Title: 13/14 PSI - Gravitational Physics Review - Lecture 10

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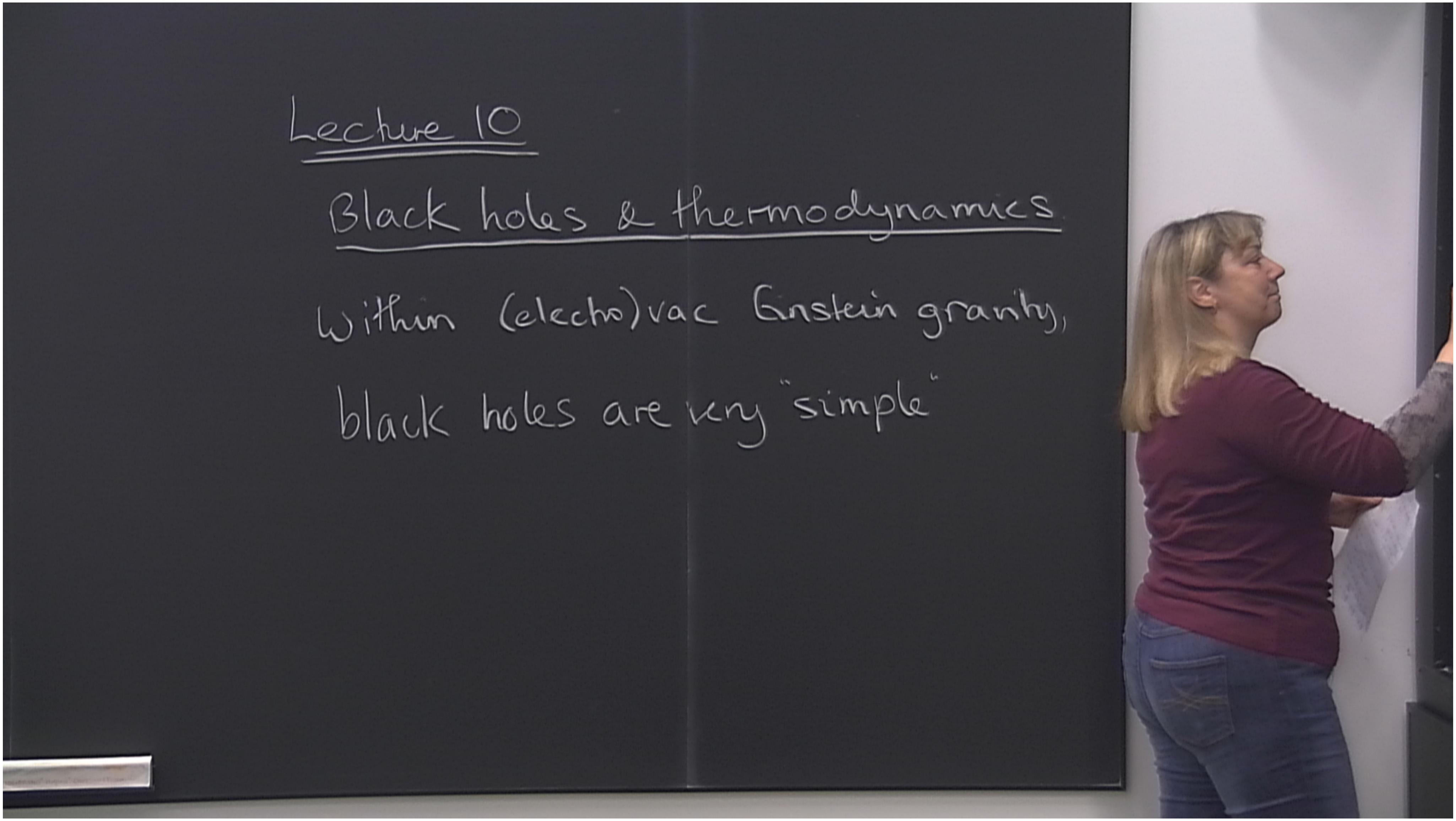
Abstract:


Lecture 10
Black holes & thermodynamics
Within (electro)vac Einstein gravity,
black holes are very "simple"

Kerr

$$ds^2 = dt^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \frac{2GMr}{\Sigma}\left(dt - a\sin^2\theta d\varphi\right)^2$$

$$= \frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\varphi\right)^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \frac{\sin^2\theta}{\Sigma}\left((r^2+a^2)d\varphi - a\,dt\right)^2$$

$\Delta = r^2 + a^2 - 2GMr$ $\qquad$ $a = J/M$ - angular momentum

$\Sigma = r^2 + a^2\cos^2\theta$ $\qquad$ $\frac{\partial}{\partial t}, \frac{\partial}{\partial \phi}$ killing vectors

Kerr

$$ds^2 = dt^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \frac{2GMr}{\Sigma}(dt - a\sin^2\theta\, d\varphi)^2 - (r^2+a^2)\sin^2\theta\, d\varphi^2$$

$$= \frac{\Delta}{\Sigma}(dt - a\sin^2\theta\, d\varphi)^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 - \frac{\sin^2\theta}{\Sigma}\left((r^2+a^2)d\varphi - a\,dt\right)^2$$

$\Delta = r^2 + a^2 - 2GMr$

$\Sigma = r^2 + a^2\cos^2\theta$

$a = J/M$ - angular momentum

$\frac{\partial}{\partial t}, \frac{\partial}{\partial \phi}$ killing vectors

$d\varphi - a\,dt)^2.$

m.

- Very few charges: M, a + Q if charged

$$\Delta = r^2 + a^2 - 2GMr + GQ^2$$

$$\text{Area} = 4\pi(r_+^2 + a^2)$$

$$r_+ = GM + \sqrt{G^2M^2 - a^2 - GQ^2}$$

Consider adding to the black hole a particle (possibly charged)

$$\delta A = 8\pi\left(r_+\delta r_+ + a\delta a\right)$$

$$= 8\pi r_+\left[G\delta M + \frac{G^2 M\delta M - a\delta a - GQ\delta Q}{r_+ - GM}\right] + 8\pi a\delta a$$

$$= 8\pi\left[\frac{G r_+^2}{r_+ - GM}\delta M - \frac{GMa\delta a}{r_+ - GM} - \frac{GQr_+\delta Q}{r_+ - GM}\right]$$

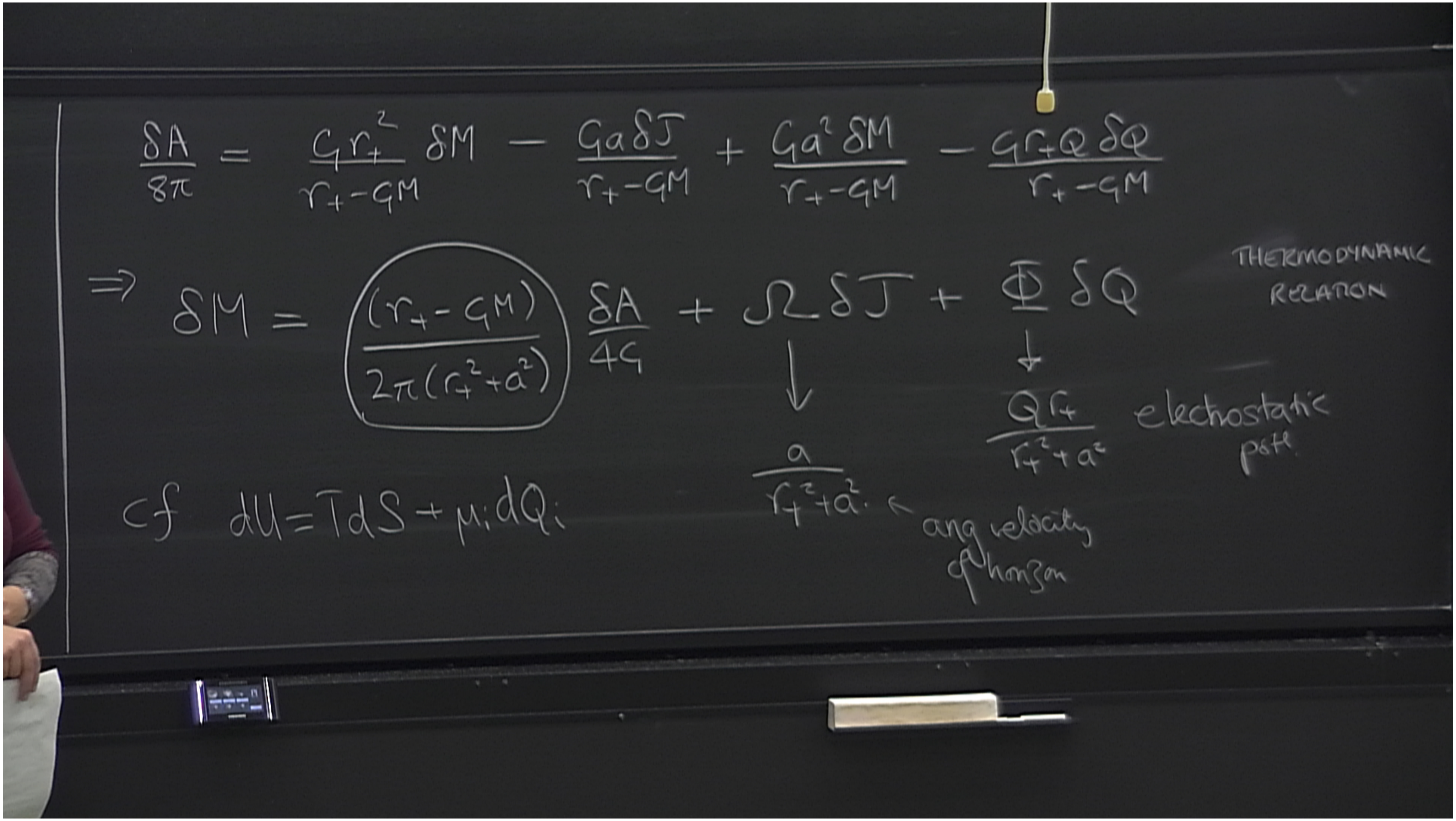
But $\delta a = \dfrac{\delta J}{M} - \dfrac{J\delta M}{M^2}$

$$\frac{\delta A}{8\pi} = \frac{G r_+^2}{r_+ - GM}\,\delta M - \frac{G a\,\delta J}{r_+ - GM} + \frac{G a^2\,\delta M}{r_+ - GM} - \frac{G r_+ Q\,\delta Q}{r_+ - GM}$$

$$\Rightarrow \quad \delta M = \frac{(r_+ - GM)}{2\pi(r_+^2 + a^2)}\,\frac{\delta A}{4G} + \Omega\,\delta J + \Phi\,\delta Q$$

$$\Omega \longrightarrow \frac{a}{r_+^2 + a^2}$$ ← ang velocity of horizon

$$\Phi \longrightarrow \frac{Q r_+}{r_+^2 + a^2}$$ electrostatic pot'l


$\frac{\delta A}{8\pi} = \frac{G r_+^2}{r_+ - GM}\delta M - \frac{G a \delta J}{r_+ - GM} + \frac{G a^2 \delta M}{r_+ - GM} - \frac{G r_+ Q \delta Q}{r_+ - GM}$
$\Rightarrow \delta M = \frac{(r_+ - GM)}{2\pi(r_+^2 + a^2)} \frac{\delta A}{4G} + \Omega \delta J + \Phi \delta Q$
THERMODYNAMIC RELATION
$\frac{a}{r_+^2 + a^2}$ ← ang velocity of horizon
$\frac{Q r_+}{r_+^2 + a^2}$ electrostatic pot.
cf $dU = TdS + \mu_i dQ_i$

$$\frac{\delta A}{8\pi} = \frac{G r_+^2}{r_+ - GM}\,\delta M - \frac{G a\,\delta J}{r_+ - GM} + \frac{G a^2\,\delta M}{r_+ - GM} - \frac{G r_+ Q\,\delta Q}{r_+ - GM}$$

$$\Rightarrow \quad \delta M = \underbrace{\frac{(r_+ - GM)}{2\pi(r_+^2 + a^2)}}_{T}\,\frac{\delta A}{4G} + \underbrace{\Omega}_{\frac{a}{r_+^2+a^2}}\,\delta J + \underbrace{\Phi}_{\frac{Q r_+}{r_+^2+a^2}}\,\delta Q$$

THERMODYNAMIC RELATION

$\frac{a}{r_+^2+a^2}$ ← ang velocity of horizon

$\frac{Q r_+}{r_+^2+a^2}$ electrostatic pot.

cf $dU = T\,dS + \mu_i\,dQ_i$

Suggests thermodynamic interpretation with

$$T = \frac{r_+ - GM}{2\pi(r_+^2 + a^2)} \xrightarrow{\text{SCH}} \frac{1}{8\pi GM}$$

$$S = \frac{A}{4G}$$

• Euclidean path integral approach

Construct partition function

$$Z \sim \mathrm{tr}\, e^{-\beta H} \qquad \beta = 1/kT$$

$$H \sim \int d^3x\, \mathcal{H} \sim \frac{1}{\beta} \int d^3x\, dt\, \mathcal{H}$$

- Euclidean path integral approach

Construct partition function

$$Z \sim \operatorname{tr} e^{-\beta H} \qquad \beta = 1/kT$$

$$H \sim \int d^3x\, \mathcal{H} \sim \frac{1}{\beta}\int d^3x\, dt\, \mathcal{H} \sim \frac{1}{\beta} I_E$$

I_E = Euclidean action

$$= \int \frac{d^4x\sqrt{g}\,R}{16\pi G} + \int \frac{d^3x\,\sqrt{h}\,K}{8\pi G}$$

- Euclidean path integral approach

Construct partition function

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I_E = Euclidean action

$$= \int \frac{d^4x\sqrt{g}\,R}{16\pi G} + \int \frac{d^3x\sqrt{h}\,K}{8\pi G}$$

for Einstein gravity.

Expect Z to be dominated by saddle point.

$$Z \sim \sum_{\substack{\text{classical} \\ \text{solns}, \beta}} e^{-I_{E,\text{class}}}$$

$R=0$ for SCH & Kerr so

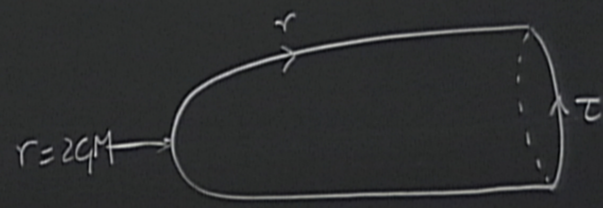
what is I_E?

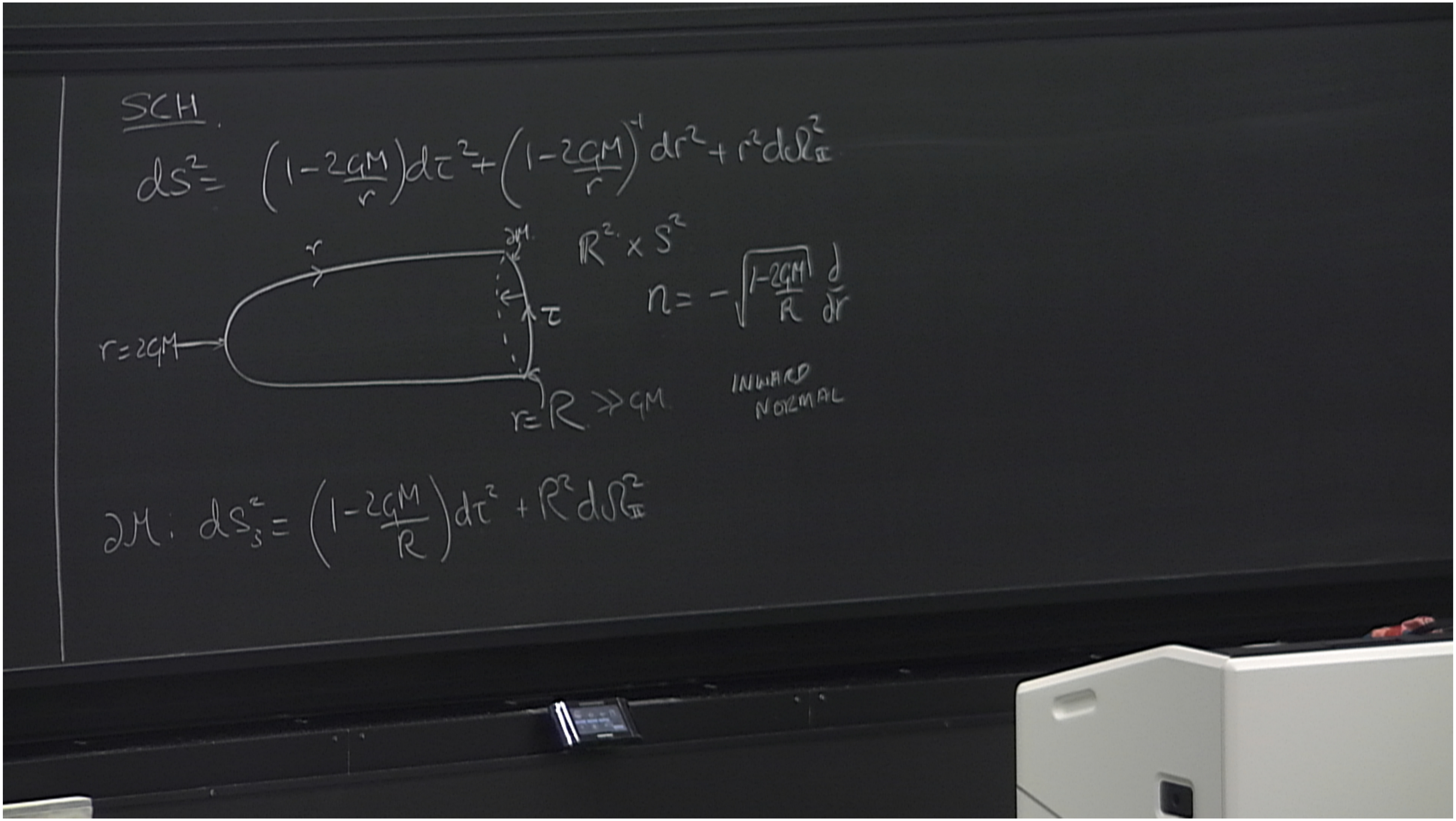
$$I_E = \int_{\partial M} d^3x \, \frac{\sqrt{h}\,K}{8\pi G}$$

SCH.

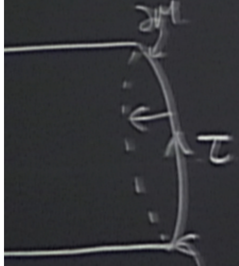
$$ds^2 = \left(1-\frac{2GM}{r}\right)d\tau^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$$

$\mathbb{R}^2 \times S^2$




SCH.
$ds^2 = \left(1-\frac{2GM}{r}\right)d\tau^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2 d\Omega_{II}^2$
∂M
r
τ
$r=2GM$
$r=R \gg GM$
$\mathbb{R}^2 \times S^2$
$n = -\sqrt{1-\frac{2GM}{R}}\,\frac{\partial}{\partial r}$
INWARD NORMAL
$\partial M: \; ds_3^2 = \left(1-\frac{2GM}{R}\right)d\tau^2 + R^2 d\Omega_{II}^2$

$$\ldots d\tau^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$$



$\mathbb{R}^2 \times S^2$

$$n = -\sqrt{1-\frac{2GM}{R}}\,\frac{\partial}{\partial r}$$

INWARD NORMAL

$r = R \gg GM$

$$\ldots\Big)\, d\tau^2 + R^2 d\Omega_{II}^2$$

$$K = \nabla_a n^a = \frac{1}{r^2}\partial_r\left(r^2 n^r\right)$$

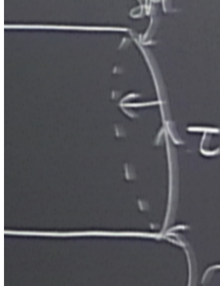
$$= -\frac{2}{R}\sqrt{1-\frac{2GM}{R}} - \frac{GM}{R^2}\frac{1}{\sqrt{1-\frac{2GM}{R}}}$$

$$\int_{\partial M} K\sqrt{h}\, \underline{d\tau\, d\theta\, d\varphi} = -4\pi\beta R^2\left[\frac{2}{R}\left(1-\frac{2GM}{R}\right) + \frac{GM}{R^2}\right]$$

$$= -4\pi\beta\left[2R^2 - 3GM\right]$$

$$\ldots \tau^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$$

∂M



$\mathbb{R}^2 \times S^2$

$$n = -\sqrt{1-\frac{2GM}{R}}\,\frac{\partial}{\partial r}$$

τ

$r = R \gg GM$

INWARD NORMAL

$$\ldots\Big) d\tau^2 + R^2 d\Omega_{II}^2$$

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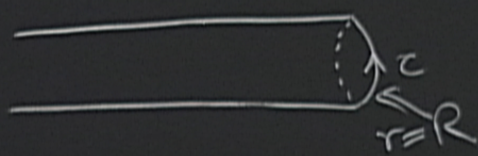
$$\int_{\partial M} K\sqrt{h}\, d\tau\, d\theta\, d\varphi = -4\pi\beta R^2\left[\frac{2}{R}\left(1-\frac{2GM}{R}\right)+\frac{GM}{R^2}\right]$$

$$= -4\pi\beta\left[2R^2 - 3GM\right]$$

$\propto R^2$ at large R!

Look at the flat space result:



$$ds^2 = \left(1 - \frac{2GM}{R}\right) d\tau^2 + dr^2 + r^2 d\Omega_{II}^2$$

same prefactor as Sch.

same periodicity.

Look at the flat space result:

$$n = -\frac{\partial}{\partial r}$$

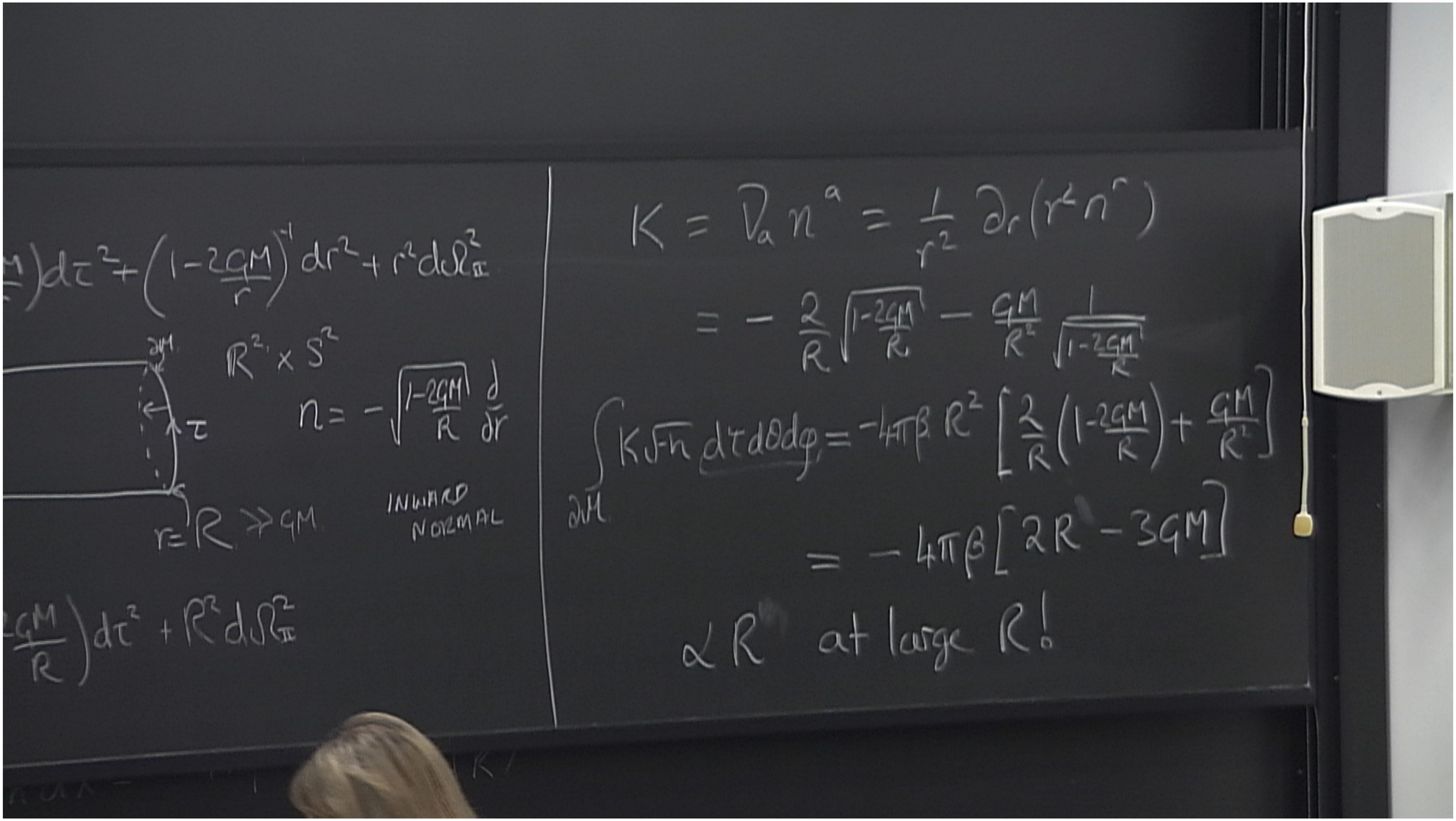
$$ds^2 = \left(1-\frac{2GM}{R}\right) d\tau^2 + dr^2 + r^2 d\Omega_{II}^2$$

same prefactor as Sch.

same periodicity.

$$K_0 = -\frac{2}{R}$$

$$\int K_0 \sqrt{h}\, d^3x = -4\pi\beta R^2 \sqrt{1-\frac{2GM}{R}}$$

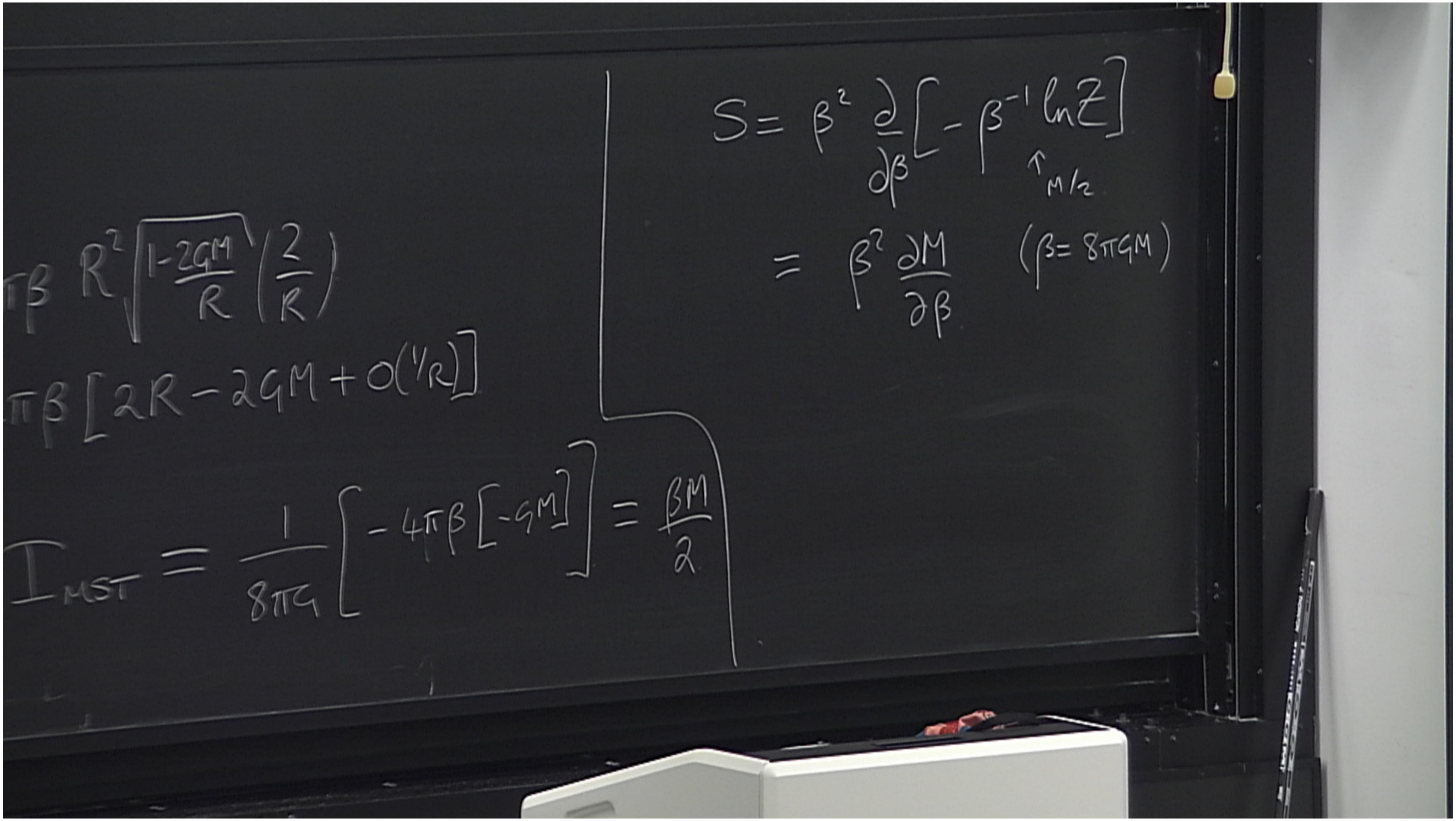

$\ldots)d\tau^2 + \left(1-\frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega_{II}^2$
∂M
$\mathbb{R}^2 \times S^2$
τ
$n = -\sqrt{1-\frac{2GM}{R}}\,\frac{d}{dr}$
$r = R \gg GM$
INWARD NORMAL
$\ldots\frac{GM}{R}\Big) d\tau^2 + R^2 d\Omega_{II}^2$
$K = \nabla_a n^a = \frac{1}{r^2}\partial_r(r^2 n^r)$
$= -\frac{2}{R}\sqrt{1-\frac{2GM}{R}} - \frac{GM}{R^2}\frac{1}{\sqrt{1-\frac{2GM}{R}}}$
$\int_{\partial M} K\sqrt{h}\, d\tau\, d\theta\, d\varphi = -4\pi\beta R^2\left[\frac{2}{R}\left(1-\frac{2GM}{R}\right) + \frac{GM}{R^2}\right]$
$= -4\pi\beta\left[2R - 3GM\right]$
$\propto R$ at large R!

$$K_0 = -\frac{2}{R}$$

$$\int K_0 \sqrt{h}\, d^3x = -4\pi\beta\, R^2 \sqrt{1-\frac{2GM}{R}} \left(\frac{2}{R}\right)$$

$$\simeq -4\pi\beta\left[2R - 2GM + O(1/R)\right]$$

Hence $I_{SCH} - I_{MST} = \frac{1}{8\pi G}\left[-4\pi\beta\left[-GM\right]\right] = \frac{\beta M}{2}$


$S = \beta^2 \frac{\partial}{\partial \beta}\left[-\beta^{-1} \ln Z\right]$
$M/2$
$= \beta^2 \frac{\partial M}{\partial \beta} \quad (\beta = 8\pi G M)$
$\pi\beta R^2 \sqrt{1-\frac{2GM}{R}}\left(\frac{2}{R}\right)$
$\pi\beta\left[2R - 2GM + O(1/R)\right]$
$I_{MST} = \frac{1}{8\pi G}\left[-4\pi\beta\left[-GM\right]\right] = \frac{\beta M}{2}$

$\propto R$ at large R!

$$\ldots \pi\beta\, R^2 \sqrt{1-\frac{2GM}{R}}\left(\frac{2}{R}\right)$$

$$\ldots \pi\beta \left[2R - 2GM + O(1/R)\right]$$

$$I_{MST} = \frac{1}{8\pi G}\left[-4\pi\beta\left[-GM\right]\right] = \frac{\beta M}{2}$$

$$S = \beta^2 \frac{\partial}{\partial\beta}\left[-\beta^{-1} \underset{\uparrow\, M/2}{\ln Z}\right]$$

$$= \frac{\beta^2}{2}\frac{\partial M}{\partial \beta} \qquad (\beta = 8\pi G M)$$

$$= \frac{\beta^2}{16\pi G} = 4\pi G M^2$$

$$= \frac{4\pi (2GM)^2}{4G}$$

- ? $S \sim \log \#$ microstates,

but classically black holes have few charges.

- $T \propto \dfrac{1}{8\pi G M}$ $\rightarrow$ negative specific heat

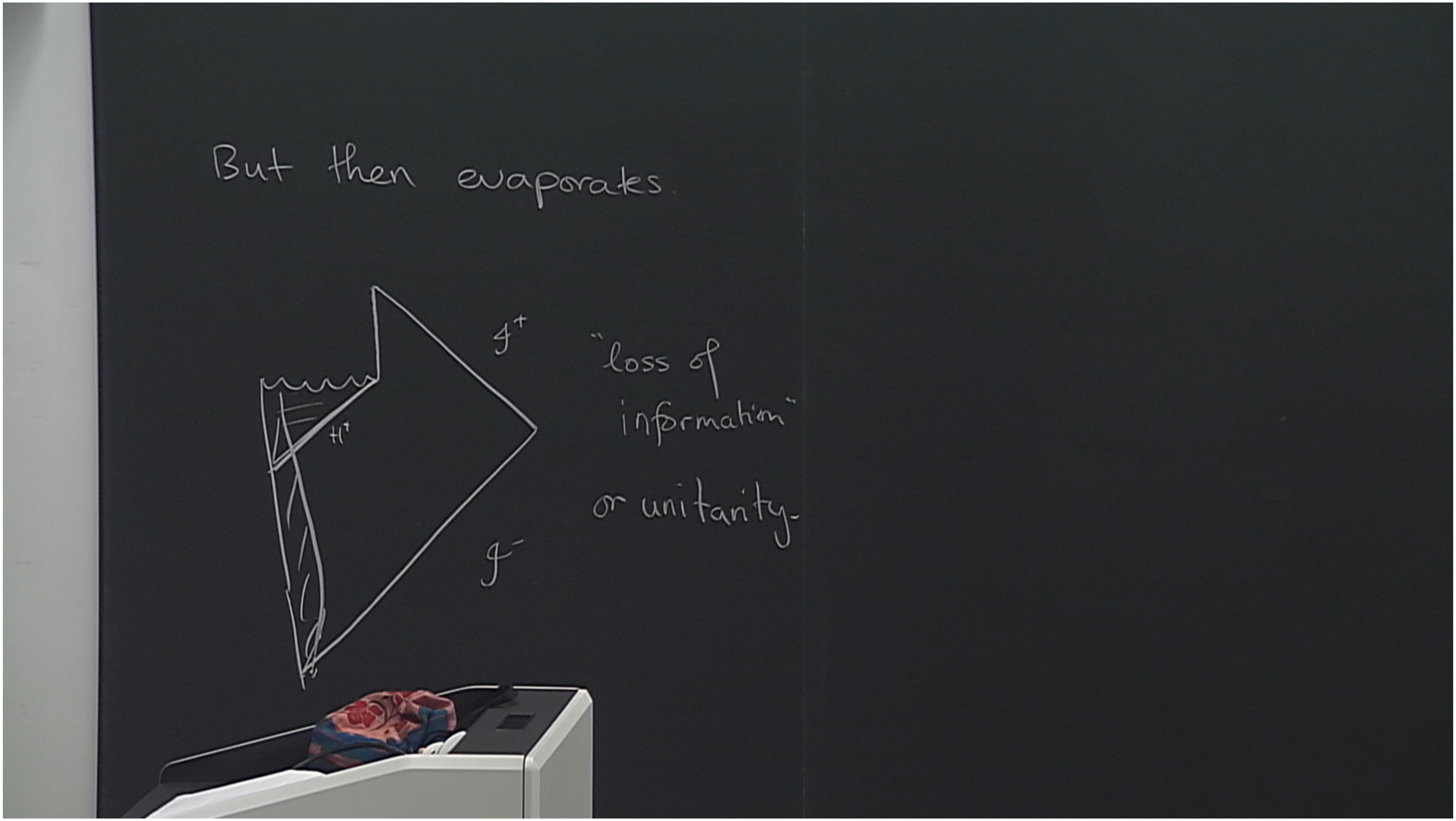
• ? $S \sim \log \#$ microstates,
but classically black holes
have few charges.

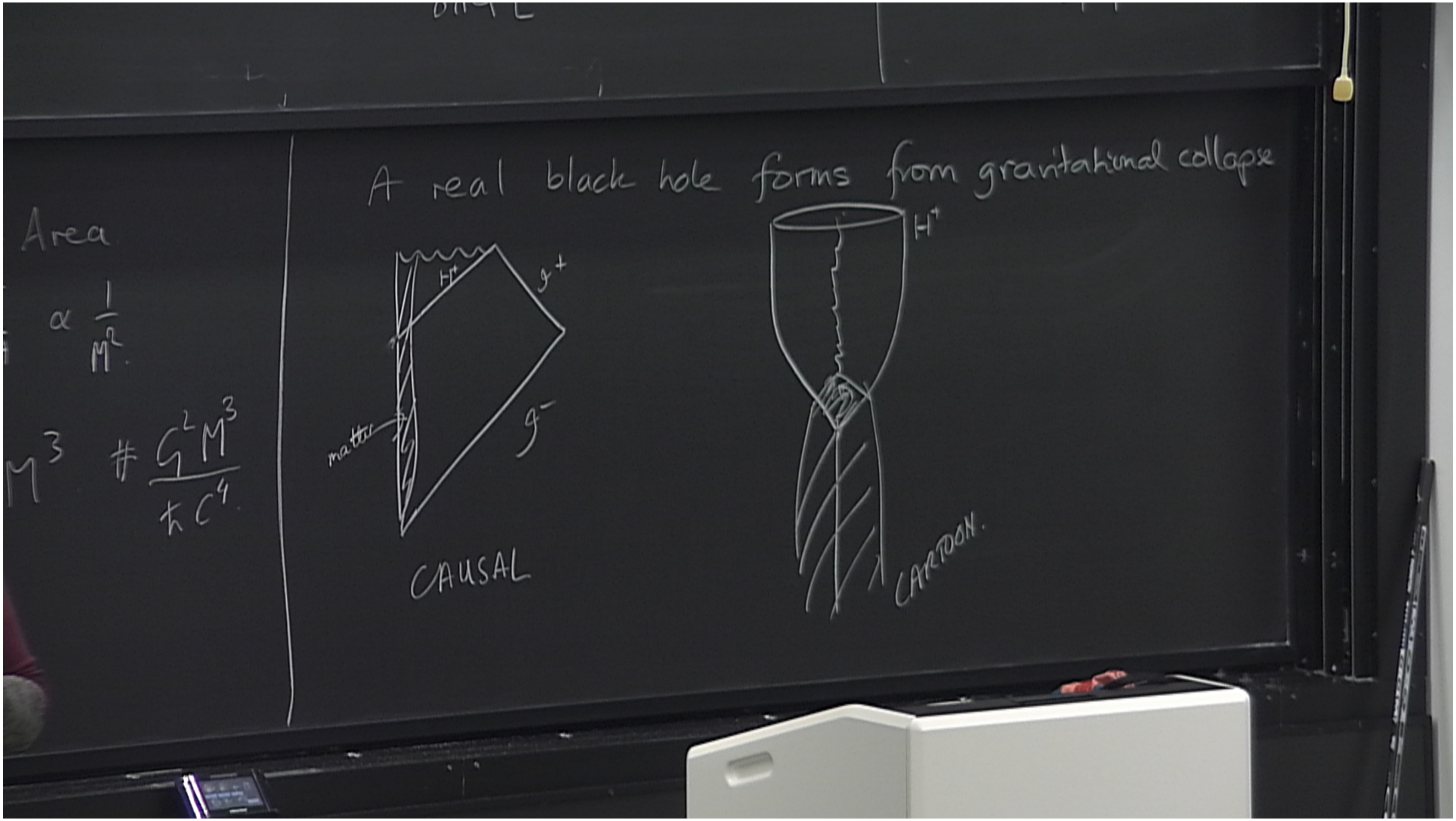
• $T \propto \dfrac{1}{8\pi G M}$ → negative specific heat

$\dot{M} \sim \sigma T^4 \times \text{Area}$

$\sim 4\pi\sigma \dfrac{(2GM)^2}{(8\pi G M)^4} \propto \dfrac{1}{M^2}$.

lifetime $\propto M^3$


But then evaporates.
$\mathscr{I}^+$
$\mathscr{H}^+$
$\mathscr{I}^-$
"loss of information"
or unitarity.



A real black hole forms from gravitational collapse
Area
matter
CAUSAL
CARTOON.