

Title: 13/14 PSI - Gravitational Physics Review - Lecture 4

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Abstract:

Lecture 4

Applying Cartan

Spherically symmetric, static.

• STATIC: Killing vector $\frac{\partial}{\partial t}$ & $t \leftrightarrow -t$
symm

Metric:

$$ds^2 = A^2(r) dt^2 - B^2(r) dr^2 - C^2(r) \underbrace{d\Omega^2}_{d\theta^2 + \sin^2\theta d\phi^2}$$

Metric:

$$ds^2 = A^2(r) dt^2 - B^2(r) dr^2 - C^2(r) \underbrace{d\Omega^2}_{d\theta^2 + \sin^2\theta d\phi^2}$$

Metric more general than nec

Apply Cartan:

① | da

① Identify orthon basis

$$\underline{\omega}^{\hat{t}} = A dt$$

$$\underline{\omega}^{\hat{\theta}} = C d\theta$$

$$\underline{\omega}^{\hat{r}} = B dr$$

$$\underline{\omega}^{\hat{\phi}} = C \sin\theta d\phi$$

$\frac{1}{r^2} \sin^2\theta d\phi^2$

$$r^2(r) dr^2 - C^2(r) \underbrace{d\Omega_{\mathbb{E}}^2}_{d\theta^2 + \sin^2\theta d\phi^2}$$

an nec

① Identify orthon basis

$$\underline{\omega}^{\hat{t}} = A \underline{dt} \quad \underline{\omega}^{\hat{\theta}} = C \underline{d\theta}$$

$$\underline{\omega}^{\hat{r}} = B \underline{dr} \quad \underline{\omega}^{\hat{\phi}} = C \sin\theta \underline{d\phi}$$

② Diff.

$$\underline{d}\omega^{\hat{t}} = A' \underline{dr} \wedge \underline{dt} = -\frac{A'}{AB} \omega^{\hat{t}} \wedge \omega^{\hat{r}}$$

$$r^2(r) dr^2 - C^2(r) \underbrace{d\Omega_{\mathbb{S}^2}}_{d\theta^2 + \sin^2\theta d\phi^2}$$

an nec.

① Identify orthon basis

$$\underline{\omega}^{\hat{t}} = A \underline{dt} \quad \underline{\omega}^{\hat{\theta}} = C \underline{d\theta}$$

$$\underline{\omega}^{\hat{r}} = B \underline{dr} \quad \underline{\omega}^{\hat{\phi}} = C \sin\theta \underline{d\phi}$$

② Diff.

$$\underline{d}\underline{\omega}^{\hat{t}} = A' \underline{dr} \wedge \underline{dt} = -\frac{A'}{AB} \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}}$$

$$\underline{d}\underline{\omega}^{\hat{r}} = 0$$

dy c/n basis

$$dt \quad \underline{\omega}^{\hat{\theta}} = C d\theta$$

$$dr \quad \underline{\omega}^{\hat{\phi}} = C \sin\theta d\phi$$

$$A' dr \wedge dt = -\frac{A'}{AB} \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}}$$

$$d\underline{\omega}^{\hat{t}} = -\frac{C'}{CB} \underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\theta}}$$

$$d\underline{\omega}^{\hat{\phi}} = C' \sin\theta dr \wedge d\phi + C \cos\theta d\theta \wedge d\phi$$

$$= -\frac{C'}{CB} \underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\theta}}$$

$$- \frac{\cot\theta}{C} \underline{\omega}^{\hat{\phi}} \wedge \underline{\omega}^{\hat{\theta}}$$

③ Read off connection:

$$\underline{d}\underline{\omega}^a + \underline{\Theta}^a{}_{b\wedge} \underline{\omega}^b = 0 \quad (\text{zero torsion})$$

ie. $\underline{d}\underline{\omega}^{\hat{a}} + \underline{\Theta}^{\hat{a}}{}_{\hat{b}\wedge} \underline{\omega}^{\hat{b}} = \frac{-A^I}{AB} \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}}$

$$\hat{r} = \frac{A^I}{AB} \underline{\omega}^{\hat{t}}$$

$$\left[\underline{d}\underline{\omega}^{\hat{a}} + \underline{\Theta}^{\hat{a}}{}_{\hat{b}\wedge} \underline{\omega}^{\hat{b}} \right]$$

0

③ (zero torsion)

$$\frac{-A'}{AB} \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}}$$

$$\underline{\theta}^{\hat{\theta}} \underline{r}^{\hat{r}} = \frac{C'}{CB} \underline{\omega}^{\hat{\theta}}$$

$$\underline{\theta}^{\hat{\phi}} \underline{r}^{\hat{r}} = \frac{C'}{CB} \underline{\omega}^{\hat{\phi}}$$

$$\underline{\theta}^{\hat{\phi}} \underline{\theta}^{\hat{\theta}} = \frac{\cot \theta}{C} \underline{\omega}^{\hat{\phi}}$$

All others vanish

④ Cartan's 2nd eqn: $\underline{\theta}^{\hat{a}} \wedge \underline{\omega}^{\hat{a}} = 0$

$\frac{C'}{CB} \underline{\omega}^{\hat{\theta}}$
 $\frac{C'}{CB} \underline{\omega}^{\hat{\phi}}$
 $\frac{C \cot \theta}{C} \underline{\omega}^{\hat{\phi}}$

ors vanish

④ Cartan's 2nd eqn: $R^a_b = d \underline{\omega}^a_b + \underline{\omega}^a_c \wedge \underline{\omega}^c_b$

$$R^{\hat{t}}_{\hat{r}} = d \underline{\omega}^{\hat{t}}_{\hat{r}} + \underbrace{\underline{\omega}^{\hat{t}}_{\hat{a}}}_{\hat{a}=\hat{r} \text{ for nonzero}} \wedge \underbrace{\underline{\omega}^{\hat{a}}_{\hat{r}}}_{\text{only nonzero for } \hat{a}=\hat{t}, \hat{\theta}, \hat{\phi}}$$

$$= d \left(\frac{A'}{B} dt \right)$$

$$= \left(\frac{A'}{B} \right)' dr \wedge dt = \frac{1}{B^2} \left(\frac{A''}{A} - \frac{A'B'}{A^2} \right) \underline{\omega}^{\hat{t}}_{\hat{r}} \wedge \underline{\omega}^{\hat{r}}_{\hat{t}}$$

$$\frac{d\omega^{\hat{r}}}{dt} = 0$$

$$\frac{\cot\theta}{c} \omega^{\hat{\phi}} \wedge \omega^{\hat{\theta}}$$

$$\frac{c'}{CB} \omega^{\hat{\theta}}$$

$$\frac{c'}{CB} \omega^{\hat{\phi}}$$

$$\frac{\cot\theta}{c} \omega^{\hat{\phi}}$$

vanish

④ Cartan's 2nd eqn: $R^a_b = d\omega^a_b + \omega^a_c \wedge \omega^c_b$

$$R^{\hat{t}}_{\hat{r}} = d\omega^{\hat{t}}_{\hat{r}} + \underbrace{\omega^{\hat{t}}_{\hat{a}}}_{\hat{a}=\hat{r} \text{ for nonzero}} \wedge \underbrace{\omega^{\hat{a}}_{\hat{r}}}_{\text{only nonzero for } \hat{a}=\hat{t}, \hat{\theta}, \hat{\phi}}$$

$$= d\left(\frac{A'}{B} dt\right)$$

$$= \left(\frac{A'}{B}\right)' dr \wedge dt = \frac{1}{B^2} \left(\frac{A''}{A} - \frac{A'B'}{AB}\right) \omega^{\hat{r}} \wedge \omega^{\hat{t}}$$

$$d\omega^{\hat{\theta}} = -\theta^{\hat{\theta}} \wedge \omega^{\hat{\theta}}$$

$$R^{\hat{\theta}} \hat{r} = \frac{1}{B^2} \left(\frac{C''}{C} - \frac{C'B'}{CB} \right) \omega^{\hat{r}} \wedge \omega^{\hat{\theta}}$$

$$R^{\hat{\phi}} \hat{r} = d \left(\frac{C'}{B} \sin \theta d\phi \right) + \theta^{\hat{\theta}} \wedge \theta^{\hat{\phi}}$$

$$\left(\frac{C'}{B} \right)' \sin \theta dr \wedge d\phi + \frac{C'}{B} \cos \theta d\theta \wedge d\phi$$

$$+ \frac{\cot \theta}{C} \omega^{\hat{\phi}} \wedge \frac{C'}{B} \omega^{\hat{\theta}} = \frac{1}{B^2} \left(\frac{C''}{C} - \frac{C'B'}{CB} \right) \omega^{\hat{r}} \wedge \omega^{\hat{\phi}}$$

$\omega^{\uparrow} \wedge \omega^{\downarrow}$
 $+ \omega^{\uparrow} \otimes \omega^{\downarrow} - \omega^{\downarrow} \otimes \omega^{\uparrow}$
 $+ \frac{c'}{B} \cos \theta d\theta d\varphi$
 $\frac{c'}{B} \omega^{\uparrow} = \frac{1}{B^2} \left(\frac{c''}{c} - \frac{c'B'}{cB} \right) \omega^{\uparrow} \wedge \omega^{\downarrow}$

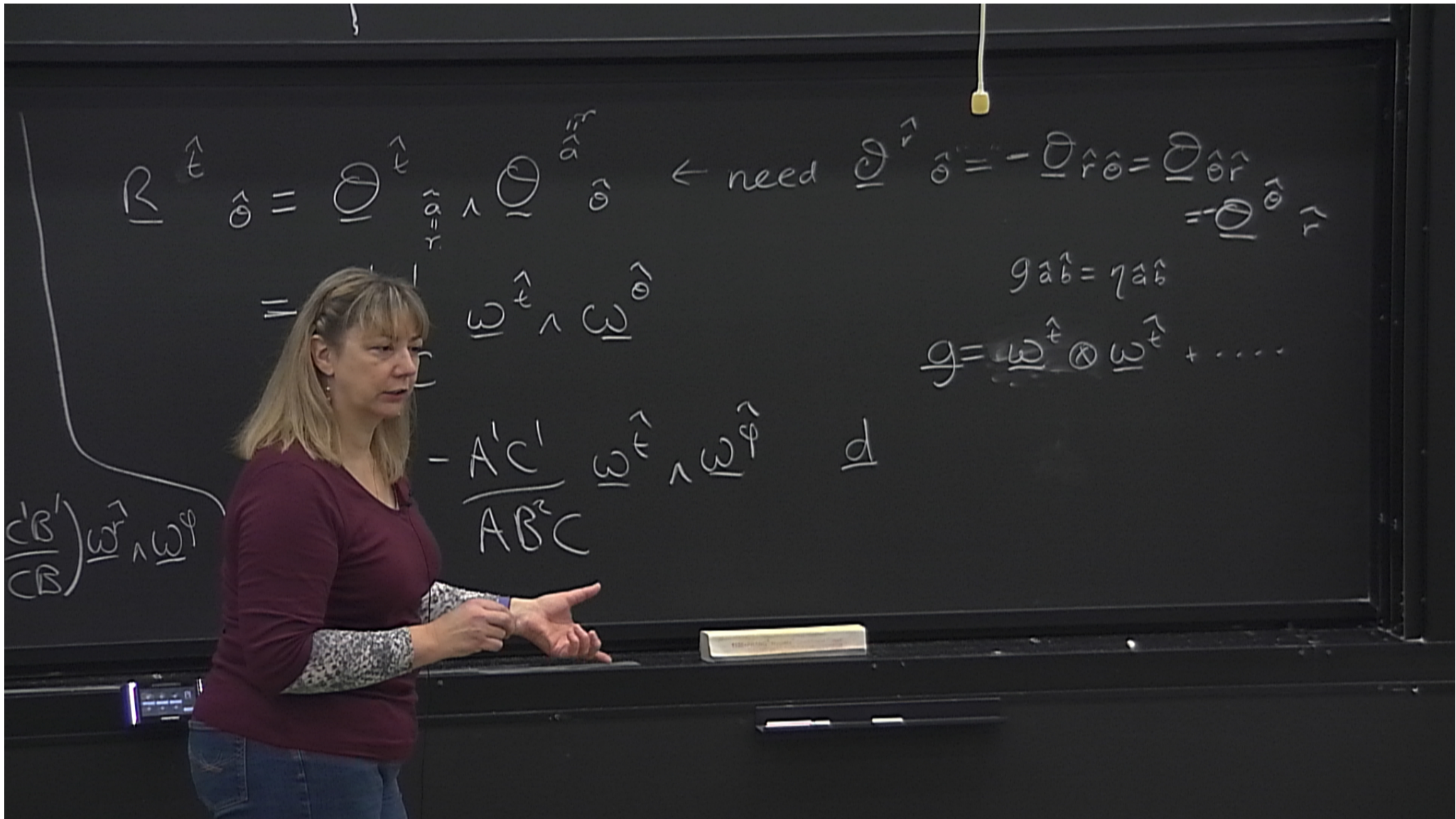
$$R^{\uparrow \downarrow} = \Theta^{\uparrow \downarrow} \wedge \Theta^{\downarrow \uparrow}$$

$$R^{\hat{t}} \hat{\theta} = \Theta^{\hat{t}} \wedge \Theta^{\hat{r}} \hat{\theta} \\ = -\frac{A'C'}{ABC} \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{\theta}}$$

← need $\underline{\partial}^{\hat{r}} \hat{\theta} = -\underline{\partial}^{\hat{t}} \hat{\theta} = \underline{\partial}^{\hat{\theta}} \hat{r} = -\underline{\partial}^{\hat{\theta}} \hat{\theta}$

$$g_{\hat{a}\hat{b}} = \eta_{\hat{a}\hat{b}}$$

$$\left(\frac{C'B'}{CB}\right) \underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\theta}}$$



$$R^{\hat{t}} \hat{o} = \underline{\underline{O}}^{\hat{t}} \wedge \underline{\underline{O}}^{\hat{r}} \hat{o} \quad \leftarrow \text{need } \underline{\underline{O}}^{\hat{r}} \hat{o} = -\underline{\underline{O}}^{\hat{r}} \hat{o} = \underline{\underline{O}}^{\hat{o}} \hat{r} = -\underline{\underline{O}}^{\hat{o}} \hat{r}$$

$$= \underline{\underline{O}}^{\hat{t}} \wedge \underline{\underline{O}}^{\hat{o}}$$

$$g_{\hat{a}\hat{b}} = \eta_{\hat{a}\hat{b}}$$

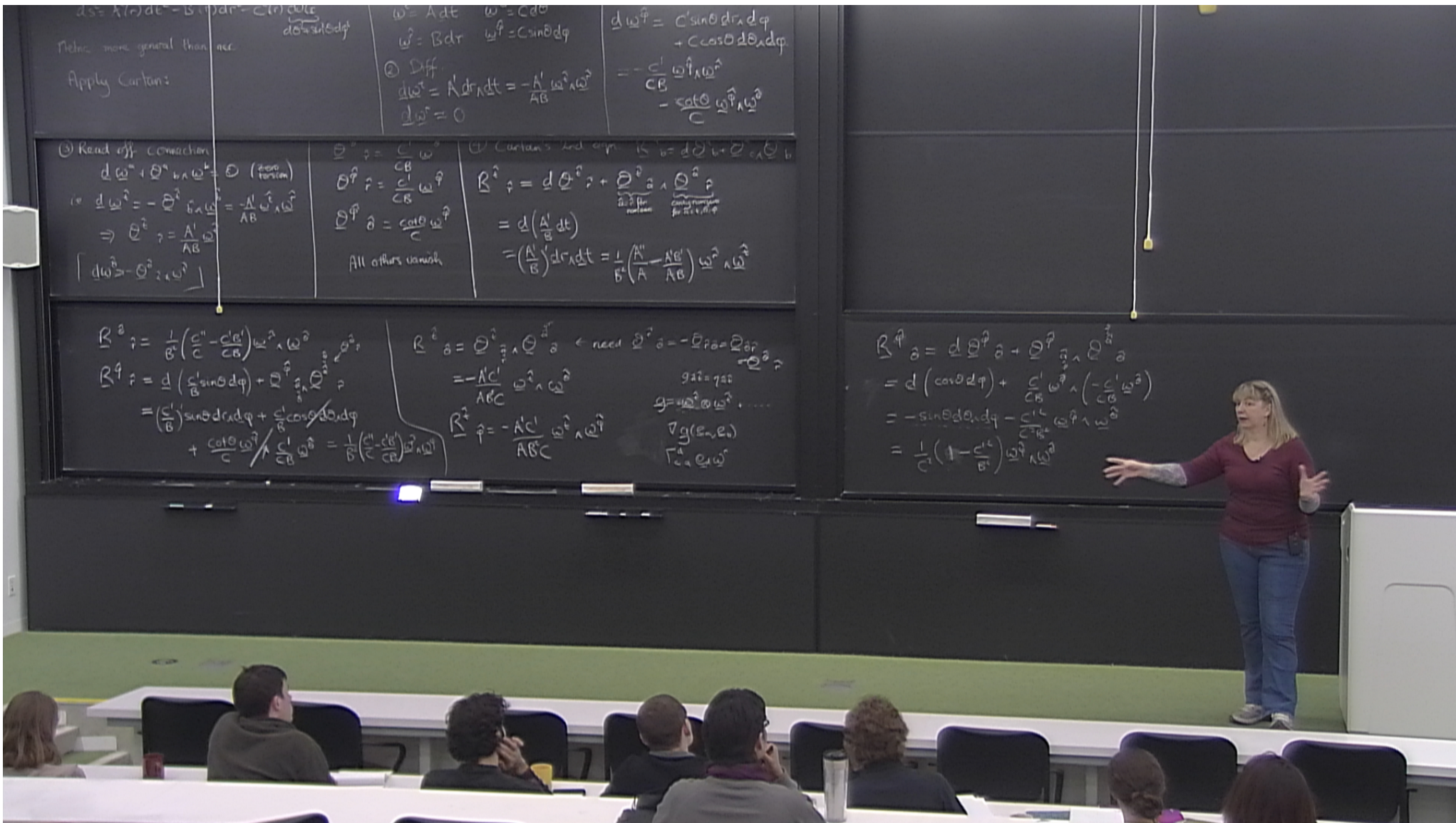
$$g = \underline{\underline{O}}^{\hat{t}} \otimes \underline{\underline{O}}^{\hat{t}} + \dots$$

$$- \frac{A'C'}{ABC} \underline{\underline{O}}^{\hat{r}} \wedge \underline{\underline{O}}^{\hat{\phi}} \quad \underline{\underline{d}}$$

$$\left(\frac{C'B'}{CB} \right) \underline{\underline{O}}^{\hat{r}} \wedge \underline{\underline{O}}^{\hat{\phi}}$$

$$\begin{aligned}
 R^{\hat{t}} \hat{\omega} &= \Theta^{\hat{t}} \wedge \Theta^{\hat{r}} \leftarrow \text{need } \partial^{\hat{r}} \hat{\omega} = -\partial^{\hat{r}} \hat{r} \hat{\omega} = \partial^{\hat{r}} \hat{\omega}^{\hat{r}} \\
 &= -\frac{A'C'}{AB^2} \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}} = -\Theta^{\hat{r}} \wedge \Theta^{\hat{t}} \\
 R^{\hat{t}} \hat{\phi} &= -\frac{A'C'}{AB^2} \underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\phi}} \\
 \left(\frac{C'B'}{CB}\right) \underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\phi}} &
 \end{aligned}$$

$$\begin{aligned}
 g_{\hat{a}\hat{b}} &= \eta_{\hat{a}\hat{b}} \\
 g &= \underline{\omega}^{\hat{t}} \otimes \underline{\omega}^{\hat{t}} + \dots \\
 \nabla g(\underline{e}_a, \underline{e}_b) & \\
 \Gamma_{ca}^d @_d \underline{\omega}^r &
 \end{aligned}$$



$ds = A(r)dr - B(\theta)d\theta - C(\phi)d\phi$
 (more general than nec.)
 Apply Cartan:
 $\omega = A dr$ $\omega = C d\theta$
 $\omega^2 = B dr$ $\omega^2 = C \sin\theta d\phi$
 ② Diff:
 $d\omega^2 = A' dr dt = -\frac{A'}{AB} \omega^2 \wedge \omega^2$
 $d\omega^2 = 0$
 $d\omega^2 = C' \sin\theta dr d\phi + C \cos\theta d\theta d\phi$
 $= -\frac{C'}{CB} \omega^2 \wedge \omega^2$
 $= -\frac{\cot\theta}{C} \omega^2 \wedge \omega^2$

③ Read off connection
 $d\omega^2 + \Omega^a{}_b \omega^b = 0$ (torsion)
 $d\omega^2 = -\Omega^2{}_1 \omega^1 = -\frac{A'}{AB} \omega^1 \wedge \omega^2$
 $\Rightarrow \Omega^2{}_1 = \frac{A'}{AB} \omega^1$
 $[d\omega^2 = -\Omega^2{}_1 \omega^1]$
 $\Omega^1{}_2 = \frac{C'}{CB} \omega^2$
 $\Omega^1{}_1 = \frac{C'}{CB} \omega^1$
 $\Omega^2{}_2 = \frac{\cot\theta}{C} \omega^2$
 All others vanish
 ④ Cartan's 2nd eqn: $R^a{}_b = d\Omega^a{}_b + \Omega^a{}_c \wedge \Omega^c{}_b$
 $R^2{}_1 = d\Omega^2{}_1 + \Omega^2{}_a \wedge \Omega^a{}_1$
 $= d\left(\frac{A'}{AB} \omega^1\right)$
 $= \left(\frac{A'}{B}\right)' dr dt = \frac{1}{B} \left(\frac{A''}{A} - \frac{A'^2}{A^2}\right) \omega^1 \wedge \omega^2$

$R^1{}_2 = \frac{1}{B} \left(\frac{C''}{C} - \frac{C'^2}{C^2}\right) \omega^2 \wedge \omega^2$
 $R^1{}_1 = d\left(\frac{C'}{CB} \sin\theta d\phi\right) + \Omega^1{}_2 \wedge \Omega^2{}_1$
 $= \left(\frac{C'}{B}\right)' \sin\theta dr d\phi + \frac{C'}{B} \cos\theta d\theta d\phi$
 $+ \frac{\cot\theta}{C} \omega^2 \wedge \frac{C'}{CB} \omega^2 = \frac{1}{B} \left(\frac{C''}{C} - \frac{C'^2}{C^2}\right) \omega^2 \wedge \omega^2$
 $R^2{}_2 = \Omega^2{}_1 \wedge \Omega^1{}_2 \leftarrow \text{need } \Omega^2{}_1 = -\Omega^1{}_2 = \Omega^2{}_1$
 $= -\frac{A'C'}{ABC} \omega^2 \wedge \omega^2$
 $R^1{}_2 = -\frac{A'C'}{ABC} \omega^2 \wedge \omega^2$

 $g_{11} = 1$
 $g_{22} = \frac{1}{B^2}$
 $\nabla g(e_1, e_1) = 0$
 $\Gamma^1_{22} = 0$

$R^1{}_2 = d\Omega^1{}_2 + \Omega^1{}_a \wedge \Omega^a{}_2$
 $= d\left(\frac{C'}{CB} \sin\theta d\phi\right) + \Omega^1{}_2 \wedge \Omega^2{}_1$
 $= -\sin\theta d\theta d\phi - \frac{C'}{CB} \cos\theta d\theta d\phi + \frac{C'}{CB} \omega^2 \wedge \omega^2$
 $= \frac{1}{C} \left(1 - \frac{C'}{B}\right) \omega^2 \wedge \omega^2$

$$\begin{aligned}
 \underline{R}^{\hat{\varphi}}_{\hat{\theta}} &= \underline{d}\underline{\theta}^{\hat{\varphi}}_{\hat{\theta}} + \underline{\theta}^{\hat{\varphi}}_{\hat{\theta}} \\
 &= \underline{d}(\cos\theta \underline{d}\varphi) + \frac{c}{B} \underline{\omega}^{\hat{\varphi}}_{\hat{\theta}} \wedge \left(\frac{-c}{B} \underline{\omega}^{\hat{\theta}}_{\hat{\theta}} \right) \\
 &= -\sin\theta \underline{d}\theta \wedge \underline{d}\varphi - \frac{c^2}{B^2} \underline{\omega}^{\hat{\varphi}}_{\hat{\theta}} \wedge \underline{\omega}^{\hat{\theta}}_{\hat{\theta}} \\
 &= \frac{1}{c^2} \left(1 - \frac{c^2}{B^2} \right) \underline{\omega}^{\hat{\varphi}}_{\hat{\theta}} \wedge \underline{\omega}^{\hat{\theta}}_{\hat{\theta}}
 \end{aligned}$$

⑤ Riemann

$R^{\hat{t}}$



Riemann: $\underline{R}^a{}_b = \frac{1}{2} R^a{}_{bcd} \underline{\omega}^c \wedge \underline{\omega}^d$

$$\hat{r} \hat{t} \hat{r} = -\frac{1}{B^2} \left(\frac{A''}{A} - \frac{A'B'}{AB} \right)$$

$$\hat{r} \hat{\theta} \hat{r} = -\frac{1}{B^2} \left(\frac{C''}{C} - \frac{C'B'}{CB} \right) = R^{\hat{r}}{}_{\hat{r}\hat{\theta}\hat{r}}$$

$$\hat{\theta} \hat{t} \hat{\theta} = -\frac{A'C'}{AB^2C} = R^{\hat{t}}{}_{\hat{\theta}\hat{t}\hat{\theta}}$$

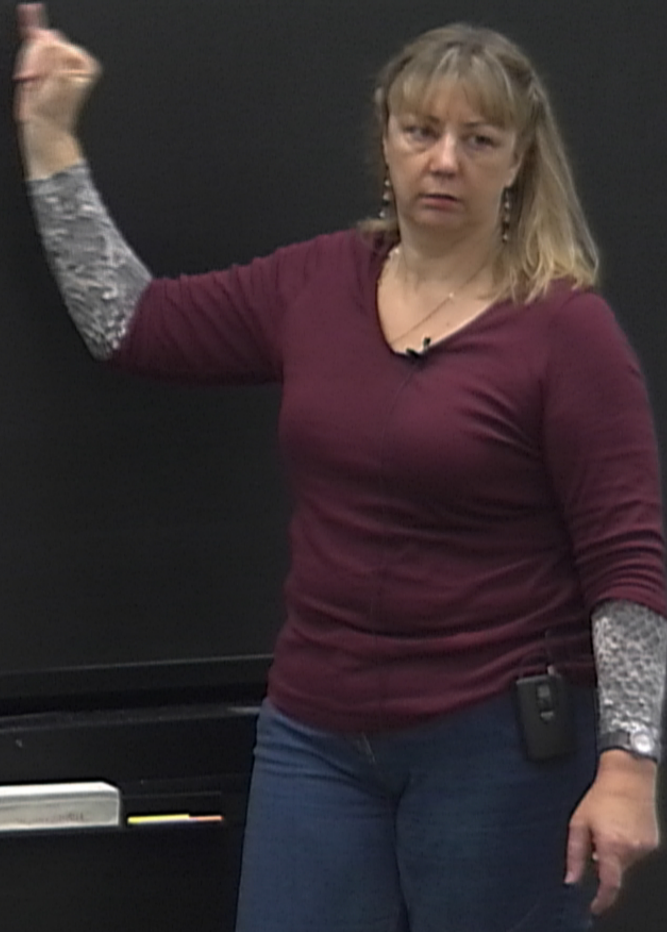
$$R^{\hat{r}}{}_{\hat{\theta}\hat{r}\hat{\theta}} = \frac{1}{C^2} \left(1 - \frac{C'^2}{B^2} \right)$$

Riemann in orthon basis

$$\underline{R} = R^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}} \underline{e}^{\hat{a}} \wedge \underline{\omega}^{\hat{b}} \wedge \underline{\omega}^{\hat{c}} \wedge \underline{\omega}^{\hat{d}}$$



$$\text{e.g. } R^t \text{ rtr} = B^2 R^{\hat{t}} \hat{t} \hat{t}$$



$$\text{e.g. } R^t{}_{rtr} = B^2 R^{\hat{t}}{}_{\hat{r}\hat{r}\hat{t}}$$

For Einstein eqns:

$$R^t{}_{t} = R^{\hat{t}}{}_{\hat{t}} =$$

$$\text{e.g. } R^t{}_{rtr} = B^2 R^{\hat{t}}{}_{\hat{t}\hat{t}\hat{t}}$$

For Einstein eqns:

$$R^t{}_{t} = R^{\hat{t}}{}_{\hat{t}} = \frac{1}{B^2} \left[\frac{A''}{A} - \frac{A'B'}{AB} + \frac{2A'C'}{AC} \right]$$

$$\text{e.g. } R^t_{\text{tr}} = B^2 R^{\hat{t}}_{\hat{t}}$$

For Einstein eqns:

$$R^t_t = R^{\hat{t}}_{\hat{t}} = \frac{1}{B^2} \left[\frac{A''}{A} - \frac{A'B'}{AB} + \frac{2AC'}{AC} \right]$$

$$R^0_0 = \frac{1}{B^2} \left[\frac{C''}{C} - \frac{C'B'}{CB} + \frac{AC'}{AC} + \frac{C'^2}{C^2} \right] -$$

$$R^A \hat{r} = \frac{1}{B^2} \left[\frac{A''}{A} + \frac{2C''}{C} - \frac{B'}{B} \right] \frac{A'}{A}$$

$$R^r = R^A \hat{r} = \frac{1}{B^2} \left[\frac{A''}{A} + \frac{2C''}{C} - \frac{B'}{B} \left(\frac{A'}{A} + \frac{2C'}{C} \right) \right]$$

$$\Rightarrow G^t_t = \frac{1}{C^2} - \frac{1}{B^2} \left(\frac{2C''}{C} - \frac{2C'B'}{CB} + \frac{C'^2}{C^2} \right)$$

$$G^r_r = \frac{1}{C^2} - \frac{1}{B^2} \left(\frac{2A'C'}{AC} + \frac{C'^2}{C^2} \right)$$

$$G^{\theta}_{\theta} = G^{\varphi}_{\varphi} = -\frac{1}{B^2} \left(\frac{A''}{A} + \frac{C''}{C} + \frac{A'C'}{AC} - \frac{B'}{B} \left(\frac{A'}{A} + \frac{C'}{C} \right) \right)$$

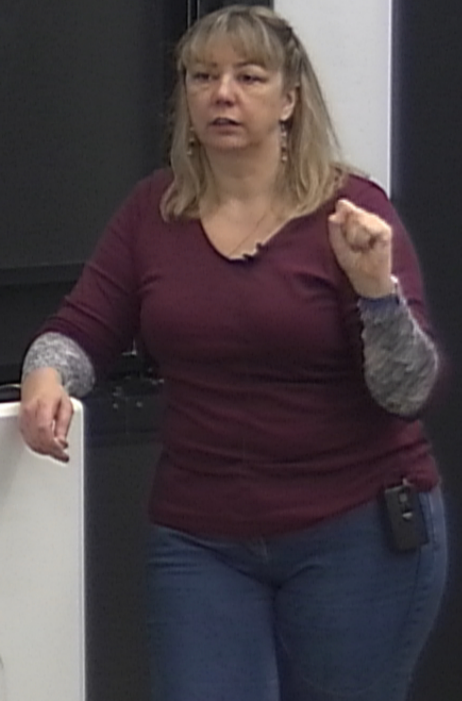
$$R_r = R^r \hat{r} = \frac{1}{B^2} \left[\frac{A''}{A} + \frac{2C''}{C} - \frac{B'}{B} \left(\frac{A'}{A} + \frac{2C'}{C} \right) \right]$$

$$\Rightarrow G^t_t = \frac{1}{C^2} - \frac{1}{B^2} \left(\frac{2C''}{C} - \frac{2C'B'}{CB} + \frac{C'^2}{C^2} \right)$$

$$G^r_r = \frac{1}{C^2} - \frac{1}{B^2} \left(\frac{2AC'}{AC} + \frac{C'^2}{C^2} \right)$$

$$G^0_0 = G^q_q = -\frac{1}{B^2} \left(\frac{A''}{A} + \frac{C''}{C} + \frac{AC'}{AC} - \frac{B'}{B} \left(\frac{A'}{A} + \frac{C'}{C} \right) \right)$$

$$\left. \frac{C'}{C} \right\}$$

$$-\frac{1}{C^2}$$


$\delta = -\partial_{\hat{r}} \hat{\theta} = \partial_{\hat{\theta}} \hat{r}$
 $= -\partial_{\hat{\theta}} \hat{r}$

$g_{\hat{a}\hat{b}} = \eta_{\hat{a}\hat{b}}$

$g = \underline{\omega}^{\hat{t}} \otimes \underline{\omega}^{\hat{t}} + \dots$

$\nabla g(\underline{e}_a, \underline{e}_b)$

$\Gamma_{ca}^d \underline{\omega}^c \underline{\omega}^a$

$K_0 = \frac{1}{B^2} \left[\frac{1}{c} - \frac{cB}{cB} + \frac{1}{c} + \frac{1}{c^2} \right] c^2$

$\Rightarrow \frac{1}{r^2} + \frac{2B'B^{-3}}{r} - \frac{B^{-2}}{r^2} = 8\pi q T_0$

$(r B^{-2})' = 8\pi q T_0 r$

B^{-2}



$\hat{\theta} = -\underline{\partial} \hat{r} \hat{\theta} = \underline{\partial} \hat{\theta} \hat{r}$
 $= -\underline{\partial} \hat{\theta} \hat{r}$
 $= \gamma \hat{a} \hat{b}$
 $\underline{\omega}^{\hat{t}} + \dots$
 (a, b)
 $\underline{\omega}^r$

$$K_{\theta} = \frac{1}{B^2} \left[\frac{1}{c} - \frac{cb}{cB} + \frac{Ac}{Ac} + \frac{c}{c^2} \right] c^2$$

$$\Rightarrow \frac{1}{r^2} + \frac{2B'B^{-3}}{r} - \frac{B^{-2}}{r^2} = 8\pi G T_{\theta}^{\theta}$$

$$(r B^{-2})' = 1 - 8\pi G T_{\theta}^{\theta} r$$

$$B^{-2} = 1 - \frac{2GM(r)}{r}$$

$$\frac{1}{r^2} - \left(1 - \frac{2GM(r)}{r}\right) \left(\frac{2A'}{Ar} + \frac{1}{r^2}\right) = -8\pi G p_r$$

$$\Rightarrow \frac{(A^2)'}{A^2} = \frac{2GM(r) + 8\pi G r^2 p_r}{r(r - 2GM(r))}$$

Cons energy-momentum

"mass
inside
r"

Cons energy-momentum

$$\nabla_a T^{ab} = 0 \quad \left(\begin{array}{l} \Gamma_{\hat{t}\hat{r}}^{\hat{t}} = \frac{A'}{AB} = \Gamma_{\hat{t}\hat{t}}^{\hat{r}} \\ \Gamma_{\hat{\theta}\hat{r}}^{\hat{\theta}} = \frac{C'}{CB} = -\Gamma_{\hat{\theta}\hat{\theta}}^{\hat{r}} \end{array} \right)$$

\hat{r} -cpt:

$$\nabla_a T^{ar} = \partial_r^{\hat{r}} P + \Gamma_{\hat{a}}^{\hat{a}} P_r$$

Cons energy-momentum

$$\nabla_a T^{ab} = 0$$

$$\left(\begin{array}{l} \Gamma_{\hat{t}\hat{t}\hat{r}}^{\hat{t}} = \frac{A'}{AB} = \Gamma_{\hat{t}\hat{t}\hat{t}}^{\hat{r}} \\ \Gamma_{\hat{\theta}\hat{r}}^{\hat{\theta}} = \frac{C'}{CB} \end{array} \right)$$

\hat{r} -cpt:

$$\nabla_a T^{a\hat{r}} = \partial_{\hat{r}} P_r + \Gamma_{\hat{a}\hat{r}}^{\hat{a}} P_r + \Gamma_{\hat{a}\hat{b}}^{\hat{r}} T^{\hat{a}\hat{b}}$$

entum

$$\Gamma_{t \hat{r}}^{\hat{t}} = \frac{A'}{AB} = \Gamma_{t \hat{t}}^{\hat{r}}$$

$$\Gamma_{\hat{\theta} \hat{r}}^{\hat{\theta}} = \frac{C'}{CB} = -\Gamma_{\hat{\theta} \hat{\theta}}^{\hat{r}}$$

\hat{r} -cpt:

$$\nabla_{\hat{a}} T^{\hat{a} \hat{r}} = \partial_{\hat{r}} P_{\hat{r}} + \Gamma_{\hat{a} \hat{r}}^{\hat{a}} P_{\hat{r}} + \Gamma_{\hat{a} \hat{b}}^{\hat{r}} T^{\hat{a} \hat{b}}$$

$$= \frac{1}{B} \left(P_{\hat{r}}' + \left(\frac{A'}{AB} + \frac{2}{rB} \right) P_{\hat{r}} + \left(\frac{A'}{AB} P - \frac{2}{rB} P_{\hat{r}} \right) \right)$$

$$= \frac{1}{B} \left(P_{\hat{r}}' \right)$$