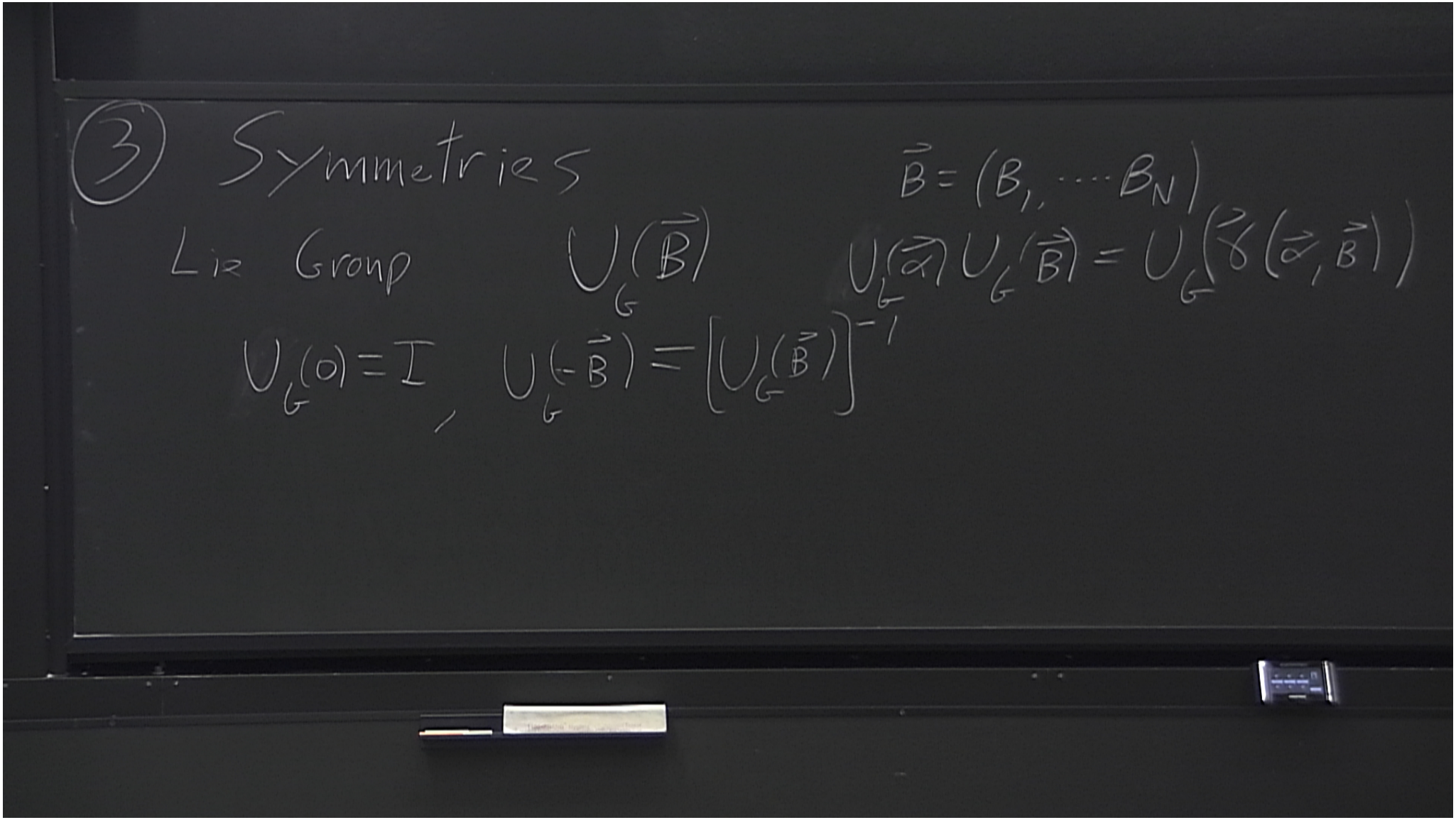
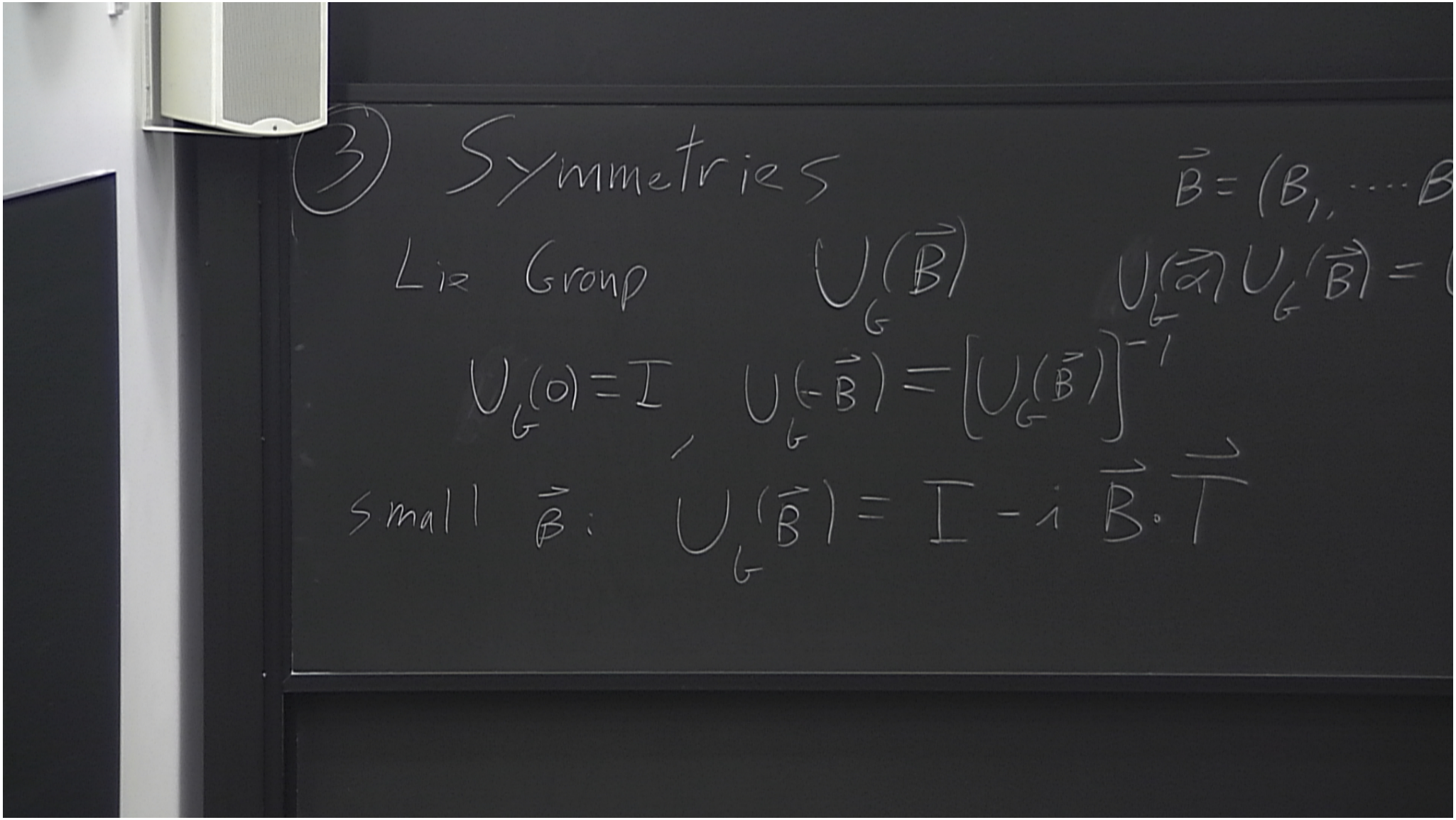
Title: 13/14 PSI - Standard Model Review - Lecture 3

Date: Jan 08, 2014 09:00 AM

URL: http://pirsa.org/14010010

Abstract:


(3) Symmetries
$\vec{B} = (B_1, \cdots B_N)$
Lie Group $U_G(\vec{B})$
$U_G(\vec{\alpha}) U_G(\vec{B}) = U_G(\vec{\gamma}(\vec{\alpha}, \vec{B}))$
$U_G(0) = I$, $U_G(-\vec{B}) = [U_G(\vec{B})]^{-1}$


(3) Symmetries
$\vec{B} = (B_1, \cdots B$
Lie Group $U_G(\vec{B})$
$U_G(\vec{a})\,U_G(\vec{B}) =$
$U_G(0) = I$, $U_G(-\vec{B}) = [U_G(\vec{B})]^{-1}$
small $\vec{B}$: $U_G(\vec{B}) = I - i\,\vec{B}\cdot\vec{T}$

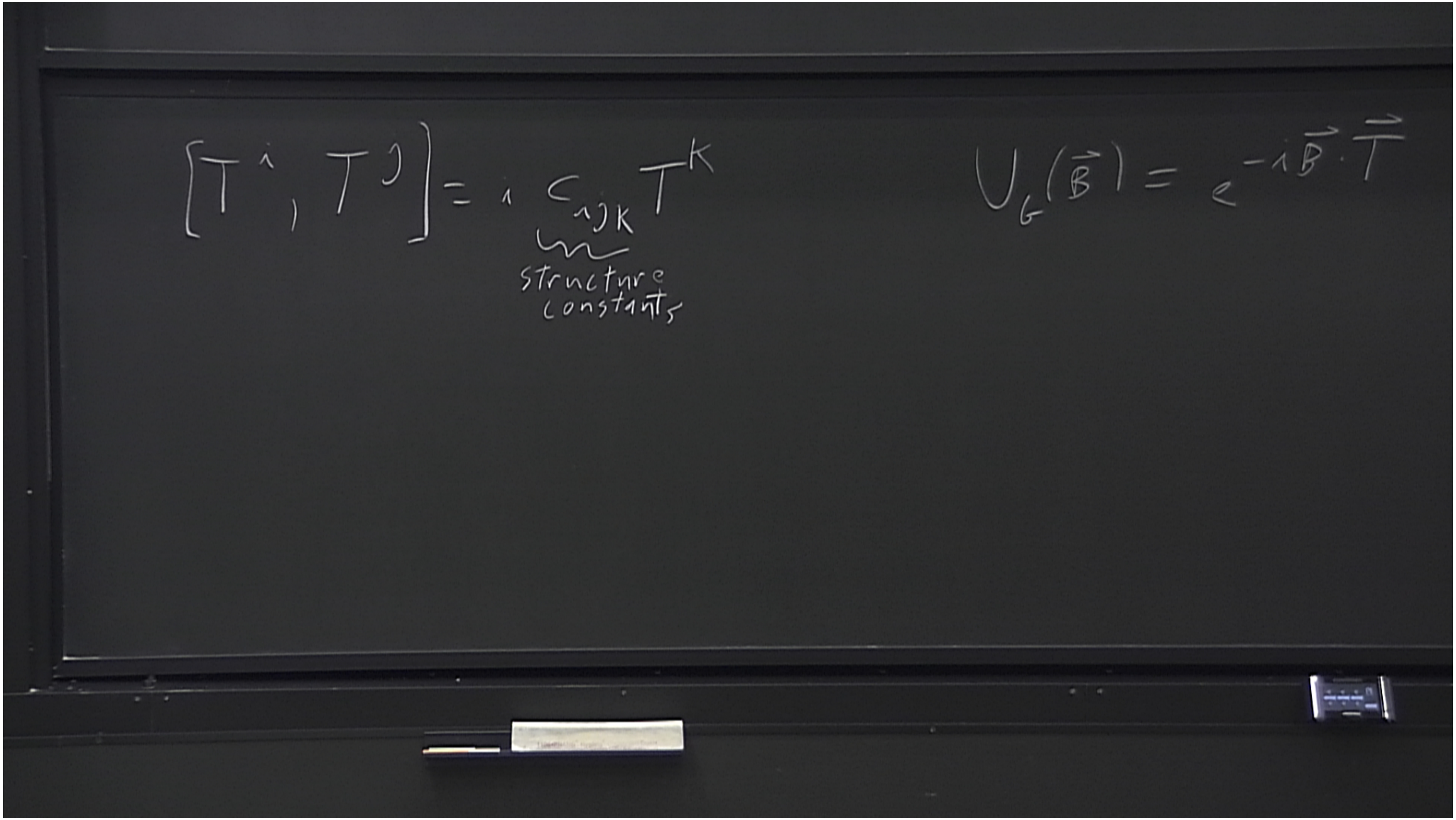
$$\vec{B} = (B_1, \cdots B_N)$$

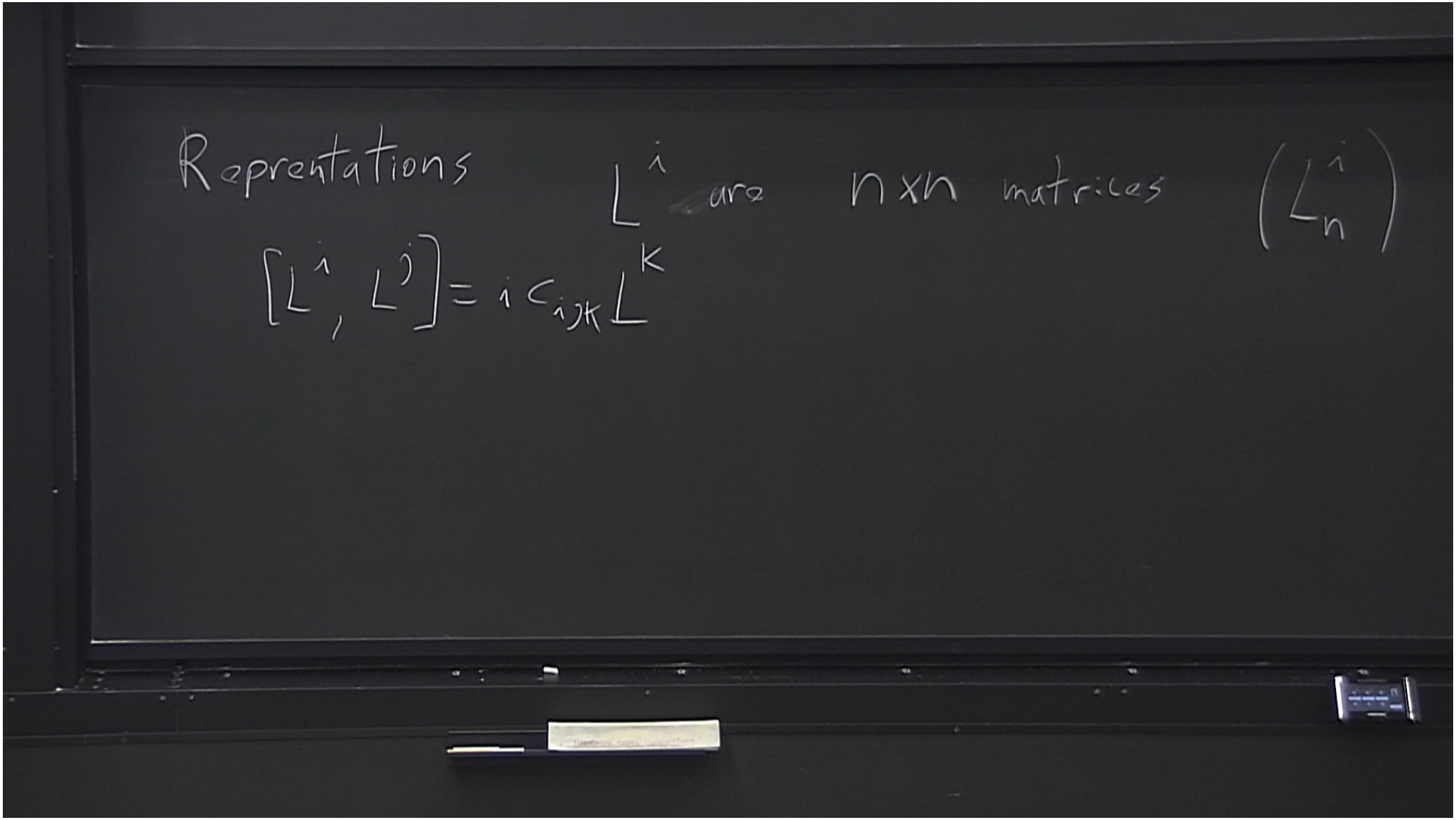
$$U_G(\vec{\alpha})\, U_G(\vec{B}) = U_G\left(\vec{\gamma}(\vec{\alpha}, \vec{B})\right)$$

$$\left.\right)\Big]^{-1}$$

$T^i \quad i = 1, \cdots, N$ are the Lie algebra (generators)

$$\vec{B} \cdot \vec{T} \qquad \vec{B} \cdot \vec{T} = \sum_{i=1}^{N} B^i T^i$$


$[T^i, T^j] = i C^{ijk} T^k$
structure constants
$U_G(\vec{B}) = e^{-i\vec{B}\cdot\vec{T}}$


Reprentations L^i are $n \times n$ matrices (L^i_n)
$[L^i, L^j] = i c_{ijk} L^k$

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for $L^i = (L^i)^\dagger \Longrightarrow$ can normalize s.t. $Tr(L^i L^j) = T(L)\,\delta_{ij}$

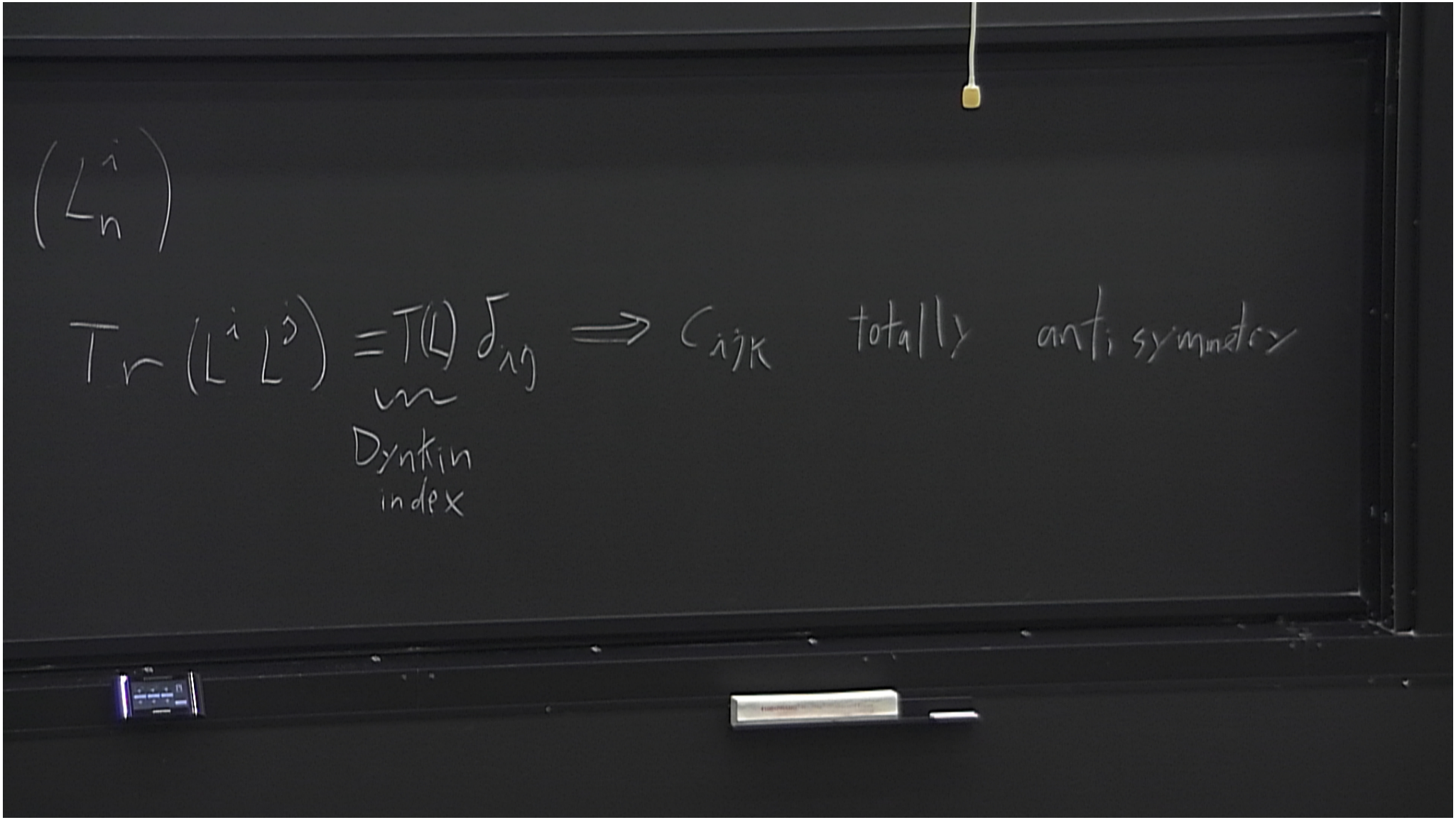
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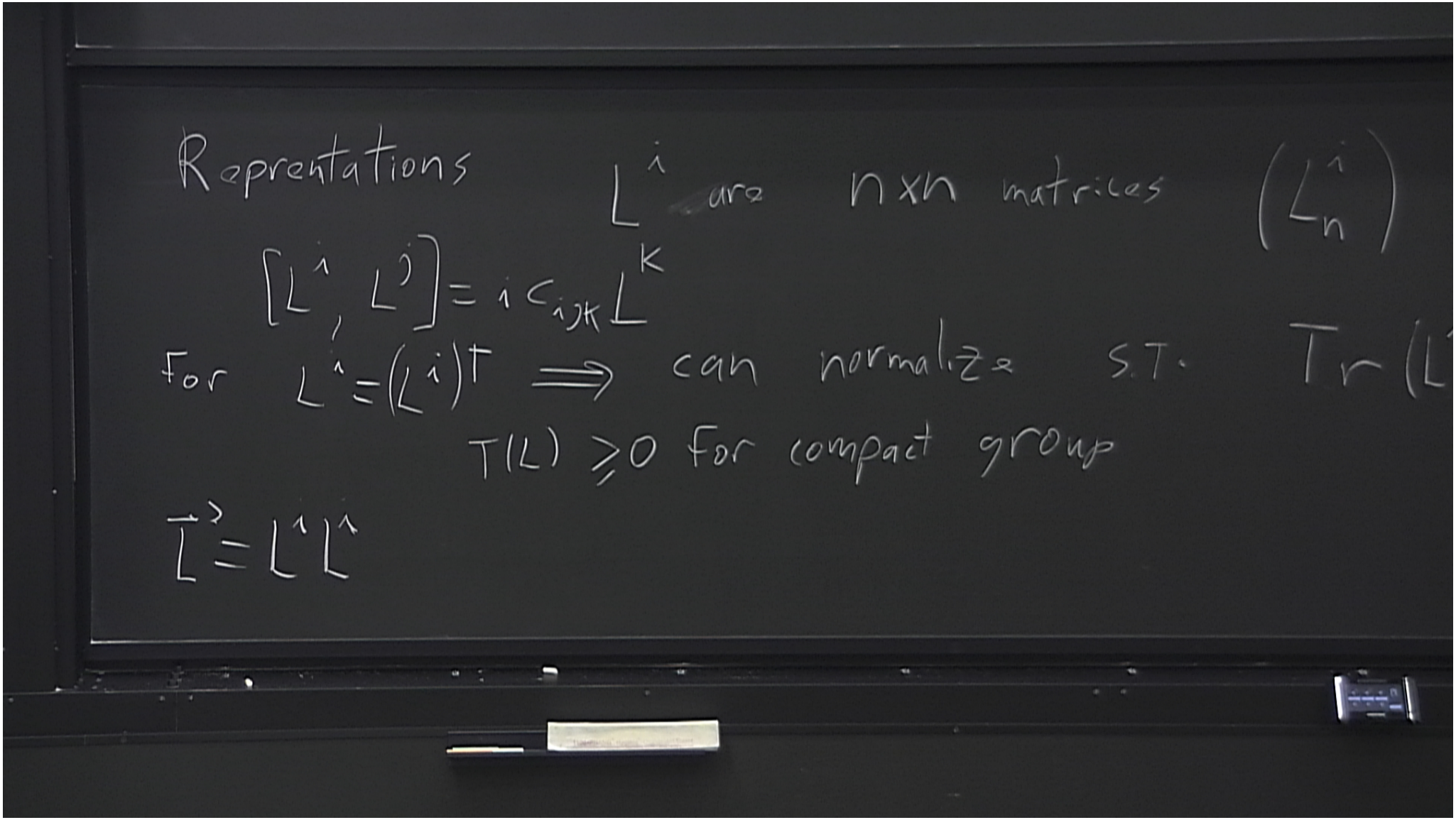
$[L^i, L^j] = i C_{ijk} L^k$

For $L^i = (L^i)^\dagger \Longrightarrow$ can normalize s.t. $Tr(L^i L^j) = T(L)\,\delta_{ij}$

$T(L) \geq 0$ for compact group

$T(L)$: Dynkin index


(L_n^i)
$Tr(L^i L^j) = T(L)\,\delta_{ij} \Rightarrow C_{ijk}$ totally antisymmetric
Dynkin index


Reprentations L^i are $n \times n$ matrices (L^i_n)
$[L^i, L^j] = i C_{ijk} L^k$
for $L^i = (L^i)^\dagger \Rightarrow$ can normalize s.t. $Tr(L$
$T(L) \geq 0$ for compact group
$\vec{L}^2 = L^i L^i$

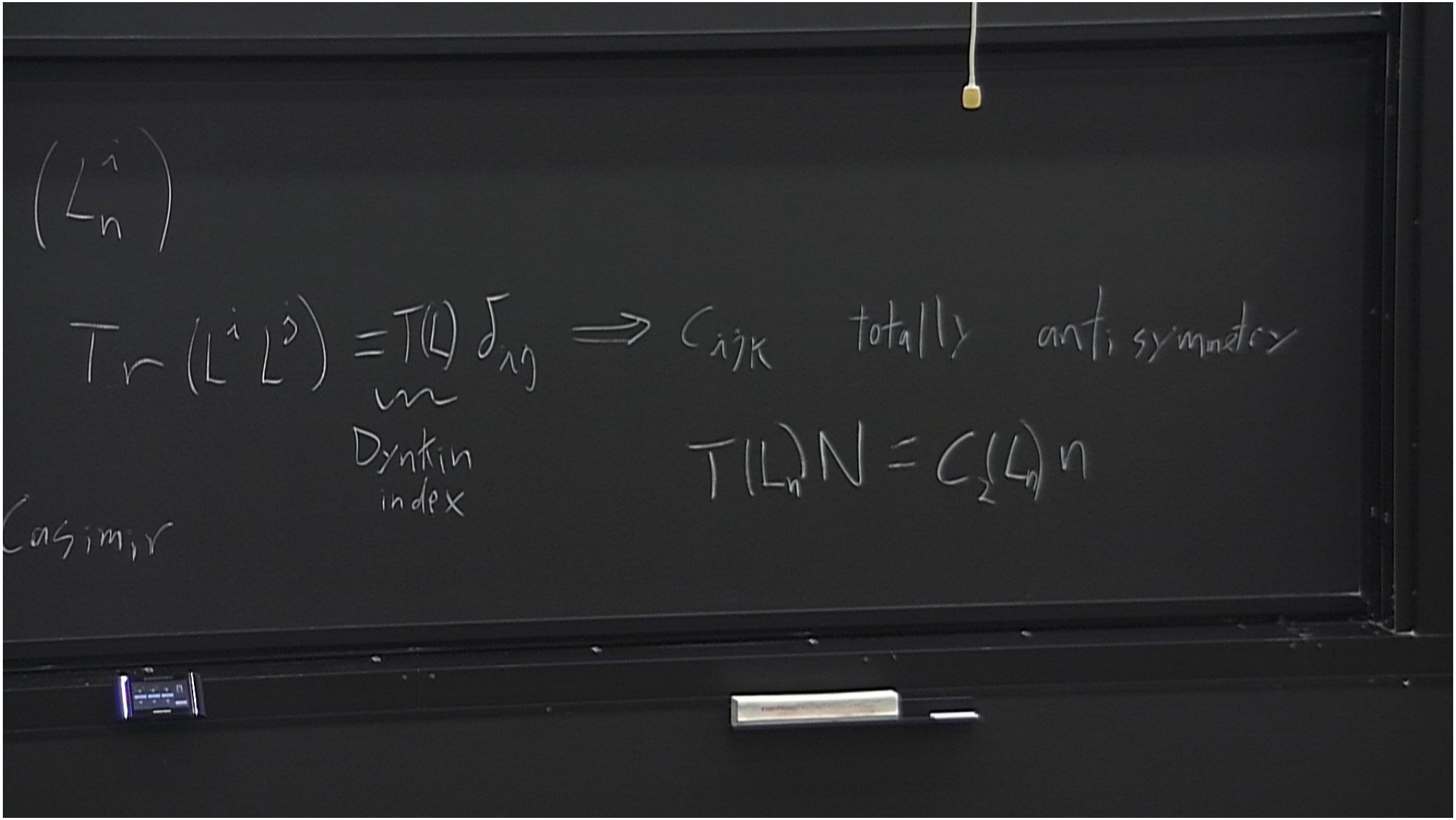
Reprentations L^i are $n \times n$ matrices (L^i_n)

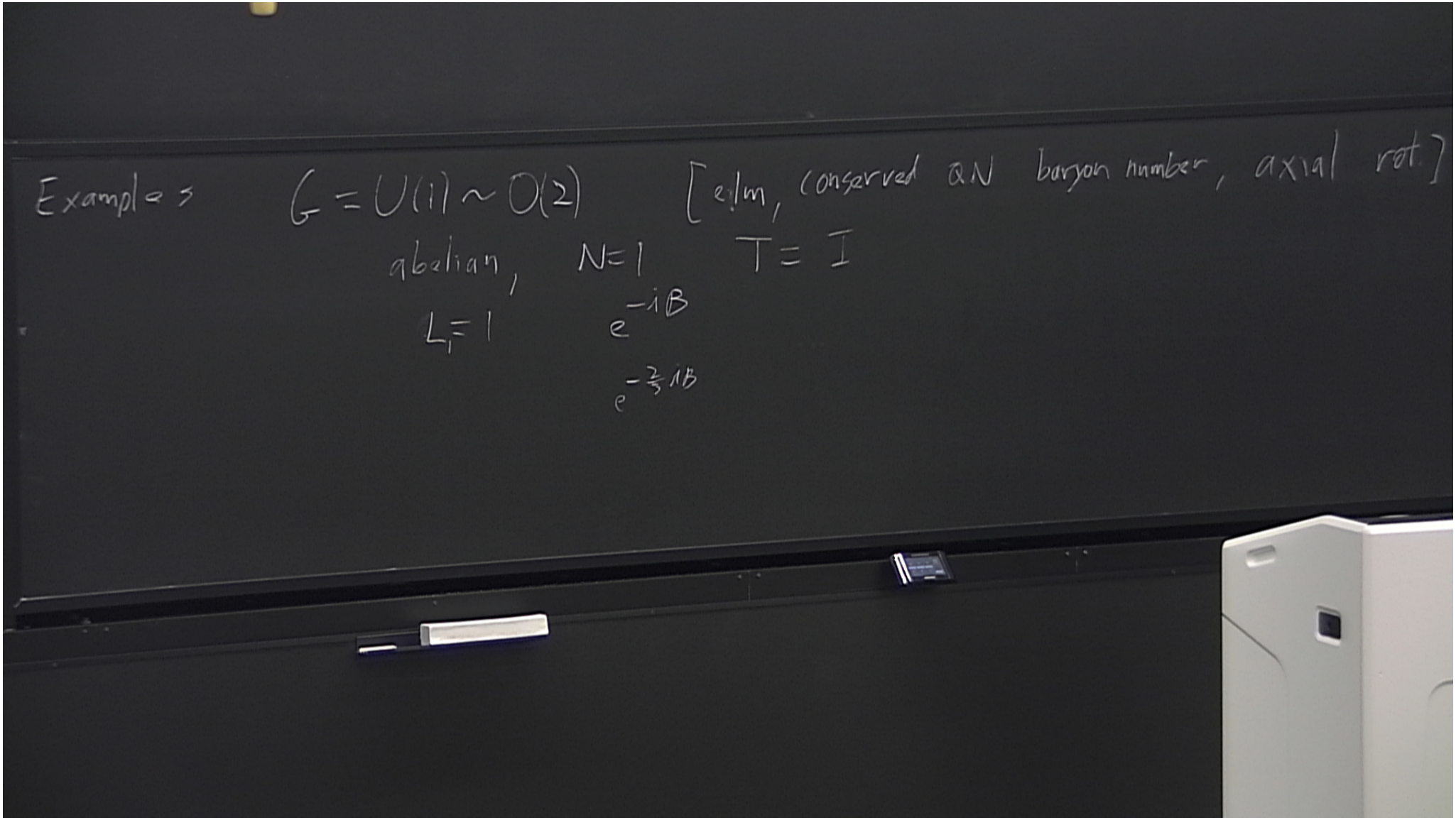
$$[L^i, L^j] = i C_{ijk} L^k$$

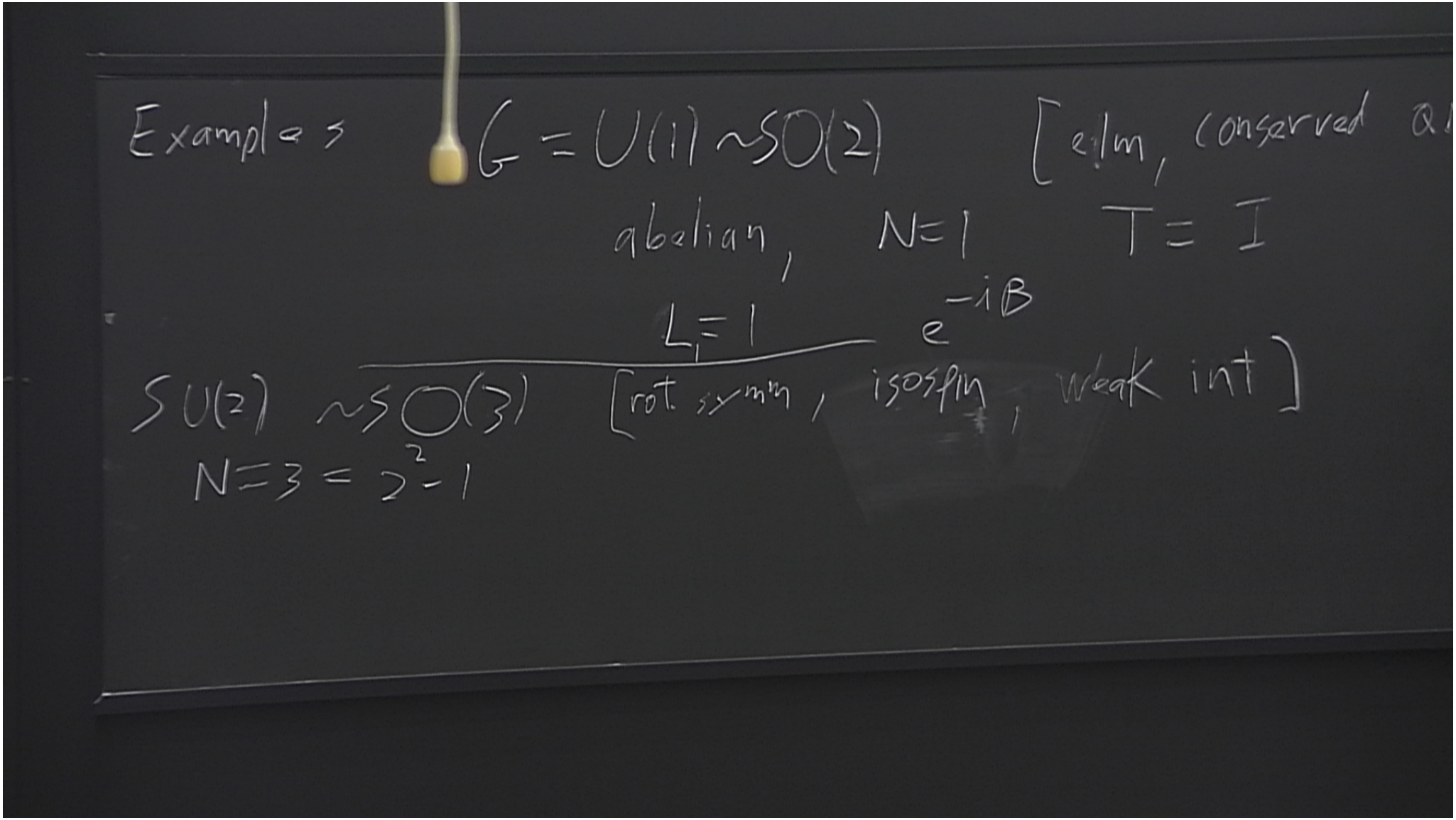
for $L^i = (L^i)^\dagger \Longrightarrow$ can normalize s.t. $\mathrm{Tr}(L^i$

$T(L) \geq 0$ for compact group

$\vec{L}^2 = L^i L^i = C_2(L)\, \mathbb{I}$ $\quad C_2(L)$ = quadratic Casimir


(L_n^i)
$Tr(L^i L^j) = T(L)\,\delta_{ij} \Longrightarrow C_{ijk}$ totally antisymmetric
Dynkin index
$T(L_n)N = C_2(L_n)\,n$
Casimir


Examples: G = U(1) ~ O(2) [e/m, conserved Q.N baryon number, axial rot.]
abelian, N=1 T = I
$L_i = 1$
e^{-iB}
$e^{-\frac{2}{3}iB}$


Examples
G = U(1) ~ SO(2)
[e/m, conserved Q
abelian, N=1
T = I
L = 1
e^{-iB}
SU(2) ~ SO(3)
[rot. symm, isospin, weak int]
N = 3 = 2^2 - 1

Examples: $G = U(1) \sim SO(2)$ [e/m, conserved Q

abelian, $N=1$ $T = I$

$L = 1$ e^{-iB}

$SU(2) \sim SO(3)$ [rot. symm, isospin, weak int]

$N = 3 = 2^2 - 1$ $\varepsilon_{123} = 1$ Defining rep: $n=2$,

$C_{ijk} = \varepsilon_{ijk}$

2) [e.m, conserved QN baryon number, axial rot.]

N=1 T=I

e^{-iB}

isospin, weak int]

Defining rep: $n=2$, $L_2^i = \frac{\tau^i}{2}$

(conserved QN baryon number, axial rot.]

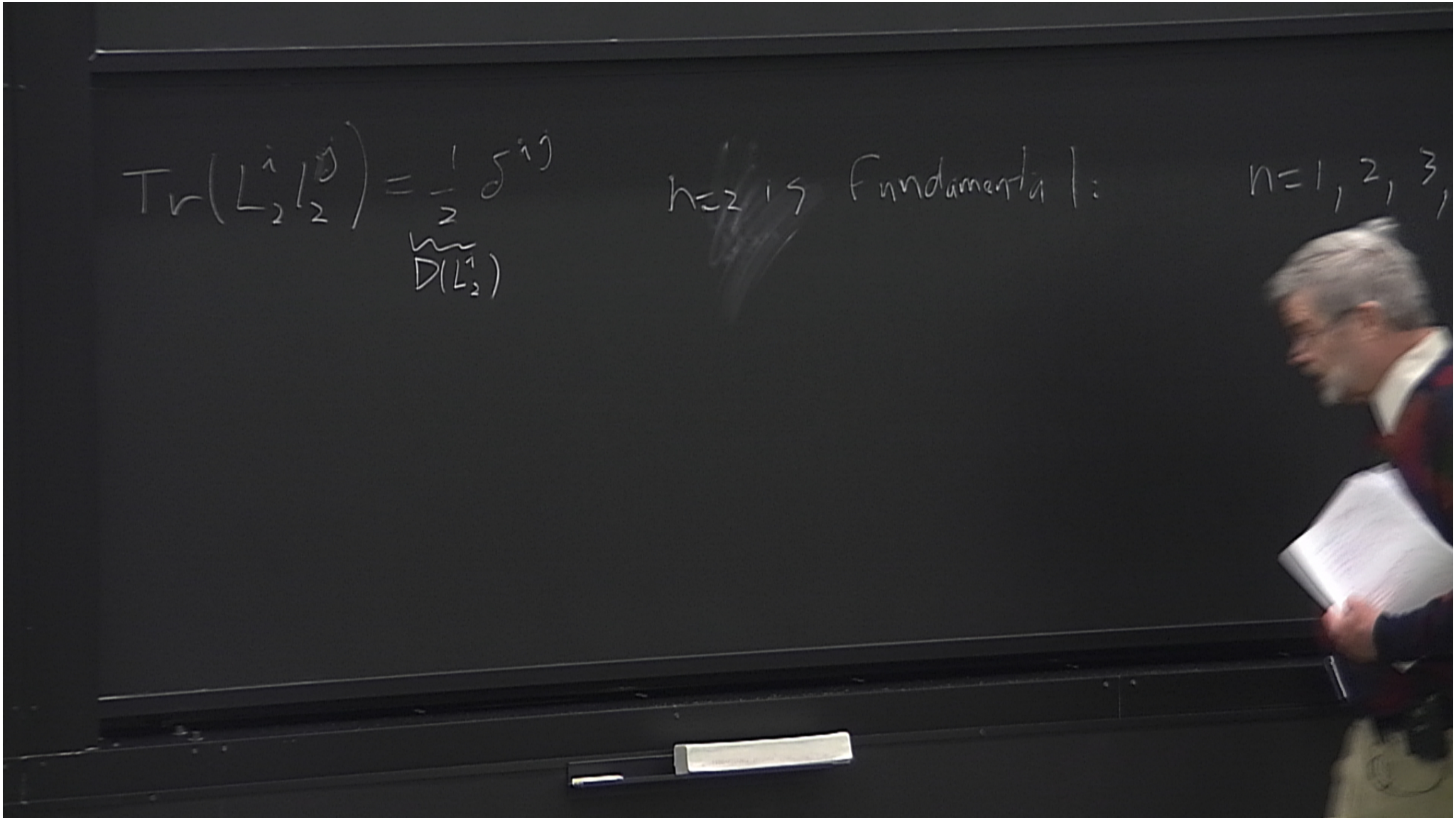
$= I$

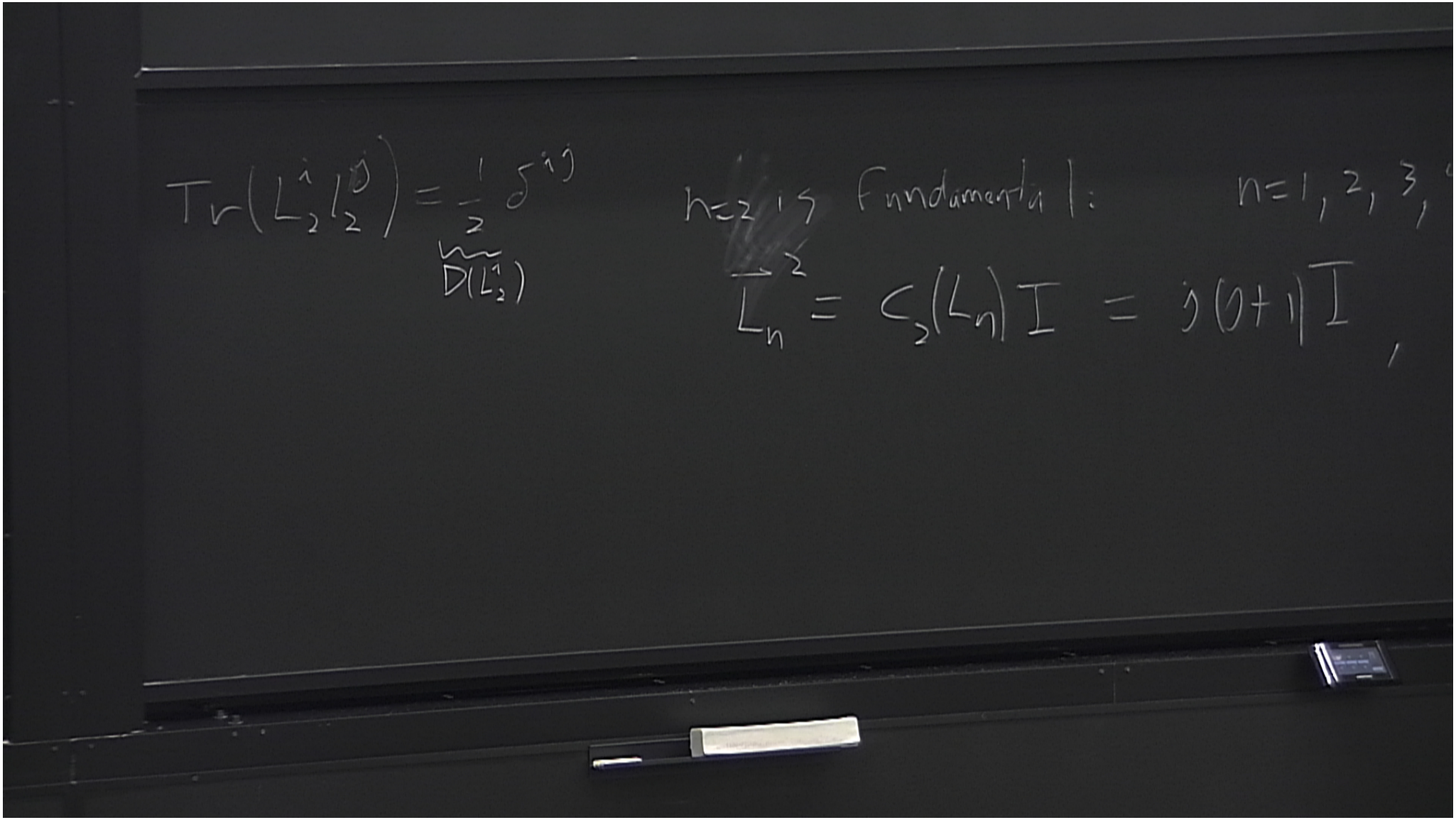
int]

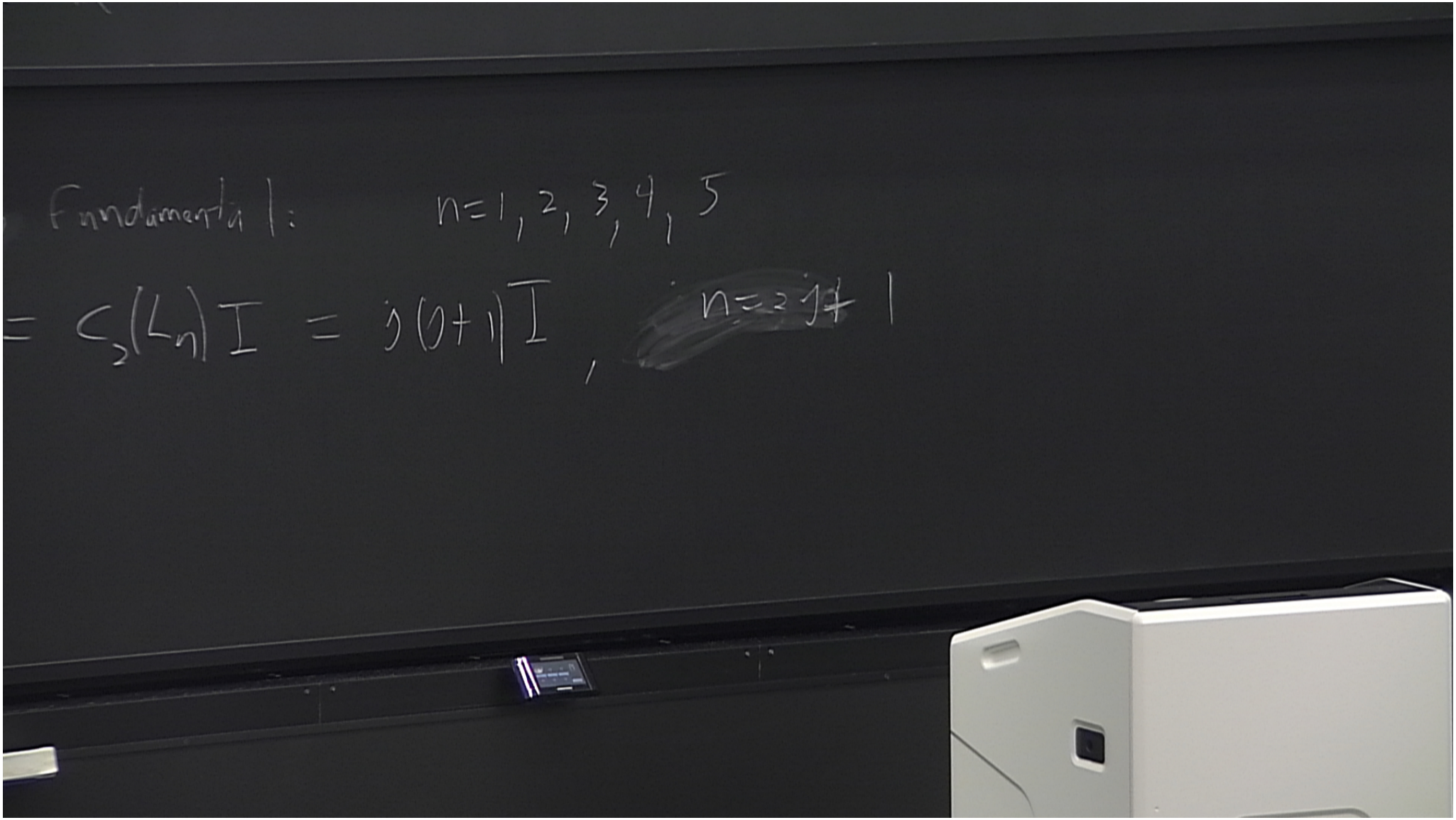
ref: $n=2$, $L_2^i = \frac{\tau^i}{2}$

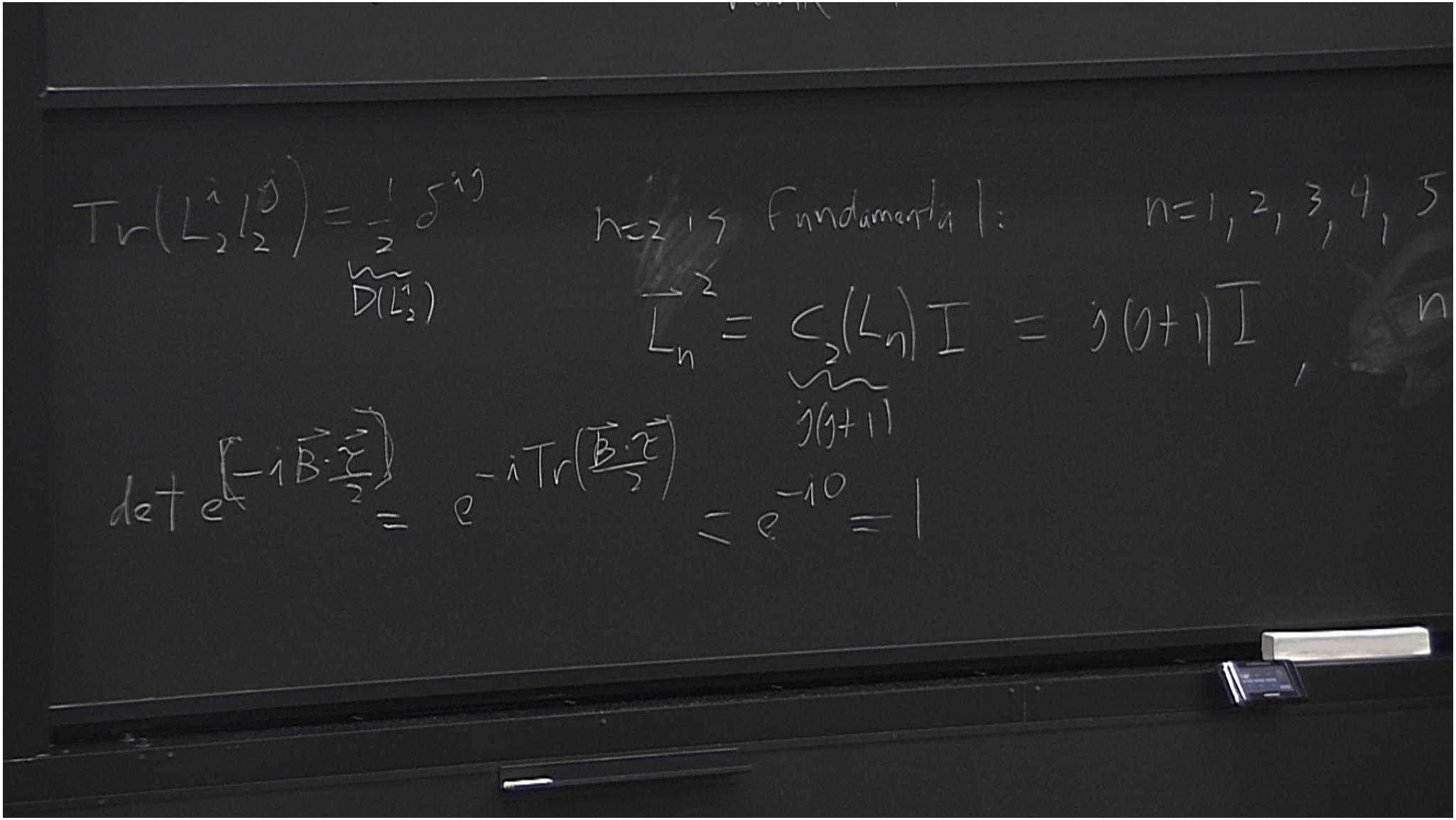
$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

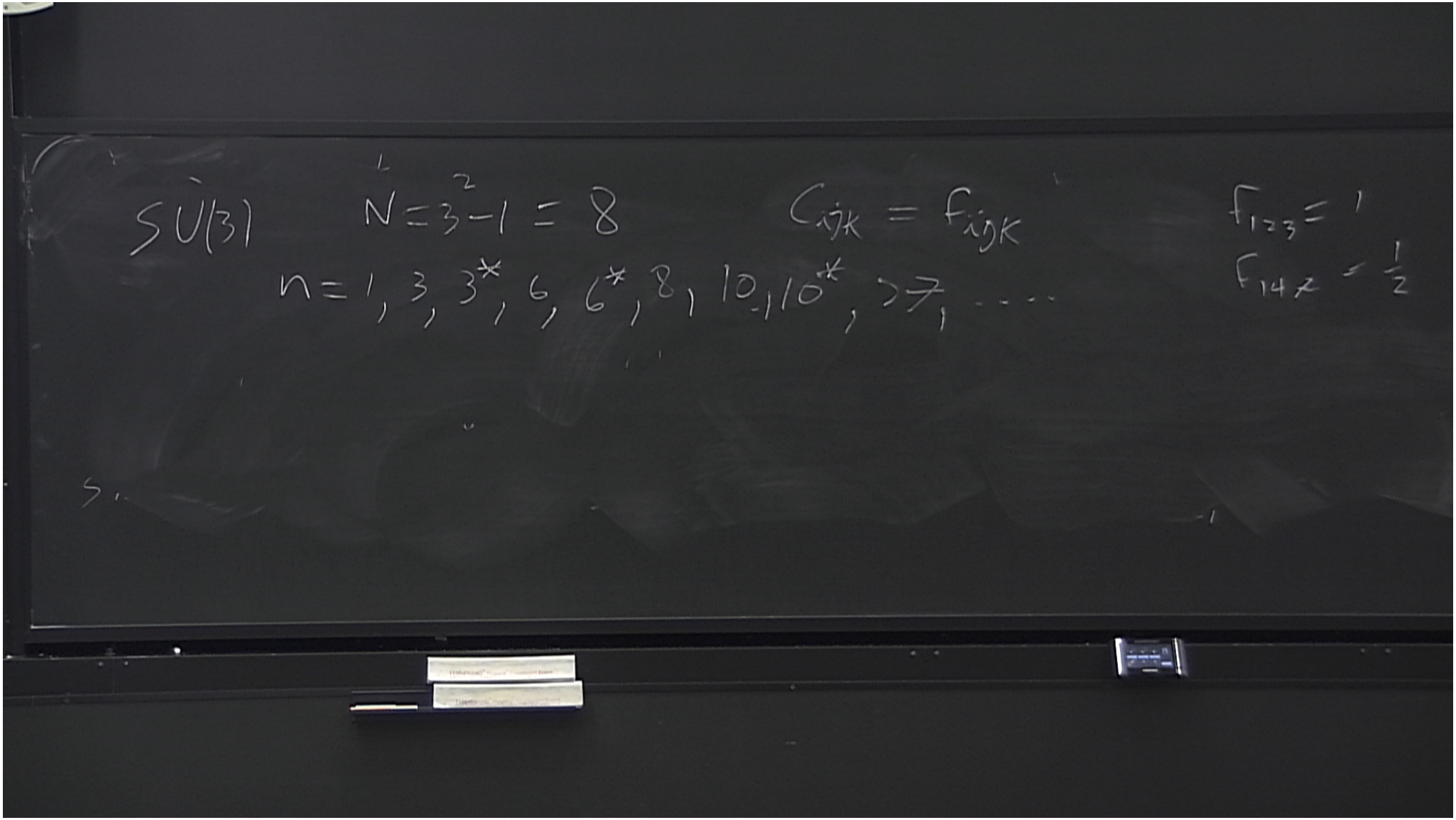
$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

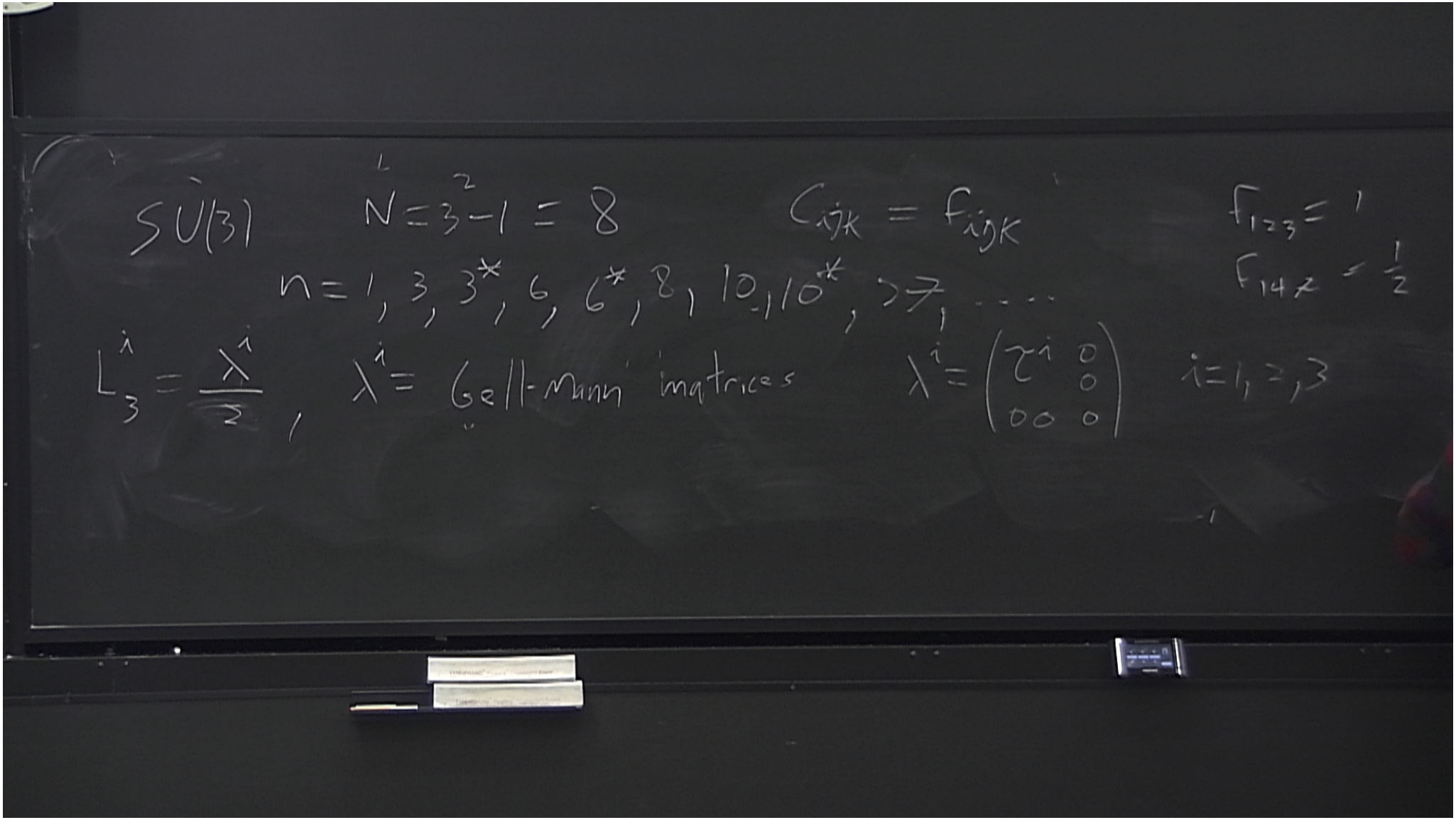

$\mathrm{Tr}(L_2^i L_2^j) = \frac{1}{2}\delta^{ij}$
$D(L_2^i)$
$n=2$ is fundamental:
$n=1, 2, 3,$

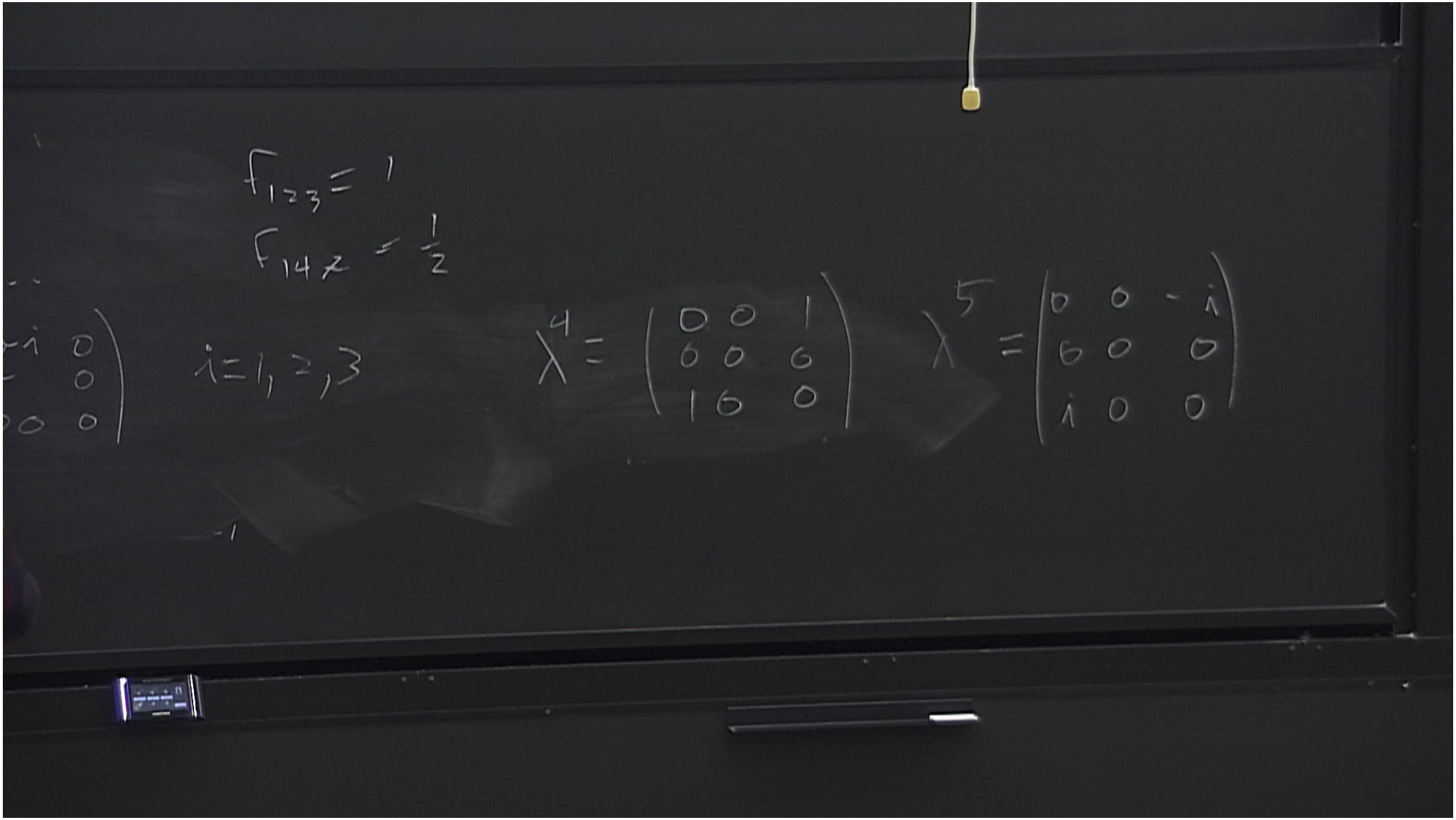

$\mathrm{Tr}(L_2^i L_2^j) = \frac{1}{2}\delta^{ij}$
$D(L_2^i)$
$n=2$ is fundamental: $n=1, 2, 3,$
$\bar{L}_n^2 = C_2(L_n)\,\mathbb{I} = j(j+1)\,\mathbb{I}$,

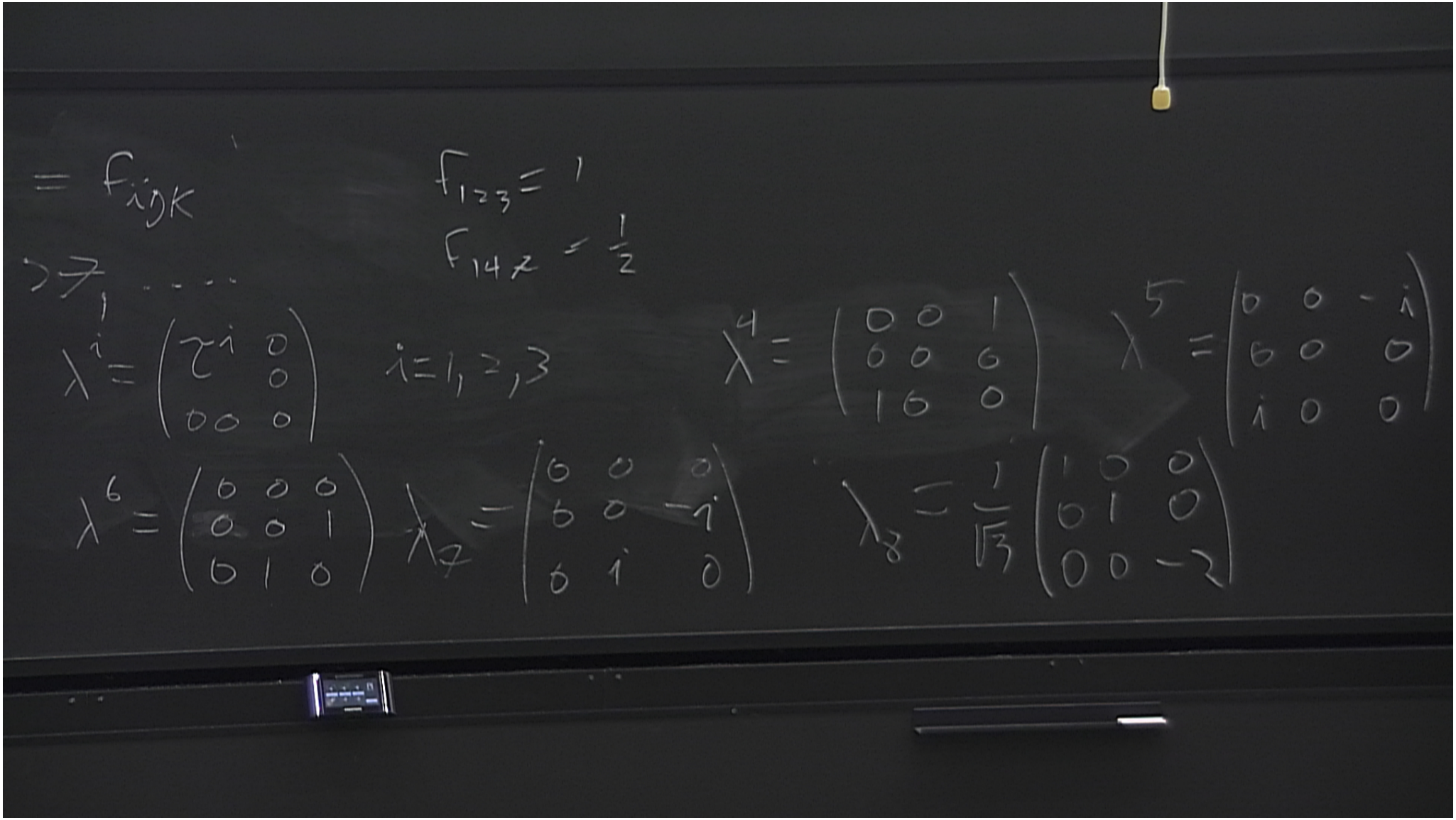

fundamental: n=1, 2, 3, 4, 5
= C2(Ln) I = j(j+1) I , n=2 j ≠ 1

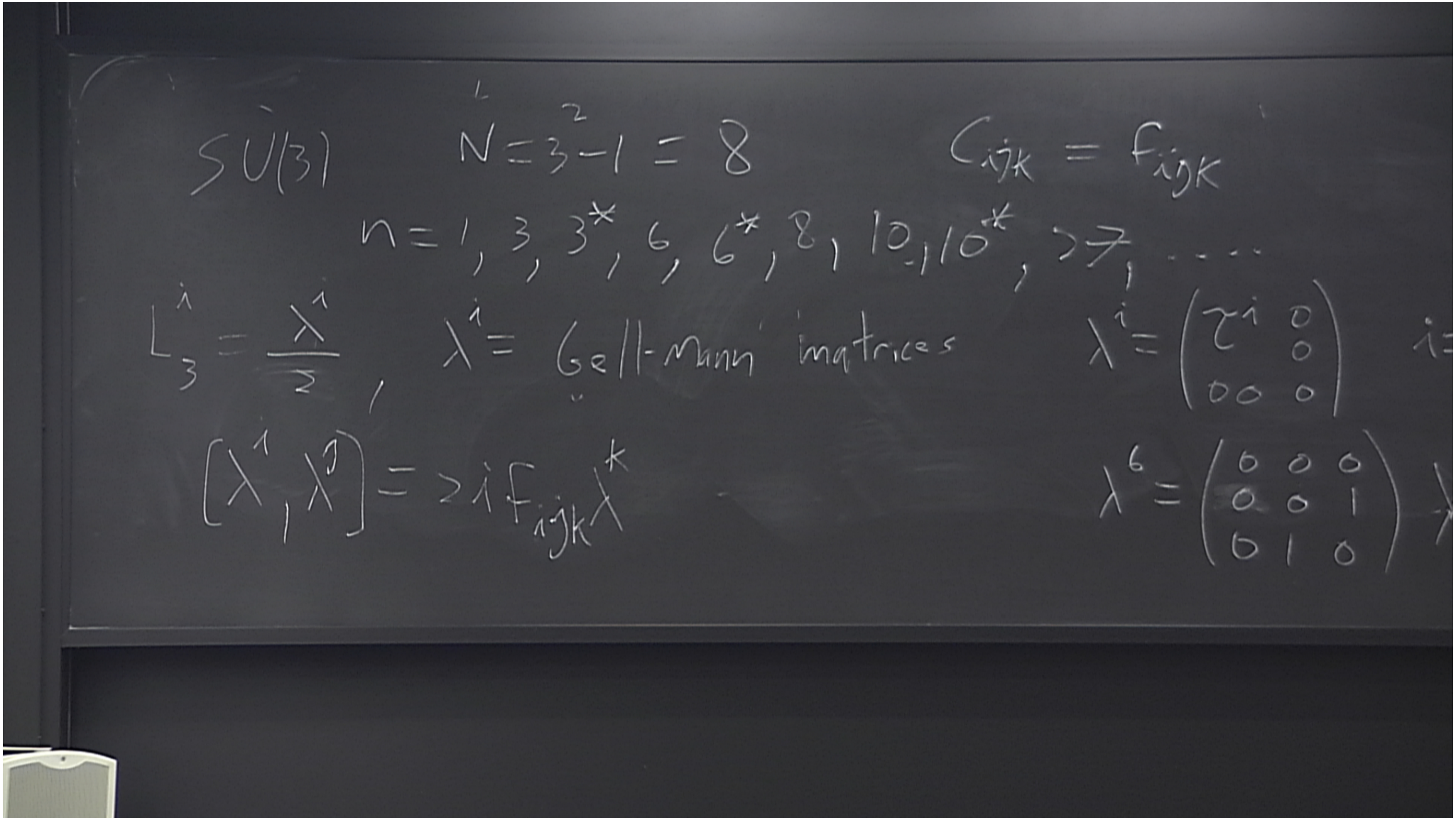

$\mathrm{Tr}(L_2^i L_2^j) = \frac{1}{2}\delta^{ij}$
$D(L_2^i)$
$n=2$ is fundamental: $n=1,2,3,4,5$
$L_n^2 = C_2(L_n)\,I = j(j+1)\,I$, n
$j(j+1)$
$\det e^{\left[-i\vec{B}\cdot\frac{\vec{\sigma}}{2}\right]} = e^{-i\mathrm{Tr}\left(\vec{B}\cdot\frac{\vec{\sigma}}{2}\right)}$
$= e^{-i0} = 1$


SU(3) N = 3^2 − 1 = 8
C_ijk = f_ijk
f_123 = 1
f_14x = 1/2
n = 1, 3, 3*, 6, 6*, 8, 10, 10*, 27, ...


SU(3)
N = 3^2 - 1 = 8
C_ijk = f_ijk
f_123 = 1
f_147 = 1/2
n = 1, 3, 3*, 6, 6*, 8, 10, 10*, 27, ...
L_3^i = λ^i / 2 , λ^i = Gell-Mann matrices
λ^i = (τ^i 0 0 / 0 0 0 / 0 0 0)
i = 1, 2, 3


$f_{123} = 1$
$f_{14x} = \frac{1}{2}$
$i = 1, 2, 3$
$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$


$= f_{ijk}$
$f_{123} = 1$
$f_{147} = \frac{1}{2}$
$\lambda^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & 0 \end{pmatrix}$ $i = 1,2,3$
$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$
$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

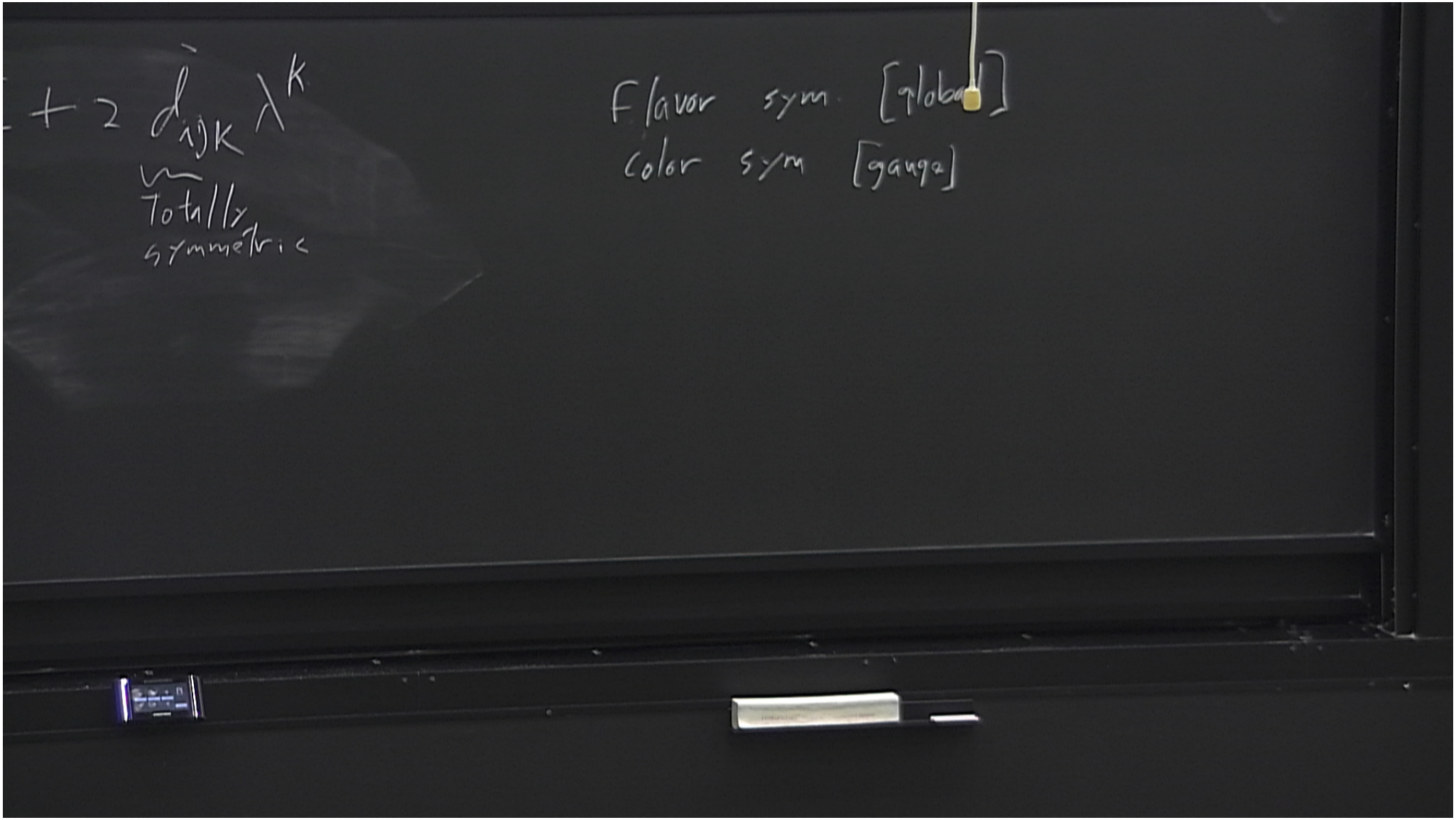

SU(3)
N = 3^2 - 1 = 8
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n = 1, 3, 3*, 6, 6*, 8, 10, 10*, 27, ...
L_3^i = λ^i / 2 , λ^i = Gell-Mann matrices
λ^i = (τ^i 0 / 0 0)
[λ^i, λ^j] = 2i f_ijk λ^k
λ^6 = (0 0 0 / 0 0 1 / 0 1 0)

$$\mathrm{Tr}\left[\frac{\lambda^i}{2}\frac{\lambda^j}{2}\right] = \frac{1}{2}\delta^{ij}$$

$T(L_3)$

$$\{\lambda^i, \lambda^j\} = \frac{4}{3}\delta^{ij} I + 2 d^{ijk}\lambda^k$$

d^{ijk} — Totally symmetric


+ 2 d_ijk λ^k
Totally symmetric
Flavor sym [global]
Color sym [gauge]

$\mathrm{Tr}\left[\frac{\lambda^i}{2}\frac{\lambda^j}{2}\right] = \frac{1}{2}\delta^{ij}$

$T(L_3)$

$\{\lambda^i, \lambda^j\} = \frac{4}{3}\delta^{ij} I + 2 d_{ijk}$

Total

symm

rank 2

$n=1$, $\vec{L}_1 = 0$ $\quad e^{-i\vec{B}\cdot\vec{L}} = 1$

$n=N$ adjoint: $\left(\hat{L}_{adj}\right)_{jk} = -i\, C_{ijk}$

$\delta^{ij} I + 2 d_{ijk} \lambda^k$

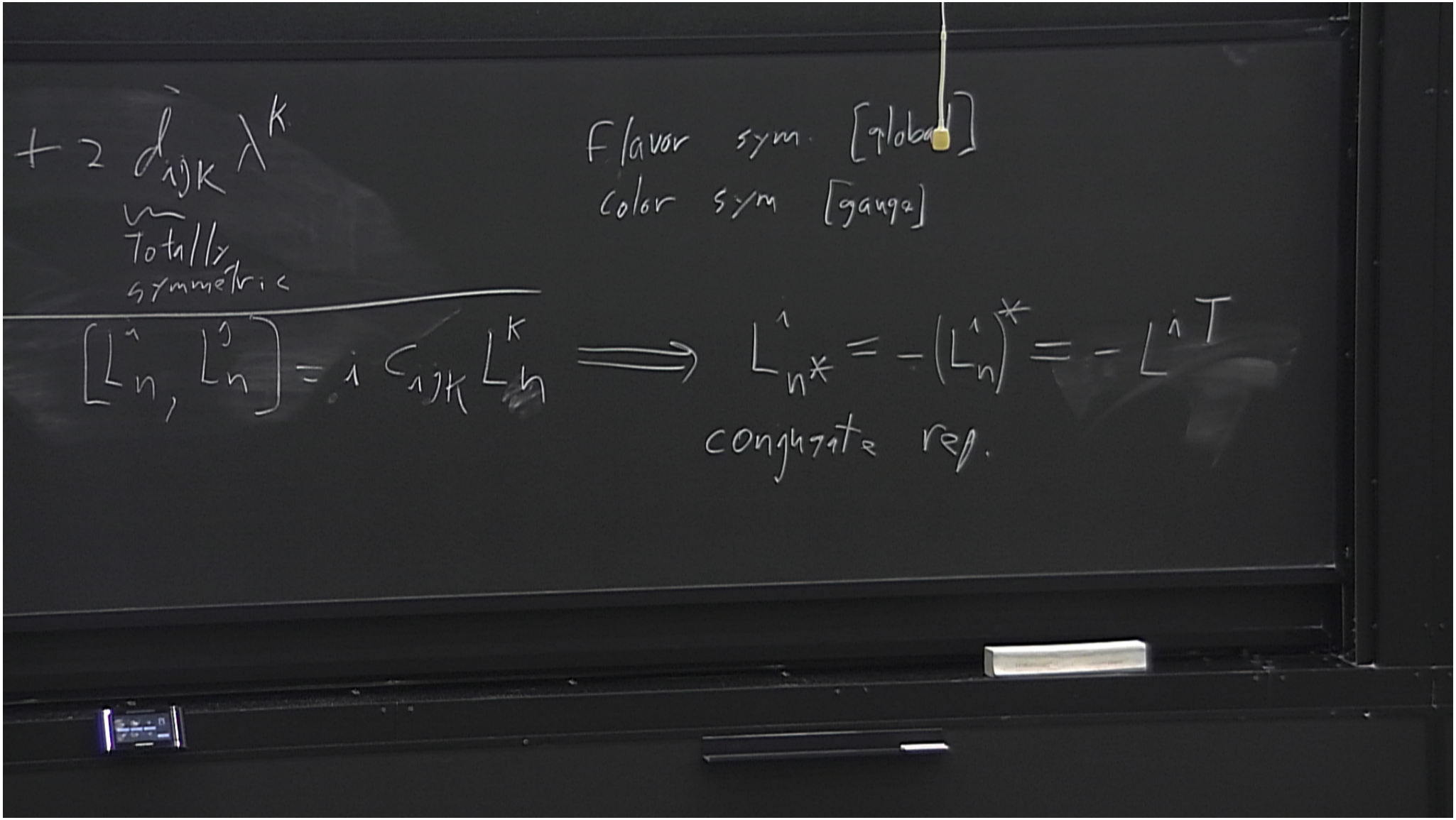
d_{ijk} Totally symmetric

Flavor sym. [global]

Color sym [gauge]

$L_n^i \quad [L_n^i, L_n^j] = i C_{ijk} L_n^k \Longrightarrow L_{n*}^i = -(L_n^i)^*$ is a rep.

conjugate rep.


$+2 d_{ijk} \lambda^k$
Totally symmetric
Flavor sym. [global]
Color sym [gauge]
$[L_n^i, L_n^j] = i C_{ijk} L_n^k \Longrightarrow L_{n*}^i = -(L_n^i)^* = -L^{iT}$
conjugate rep.

$\frac{4}{3}\delta^{ij} I + 2 d_{ijk} \lambda^k$

Totally symmetric

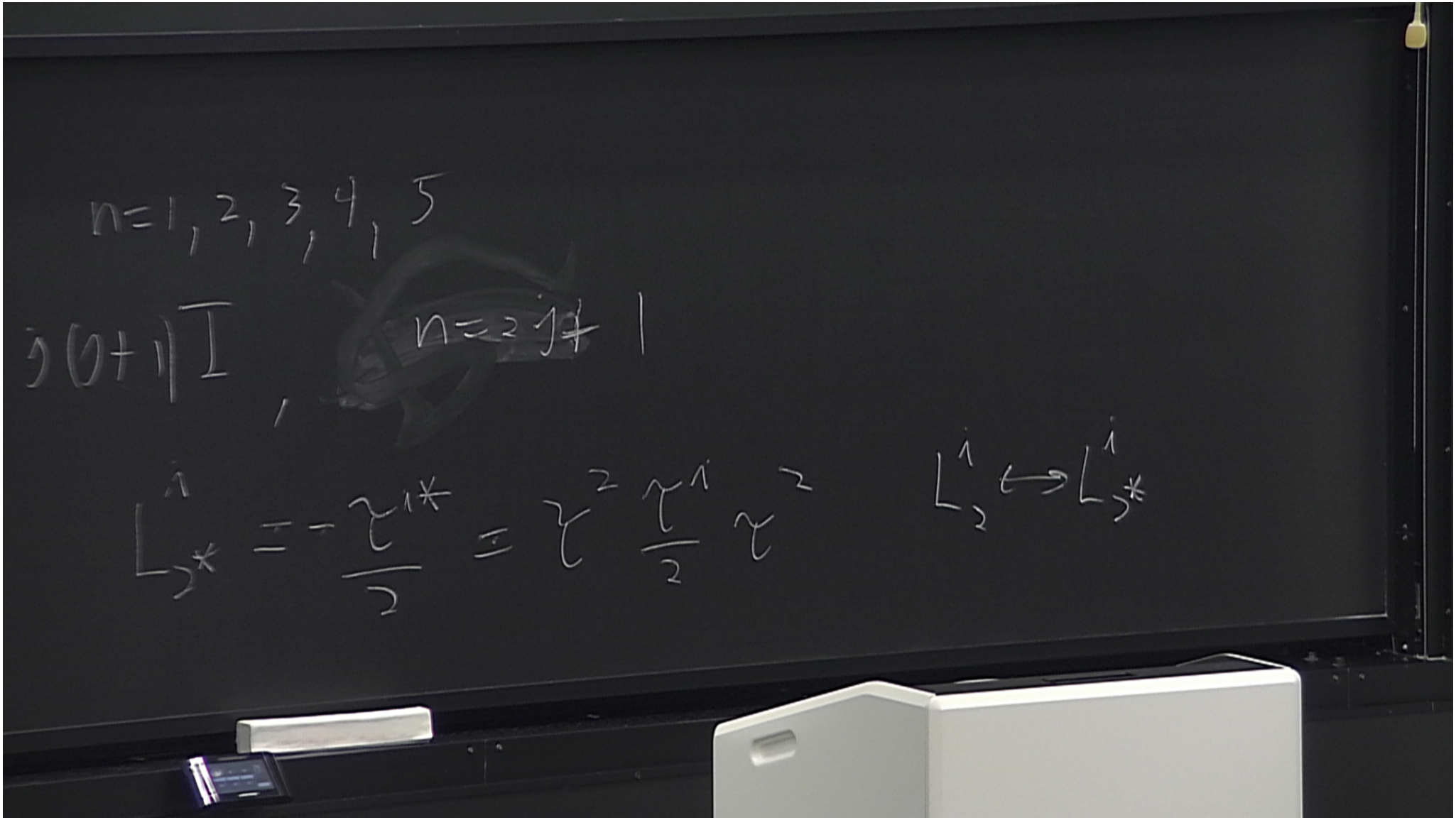
Flavor sym [global]

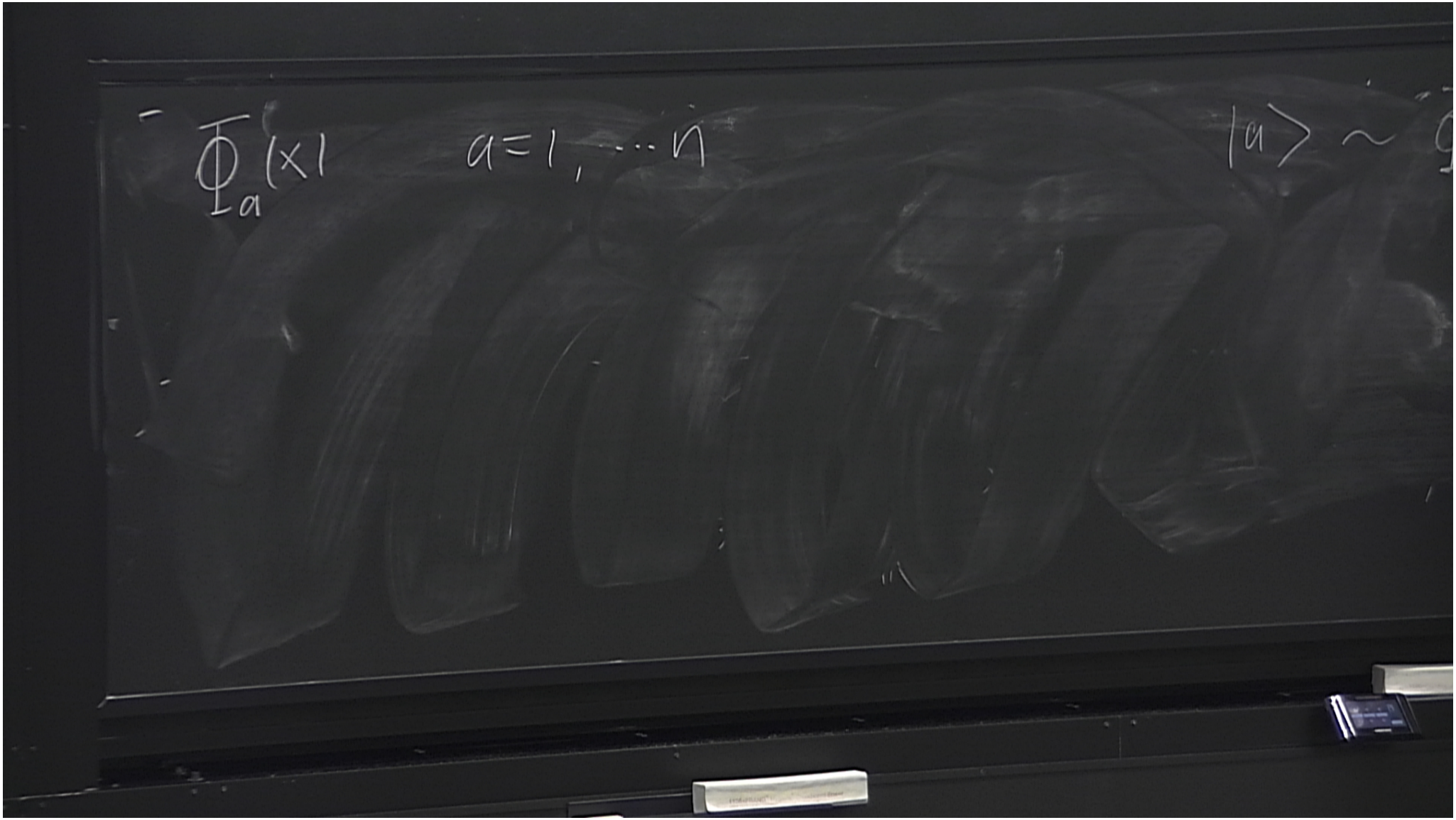
Color sym [gauge]

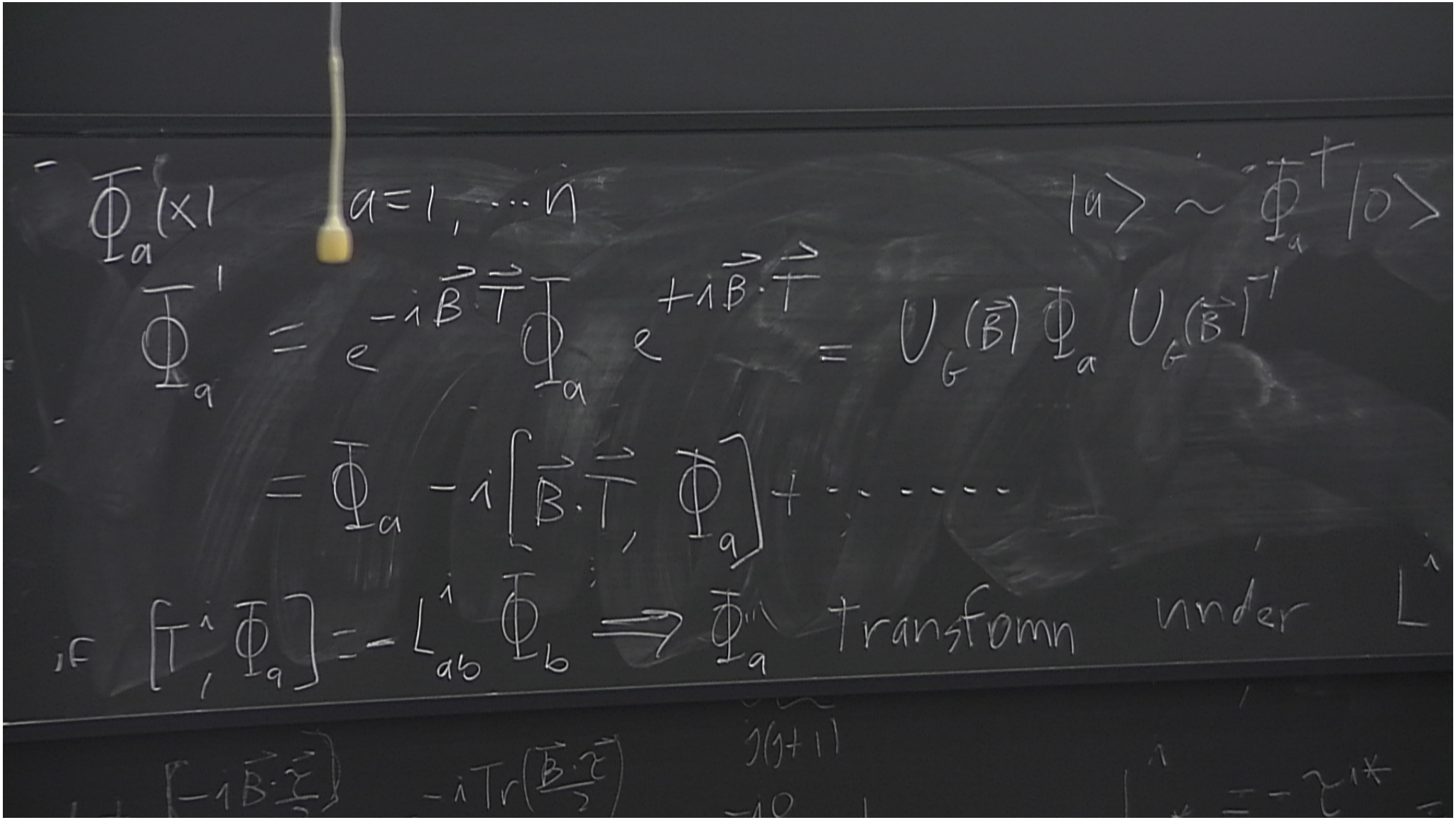
$L_n^i \quad [L_n^i, L_n^j] = i C_{ijk} L_n^k \Longrightarrow L_{n*}^i = -(L_n^i)^* = -L^{iT}$

conjugate rep.

if $L_{n*}^i = U L_n^i U^{-1} \Longrightarrow L_n^i$ is real


n=1, 2, 3, 4, 5
n=2 j≠1


$\Phi_a(x)$ $a=1,\cdots n$
$|a\rangle \sim$


$\Phi_a(x)$ $a=1,\cdots n$
$|a\rangle \sim \Phi_a^\dagger |0\rangle$
$\Phi_a' = e^{-iB\cdot T}\Phi_a e^{+iB\cdot T} = U_G(B)\Phi_a U_G(B)^{-1}$
$= \Phi_a - i[B\cdot T, \Phi_a] + \cdots\cdots$
if $[T_i, \Phi_a] = -L^i_{ab}\Phi_b \Rightarrow \Phi_a$ transfomn under L^i

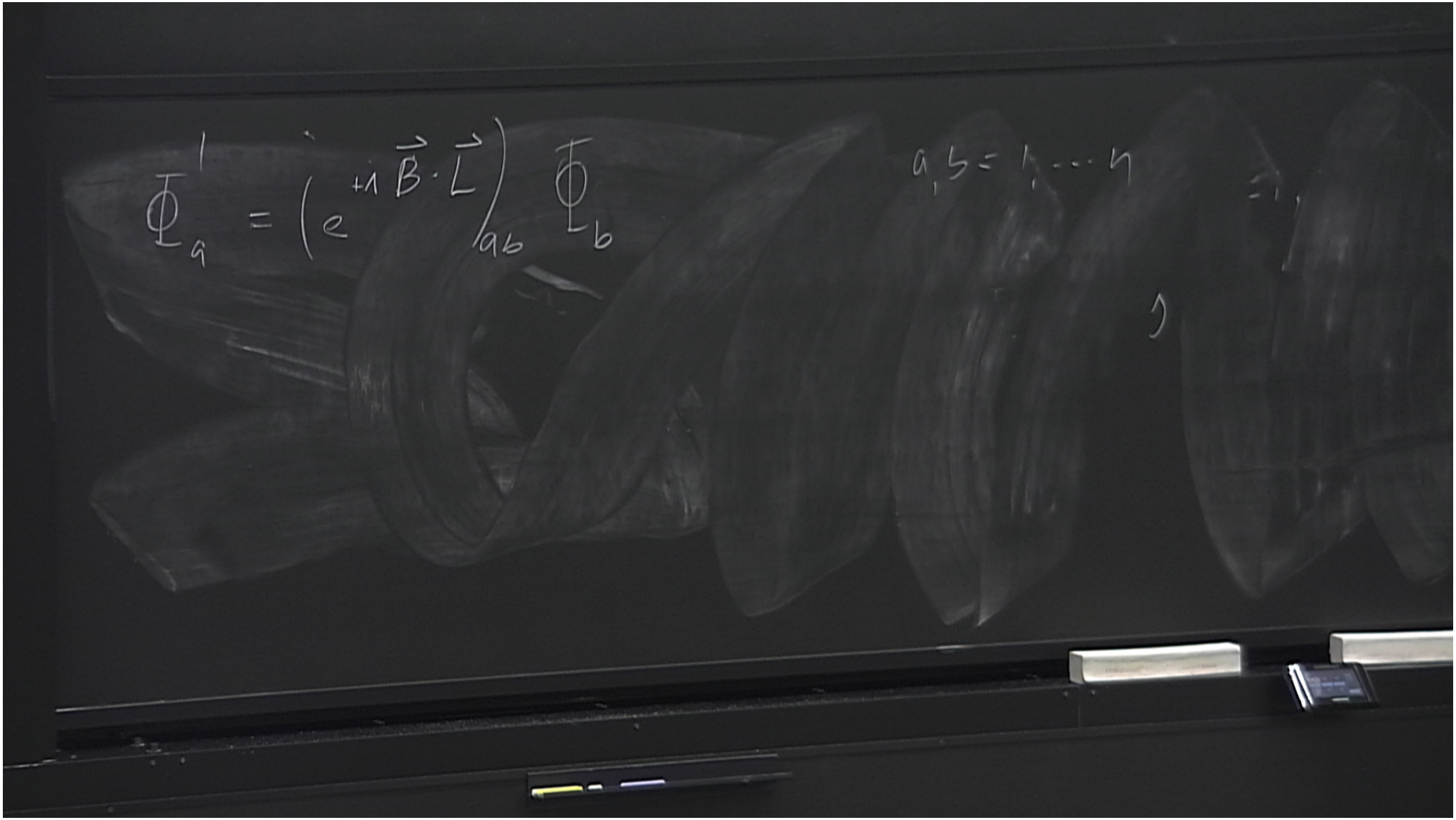
$\Phi_a(x)$ $\quad a = 1, \cdots n$

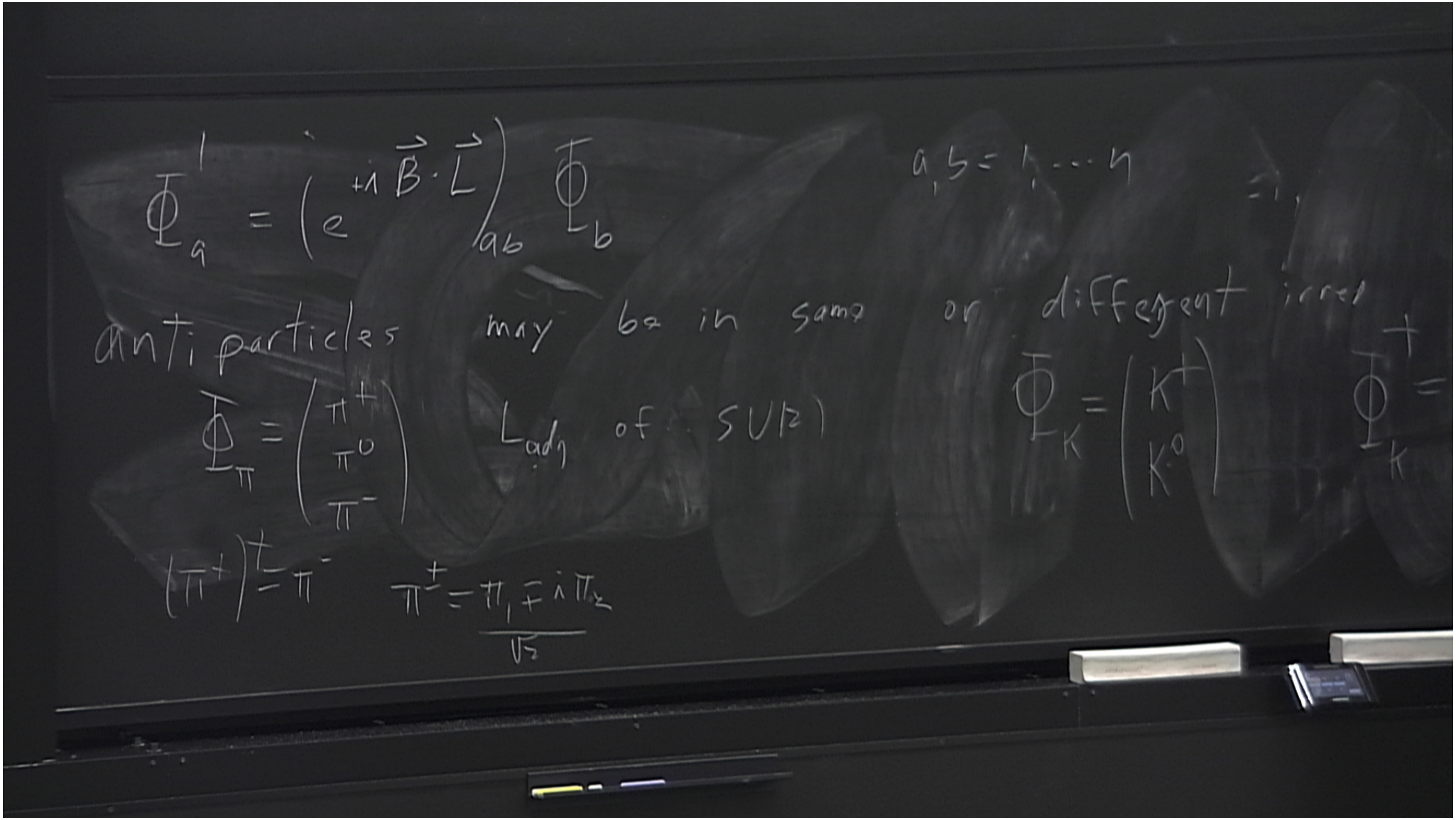
$|a\rangle \sim \Phi_a^\dagger |0\rangle$

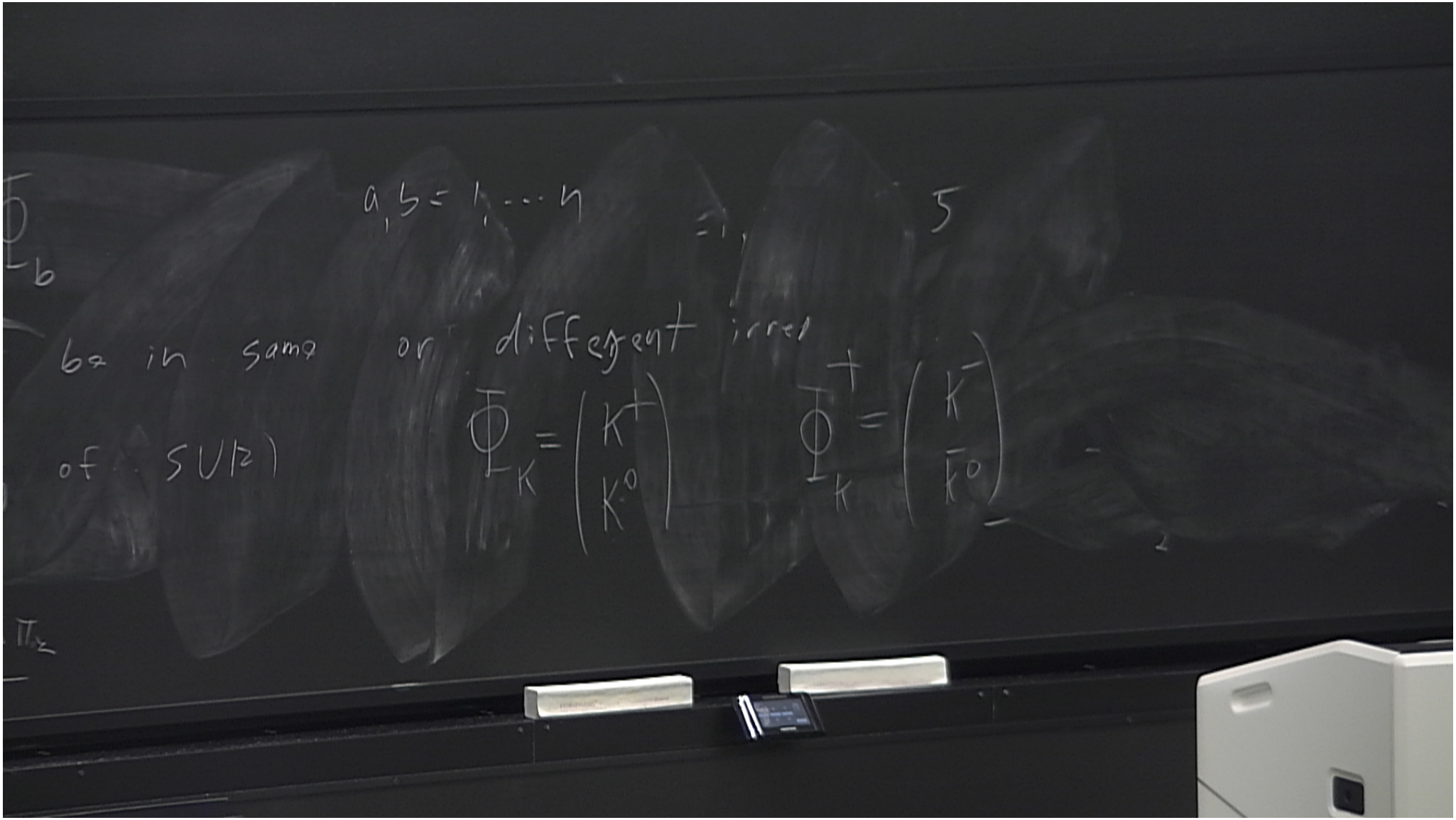
$$\Phi_a' = e^{-i\vec{B}\cdot\vec{T}} \Phi_a \, e^{+i\vec{B}\cdot\vec{T}} = U_G(\vec{B})\, \Phi_a \, U_G(\vec{B})^{-1}$$

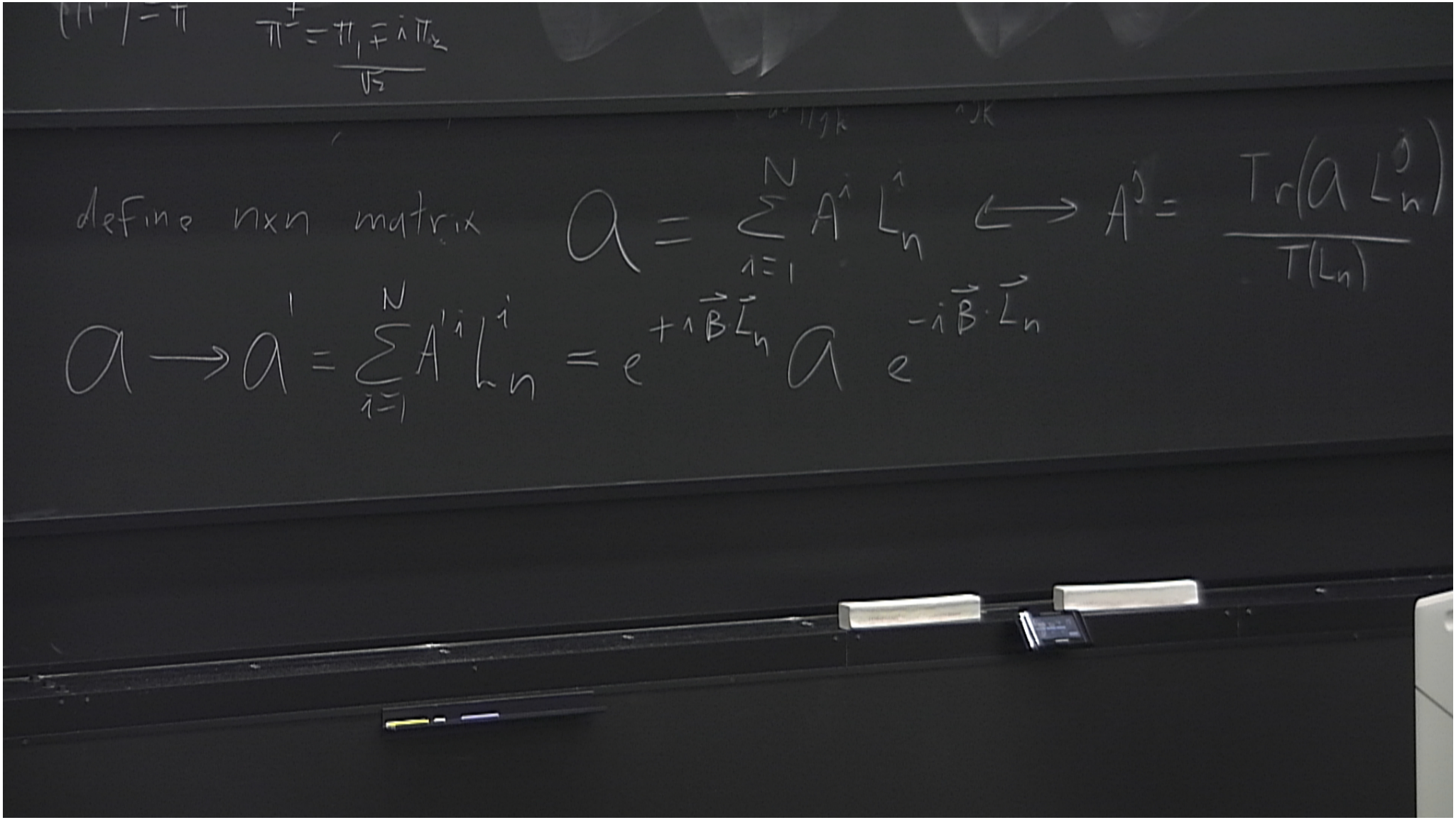
$$= \Phi_a - i\left[\vec{B}\cdot\vec{T}, \Phi_a\right] + \cdots\cdots$$

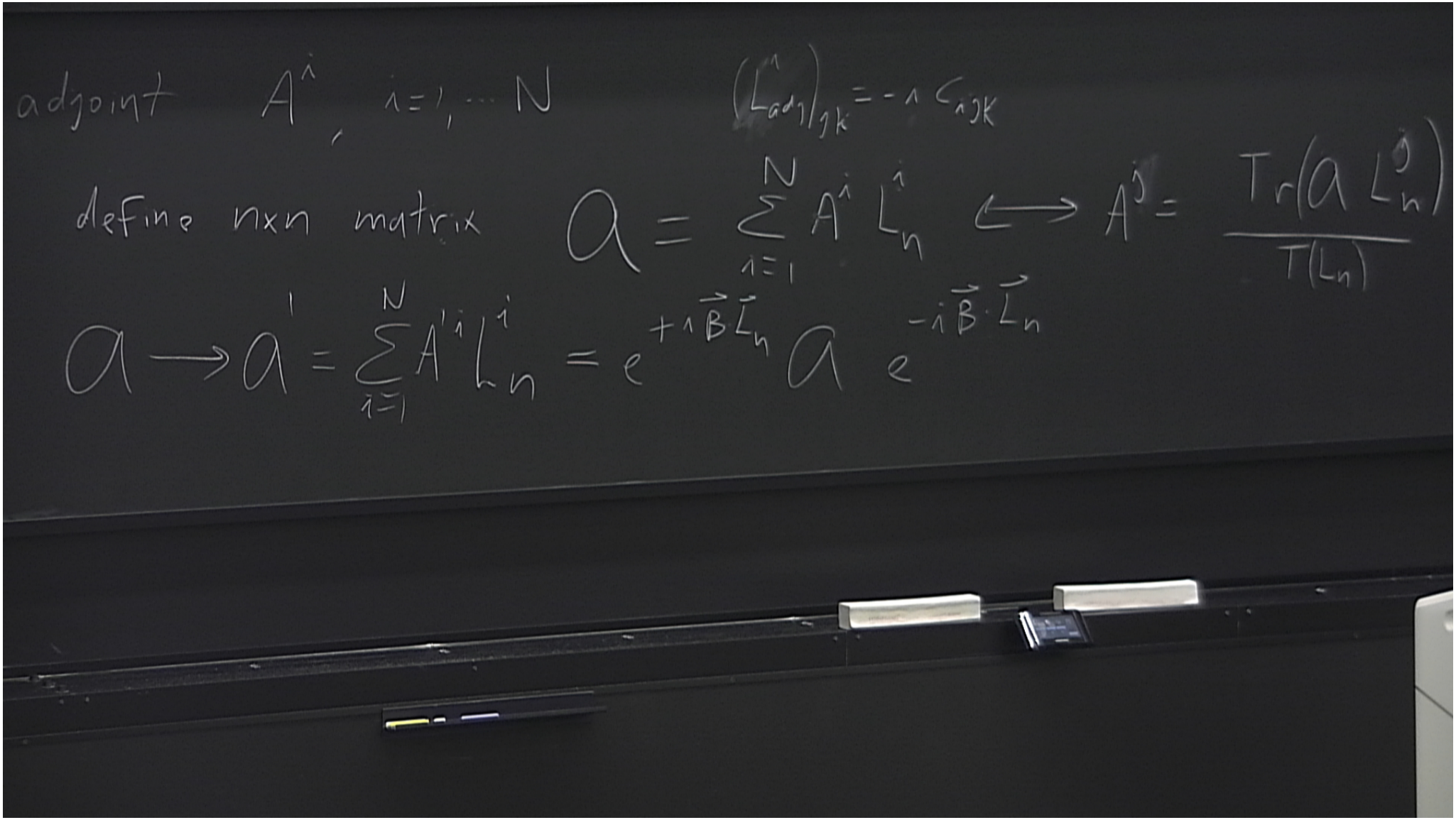
IF $\left[\hat{T}, \Phi_a\right] = -(\hat{L}_n)_{ab}\, \Phi_b \Rightarrow \Phi_a$ transfomn under $\hat{L}_n$


$\Phi'_a = \left(e^{+i\vec{B}\cdot\vec{L}}\right)_{ab} \Phi_b$
$a,b = 1,\dots n$


$\Phi'_a = \left(e^{+i\vec{B}\cdot\vec{L}}\right)_{ab} \Phi_b$
$a,b = 1,\dots n$
antiparticles may be in same or different irrep
$\Phi_\pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$
Lagrangian of SU(2)
$\Phi_K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$
$\Phi_K^\dagger =$
$(\pi^+)^\dagger = \pi^-$
$\pi^\pm = \frac{\pi_1 \mp i\pi_2}{\sqrt{2}}$


$a, b = 1, \dots n$
$= 1, \dots 5$
b_a in same or different irrep
of SU(2)
$\Phi_K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$
$\bar{\Phi}_K = \begin{pmatrix} K^- \\ \bar{K}^0 \end{pmatrix}$


define nxn matrix
$a = \sum_{i=1}^{N} A^i L_n^i \longleftrightarrow A^j = \frac{Tr(a L_n^j)}{T(L_n)}$
$a \to a' = \sum_{i=1}^{N} A'^i L_n^i = e^{+i\vec{B}\vec{L}_n} a\, e^{-i\vec{B}\cdot\vec{L}_n}$

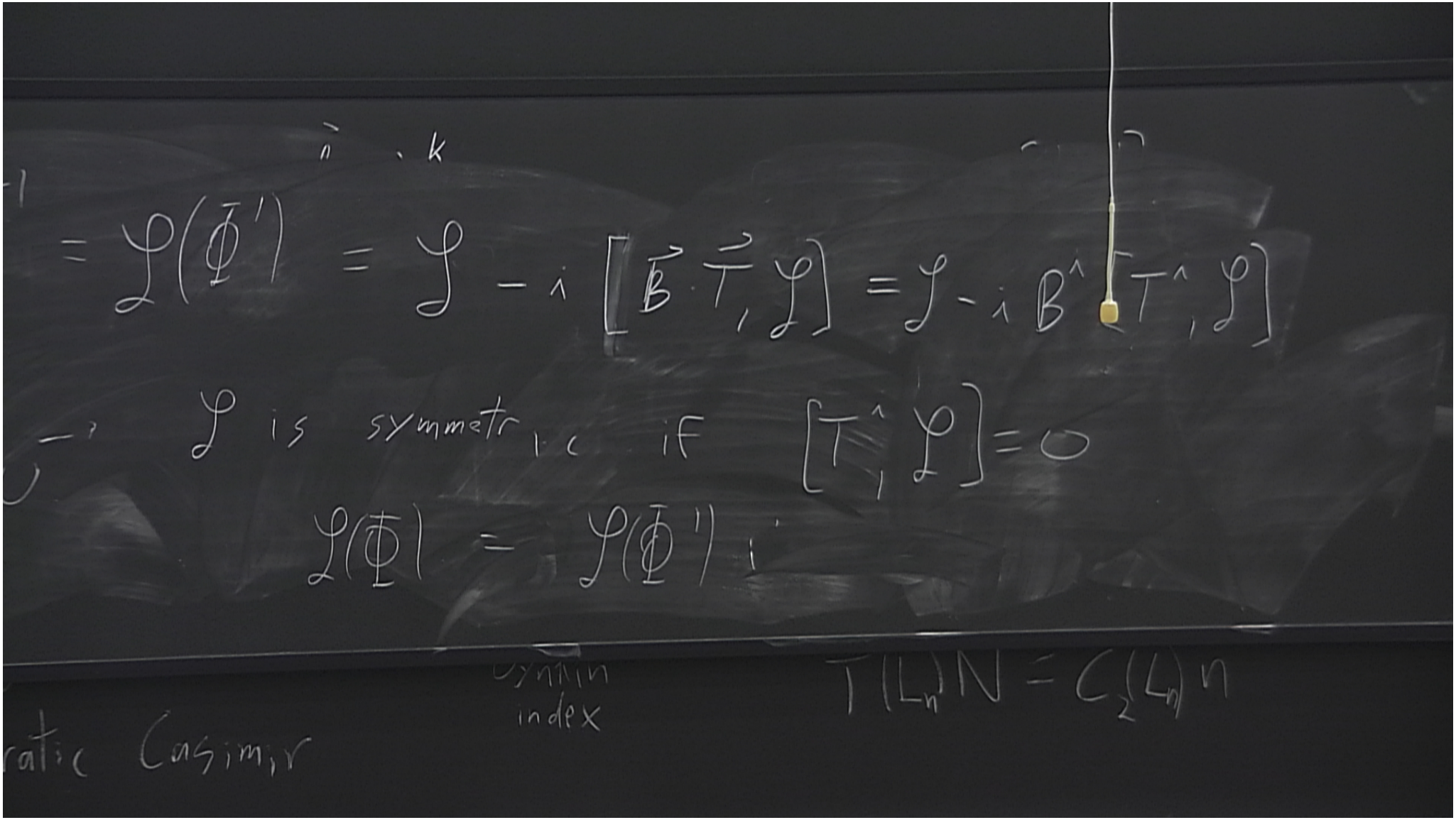

adjoint A^i, $i=1,\dots N$
$(L^i_{adj})_{jk} = -i C_{ijk}$
define nxn matrix $a = \sum_{i=1}^{N} A^i L^i_n \longleftrightarrow A^j = \frac{Tr(a L^j_n)}{T(L_n)}$
$a \to a' = \sum_{i=1}^{N} A'^i L^i_n = e^{+i\vec{B}\vec{L}_n} a\, e^{-i\vec{B}\cdot\vec{L}_n}$

$$\mathcal{L}(\vec{\Phi}) \longrightarrow \mathcal{L}' = U_L(\vec{B})\, \mathcal{L}\, U_L(\vec{B})^{-1} = \mathcal{L}(\vec{\Phi}')$$

$$U \Phi_a \Phi_b U^{-1} = U \Phi U^{-1} U \Phi U^{-1}$$

$$= \Phi'_a \Phi'_b$$

$$\vec{L}^2 = L^i L^i = C_2(L)\, \mathbb{I} \qquad C_2(L) = \text{quadratic Casimir}$$


$\mathcal{L}(\Phi') = \mathcal{L} - i[\vec{B}\cdot\vec{T}, \mathcal{L}] = \mathcal{L} - i\hat{B}\cdot[\hat{T}, \mathcal{L}]$
$\mathcal{L}$ is symmetric if $[\hat{T}, \mathcal{L}] = 0$
$\mathcal{L}(\Phi) = \mathcal{L}(\Phi')$
$T(L_n)N = C_2(L_n)n$
index

$\mathcal{L}(\Phi) = \mathcal{L}(\Phi')$

Noether current

$$\partial_\mu J^\mu = 0$$

$$Q = \int d^3\vec{x}\, J^0(\vec{x})$$

$$J^\mu = \bar{\psi}_a \gamma^\mu L^{\wedge}_{ab} \psi_b = \bar{\psi} \gamma^\mu L^{\wedge} \psi$$