

Title: 13/14 PSI - Cosmology Review - Lecture 3

Date: Jan 29, 2014 11:30 AM

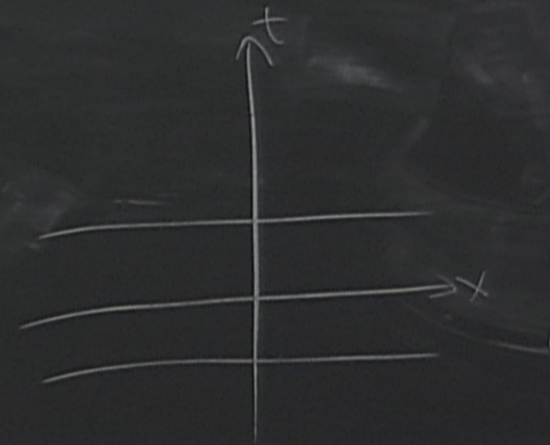
URL: <http://pirsa.org/14010067>

Abstract:

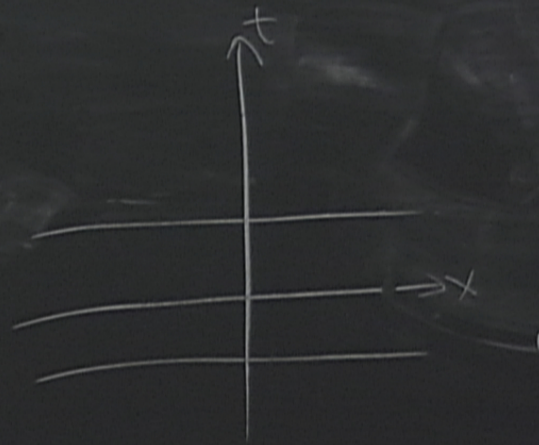
- 1) Max Sym
- 2) FRW
- 3) BH
 - Schwarzschild
 - Kerr

$$10 = 4 + 3 + 3$$

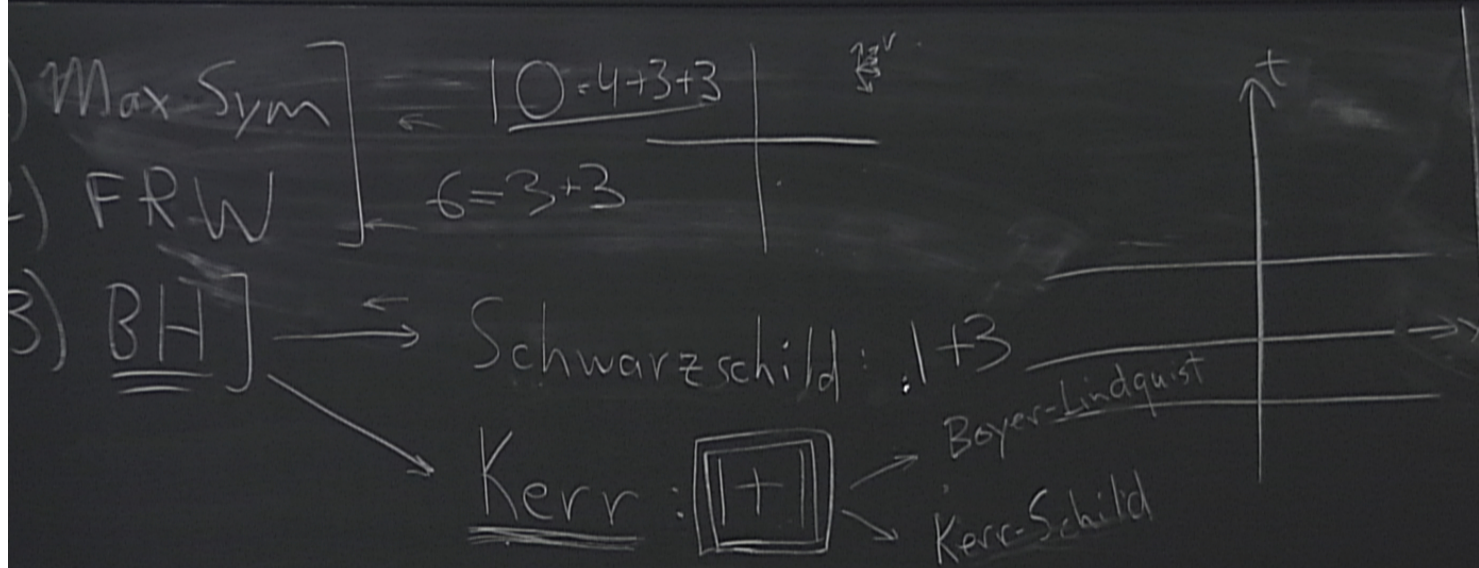
2×3



- 1) Max Sym $\left[\begin{array}{l} 10 = 4 + 3 + 3 \end{array} \right]$
- 2) FRW $\left[\begin{array}{l} 6 = 3 + 3 \end{array} \right]$
- 3) BH $\left[\begin{array}{l} \rightarrow \text{Schwarzschild} \\ \rightarrow \text{Kerr} \end{array} \right]$



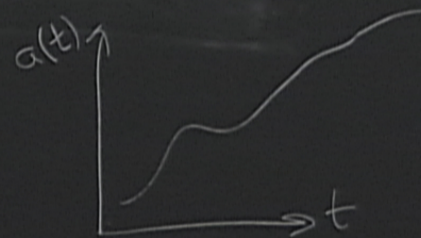
1) Max Sym $\left[\begin{array}{c|c} 10 = 4+3+3 & 2+2+2 \end{array} \right]$
 2) FRW $\left[\begin{array}{c|c} 6 = 3+3 & \cdot \end{array} \right]$
 3) BH $\left[\begin{array}{c} \rightarrow \text{Schwarzschild: } 1+3 \\ \searrow \text{Kerr: } \boxed{\boxed{1+1}} \end{array} \right]$



FRW

$g_{ab}^{(p,q)K}$

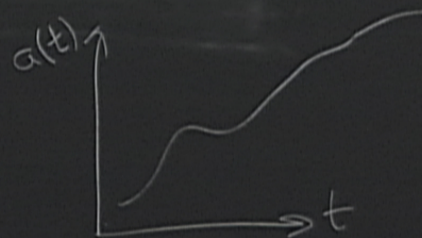
$$ds^2 = -dt^2 + a^2(t) \left[g_{ab}^{(3,0)K} dx^a dx^b \right] = -dt^2 + a^2(t) \left[\delta_{ij} + \frac{K x_i x_j}{\rho^2 - K x^2} \right] dx^i dx^j$$



FRW

$g_{ab}^{(p,q)K}$

$$ds^2 = -dt^2 + a^2(t) \left[g_{ab}^{(3,0)K} dx^a dx^b \right] = -dt^2 + \tilde{a}^2(t) \left[\delta_{ij} + \frac{K \tilde{x}_i \tilde{x}_j}{1 - K \tilde{x}^2} \right] d\tilde{x}^i d\tilde{x}^j$$



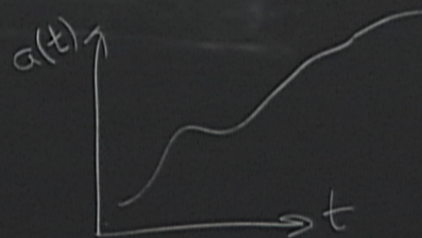
$$x^i = \int \tilde{x}^i$$

$$a = \frac{1}{f} \tilde{a}$$

FRW

$g_{ab}^{(p,q)K}$

$$ds^2 = -dt^2 + a^2(t) \left[g_{ab}^{(3,0)K} dx^a dx^b \right] = -dt^2 + \tilde{a}^2(t) \left[\delta_{ij} + \frac{K \tilde{x}_i \tilde{x}_j}{1 - K \tilde{x}^2} \right] d\tilde{x}^i d\tilde{x}^j$$



$$x^i = \int \tilde{x}^i$$

$$a = \frac{1}{f} \tilde{a}$$

$$\begin{aligned}x^1 &= r \sin \theta \cos \varphi \\x^2 &= r \sin \theta \sin \varphi \\x^3 &= r \cos \theta\end{aligned}$$

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

$$\begin{aligned}x^1 &= r \sin \theta \cos \varphi \\x^2 &= r \sin \theta \sin \varphi \\x^3 &= r \cos \theta\end{aligned}$$

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$$

$$+ a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

$$d\Omega^2 = d\Theta^2 + \sin^2 \Theta d\varphi^2$$

$$r = S_K(\chi) = \begin{cases} \sin \chi & K=+1 \\ \chi & K=0 \\ \sinh \chi & K=-1 \end{cases}$$

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2 \right]$$

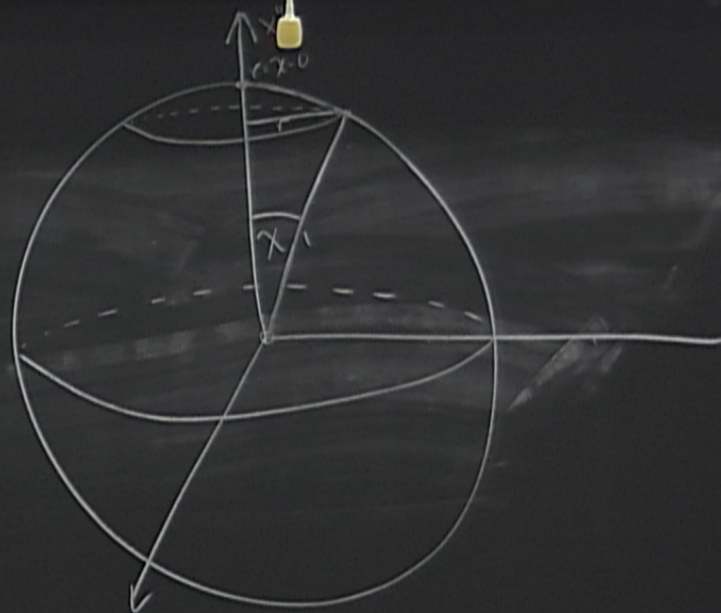
$$= -dt^2 + a^2(t) \left[dx^2 + S_K^2(x) d\Omega^2 \right]$$

$$d\Omega^2 = d\Theta^2 + \sin^2\Theta d\varphi^2$$

$$r = S_K(x) = \begin{cases} \sin x & K=+1 \\ x & K=0 \\ \sinh x & K=-1 \end{cases}$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$$

$$r = S_K(\chi) = \begin{cases} \sin \chi & K=+1 \\ \chi & K=0 \\ \sinh \chi & K=-1 \end{cases}$$



$$\begin{aligned} & r \sin \theta \cos \varphi \\ & r \sin \theta \sin \varphi \\ & r \cos \theta \end{aligned}$$

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2 \right]$$

$$= -dt^2 + a^2(t) [dx^2 + S_K^2(x) d\Omega^2]$$

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2$$

$$r = S_K(x) = \begin{cases} \sin x \\ x \\ \sinh x \end{cases}$$

$$t = a d\eta$$

$$= -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right]$$

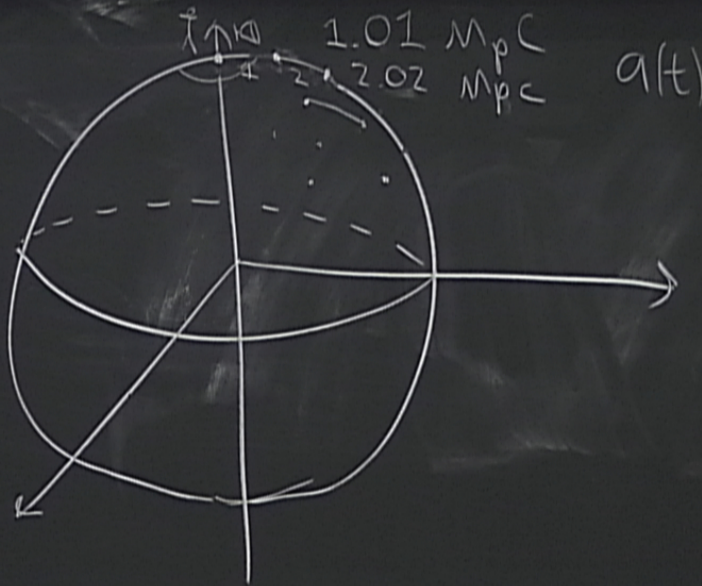
$$= -dt^2 + a^2(t) \left[dx^2 + S_K^2(x) d\Omega^2 \right]$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$$

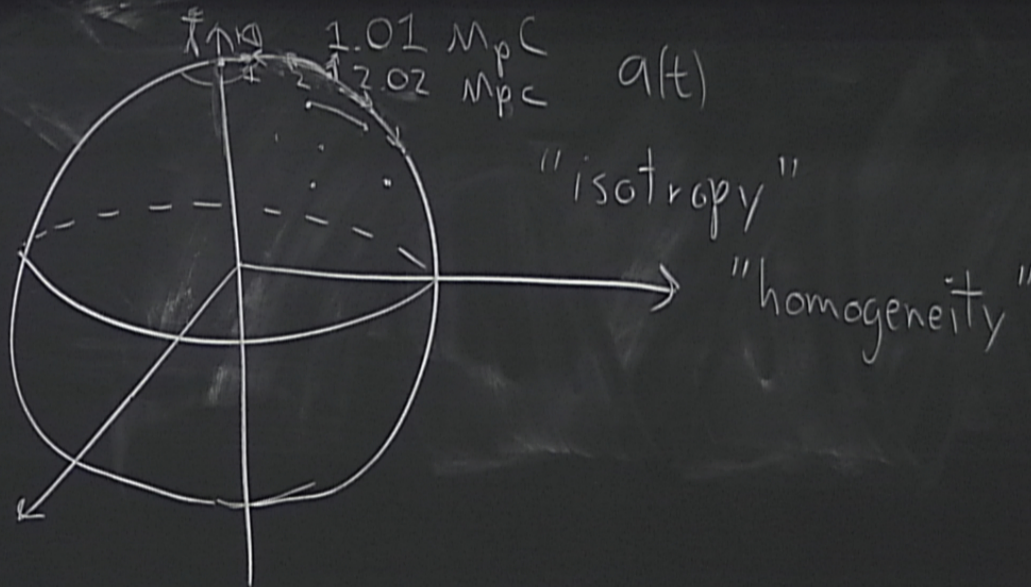
$$r = S_K(x) = \begin{cases} \sin x & K=+1 \\ x & K=0 \\ \sinh x & K=-1 \end{cases}$$

$$dt = a d\eta \Rightarrow \eta - \eta_0 = \int_{t_0}^t \frac{dt'}{a(t')}$$

$$K=0$$



$$K=0$$



atwistic
n-Relativistic

$$U^{\mu} \nabla_{\mu} U^{\nu} = 0$$

$$\begin{aligned} ' &= \frac{d}{d\lambda} \\ &= \frac{d}{dt} \end{aligned}$$

$$\begin{aligned} x^{\mu}(\lambda) \\ p^{\mu} = \frac{dx^{\mu}}{d\lambda} \quad T = m\lambda \end{aligned}$$

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t) g_{ij}^{(3,0)K} dx^i dx^j \\ L &= \frac{1}{2} \left[-\dot{t}^2 + a^2(t) g_{ij}^{(3,0)K} \dot{x}^i \dot{x}^j \right] \end{aligned}$$

$$\frac{\partial L}{\partial t} = \left(\frac{\partial L}{\partial t'} \right)' = a \dot{a} g_{ij}^{(3,0)K} p^i p^j = \frac{\dot{a}}{a} \underbrace{a^2 g_{ij}^{(3,0)K}}_{H p^2} p^i p^j = -p^0'$$

$$-p^{0'} = H P^2 \quad m^2 = g_{\mu\nu} p^\mu p^\nu = -(p^0)^2 + \underbrace{g_{ij} p^i p^j}_{p^2} = -(p^0)^2 + P^2 \Rightarrow 0 = -2p^0 dp^0 + 2P dP$$

$$\begin{aligned}
 -p^{0'} &= H P^2 & m^2 &= g_{\mu\nu} p^\mu p^\nu = -(p^0)^2 + \underbrace{g_{ij} p^i p^j}_{P^2} = -(p^0)^2 + P^2 \Rightarrow 0 = -2p^0 dp^0 + 2P dP \\
 &= -\frac{P P'}{p^0} = -\cancel{P} \frac{dP}{dt} = \frac{1}{a} \frac{da}{dt} P^2 \Rightarrow \frac{dP}{P} = -\frac{da}{a} \Rightarrow \ln P = -\ln a + C \\
 & & & P \propto \frac{1}{a}
 \end{aligned}$$

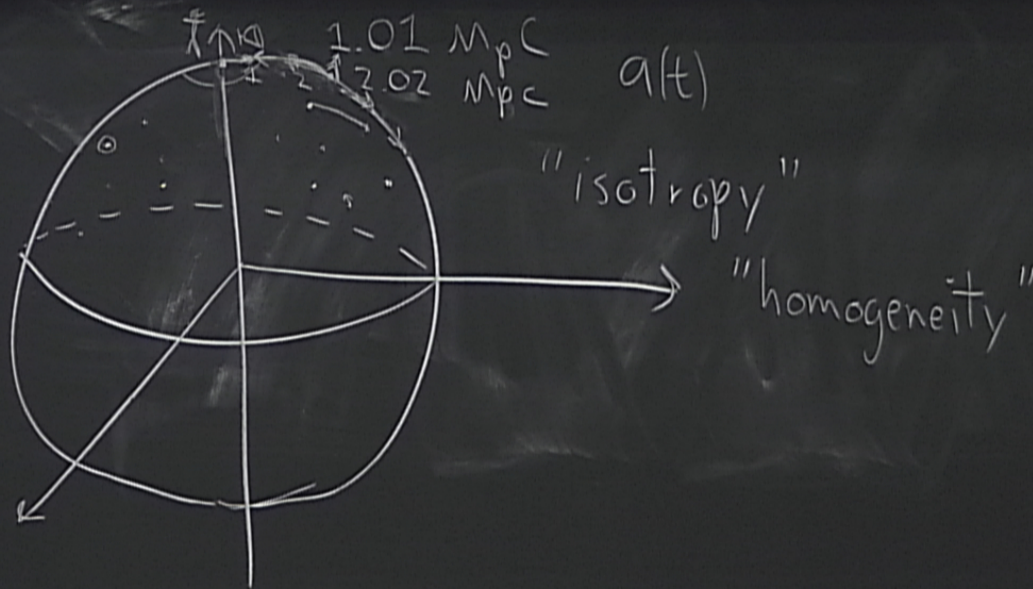
$$-p^{\circ'} = H P^2 \quad m^2 = g_{\mu\nu} p^\mu p^\nu = -(p^{\circ'})^2 + \underbrace{g_{ij} p^i p^j}_{P^2} = -(p^{\circ'})^2 + P^2 \Rightarrow 0 = -2p^{\circ'} \frac{dp^{\circ'}}{d\lambda} + 2P \frac{dP}{d\lambda}$$

$$= -\frac{P p^{\circ'}}{p^{\circ'}} = -\cancel{P} \frac{dP}{dt} = \frac{1}{a} \frac{da}{dt} P^{\circ'} \Rightarrow \frac{dP}{P} = -\frac{da}{a} \Rightarrow \ln P = -\ln a + C$$

$P \propto \frac{1}{a}$

$$P = m v \quad v \propto \frac{1}{a}$$

"co
coords" $k=0$
"comoving observer"



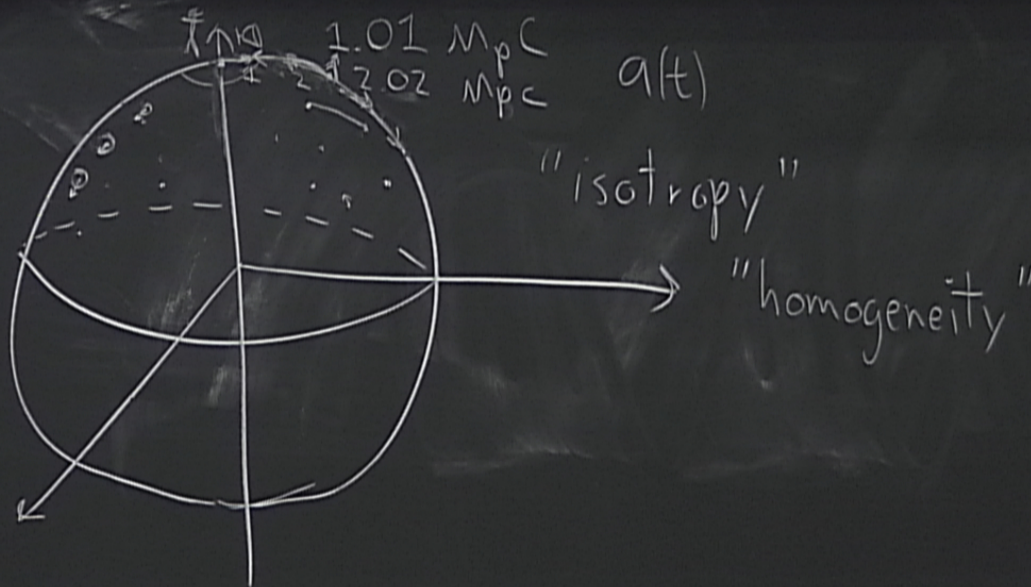
$$p \propto \frac{1}{a}$$

Relativistic
Non-Relativistic

$$x^{\mu}(\tau)$$

$$p^{\mu} =$$

"co
coords" $K=0$
"comoving observer"



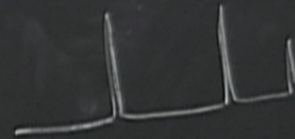
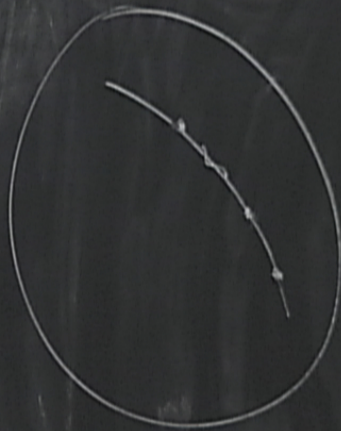
$$p \propto \frac{1}{a}$$

Relativistic
Non-Relativistic

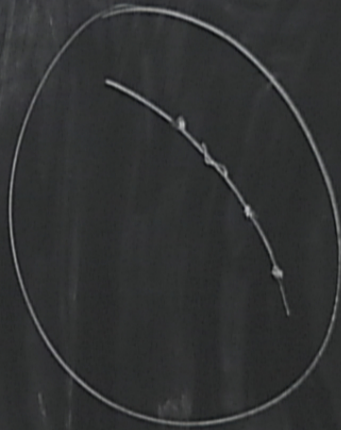
$$x^{\mu}(\tau)$$

$$p^{\mu} =$$

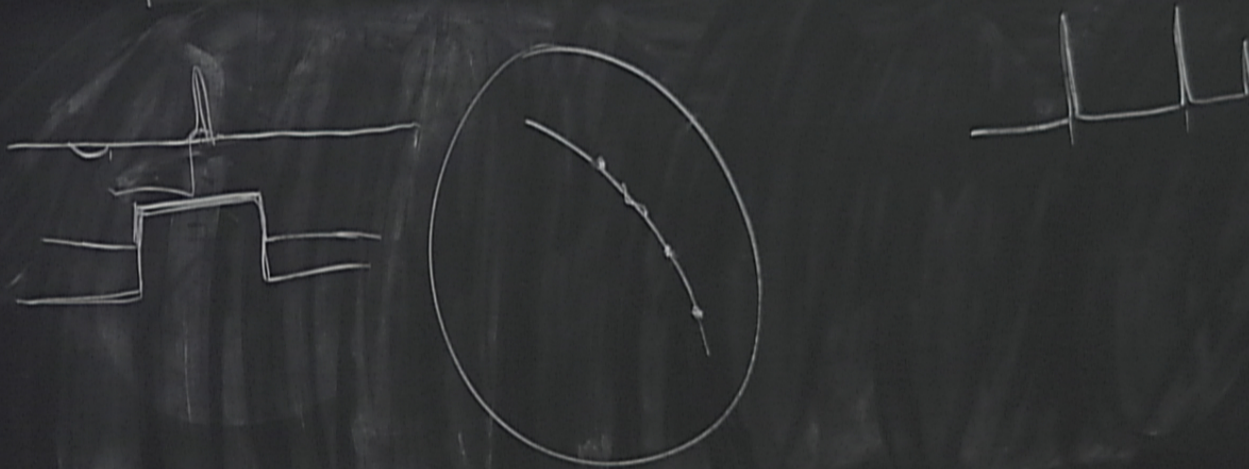
$$p = E \quad E \quad \omega \propto \frac{1}{a} \quad \lambda \propto a$$



$$p = E \quad E \quad \omega \propto \frac{1}{a} \quad \lambda \propto a$$



$$p = E \quad E \quad \omega \propto \frac{1}{a} \quad \lambda \propto a$$



$$S = -m \int \left(\frac{d\vec{t}}{d\lambda} \right) d\lambda$$

$\nwarrow ds^2$
 $\downarrow$
 L