

Title: 13/14 PSI - Gravitational Physics Review - Lecture 5

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Abstract:

Lecture 5 Causal Structure of Black Holes

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Causal Structure of Black Holes

Recap: $ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - r^2 d\Omega^2$

Lecture 5 Causal Structure of Black Holes

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E-egns $(r B^{-2})' = 1 - 8\pi G \rho_0 r^2$

Lecture 5 Causal Structure of Black Holes

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E-egns $(r B^{-2})' = 1 - 8\pi G T^0_0 r^2$

$$\frac{A'}{A} + \frac{B'}{B} = 8\pi G (T^0_0 - T^r_r)$$

Lecture 5 Causal Structure of Black Holes

Recap: $ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - r^2 d\Omega^2$

E-eqns $(r B^{-2})' = 1 - 8\pi G \rho$

$$\frac{A'}{A} + \frac{B'}{B} = 8\pi G (T^t_t)$$

Sch: $A^2 = B^{-2} =$

Lecture 5 Causal Structure of Black Holes

Recap: $ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - r^2 d\Omega_{\mathbb{H}}^2$

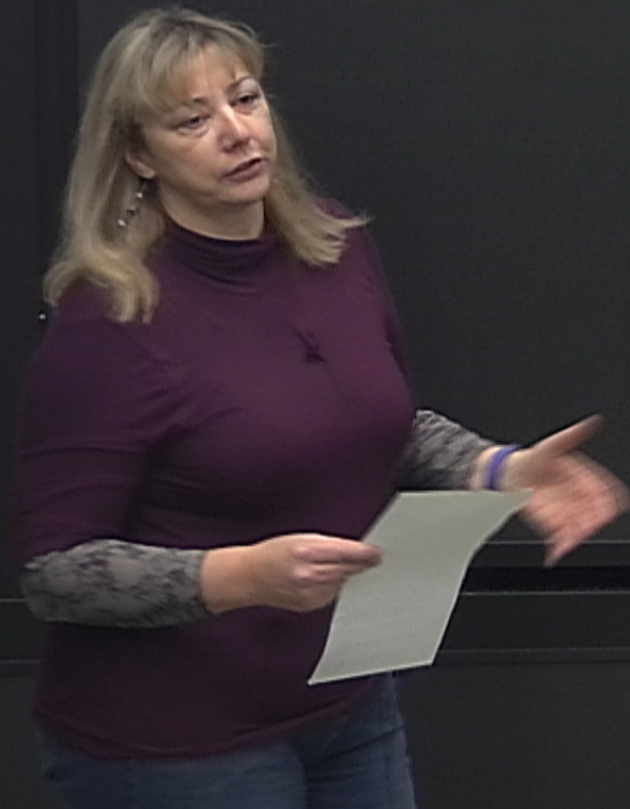
E-eqns $(r B^{-2})' = 1 - 8\pi G T^0_0 r^2$

$$\frac{A'}{A} + \frac{B'}{B} = 8\pi G (T^0_0 - T^r_r) (B^2)$$

Sch: $A^2 = B^{-2} = 1 - \frac{2GM}{r}$

+ Adding a cosmological

+ Adding a cosmological constant, Λ_{gas}



+ Adding a cosmological constant, Λ_{gas}

$$(r B^2 - 1 - \Lambda r^2)$$

$$\Rightarrow R = 2C$$

+ Adding a cosmological constant, Λ_{gal}

$$(r B^{-2}) = 1 - \Lambda r^2$$

$$\Rightarrow B^{-2} = 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 = A^2$$

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• $\Lambda > 0$ de-Sitter

+ Adding a cosmological constant, Λ_{gal}

$$(rB^{-2}) = 1 - \Lambda r^2$$

$$\Rightarrow = 1 - \frac{2GM}{r} - \frac{\Lambda}{3} r^2 = A^2$$

de-Sitter

$$r \approx 2GM$$

$$\text{AND } r \approx \sqrt{\frac{3}{\Lambda}}$$

+ Adding a cosmological constant, Λ_{gub}

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* $\Lambda > 0$ de-Sitter

$A^2 \rightarrow 0$ at

$r \approx 2GM$ +
AND $r \approx \sqrt{\frac{3}{\Lambda}}$

BLACK HOLE
EVENT HORIZON

COSMOLOGICAL
HORIZON

+ Adding a cosmological constant, Λ_{gab}

$$(r B^{-2}) = 1 - \Lambda r^2$$

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BLACK HOLE
EVENT HORIZON

AND $r \approx \sqrt{\frac{3}{\Lambda}}$

COSMOLOGICAL
HORIZON.

stant, Λ_{gab}

dS has several common co-e

BLACK HOLE
EVENT HORIZON

COSMOLOGICAL
HORIZON

stant, Λ_{gab} | dS has several common co-ord systems

$$ds^2 =$$

BLACK HOLE
EVENT HORIZON

COSMOLOGICAL
HORIZON

Λ_{gib}

dS has several common co-ord systems $(M \rightarrow 0)$

$$ds^2 = (1-r^2)dt^2 - (1-r^2)^{-1}$$

HOLE
HORIZON

CAL
HORIZON.

Λ_{gub} dS has several common co-ord systems $(M \rightarrow 0)$
(STATIC PATCH)
$$ds^2 = (-r^2)dt^2 - (1-r^2)^{-1}dr^2 - r^2 d\Omega_{\text{II}}^2$$
$$= d\tau^2 - \cosh^2 \tau d\Omega_{\text{III}}^2$$

gab. dS has several common co-ord systems $(M \rightarrow 0)$
 $ds^2 = (-r^2)dt^2 - (1-r^2)^{-1}dr^2 - r^2 d\Omega_{II}^2$ (STATIC PATCH)
 $= dT^2 - \cosh^2 \tau d\Omega_{III}^2$ (GLOBAL COORDS)

dS has several common co-ord systems $(M \rightarrow 0)$
 $ds^2 = (-r^2)dt^2 - (1-r^2)^{-1}dr^2 - r^2 d\Omega_{II}^2$ (STATIC PATCH)
 $= dT^2 - \cosh^2 T d\Omega_{III}^2$ (GLOBAL EDS)
 $= dT - e^{2T} d\underline{x}^2$ - FLAT UNIVERSE - HARD EXP

$\Lambda < 0$ anti-de Sitter (ads) space

$$ds^2 = A^2 dt^2 - \frac{dr^2}{A^2} - r^2 d\Omega_{\kappa}^2 \quad \begin{array}{l} \kappa=1 \quad S^2 \\ \kappa=0 \quad \mathbb{R}^2 \\ \kappa=-1 \quad H^2 \end{array}$$

$R^2 = -\Delta$

$+ \boxed{\frac{R^2}{r^2}} - 1 \text{ HORIZON}$
 $r \approx 2GM$

$\Lambda < 0$ anti-de Sitter (ads) space

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$R^2 = -\frac{\Lambda}{3} \downarrow$

$$\left(\text{circle with a slash} \right) - \frac{2GM}{r} + \boxed{R^2 r^2} - 1 \text{ HORIZON}$$

$r \approx 2GM$

$\rightarrow \kappa$

$+ve$

In ads have spherical, planar + hyperbolic black holes.

$\Lambda < 0$ anti-de Sitter (adS) space

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$r \approx 2GM$

$\rightarrow \kappa$

tr

In adS have spherical, planar + hyperbolic black holes.

dS) space

$\kappa=1$ S^2
 $\kappa=0$
 $\kappa=-1$

Causal structure of SCH

$ds)$ space

$$\begin{array}{ll} \kappa=1 & S^2 \\ \kappa=0 & \mathbb{R}^2 \\ \kappa=-1 & H^2 \end{array}$$

ON

M

anar + hyperbolic
black holes

Causal structure of SCH

$$ds^2 = \left(1 - 2\frac{GM}{r}\right) dt^2 - \left(1 - 2\frac{GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

ds) space

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ds) space

$$\begin{aligned} \kappa=1 & \quad S^2 \\ \kappa= & \quad R^2 \\ & \quad t^2 \end{aligned}$$

holic

Causal structure of SCH

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

Tortoise co-ord

$$r^* = \int \frac{dr}{1 - \frac{2GM}{r}}$$

ds) space

$$\begin{array}{ll} \kappa=1 & S^2 \\ \kappa=0 & \mathbb{R}^2 \\ \kappa=-1 & H^2 \end{array}$$

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$$r^* = \int \frac{dr}{1 - \frac{2GM}{r}} = r + 2GM \ln \left(\frac{r - 2GM}{2GM} \right)$$

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null rays have $t = \pm r^*$

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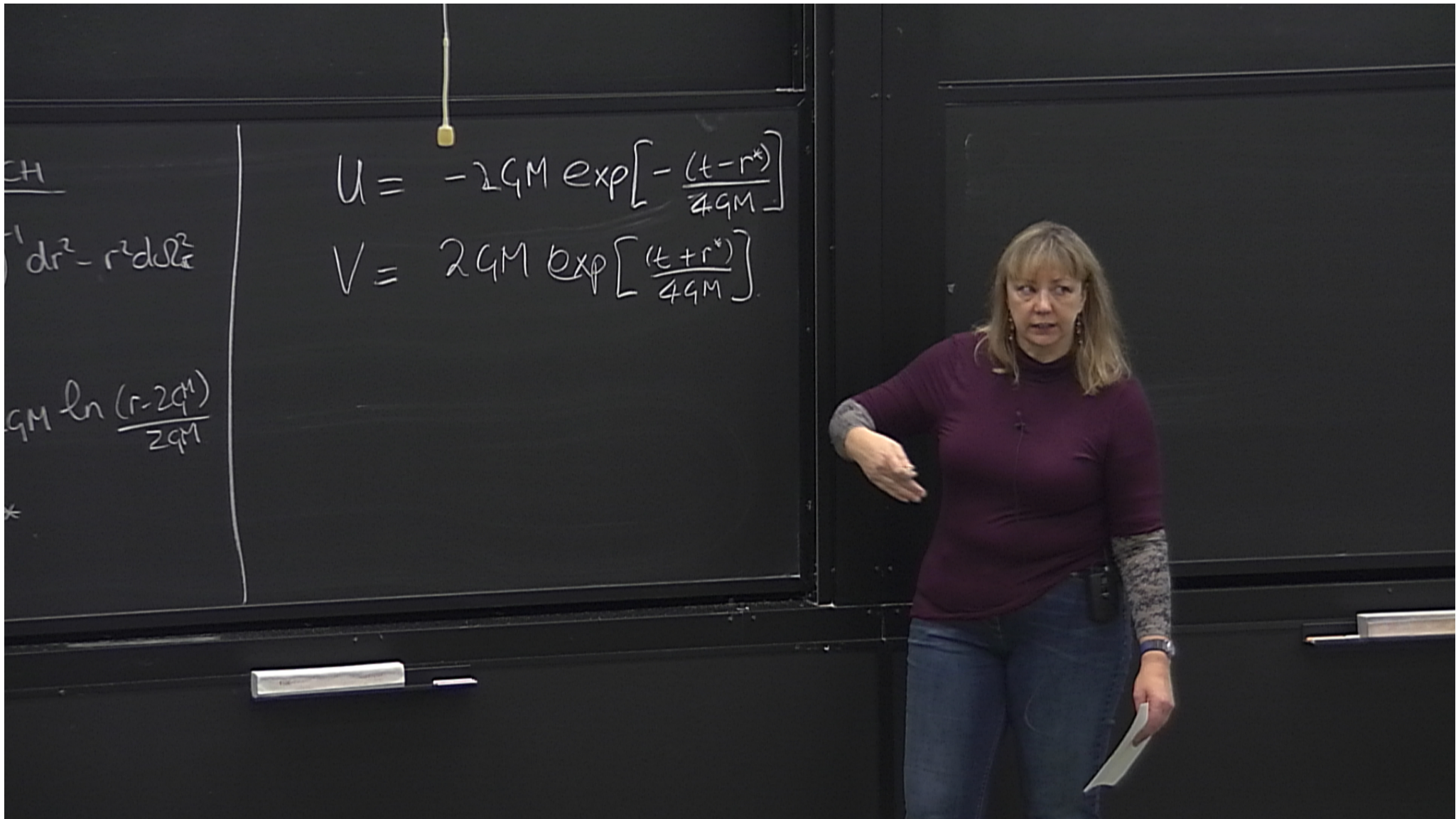
Tortoise co-ord

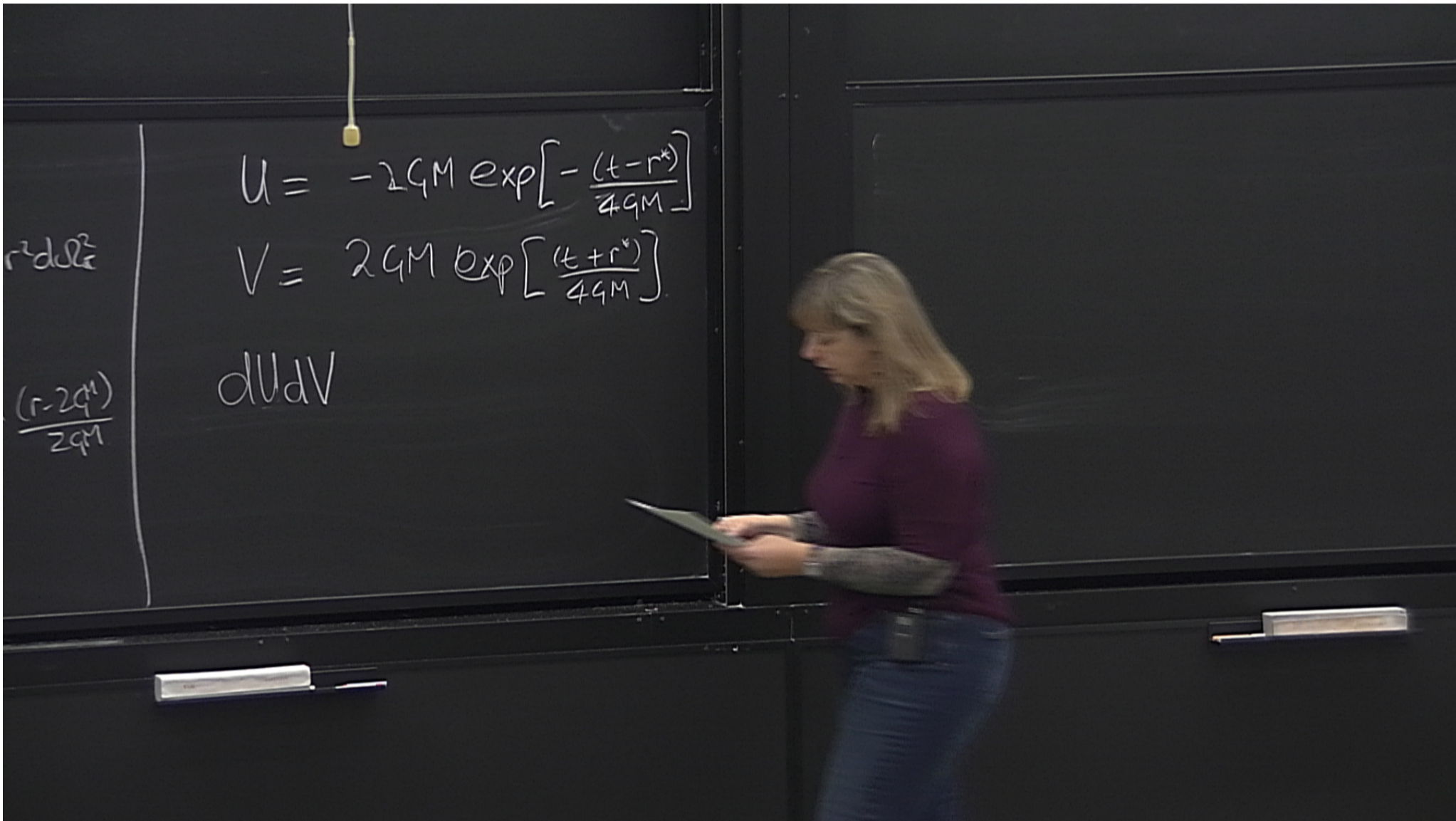
$$r^* = \int \frac{dr}{1 - \frac{2GM}{r}} = r + 2GM \ln \left(\frac{r - 2GM}{2GM} \right)$$

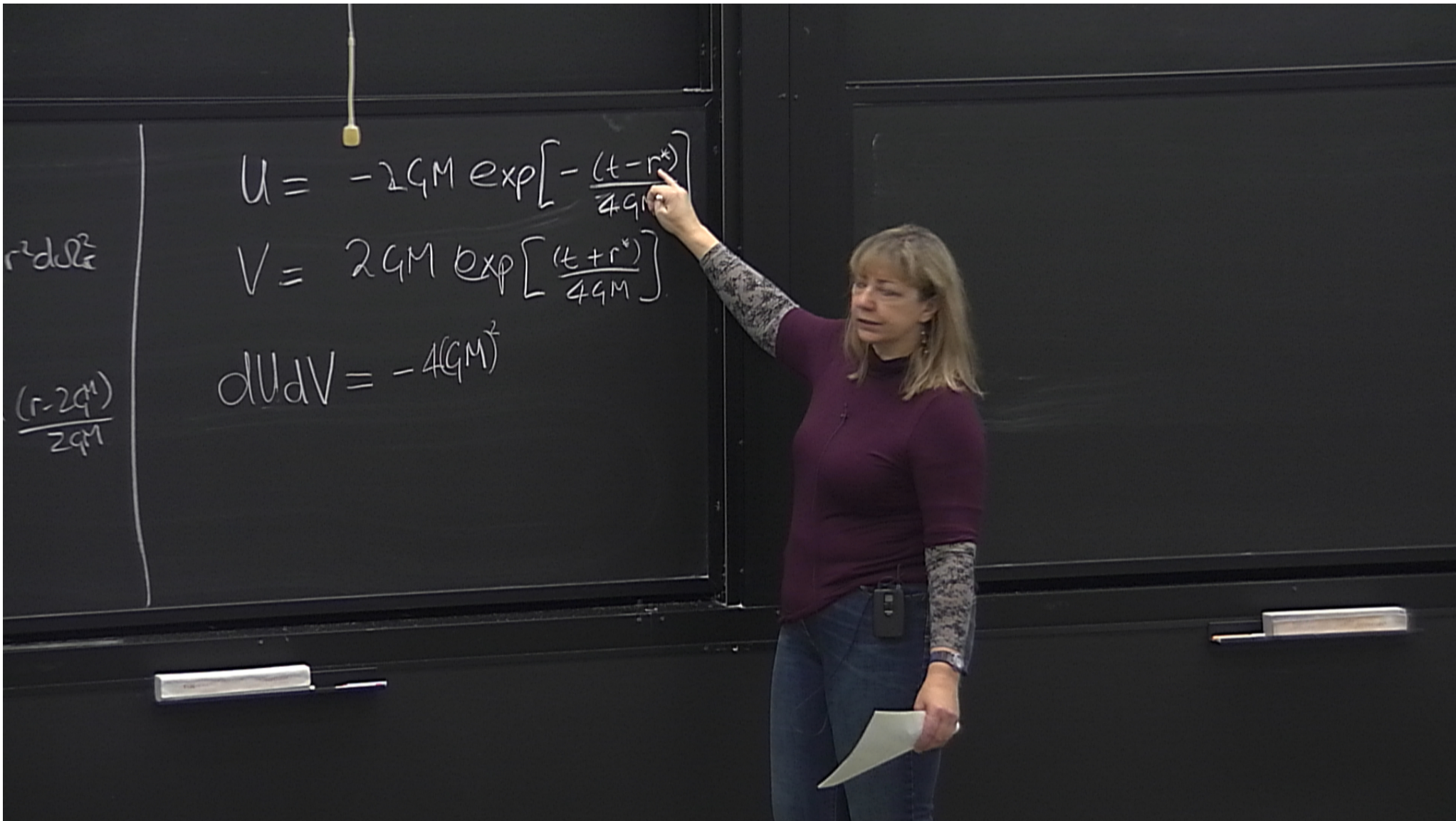
null rays have $t = \pm r^*$

$$U = -2GM \exp\left[-\frac{(t - r^*)}{4GM}\right]$$

$$V = 2GM \exp\left[\frac{(t + r^*)}{4GM}\right]$$







$$U = -2GM \exp\left[-\frac{(t-r^*)}{4GM}\right]$$

$$V = 2GM \exp\left[\frac{(t+r^*)}{4GM}\right]$$

$$dV = -4(GM)^2 UV (dt^2 - dr^{*2})$$

usual structure of SCH

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2$$

raise co-ord

$$r^* = \int \frac{dr}{1 - \frac{2GM}{r}} = r + 2GM \ln \frac{(r - 2GM)}{2GM}$$

all rays have $t = \pm r^*$

$$U = -2GM \exp\left[-\frac{(t - r^*)}{4GM}\right]$$

$$V = 2GM \exp\left[\frac{(t + r^*)}{4GM}\right]$$

$$dU dV = \frac{1}{(4GM)^2} UV (dt^2 - dr^{*2})$$

$\rightarrow R$. In adS have spherical, planar + hyperbolic
black holes.

null rays have +

$$e^{r^*/2GM} = e^{r/2GM} \frac{(r-2GM)}{2GM}$$

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$$e^{r^*/2GM} = e^{r/2GM} \frac{(r-2GM)}{2GM}$$

$$d = (GM)^2$$

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$$ds^2 = (GM)^2 e^{r/2GM}$$

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$$e^{r^*/2GM} = e^{r/2GM} \frac{(r-2GM)}{2GM}$$

$$ds^2 = \frac{4}{2GMr} du dv e^{-r/2GM} - r^2(u,v) d\Omega_{\mathbb{H}}^2$$

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Now regular

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Now regular at $2GM$.

KRUSKAL
COORDS

spherical, planar + hyperbolic
black holes

null rays have $t = \pm r^*$

$$UV = -(2GM)^2 e^{\pm \frac{r}{2GM}}$$

$$r = \text{const} \leftrightarrow UV = \text{const}$$

$$\frac{2GM}{r}$$

$$2GM$$

$$-r^2(u,v) d\Omega_{\mathbb{H}}^2$$

KRUSKAL
COORDS

spherical, planar + hyperbolic
black holes

null rays have $t = \pm r^*$

$$UV = -(2GM)^2 e^{\pm 2\pi r^*/4M}$$

$$r = \text{const} \leftrightarrow UV = \text{const}$$

$$t = \text{const}$$

$$-r^2(u,v) d\Omega_{\mathbb{H}}^2$$

KRUSKAL
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null rays have $t = \pm r^*$

$$UV = -(2GM)^2 e^{\dots}$$

$$r = \text{const} \leftrightarrow UV = \text{const}$$

$$t = \text{const} \quad U/V = \text{const}$$

$$r \rightarrow 2GM$$

$$UV$$

$$-r^2(u,v) d\Omega_{\mathbb{H}}^2$$

KRUSKAL
COORDS

spherical, planar + hyperbolic
black holes

null rays have $t = \pm r^*$

$$UV = -(2GM)^2 e^{\pm \dots}$$

$$r = \text{const} \leftrightarrow UV = \text{const}$$

$$t = \text{const} \leftrightarrow U/V = \text{const}$$

$$r \rightarrow 2GM \leftrightarrow UV \rightarrow 0$$

$$r = 0 \leftrightarrow UV = (2GM)^2$$

$$-r^2(u,v) d\Omega_{\mathbb{H}}^2$$

KRUSKAL
COORDS

ys have $t = \pm r^*$

$$UV = -(2GM)^2 \exp[r/2GM]$$

$$\Leftrightarrow UV = \text{const}$$

$$\Leftrightarrow u/v = \text{const}$$

$$\Leftrightarrow UV \rightarrow 0$$

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ys have $t = \pm r^*$

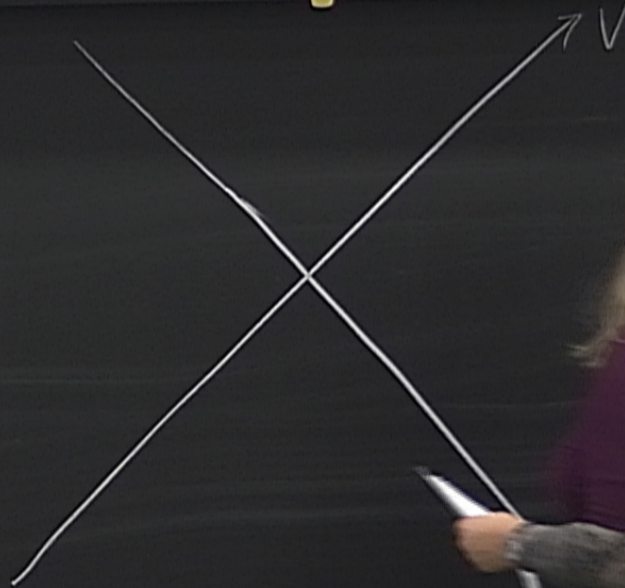
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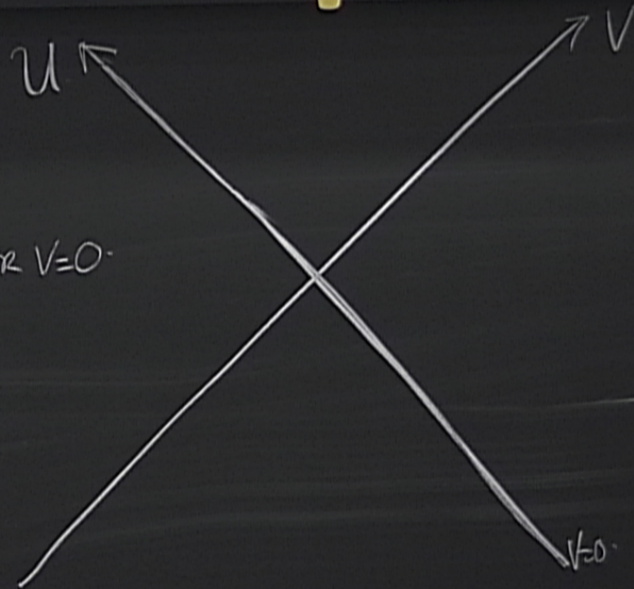
$$\Leftrightarrow UV = \text{const}$$

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$U=0$ or $V=0$



ys have $t = \pm r^*$

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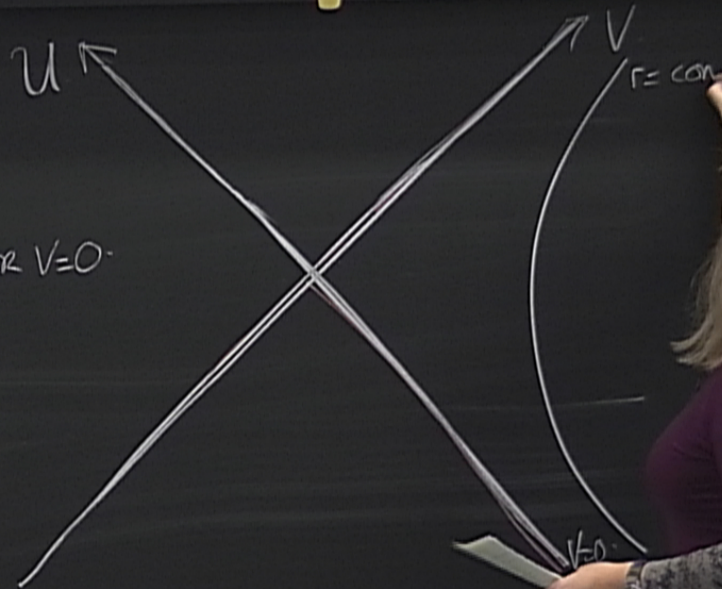
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ys have $t = \pm r^*$

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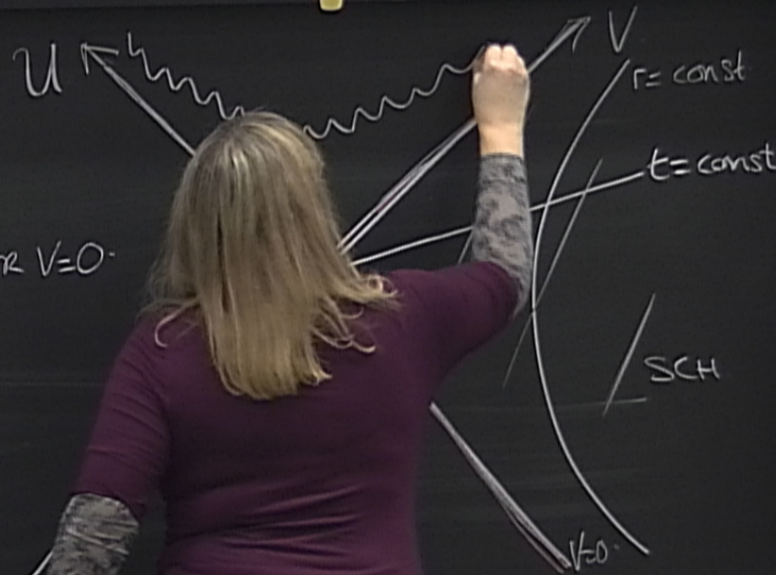
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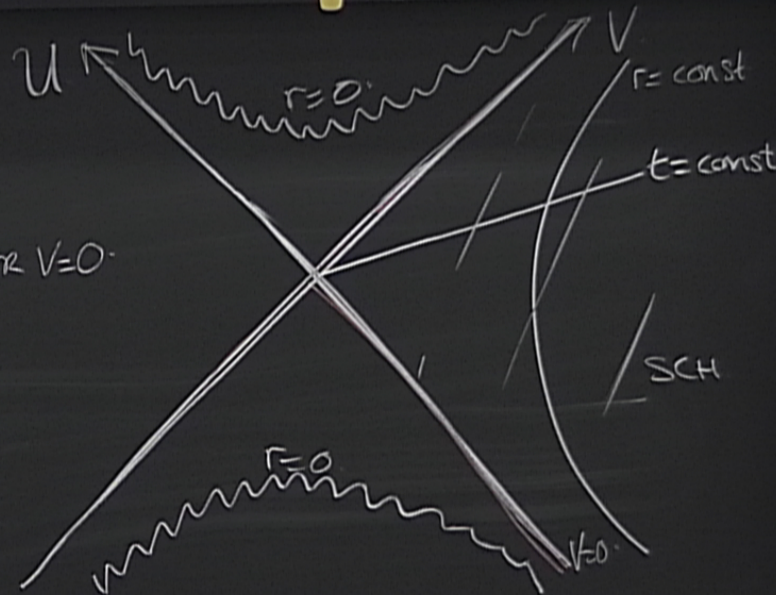
$$\Leftrightarrow UV = \text{const}$$

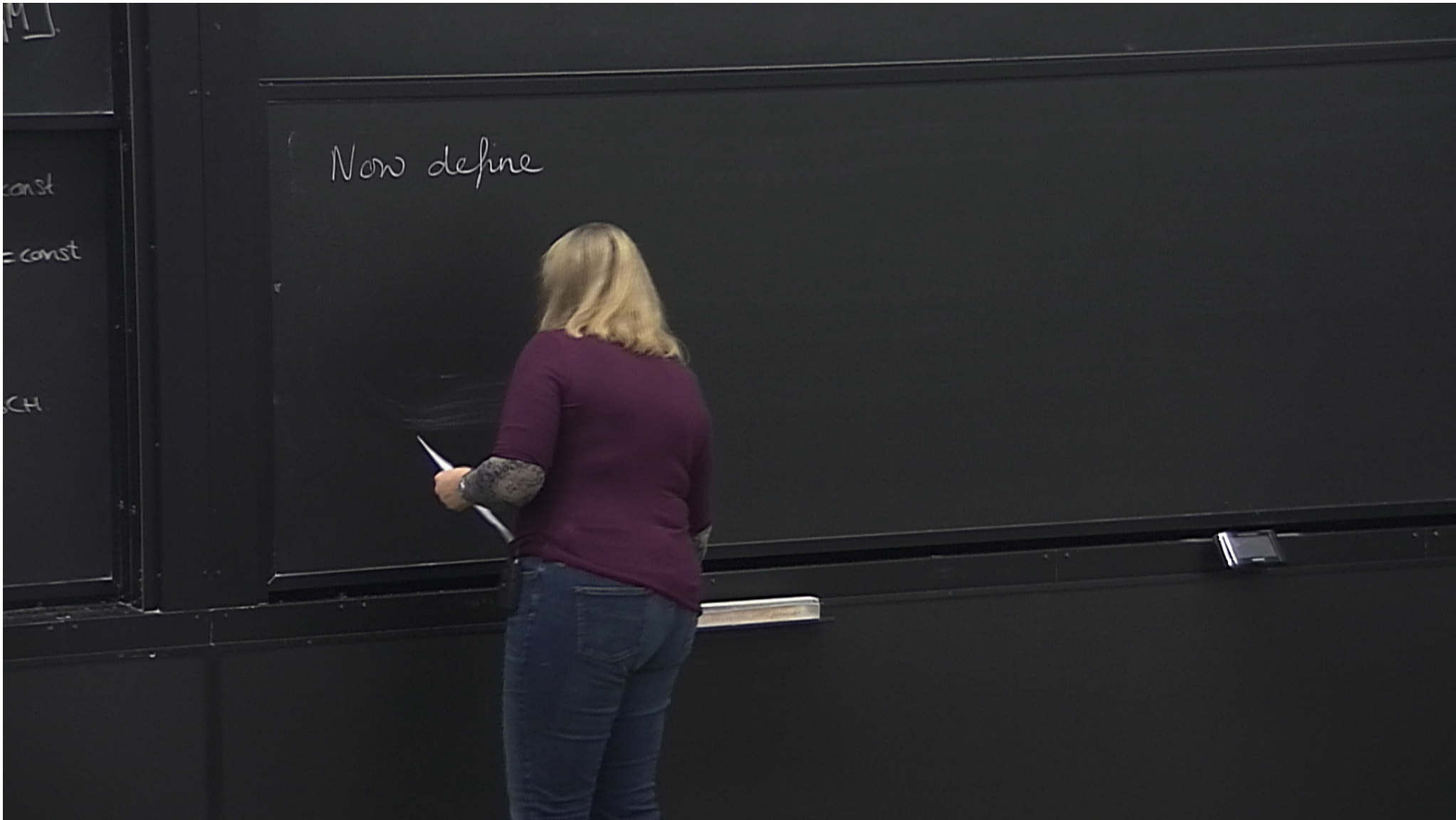
$$\Leftrightarrow U/V = \text{const}$$

$$\Leftrightarrow UV \rightarrow 0$$

$$\Leftrightarrow UV = (2GM)^2$$

$U=0$ or $V=0$





Now define $p = \arctan \sqrt{2} q M$
 $q = \arctan U / 2 q M$

Now define $p = \arctan V/2GM$
 $q = \arctan U/2GM$

$$V=0 \Leftrightarrow p=0$$

$$V=2GM \Leftrightarrow p=\pi/4$$

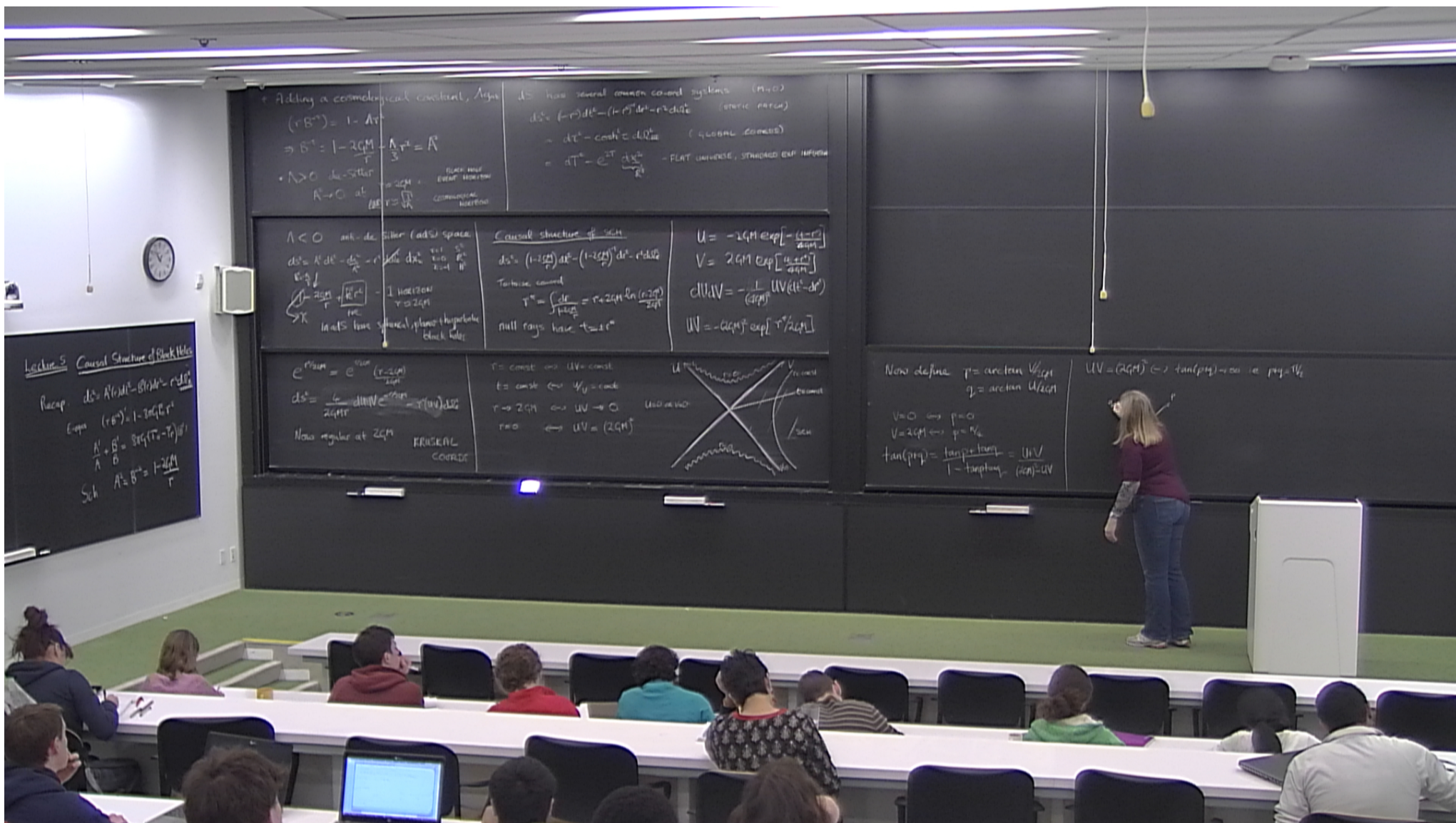
$$\tan(p+q) = \frac{\tan p + \tan q}{1 - \tan p \tan q}$$

Now define $p = \arctan V/2GM$
 $q = \arctan U/2GM$

$$V=0 \Leftrightarrow p=0$$

$$V=2GM \Leftrightarrow p=\pi/4$$

$$\tan(p+q) = \frac{\tan p + \tan q}{1 - \tan p \tan q} = \frac{U+V}{(2GM)^2 - UV}$$



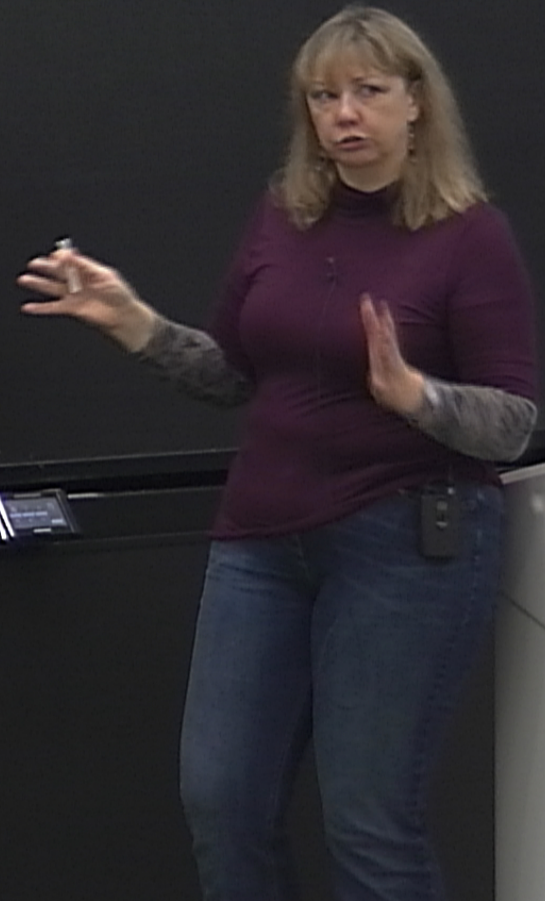
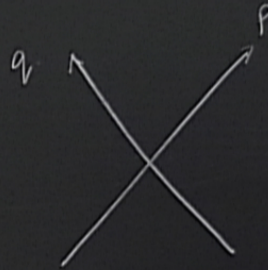
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$$UV = (2GM)^2 \Leftrightarrow \tan(p+q) \rightarrow \infty \text{ ie } p+q = \pi/2$$



$$p = \arctan V/2GM$$

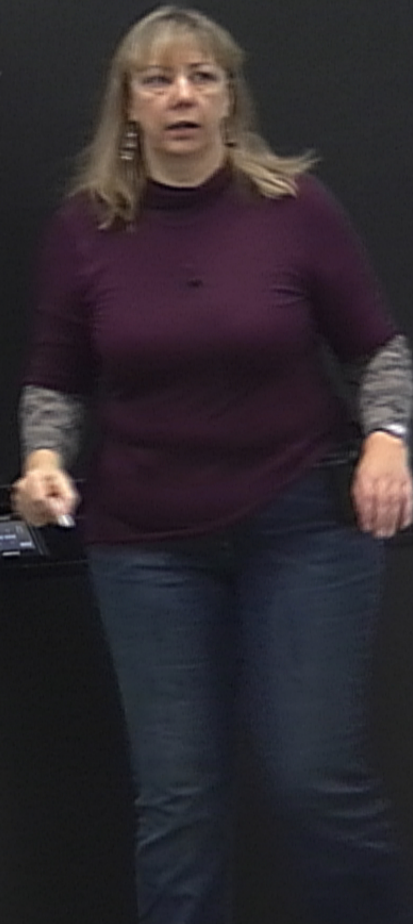
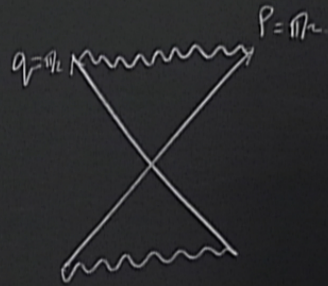
$$q = \arctan U/2GM$$

$$p = 0$$

$$p = \pi/4$$

$$\frac{\tan p + \tan q}{1 - \tan p \tan q} = \frac{U+V}{(2GM)^2 - UV}$$

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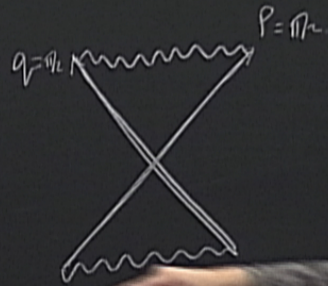
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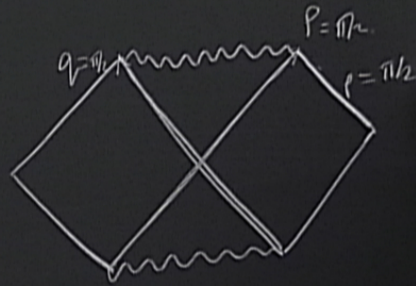
$$q = \arctan U/2GM$$

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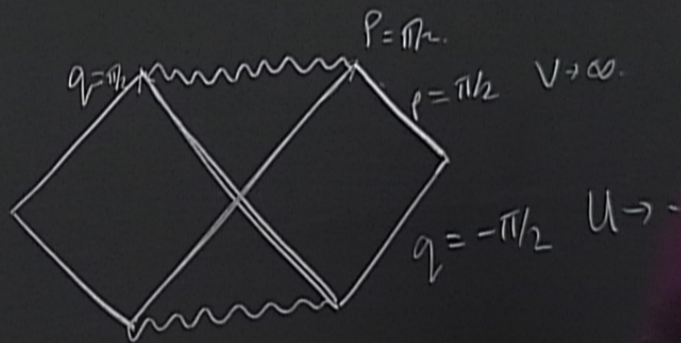
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$$\arctan V/2GM$$

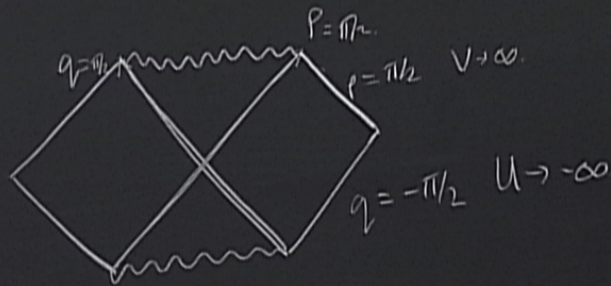
$$\arctan U/2GM$$

$$UV = (2GM)^2 \Leftrightarrow \tan(p+q) \rightarrow \infty \text{ ie } p+q = \pi/2$$



$$q = \frac{U+V}{(2GM)^2 - UV}$$

$$UV = (2GM)^2 \hookrightarrow \tan(p+q) \rightarrow \infty \text{ i.e. } p+q = \pi/2$$



$(p+q) \rightarrow \infty$ ie $p+q = \pi/2$

$V \rightarrow \infty$

$\pi/2$ $U \rightarrow -\infty$

PENROSE-CARTER
DIAGRAM.

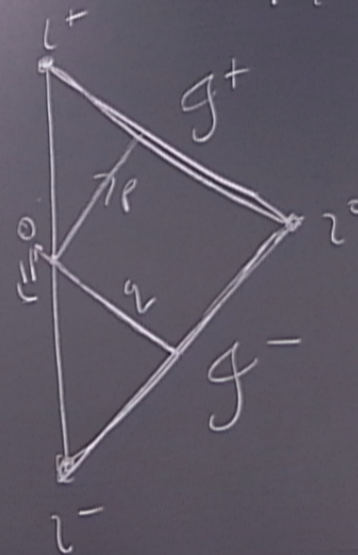
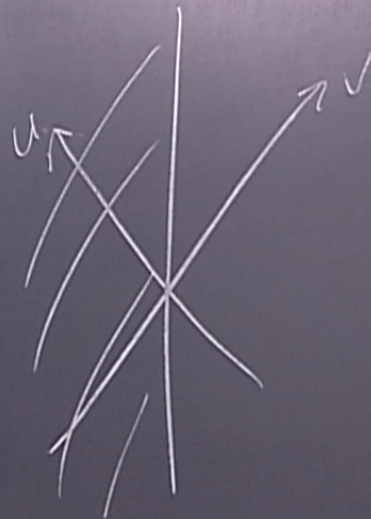
Minkowski Spacetime

$$dt^2 - dr^2 - r^2 d\Omega^2$$

$$u = t - r$$

$$v = t + r$$

$$r > 0 \quad v - u > 0$$



(S^2 suppressed)

l^+ - future timelike infinity

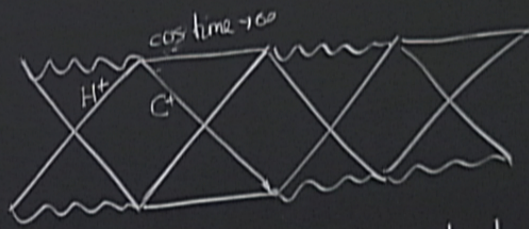
l^- - past " "

l^0 - spacelike infinity.

$\mathcal{I}^+$ = "scri" plus = future null inf.

$\mathcal{I}^-$ = past " "

SCH - DS



"small r"
black hole

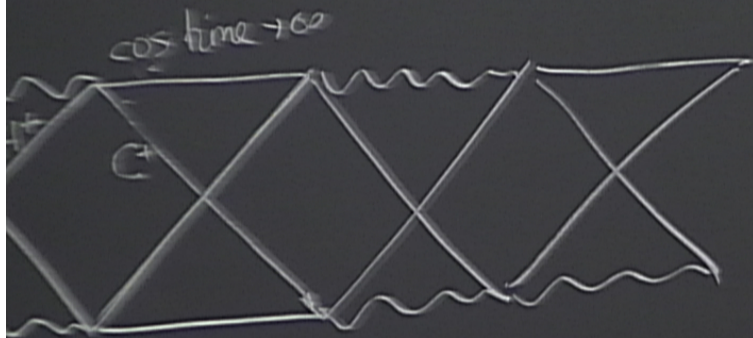
Use Kruskal
style coords to
extend across
horizons, cannot do
all simultaneously

SCH - ADS



← boundary of adS

- DS



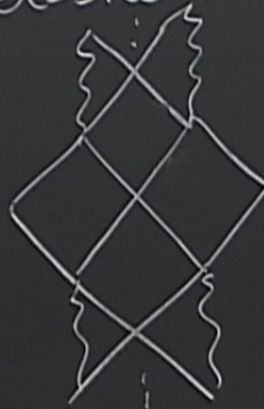
"hole"
Use Kruskal
style coords to
extend across
horizons, cannot do
all simultaneously.

SCH - ADS



← boundary of adS

Reissner-Nordstrom:



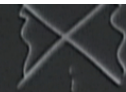
Euclidean "black hole" ?

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

? $r \rightarrow 2GM$ $ds^2 = \left(\frac{r-2GM}{r} dt^2 + \frac{r dr^2}{r-2GM} \right)$

Define $\rho^2 = \lambda(r-2GM)$

horizons, cannot do
all simultaneously



Define $p^2 = \lambda(r - 2GM)$

$$2p dp = \lambda dr$$

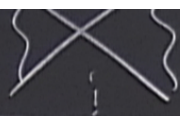
$$\& \quad ds_{\text{II}}^2 = \frac{p^2}{p^2 + 2GM\lambda} dt^2 + \frac{p^2 + 2GM\lambda}{\cancel{p^2}} \frac{4p^2 dp^2}{\lambda^2}$$

$$\checkmark \quad p \rightarrow 0 \quad \frac{p^2}{2GM\lambda} dt^2 + \frac{8GM}{\lambda} dp^2$$

Choose $\lambda = 8GM$, $\Theta = \frac{t}{4GM}$

$$ds_{\text{II}}^2 = p^2 d\Theta^2 + dp^2 \quad \text{ie. } p \rightarrow 0 \text{ in } \mathbb{R}^2$$

all simultaneously



Choose $\lambda = 8GM$, $\Theta = \frac{t}{4GM}$

$$ds^2 = p^2 d\Theta^2 + dp^2 \quad \text{ie. } p \rightarrow 0 \text{ in } \mathbb{R}^2$$

- Regular, provided $\Delta\Theta = 2\pi$.

$$\frac{+2GM}{\lambda^2} \quad \frac{4p^2 dp^2}{\lambda^2}$$

$$\frac{8GM}{\lambda} dp^2$$

all simultaneously

~~1~~

Choose $\lambda = 8GM$, $\Theta = t/4GM$.

$$ds^2 = p^2 d\Theta^2 + dp^2 \quad \text{ie. } p \rightarrow 0 \text{ in } \mathbb{R}^2$$

- Regular, provided $\Delta\Theta = 2\pi$.

$$\text{ie. } \Delta t = 8\pi GM.$$

$$\frac{+2GM}{\lambda^2} \quad \frac{4p^2 dp^2}{\lambda^2}$$

$$\frac{8GM}{\lambda} dp^2$$

In thermal field theory,
periodic Euclidean time
corresponds to finite Temp.

$$T = 1/\beta = \frac{1}{8\pi G M} \rightarrow \frac{\hbar c^3}{8\pi G M R_B}$$

'74 Hawking temperature.