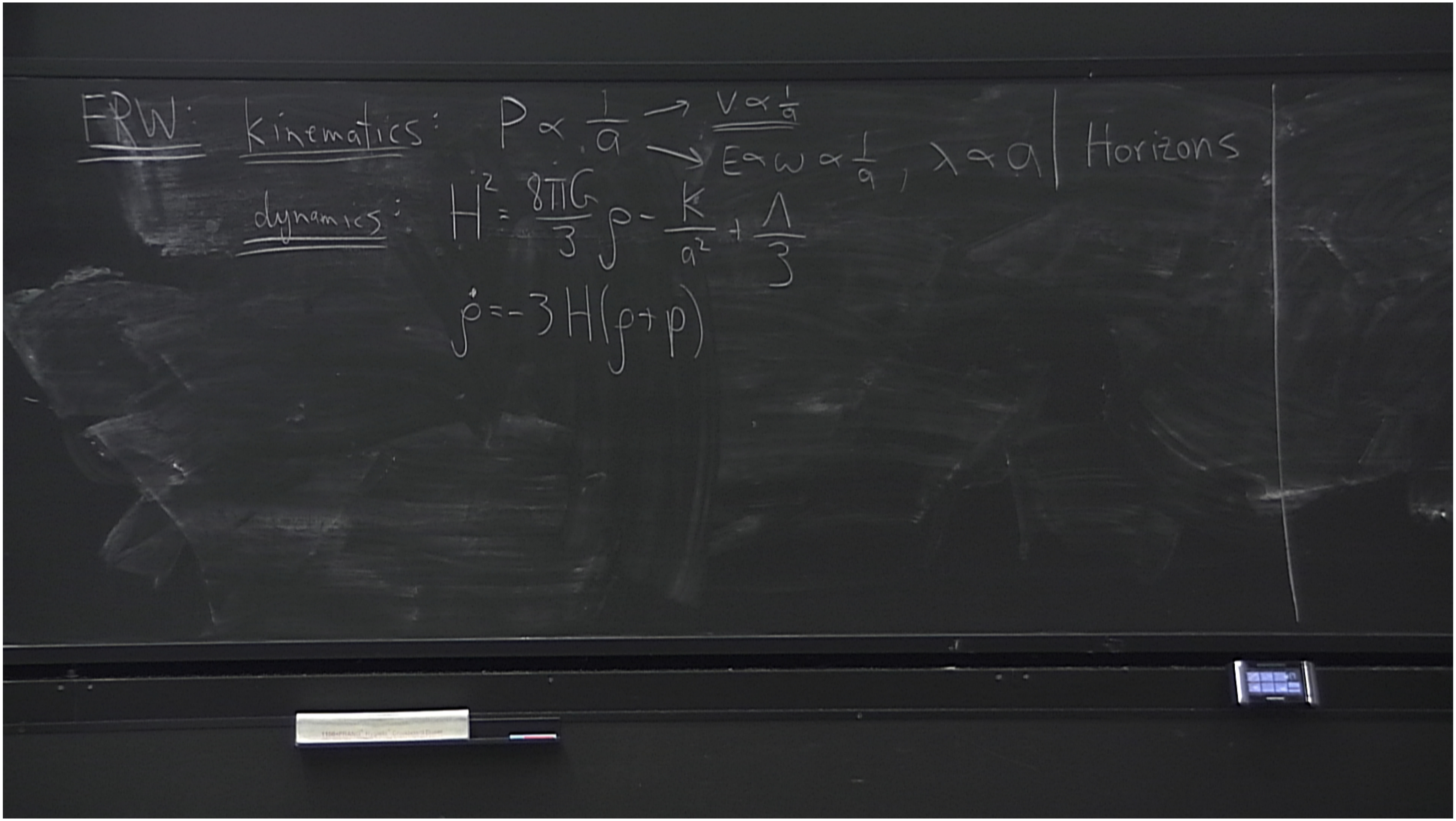
Title: 13/14 PSI - Cosmology Review - Lecture 4

Date: Jan 31, 2014 11:30 AM

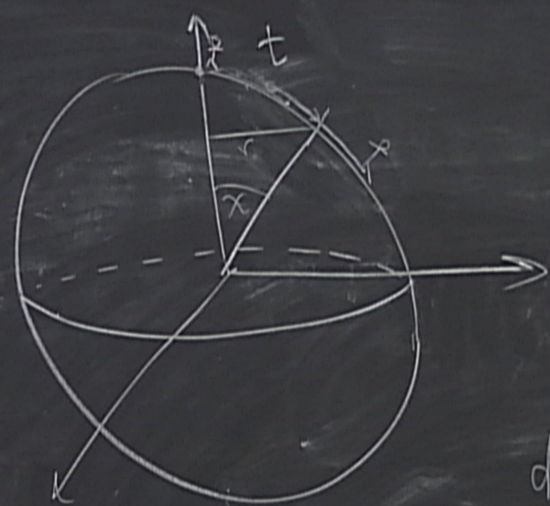
URL: http://pirsa.org/14010068

Abstract:



FRW:
kinematics: $p \propto \frac{1}{a} \to v \propto \frac{1}{a}$
$\to E \propto \omega \propto \frac{1}{a}, \lambda \propto a$
Horizons
dynamics: $H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$
$\dot{\rho} = -3H(\rho + p)$

...zons



$$a(t) \propto t^p = a_0 \left(\frac{t}{t_0}\right)^p \qquad a_0, t_0, p > 0$$

$$t, \eta$$

$$t: -\infty \to 0, \quad \underline{t: 0 \to \infty} \qquad r, \chi$$

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\chi^2 + \cancel{S_k^2(\chi)\, d\Omega^2}\right]$$

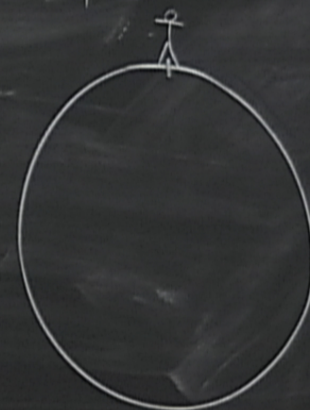
$$ds^2 = 0 \qquad d\eta = \pm d\chi$$

$$a = a_0\left(\frac{t}{t_0}\right)^p = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$$

$$d\eta = \frac{dt}{a(t)} \to \eta \propto t^{1-p} \qquad t \propto \eta^{\frac{1}{1-p}}$$

$\dot{a} > 0$ $\quad$ $\ddot{a} \to > 0$ $\quad$ $p > 1 \leftarrow$

$\qquad\qquad\quad < 0$ $\quad$ $0 < p < 1 \leftarrow$



d

$$\boxed{\begin{array}{lll} 0 < p < 1 & (\text{decel}): & \eta: 0 \to +\infty \\ p > 1 & (\text{accel}): & \eta: -\infty \to 0 \end{array}}$$

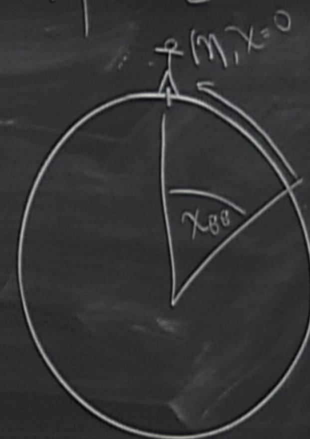
$$a = a_0\left(\frac{t}{t_0}\right)^p = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$$

$$d\eta = \frac{dt}{a(t)} \to \eta \propto t^{1-p} \qquad t \propto \eta^{\frac{1}{1-p}}$$

$\dot{a} > 0 \qquad \ddot{a} \to >0 \qquad p>1$

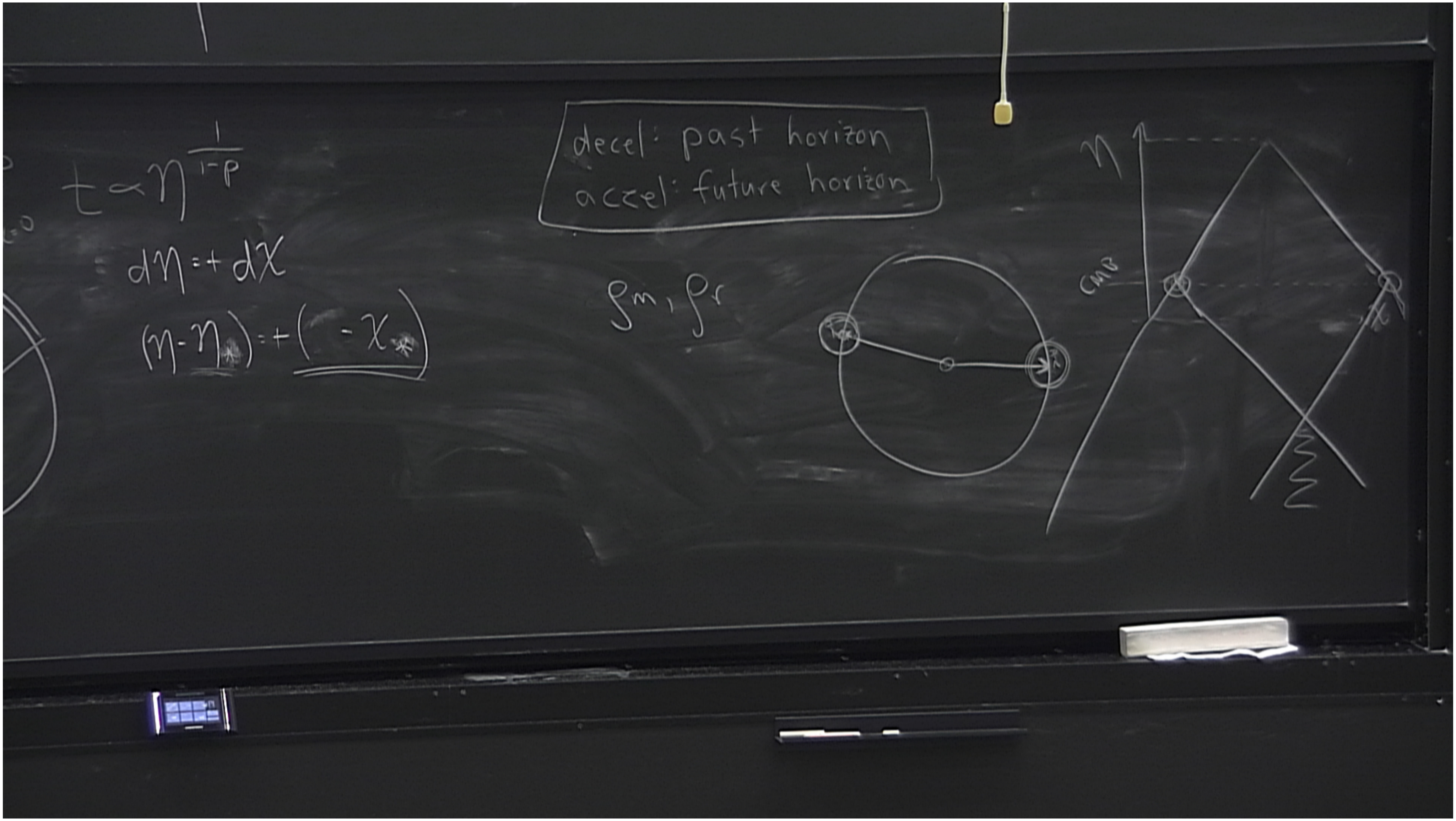
$\qquad\qquad\qquad <0 \qquad 0<p<1$

$$\boxed{\begin{array}{ll} 0<p<1 \ (\text{decel}): & \eta: 0 \to +\infty \\ p>1 \ (\text{accel}): & \eta: -\infty \to 0 \end{array}}$$



$$d\eta = -d\chi$$

$$(\eta - \eta_{BB}) = -(\quad - \chi_{BB})$$


$t \propto \eta^{\frac{1}{1-p}}$
$d\eta = \pm d\chi$
$(\eta - \eta_*) = \pm(\ \ - \chi_*)$
decel: past horizon
accel: future horizon
ρ_m, ρ_r
η
CMB
χ

Dynamics of FRW:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$$

$$\dot{\rho} = -3H(\rho + p)$$

$$\rho_{crit} \equiv \frac{3H^2}{8\pi G_N}$$

$$G_{\mu\nu} + \cancel{\Lambda g_{\mu\nu}} = 8\pi G_N T_{\mu\nu} - \underline{\Lambda g_{\mu\nu}} = 8\pi G_N \tilde{T}_{\mu\nu} \quad T_{\mu\nu} + T^{vac}_{\mu\nu}$$

$$\frac{\Lambda}{8\pi G_N} = \rho_{vac}$$

$\rho = \rho_{crit}$ $K=0$ (flat U.)

$\rho > \rho_{crit}$ $K=+1$ (closed U.)

$\rho < \rho_{crit}$ $K=-1$ (open)

$$\underline{\rho = \rho_{crit}(1 \pm 0.01)}$$

$$T_{\mu\nu} - \underline{\Lambda g_{\mu\nu}} = 8\pi G_N \tilde{T}_{\mu\nu} \qquad T_{\mu\nu} + T^{vac}_{\mu\nu}$$

$$\underline{\rho = \rho_{crit}(1 \pm 0.01)}$$

flat U.)

closed U.)

open)

$$p = p(\rho)$$

$$p = w\rho$$

$$\dot\rho = -3H(\rho + p) = -3(1+w)H\rho$$

$$\frac{\dot\rho}{\rho} = -3(1+w)\frac{\dot a}{a}$$

$$\ln\rho = -3(1+w)\ln a + C$$

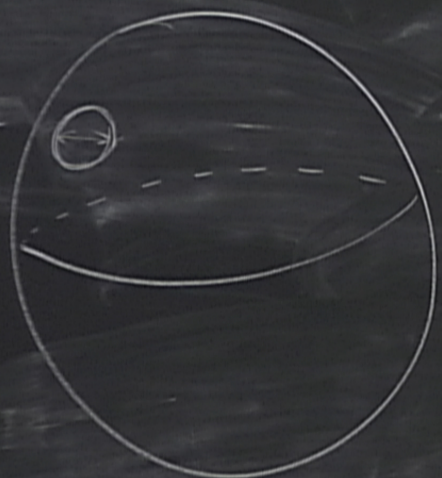
$$\boxed{\rho \propto a^{-3(1+w)}}$$

radiation $w = \frac{1}{3} \Rightarrow \rho \propto a^{-4}$

matter $w = 0 \Rightarrow \rho \propto a^{-3}$

vacuum E. $w = -1 \Rightarrow \rho \propto a^{0}$

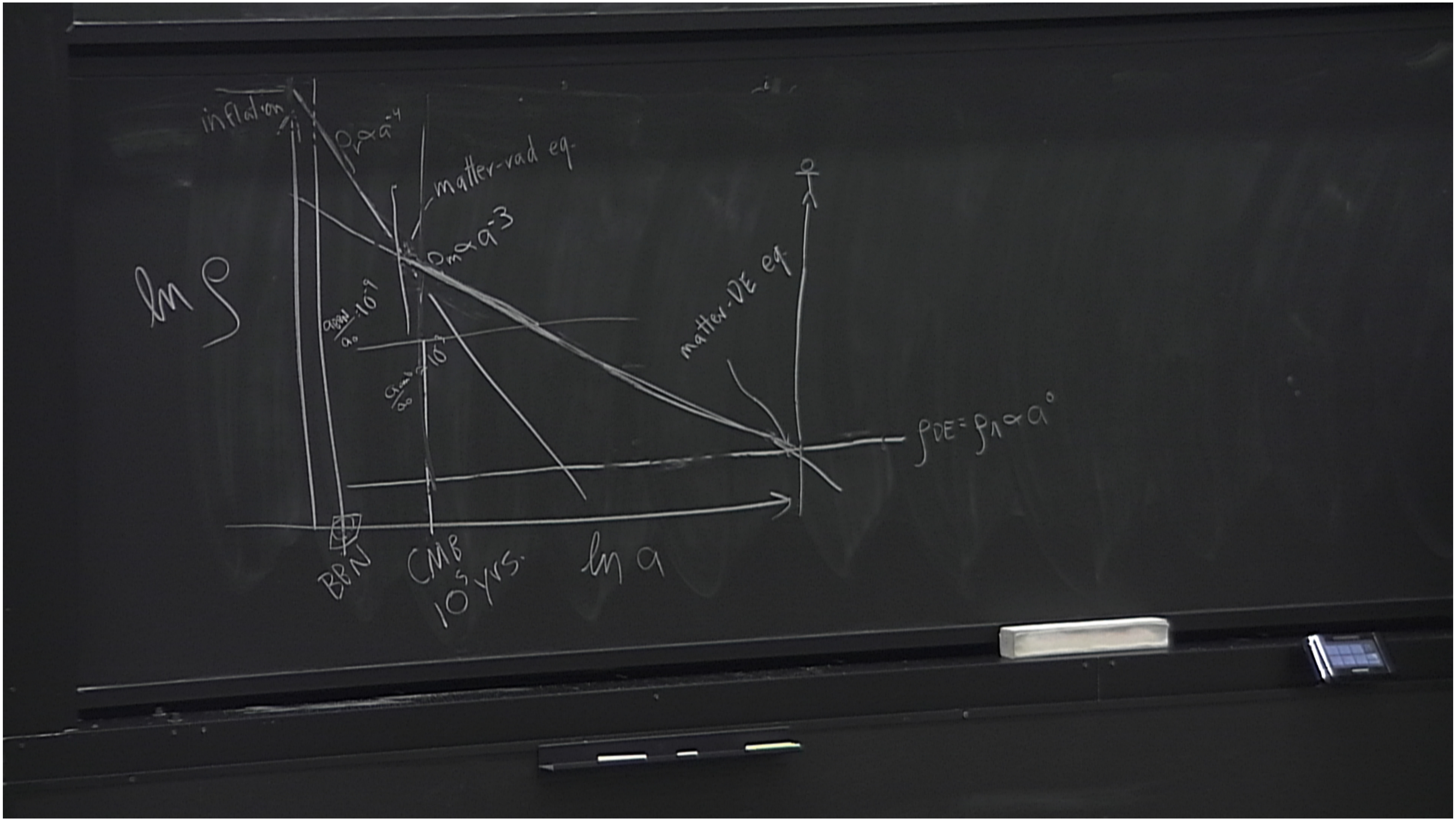
$$dE = dQ - P\,dV$$



$$d(\rho V) = -w\rho\, dV = \rho\, dV + V\, d\rho$$

$$V\, d\rho = -(1+w)\rho\, dV$$

$$\frac{d\rho}{\rho} = -(1+w)\frac{dV}{V} \Longrightarrow \boxed{\rho \propto V^{-(1+w)} = a^{-3(1+w)}}$$



inflation
$\rho_r \propto a^{-4}$
matter-rad eq.
$\rho_m \propto a^{-3}$
$\ln \rho$
matter-DE eq.
$\rho_{DE} = \rho_\Lambda \propto a^0$
BBN
CMB
10^5 yrs.
$\ln a$