

Title: 13/14 PSI - Gravitational Physics Review - Lecture 7

Date: Jan 14, 2014 10:15 AM

URL: <http://pirsa.org/14010035>

Abstract:

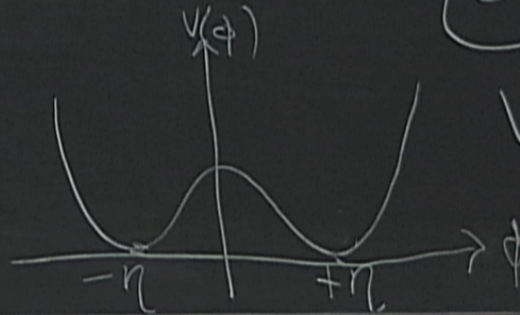
Lecture 7

Gravity & nonperturbative
field theory

• DOMAIN WALL

Consider a scalar field:

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - \underbrace{\frac{\lambda}{2}(\phi^2 - \eta^2)}_{V(\phi)}$$



$$\phi\text{-e.o.m} \quad \square\phi + 2\lambda\phi(\phi^2 - \eta^2) = 0$$

Look for a soln with

$$\phi \rightarrow \pm\eta \quad \text{as} \quad z \rightarrow \pm\infty$$

- no gravity z usual cartesian coord

$$\phi\text{-e.o.m} \quad \square\phi + 2\lambda\phi(\phi^2 - \eta^2) = 0$$

Look for a soln with

$$\phi \rightarrow \pm\eta \quad \text{as } z \rightarrow \pm\infty$$

- no gravity z usual cartesian coord

$$-\phi'' + 2\lambda\phi(\phi^2 - \eta^2) = 0$$

$$-\frac{1}{2}\phi'^2 + \frac{\lambda}{2}(\phi^2 - \eta^2)^2 = \text{const.}$$

$$\square \phi + 2\lambda \phi (\phi^2 - \eta^2) = 0$$

for a soln with

$$\phi \rightarrow \pm \eta \quad \text{as } z \rightarrow \pm \infty.$$

z usual cartesian coord

$$\phi'' + 2\lambda \phi (\phi^2 - \eta^2) = 0$$

$$\frac{1}{2} \phi'^2 + \frac{\lambda}{2} (\phi^2 - \eta^2)^2 = \text{const.}$$

$$\phi = \eta \tanh(\sqrt{\lambda} \eta z)$$

$$\phi' = \sqrt{\lambda} \eta^2 \operatorname{sech}^2(\sqrt{\lambda} \eta z)$$

$$= \sqrt{\lambda} (\eta^2 - \phi^2)$$

$$= \sqrt{2V}$$

$$\square \phi + 2\lambda \phi (\phi^2 - \eta^2) = 0$$

for a soln with

$$\phi \rightarrow \pm \eta \quad \text{as } z \rightarrow \pm \infty$$

z usual cartesian coord

$$\phi'' + 2\lambda \phi (\phi^2 - \eta^2) = 0$$

$$\frac{1}{2} \phi'^2 + \frac{\lambda}{2} (\phi^2 - \eta^2)^2 = \text{const}$$

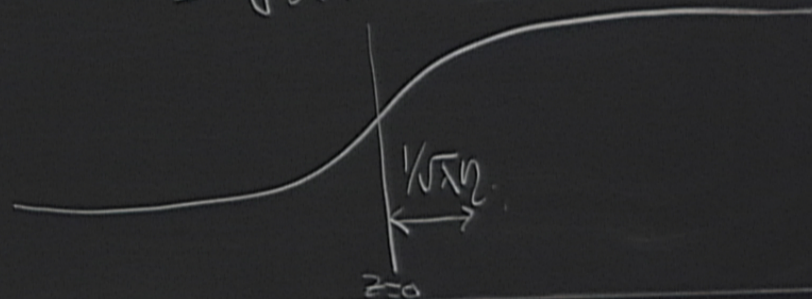
$$\phi = \eta \tanh(\sqrt{\lambda} \eta z)$$

$$\phi' = \sqrt{\lambda} \eta^2 \operatorname{sech}^2(\sqrt{\lambda} \eta z)$$

$$= \sqrt{\lambda} (\eta^2 - \phi^2)$$

$$= \sqrt{2V}$$

"kink"

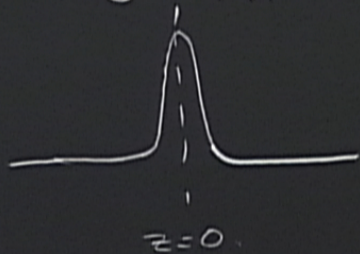


$$\text{kink width} \sim 1/\sqrt{\lambda}\eta \sim m_\phi^{-1}$$

$$\begin{aligned} T_{\mu\nu} &= \phi_{,\mu} \phi_{,\nu} - g_{\mu\nu} \mathcal{L}_\phi \\ &= \lambda\eta^4 \text{sech}^4(\sqrt{\lambda}\eta z) \delta_\mu^z \delta_\nu^z \\ &\quad - g_{\mu\nu} \left(-\frac{\phi'^2}{2} - V \right) \\ &= \lambda\eta^4 \text{sech}^4(\sqrt{\lambda}\eta z) [\delta_\mu^z \delta_\nu^z + g_{\mu\nu}] \end{aligned}$$

$$\rightarrow T_z^z = 0.$$

$$T_0^0 = T_x^x = T_y^y = \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z)$$



$$\sim (\eta^2) \delta(z) (\lambda \eta^4)$$

Strongly localized, T_{no} mostly vacuum.

$$\int T_0^0 dz = \int \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z) dz = \frac{4}{3} \sqrt{\lambda} \eta^3.$$

= Energy per unit area
Finite, but $z=0$ has ∞

$$T_y = \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z)$$

$$\sim (\eta^2) \delta(z) (\lambda \eta^4)$$

alized, $T_{\mu\nu}$ mostly vacuum.

$$\int \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z) dz = \frac{4}{3} \sqrt{\lambda} \eta^3$$

= Energy per unit area

Finite, but $z=0$ has ∞ area.

N.B Soln here written for centre of kink
at $z=0$, $\phi \rightarrow \eta \tanh[\sqrt{\lambda} \eta (z-z_0)]$ the
general kink.

$$y = \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z)$$

$$\sim (\eta^2) \delta(z) (\lambda \eta^4)$$

alized, T_{no} mostly vacuum.

$$\int \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z) dz = \frac{4}{3} \sqrt{\lambda} \eta^3$$

= Energy per unit area

Finite, but $z=0$ has ∞ area.

N.B. Soln here written for centre of kink
at $z=0$, $\phi \rightarrow \eta \tanh[\sqrt{\lambda} \eta (z-z_0)]$ the
general kink



$$= \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z) [\delta_\mu^z \delta_\nu^z + g_{\mu\nu}]$$

$$\int T_0 dz = \int \lambda \eta^4 \operatorname{sech}^4(\sqrt{\lambda} \eta z) dz = \frac{4}{3} \sqrt{\lambda} \eta$$

④ Granth

Kink has t-x-y Lorentz symmetry $\rightarrow$ suggests constant curvature t-x-y spacetime.

c.f. FRW

Consider

$$D \rightarrow ds^2 = A^2(z) \overset{(D-1)}{\gamma}_{\mu\nu} dx^\mu dx^\nu - dz^2$$

"warped compactification"

Cartan: $\underline{\omega}^z = \underline{dz}$ $\underline{\omega}^a = A(z) \underline{\omega}_0^a$
 $= A(z) e^a_\mu dx^\mu$

$$d\underline{\omega}^a = \frac{A'}{A} \underline{\omega}^z \wedge \underline{\omega}^a - A \Theta^a_{0}{}^b \underline{\omega}_0^b$$

$-dz^2$
 achfication

$$\begin{aligned} \underline{\omega}^a &= A(z) \underline{\omega}_0^a \\ &= A(z) e^a_\mu dx^\mu \end{aligned}$$

$$\underline{\omega}_0^a \wedge \underline{\omega}_0^b$$

$$\underline{\omega}^a_b = \underline{\omega}_0^a_b \quad \underline{\omega}^a_z = \frac{A'}{A} \underline{\omega}^a$$

$$\begin{aligned} \underline{R}^a_b &= d \underline{\omega}_0^a_b + \underline{\omega}_0^a_c \wedge \underline{\omega}_0^c_b \\ &\quad + \underline{\omega}^a_z \wedge \underline{\omega}^z_b \end{aligned}$$

$-dz^2$
 achfication

$$\underline{\omega}^a = A(z) \underline{\omega}_0^a$$

$$= A(z) e^a_\mu dx^\mu$$

$$\underline{\omega}_0^a \wedge \underline{\omega}_0^b$$

$$\underline{\omega}^a_b = \underline{\omega}_0^a_b \quad \underline{\omega}^a_z = \frac{A'}{A} \underline{\omega}^a$$

$$\underline{R}^a_b = d \underline{\omega}_0^a_b + \underline{\omega}_0^a_c \wedge \underline{\omega}_0^c_b \leftarrow \underline{R}_0^a_b$$

$$+ \underline{\omega}^a_z \wedge \underline{\omega}^z_b$$

$$= \underline{R}_0^a_b + \underline{\omega}^a_z \wedge \underline{\omega}^z_z \eta_{cb}$$

$$= \frac{1}{2} R_0^a{}_{bcd} \underline{\omega}_0^c \wedge \underline{\omega}_0^d + \left(\frac{A'}{A}\right)^2 \eta_{bc} \underline{\omega}^a \wedge \underline{\omega}^c \Rightarrow R^{\mu\nu}{}_{\lambda\tau}$$

$$\underline{R}^a{}_z = \frac{A''}{A} \underline{\omega}^z \wedge \underline{\omega}^a - \frac{A'}{A} \cancel{\underline{\omega}_0^a} \wedge \underline{\omega}^b + \cancel{\underline{\omega}_0^a} \wedge \underline{\omega}_z^b$$

$$R^a{}_{bcd} = \frac{1}{A^2} R_0^a{}_{bcd} + \left(\frac{A'}{A}\right)^2 [\delta_c^a \eta_{bd} - \delta_d^a \eta_{bc}]$$

$$= \frac{1}{2} R_0^a{}_{bcd} \underline{\omega}_0^c \wedge \underline{\omega}_0^d + \left(\frac{A'}{A}\right)^2 \eta_{bc} \underline{\omega}^a \wedge \underline{\omega}^c$$

$$\Rightarrow R^{\mu\nu}{}_{\lambda\tau}$$

$$R^{\mu z}{}_{\nu z} =$$

$$\underline{R}^a{}_z = \frac{A''}{A} \underline{\omega}^z \wedge \underline{\omega}^a - \frac{A'}{A} \cancel{\underline{\omega}_0^a} \wedge \underline{\omega}^b$$

$$+ \cancel{\underline{\omega}_0^a} \wedge \underline{\omega}_0^b$$

$$R^a{}_{bcd} = \frac{1}{A^2} R_0^a{}_{bcd} + \left(\frac{A'}{A}\right)^2 [\delta_c^a \eta_{bd} - \delta_d^a \eta_{bc}]$$

$$\Rightarrow R^{\mu\nu}_{\lambda\tau} = \frac{1}{A^2} R_0^{\mu\nu}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2 [\delta^\mu_\lambda \delta^\nu_\tau - \delta^\mu_\tau \delta^\nu_\lambda]$$

$$R^{\mu z}_{\nu z} = \frac{A''}{A} \delta^\mu_\nu$$

$$\Rightarrow R^z_z = (D-1) \frac{A''}{A} \rightarrow 3 \frac{A''}{A}$$

$$R^M_\nu = \frac{A''}{A} \delta^M_\nu + \frac{R_0^M}{A^2} + (D-2) \left(\frac{A'}{A}\right)^2 \delta^M_\nu \rightarrow \left[\frac{A''}{A} + 2\left(\frac{A'}{A}\right)^2 + \frac{\kappa}{l^2 A^2} \right] \delta^M_\nu$$

$\gamma_{\mu\nu}$
 const
 curvature
 ↓

Our Einstein eqns

$$G^0_0 = \frac{\kappa}{e^2 A^2} - \frac{A'^2}{A^2} - \frac{2A''}{A} = 8\pi G (V + \frac{1}{2}\dot{\phi}^2)$$

$$G^z_z = \frac{3\kappa}{e^2 A^2} - 3\frac{A'^2}{A^2} = 8\pi G (V - \frac{1}{2}\dot{\phi}^2)$$

$$\square\phi = -\frac{1}{A^3}\partial_z A^3\partial_z\phi = -2\lambda\phi(\phi^2 - \eta^2)$$

Our Einstein eqns $\phi = \phi(z)$

$$G^0_0 = \frac{\kappa}{e^2 A^2} - \frac{A'^2}{A^2} - \frac{2A''}{A} = 8\pi G (V + \frac{1}{2}\phi'^2) \cosh^4$$

$$G^z_z = \frac{3\kappa}{e^2 A^2} - 3\frac{A'^2}{A^2} = 8\pi G (V - \frac{1}{2}\phi'^2) = 0 \text{ f.s.}$$

$$\square \phi = -\frac{1}{A^3} \partial_z A^3 \partial_z \phi = -2\lambda \phi (\phi^2 - \eta^2)$$

$\underbrace{-\phi'' - 3\frac{A'}{A}\phi'}_{\text{from } \partial_z A^3 \partial_z \phi}$

Suppose $8\pi G\eta^2 \ll 1$ ($\frac{\eta}{M_p} \ll 1$)

Then

$$8\pi G V = \underbrace{O(8\pi G\eta^2)}_{\in} \times \underbrace{\lambda\eta^2}_{\text{width}^2} \leftarrow \begin{array}{l} \text{sets} \\ \text{length} \\ \text{scale} \end{array}$$

- Solve perturbatively.

Suppose $8\pi G\eta^2 \ll 1$ ($\frac{\eta}{M_p} \ll 1$)

Then

$$8\pi G V = \underbrace{O(8\pi G\eta^2)}_{\epsilon} \times \underbrace{\lambda\eta^2}_{\text{width}^2} \leftarrow \begin{array}{l} \text{sets} \\ \text{length} \\ \text{scale} \end{array}$$

• Solve perturbatively.

$$A(z) = 1 + o(\epsilon)$$

$$\phi = \eta \tanh(\sqrt{\lambda}\eta z) + o(\epsilon).$$

Away from wall $T^M_\nu = 0$

$$\Rightarrow A'' = 0$$

$$\& A'^2 = \frac{\kappa}{l^2}$$

Suppose $8\pi G\eta^2 \ll 1$ ($\frac{l}{M_p} \ll 1$)

Then

$$8\pi G V = \underbrace{O(8\pi G\eta^2)}_{\epsilon} \times \underbrace{\lambda\eta^2}_{\text{width}^2} \leftarrow \text{sets length scale}$$

• Solve perturbatively.

$$A(z) = 1 + o(\epsilon)$$

$$\phi = \eta \tanh(\sqrt{\lambda}\eta z) + o(\epsilon).$$

Away from wall $T^M_\nu = 0$

$$\Rightarrow A'' = 0$$

$$\& A^{12} = \frac{\kappa}{l^2} \rightarrow \kappa > 0$$

$$\gamma_{\mu\nu}(-) dS$$

$$A = 1 \pm z/l$$

Use kink

$$-\phi'' - 3\frac{A'}{A}\phi'$$

$$A'' = -\epsilon \frac{\lambda \eta^2}{2} \operatorname{sech}^4(\sqrt{\lambda} \eta z)$$

$$A' = -\epsilon \sqrt{\lambda} \eta \left(\tanh(\sqrt{\lambda} \eta z) - \frac{1}{3} \tanh^3(\sqrt{\lambda} \eta z) \right)$$

set $A'(0) = 0$ by symmetry

$$\rightarrow l = 3/2\epsilon\sqrt{\lambda}\eta$$

$$\phi = \eta \tanh(\sqrt{\lambda} \eta z)$$

$$A(z) = 1 - \epsilon \left[\frac{2}{3} \log \cosh(\sqrt{\lambda} \eta z) - \frac{1}{6} \operatorname{sech}^2(\sqrt{\lambda} \eta z) + \frac{2}{3} \log 2 \right]$$

$$\sqrt{\lambda} \eta z \gg 1 \quad \hookrightarrow \quad 1 - \frac{2}{3} \epsilon (\pm \sqrt{\lambda} \eta z) \quad ? \quad z \rightarrow \pm l$$

$$= \Lambda \eta^4 \operatorname{sech}^4(\sqrt{\Lambda} \eta z) [\partial_\mu \partial_\nu + g_{\mu\nu}] \quad \left| \quad \int T_0 dz = \int \Lambda \eta^4 \operatorname{sech}^4(\sqrt{\Lambda} \eta z) dz = \frac{1}{3} \right.$$

Looks like horizon or singularity?

In fact a horizon as

$$ds^2 = \left(1 - \frac{r_e}{r}\right)^2 (dt^2 - \cosh^2 t d\Omega_e^2) - dz^2$$

is MST in strange coords

