

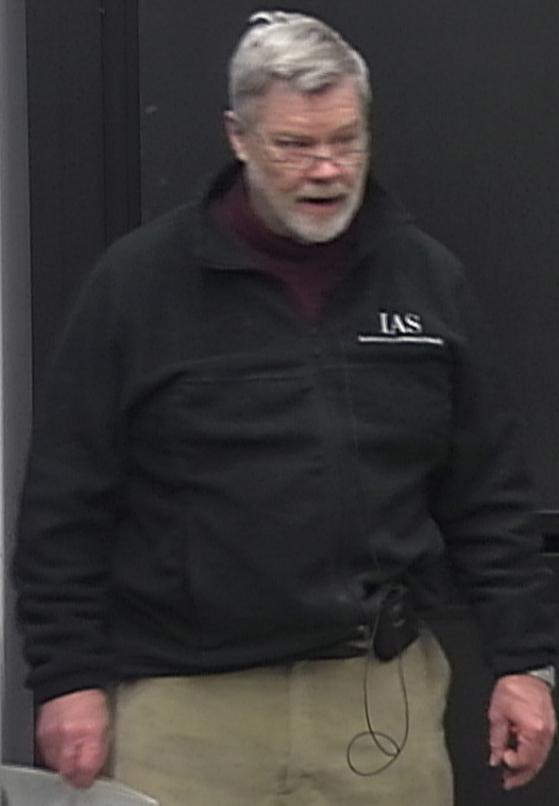
Title: 13/14 PSI - Standard Model Review - Lecture 5

Date: Jan 10, 2014 09:00 AM

URL: <http://pirsa.org/14010012>

Abstract:

⑤ a) Spontaneous Symmetry Breaking
b) Non-abelian Gauge Theories



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(4) Non-abelian Gauge Theories

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Exact symmetry: $[T^i, \mathcal{L}] = 0$

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Exact sym: $[T^i, \mathcal{L}] = 0, T^i |0\rangle = 0$

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 $e^{-i\vec{B} \cdot \vec{T}} |0\rangle = |0\rangle$

⑤ a) Spontaneous Symmetry Breaking
 (4) Non-abelian Gauge Theories

Exact symmetry: $[T^i, \mathcal{L}] = 0$, $T^i |0\rangle = 0$
eqns of motion Vacuum
 $e^{-i\vec{B} \cdot \vec{T}} |0\rangle = |0\rangle$

Non-abelian Gauge Theories

Exact symmetry:

$$\underbrace{[T^i, \mathcal{L}]}_{\text{eqns of motion}} = 0, \quad T^i |0\rangle = 0$$

Explicit breaking:

$$e^{-i\vec{B}\cdot\vec{T}} |0\rangle = |0\rangle$$

vacuum

Non-abelian Gauge Theories

Exact symmetry: $\underbrace{[T^i, \mathcal{L}]}_{\text{eqns of motion}} = 0, \quad T^i |0\rangle = 0$
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Explicit breaking: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1$

4 Non-abelian Gauge Theories

Exact symmetry: $\underbrace{[T^i, \mathcal{L}]}_{\text{eqns of motion}} = 0, \quad T^i |0\rangle = 0$

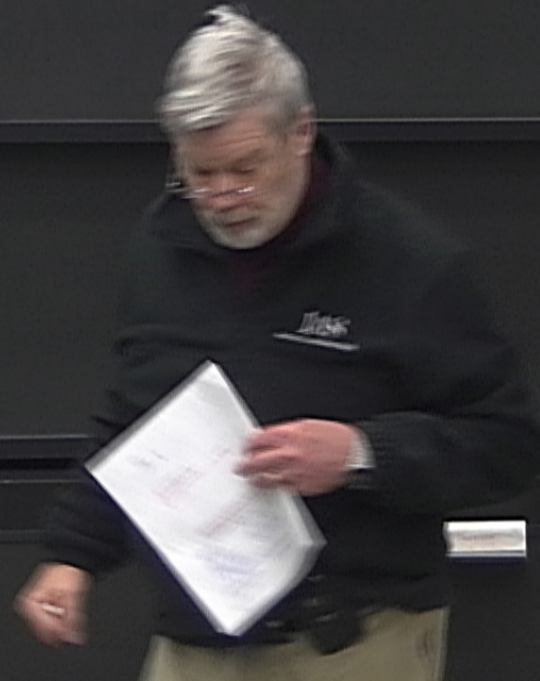
$$e^{\underbrace{-i\vec{B} \cdot \vec{T}}_{\text{vacuum}}} |0\rangle = |0\rangle$$

Explicit breaking: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1, \quad [T^i, \mathcal{L}_0] = 0$

4 Non-abelian Gauge Theories

Exact symmetry: $\underbrace{[T^i, \mathcal{L}]}_{\text{eqns of motion}} = 0, T^i |0\rangle = 0$
 $\underbrace{e^{-i\vec{B}\cdot\vec{T}} |0\rangle}_{\text{vacuum}} = |0\rangle$

Explicit breaking: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1, [T^i, \mathcal{L}_0] = 0, [T^i, \mathcal{L}_1] \neq 0$

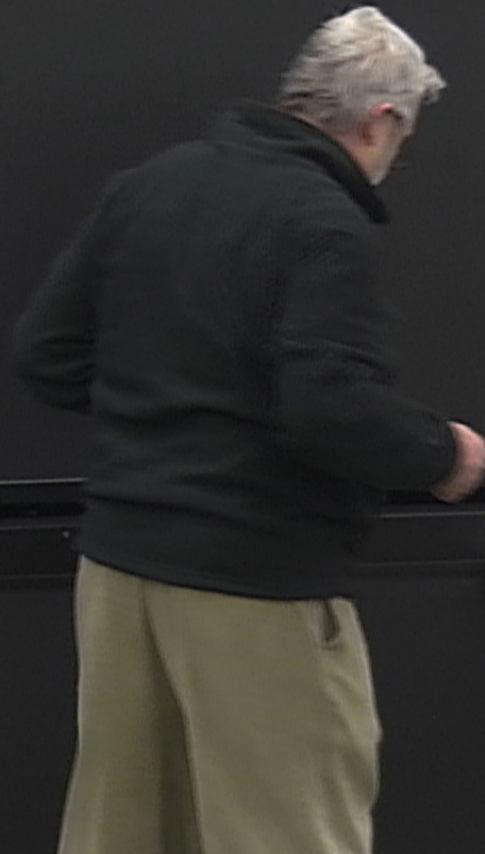


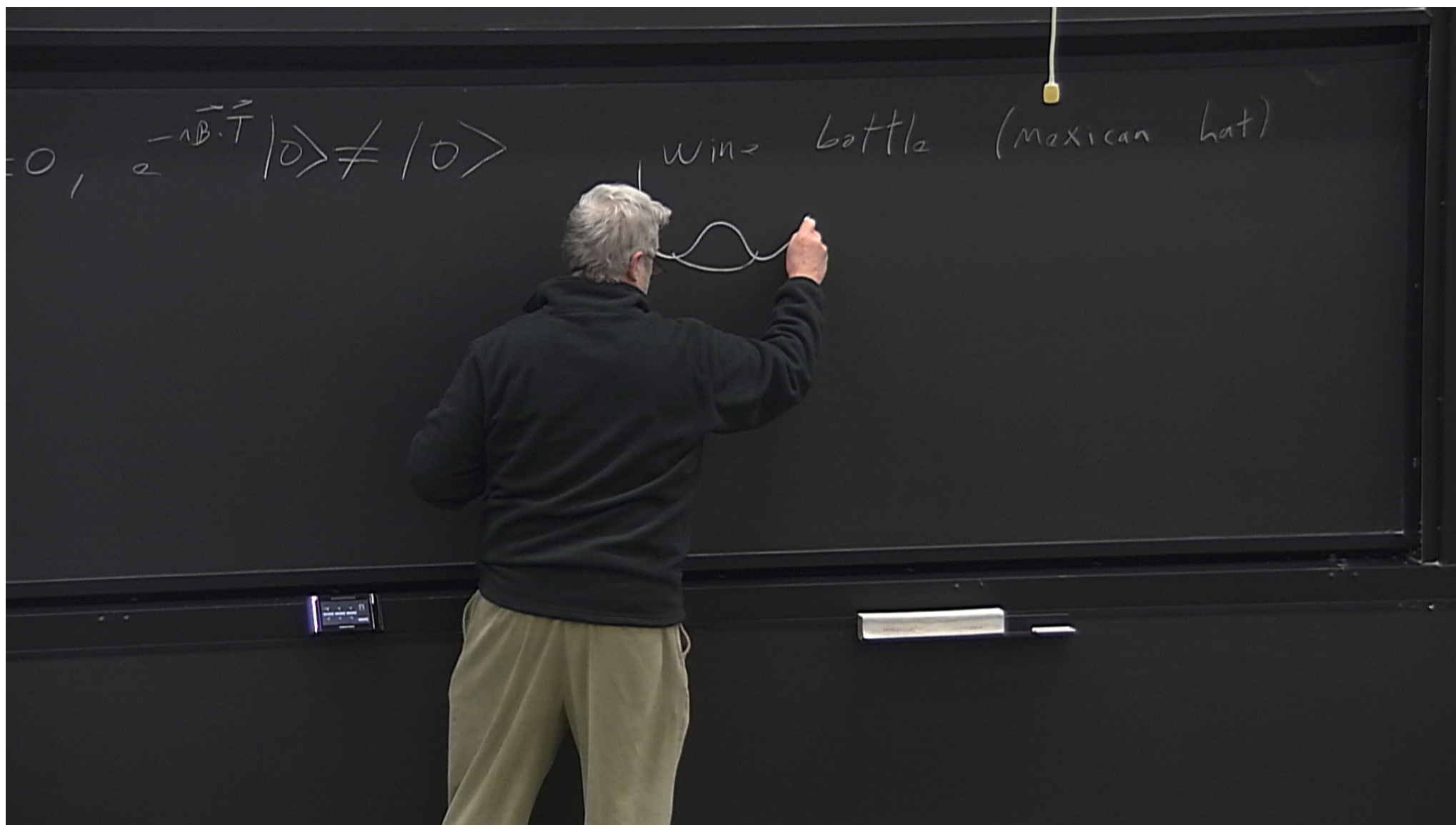
Spontaneous breaking: $T^i|0\rangle \neq 0,$

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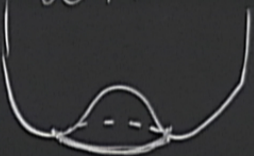
$$\neq 0, e^{-i\vec{B} \cdot \vec{T}} |0\rangle \neq |0\rangle$$

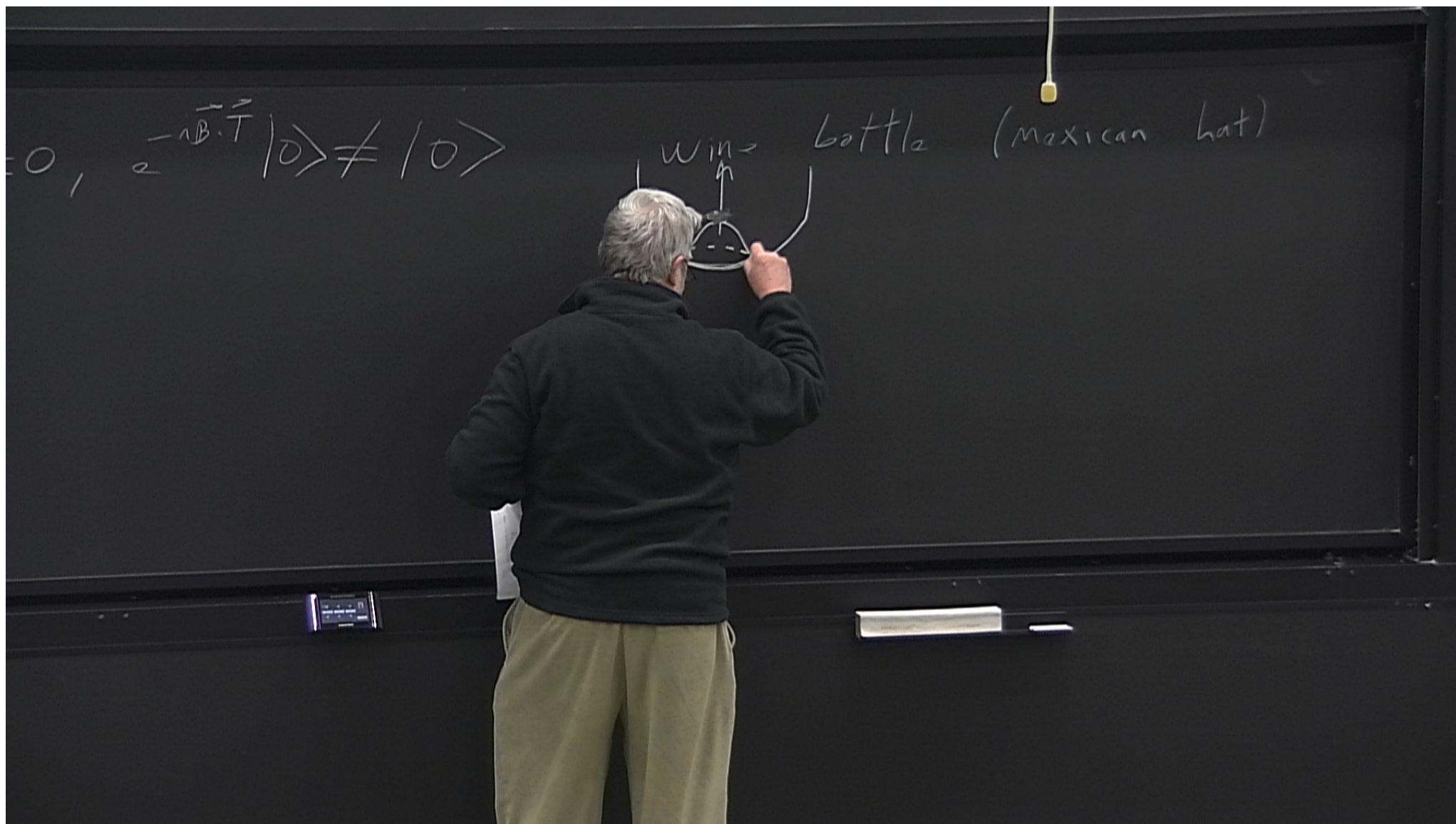


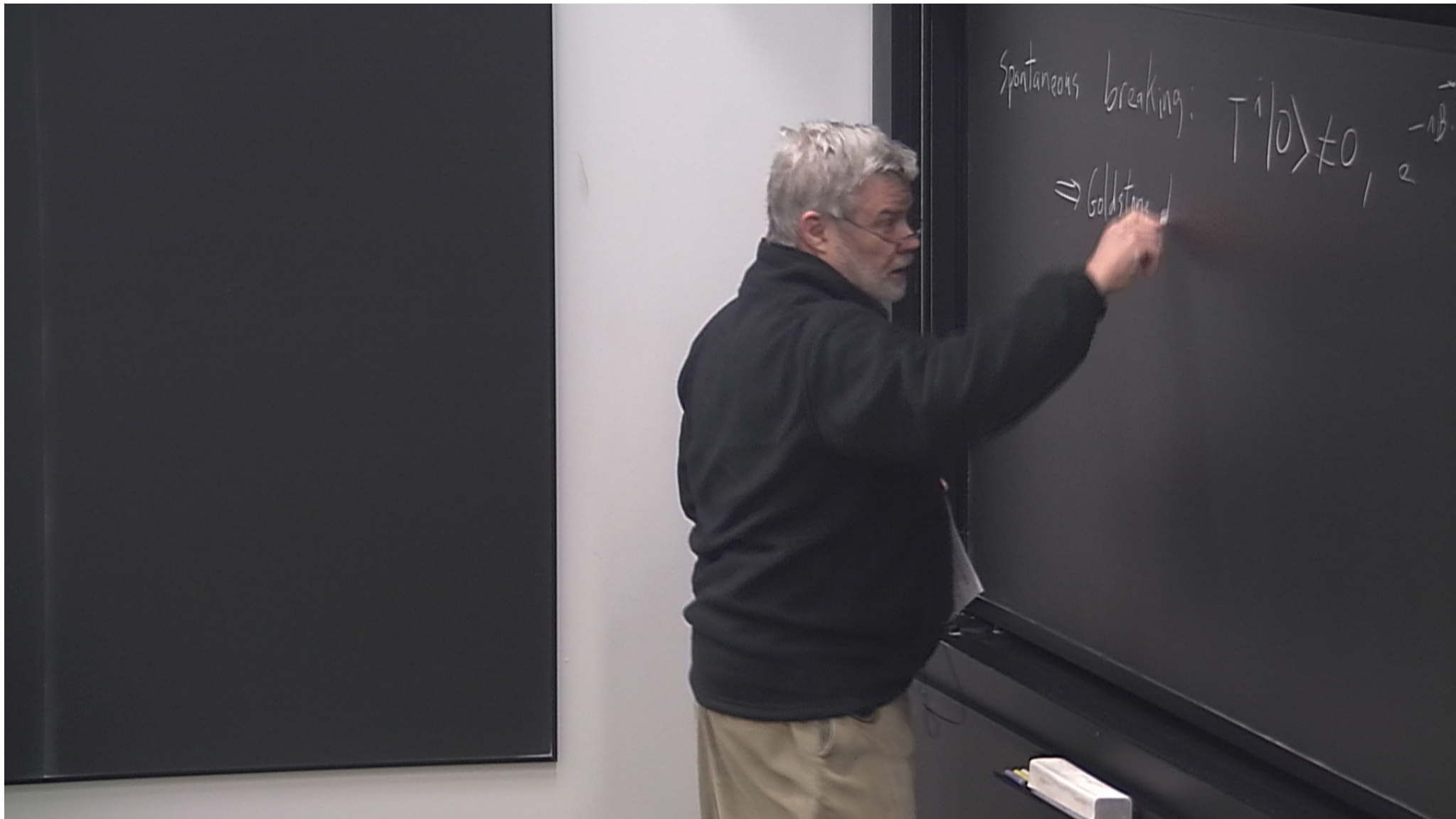


$$= 0, \quad e^{-\vec{n} \cdot \vec{B} \cdot \vec{T}} |0\rangle \neq |0\rangle$$

win = bottle (mexican hat)







Spontaneous breaking: $T|0\rangle \neq 0$, $-iB$
 $\Rightarrow$ Goldstone

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$[T^i, \mathcal{L}] = 0 \Rightarrow$
Goldstone
alternative

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$[T^i, L]$

Goldstone
alternative

$\left\{ \begin{array}{l} T^i |0\rangle = 0 \Rightarrow \text{Wigner-Weyl} \end{array} \right.$

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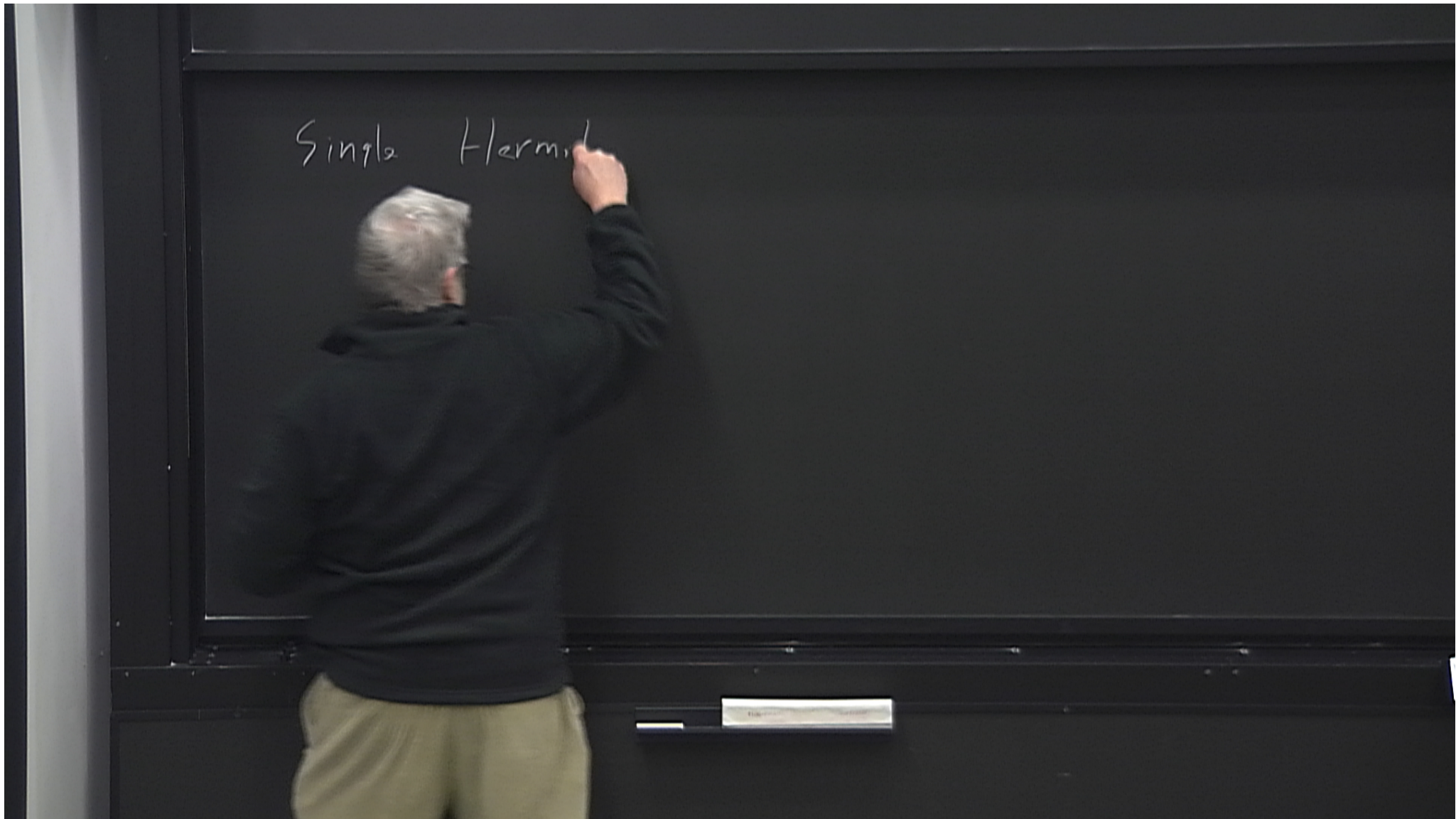
$[T^i, \mathcal{L}] = 0 \Rightarrow$
Goldstone
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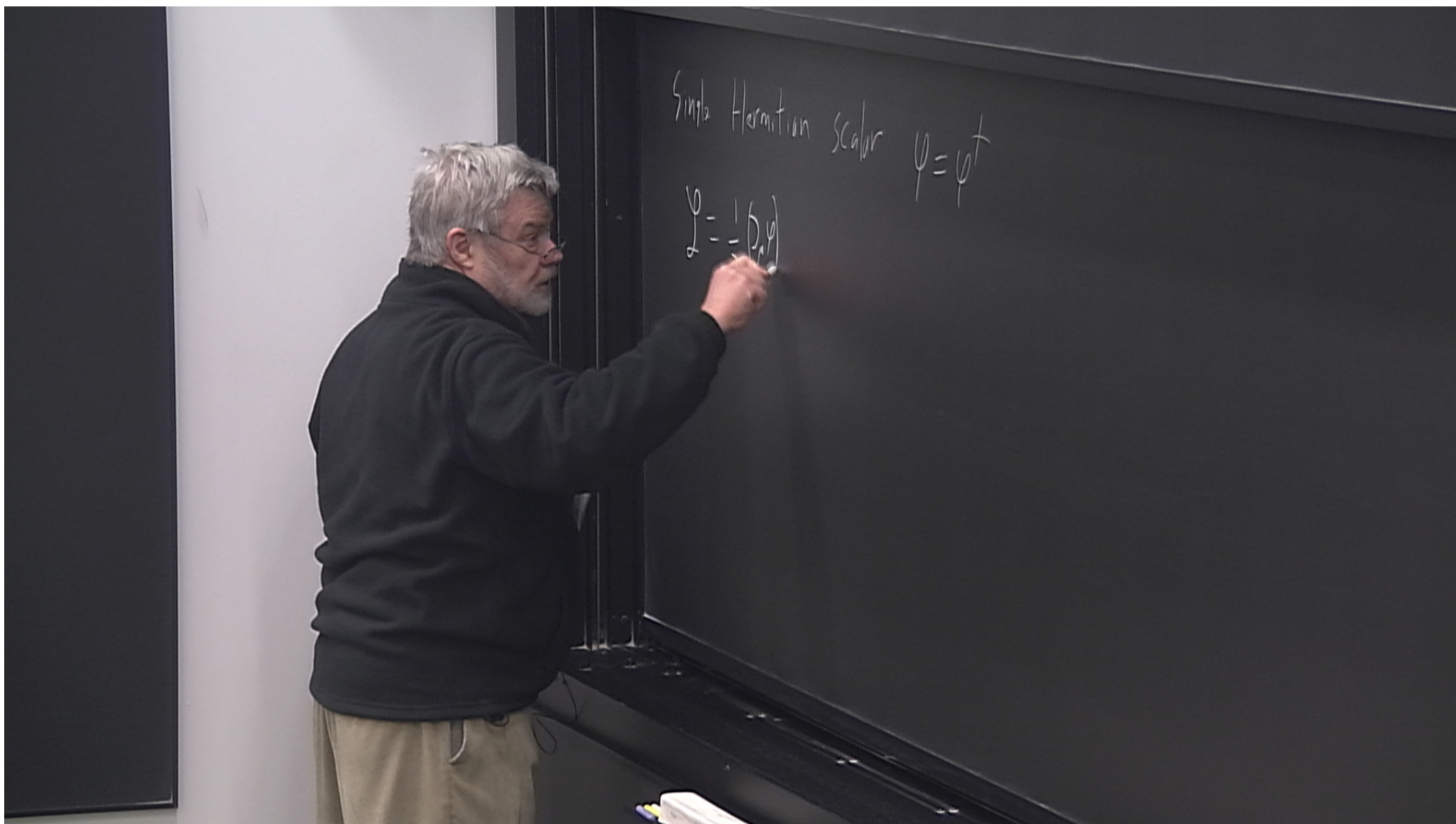
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 $\Rightarrow$ Goldstone boson (massless spin 0 particle)

$[T^i, \mathcal{L}] = 0 \Rightarrow$
 Goldstone
 alternative

$\left\{ \begin{array}{l} T^i|0\rangle = 0 \Rightarrow \text{Wigner-Weyl} \\ T^i|0\rangle \neq 0 \Rightarrow \text{Goldstone (Nambu)} \end{array} \right.$



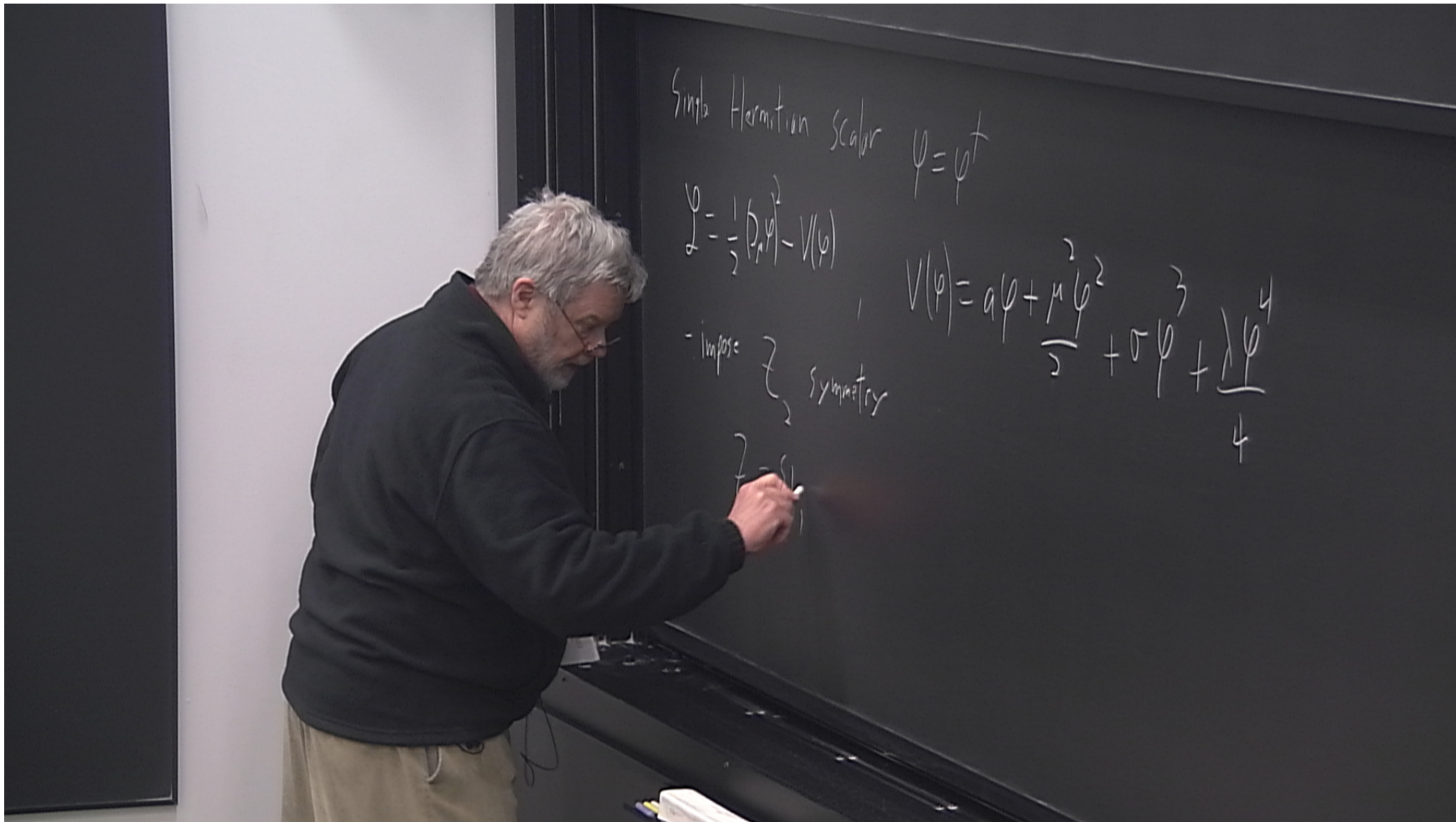


Single Hermitian scalar $\varphi = \varphi^\dagger$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi), \quad V(\varphi) = \frac{1}{2} m^2 \varphi^2 + \frac{\lambda}{4} \varphi^4$$

Single Hermitian scalar $\varphi = \varphi^\dagger$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi), \quad V(\varphi) = a\varphi + \frac{\mu^2}{2} \varphi^2 + \sigma \varphi^3 + \frac{\lambda}{4} \varphi^4$$



Single Hermitian scalar $\varphi = \varphi^\dagger$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi), \quad V(\varphi) = \cancel{a\varphi} + \frac{\mu^2}{2} \varphi^2 + \cancel{\sigma\varphi^3} + \frac{\lambda}{4} \varphi^4$$

- impose $\mathbb{Z}_2$ symmetry: $\varphi \rightarrow -\varphi \Rightarrow a = \sigma = 0 \quad V(\varphi) = \frac{\mu^2 \varphi^2}{2} + \frac{\lambda}{4} \varphi^4$

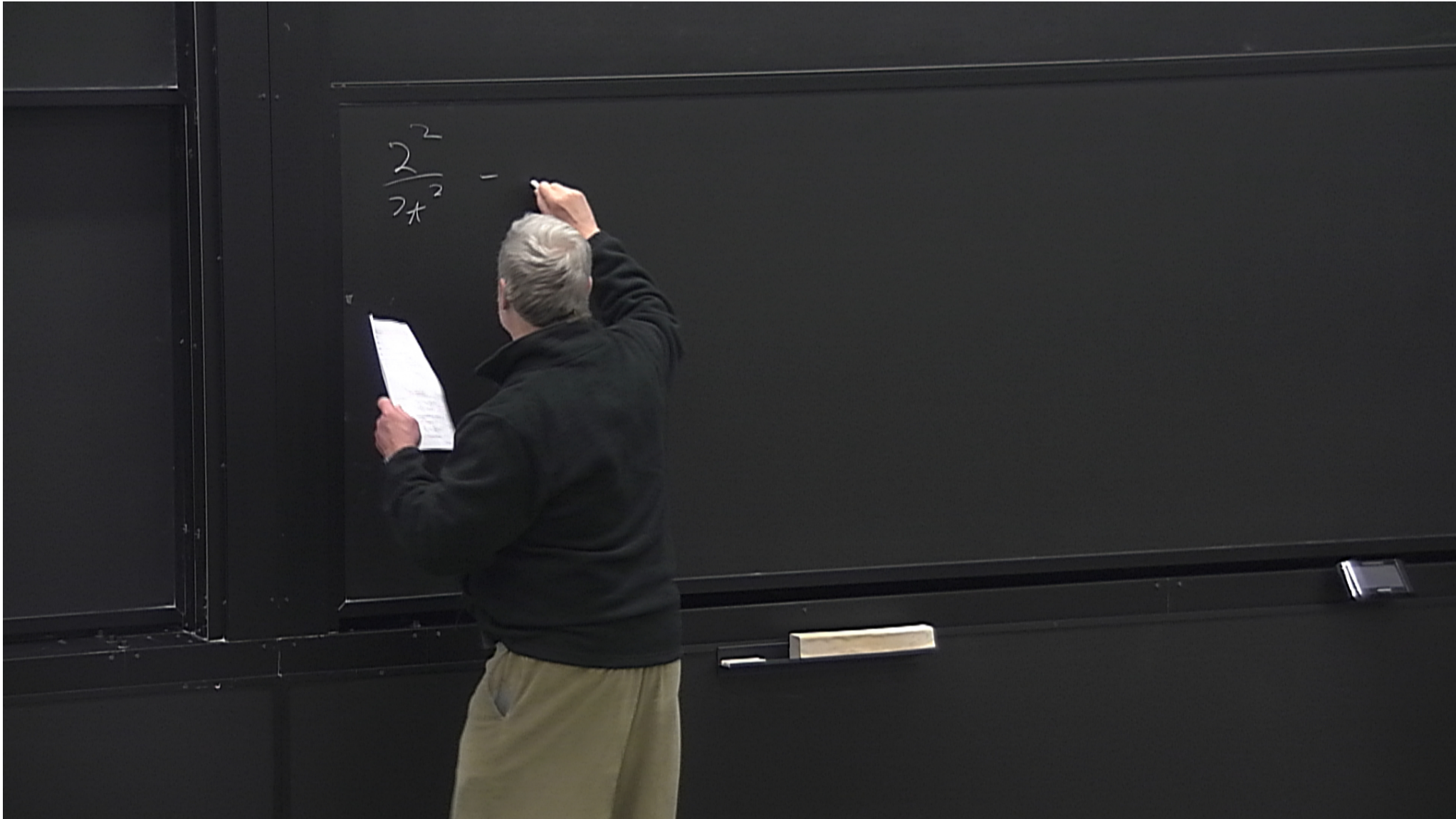
$$\mathbb{Z}_2 = \{1, -1\}$$

Single Hermitian scalar $\varphi = \varphi^\dagger$

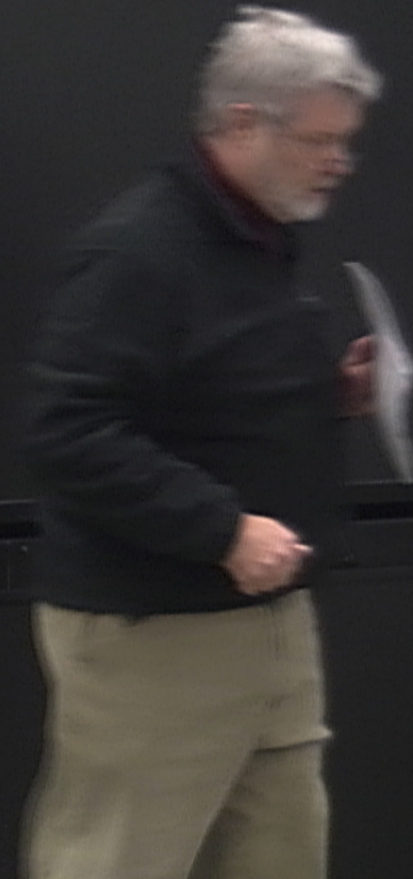
$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi)^2 - V(\varphi), \quad V(\varphi) = \cancel{a\varphi} + \frac{m^2}{2} \varphi^2 + \cancel{\sigma\varphi^3} + \frac{\lambda}{4} \varphi^4$$

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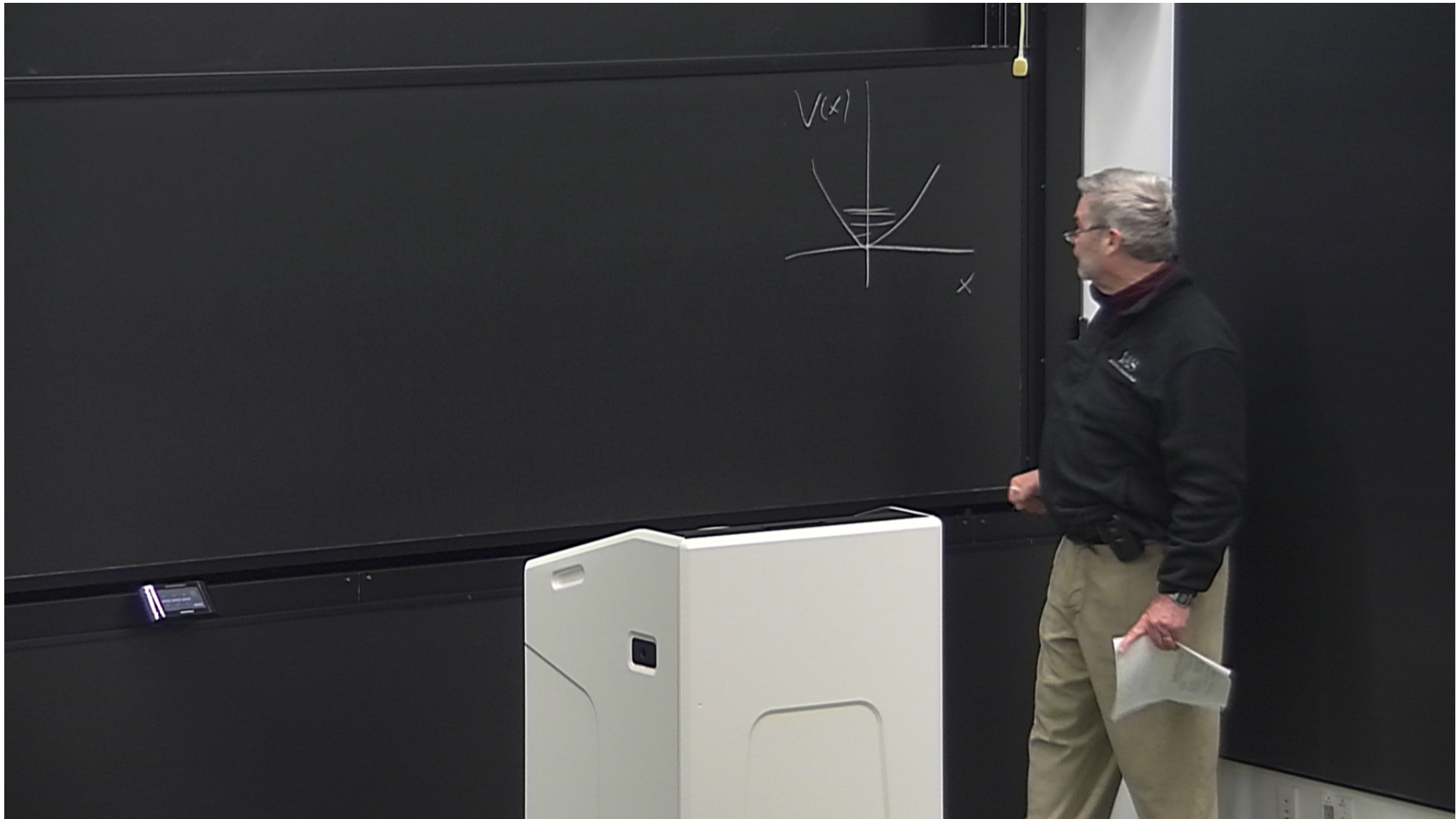
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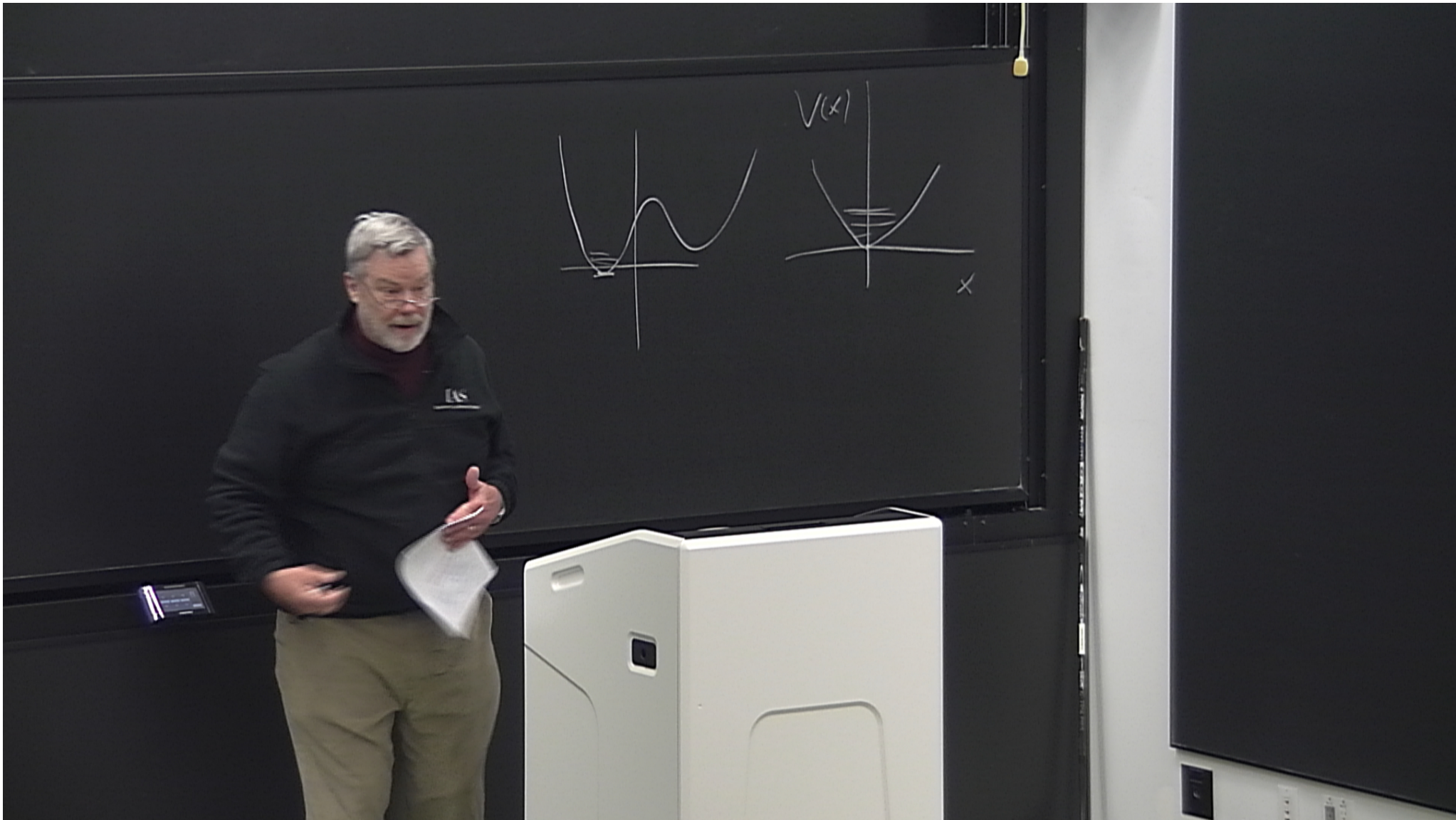


$$\left[\frac{\hbar^2}{2m} \nabla^2 - V \right] \psi = -\frac{\partial V}{\partial \psi} = -[m^2 c^2 + \lambda \psi^2] \psi$$



$$\left[\frac{\hbar^2}{2m} \nabla^2 - V \right] \psi = -\frac{\partial V}{\partial \psi} = -[\mu^2 + \lambda \psi^2] \psi$$





$$\left[\frac{\hbar^2}{2m} \nabla^2 - V \right] \psi = -\frac{E}{\hbar^2} \psi = -\left[\frac{\hbar^2}{2m} \nabla^2 + V \right] \psi$$

$$\langle 0 | \psi | 0 \rangle = \langle \psi \rangle$$

$$\left[\frac{\hbar^2}{2m} \nabla^2 - V \right] \psi = -\frac{\partial V}{\partial \psi} = -[\mu^2 + \lambda \psi^2] \psi$$

$$\langle 0 | \psi | 0 \rangle = \langle \psi \rangle = \psi_{\text{class}}$$

$$\left[\frac{\partial^2}{\partial x^2} - \nabla^2 \right] \psi = -\frac{\partial V}{\partial \psi} = -[\mu^2 + \lambda \psi^2] \psi$$

$$\langle 0 | \psi | 0 \rangle = \langle \psi \rangle = \psi_{\text{class}}$$

$$\frac{\partial}{\partial x} \langle \psi \rangle = \frac{\partial}{\partial x^i} \langle \psi \rangle = 0$$

$$\left[\frac{\hbar^2}{2m} \nabla^2 - V \right] \psi = -\frac{E}{\hbar^2} \psi = -\left[\mu^2 + \lambda \psi^2 \right] \psi$$

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$$\frac{\hbar}{2\pi} \langle p \rangle = \frac{\hbar}{2\pi} \langle p \rangle = 0$$

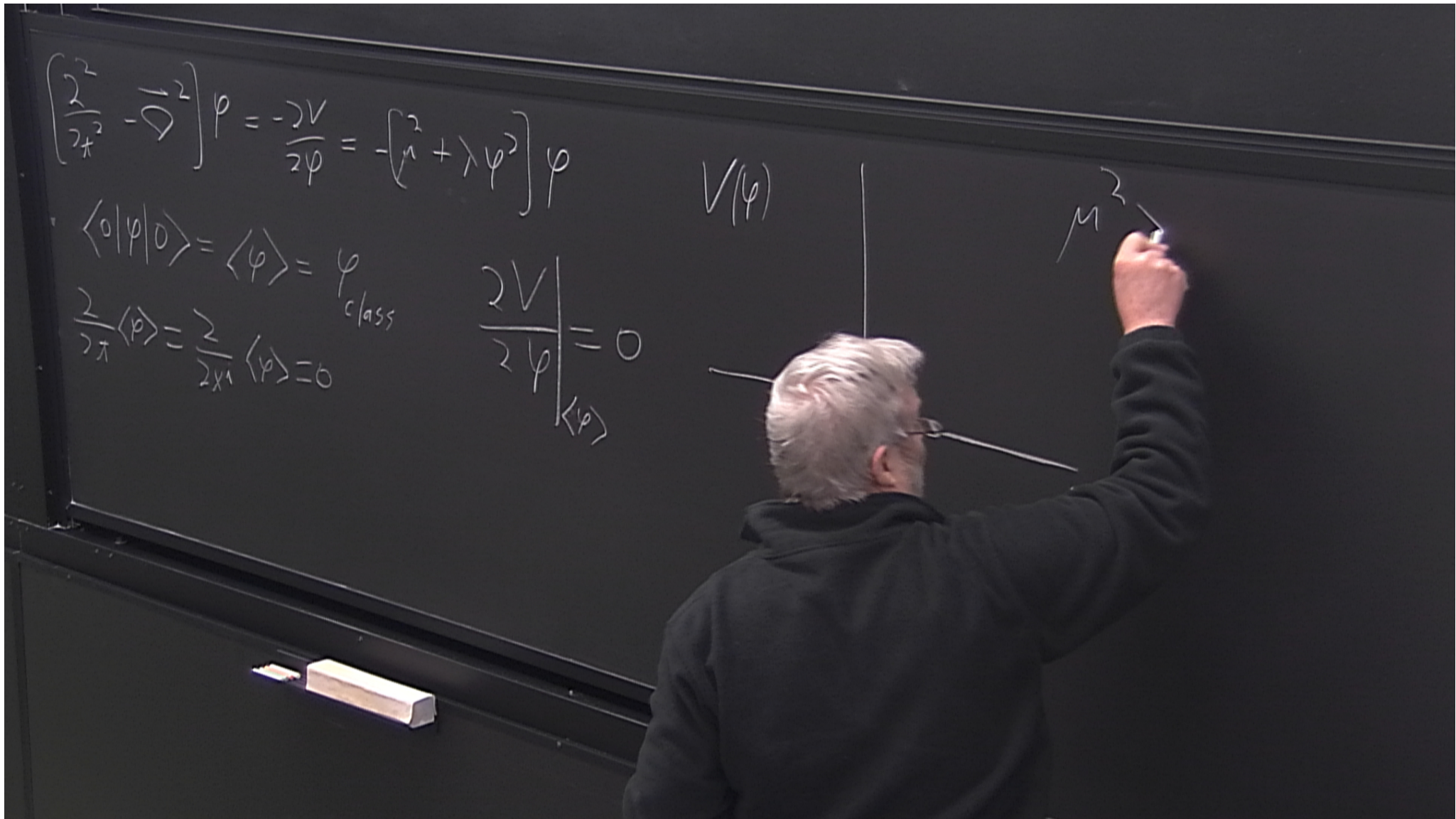
$$\frac{\hbar^2}{2m}$$

$$\left[\frac{\hbar^2}{2m} \nabla^2 - V \right] \psi = -\frac{E}{\hbar^2} \psi = -\left[\mu^2 + \lambda \psi^2 \right] \psi$$

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$$\frac{\hbar^2}{2m} \langle \psi \rangle = \frac{\hbar^2}{2m} \langle \psi \rangle = 0$$

$$\frac{\partial V}{\partial \psi} \Big|_{\langle \psi \rangle} = 0$$



$$\left(\frac{\nabla^2}{2\mu} - V \right) \psi = -\frac{\partial V}{\partial \psi} = -\left[\mu^2 + \lambda \psi^2 \right] \psi$$

$$V(\psi)$$

$$\langle 0 | \psi | 0 \rangle = \langle \psi \rangle = \psi_{\text{class}}$$

$$\frac{\nabla}{\nabla x} \langle \psi \rangle = \frac{\nabla}{\nabla x} \langle \psi \rangle = 0$$

$$\frac{\partial V}{\partial \psi} \Big|_{\langle \psi \rangle} = 0$$

$$\mu^2 > 0$$

$$\left(\frac{\nabla^2}{2\mu^2} - \nabla^2 \right) \psi = -\frac{\partial V}{\partial \psi} = -\left[\mu^2 + \lambda \psi^2 \right] \psi$$

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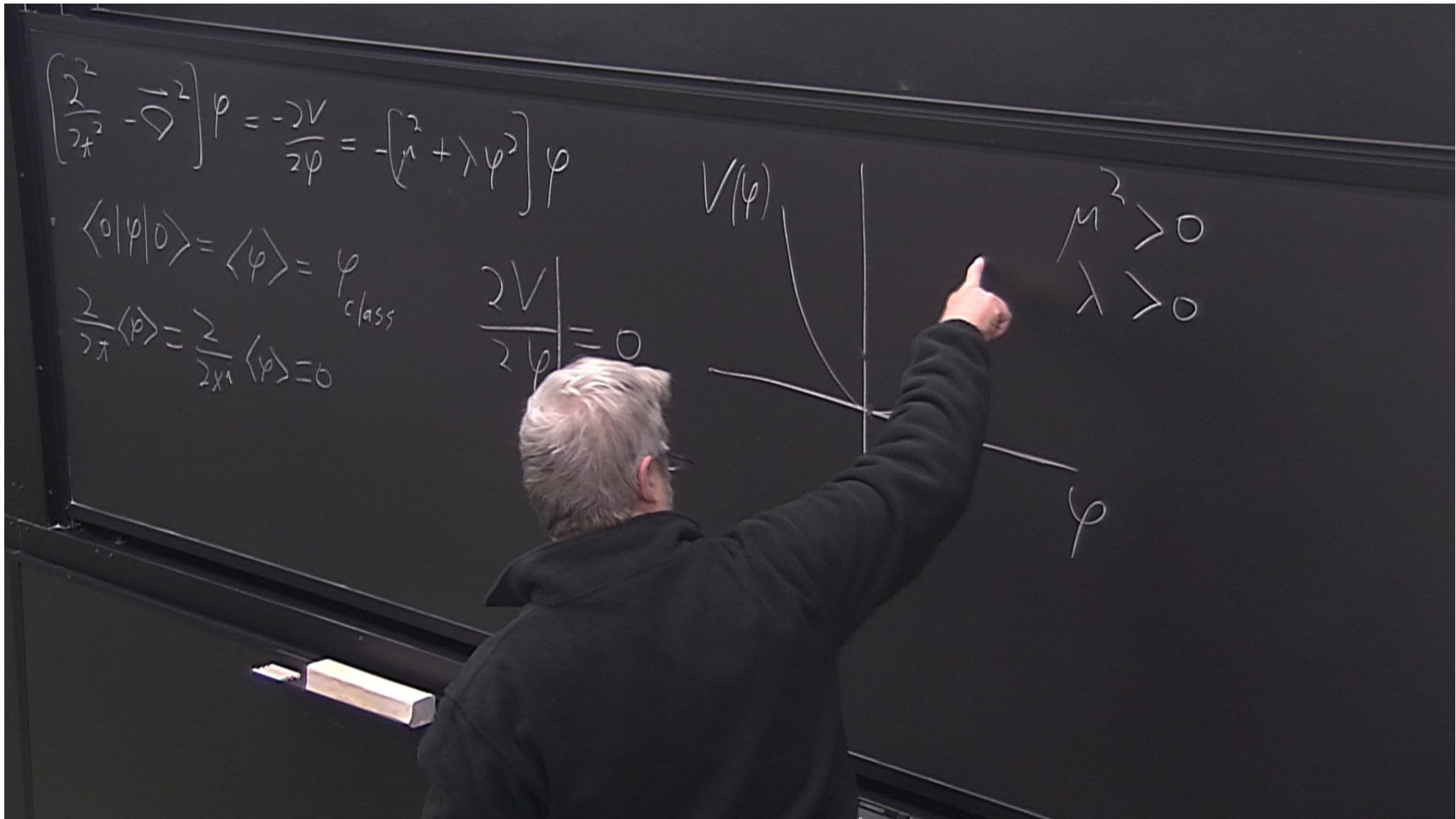
$$\frac{\partial V}{\partial \psi} \Big|_{\langle \psi \rangle} = 0$$

$V(\psi)$

$$\mu^2 > 0$$

$$\lambda > 0$$

ψ

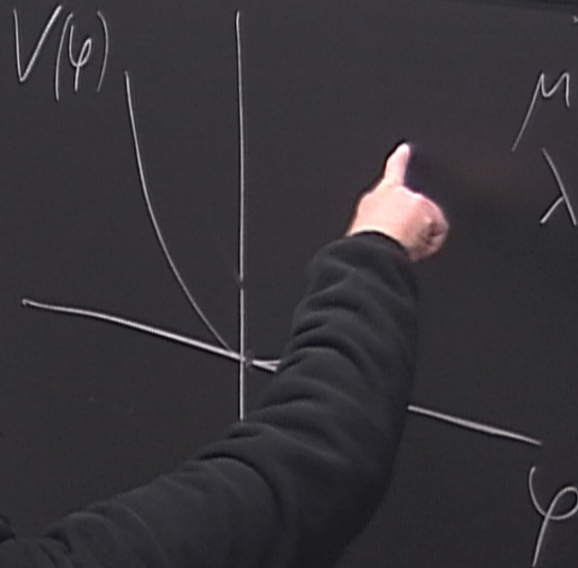


$$\left(\frac{\nabla^2}{2\hbar^2} - \nabla^2 \right) \psi = -\frac{\partial V}{\partial \psi} = -\left[\frac{\nabla^2}{2\hbar^2} + \lambda \psi^2 \right] \psi$$

$$\langle 0 | \psi | 0 \rangle = \langle \psi \rangle = \psi_{\text{class}}$$

$$\frac{\nabla^2}{2\hbar^2} \langle \psi \rangle = \frac{\nabla^2}{2\hbar^2} \langle \psi \rangle = 0$$

$$\frac{\partial V}{\partial \psi} = 0$$



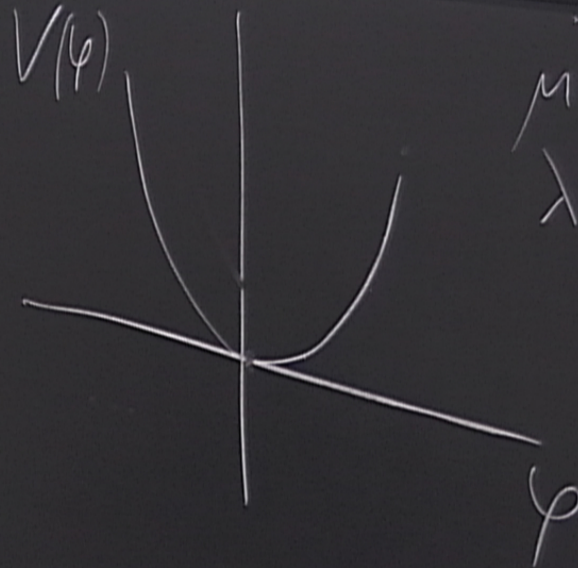
$$\mu^2 > 0$$
$$\lambda > 0$$

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 \right) \varphi = -\frac{\partial V}{\partial \varphi} = -[m^2 + \lambda \varphi^2] \varphi$$

$$\langle 0 | \varphi | 0 \rangle = \langle \varphi \rangle = \varphi_{\text{class}}$$

$$\frac{\partial}{\partial \pi} \langle \varphi \rangle = 0$$

$$\left. \frac{\partial V}{\partial \varphi} \right|_{\langle \varphi \rangle} = 0$$



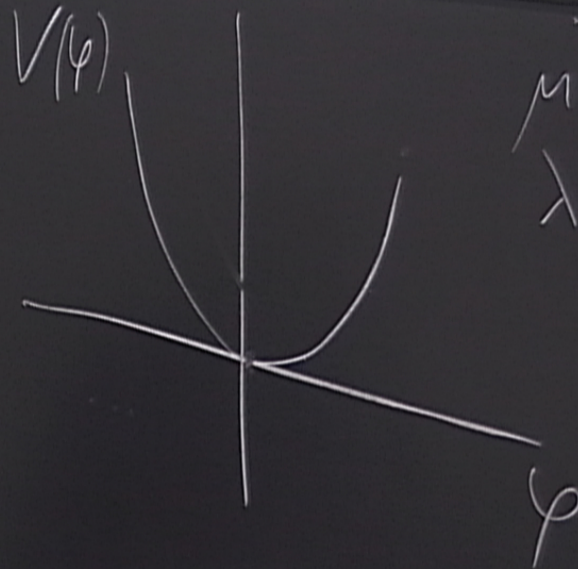
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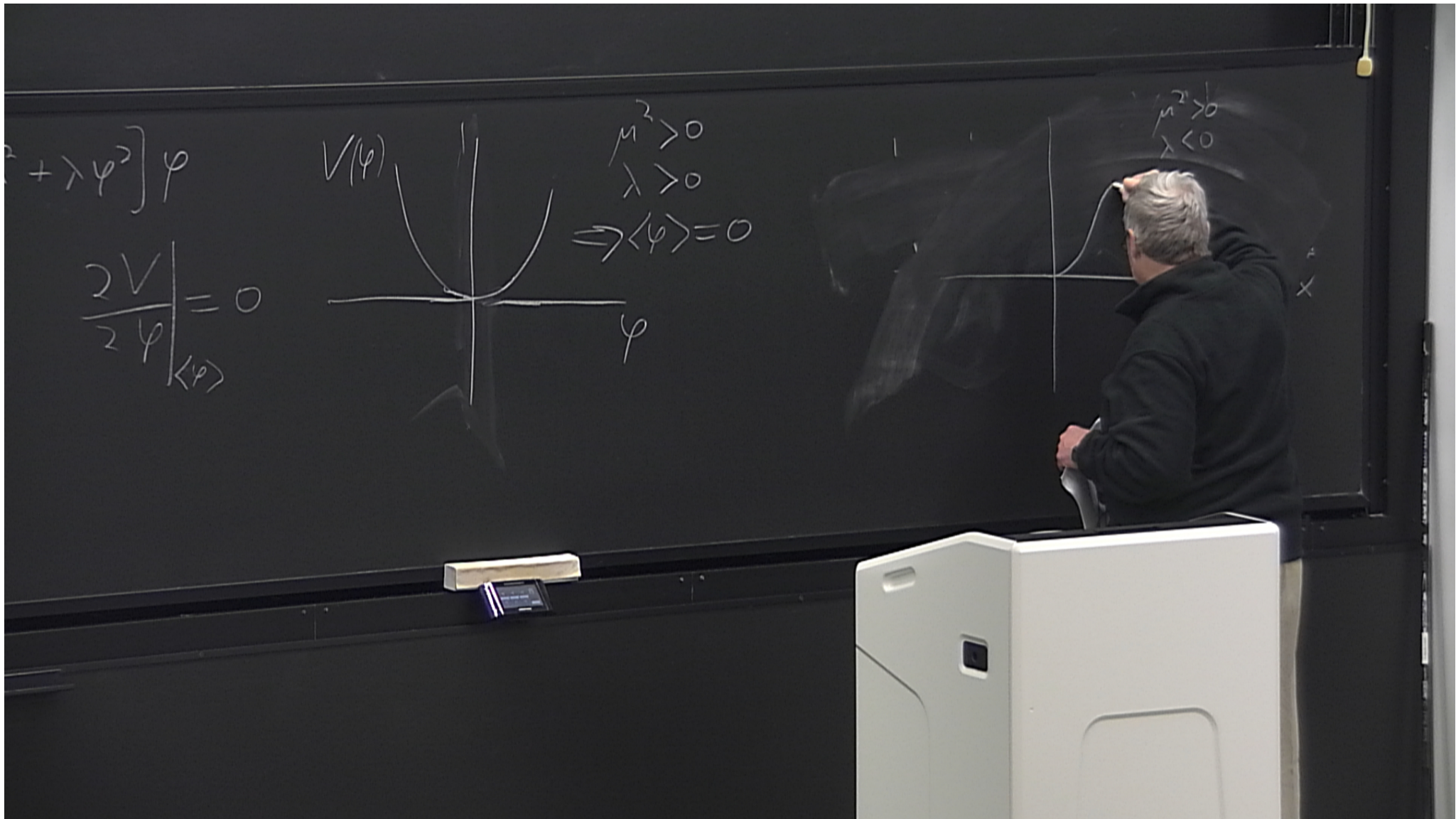
$$\langle 0 | \varphi | 0 \rangle = \langle \varphi \rangle = \varphi_{\text{class}}$$

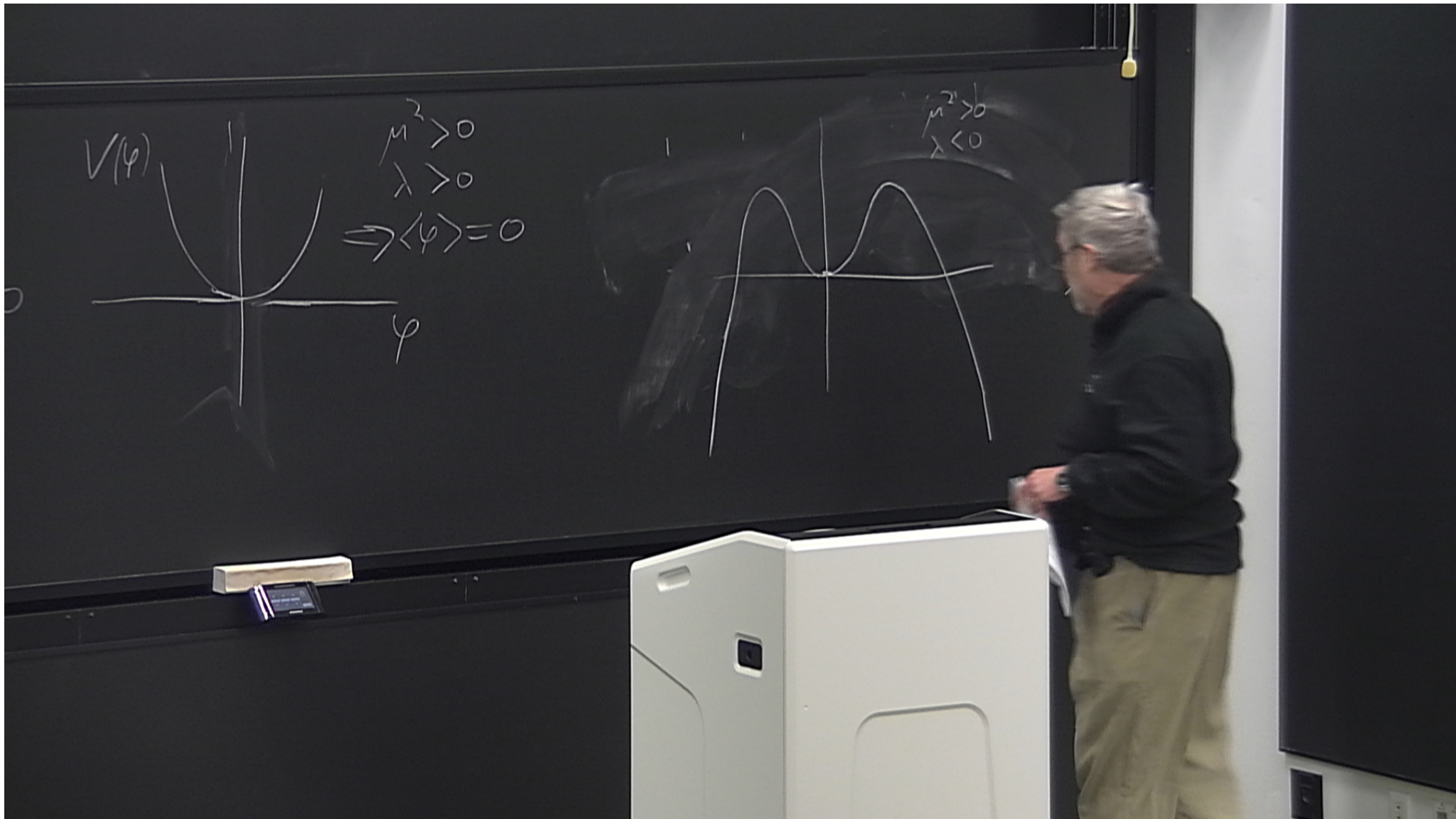
$$\frac{\partial}{\partial t} \langle p \rangle = \frac{\partial}{\partial x} \langle p \rangle = 0$$

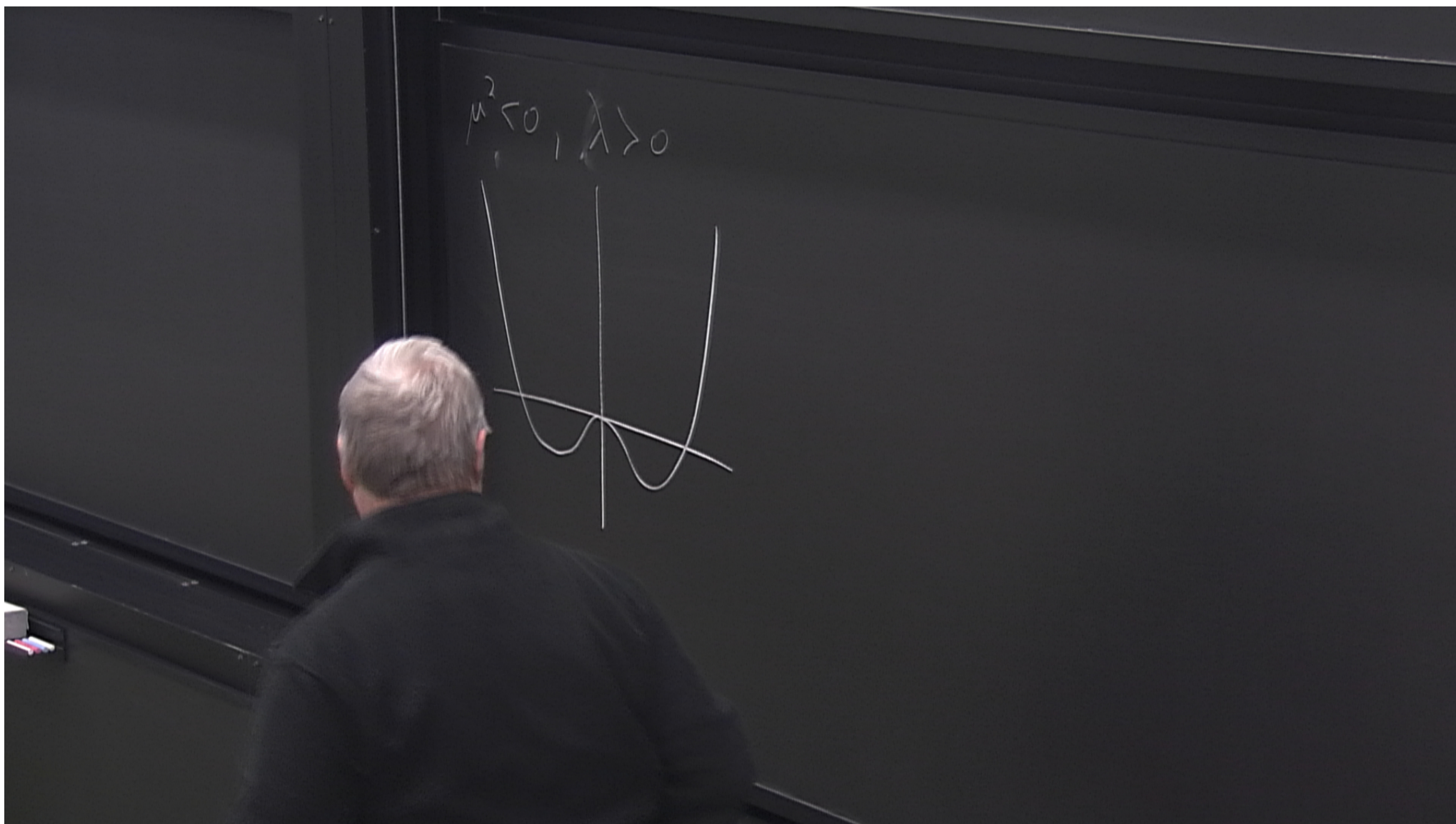


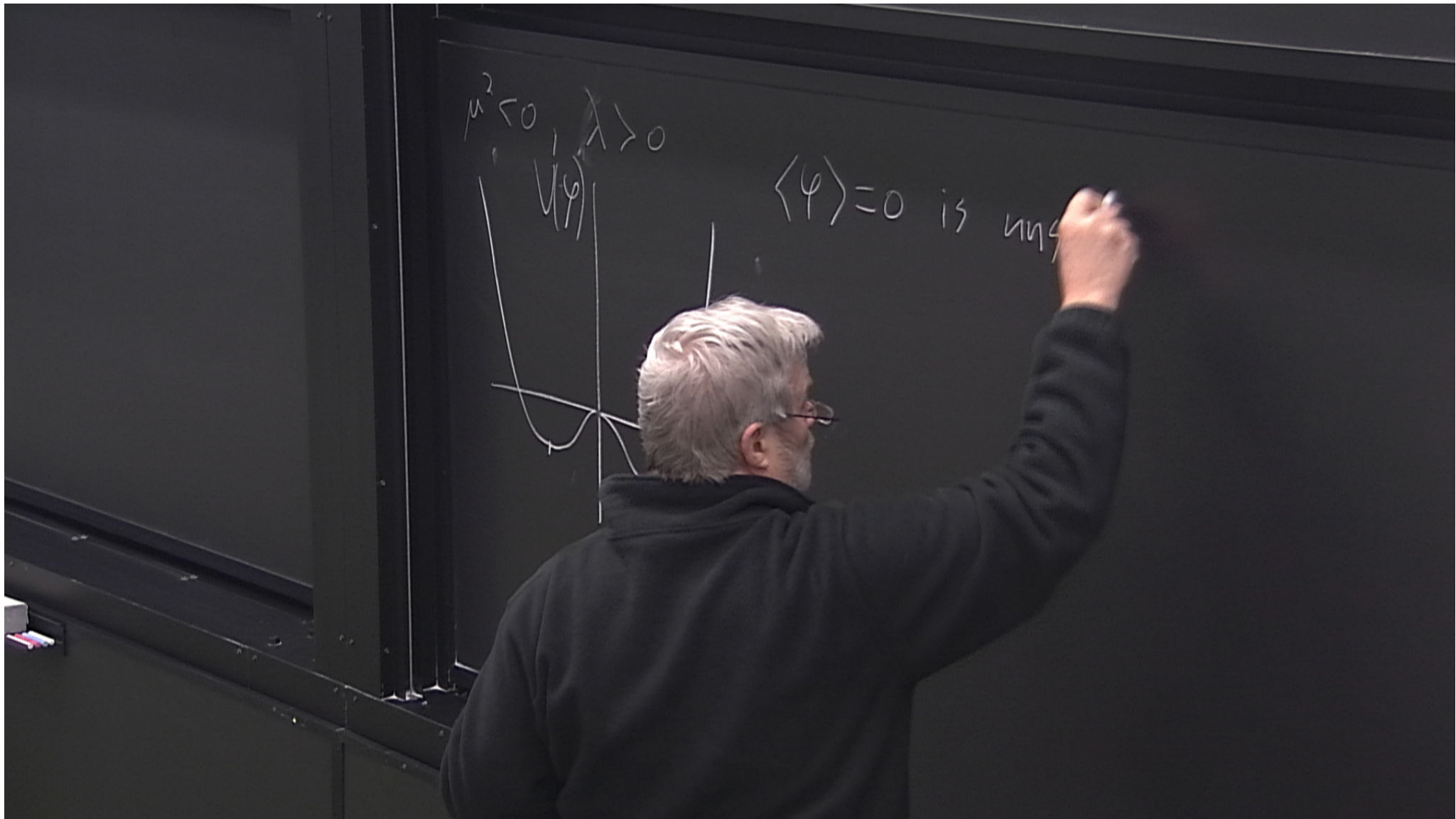
$$\mu^2 > 0$$

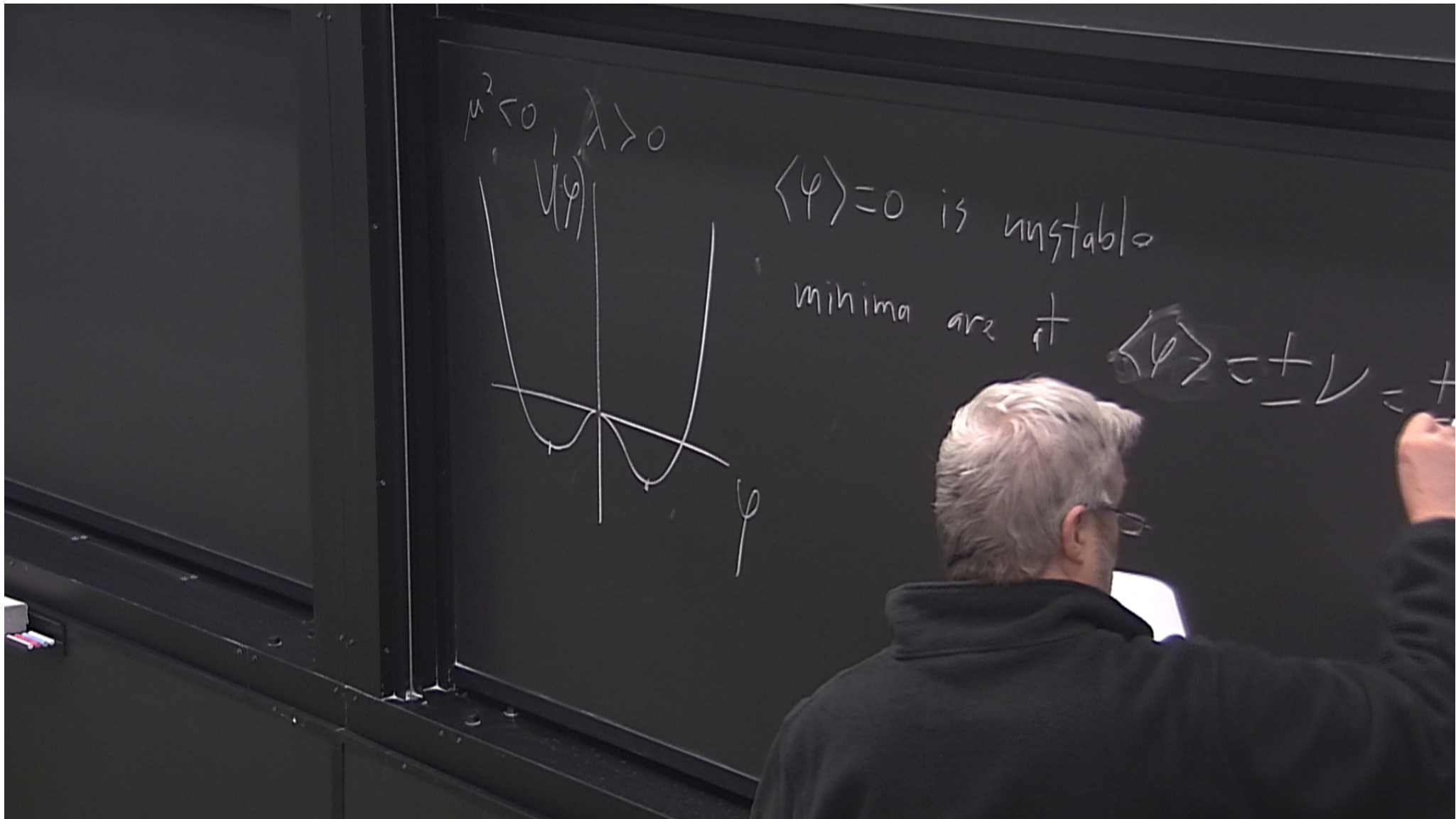
$$\lambda > 0$$



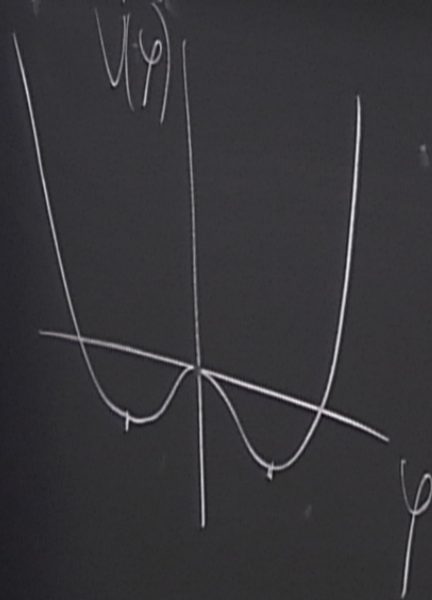








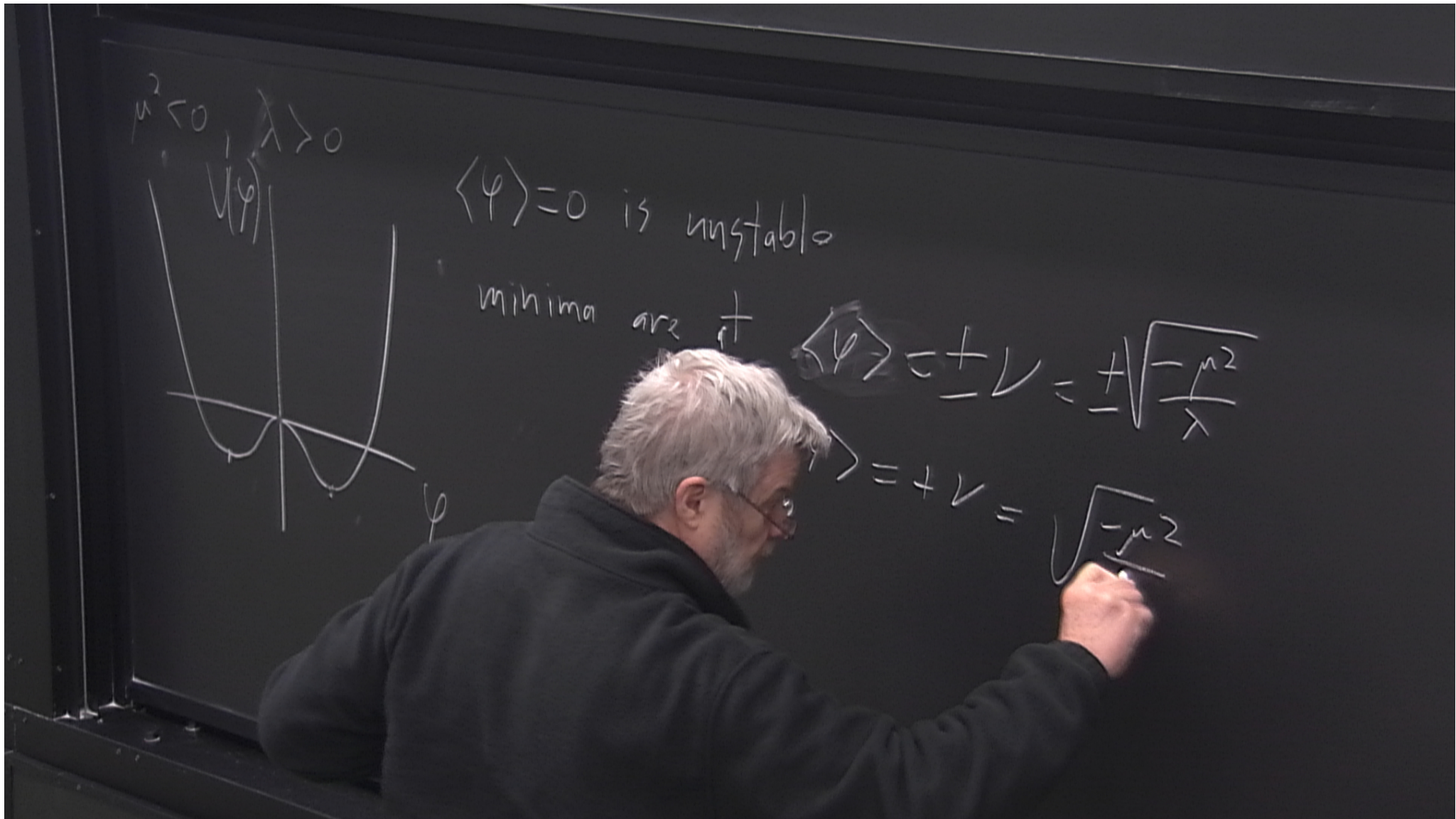
$$\mu^2 < 0, \lambda > 0$$



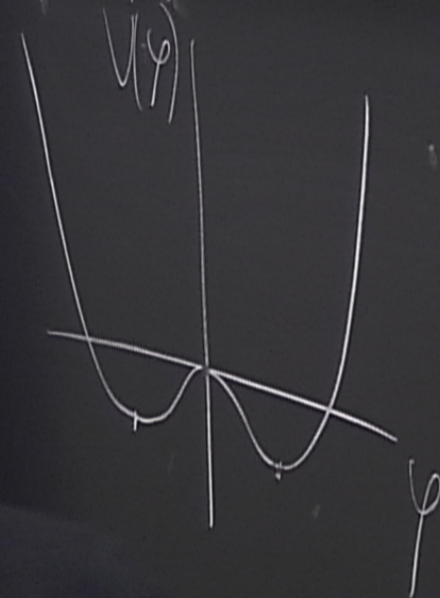
$\langle \phi \rangle = 0$ is unstable

minima are at

$$\langle \phi \rangle = \pm v = \pm \sqrt{\frac{-\mu^2}{\lambda}}$$



$$\mu^2 < 0, \lambda > 0$$



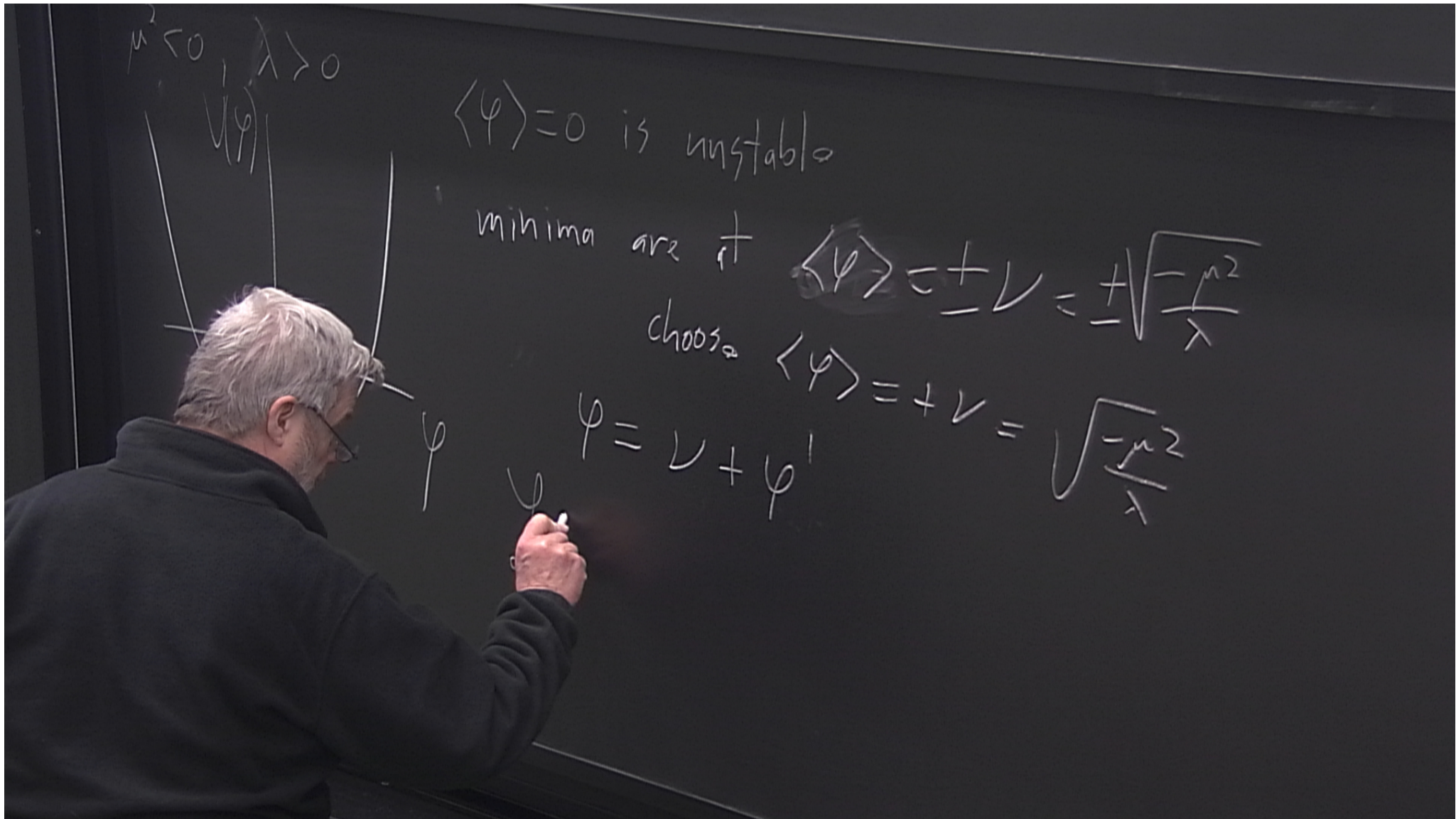
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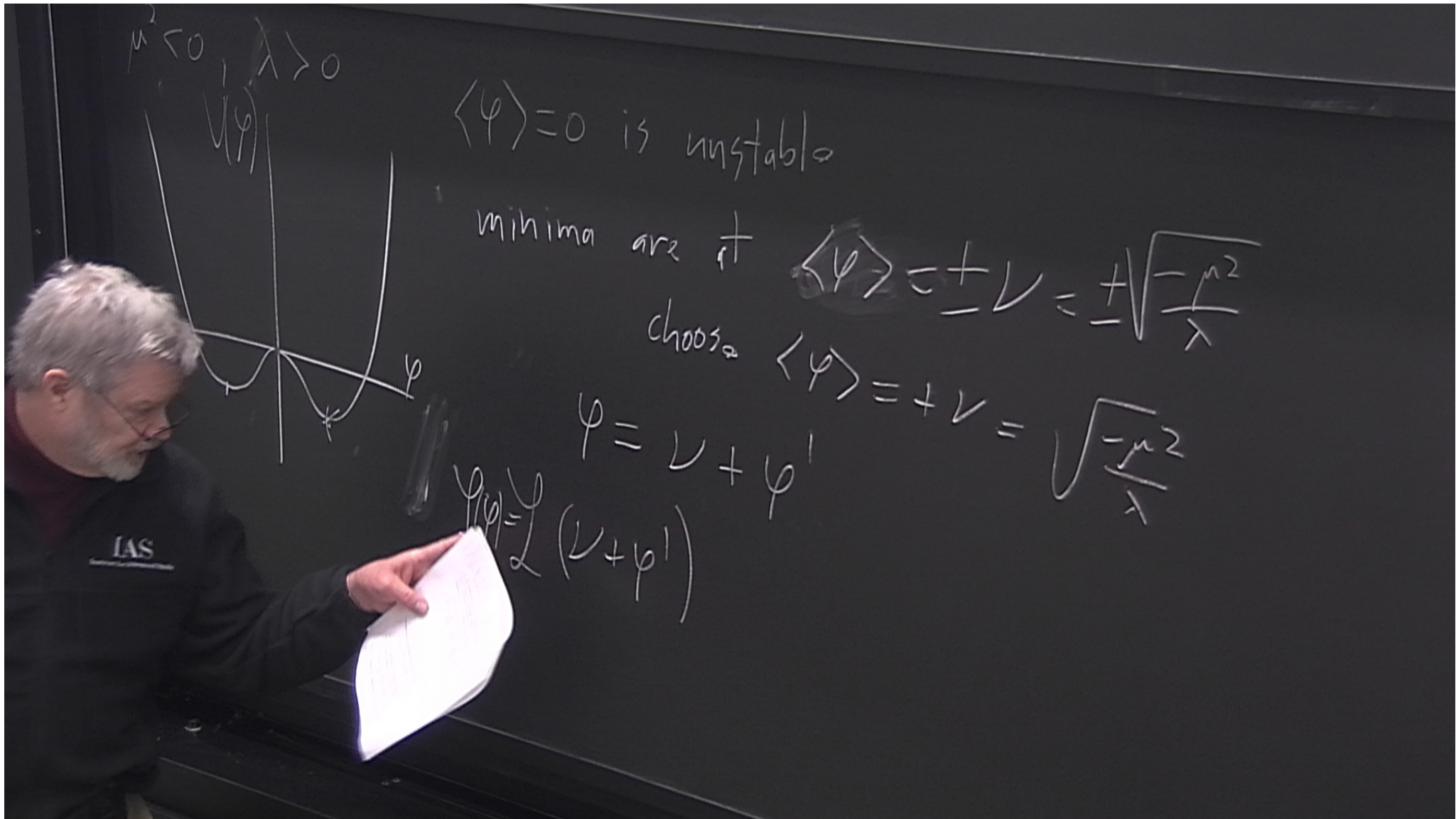
minima are at

$$\langle \phi \rangle = \pm v = \pm \sqrt{\frac{-\mu^2}{\lambda}}$$

choose

$$\langle \phi \rangle = +v = \sqrt{\frac{-\mu^2}{\lambda}}$$





$$\pm \sqrt{\frac{\mu^2}{\lambda}}$$

$$\sqrt{\frac{\mu^2}{\lambda}}$$

$$V(\varphi')$$

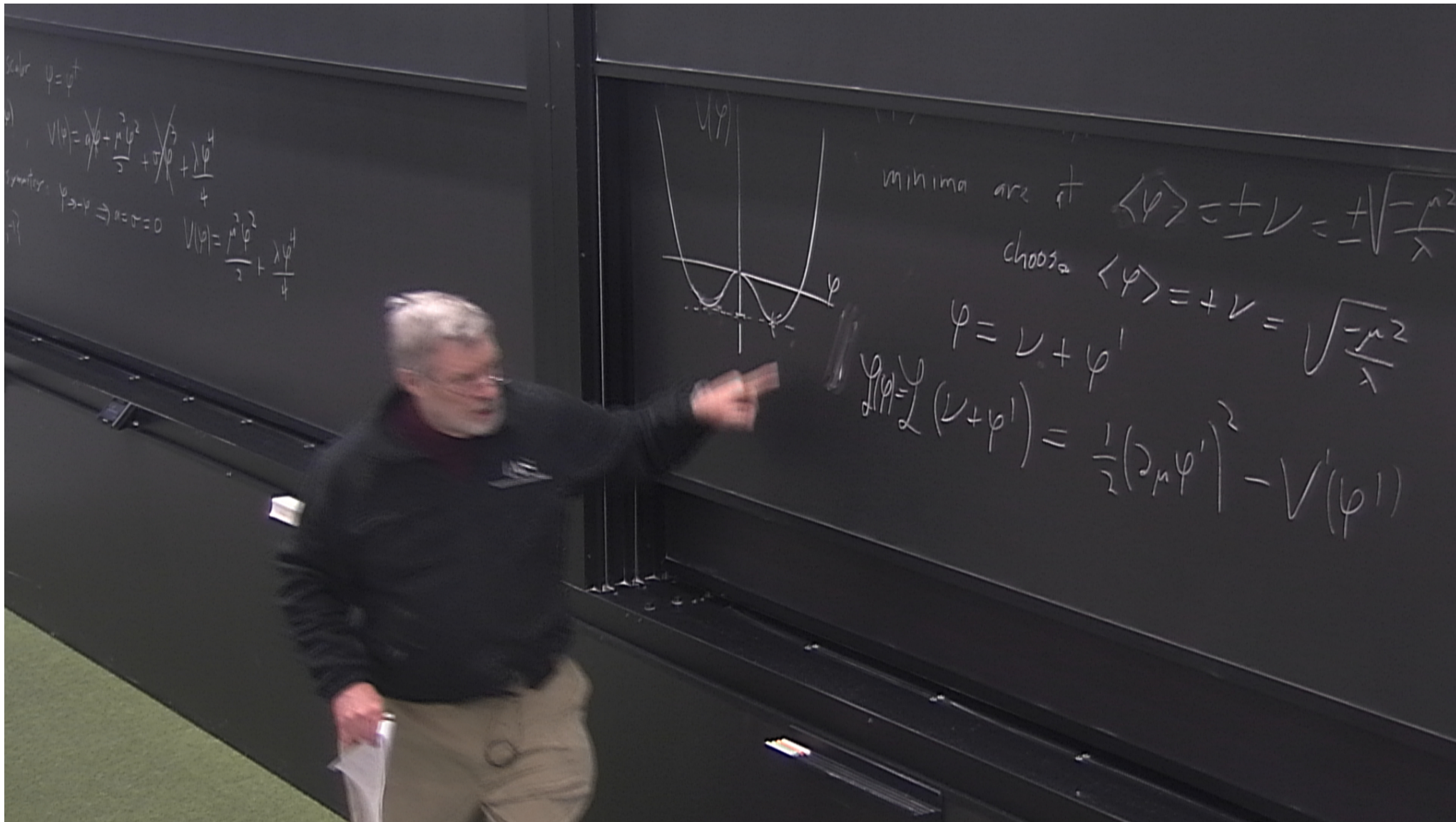
$$V(\varphi') = \frac{-\mu^4}{4\lambda} - \mu^2 \varphi'^2 + \lambda \varphi'^3 +$$

$$\pm \sqrt{-\frac{\mu^2}{\lambda}}$$

$$\sqrt{-\frac{\mu^2}{\lambda}}$$

$$V'(\varphi')$$

$$V(\varphi') = \frac{-\mu^4}{4\lambda} - \mu^2 \varphi'^2 + \lambda \varphi'^3 + \frac{\lambda}{4} (\varphi')^4$$



$$V(\varphi') = \frac{-\mu^4}{4\lambda} - \underbrace{\mu^2}_{\mu^2/6} \varphi'^2 + \lambda \varphi'^3 + \frac{\lambda}{4} (\varphi')^4$$

$$V(\varphi') = \frac{-\mu^4}{4\lambda} - \underbrace{\mu^2}_{\mu^2/6} \varphi'^2 + \lambda \varphi'^3 + \frac{\lambda}{4} |\varphi'|^4$$

$$\text{re st } \langle \varphi \rangle = \pm v = \pm \sqrt{\frac{-\mu^2}{\lambda}}$$

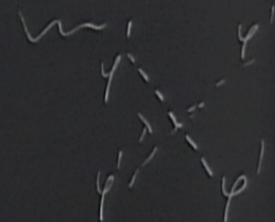
$$\text{choose } \langle \varphi \rangle = +v = \sqrt{\frac{-\mu^2}{\lambda}}$$

$$v + \varphi'$$

$$L = \frac{1}{2}(\partial_\mu \varphi')^2 - V(\varphi')$$

$$V(\varphi') = \frac{-\mu^4}{4\lambda} - \mu^2 \varphi'^2 + \lambda v \varphi'^3 + \frac{\lambda}{4} \varphi'^4$$

$\mu_{\varphi}^{12} = -$ induced cubic φ'



$$\text{or } \langle \varphi \rangle = \pm v = \pm \sqrt{\frac{-\mu^2}{\lambda}}$$

$$\text{choose } \langle \varphi \rangle = +v = \sqrt{\frac{-\mu^2}{\lambda}}$$

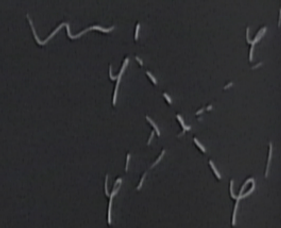
$$L + \varphi'$$

$$L' = \frac{1}{2}(\partial_\mu \varphi')^2 - V(\varphi')$$

$$V(\varphi') = \frac{-\mu^4}{4\lambda}$$

$$-\mu^2 \varphi'^2 + \lambda v \varphi'^3 + \frac{\lambda}{4} (\varphi')^4$$

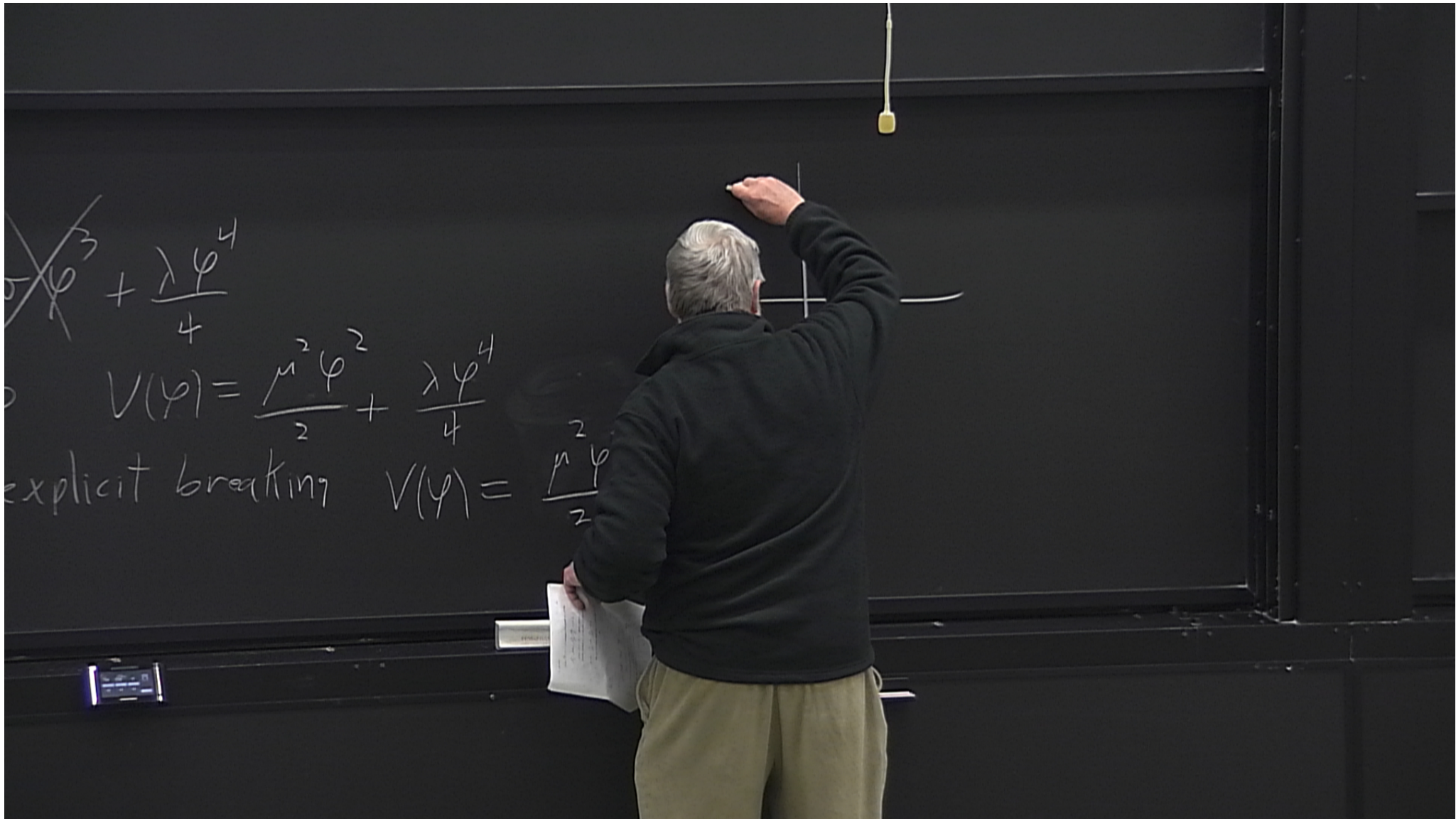
$\mu_{\varphi'}^2 = -2\mu^2 > 0$ induced cubic

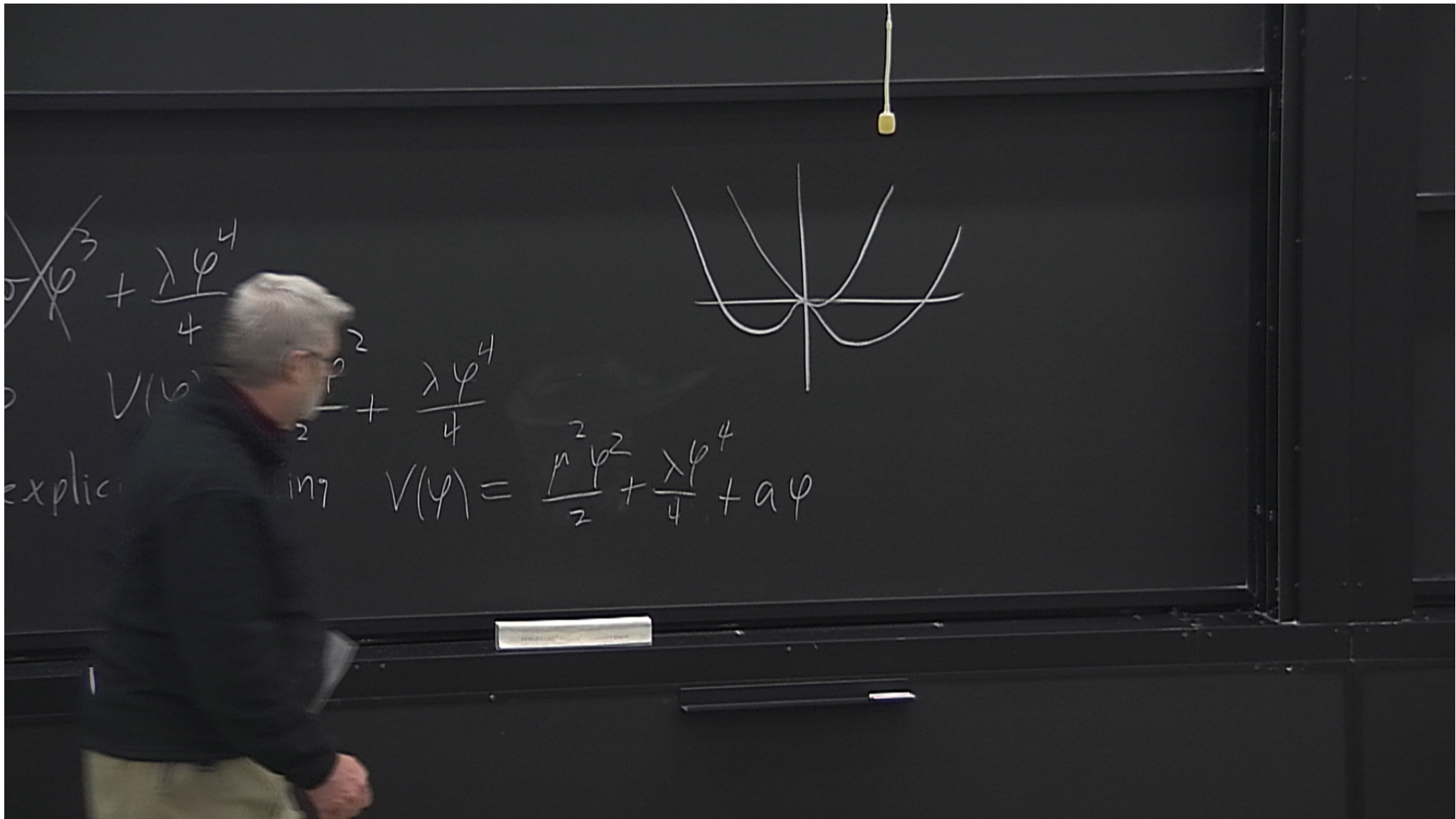


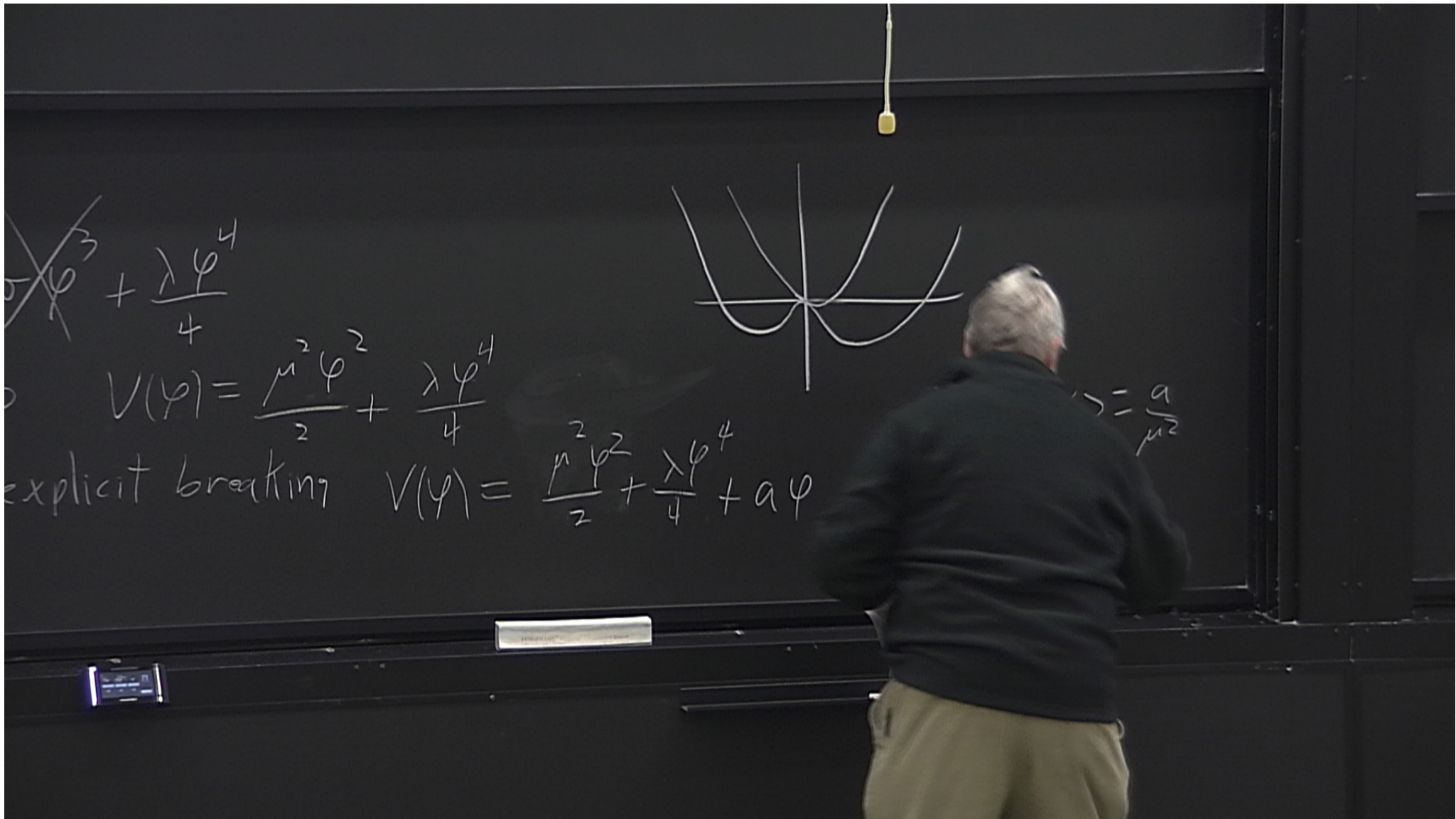
$$\cancel{\phi^3} + \frac{\lambda \phi^4}{4}$$

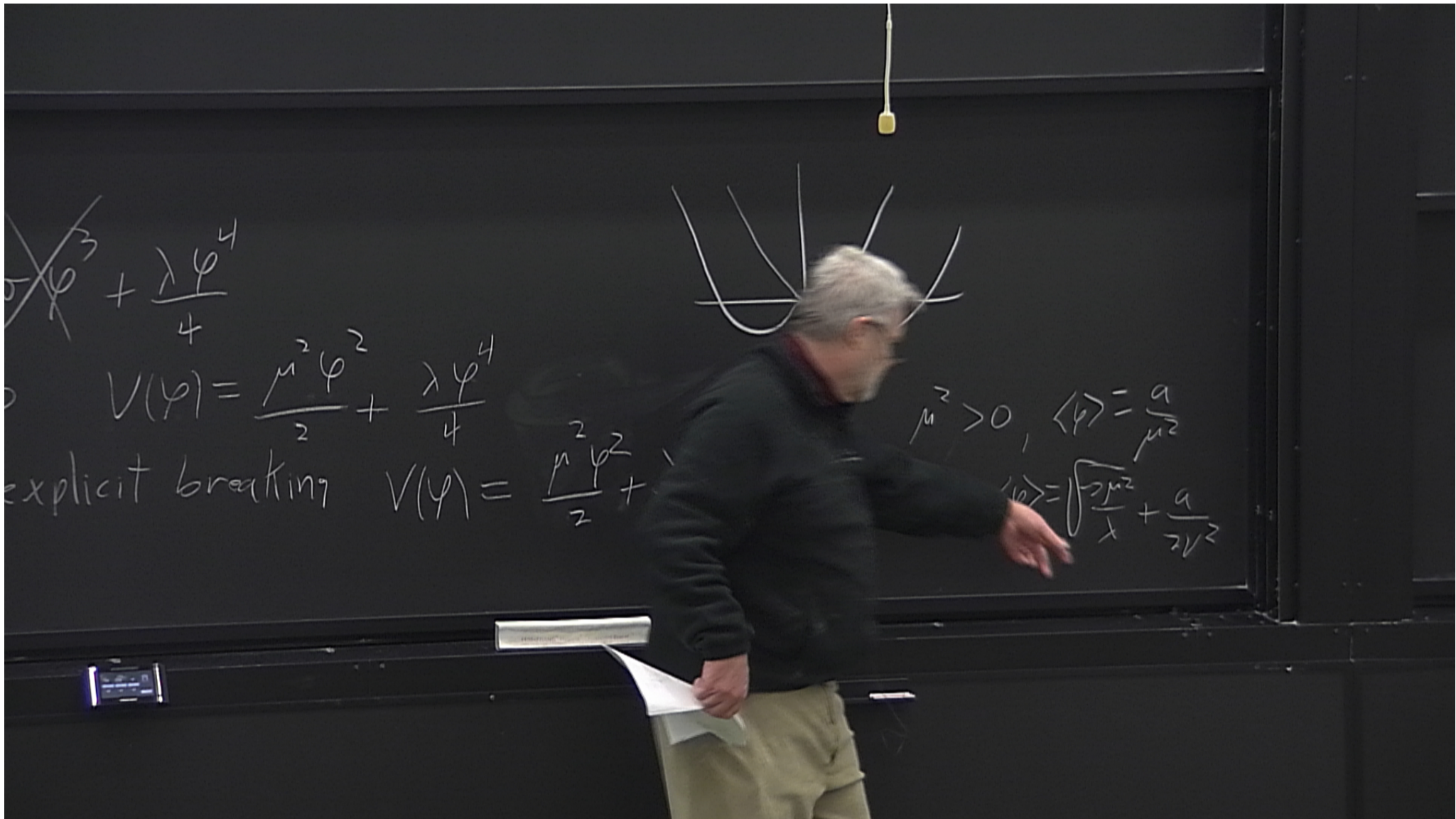
$$V(\phi) = \frac{\mu^2 \phi^2}{2} + \frac{\lambda \phi^4}{4}$$

explicit breaking $V(\phi) = \frac{\mu^2 \phi^2}{2} + \frac{\lambda \phi^4}{4} + a\phi$









$$\cancel{\phi^3} + \frac{\lambda \phi^4}{4}$$

$$V(\phi) = \frac{\mu^2 \phi^2}{2} + \frac{\lambda \phi^4}{4}$$

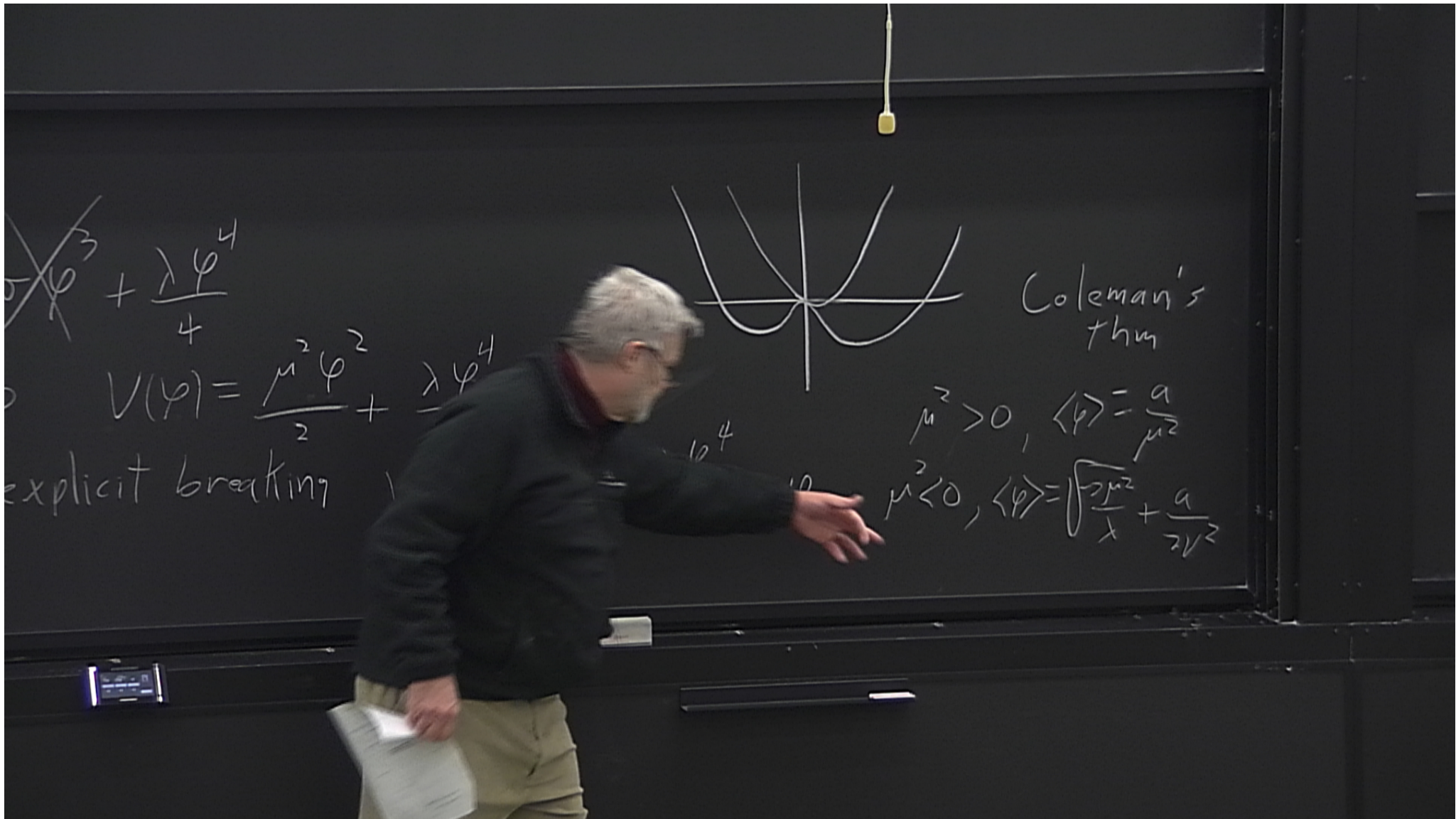
explicit breaking

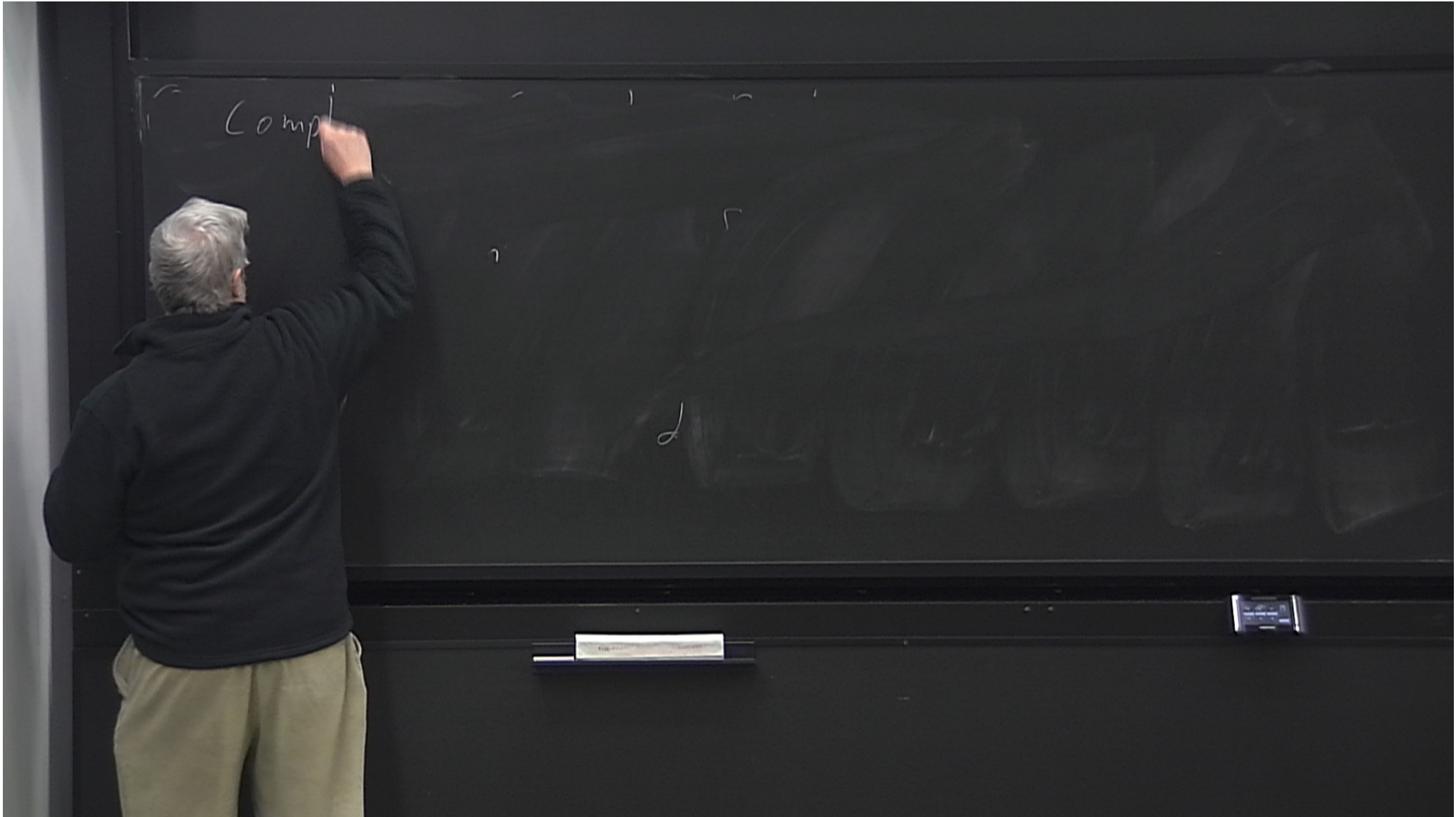
$$V(\phi) = \frac{\mu^2 \phi^2}{2} +$$

$$\mu^2 > 0, \langle \phi \rangle = \frac{a}{\mu^2}$$

$$\langle \phi \rangle = \sqrt{\frac{\mu^2}{\lambda}} + \frac{a}{2\mu^2}$$







Complex skalar $\psi \neq \psi^\dagger$

$$\mathcal{L} = (\partial^\mu \psi)^\dagger \partial_\mu \psi - V(\psi)$$

$$V(\psi) = m^2 \psi^\dagger \psi$$

Complex scalar $\dot{\varphi} \neq \varphi^\dagger$, $\lambda > 0$
 $\mathcal{L} = (\partial^\mu \varphi)^\dagger \partial_\mu \varphi - V(\varphi)$, $V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$

Complex scalar $\psi \neq \psi^\dagger$, $\lambda > 0$

$$\mathcal{L} = (\partial^\mu \psi)^\dagger \partial_\mu \psi - V(\psi)$$

$$V(\psi) = \mu^2 \psi^\dagger \psi + \lambda (\psi^\dagger \psi)^2$$

Complex scalar $\psi \neq \psi^\dagger$, $\lambda > 0$

$$\mathcal{L} = (\partial^\mu \psi)^\dagger \partial_\mu \psi - V(\psi), \quad V(\psi) = \mu^2 \psi^\dagger \psi + \lambda (\psi^\dagger \psi)^2$$

$U(1): \quad \psi \rightarrow e^{iB} \psi, \quad \psi^\dagger \rightarrow \psi^\dagger e^{-iB}$

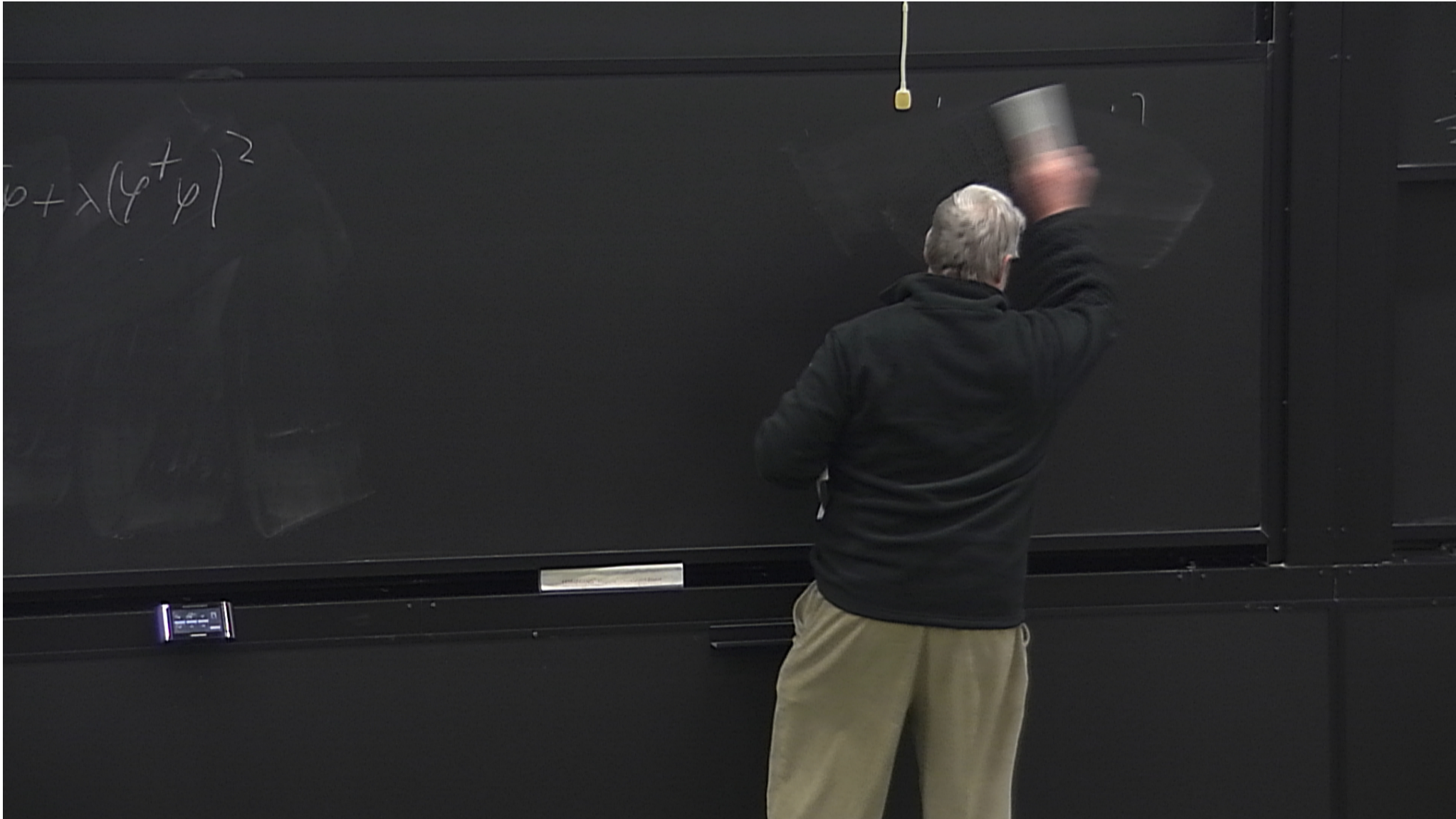
Complex scalar $\dot{\varphi} \neq \varphi^\dagger$, $\lambda > 0$

$$\mathcal{L} = (\partial^\mu \varphi)^\dagger \partial_\mu \varphi - V(\varphi)$$

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

$U(1)$:

$$\varphi \rightarrow e^{i\beta} \varphi, \quad \varphi^\dagger \rightarrow \varphi^\dagger e^{-i\beta}$$



Complex scalar $\dot{\varphi} \neq \varphi^\dagger$, $\lambda > 0$

$$\varphi = (\varphi^0, \vec{\varphi})^T, \quad V(\varphi) = m^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

$U(1): \quad \varphi \rightarrow e^{i\beta} \varphi, \quad \varphi^\dagger \rightarrow \varphi^\dagger e^{-i\beta}$

Complex scalar $\varphi \neq \varphi^\dagger$, $\lambda > 0$

$$\mathcal{L} = (\partial^\mu \varphi)^\dagger \partial_\mu \varphi - V(\varphi), \quad V(\varphi) = m^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2$$

$U(1): \quad \varphi \rightarrow e^{i\beta} \varphi, \quad \varphi^\dagger \rightarrow \varphi^\dagger e^{-i\beta}$

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

$$\varphi^\dagger = \frac{\varphi_1 - i\varphi_2}{\sqrt{2}}$$

Complex scalar $\psi \neq \psi^\dagger$, $\lambda > 0$

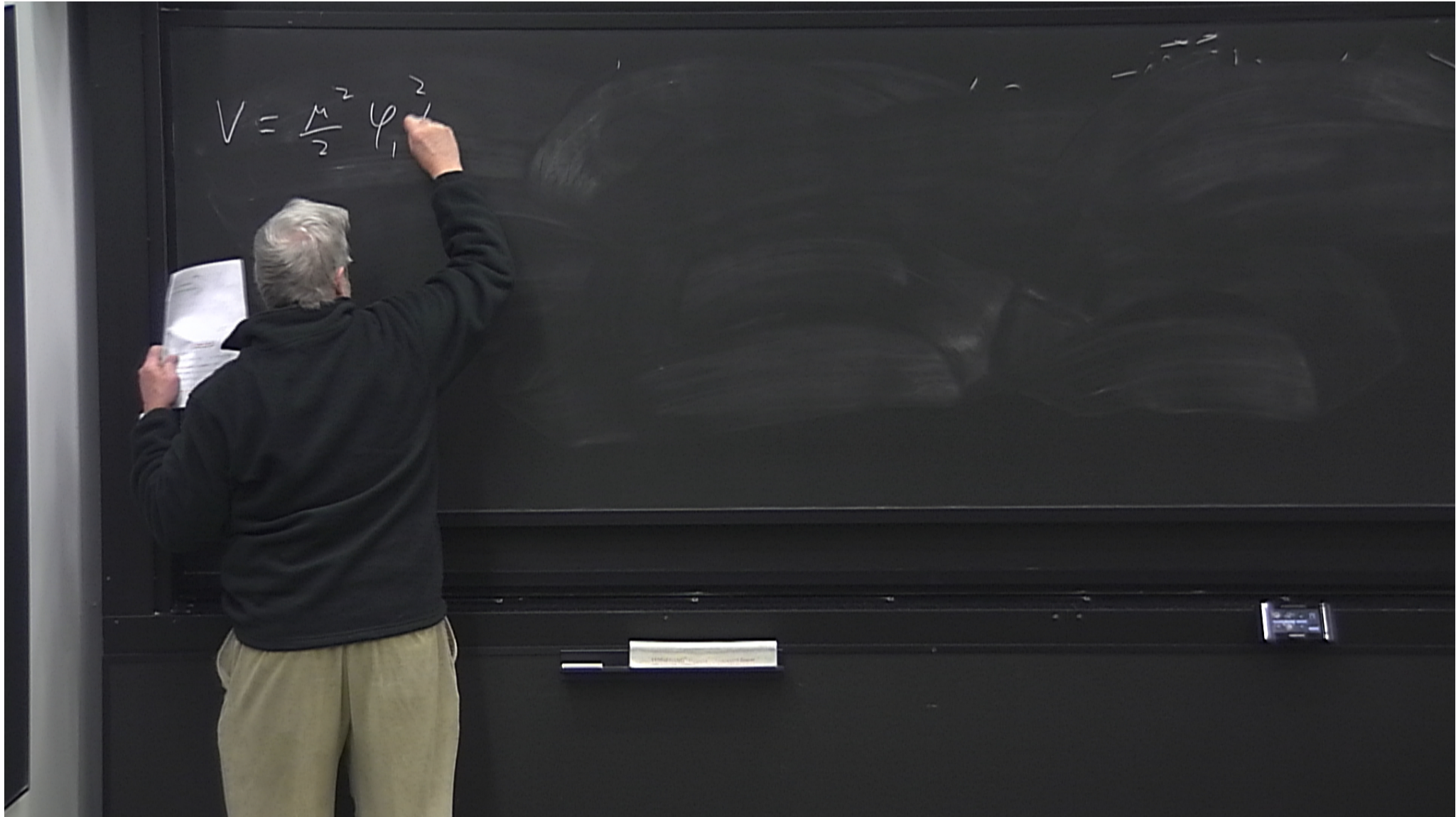
$$\mathcal{L} = (\partial^\mu \psi)^\dagger \partial_\mu \psi - V(\psi), \quad V(\psi) = m^2 \psi^\dagger \psi + \lambda (\psi^\dagger \psi)^2$$

$U(1): \quad \psi \rightarrow e^{i\theta} \psi, \quad \psi^\dagger \rightarrow \psi^\dagger e^{-i\theta}$

$$\psi = \frac{\psi_1 + i\psi_2}{\sqrt{2}}$$

$$\psi^\dagger = \frac{\psi_1 - i\psi_2}{\sqrt{2}}$$

$$\mathcal{L} = \frac{1}{2} \left[(\partial_\mu \psi_1)^2 + (\partial_\mu \psi_2)^2 \right] - V(\psi)$$



$$V = \frac{M^2}{2}(\varphi_1^2 + \varphi_2^2) + \frac{\lambda'}{4}(\varphi_1^2 + \varphi_2^2)^2$$

$$V = \frac{m^2}{2} (\varphi_1^2 + \varphi_2^2) + \frac{\lambda'}{4} (\varphi_1^2 + \varphi_2^2)^2 \Rightarrow 'O(2)'$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \cos \beta$$

$$V = \frac{M^2}{2}(\varphi_1^2 + \varphi_2^2) + \frac{\lambda}{4}(\varphi_1^2 + \varphi_2^2)^2 \Rightarrow 'O(2)$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

$$V = \frac{M^2}{2}(\varphi_1^2 + \varphi_2^2) + \frac{\lambda'}{4}(\varphi_1^2 + \varphi_2^2)^2 \Rightarrow 'O(2)' - \text{if } \lambda' > 0$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

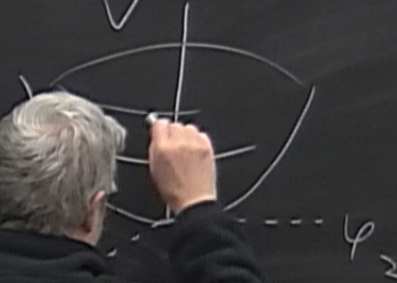
$$V = \frac{M^2}{2}(\varphi_1^2 + \varphi_2^2) + \frac{\lambda'}{4}(\varphi_1^2 + \varphi_2^2)^2 \Rightarrow 'O(2)'_{\sqrt{V}} \quad \lambda'^2 > 0$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$



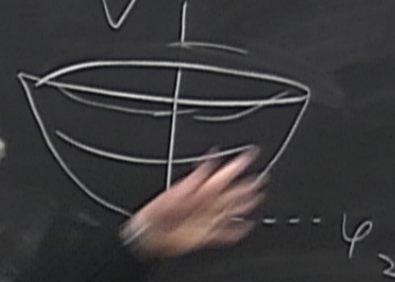
$$V = \frac{\mu^2}{2}(\varphi_1^2 + \varphi_2^2) + \frac{\lambda}{4}(\varphi_1^2 + \varphi_2^2)^2 \Rightarrow \text{'O(2)'} \quad \mu^2 > 0$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$



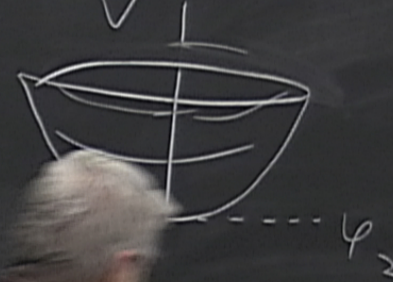
$$V = \frac{M^2}{2}(\varphi_1^2 + \varphi_2^2) + \frac{\lambda'}{4}(\varphi_1^2 + \varphi_2^2)^2 \Rightarrow 'O(2)' \quad \mu^2 > 0$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$

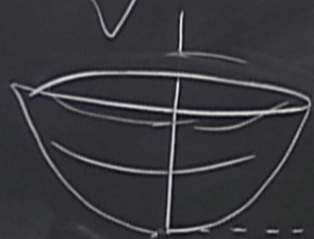


$$V = \frac{M^2}{2} (\varphi_1^2 + \varphi_2^2) + \frac{\lambda'}{4} (\varphi_1^2 + \varphi_2^2)^2 \Rightarrow 'O(2)' \quad M^2 > 0$$

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \cos B & -\sin B \\ \sin B & \cos B \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$



$\Rightarrow O(2)$



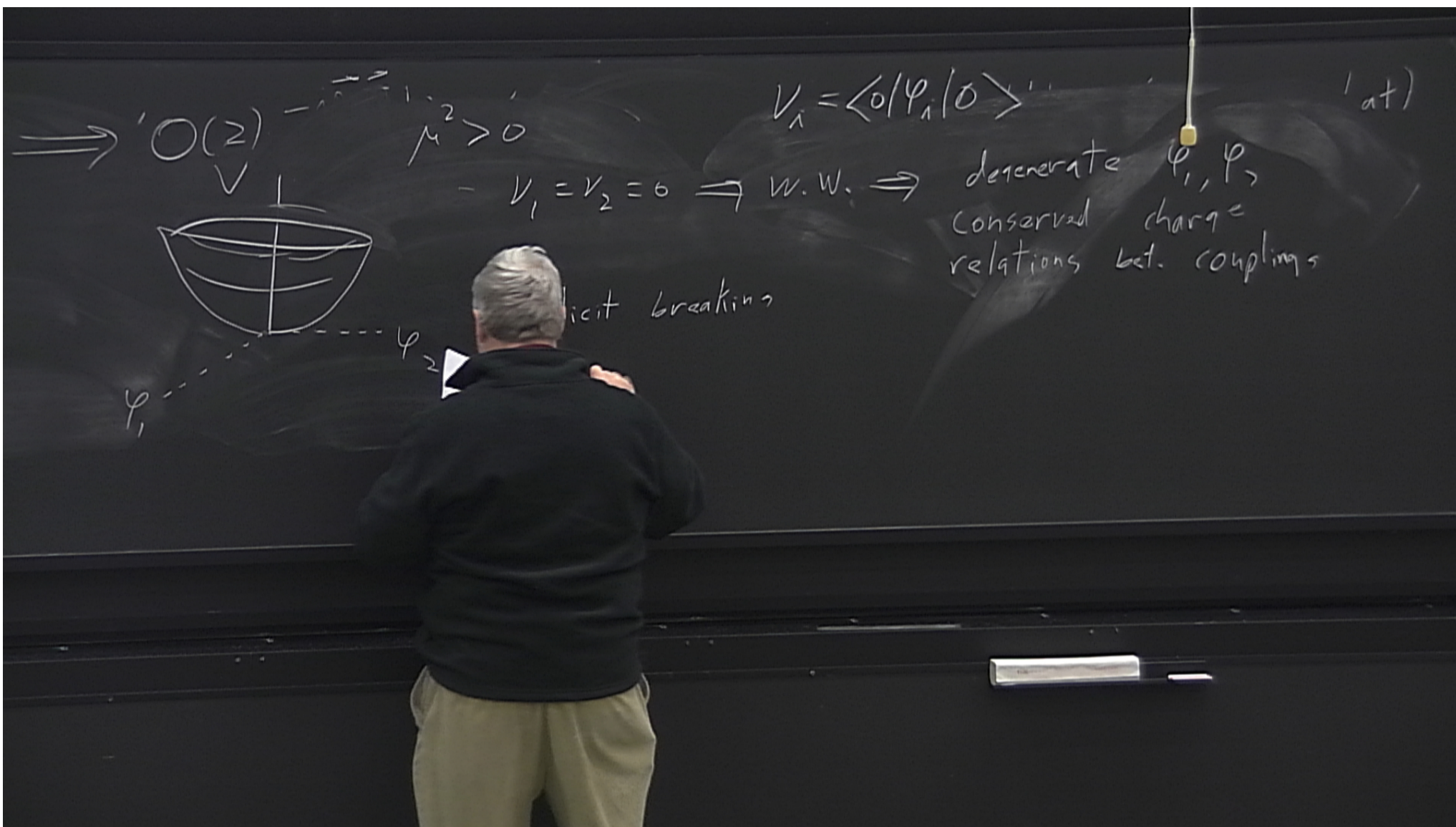
φ_1 φ_2

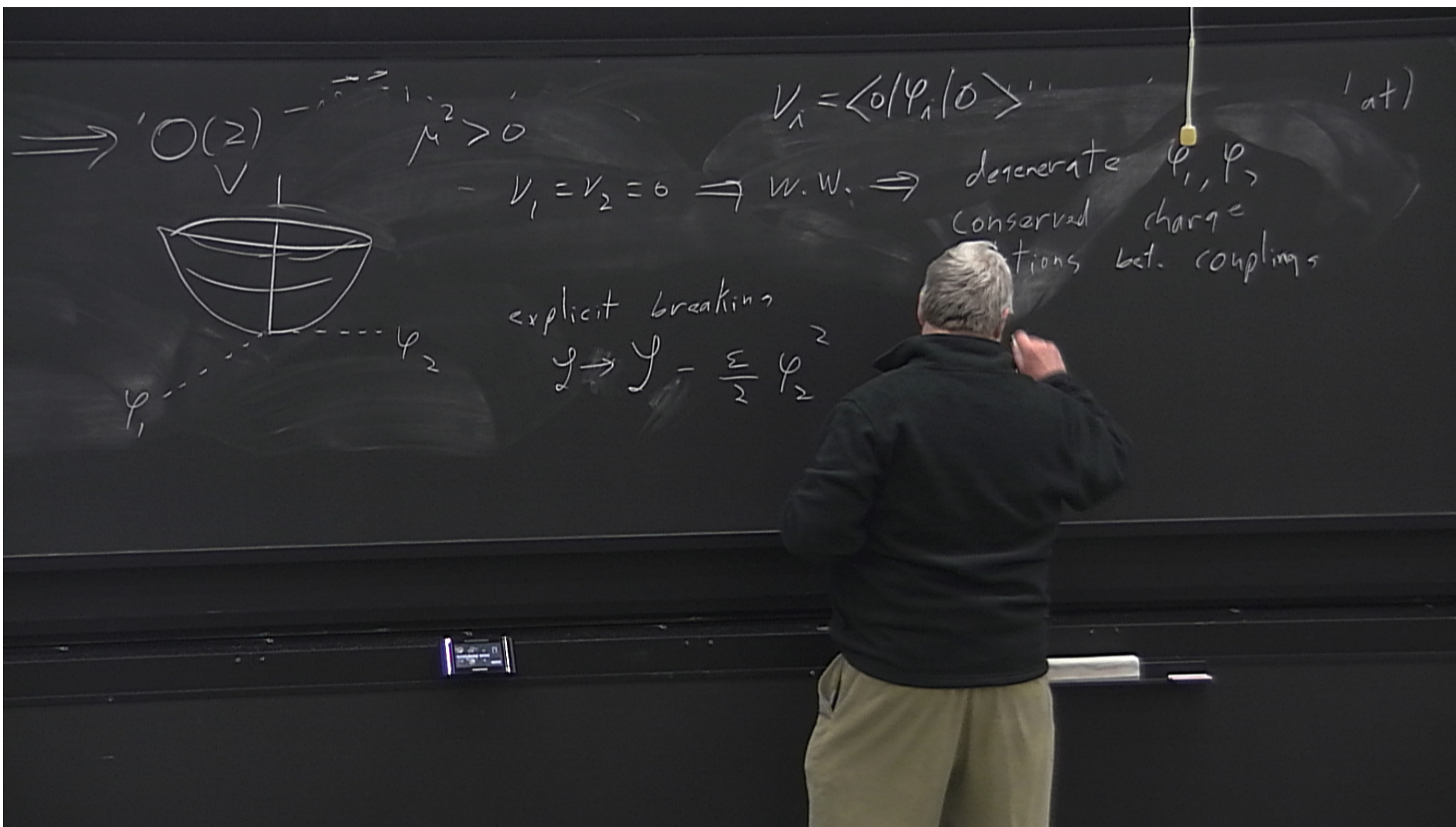
$$M^2 > 0$$

$V_1 = V_2 = 0 \Rightarrow$ w.w. $\Rightarrow$ degenerate φ_1, φ_2
 Conserved charge relations bet. coupling

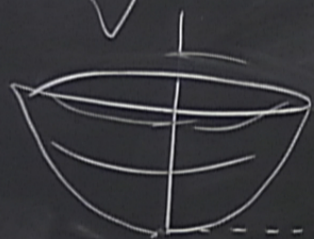
$$V_i = \langle 0 | \varphi_i | 0 \rangle$$

(at)





$\Rightarrow O(2)$



$$\mu^2 > 0$$

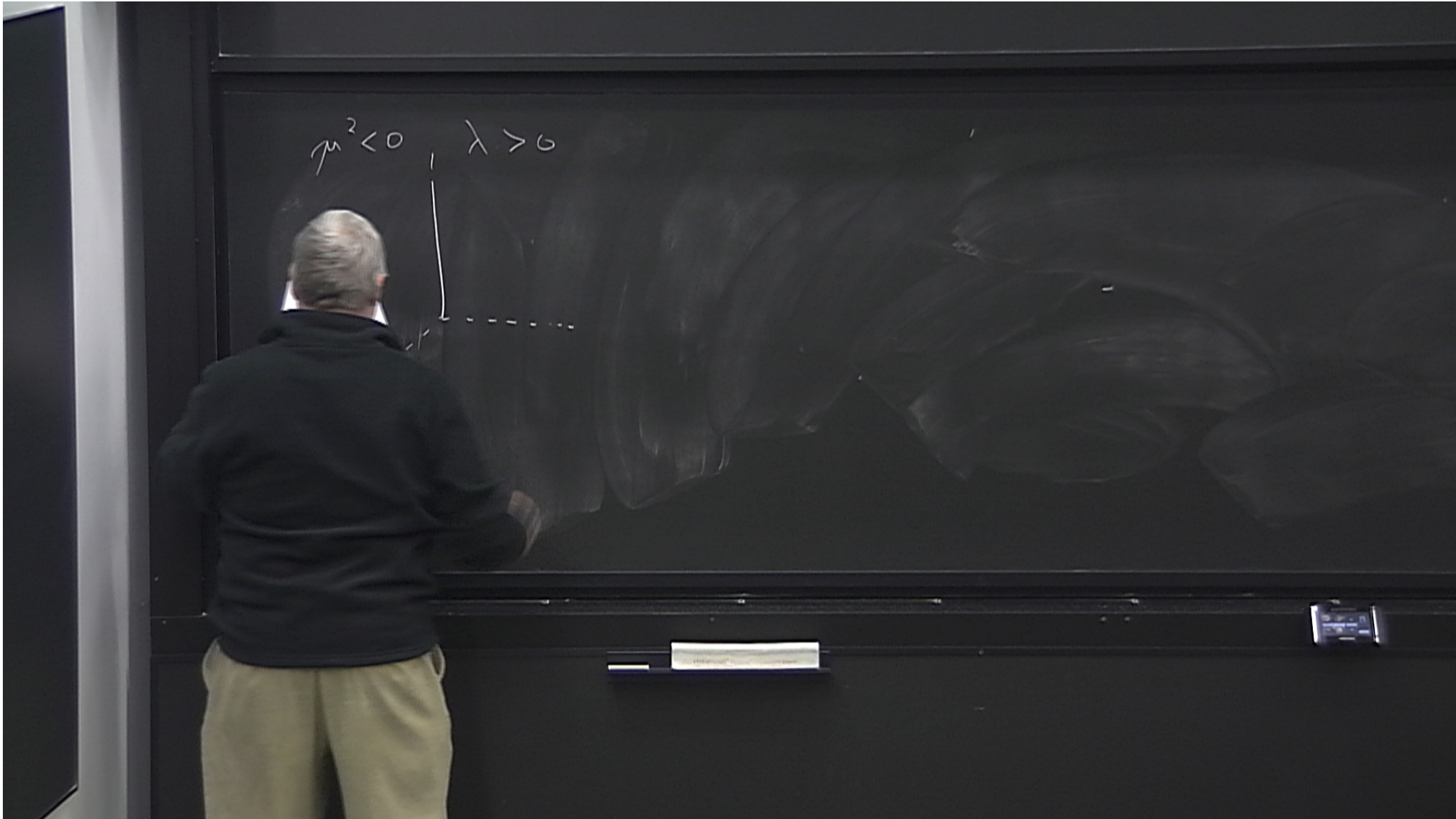
$$V_i = \langle 0 | \varphi_i | 0 \rangle$$

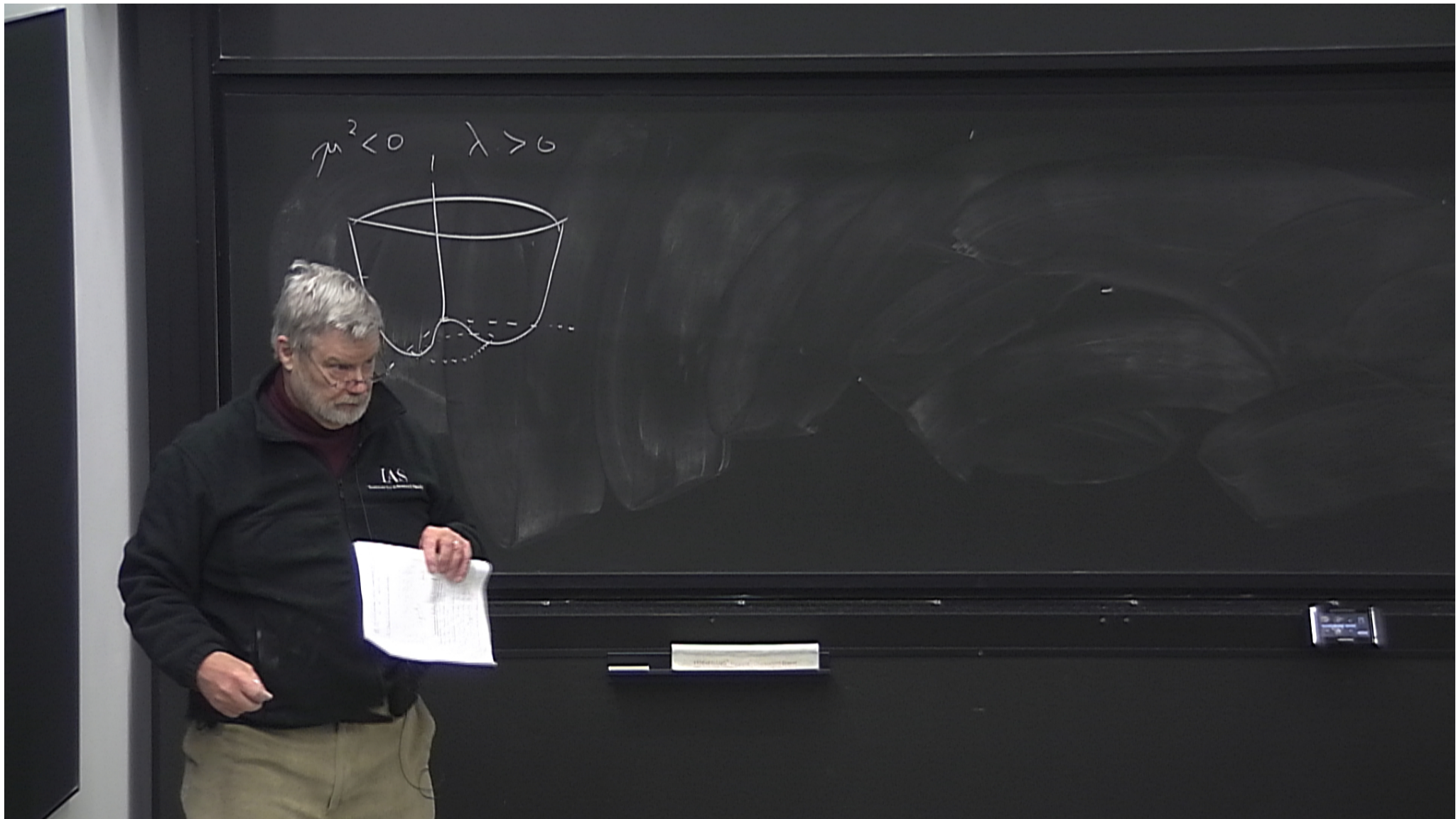
$V_1 = V_2 = 0 \Rightarrow$ w.w. $\Rightarrow$ degenerate φ_1, φ_2
conserved charge relations bet. couplings

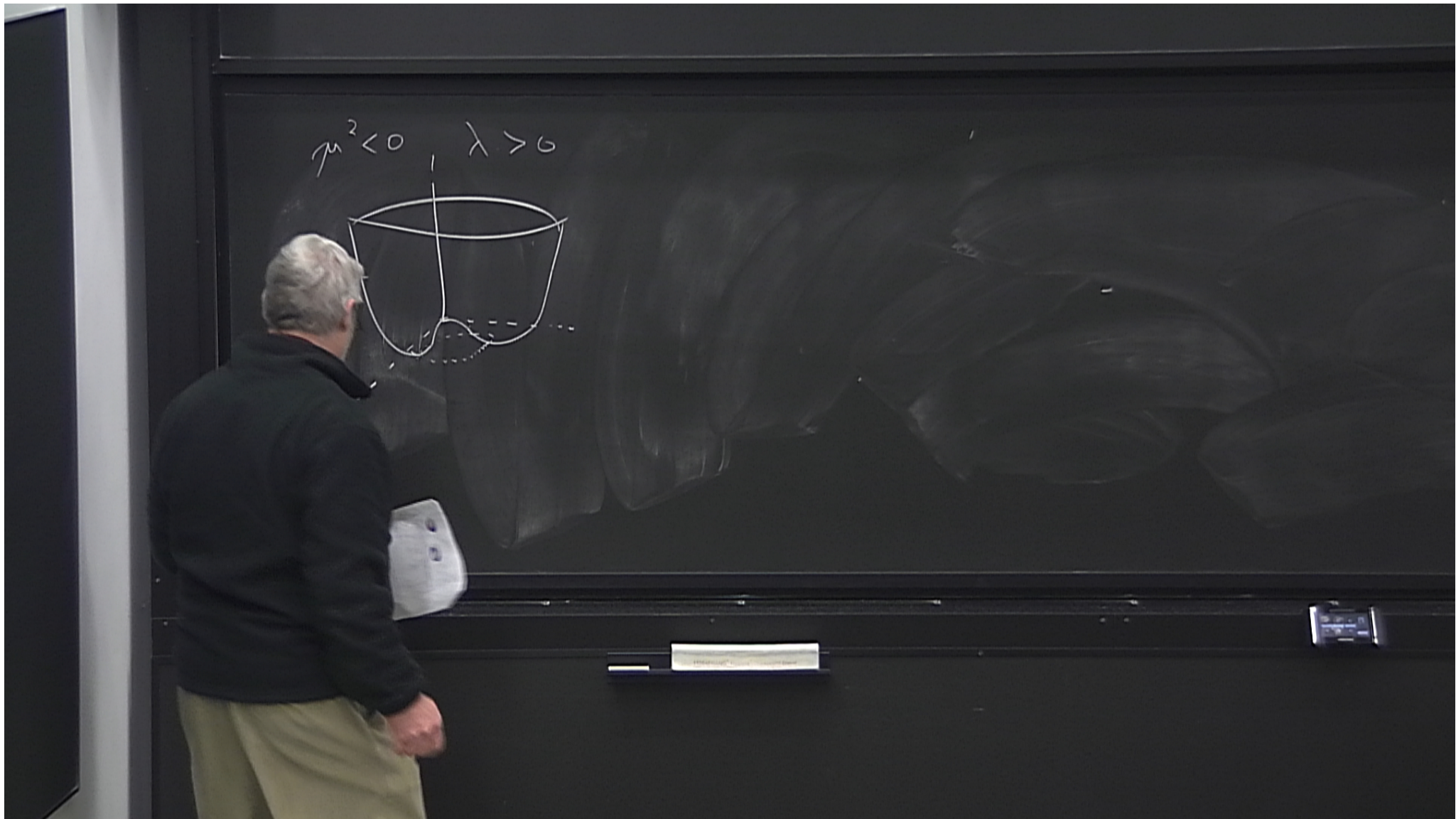
explicit breaking

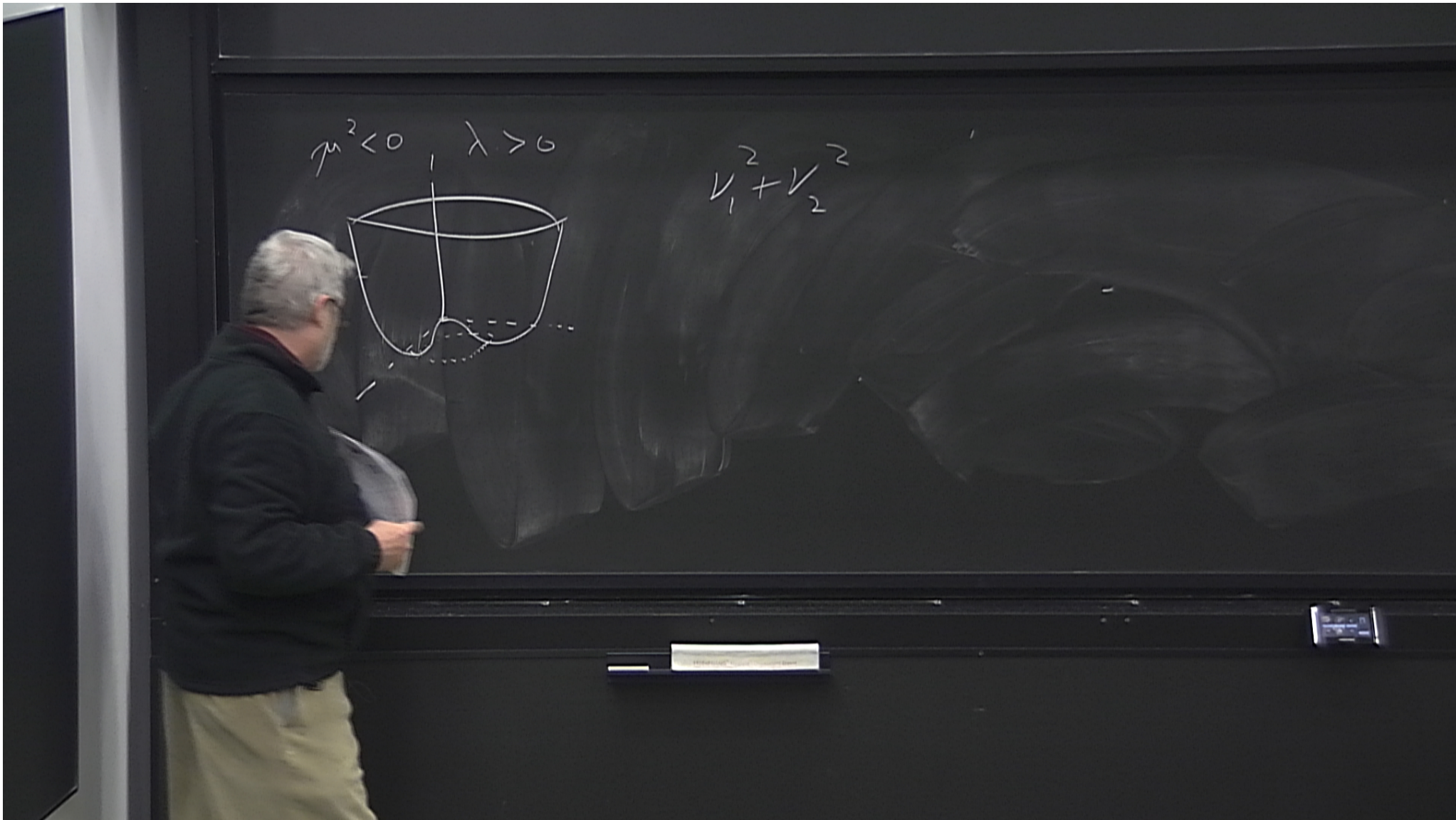
$$\mathcal{L} \rightarrow \mathcal{L} - \frac{\varepsilon}{2} \varphi_2^2$$

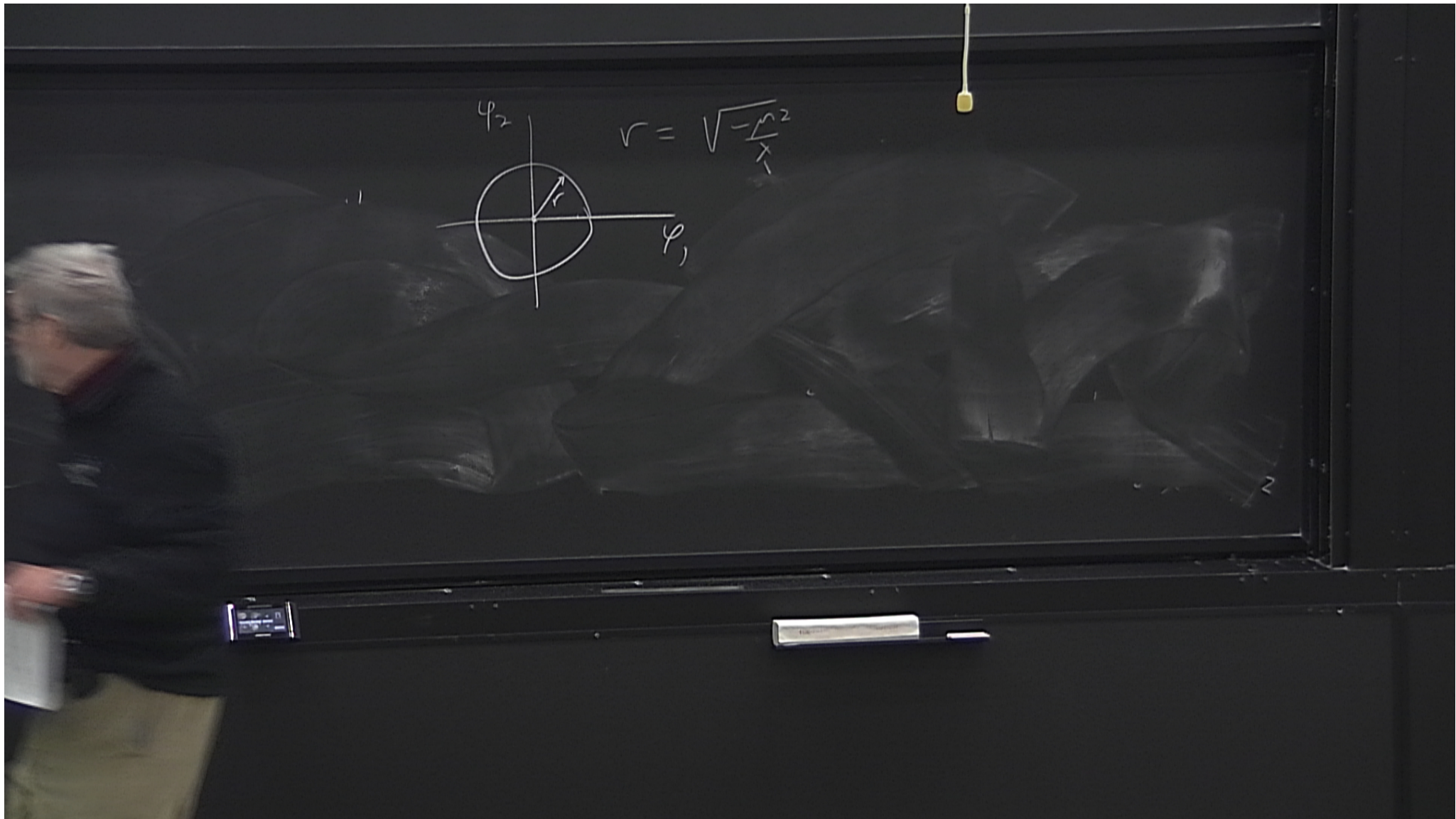
$$\Rightarrow \begin{aligned} m_1^2 &= \mu^2 \\ m_2^2 &= \mu^2 + \varepsilon \end{aligned}$$



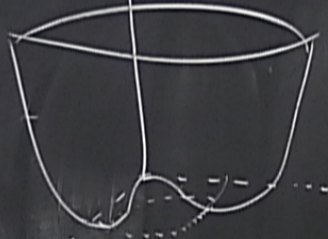




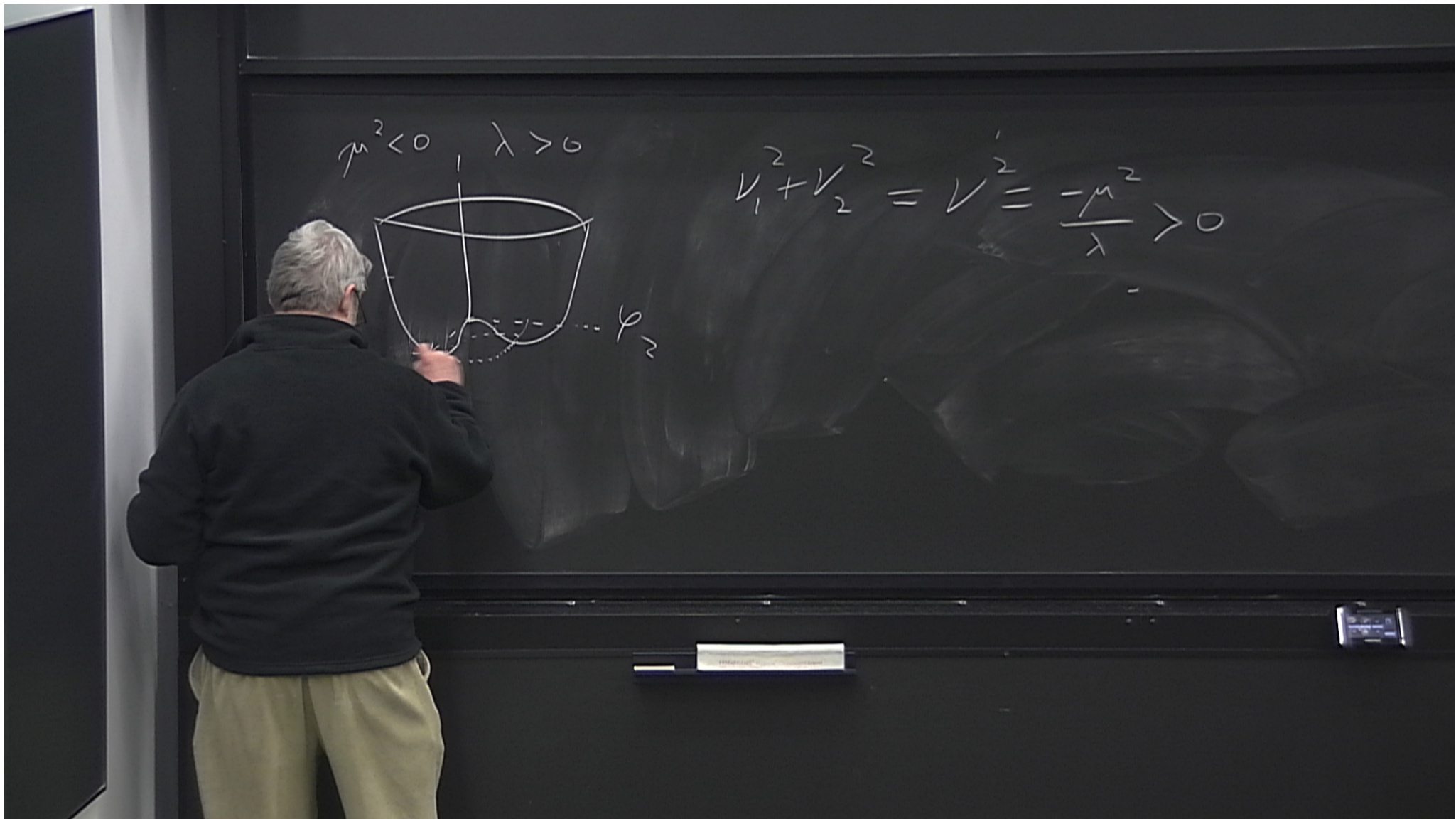


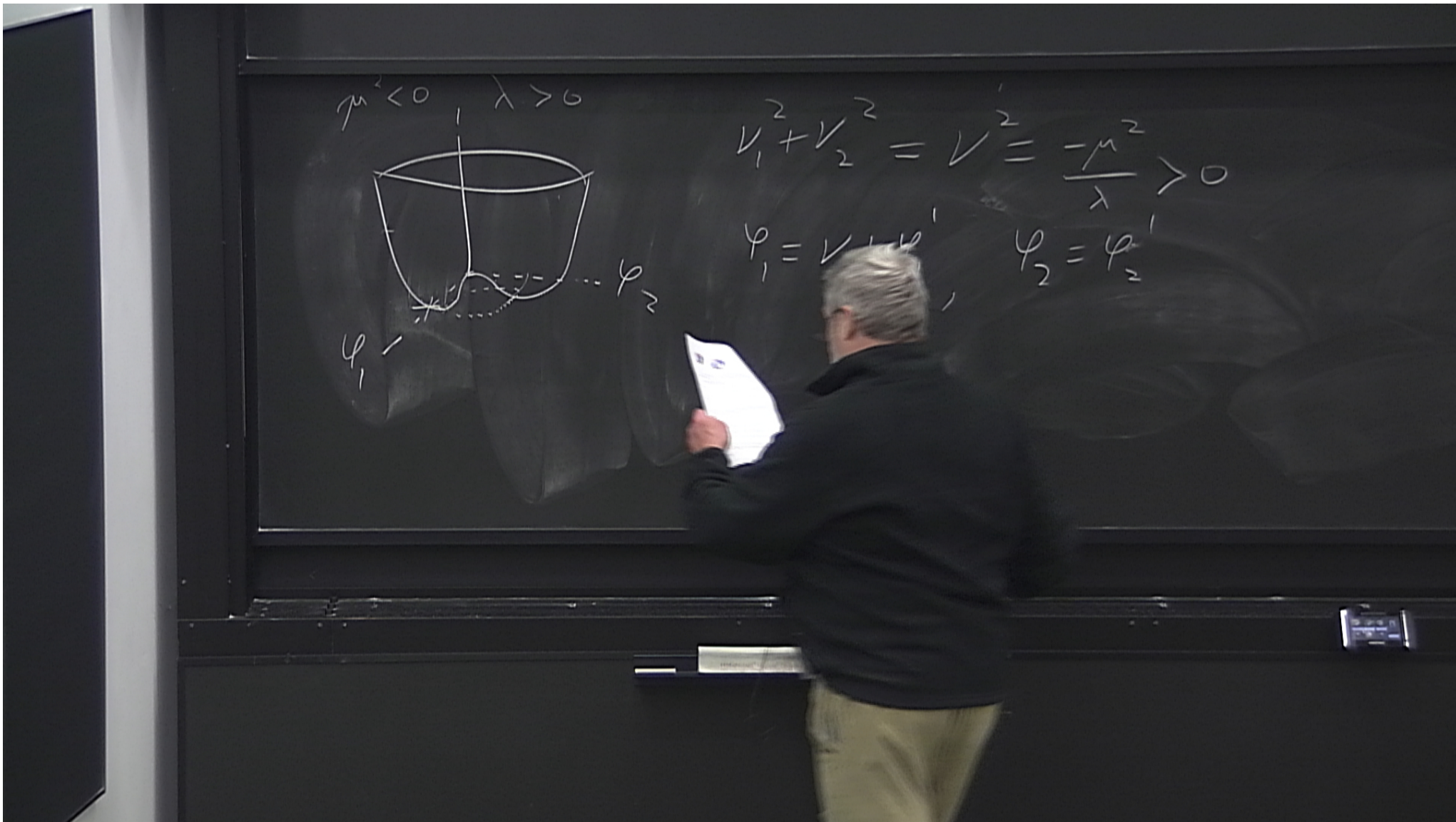


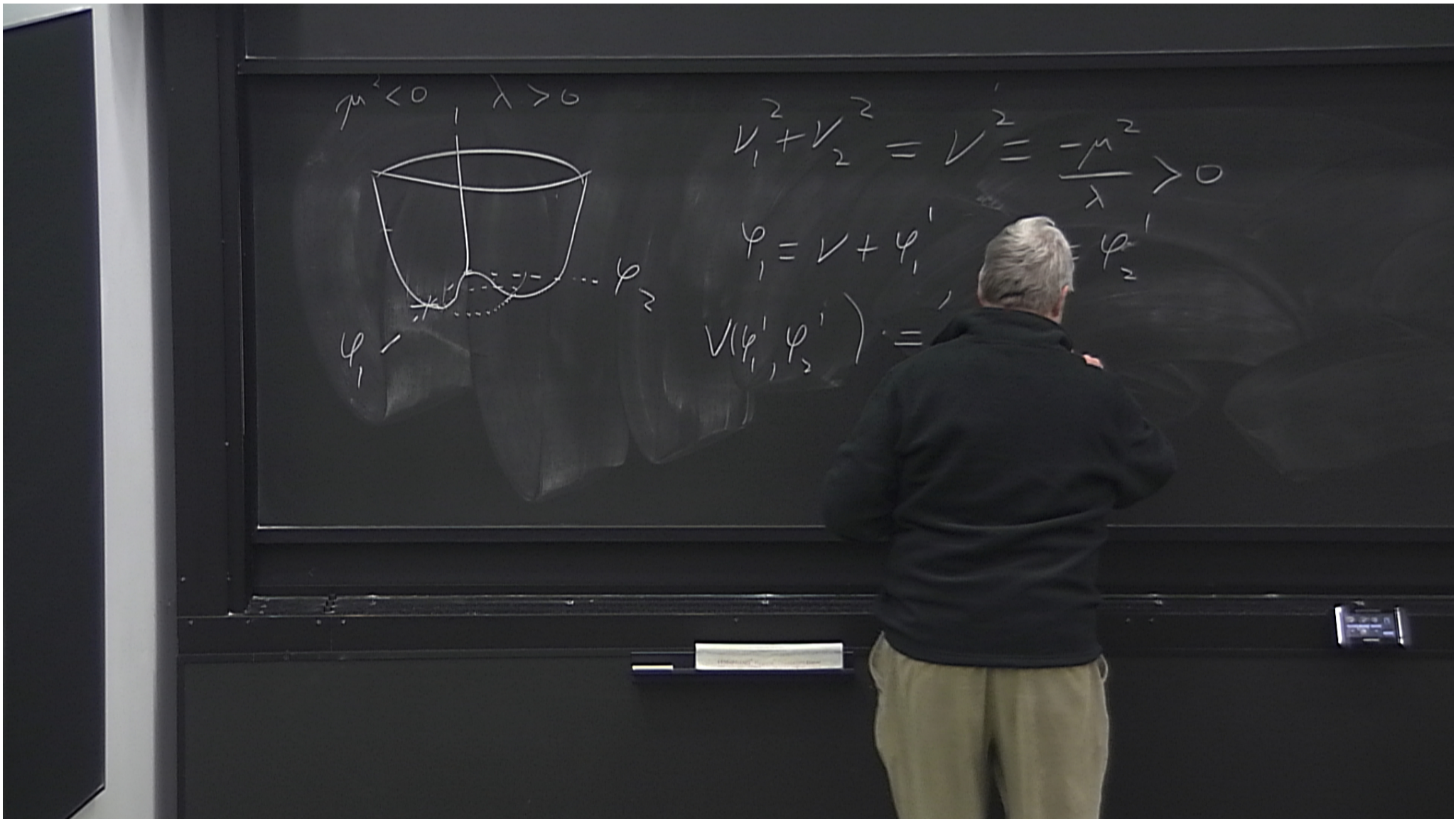
$$m^2 < 0, \lambda > 0$$



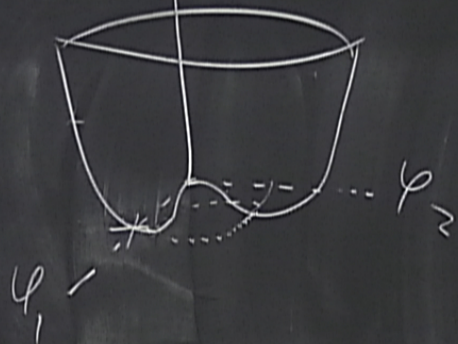
$$V_1^2 + V_2^2 = V^2 = \frac{-m^2}{\lambda} > 0$$







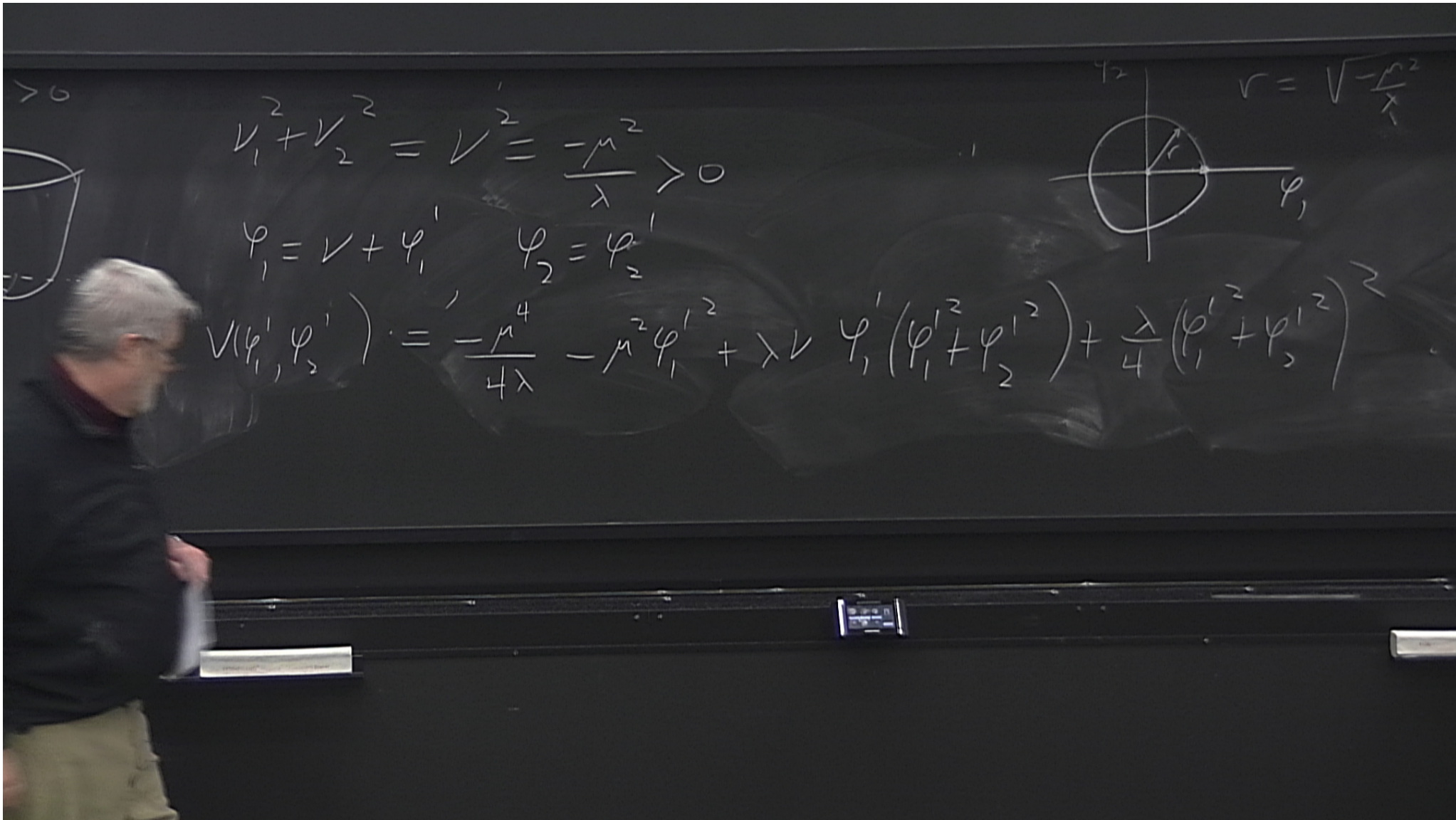
$$\mu^2 < 0, \lambda > 0$$



$$V_1^2 + V_2^2 = V^2 = \frac{-\mu^2}{\lambda} > 0$$

$$\varphi_1 = V + \varphi_1', \quad \varphi_2 = \varphi_2'$$

$$V(\varphi_1', \varphi_2') = \frac{-\mu^4}{4\lambda} - \mu^2 \varphi_1'^2$$



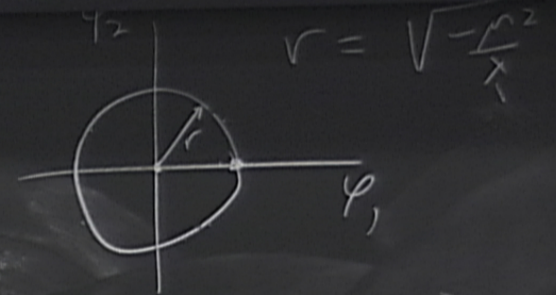
$$V_1^2 + V_2^2 = V^2 = -\frac{\mu^2}{\lambda} > 0$$

$$\varphi_1 = V + \varphi_1', \quad \varphi_2 = \varphi_2'$$

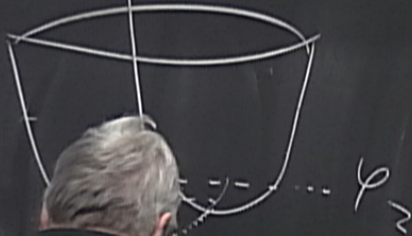
$$V(\varphi_1', \varphi_2') = -\frac{\mu^4}{4\lambda} - \mu^2 \varphi_1'^2 + \lambda V \varphi_1' (\varphi_1'^2 + \varphi_2'^2) + \frac{\lambda}{4} (\varphi_1'^2 + \varphi_2'^2)^2$$

$$\mu_1^2 = -\mu^2 > 0$$

$$V_{10}$$



$$\mu^2 < 0, \lambda > 0$$



$$V_1^2 + V_2^2 = V^2 = \frac{-\mu^2}{\lambda} > 0$$

$$\varphi_1 = V + \varphi_1', \quad \varphi_2 = \varphi_2'$$

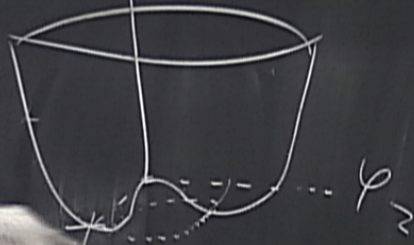
$$V(\varphi_1', \varphi_2') = \frac{-\mu^4}{4\lambda} - \mu^2 \varphi_1'^2 + \lambda V \varphi_1' (\varphi_1'^2 + \varphi_2'^2)$$

$$m_2^2 = 0$$

$$m_1^2 = -\mu^2 > 0$$

$$V_{10}$$

$$\mu < 0 \quad \lambda > 0$$



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$$V_1^2 + V_2^2 = V^2 = \frac{-\mu^2}{\lambda} > 0$$

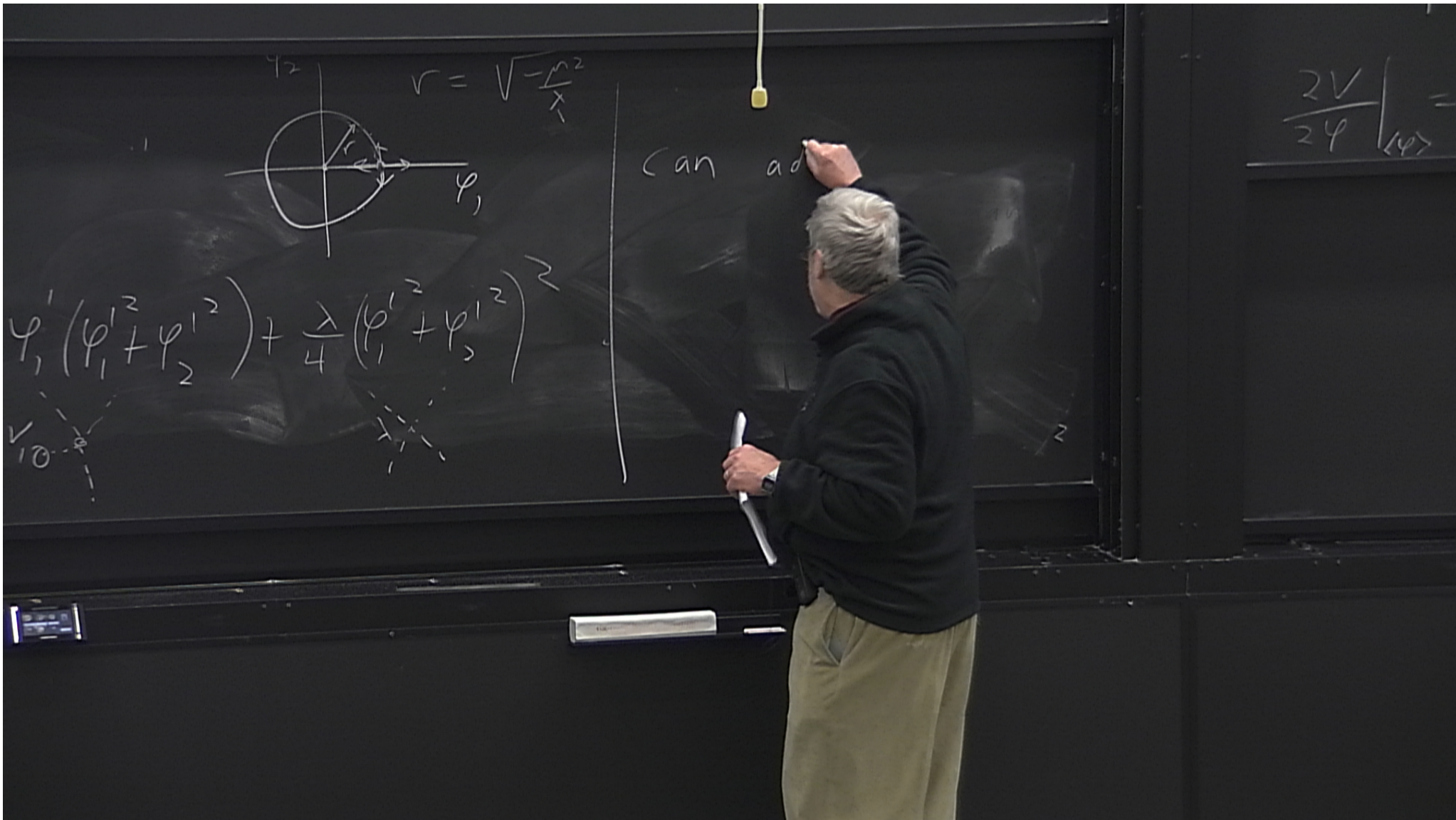
$$\varphi_1 = V + \varphi_1', \quad \varphi_2 = \varphi_2'$$

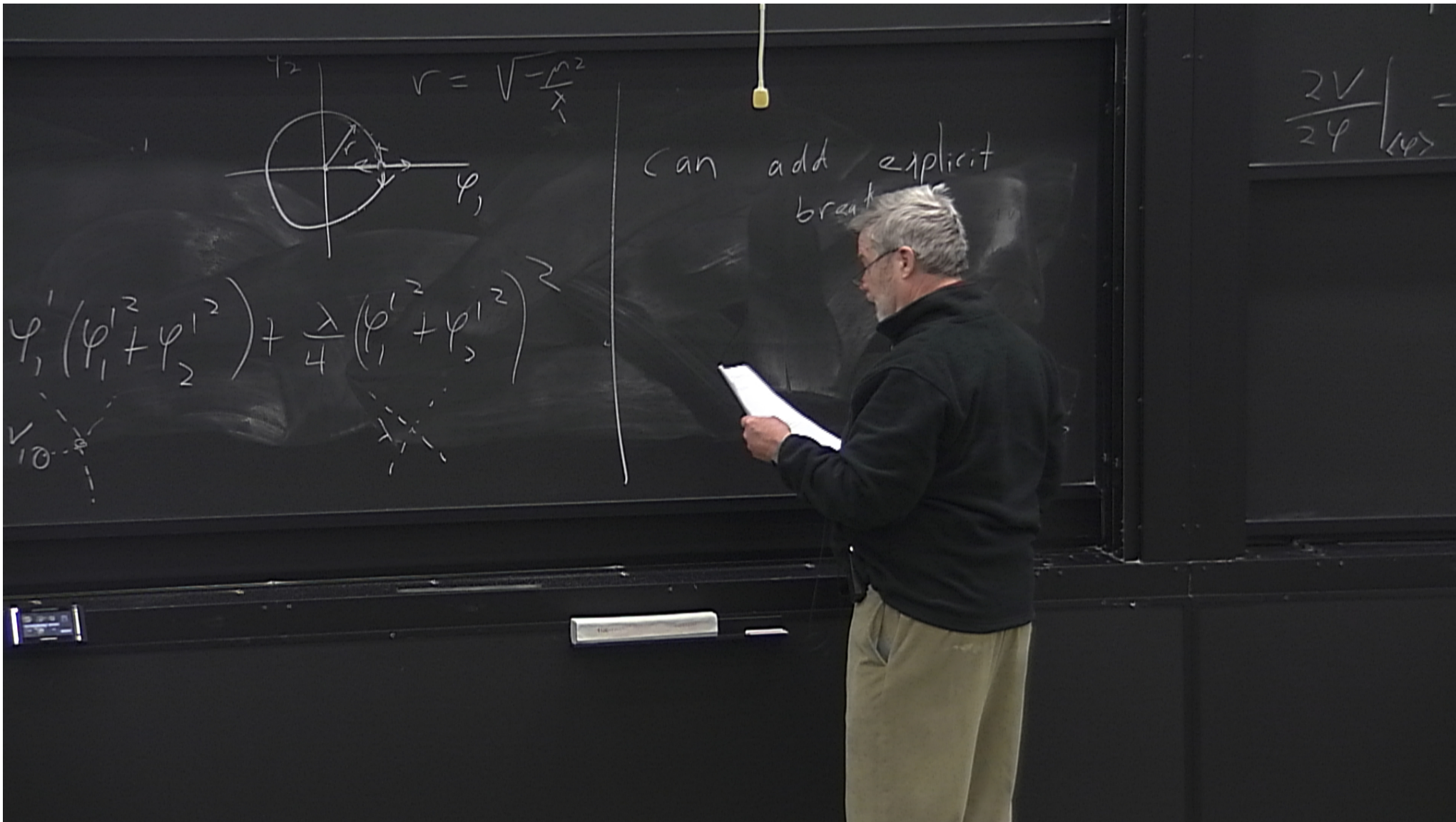
$$V(\varphi_1', \varphi_2') = \frac{-\mu^4}{4\lambda} - \mu^2 \varphi_1'^2 + \lambda V \varphi_1' (\varphi_1'^2 + \varphi_2'^2)$$

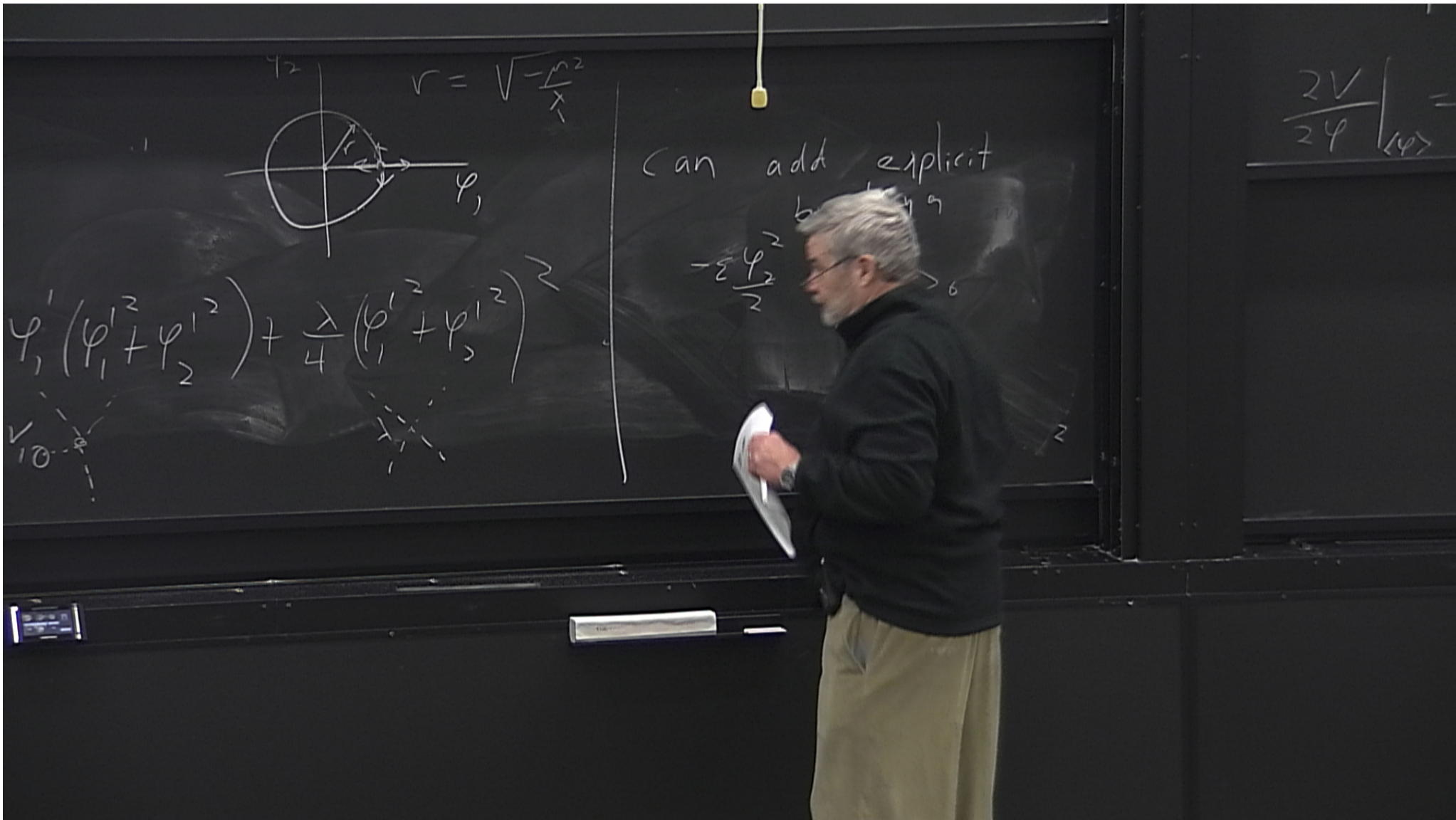
$$m_2^2 = 0$$

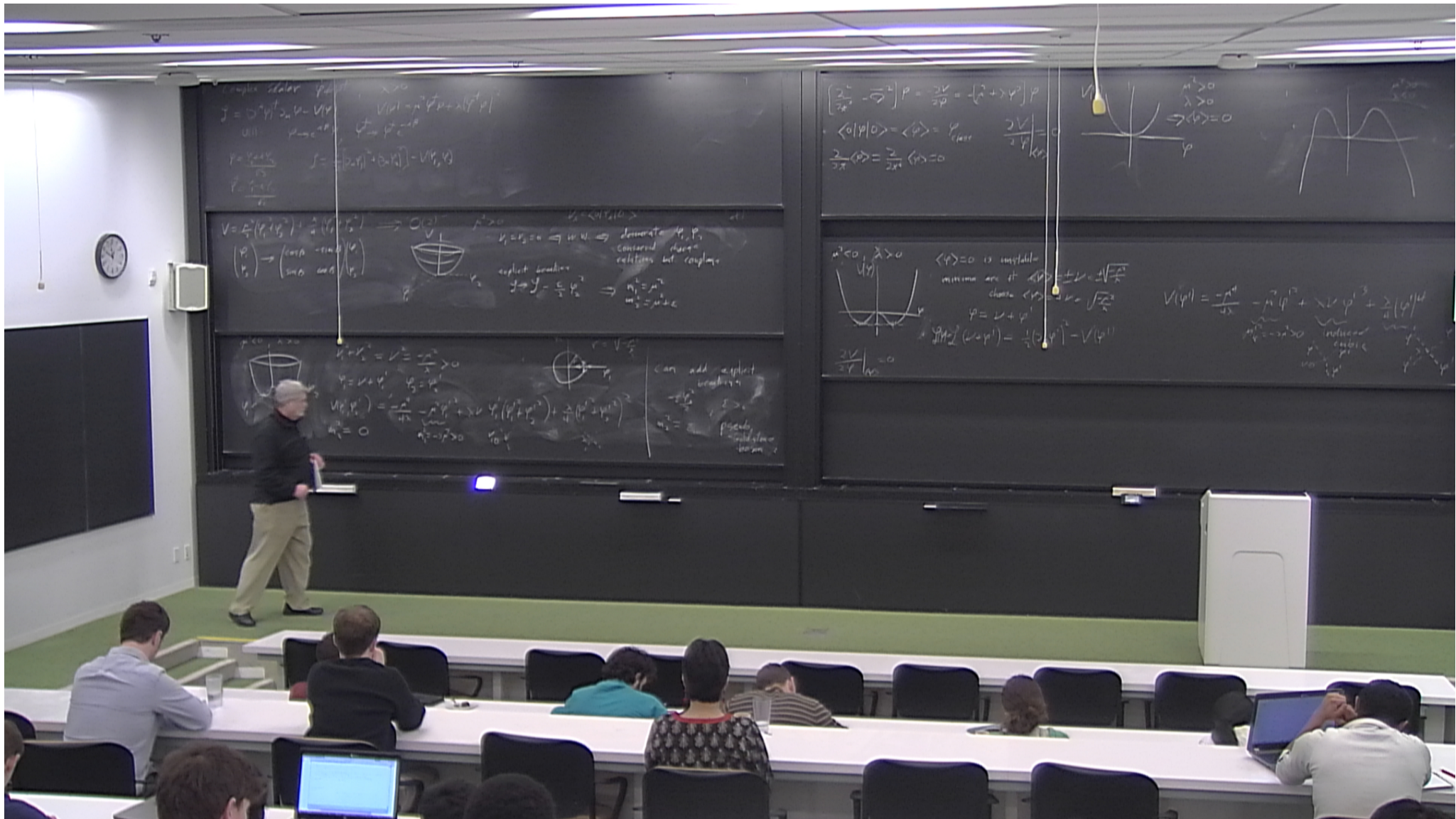
$$m_1^2 = -\mu^2 > 0$$

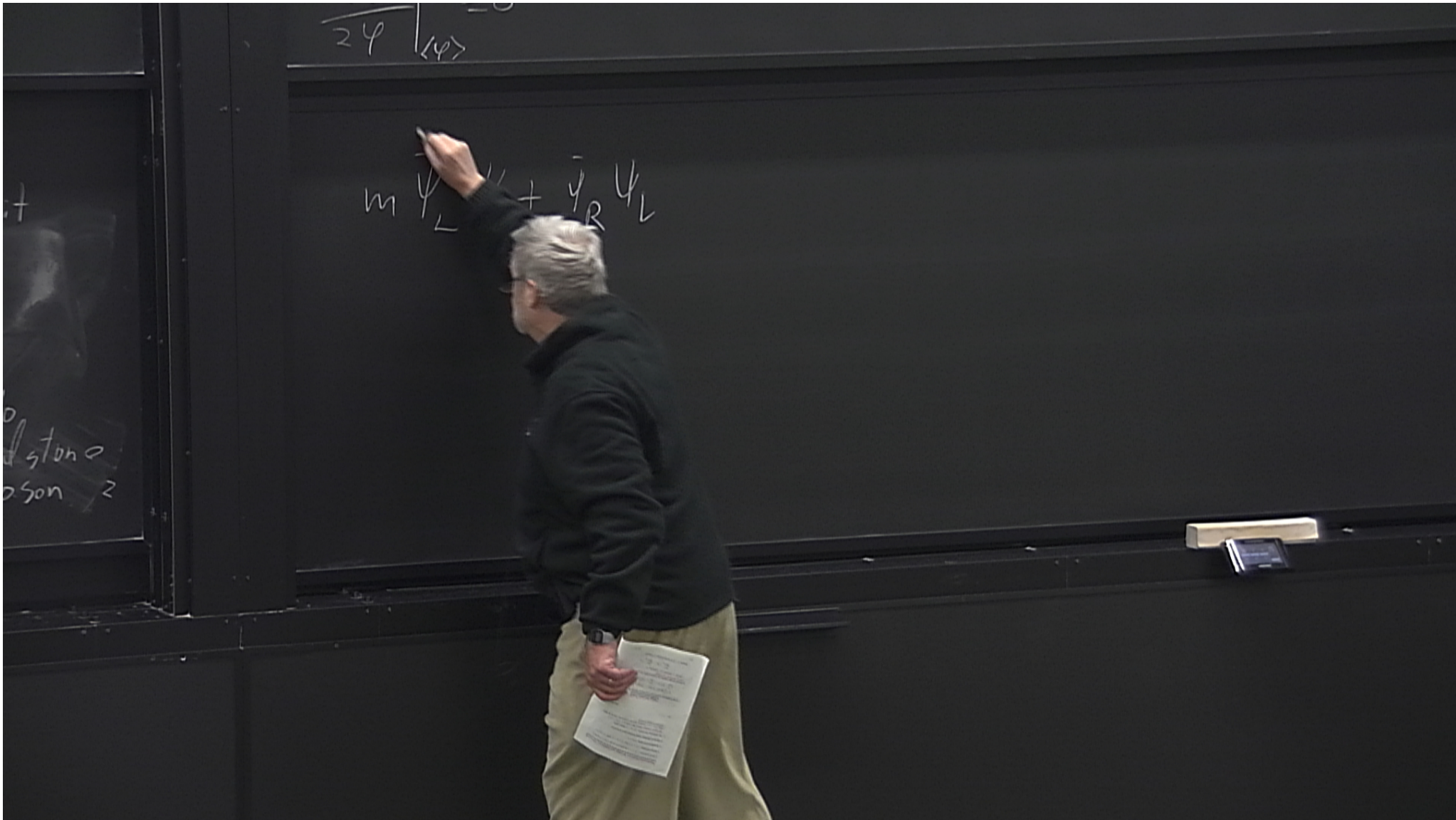
$$V_{10}$$

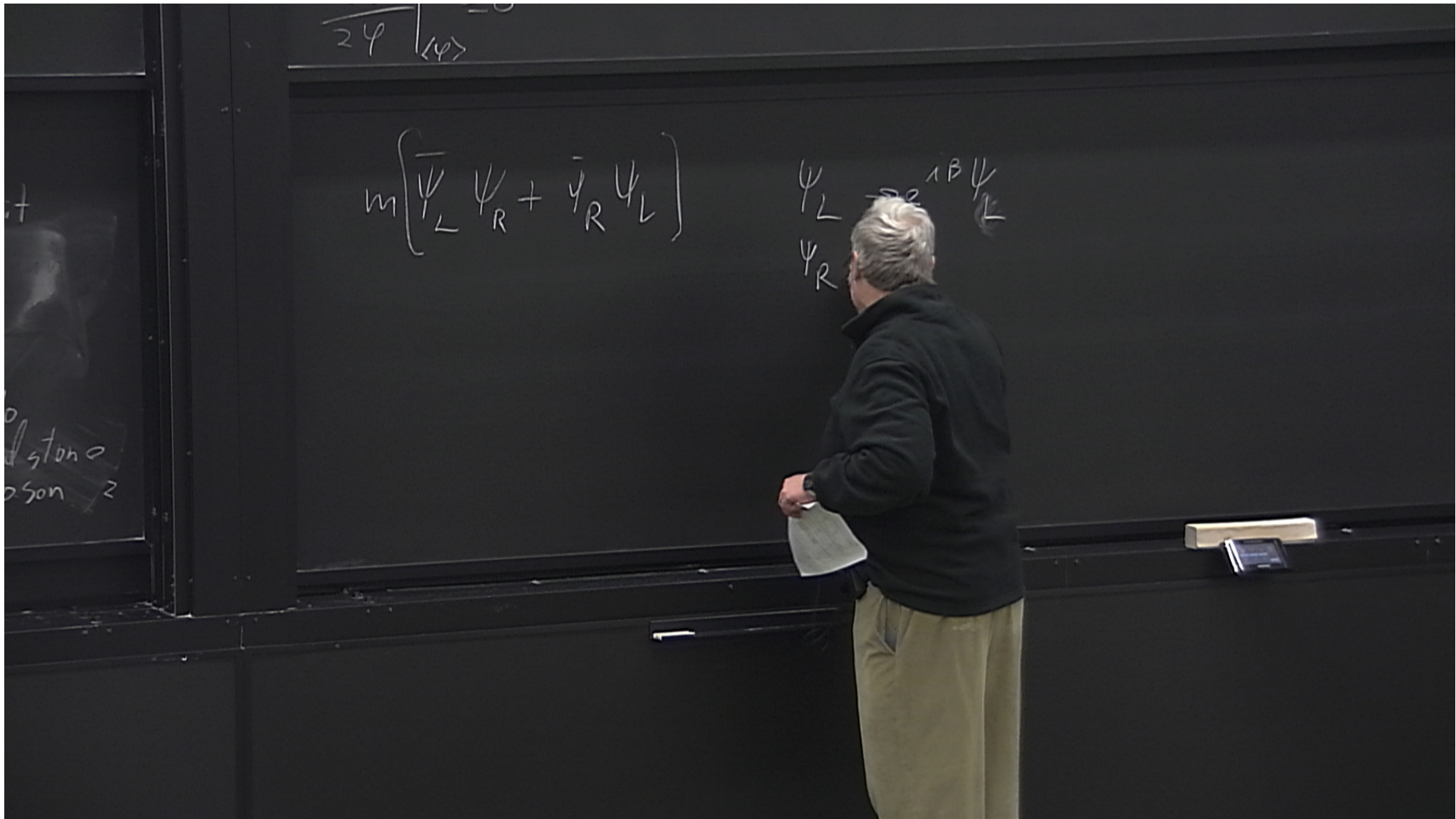


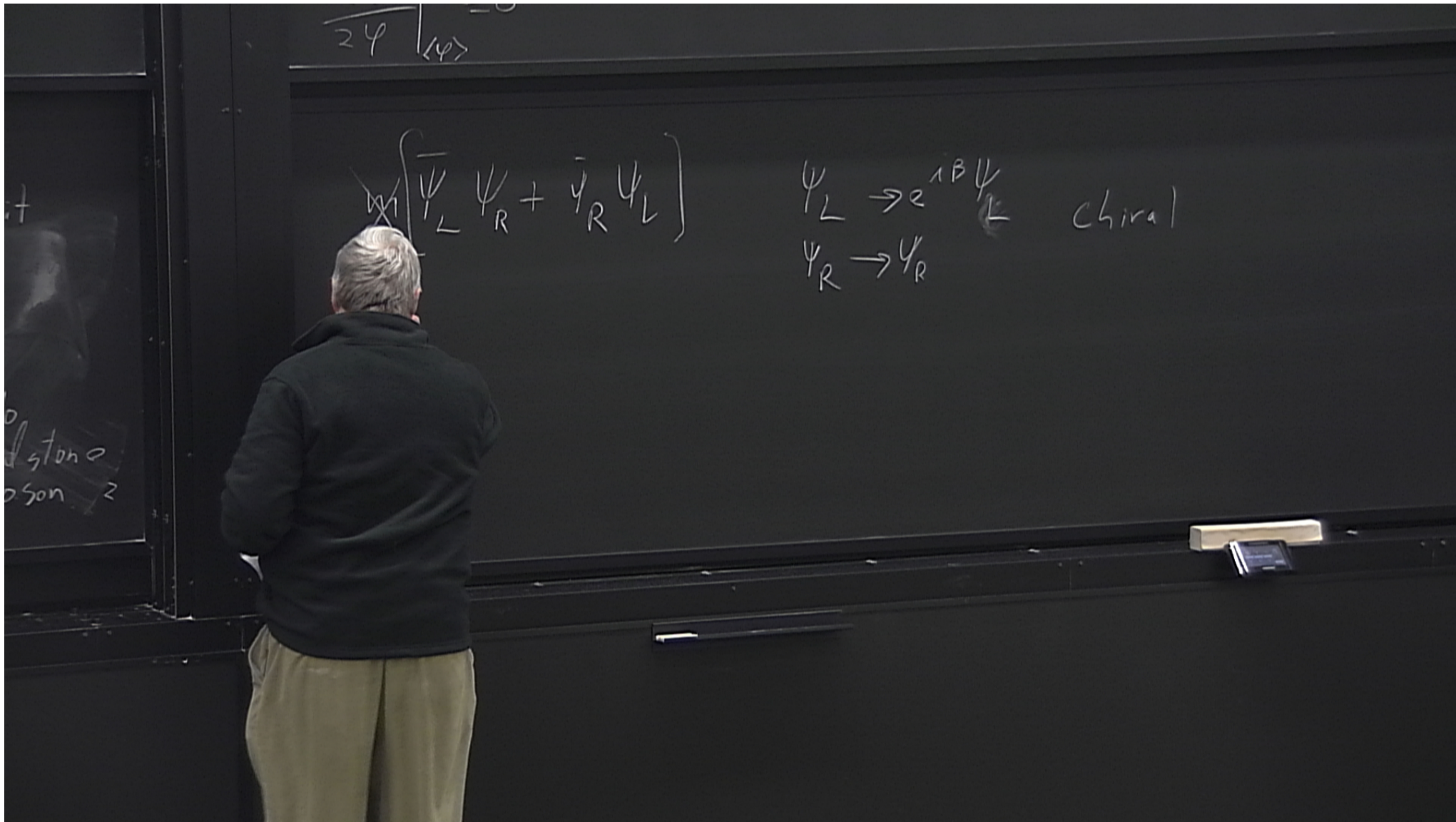










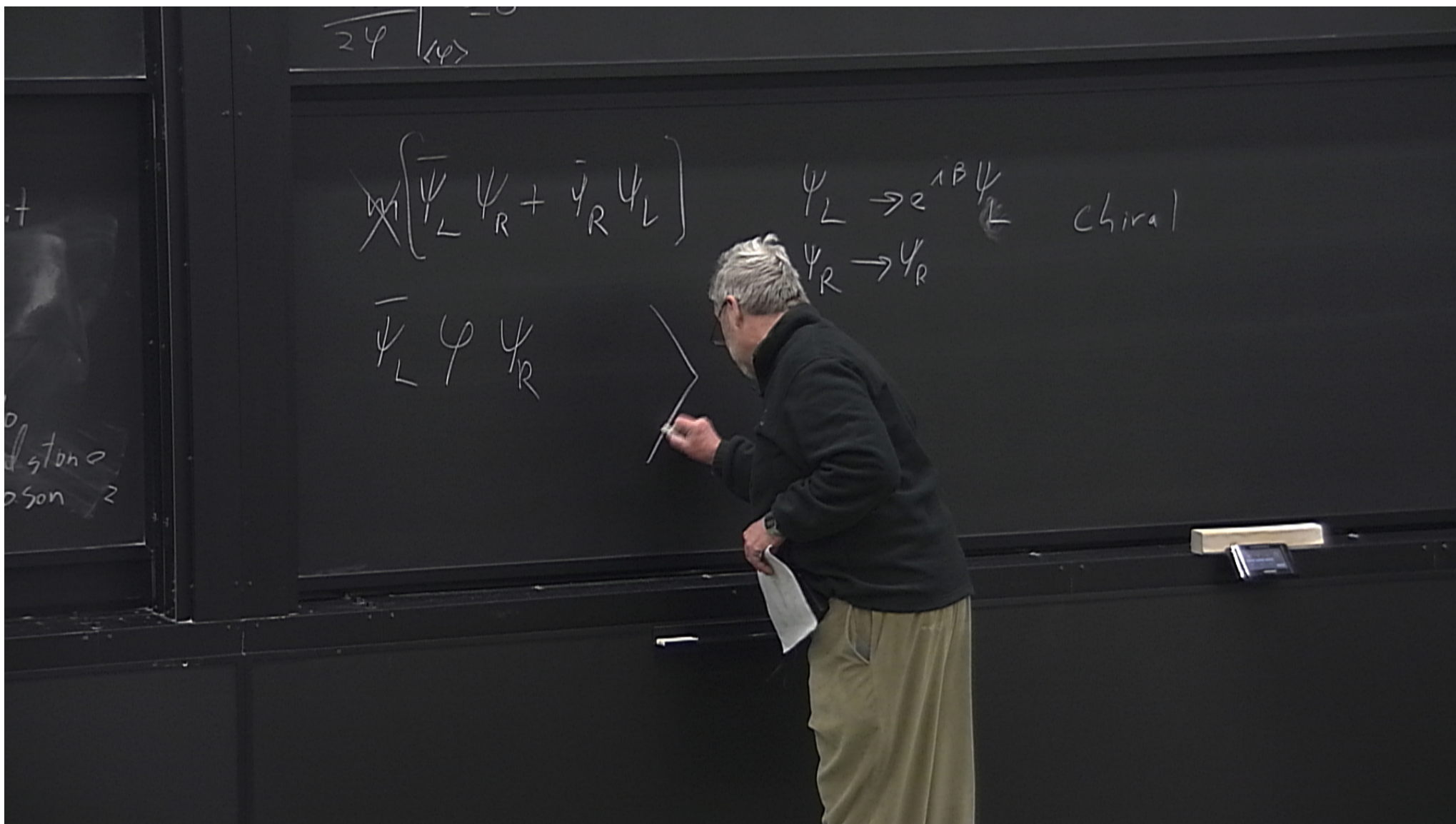


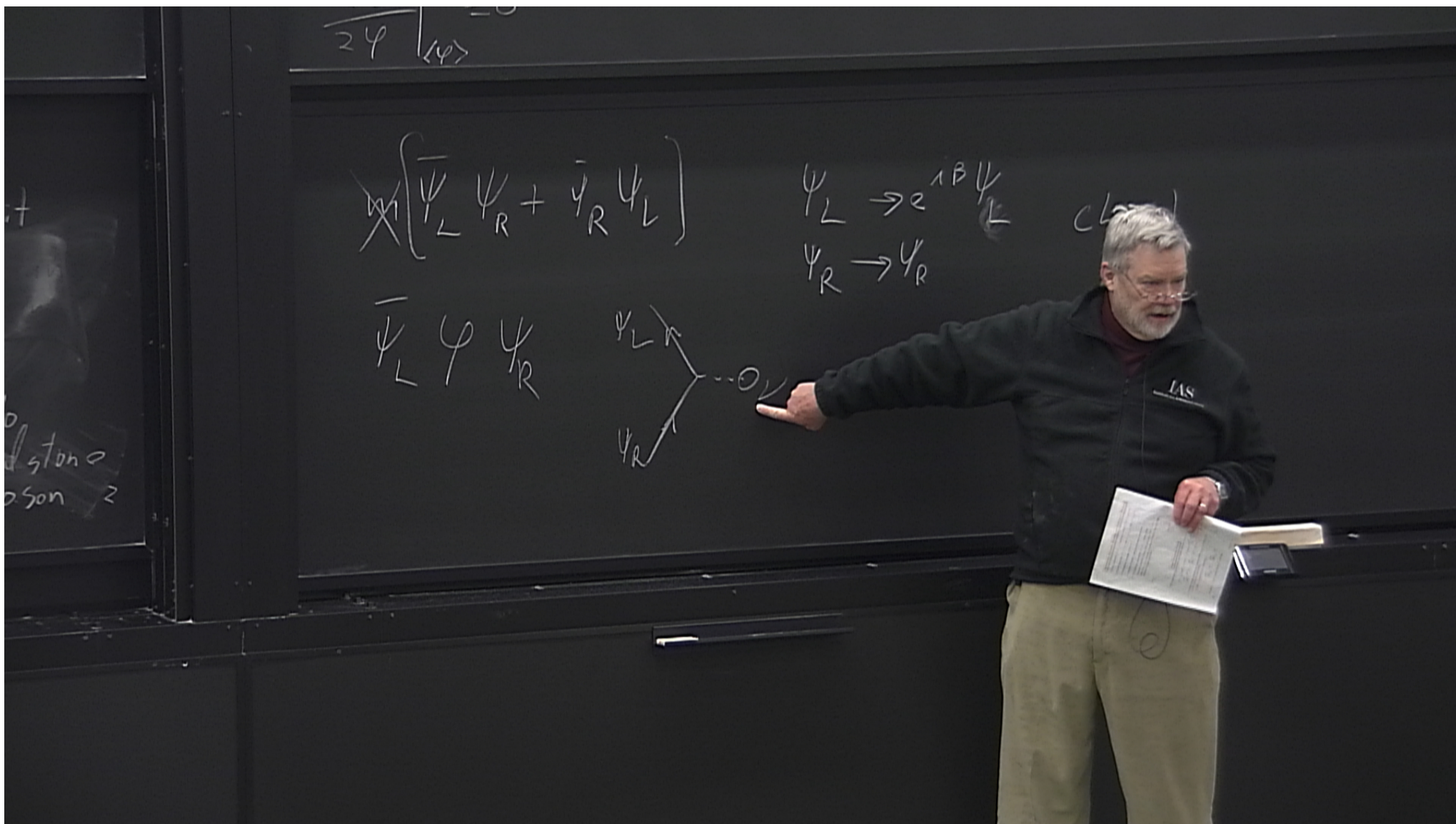
$$\overline{\psi} \psi$$

$$\cancel{\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L}$$

$$\begin{aligned} \psi_L &\rightarrow e^{iB} \psi_L \\ \psi_R &\rightarrow \psi_R \end{aligned} \quad \text{Chiral}$$

$$\overline{\psi}_L \psi \psi_R$$



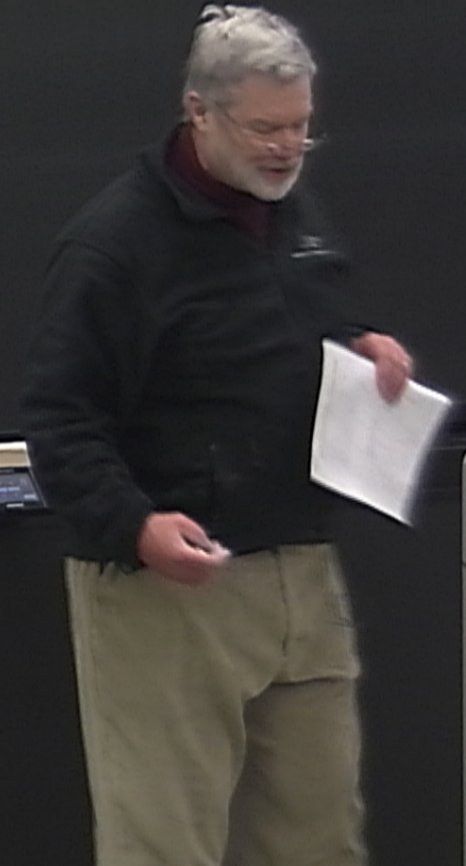
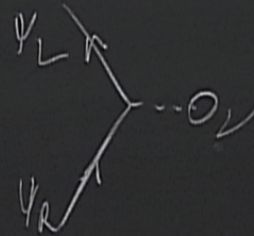


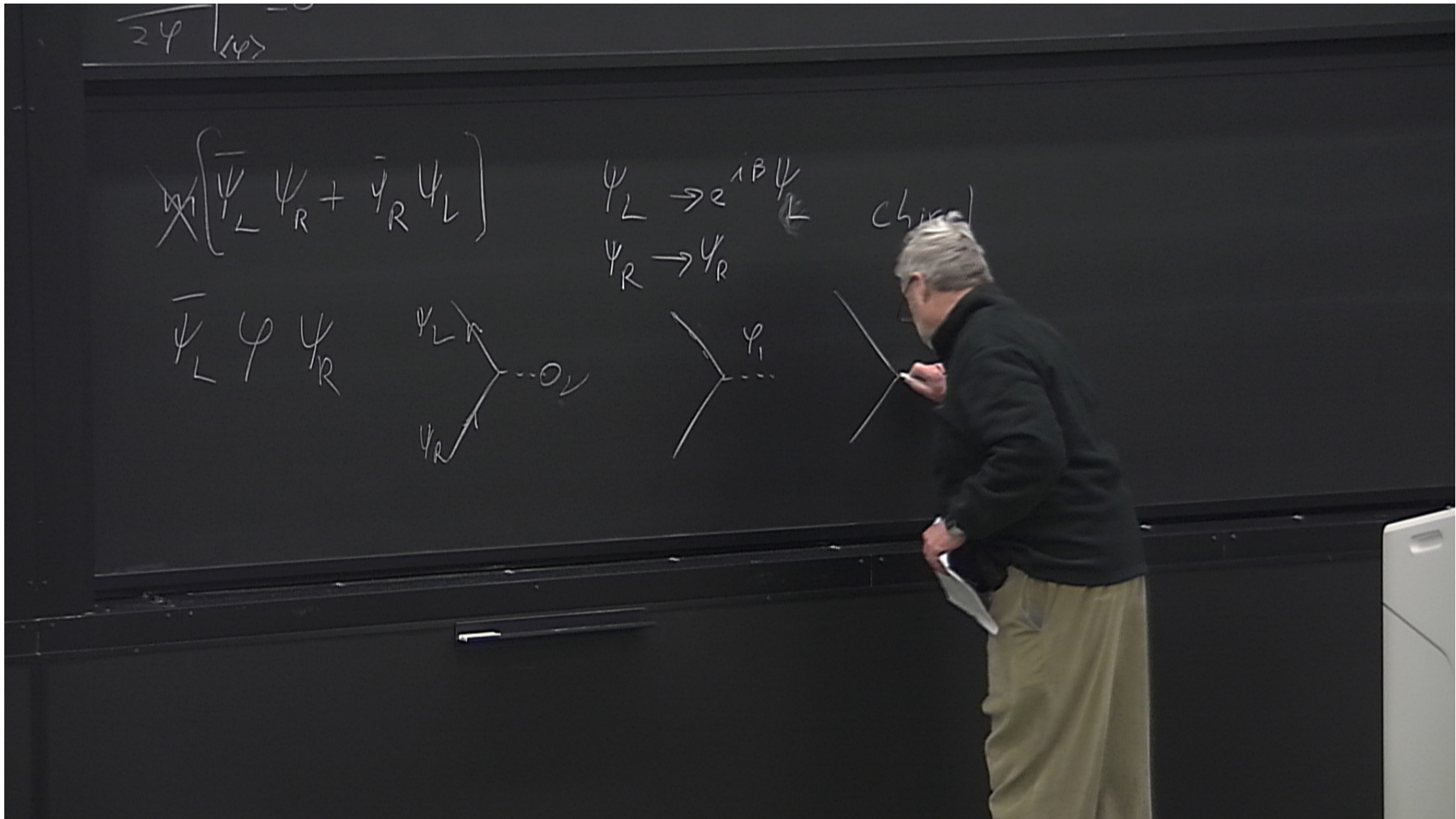
$$\overline{2\psi} \quad | \psi \rangle$$

$$\cancel{\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L}$$

$$\begin{aligned} \psi_L &\rightarrow e^{iB} \psi_L \\ \psi_R &\rightarrow \psi_R \end{aligned} \quad \text{Chiral}$$

$$\overline{\psi}_L \psi \psi_R$$





$$\cancel{m_1 \left[\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right]}$$

$$\begin{aligned} \psi_L &\rightarrow e^{iB} \psi_L \\ \psi_R &\rightarrow \psi_R \end{aligned} \quad \text{Chiral}$$

$$\bar{\psi}_L \psi \psi_R$$

