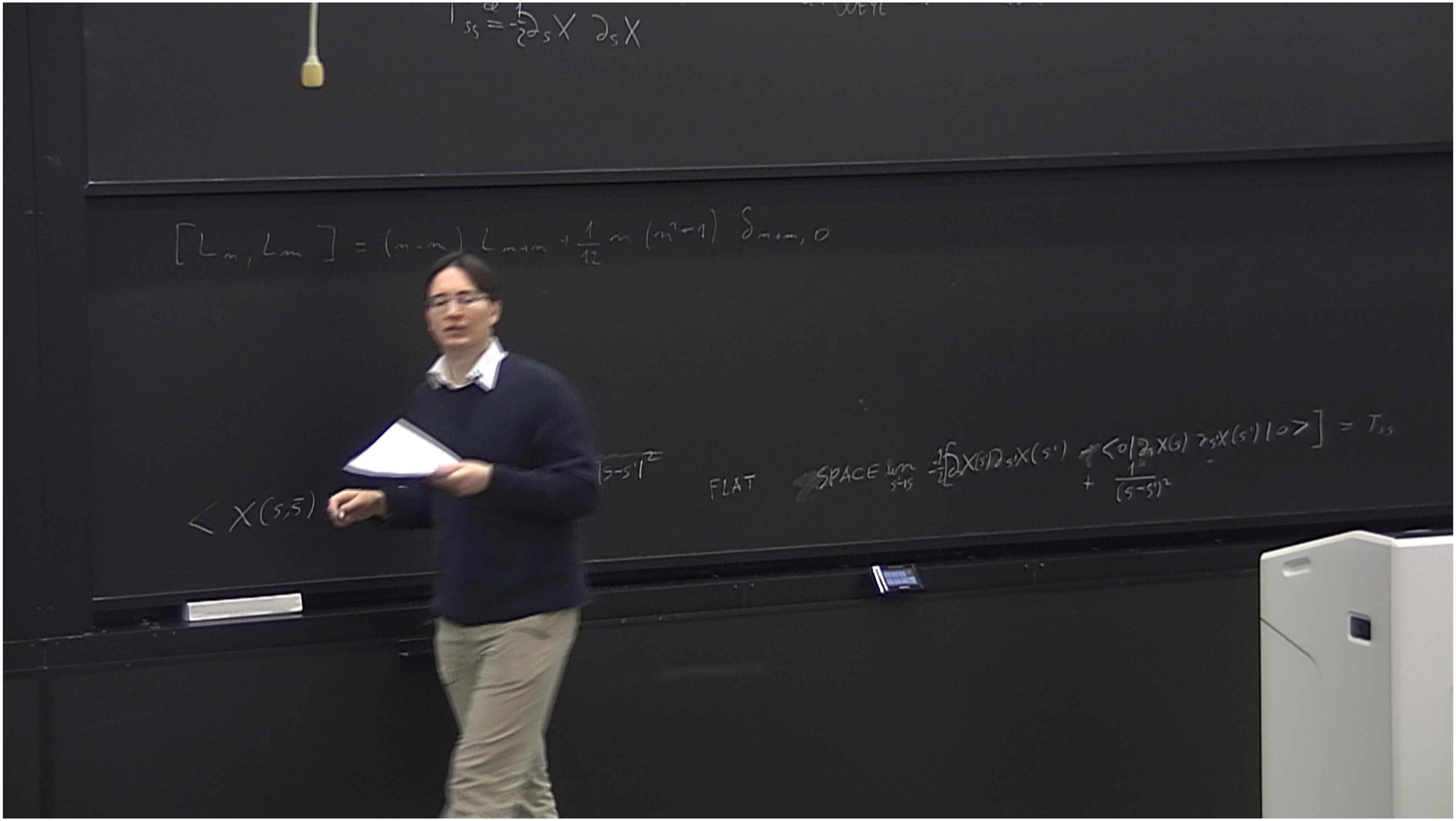
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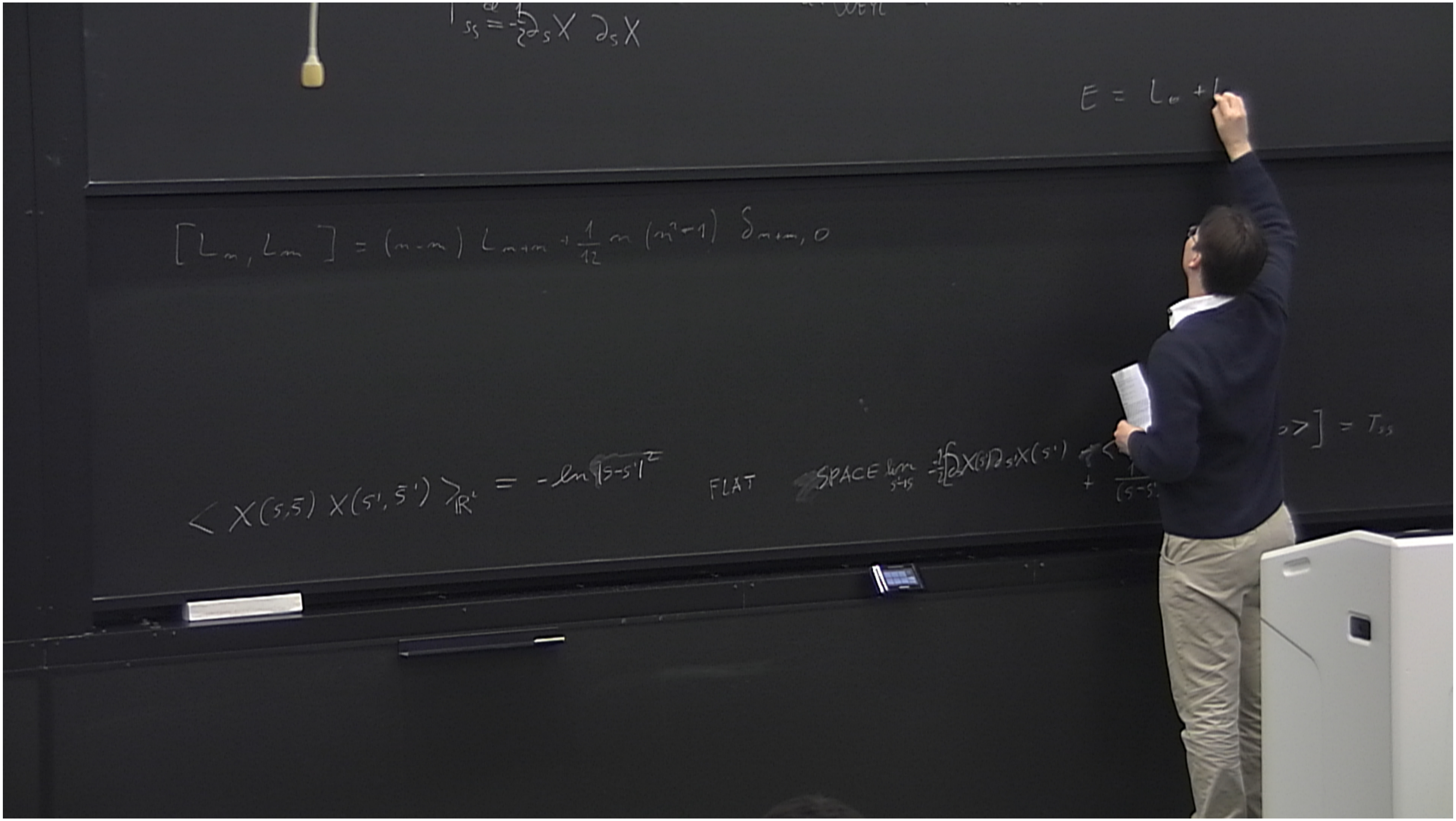
$E = L_0 + L$
$[L_n, L_m] = (n-m) L_{n+m} + \frac{1}{12} n (n^2-1) \delta_{n+m,0}$
$\langle X(s,\bar{s}) X(s',\bar{s}') \rangle_{\mathbb{R}^2} = -\ln |s-s'|^2$
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