

Title: 13/14 PSI - Standard Model Review - Lecture 4

Date: Jan 09, 2014 09:00 AM

URL: <http://pirsa.org/14010011>

Abstract:

# ④ Global symmetries of strong interactions

$p$  (938.3)

$n$  (939.6)

$$B=1$$

$\pi^\pm$  (139.6)

$\pi^0$  (135.0)

$$B=0$$

$K^\pm$  (493.7)

$K^0$  (497.6)

$$B=0$$



Isospin: approximate global  $SU(2)$ , broken at

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$$\pi^\pm = \frac{\pi_1 \mp i \pi_2}{\sqrt{2}}$$

- broken by  $\sqrt{2}$  electromagnetism
- broken by  $(u, d)$  mass difference

- broken by  $U(1)^2$  electromagnetism
- broken by  $(u, d)$  mass difference

Yukawa model

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \quad \pi = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} \quad \pi_i = \pi_i^+$$



- broken by  $U(1)$  electromagnetism
- broken by  $(u, d)$  mass difference

Yukawa model

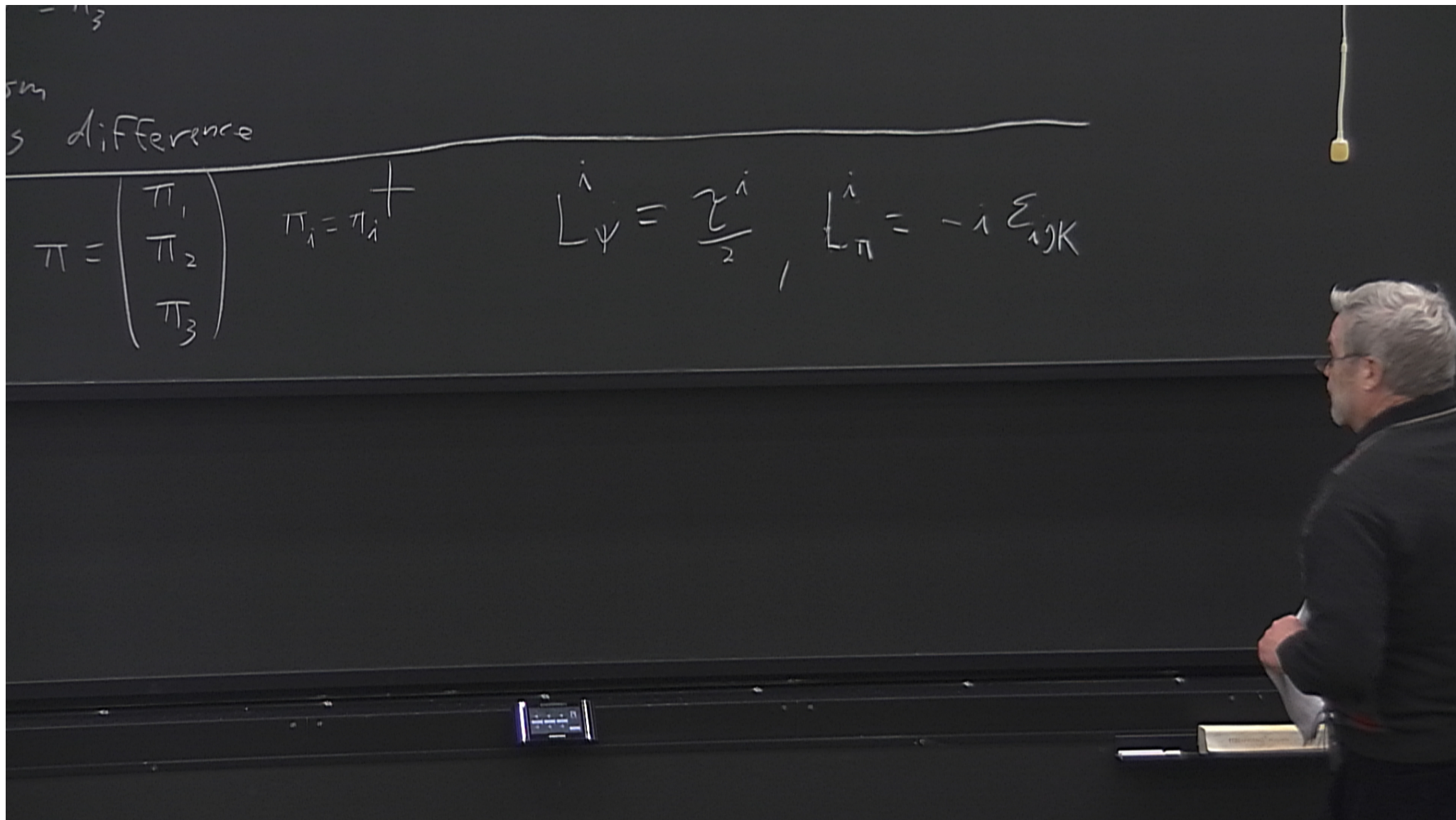
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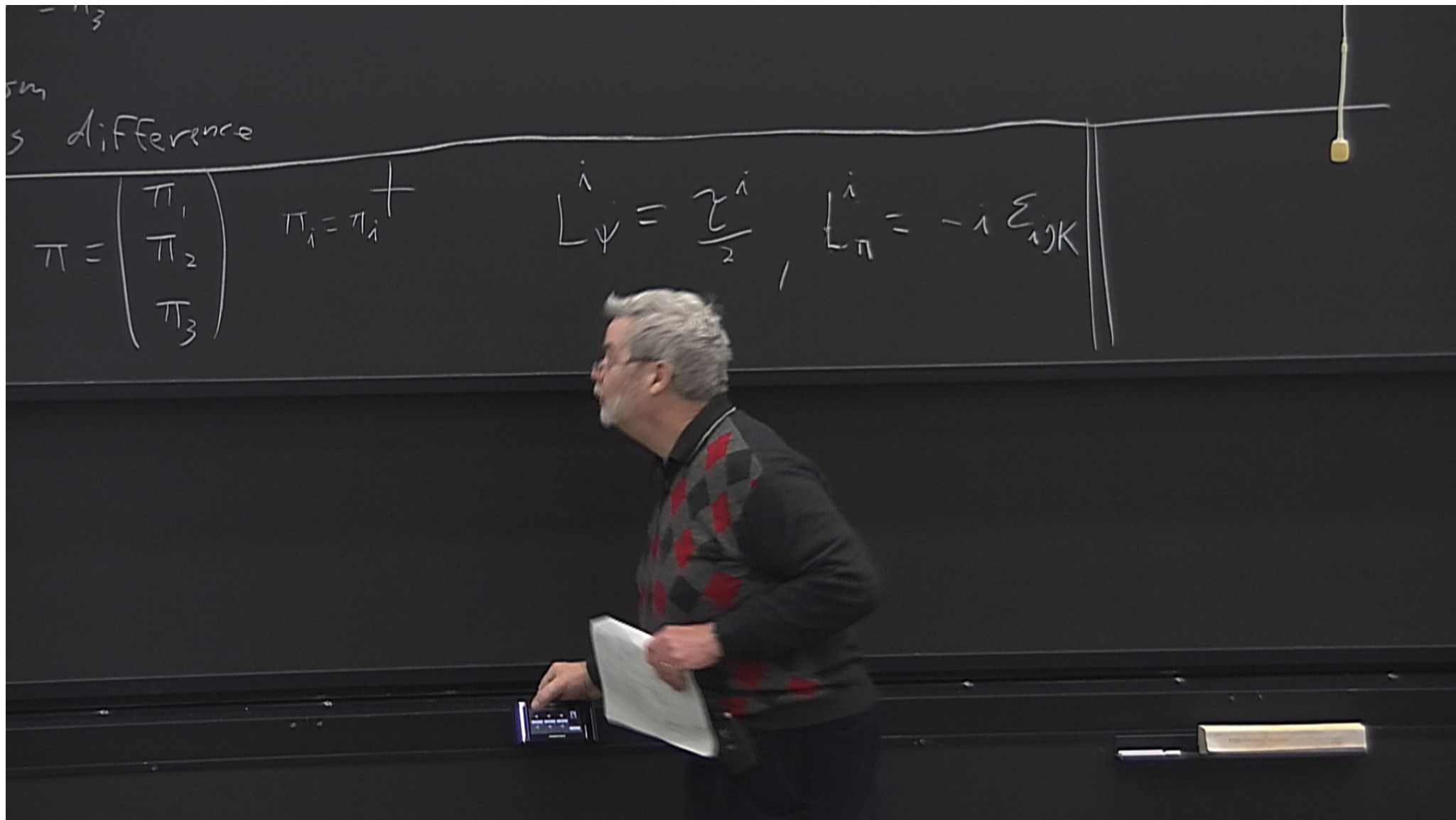
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Yukawa model

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \quad \pi = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} \quad \pi_i = \pi_i^\dagger$$









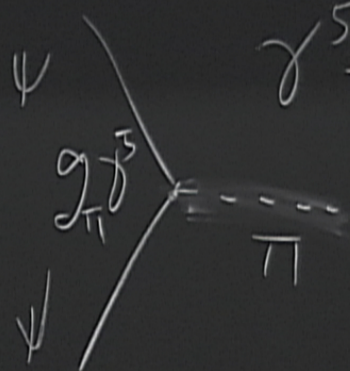
$$\mathcal{L}_0 = \bar{\Psi} \left[ i \not{\partial} - m I + i g_{\pi} \gamma^5 \vec{\pi} \cdot \vec{\tau} \right] \Psi + \frac{1}{2} \left( \partial_{\mu} \vec{\pi} \right)^2$$

(14)  $(\pi_3)$

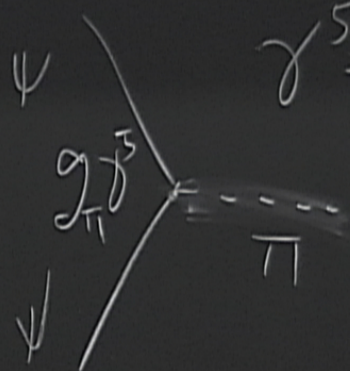
$$\not{p} - m I + i g_\pi \not{\vec{\pi}} \cdot \vec{\sigma} \psi + \frac{1}{2} \left[ \left( \partial_\mu \vec{\pi} \right)^2 - \mu^2 \vec{\pi}^2 \right] - \lambda \left( \vec{\pi}^2 \right)^2$$



$$\left[ -\mu^2 \frac{\pi^2}{\pi^2} \right] - \lambda \left( \frac{\pi^2}{\pi^2} \right)^2$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$


$$\left[ -\mu^2 \frac{\vec{\pi}^2}{\pi^2} \right] - \lambda \left( \frac{\vec{\pi}^2}{\pi^2} \right)^2$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$




$$\mathcal{L}_0 = \bar{\psi} \left[ i \not{\partial} - m I + i g_{\pi} \gamma^5 \vec{\pi} \cdot \vec{\tau} \right] \psi + \frac{1}{2} \left[ (\partial_{\mu} \vec{\pi})^2 - m^2 \vec{\pi}^2 \right]$$

$$\frac{\vec{\pi} \cdot \vec{\tau}}{\sqrt{2}} = \sum_{i=1}^3 \frac{\pi^i \tau^i}{\sqrt{2}} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}$$

$$\begin{bmatrix} T^{\wedge} & \varphi \\ 1 & \phi \end{bmatrix} = 0$$



$$[T^a, \psi] = 0$$

$$\psi \rightarrow e^{+i\vec{B} \cdot \vec{\tau}} \psi$$

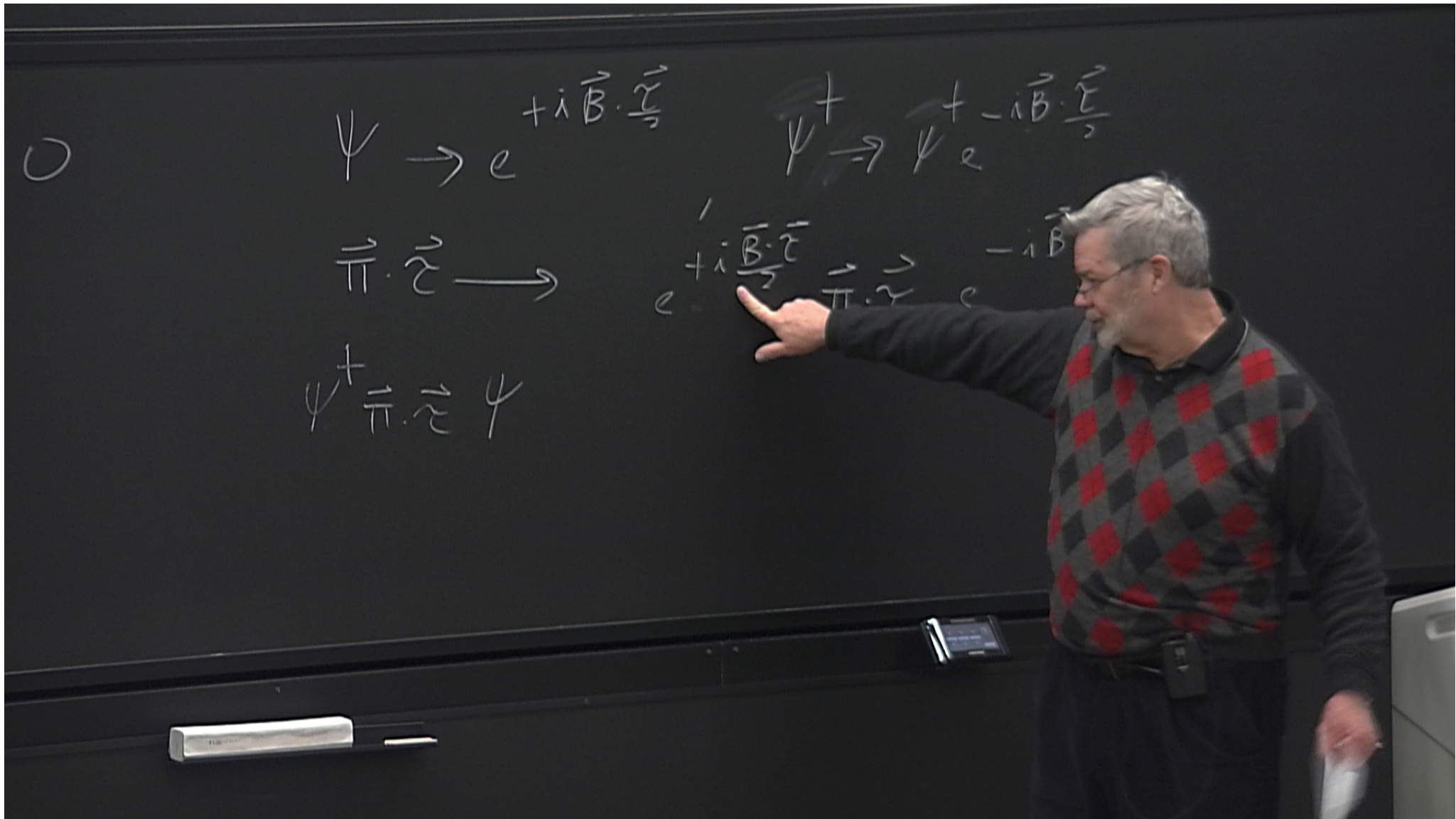
$$\bar{\psi}$$

$$[T_1, \psi] = 0$$

$$\psi \rightarrow e^{+i\vec{B} \cdot \vec{\tau}} \psi$$

$$\bar{\psi} \rightarrow \bar{\psi} e^{-i\vec{B} \cdot \vec{\tau}}$$





$$\begin{aligned}
 O \quad & \psi \rightarrow e^{+i\vec{B}\cdot\vec{\tau}/2} \quad \psi^\dagger \rightarrow \psi^\dagger e^{-i\vec{B}\cdot\vec{\tau}/2} \\
 & \vec{\pi}\cdot\vec{\tau} \longrightarrow e^{+i\vec{B}\cdot\vec{\tau}/2} \vec{\pi}\cdot\vec{\tau} e^{-i\vec{B}\cdot\vec{\tau}/2} \\
 & \psi^\dagger \vec{\pi}\cdot\vec{\tau} \psi
 \end{aligned}$$

0

$$\psi \rightarrow e^{+i\vec{B} \cdot \vec{\tau}} \psi$$

$$\psi^\dagger \rightarrow \psi^\dagger e^{-i\vec{B} \cdot \vec{\tau}}$$

$$(\vec{\pi} \cdot \vec{\tau})^2 \rightarrow$$

$$\vec{\pi} \cdot \vec{\tau} \rightarrow e^{+i\vec{B} \cdot \vec{\tau}} \vec{\pi} \cdot \vec{\tau} e^{-i\vec{B} \cdot \vec{\tau}}$$

$$\psi^\dagger \vec{\pi} \cdot \vec{\tau} \psi$$



$$\psi \rightarrow \psi e^{+i\vec{B} \cdot \frac{\vec{\tau}}{2}}$$

$$e^{+i\vec{B} \cdot \frac{\vec{\tau}}{2}} \vec{\pi} \cdot \vec{\tau} e^{-i\vec{B} \cdot \frac{\vec{\tau}}{2}}$$

$$(\vec{\pi} \cdot \vec{\tau})^2 \rightarrow e^{+i\vec{B} \cdot \frac{\vec{\tau}}{2}} (\vec{\pi} \cdot \vec{\tau})^2 e^{-i\vec{B} \cdot \frac{\vec{\tau}}{2}}$$

$$\psi \rightarrow \psi e^{+i\vec{B} \cdot \frac{\vec{\tau}}{2}}$$

$$e^{+i\vec{B} \cdot \frac{\vec{\tau}}{2}} \vec{\pi} \cdot \vec{\tau} e^{-i\vec{B} \cdot \frac{\vec{\tau}}{2}}$$

$$(\vec{\pi} \cdot \vec{\tau})^2 \rightarrow e^{+i\vec{B} \cdot \frac{\vec{\tau}}{2}} (\vec{\pi} \cdot \vec{\tau})^2 e^{-i\vec{B} \cdot \frac{\vec{\tau}}{2}}$$



Symmetry  $SU(2) \times U(1)_B$

$$\psi \rightarrow e^{i\alpha} \psi, \quad \psi^\dagger \rightarrow \psi^\dagger e^{-i\alpha}, \quad \pi^i \rightarrow \pi^i$$

$$\Phi \rightarrow e^{iB\alpha} \Phi$$

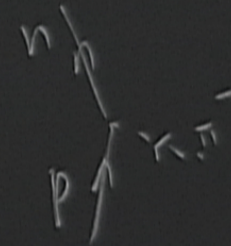
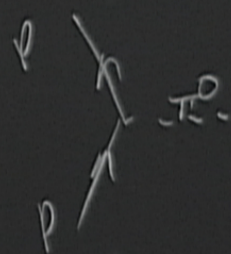
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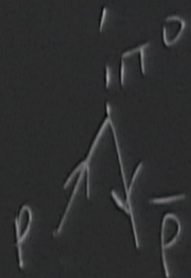
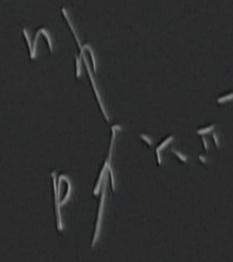
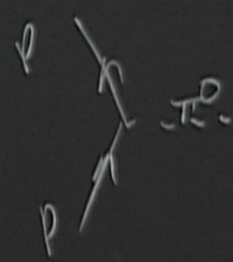
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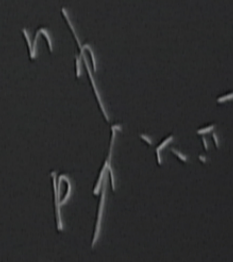
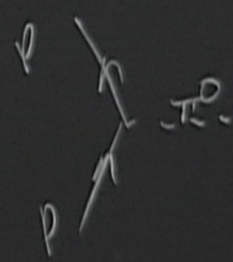
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expl



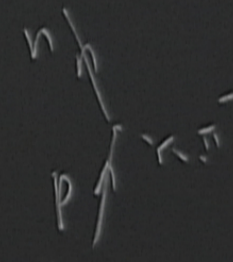
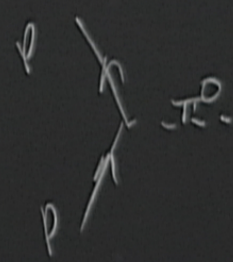
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$$\psi \rightarrow e^{i\alpha} \psi$$

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$$\pi^i \rightarrow \pi^i$$

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explicit

$$\mathcal{L} =$$

$$\mathcal{L}_1$$

$$SU(2) \times U(1)_B \longrightarrow U(1)_{T^3} \times U(1)_B$$

$T^1, T^2$  broken

$T^3, B$  unbroken

$$\bar{\Psi} \not{D} \Psi = (\bar{\Psi}_{\text{red}} \quad \bar{\Psi}_{\text{ind}})$$



Space reflection  $P$  (parity)

$$\vec{x} \rightarrow \vec{x} \quad \vec{p} \rightarrow -\vec{p}$$

vector

(parity)

$\rightarrow -\vec{P}$

$$\vec{L} = \vec{x} \times \vec{p} \rightarrow +\vec{L}$$

axial      vectors      [pseudo vectors]



Space reflection  $P$  (parity)  $\bar{\psi} \gamma_m \psi$

$$\vec{x} \rightarrow -\vec{x} \quad \vec{p} \rightarrow -\vec{p}$$

$$\vec{L} = \vec{x} \times \vec{p} \rightarrow \vec{L}$$

axial

$\bar{\psi}$

Vector

$$\vec{x}^2 \rightarrow +\vec{x}^2 \quad \vec{p}^2 \rightarrow +\vec{p}^2$$

Scalars

$$\vec{L} \cdot \vec{p} \rightarrow -\vec{L} \cdot \vec{p}$$

pseudo scalar



$$\vec{L} = \vec{x} \times \vec{p} \rightarrow + \vec{L}$$

axial

vectors [Pseudo vectors]

$$\bar{\psi} \gamma^5 \psi$$

$$\rightarrow -\vec{L} \cdot \vec{p}$$

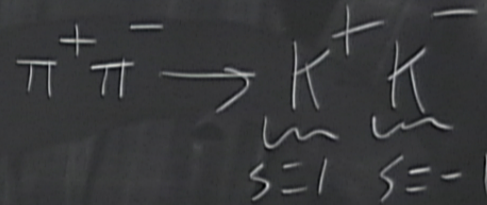
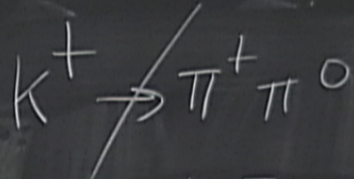
pseudo scalar

$\pi$  are pseudoscalars,  $\bar{\psi} \gamma^5 \psi$  is a pseudoscalar



Strangeness

associated production



spin  $\frac{1}{2}$  (hyperons)

[8-fold way  
Gell-Mann, Ne'eman  
 $SU(3)$   $T^3$   $T^8$

$\Sigma^-$

$n$

$p$

$\Sigma^-$

$\Sigma^0$   
 $\Lambda$

$\Sigma^+$

$\rightarrow I_3$

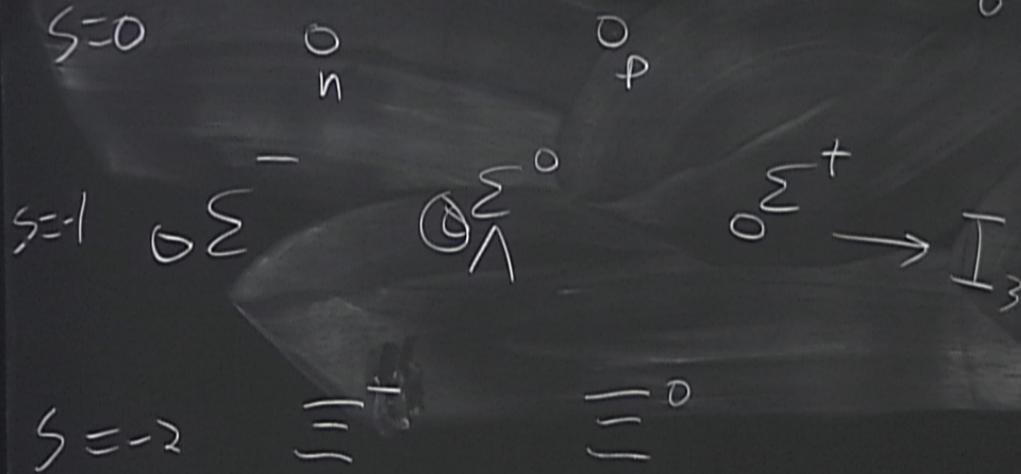
$\equiv$

$\equiv$



spin  $\frac{1}{2}$  (hyperons)

[8-fold way  
Gell-Mann, Ne'eman  
 $SU(3)$ :  $T^3$   $T^8$   
baryon octet



aman  
T8

$S=+1$   $\bar{0}K^0$   $0K^+$

$S=0$   $0-\pi$   $0\pi^0, \eta$   $0\pi^-$

$S=-1$   $K^-$   $\bar{K}^0$



$$S=+1 \quad \bar{0}K^0 \quad 0K^+$$

$Y = \text{strong hypercharge}$

$$S=0 \quad 0^-_{\pi} \quad \pi^0, n \quad \pi^-$$

$$Y = B + S$$

$$S=-1 \quad K^- \quad \bar{K}^0$$

$$I_3 = T^3, \quad Y = \frac{2}{\sqrt{3}} T^8$$

$$B=0$$

$$S=+1 \quad \bar{0}K^0 \quad 0K^+$$

$Y = \text{strong hypercharge}$

$$S=0 \quad 0^-_{\pi} \quad \pi^0, n \quad \pi^-$$

$$Y = B + S$$

$$S=-1 \quad K^- \quad \bar{K}^0$$

$$I_3 = T^3 \quad Y = \frac{2}{\sqrt{3}} T^8$$

$$B=0$$

$$Q = I_3 + \frac{Y}{2}$$



$\frac{1}{2}$  (hyperons)

$n$

$p$

$\Sigma^0$   
 $\Lambda$

$\Sigma^+$

$\rightarrow I_3$

masses degenerate  
to  $\sim 25\%$

$\Xi^-$

$\Xi^0$

$B=1$

[8-Fold way  
Gell-Mann, Ne'eman  
 $SU(3)$ :  $T^3$   $T^8$   
baryon octet

$S=+1$   $K^0$

$S=0$   $\pi^-$

$\pi^0, \eta$

$S=-1$

pseudo scalar  
mesons

$K^-$

$B=0$

Gell-mann, Zweig

$$\begin{pmatrix} u \\ d \end{pmatrix}$$

$SU(2)$  doublet

$s$   
singlet

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

3

$$q_u = \frac{2}{3}, \quad q_d = q_s = -\frac{1}{3}$$



Gell-mann, Zweig

$\begin{pmatrix} u \\ d \end{pmatrix}$   $S$  singlet  
 $SU(2)$  doublet

$$q_u = \frac{2}{3} \quad q_d = q_s = -\frac{1}{3}$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \longrightarrow e^{i \vec{B} \cdot \frac{\vec{\lambda}}{2}} q$$

$$Q = \frac{\lambda^3}{2} + \frac{\lambda^8}{2\sqrt{3}} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

$$Y = \frac{\lambda^8}{\sqrt{3}} = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \rightarrow e$$

$$i \vec{B} \cdot \frac{\vec{\lambda}}{2} q$$

$$y = \frac{1}{3} \quad 0^d \quad 0^u$$

$$y = -\frac{2}{3} \quad 0^s$$

$$0^3$$

$$5/3$$

$$0^d$$

$$3^x$$

$$y = \frac{\lambda^8}{\sqrt{3}} = \frac{1}{3} \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$$



$$\mathcal{L}_{\text{quarks}} = \sum_{a=u,d,s} \bar{q}_a (i \not{D} - m_a) q_a = \underbrace{\bar{q} (i \not{D} - m_0) q}_{\mathcal{L}_0} - m_3 q$$

$$\sum_{i,d,s} \bar{q}_a (i \not{\lambda} - m_a) q_a = \underbrace{\bar{q} (i \not{\lambda} - m_0) q}_{\mathcal{L}_0} - \underbrace{m_3 \bar{q} \lambda^3 q}_{-\mathcal{E}_3 \mathcal{L}_3} - \underbrace{m_8 \bar{q} \lambda^8 q}_{-\mathcal{E}_8 \mathcal{L}_8}$$

$$m_u = m_0 + m_3 + \frac{m_8}{\sqrt{3}}$$

$$m_d = m_0 - m_3 + \frac{m_8}{\sqrt{3}}$$

$$m_s = m_0 - 2 \frac{m_3}{\sqrt{3}}$$



$$\sum_{i,d,s} \bar{q}_a (i \not{\lambda} - m_a) q_a = \underbrace{\bar{q} (i \not{\lambda} - m_0) q}_{\mathcal{L}_0} - \underbrace{m_3 \bar{q} \lambda^3 q}_{-\mathcal{E}_3 \mathcal{L}_3} - \underbrace{m_8 \bar{q} \lambda^8 q}_{-\mathcal{E}_8 \mathcal{L}_8}$$

$$m_u = m_0 + m_3 + \frac{m_8}{\sqrt{3}}$$

$$m_d = m_0 - m_3 + \frac{m_8}{\sqrt{3}}$$

$$m_s = m_0 - 2 \frac{m_3}{\sqrt{3}}$$

Chiral symmetry

$$\mathcal{L} = \bar{\Psi} (i \not{\partial} - m) \Psi$$

$$\Psi_L = P_L \Psi$$

$$\Psi_R = P_R \Psi$$

$$P_L =$$



Chiral symmetry

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - m)\Psi$$

$$\Psi = \Psi_L + \Psi_R$$

$$\Psi_L = P_L \Psi$$

$$\Psi_R = P_R \Psi$$

Weyl spinors

$$P_L = \frac{1 - \gamma^5}{2} \quad P_R = \frac{1 + \gamma^5}{2}$$

right hand



symmetry

$$\psi_L = P_L \psi$$

$$\psi_R = P_R \psi$$

Weyl spinors

$\psi_R$

Chirality

$$P_L = \frac{1 - \gamma^5}{2}$$

$$P_R = \frac{1 + \gamma^5}{2}$$

left  
helicity

right  
helicity

$$P_L^2 = P_L$$

$$P_R^2 = P_R$$

$$P_L P_R = 0$$

$$\mathcal{L} = \bar{\psi}_L i \not{\partial} \psi_L + \bar{\psi}_R i \not{\partial} \psi_R - m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$



symmetry

$$\psi_L = P_L \psi$$

$$\psi_R = P_R \psi$$

Weyl spinors

$\psi_R$

Chirality

$$P_L = \frac{1 - \gamma_5}{2}$$

$$P_R = \frac{1 + \gamma_5}{2}$$

left helicity

right helicity

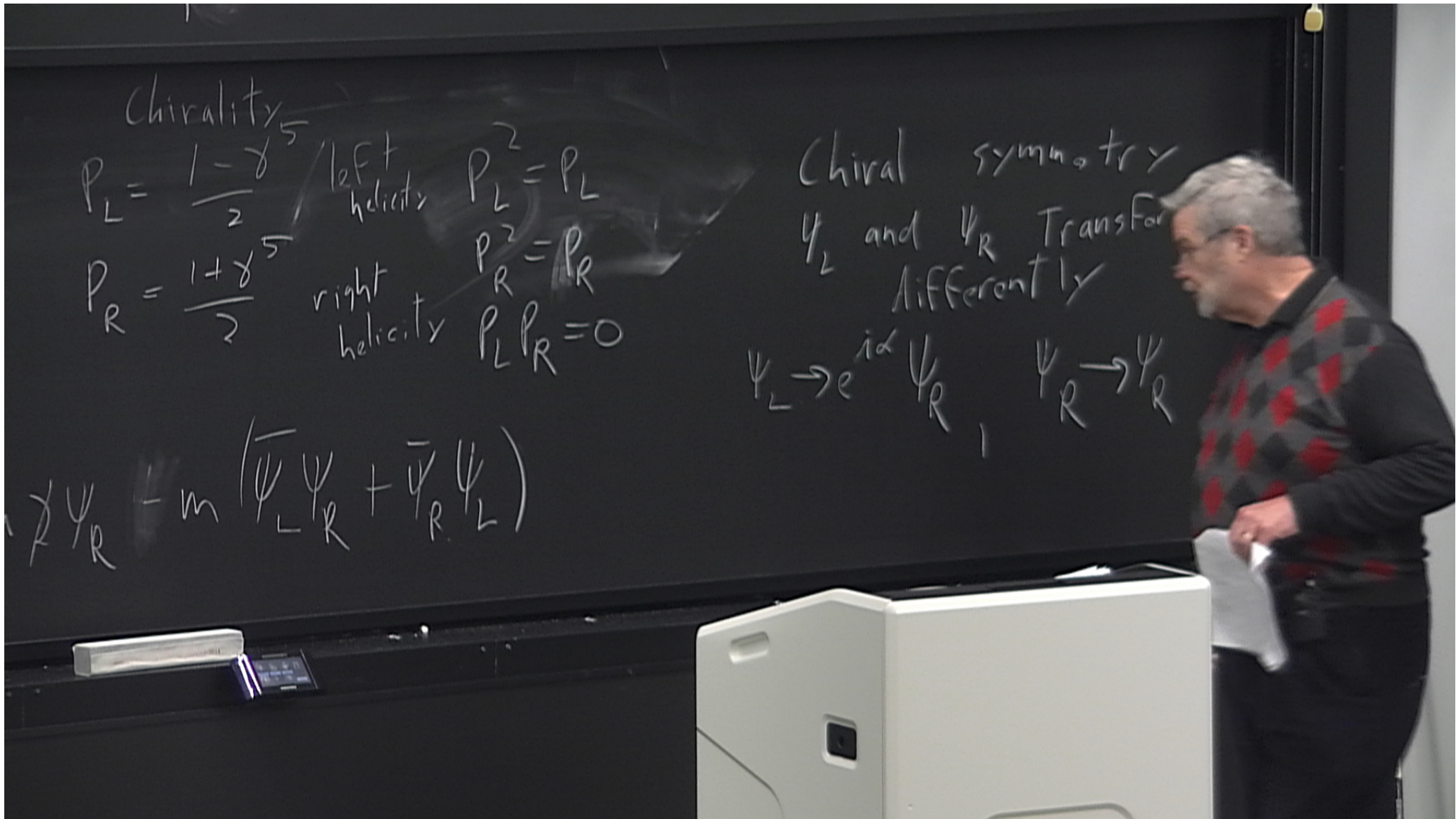
$$P_L^2 = P_L$$

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$$\mathcal{L} = \bar{\psi}_L i \not{\partial} \psi_L + \bar{\psi}_R i \not{\partial} \psi_R - m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$





Chirality

$$P_L = \frac{1 - \gamma^5}{2}$$

left  
helicity

$$P_L^2 = P_L$$

$$P_R = \frac{1 + \gamma^5}{2}$$

right  
helicity

$$P_R^2 = P_R$$

$$P_L P_R = 0$$

Chiral symmetry

$\psi_L$  and  $\psi_R$  Transform  
Differently

$$\psi_L \rightarrow e^{i\alpha} \psi_L, \quad \psi_R \rightarrow \psi_R$$

$$\bar{\psi} \psi = m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$



$$P_L = \frac{1-\gamma^5}{2} \quad \text{left helicity} \quad P_L^2 = P_L$$

$$P_R = \frac{1+\gamma^5}{2} \quad \text{right helicity} \quad P_R^2 = P_R$$

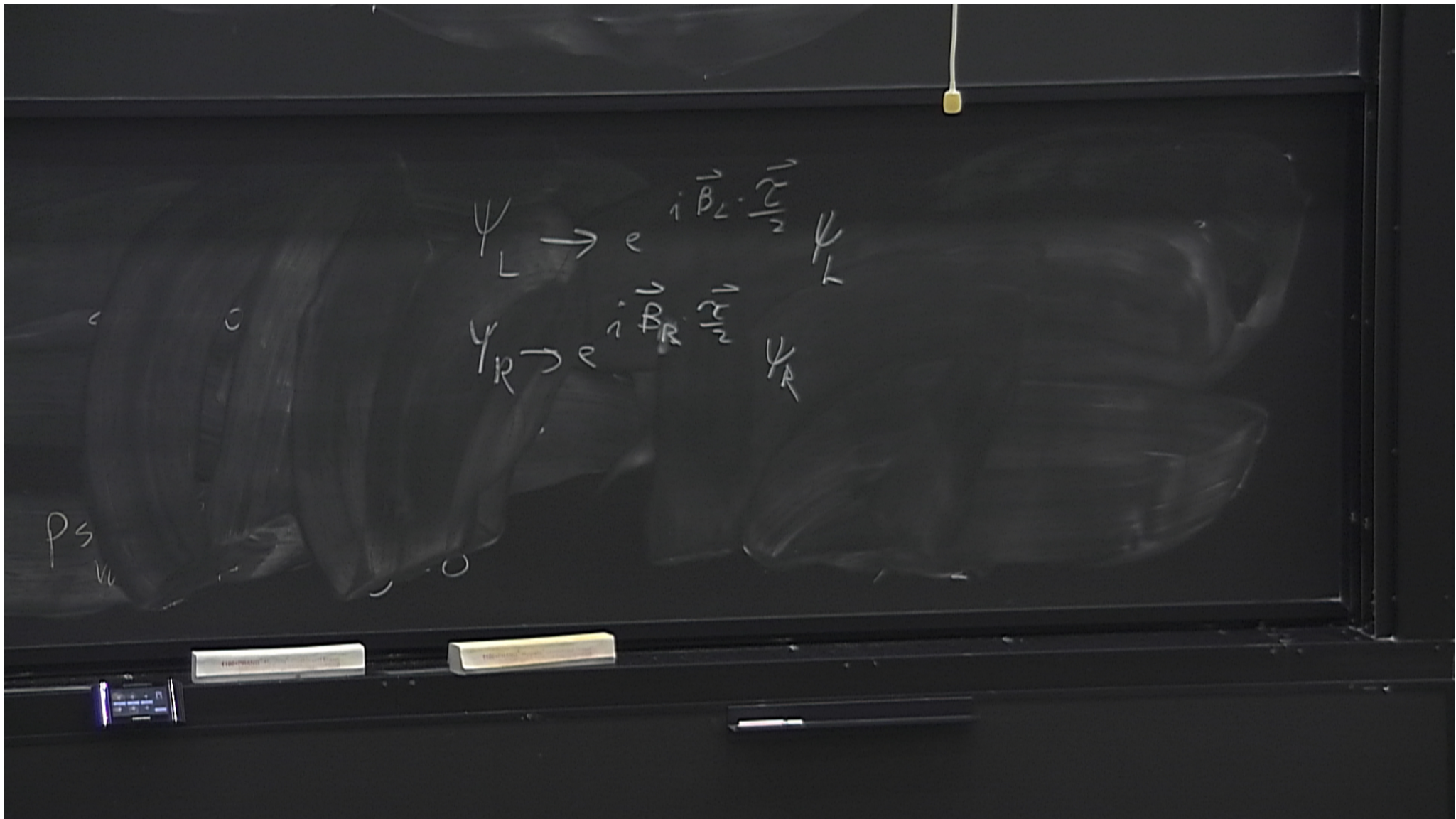
$$P_L P_R = 0$$

Chiral symmetry  
 $\psi_L$  and  $\psi_R$  Transform  
 Differently

$$\psi_L \rightarrow e^{i\alpha} \psi_L, \quad \psi_R \rightarrow \psi_R$$

$$\bar{\psi} \psi = m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

no scaling





$$P_L = \frac{1-\gamma^5}{2} \quad \text{left helicity} \quad P_L^2 = P_L$$

$$P_R = \frac{1+\gamma^5}{2} \quad \text{right helicity} \quad P_R^2 = P_R$$

$$P_L P_R = 0$$

$$\psi_R = m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

Chiral symmetry  
 $\psi_L$  and  $\psi_R$  Transform  
 Differently

$$\psi_L \rightarrow e^{i\alpha} \psi_L, \quad \psi_R \rightarrow \psi_R$$

$$\Leftrightarrow m=0$$