Title: 13/14 PSI - Standard Model Review - Lecture 9

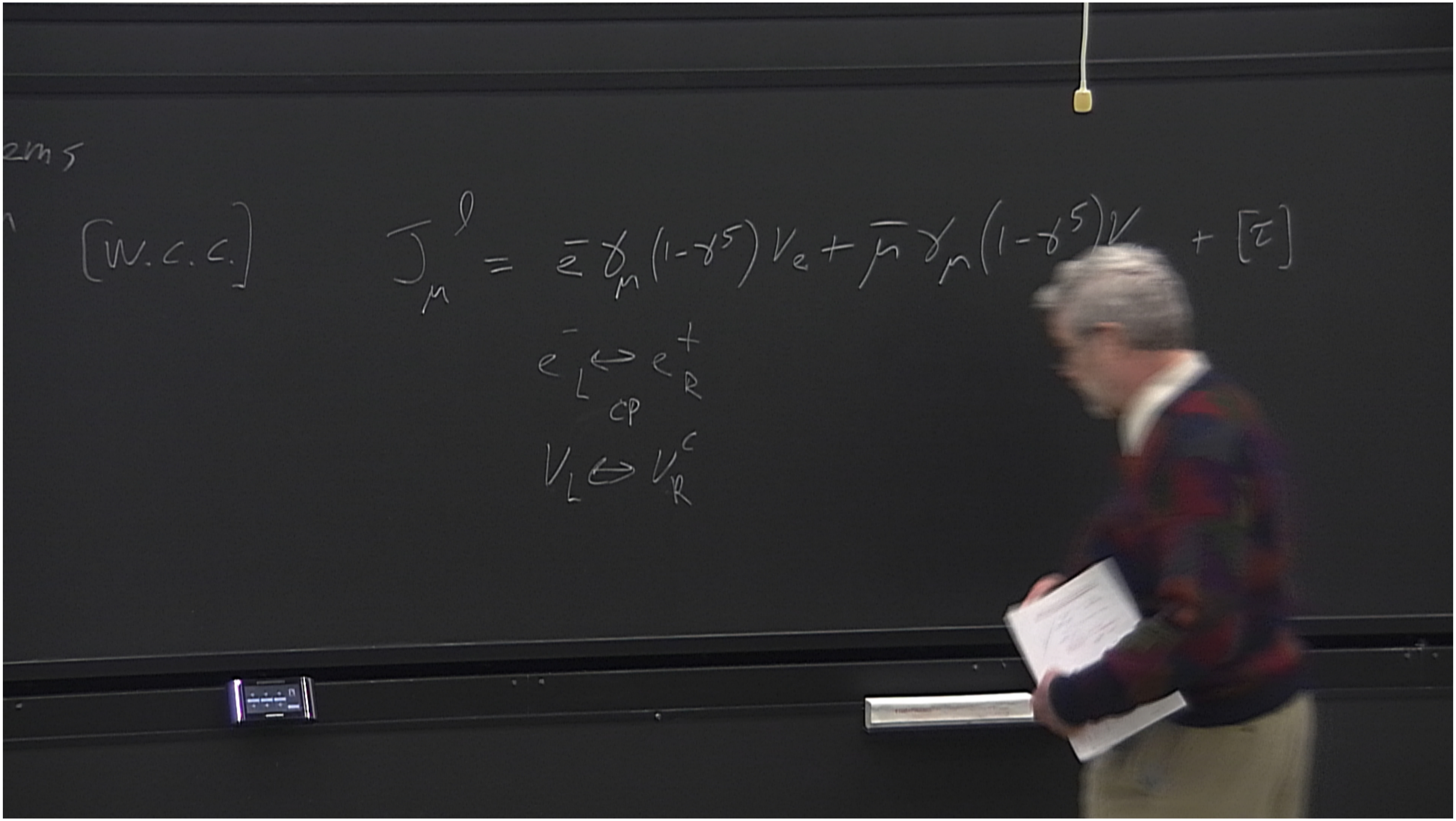
Date: Jan 16, 2014 09:00 AM

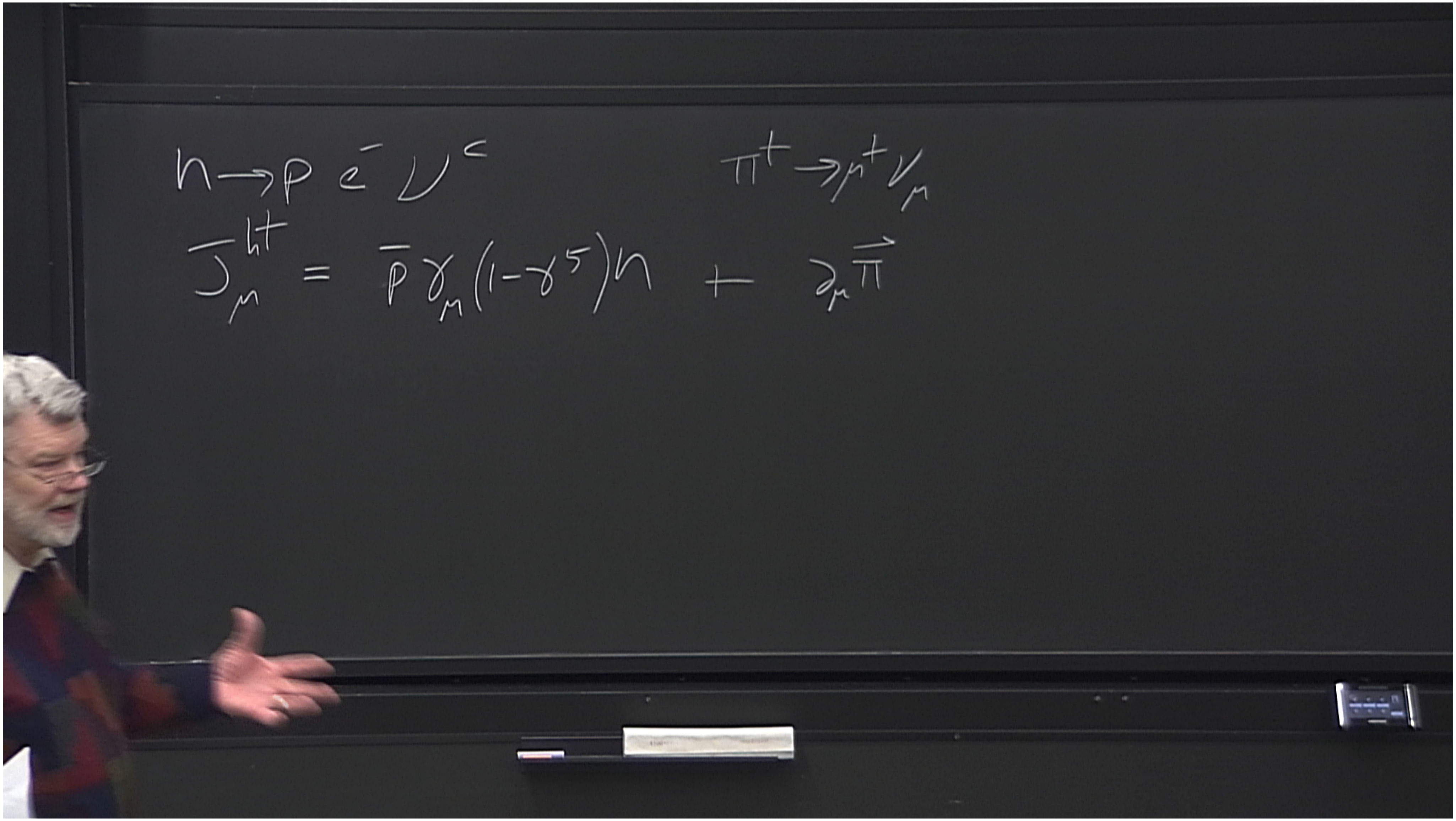
URL: http://pirsa.org/14010018

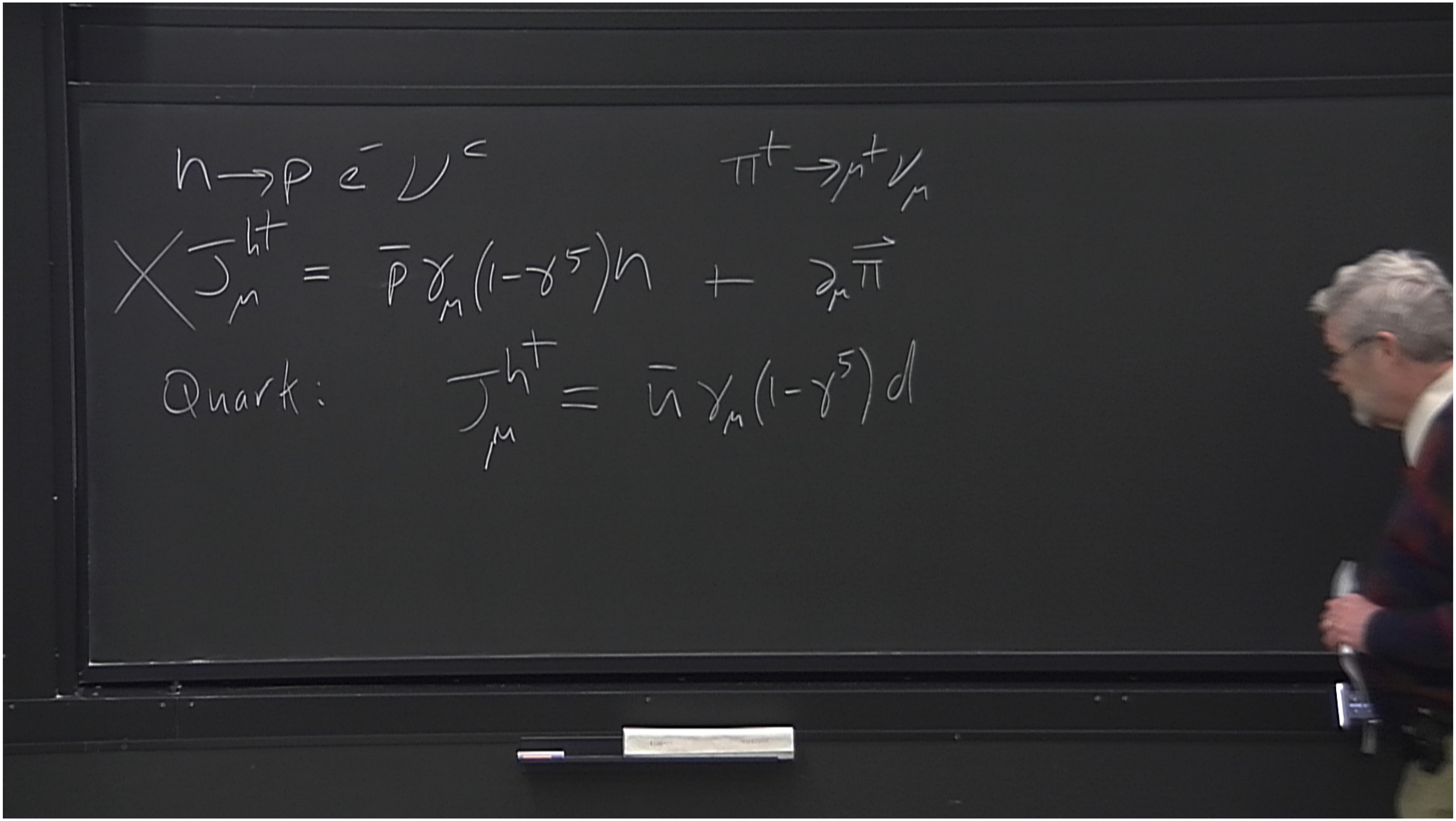
Abstract:

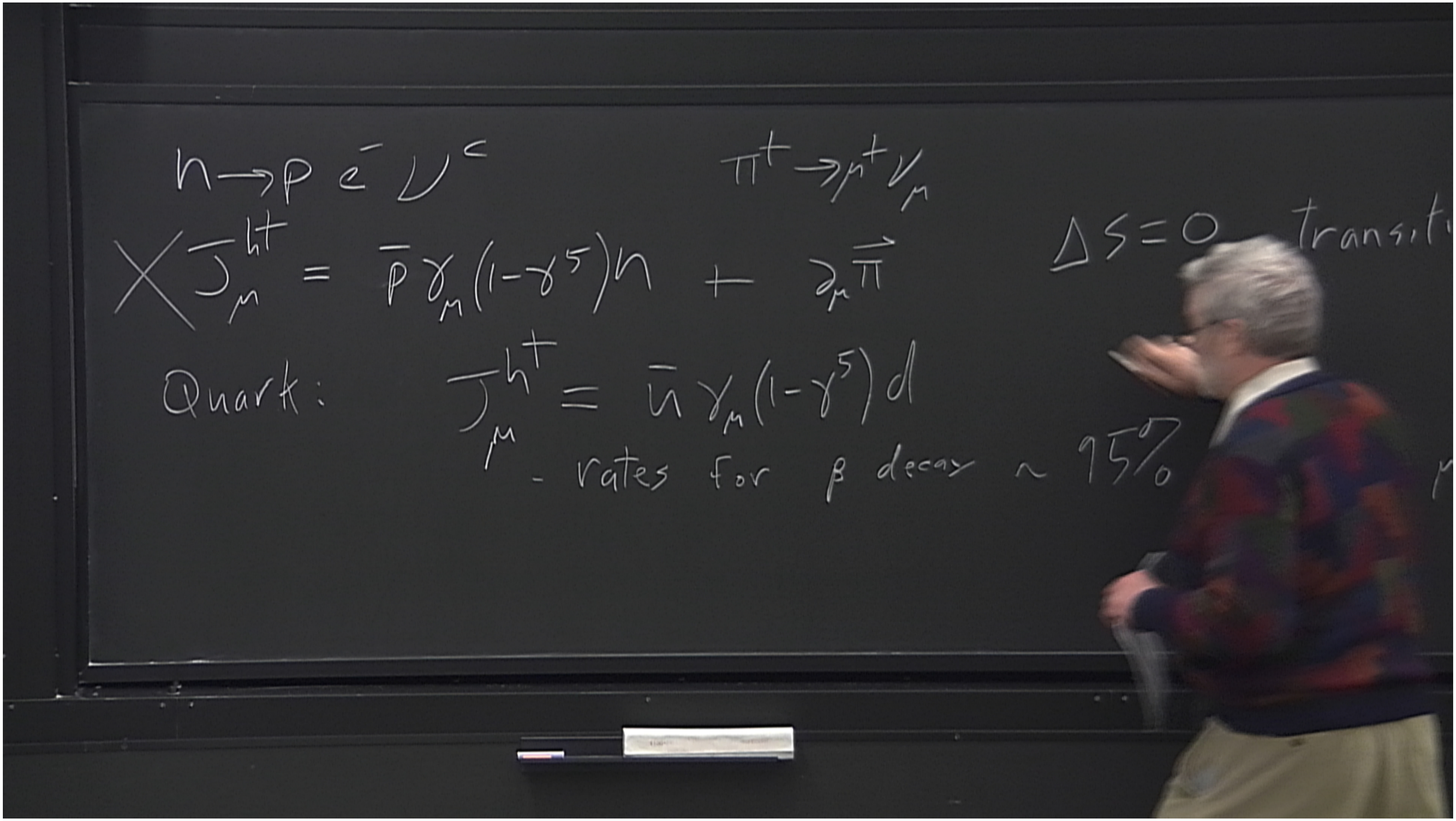
9 Fermi Theory: semi-leptonic; problems

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} J_\mu^{+} J^\mu \qquad J_\mu = J_\mu^{l} + J_\mu^{h}$$


ems
[W.C.C.]
$J_\mu^\ell = \bar{e}\gamma_\mu(1-\gamma^5)\nu_e + \bar{\mu}\gamma_\mu(1-\gamma^5)\nu + [\tau]$
$e^-_L \leftrightarrow e^+_R$
CP
$\nu_L \leftrightarrow \nu^c_R$


$n \to p\, e^- \nu^c$
$\pi^+ \to \mu^+ \nu_\mu$
$J_\mu^{wk} = \bar{p}\gamma_\mu(1-\gamma^5)n + \partial_\mu \vec{\pi}$


$n \to p\, e^- \nu^c$
$\pi^+ \to \mu^+ \nu_\mu$
$J_\mu^{h+} = \bar{p}\gamma_\mu(1-\gamma^5)n + \partial_\mu \vec{\pi}$
Quark: $J_\mu^{h+} = \bar{u}\gamma_\mu(1-\gamma^5)d$


$n \to p\, e^- \nu^c$
$\pi^+ \to \mu^+ \nu_\mu$
$J^{h+}_\mu = \bar{p}\gamma_\mu(1-\gamma^5)n + \partial_\mu \vec{\pi}$
$\Delta S = 0$
Quark: $J^{h+}_\mu = \bar{u}\gamma_\mu(1-\gamma^5)d$
- rates for β decay $\sim 95\%$

$p\, e^- \nu^c$ $\qquad \pi^+ \to \mu^+ \nu_\mu$

$$= \bar{p}\gamma_\mu(1-\gamma^5)n + \partial_\mu \vec{\pi}$$

$\Delta S = 0$ transitions

k: $J_\mu^{h+} = \bar{u}\gamma_\mu(1-\gamma^5)d$

- rates for β decay $\sim 95\%$ rates for μ decay

$\Delta S \neq$

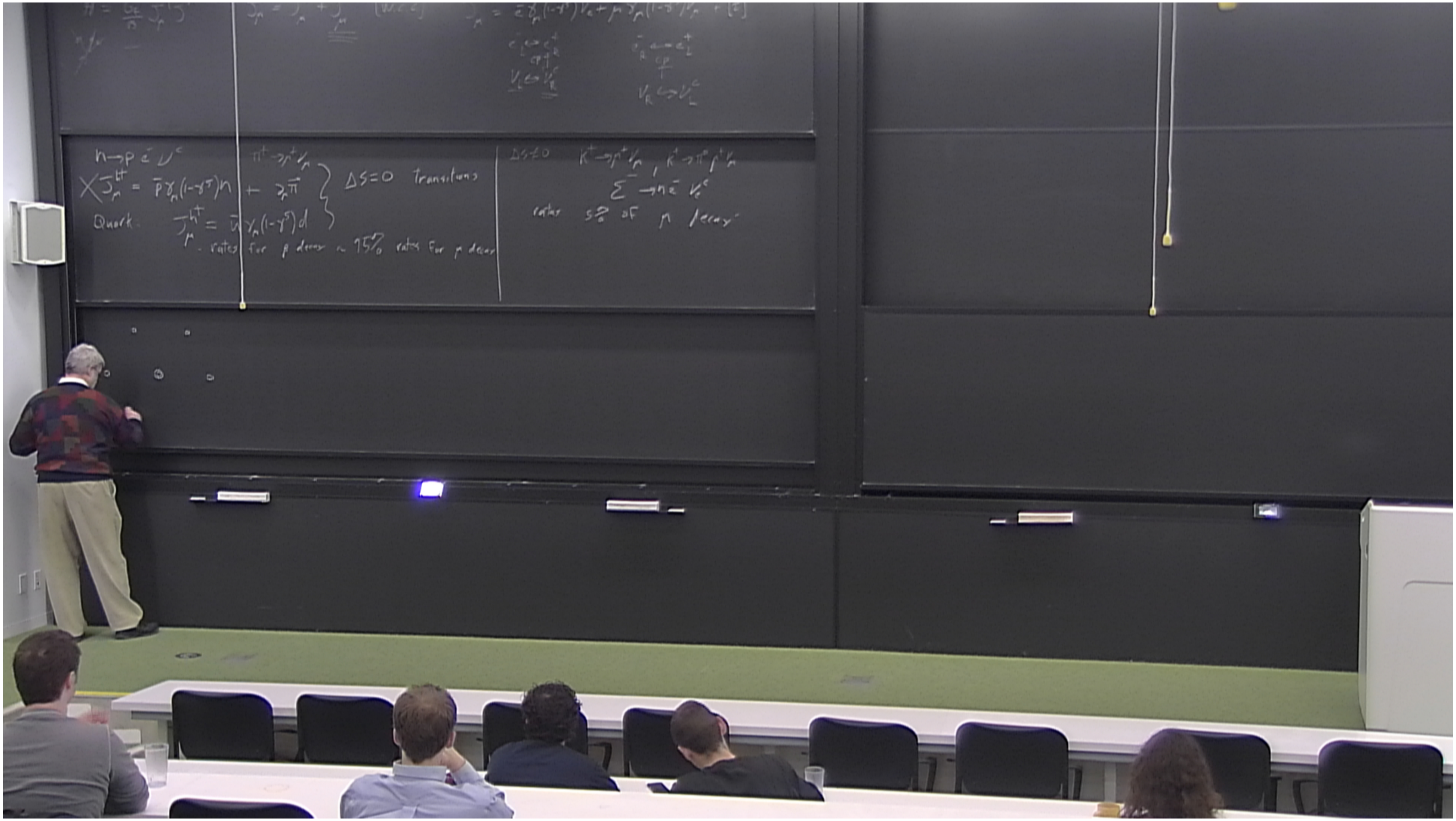
0 transitions

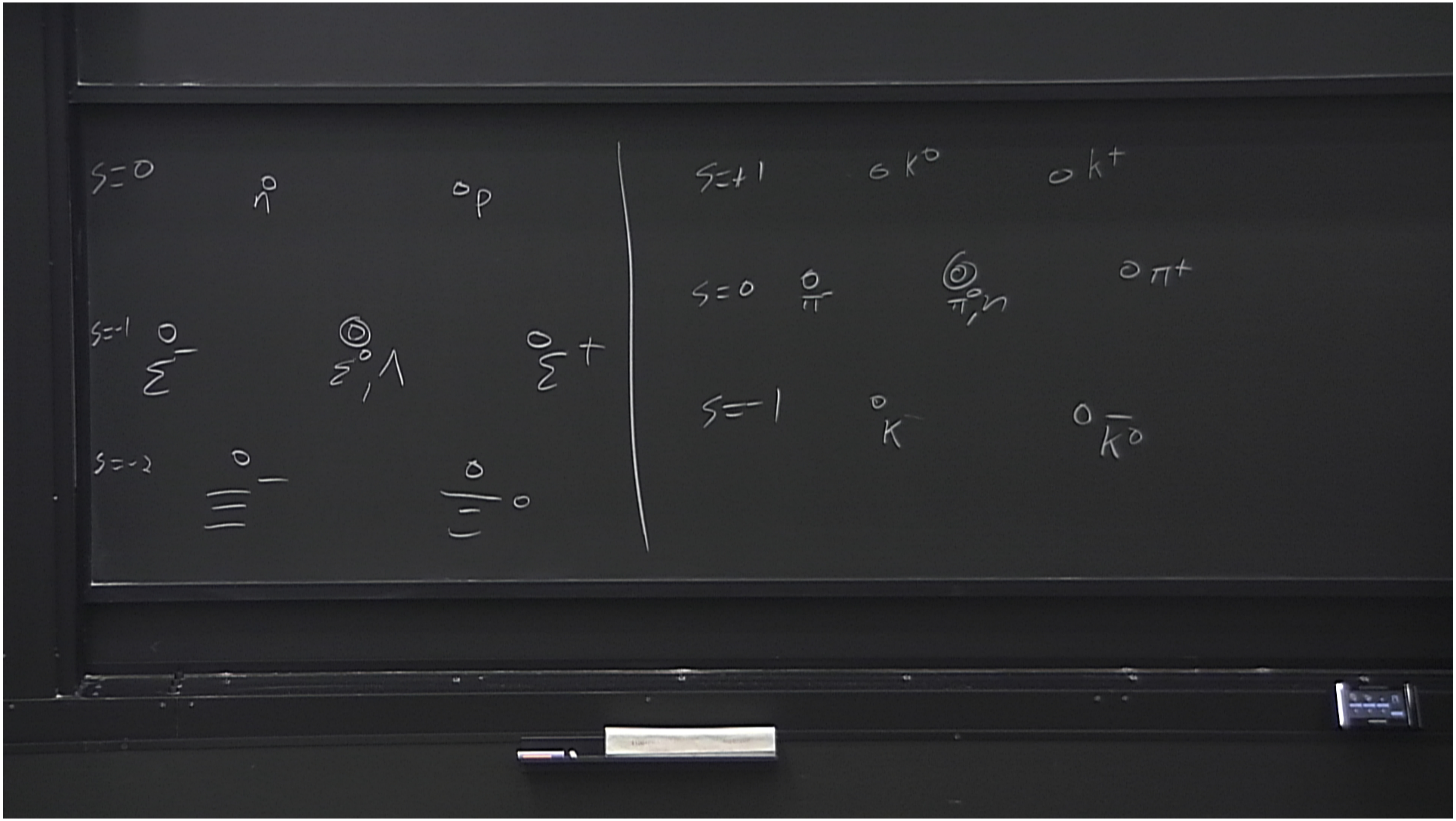
rates for μ decay

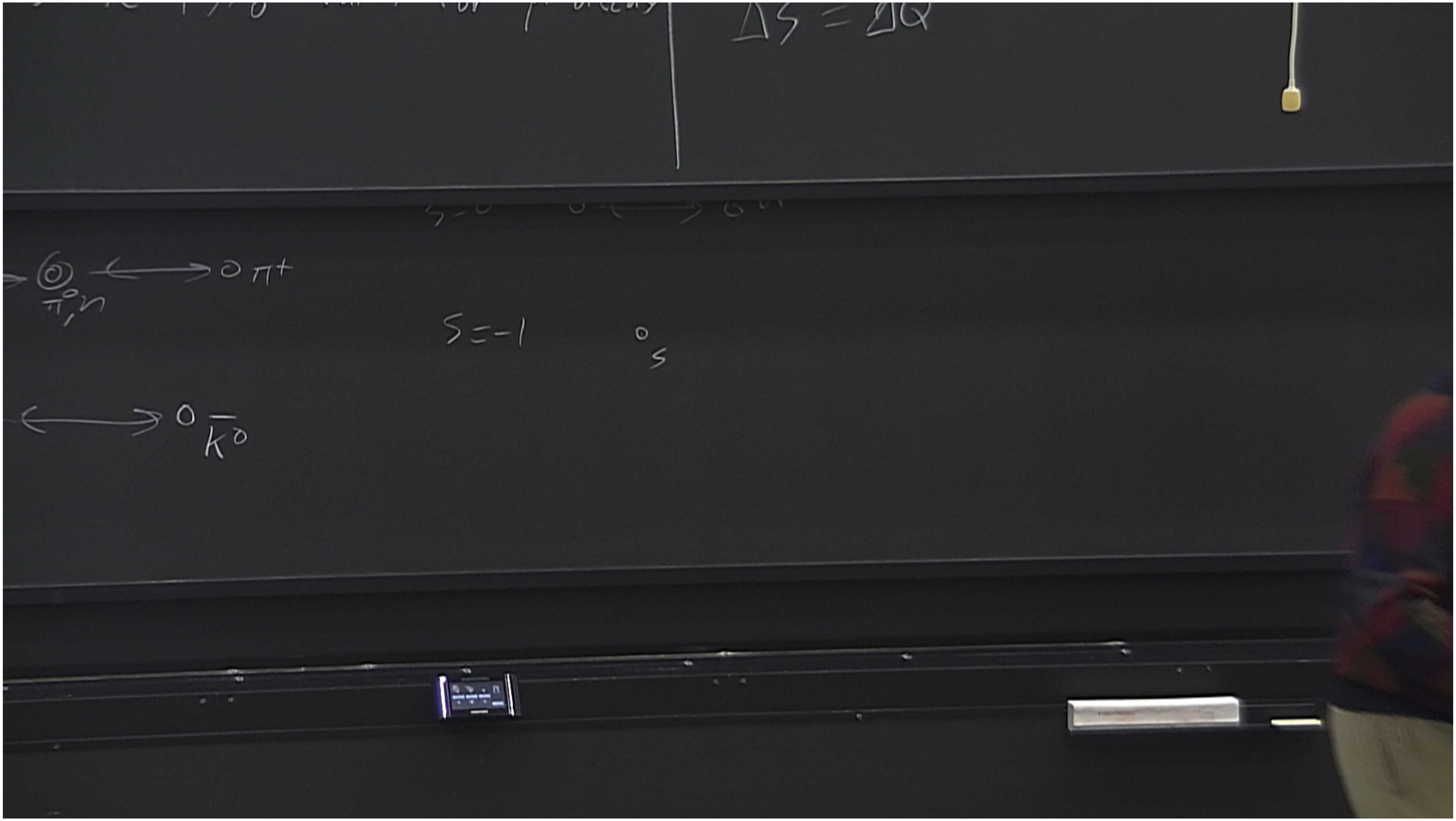
$\Delta S \neq 0$ $\quad K^+ \to \mu^+ \nu_\mu$, $K^+ \to \pi^0 \mu^+ \nu_\mu$

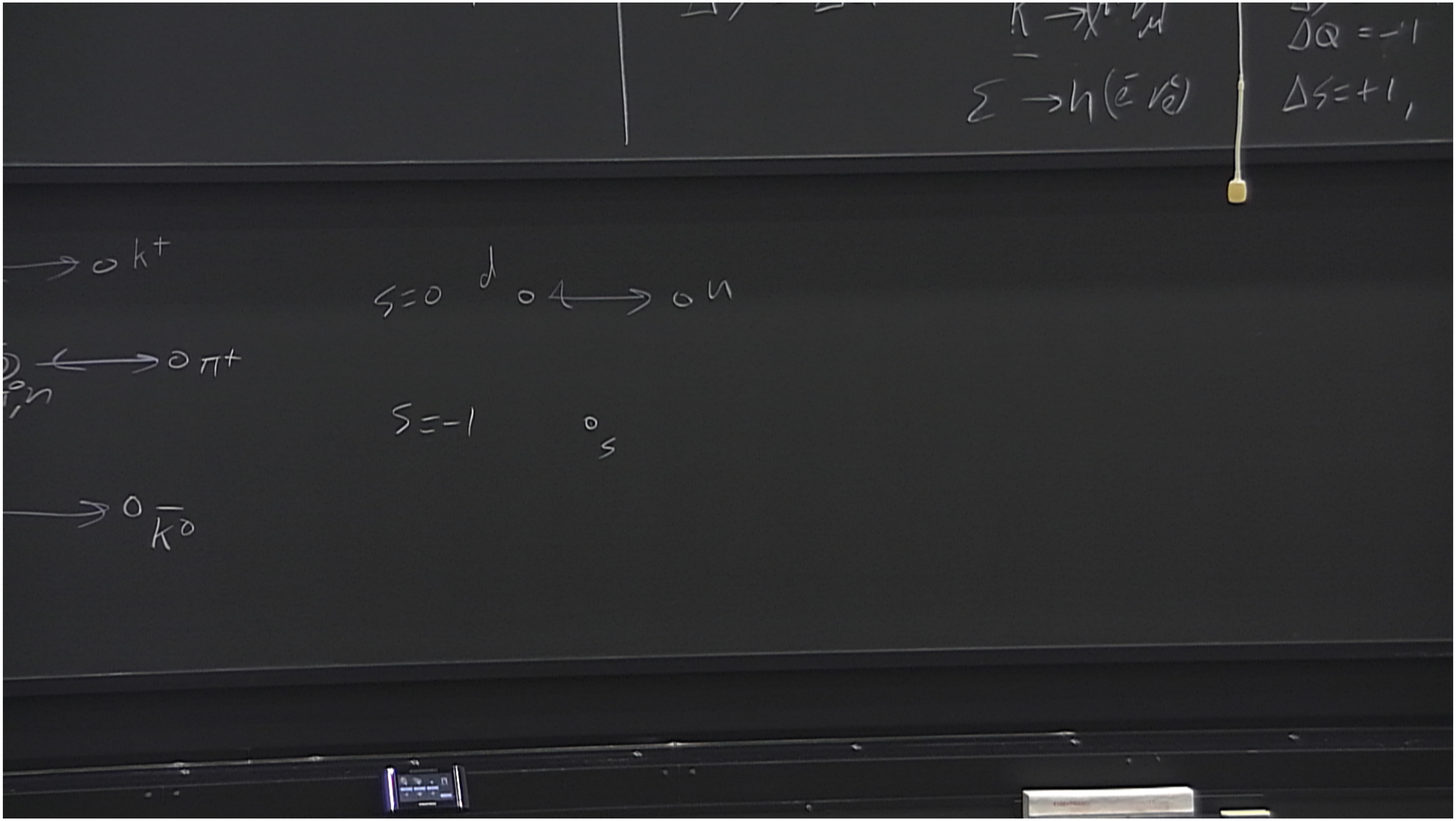
$\Sigma^- \to n e^- \bar{\nu}_e$

rates 5% of μ decay


ΔS=0 transitions
Quark
rates for μ decay


S=0 n p
S=-1 Σ⁻ Σ⁰, Λ Σ⁺
S=-2 Ξ⁻ Ξ⁰
S=+1 K⁰ K⁺
S=0 π⁻ π⁰, η π⁺
S=-1 K⁻ K̄⁰


ΔS = ΔQ
S = -1


ΔQ = -1
Σ → n(e⁻ ν̄e)
ΔS = +1,
→ o K⁺
S=0 d o ⟷ o u
⟷ o π⁺
S=-1 o s
→ o K̄⁰

$K \to \pi^0 \mu \nu_\mu$

$\bar{\Sigma} \to n(\bar{e}\,\bar{\nu}_e)$

$\Delta Q = -1$

$\Delta S = +1, \quad \Delta Q = +1$

$S = 0 \qquad {}^{\circ}d \longleftrightarrow {}^{\circ}u$

$S = -1 \qquad {}^{\circ}s$

$\Sigma^+ \not\to n e^+ \nu_e$ not observed

$\Delta S = +1, \ \Delta Q = -1$

$\Delta S = -\Delta Q$

$\Sigma^+ \not\to p e^+ e^-$ WNC

$\Delta S = +1$

$\Delta Q = 0$

$\Sigma \to n(e\,\nu_e)$ $\quad \Delta S = +1, \quad \Delta Q = \ldots$

$S=0 \qquad d \; \circ \longleftrightarrow \circ \; u$

$S=-1 \qquad \circ \; s$

$\Delta S = \Delta Q$

$\cancel{\Sigma^+ \to n e^+ \nu_e}$ not observed

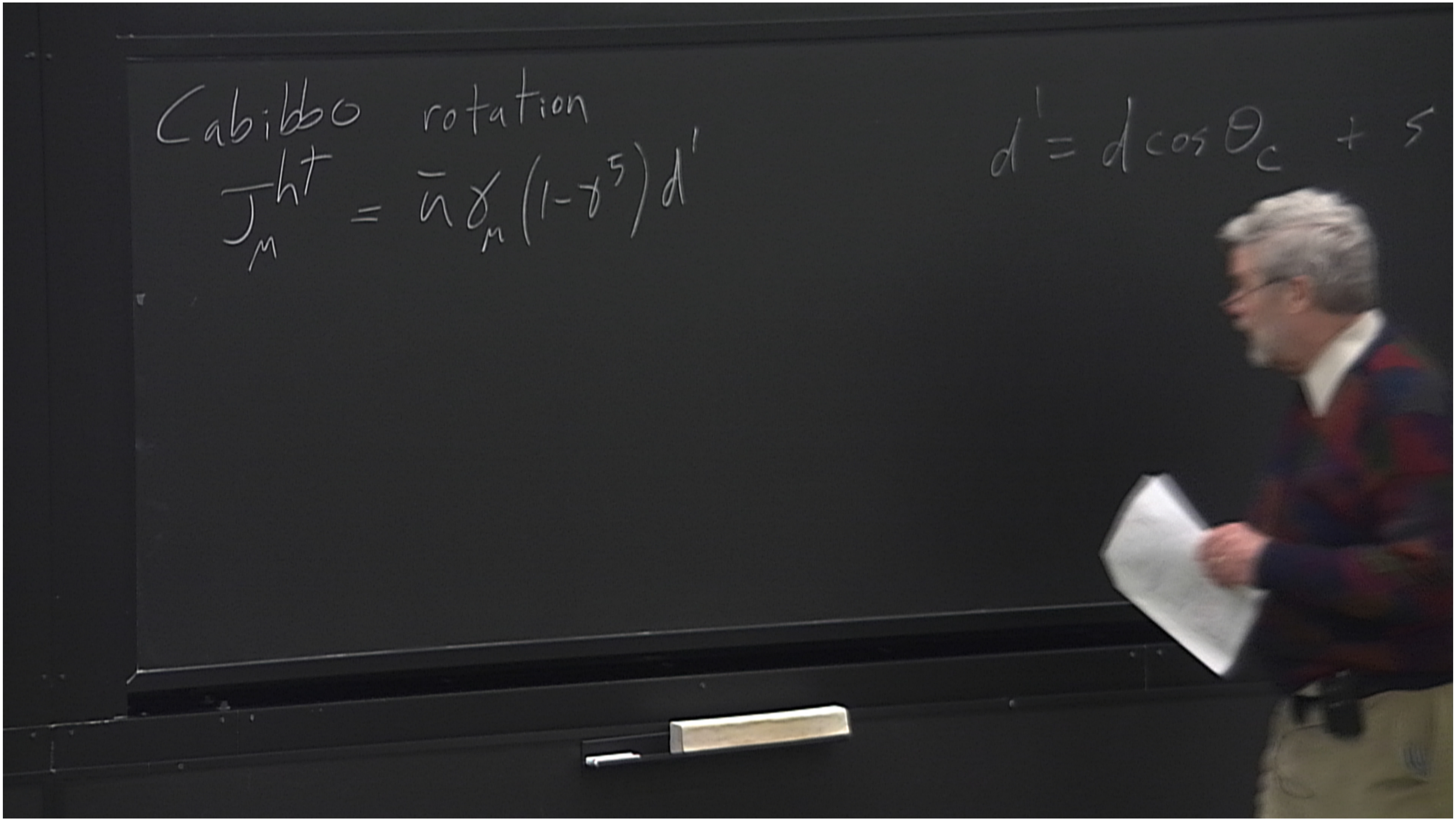
$\Delta S = +1, \; \Delta Q = -1, \; \Delta S = -\Delta Q$

$\left\{ \begin{array}{l} \text{SCNC} \\ \text{FCNC} \end{array} \right.$

$\cancel{\Sigma^+ \to p e^+ e^-}$ $\quad \underline{\text{WNC}}$

$\Delta S = +1$

$\Delta Q = 0$


Cabibbo rotation
$J_\mu^{h+} = \bar{u}\gamma_\mu(1-\gamma^5)d'$
$d' = d\cos\theta_c + s$

Cabibbo rotation

$$J_\mu^{h+} = \bar{u}\gamma_\mu(1-\gamma^5)d'$$

$$d' = d\cos\theta_c + s$$

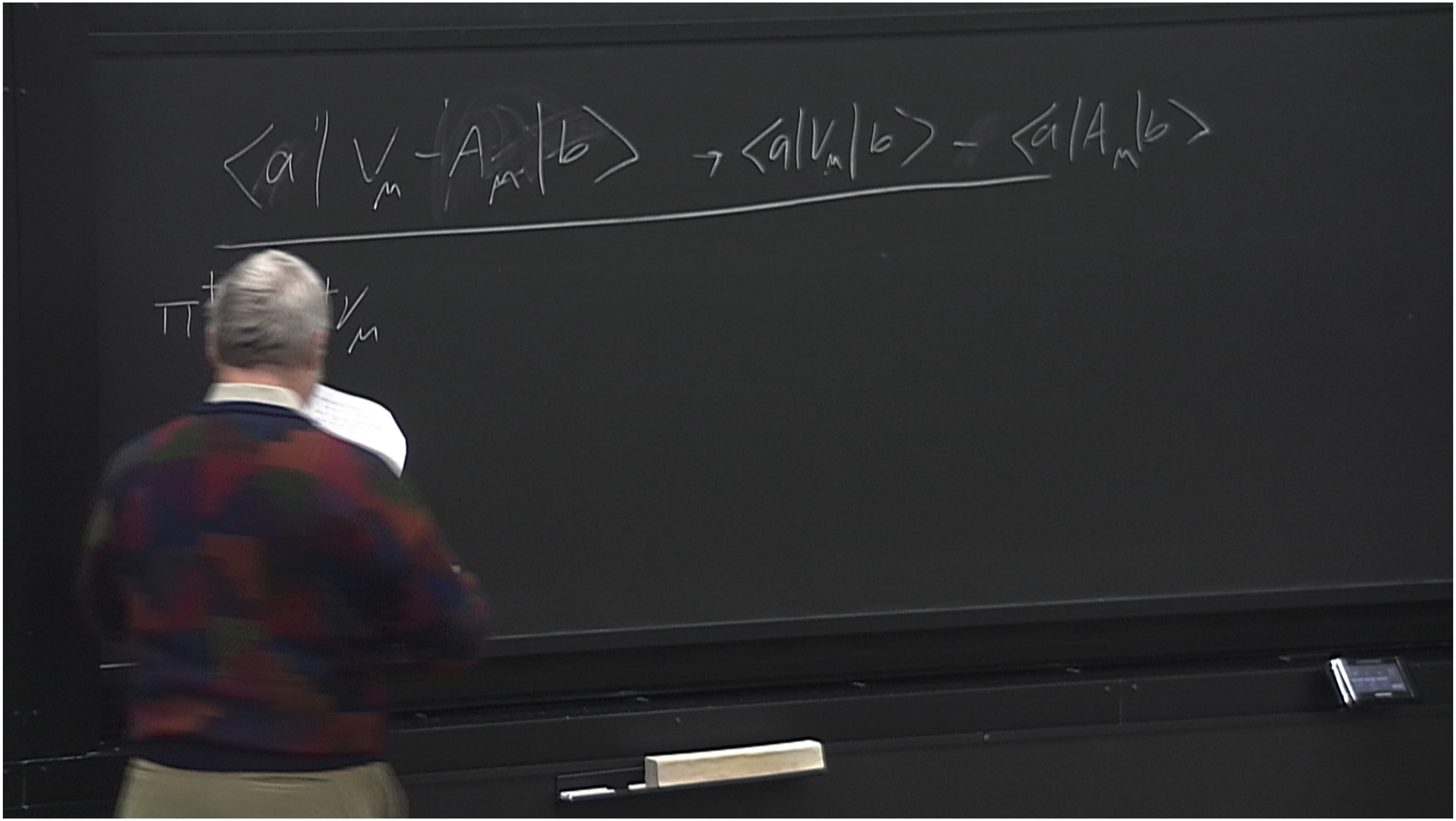
$$\sin\theta_c$$

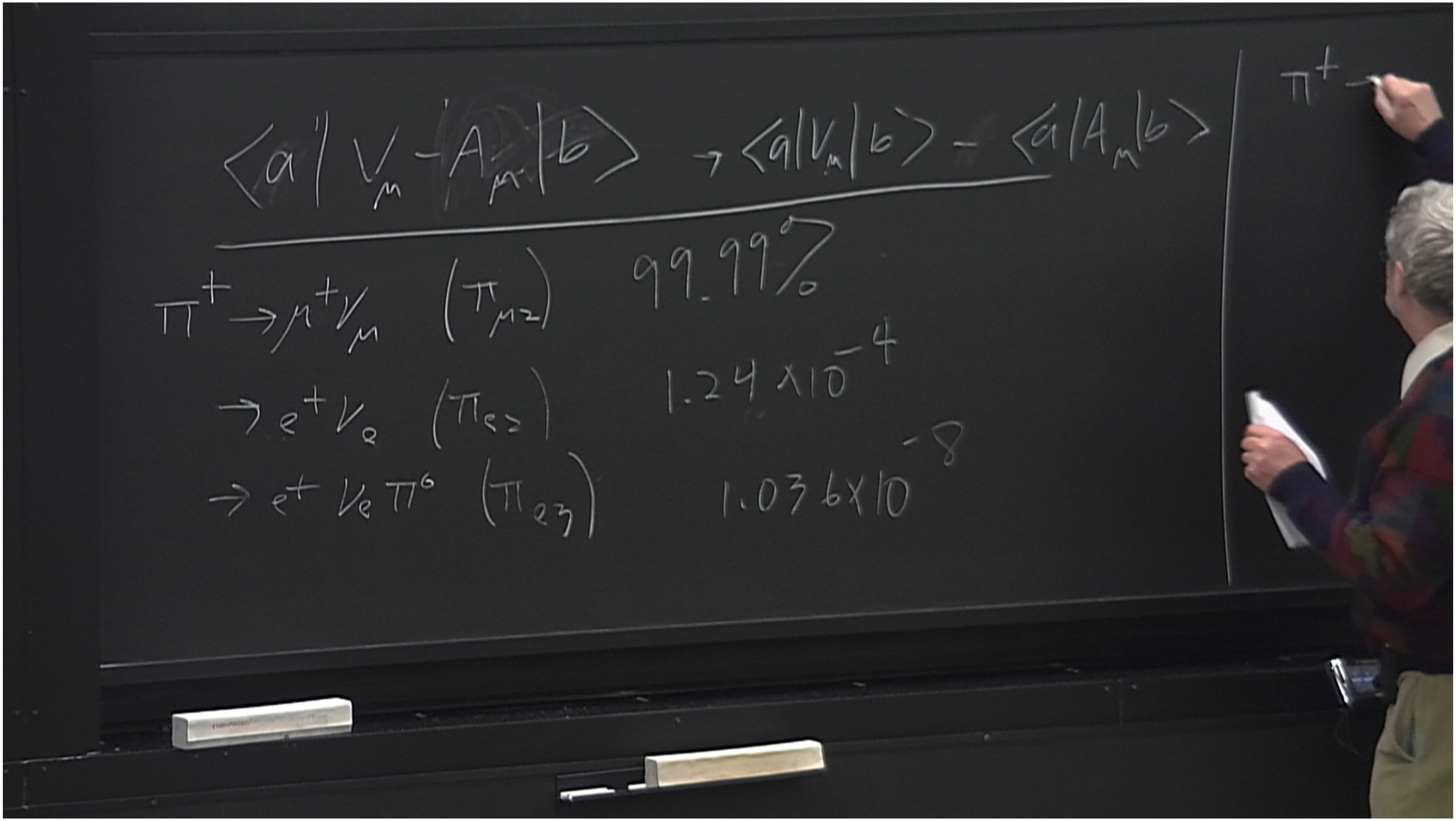
θ_c = Cabibbo angle

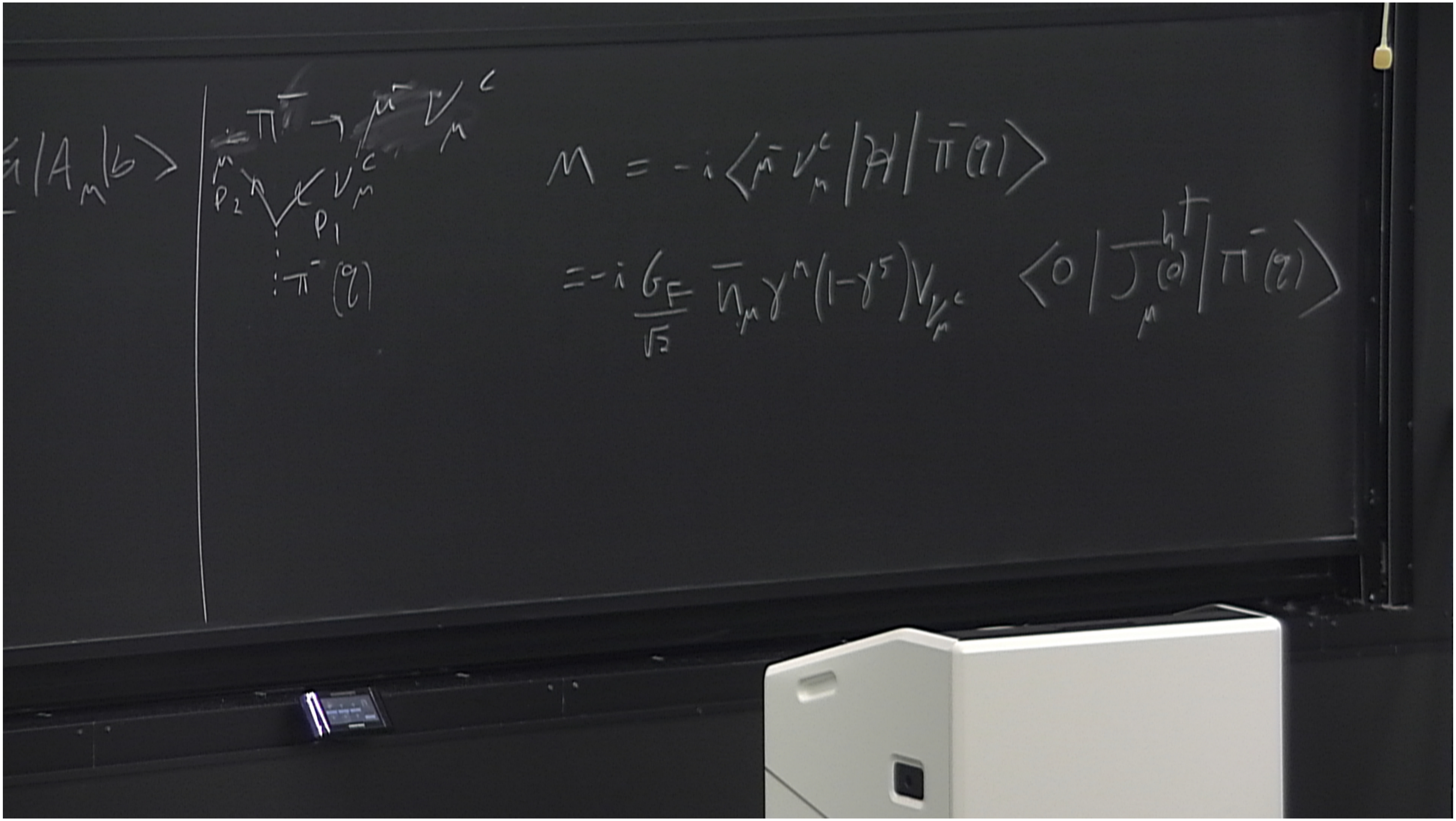
mismatch

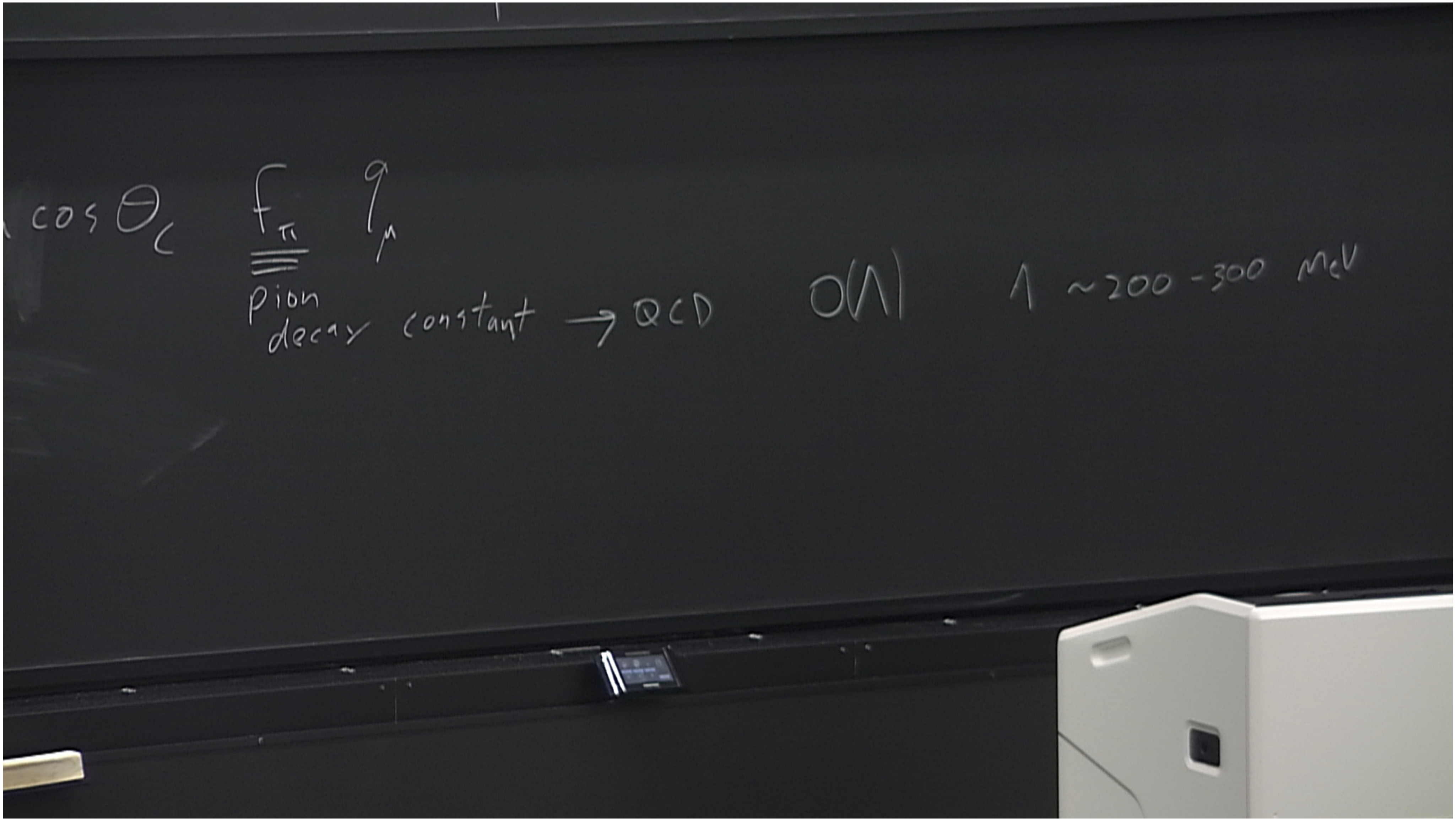
$$= \cos\theta_c\, \bar{u}\gamma_\mu(1-\gamma_5)d + \sin\theta_c\, \bar{u}\gamma_\mu(1-\gamma^5)s$$

$$s' = -\sin\theta_c\, d + \cos\theta_c\, s$$


$\langle a' | V_\mu - A_\mu | b \rangle \to \langle a | V_\mu | b \rangle - \langle a | A_\mu | b \rangle$


$\langle a' | V_\mu - A_\mu | b \rangle \rightarrow \langle a | V_\mu | b \rangle - \langle a | A_\mu | b \rangle$
$\pi^+ \rightarrow \mu^+ \nu_\mu$ $(\pi_{\mu 2})$ 99.99%
$\rightarrow e^+ \nu_e$ (π_{e2}) 1.24×10^{-4}
$\rightarrow e^+ \nu_e \pi^0$ (π_{e3}) 1.036×10^{-8}
$\pi^+ \rightarrow$

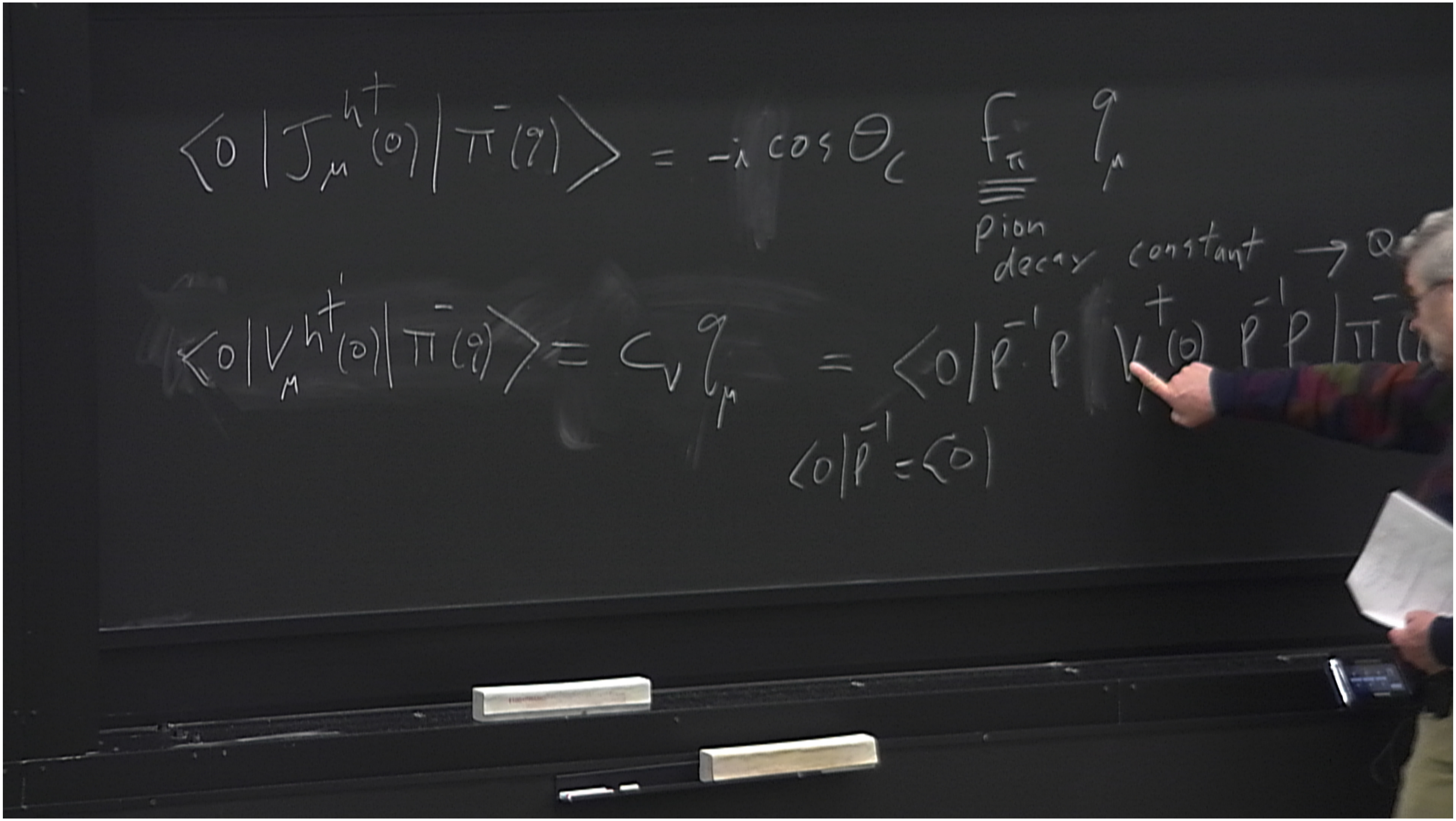

$|A_\mu|b\rangle$
$\pi^- \to \mu^- \nu_\mu^c$
μ^- ν_μ^c
p_2 p_1
$\pi^-(q)$
$\mathcal{M} = -i\langle \mu^- \nu_\mu^c | \mathcal{H} | \pi^-(q)\rangle$
$= -i \frac{G_F}{\sqrt{2}} \bar{u}_\mu \gamma^\mu (1-\gamma^5) v_{\nu_\mu^c} \langle 0 | J_\mu^{h\dagger}(0) | \pi^-(q)\rangle$


cos θ_C
f_π
g_μ
pion decay constant → QCD
O(Λ)
Λ ~ 200 - 300 MeV

$$\langle 0 | J_\mu^{h+}(0) | \pi^-(q) \rangle = -i \cos\theta_c \, \underline{\underline{\underline{f_\pi}}} \, q_\mu$$

pion
decay constant $\rightarrow$ QCD

$$\langle 0 | V_\mu^{h+}(0) | \pi^-(q) \rangle = 0$$


$\langle 0 | J_\mu^{h^+}(0) | \pi^-(q) \rangle = -i \cos\theta_c \; f_\pi \; q_\mu$
pion
decay constant → Q
$\langle 0 | V_\mu^{h^+}(0) | \pi^-(q) \rangle = C_V q_\mu = \langle 0 | P^{-1} P \; V^+_\mu(0) \; P^{-1} P | \pi^-$
$\langle 0 | P^{-1} = \langle 0 |$

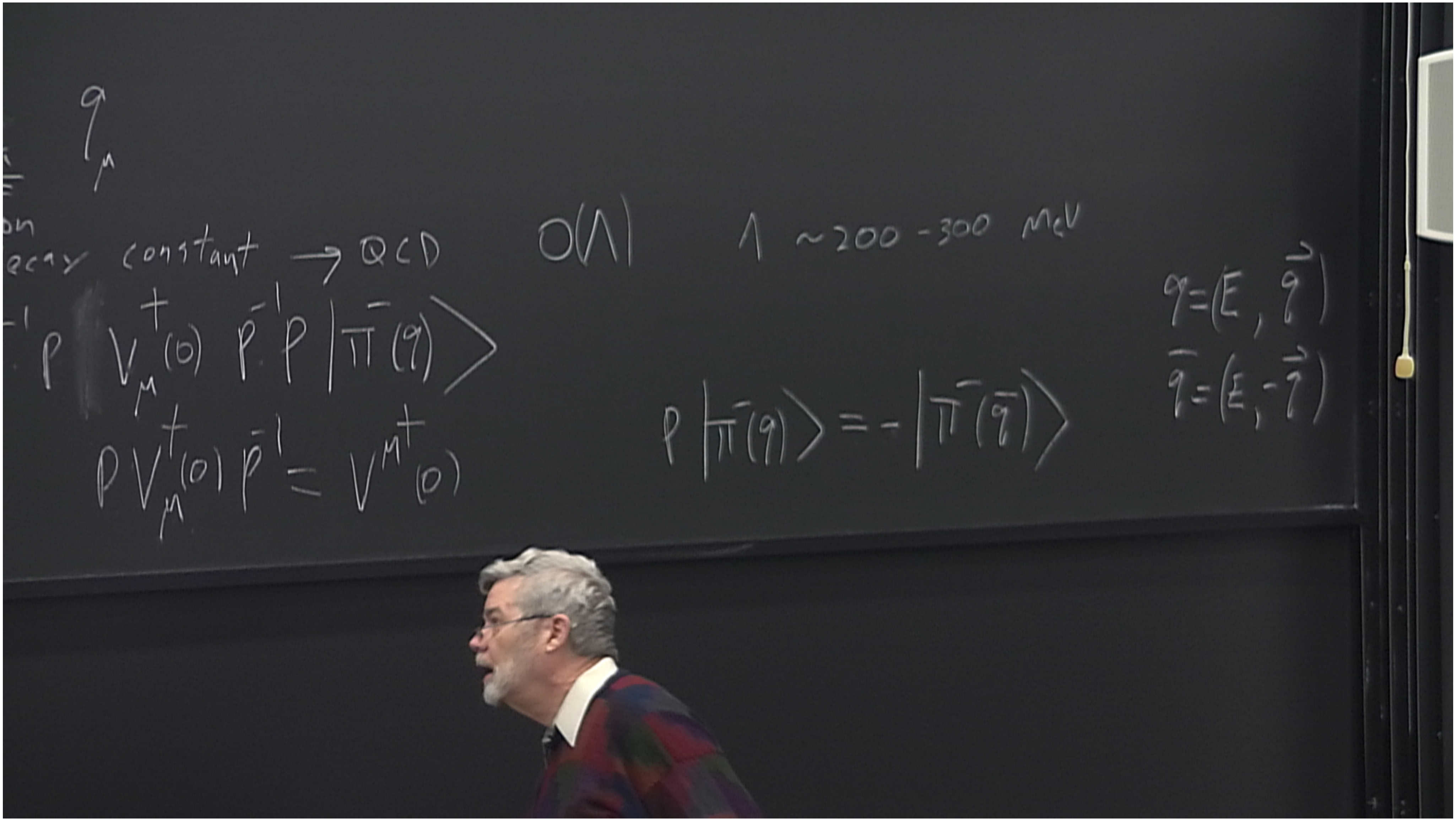
$$= -i \cos\theta_C \; \underline{\underline{f_\pi}} \; q_\mu$$

pion decay constant $\rightarrow$ QCD $\qquad O(\Lambda) \qquad \Lambda \sim 200-300$

$$C_V q_\mu = \langle 0 | P^{-1} P \, V_\mu^\dagger(0) \, P^{-1} P | \pi^-(q) \rangle$$

$$\langle 0 | P^{-1} = \langle 0 | \qquad P V_\mu^\dagger(0) P^{-1} = V^{\mu\dagger}(0)$$

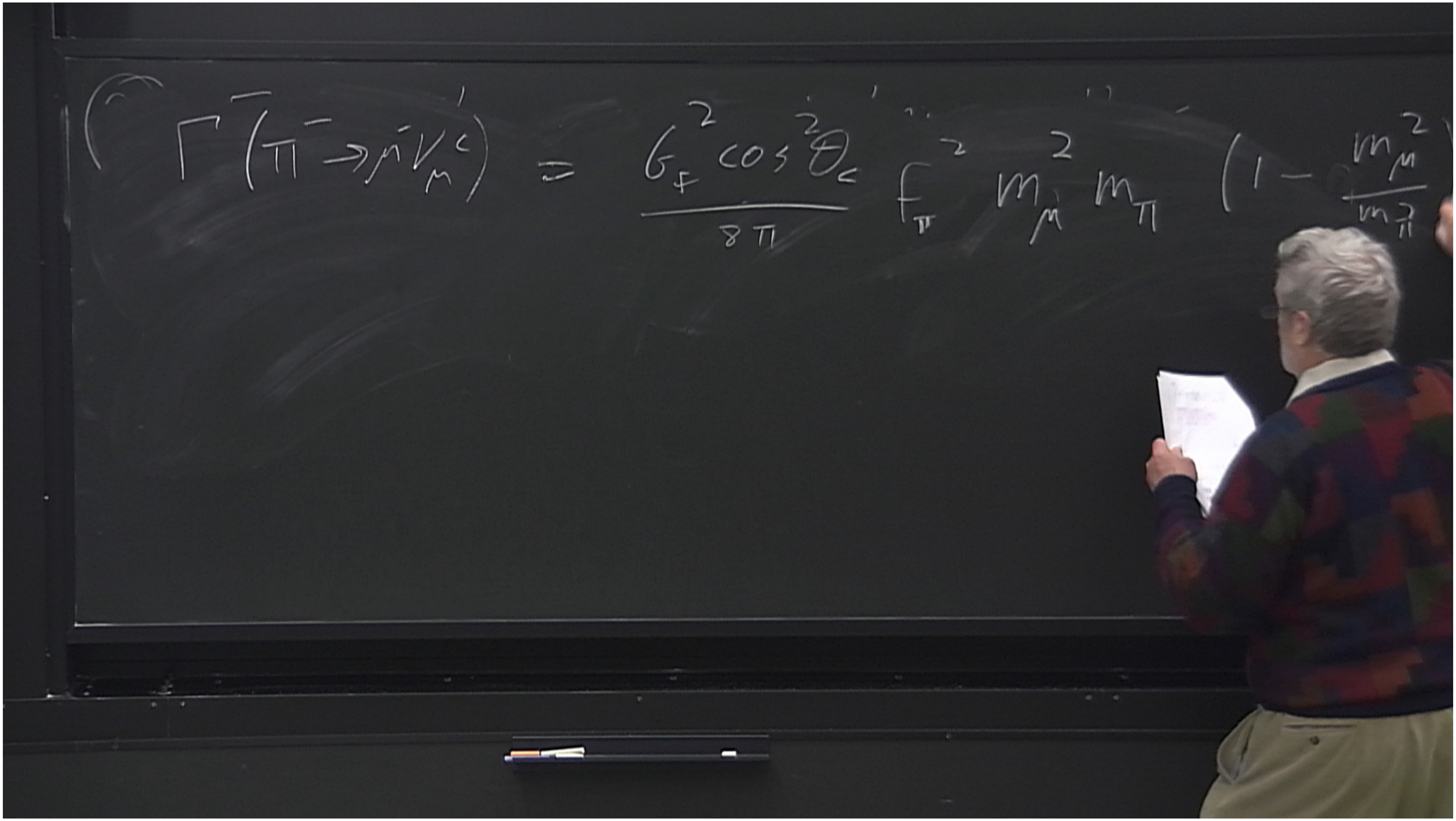
$$P | \pi^-(q) \rangle = - | \pi^-$$


q_μ
on
decay constant → QCD
$O(\Lambda)$
$\Lambda \sim 200-300$ MeV
$P \, V_\mu^+(0) \, P^{-1} P \, |\pi^-(q)\rangle$
$q = (E, \vec{q})$
$\bar{q} = (E, -\vec{q})$
$P \, |\pi^-(q)\rangle = - |\pi^-(\bar{q})\rangle$
$P V_\mu^+(0) P^{-1} = V^{\mu +}(0)$

$$C_V\, q_\mu = -\langle 0 | V^{\mu +}(0) | \pi^-(\bar{q}) \rangle = -\, C_V\, \bar{q}^\mu$$

$$\pi(\bar{q})\rangle = -c_V\,\bar{q}^\mu = -c_V\,q_\mu \qquad \bar{q}^\mu = q_\mu$$

$$\Rightarrow c_V = 0$$


$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) = \frac{G_F^2 \cos^2\theta_c}{8\pi} f_\pi^2 \, m_\mu^2 \, m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right.$

$$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) = \underbrace{\frac{G_F^2 \cos^2\theta_c}{8\pi} f_\pi^2 m_\mu^2}_{|M|^2} \; \underbrace{m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}_{\text{phase space}}$$

$\cos\theta_c$ known from β decay

$$f_\pi \sim 130 \text{ MeV}$$

$$\frac{\Gamma_{e2}}{\Gamma_{\mu 2}} = \frac{m_e^2}{m_\mu^2} \frac{\left(1-\frac{m_e^2}{m_\pi^2}\right)^2}{\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2} = 1.24 \times 10^{-4}$$

$C_V g_\mu$

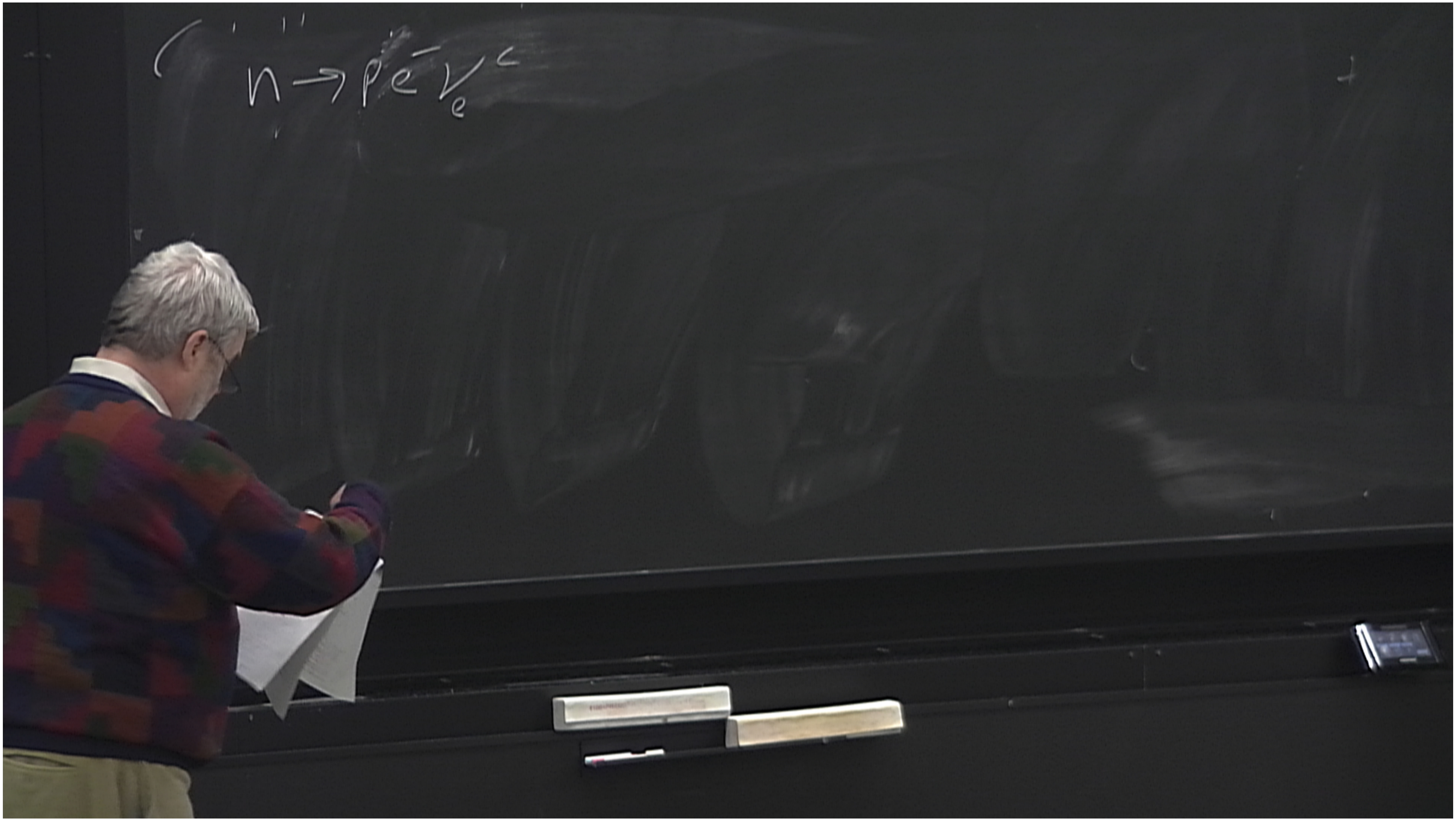
$$\Gamma(\pi^- \to \mu^- \nu_\mu^c) = \underbrace{\frac{G_F^2 \cos^2\theta_c}{8\pi} f_\pi^2 \, m_\mu^2}_{|M|^2} \, \underbrace{m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)}_{\text{phase spa}}$$

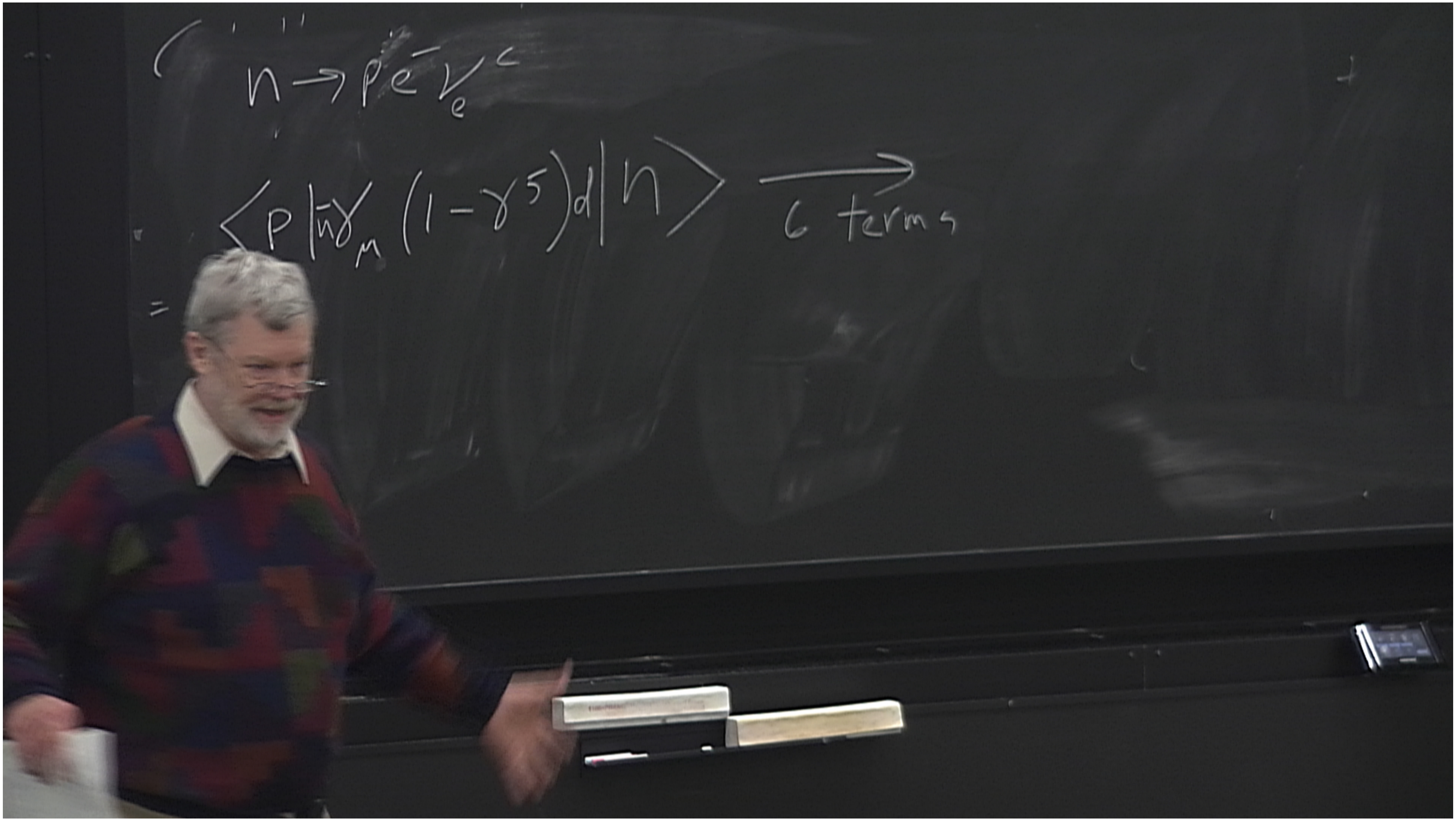
$\cos\theta_c$ known from β decay

$f_\pi \sim 130\ \mathrm{MeV}$

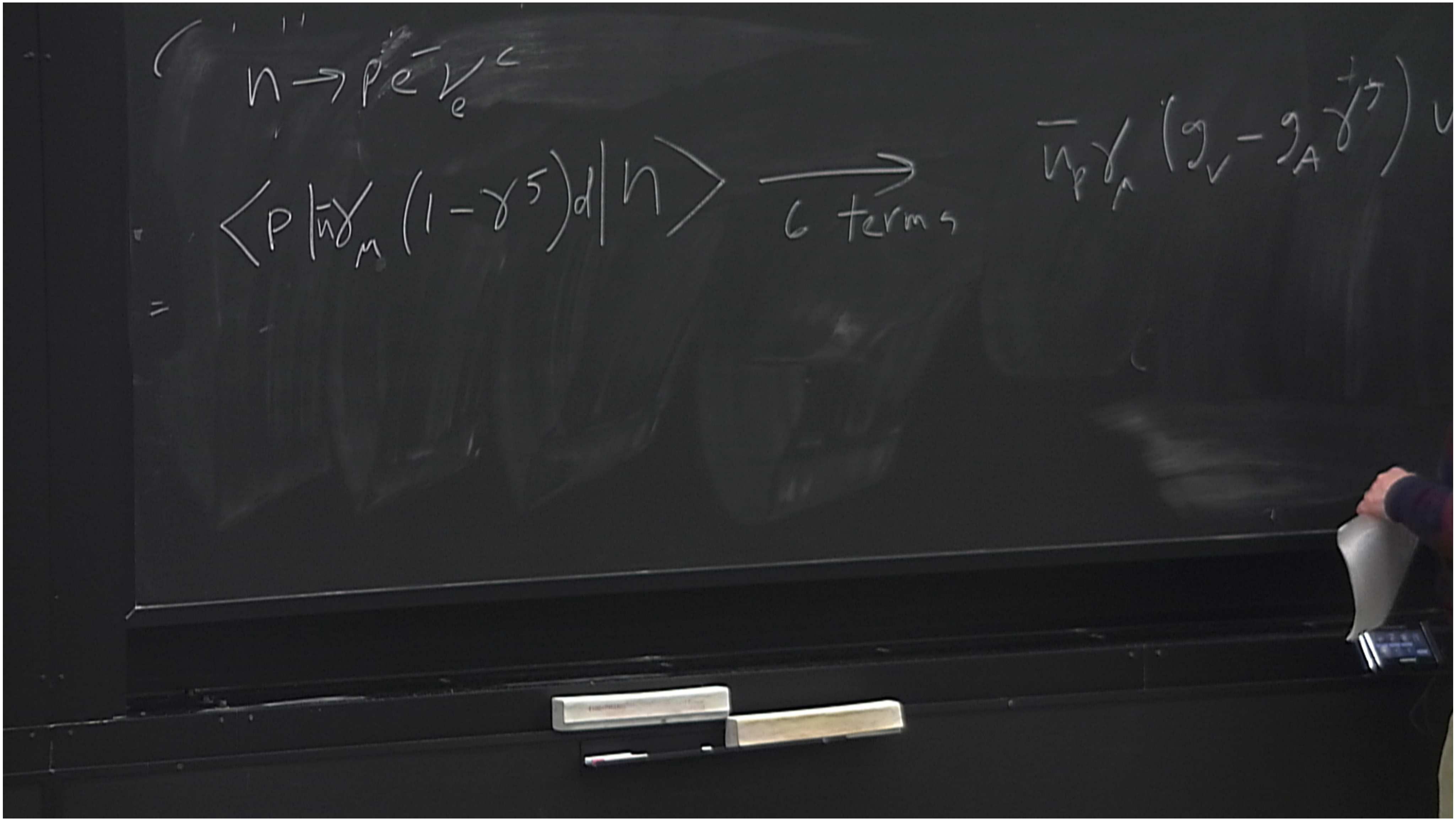
e_L^-, ν_R^c

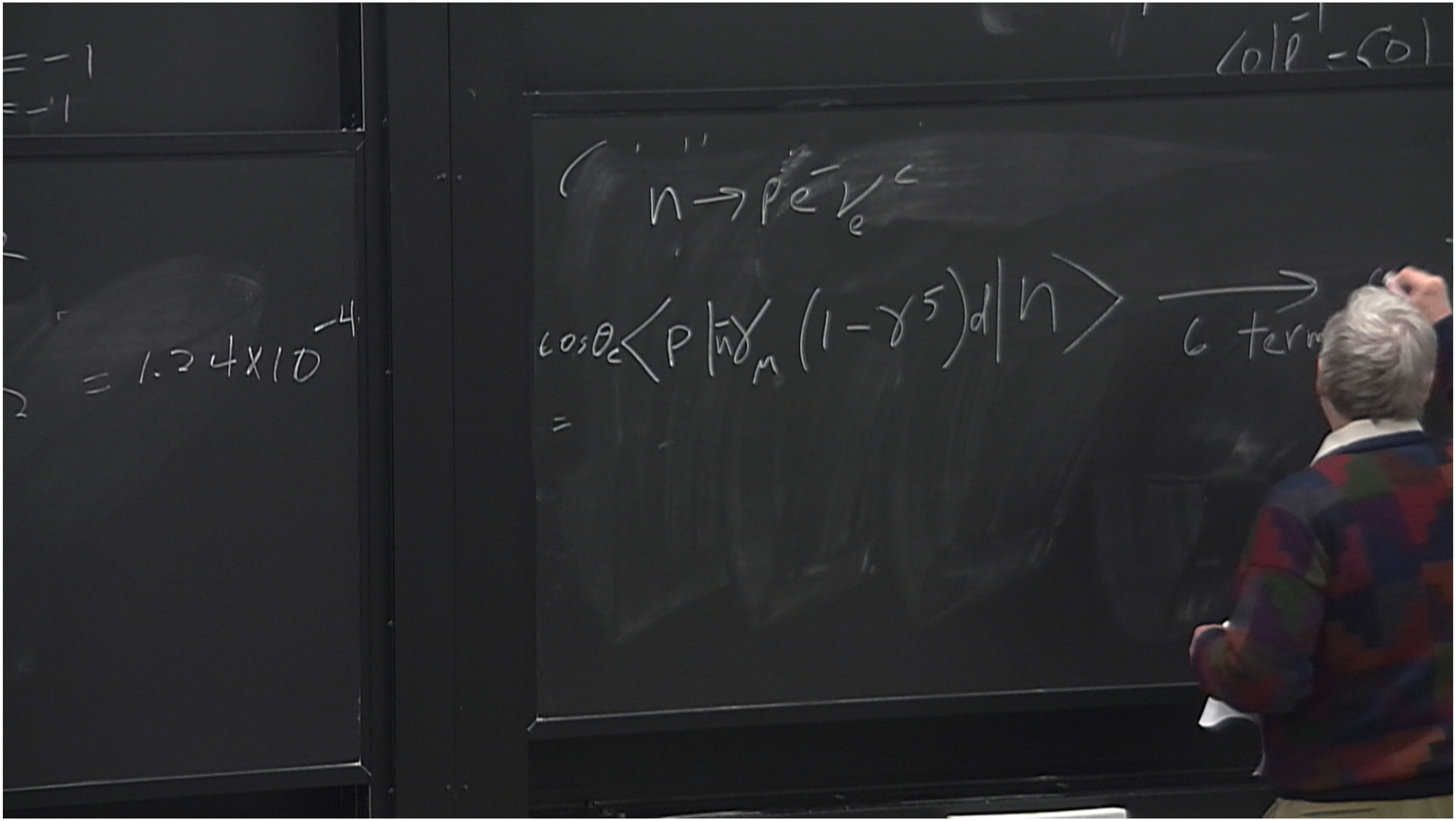
$\pi^+ \to \pi^0 e^+ \nu_e$


$n \rightarrow p e^- \bar{\nu}_e$



$n \to p e^- \bar{\nu}_e$
$\langle p | \bar{u} \gamma_\mu (1-\gamma^5) d | n \rangle \xrightarrow[6 \text{ terms}]{}$


$n \to p e^- \bar{\nu}_e$
$\langle p | \bar{u} \gamma_\mu (1-\gamma^5) d | n \rangle \xrightarrow{\text{6 terms}}$
$\bar{u}_p \gamma_\mu (g_V - g_A \gamma^5) u$

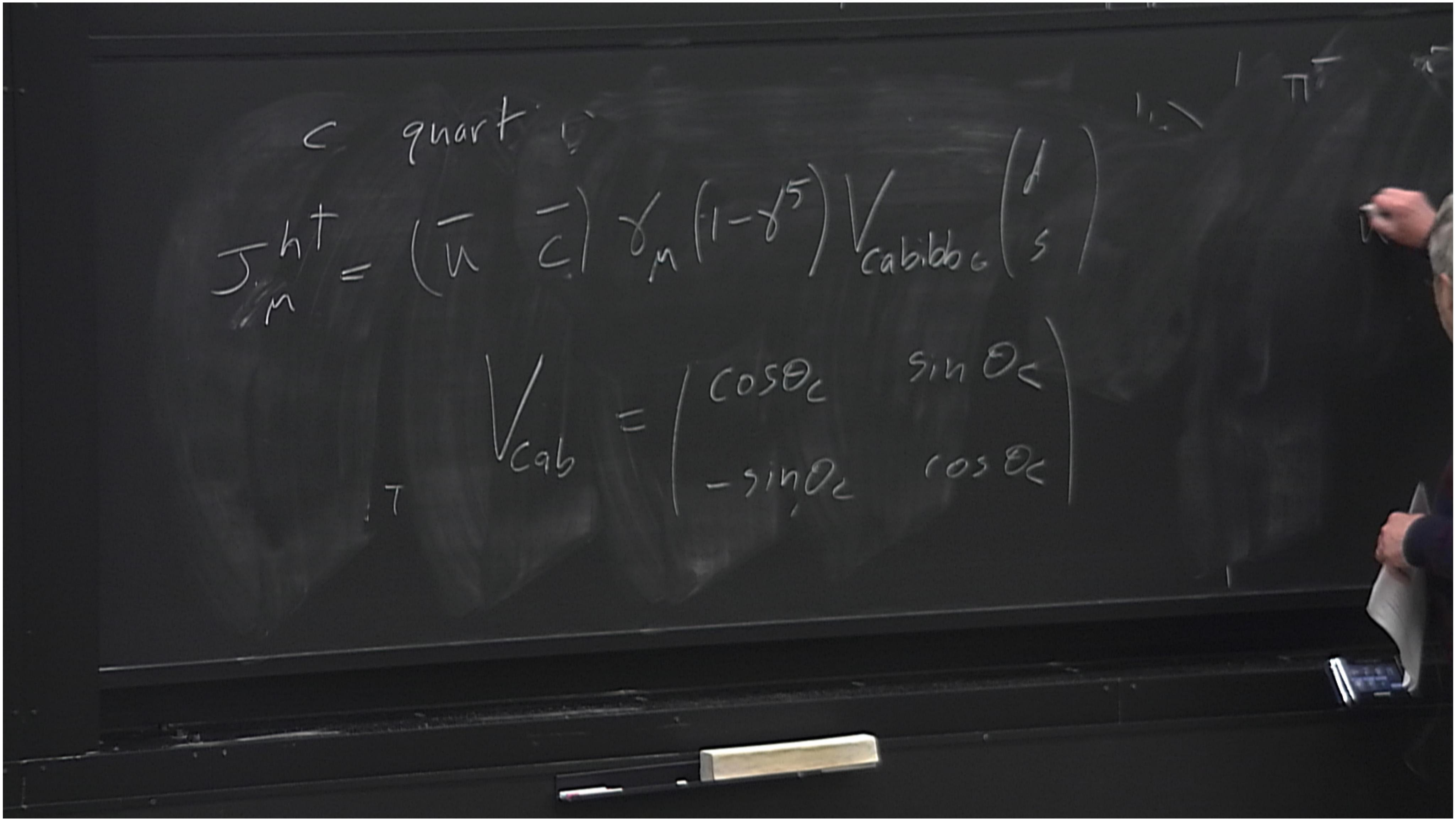

$n \to p e^- \bar{\nu}_e$
$\cos\theta_c \langle p | \bar{u}\gamma_\mu (1-\gamma^5) d | n \rangle \longrightarrow$
6 term
$= 1.34 \times 10^{-4}$

$$n \to p\, e^- \bar{\nu}_e$$

$$\langle p | \bar{u} \gamma_\mu (1-\gamma^5) d | n \rangle \xrightarrow[\text{6 terms}]{} \cos\theta_c \, \bar{u}_p \gamma_\mu (g_V - g_A \gamma^5) u_n$$

isospin: $g_V = 1$

$\Delta S = 0$


c quark
$J_\mu^{h\dagger} = (\bar{u} \ \ \bar{c})\, \gamma_\mu (1-\gamma^5) V_{cabibbo} \begin{pmatrix} d \\ s \end{pmatrix}$
$V_{cab} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$