

Title: 13/14 PSI - Foundations of Quantum Mechanics - Lecture 15

Date: Jan 24, 2014 11:30 AM

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Abstract:

Regrets. I have a few.

Gleason's theorem.

Bell's theorem.

PIRSA — QBism/Copenhagen

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Gleason's theorem.

Bell's theorem.

PIRSA — QBism/Copenhagen

Weak measurements.

PR boxes

Fapeen Rohlich

Tirolson bound $2\sqrt{2}$

Sorkin measure theory.

Feynman Rohlich

Tsirelson.)

The shape of Quantum state space

Know $K_n = N_n^2$ Convex set of this dimension

Normalised state, $K_n - 1 = N_n^2 - 1$

Feynman Rohlich

Tsirelson.)

The shape of Quantum state space

Know $K_n = N_n^2$ Convex set of this dimension

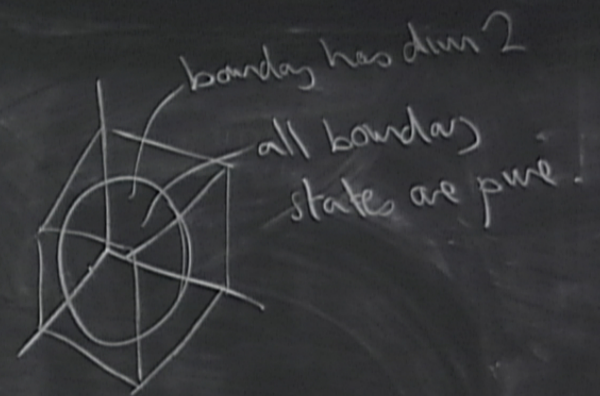
Normalised states $K_n - 1 = N_n^2 - 1$

boundary of normalised states $K_n - 2 = N_n^2 - 2$

Consider $N=2$

$$\dim \text{of boundary} = N^2 - 2 = 2$$

$$\dim \text{ of pure state space} = L = 2N - 2 = 2$$



boundary has dim 2

all boundary
states are pure.

If $N > 2$

$$\text{Boundary dim} = \underbrace{N^2 - 2}_{\text{grows as } N^2} > \underbrace{2N - 2}_{\text{grows as } N} = L$$

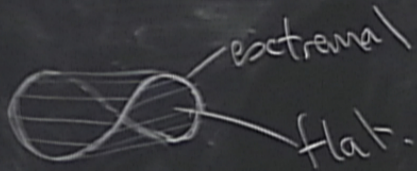
$\Rightarrow$ there are flat parts to boundary.

$$\hat{\rho} = \sum_{n \in S} \lambda_n |n\rangle\langle n|$$

$\lambda_n \geq 0 \quad \sum \lambda_n = 1$

S is missing one
of $n=1$ to N .

3D case with properties like this



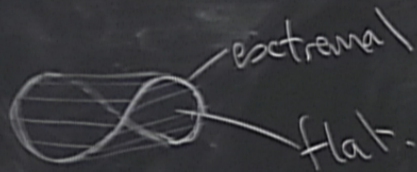
Adapted from Appleby, Ericsson, Fuchs.

State is pure iff

$$\rho = \rho^2$$

$$\rho = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} \Leftrightarrow \rho = \begin{pmatrix} p_1 \\ \vdots \\ p_k \end{pmatrix} \quad K=N^2$$

3D case with properties like this



Adapted from Appleby, Ericsson, Fuchs.

State is pure iff

$$\rho = \rho^2$$

$$\rho = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

linear $\Leftrightarrow$ $P =$

$$\begin{pmatrix} p_1 \\ \vdots \\ p_k \\ \vdots \\ p_K \end{pmatrix}$$

$$K = N^2$$

$\Rightarrow$

from Appleby, Ericsson, Fuchs.

is pure iff

$$P = P^2$$

linear

$$\Leftrightarrow P =$$

$$\begin{pmatrix} p_1 \\ \vdots \\ p_k \\ \vdots \\ p_K \end{pmatrix}$$

$$K = N^2$$

$$\Rightarrow p_k = \text{quadratic fn}(\{p_k\})$$
$$= q_k(\{p_k\})$$

on, Fuchs.

$$\Rightarrow P_k = \text{quadratic in } \{P_{k'}\} \\ = q_k(\{P_{k'}\})$$

have
 N^2
equations.

$$K = N^2$$

There is a theorem (Jones & Linden 2004)

State is pure iff

$$\text{tr}(\rho^2) = \text{tr}(\rho^3) = 1$$

remarkable result.

There is a theorem (Jones & Linden 2004)

State is pure iff

$$\text{tr}(\rho^2) = \text{tr}(\rho^3) = 1$$

remarkable result. Rescued from obscurity
by Fuchs.

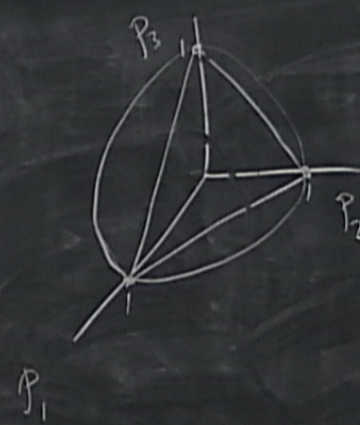
State in pure iff

$$\text{quadratic tn}(\{p_k\}) = 1$$

$$\text{cubic tn}(\{p_k\}) = 1$$

Only two equations.

C.f. classical case.



$$\sum_k p_k = 1$$

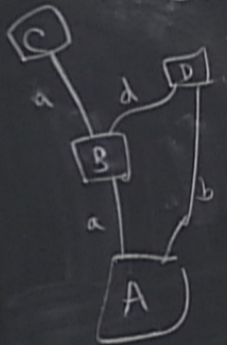
$$\text{quadratic tn}(\{p_k\}) = 1$$

Fuchs.

Boundary of normalised states

$$K_a - 2 = N_a - 2$$

Retormulation of QT (the operator tensor reformulation)



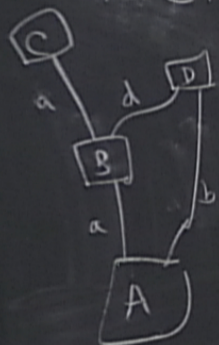
$$\text{Prob} \left(A_{12}^{a,b_2} B_{a_1}^{a_3 d_4} C_{a_3} D_{d_1 b_2}^{d_4 b_2} \right) = A_{12}^{a,b_2} B_{a_1}^{a_3 d_4} C_{a_3} D_{d_1 b_2}^{d_4 b_2}$$

very general.

Boundary of normalised states

$$K_a - 2 = N_a - 2$$

Retormulation of QT (the operator tensor reformulation)



$$\text{Prob}(A^{a_1 b_2} B^{a_2 d_3} C^{a_3} D^{d_4 b_2}) = A^{a_1 b_2} B^{a_2 d_3} C^{a_3} D^{d_4 b_2}$$

very general.

$$= \hat{A}^{a_1 b_2} \hat{B}^{a_2 d_3} \hat{C}^{a_3} \hat{D}^{d_4 b_2}$$

$$-2 = N_n - 2$$

...mutation)

...very general.

$$C_{a_3} D_{d_4 b_2}$$

$\updownarrow$ $\mathbb{C}$ -linear map.

$$C_{a_3} D_{d_4 b_2}$$

$$\sum_{a_1}^{a_3 d_4}$$

is an Hermitian operator on

$$H_{a_1}^{\otimes a_3} \otimes H_{d_4}^{\otimes d_4}$$

where H_{a_1} and H_{d_4} have dim N_{a_1}

$$\hat{A}_{a_3 b_2} \hat{B}_{c_3 a_4}$$

partial trace

The constraints on physical operators.

$$(i) \quad 0 \leq \hat{B}_{a_1}^{b_2 T}$$

corresponds to $0 \leq \text{prob}$

$$(ii) \quad \hat{B}_{a_1}^{b_2} \hat{I}_{b_2} \leq I$$

corresponds to $\text{prob} \leq 1$

constraints on physical operators.

$$0 \leq \hat{B}_{a_1}^{b_2^T}$$

corresponds to $0 \leq \text{prob}$

$$\hat{B}_{a_1}^{b_2} \hat{I}_{b_2} \leq 1$$

corresponds to $\text{prob} \leq 1$



$$K=4$$

$$P_{2+} + P_{2-} = 1$$