Title: 13/14 PSI - Standard Model Review - Lecture 2

Date: Jan 07, 2014 09:00 AM

URL: http://pirsa.org/14010009

Abstract:

1. Introduction to SM
2. QED

Strong

hadrons: $p, n, \pi^{\pm}, \pi^{0}$

QCD: quarks

nuclear binding energy in stars

① Introduction to SM

② QED

Strong

hadrons: $p, n, \pi^{\pm}, \pi^{0}$

QCD: quarks

nuclear binding energy in stars

Electromagnetic

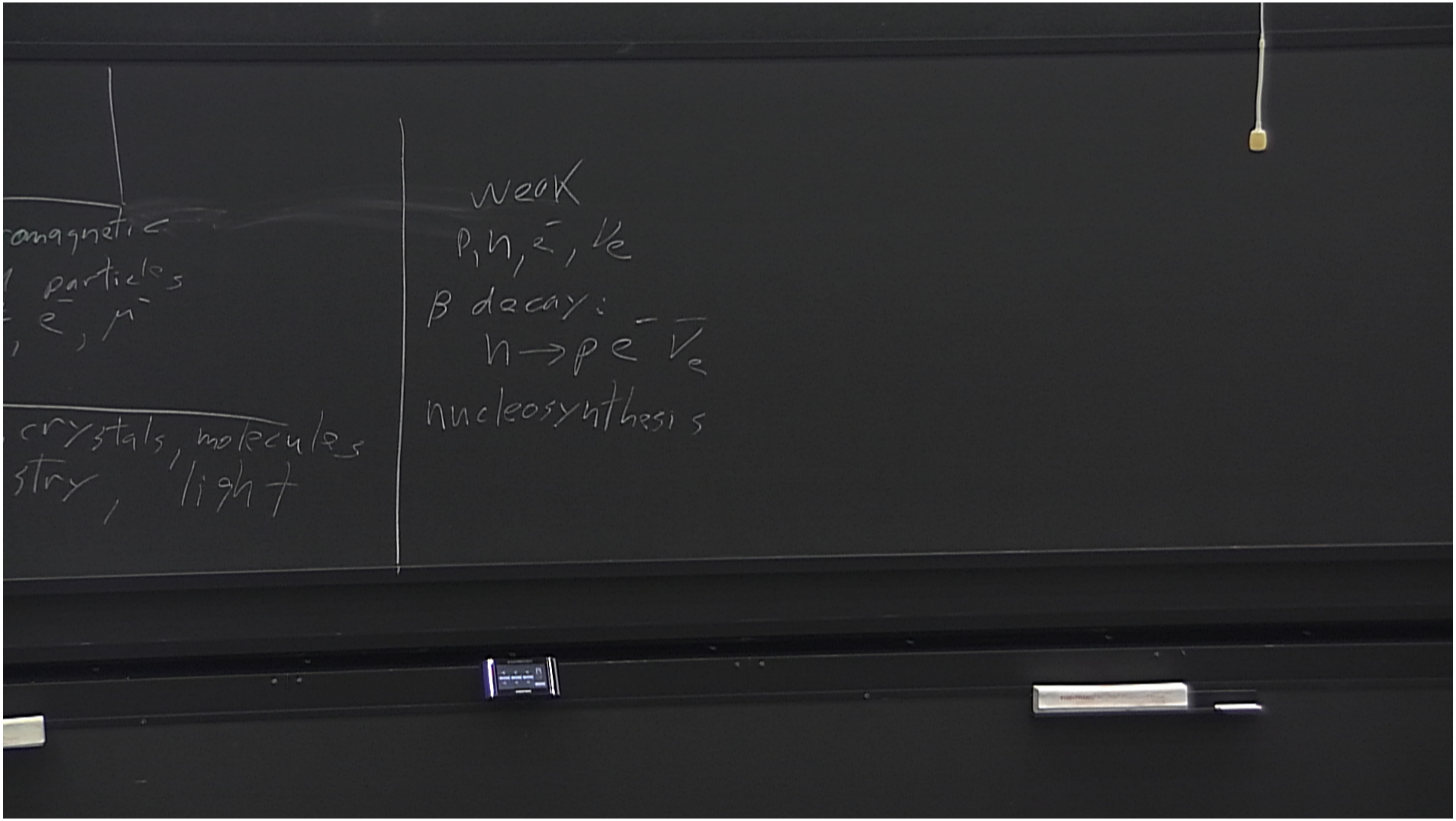
charged particles

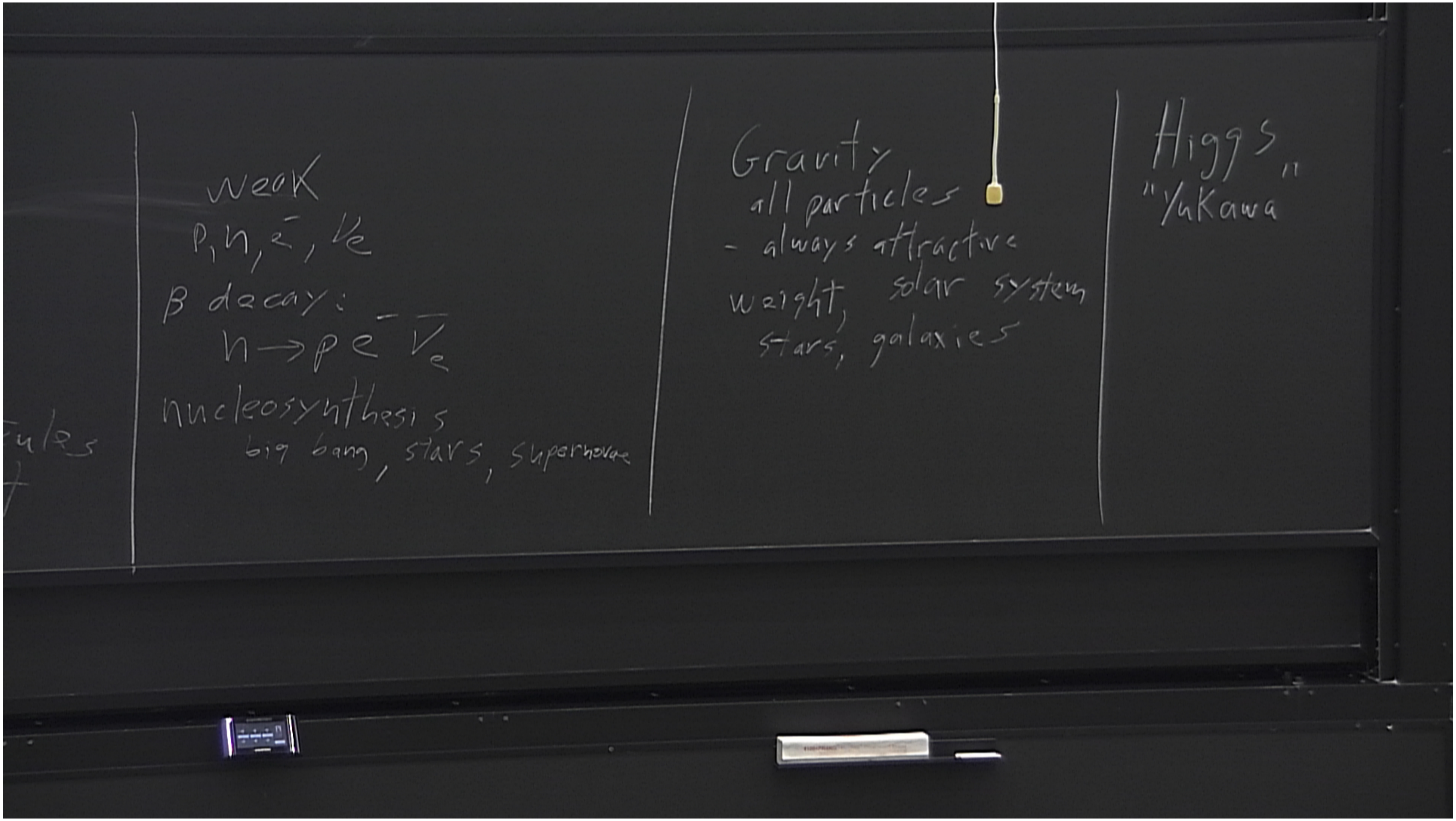
$p, \pi^{\pm}, e^{-}, \mu^{-}$

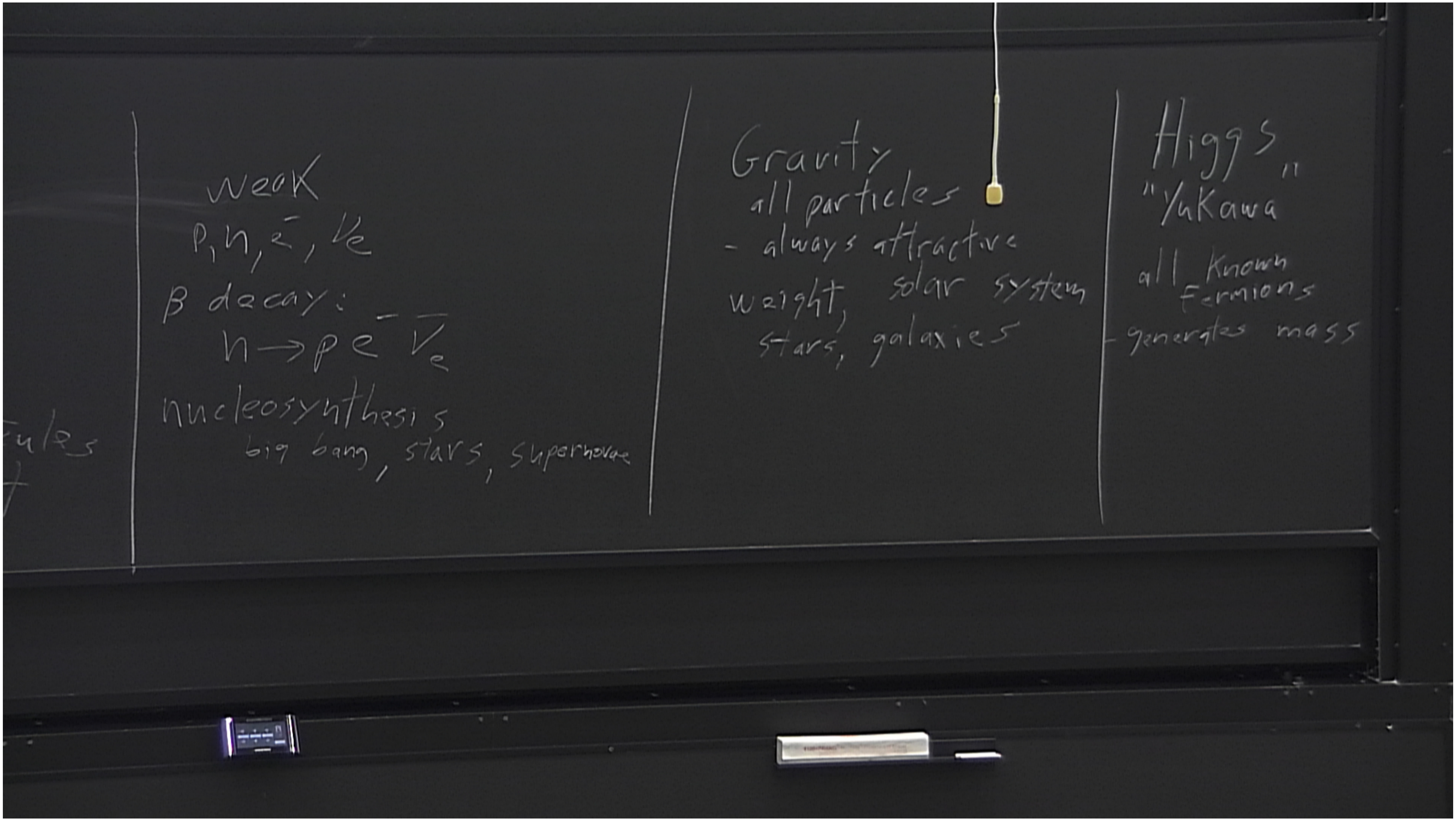
atoms, crystals, molecules

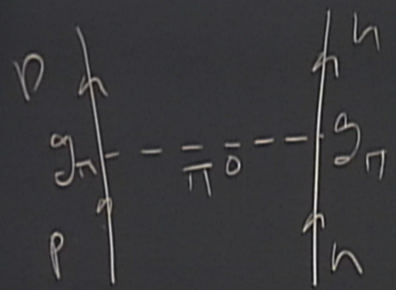
chemistry, light

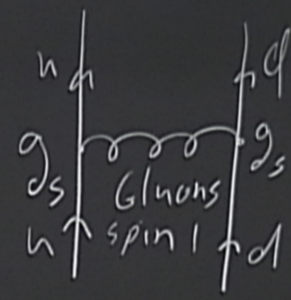
weak


omagnetic
particles
crystals, molecules
stry, light
weak
p, n, e⁻, νe
β decay:
n → p e⁻ ν̄e
nucleosynthesis


weak
p, n, e⁻, νe
β decay:
n → p e⁻ ν̄e
nucleosynthesis
big bang, stars, supernovae
Gravity
all particles
- always attractive
weight, solar system
stars, galaxies
Higgs
"Yukawa"


ules
weak
p, n, e⁻, νe
β decay:
n → p e⁻ ν̄e
nucleosynthesis
big bang, stars, supernovae
Gravity
all particles
- always attractive
weight, solar system
stars, galaxies
Higgs
"Yukawa"
all known fermions
- generates mass





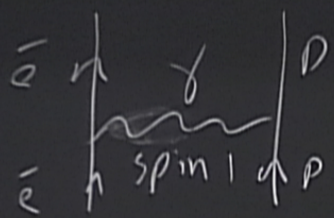
$$V = \frac{g_\pi^2 \, e^{-m_\pi r}}{r}$$

$$\frac{g_s^2}{4\pi} \sim 0.10$$

[at $E \sim 100$ GeV]

strength $\frac{g_\pi^2}{4\pi} \sim 14$

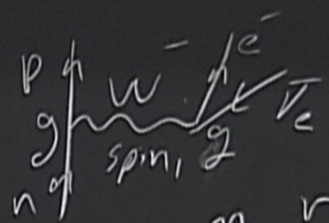
range: $\frac{1}{m_\pi} \sim 10^{-13}$ cm $= 1$ fm



$$V = \frac{e^2}{r}$$

$$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

range ∞

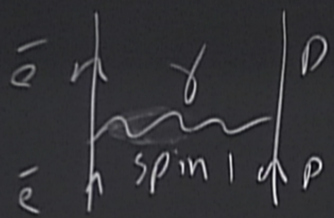


$$g^2 \frac{e^{-M_W r}}{r}$$

$$\frac{g^2 E^2}{M_W^2} \sim 10^{-11}$$ For $E \sim 1$ MeV

$M_W \sim 80$ GeV

range $\frac{1}{M_W} \sim 10^{-16}$ cm



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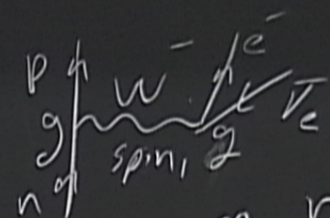
range ∞

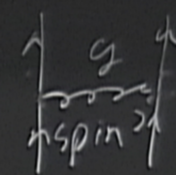


$$g^2 \frac{e^{-M_W r}}{r}$$

$$\frac{g^2 E^2}{M_W^2} \sim 10^{-11} \quad \text{For } E \sim 1 \text{ MeV}$$

$M_W \sim 80$ GeV

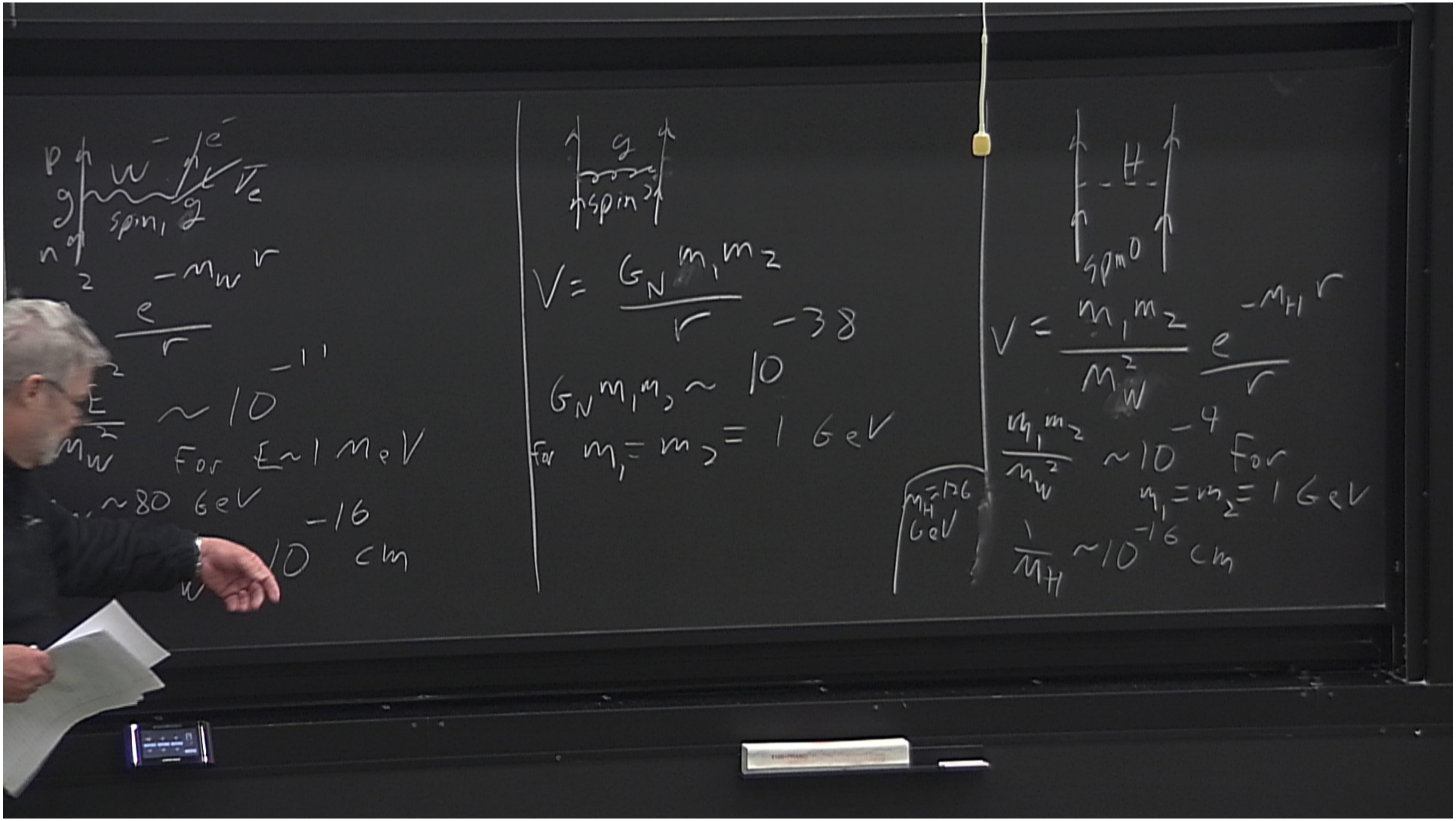
range $\frac{1}{M_W} \sim 10^{-16}$ cm

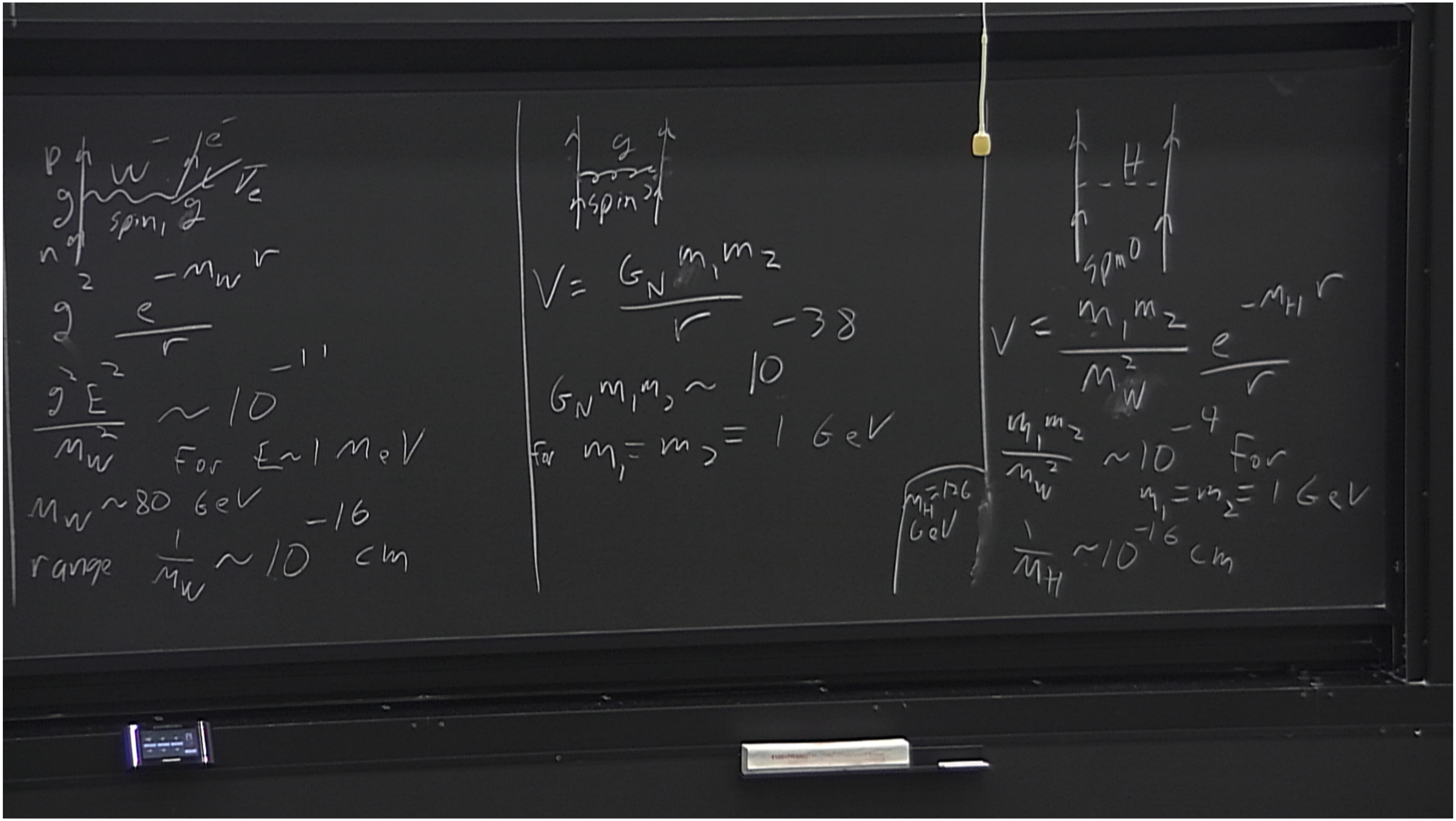


$$V = \frac{G_N m_1 m_2}{r}$$

$$G_N m_1 m_2 \sim 10^{-38}$$

for $m_1 = m_2 = 1$ GeV


p
n
W⁻
e⁻
ν̄e
g
spin 1
g
e^{-m_W r} / r
~ 10^{-11}
For E ~ 1 MeV
~ 80 GeV
10^{-16} cm
g
spin 2
V = G_N m_1 m_2 / r
G_N m_1 m_2 ~ 10^{-38}
for m_1 = m_2 = 1 GeV
H
spin 0
V = (m_1 m_2 / M_W^2) e^{-m_H r} / r
m_1 m_2 / m_W^2 ~ 10^{-4}
for m_1 = m_2 = 1 GeV
1/m_H ~ 10^{-16} cm
m_H = 126 GeV


p
n
W^-
e^-
$\bar{\nu}_e$
g
spin 1
$g^2 \frac{e^{-m_W r}}{r}$
$\frac{g^2 E^2}{m_W^2} \sim 10^{-11}$
For $E \sim 1$ MeV
$m_W \sim 80$ GeV
range $\frac{1}{m_W} \sim 10^{-16}$ cm
g
spin 2
$V = \frac{G_N m_1 m_2}{r}$
$G_N m_1 m_2 \sim 10^{-38}$
for $m_1 = m_2 = 1$ GeV
H
spin 0
$V = \frac{m_1 m_2}{M_W^2} \frac{e^{-m_H r}}{r}$
$\frac{m_1 m_2}{m_W^2} \sim 10^{-4}$
For $m_1 = m_2 = 1$ GeV
$m_H = 126$ GeV
$\frac{1}{m_H} \sim 10^{-16}$ cm

QED

Free electron of charge $q_{e^-} = -e$

$$E_p = \sqrt{\vec{p}^{\,2} + m^2}$$

$$\psi(x) = \int \frac{d^3\vec{p}}{(2\pi)^3\, 2E_p}$$

$$\mathcal{L}_0 = \bar{\psi}\,(i\not{\partial} - m)\,\psi$$

ψ = 4 component spinor ψ_a

$$\not{\partial} = \gamma^\mu \frac{\partial}{\partial x^\mu} = \gamma^\mu \partial_\mu \qquad \gamma^\mu = 4\times 4 \text{ Di}$$

$$\bar{\psi} = \psi^\dagger \gamma^0 = \text{Dirac adjoint}$$

$$\mathcal{L}_0 = \bar{\psi}\,(i\not{\partial} - m)\,\psi$$

ψ = 4 component spinor ψ_α

$$\not{\partial} = \gamma^\mu \frac{\partial}{\partial x^\mu} = \gamma^\mu \partial_\mu$$

γ^μ = 4x4 Dirac matrices ; $\{\gamma^\mu, \gamma^\nu$

$\bar{\psi} = \psi^\dagger \gamma^0$ = Dirac adjoint

$$= \sum_{s=1}^{2} \left[u(p,s)\, a(\vec{p},s)\, e^{-ip\cdot x} + V(\vec{p},s)\, b^\dagger(\vec{p},s)\, e^{+ip\cdot x} \right]$$

$$g_{\mu\nu} = (1,-1,-1,-1)$$

$= 4\times 4$ Dirac matrices; $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$, $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$

$= \gamma_\mu$

$$, s)\, b^\dagger(\vec{p}, s)\, e^{+ip\cdot x} \Big]$$

$$(\not p - m)\, u(\vec p, s) = (\not p + m)\, v(\vec p, s) = 0$$

$$a^\dagger(\vec p, s)|0\rangle = |e^-(\vec p, s)\rangle \qquad b^\dagger(\vec p, s)|0\rangle = |e^+(\vec p, s)\rangle$$

Global U(1) symmetry: $\psi(x) \longrightarrow \psi'(x) = e^{-i\beta}\,\psi(x)$

$$\psi^\dagger(x) \rightarrow \psi'^\dagger(x) = e^{+i\beta}\,\psi^\dagger(x)$$

$B = \text{real}$

$e^{\dagger}(\vec{p}, s)\rangle$

$$\mathcal{L}_0(\psi, \psi^\dagger) = \mathcal{L}_0(\psi', \psi'^\dagger)$$

invariance

$= e^{-iB}\psi(x)$

$= e^{+iB}\psi^\dagger(x)$

$$(\not p - m)\, u(\vec p, s) = (\not p + m)\, v(\vec p, s) = 0$$

$$a^\dagger(\vec p, s)|0\rangle = |e^-(\vec p, s)\rangle \qquad b^\dagger(\vec p, s)|0\rangle = |e^+(\vec p, s)\rangle$$

Global U(1) symmetry: $\psi(x) \longrightarrow \psi'(x) = e^{-iB}\,\psi(x)$

$U(1) = 1\times 1$ unitary matrices $\quad \psi^\dagger(x) \to \psi'^\dagger(x) = e^{+iB}\,\psi^\dagger(x)$

$$e^{-i\beta} e^{-i\alpha} = e^{-i(\beta+\alpha)}$$

$\beta = \text{real}$

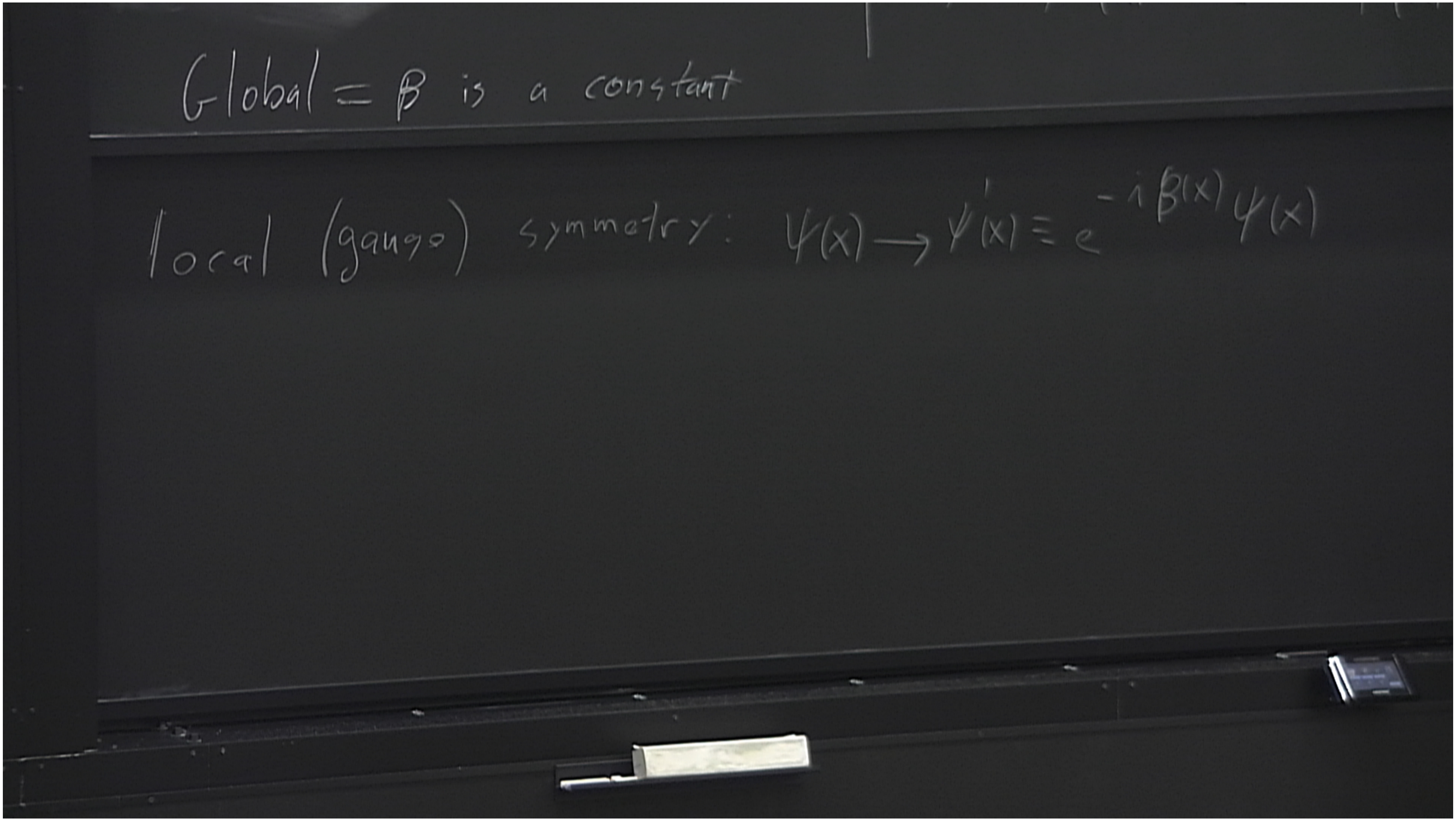
$= |e^{+}(\vec{p}, s)\rangle$

$\psi(x) = e^{-i\beta} \psi(x)$

$\mathcal{L}_0(\psi, \psi^{+}) = \mathcal{L}_0$

$^{+}(x) = e^{+i\beta} \psi^{+}(x)$

invaria


Global = β is a constant
local (gauge) symmetry: $\psi(x) \to \psi'(x) \equiv e^{-i\beta(x)}\psi(x)$

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$\mathcal{L}_0$ is <u>not</u> invariant. $\mathcal{L}_0(\psi', \psi'^{\dagger}) \neq \mathcal{L}_0(\psi, \psi^{\dagger})$

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local (gauge) symmetry: $\psi(x) \to \psi'(x) \equiv e^{-i\beta(x)}\psi(x)$

$\mathcal{L}_0$ is <u>not</u> invariant. $\mathcal{L}_0(\psi', \psi'^{\dagger}) \neq \mathcal{L}_0(\psi, \psi^{\dagger})$

$$\mathcal{L}_0(\psi') = \overline{\psi'}\left[i\not{\partial}_\mu - m\right]\psi'(x) = \overline{\psi}\, e^{+i\beta(x)}\left[i\not{\partial}_\mu - m\right] e^{-i\beta(x)}\psi$$

$$= e \quad \psi(x)$$

$$\psi, \psi^\dagger)$$

$$- m \Big] e^{-i\beta(x)} \psi = \bar{\psi} \Big[i \not{\partial} - m + \gamma^\mu (\partial_\mu \beta(x)) \Big] \psi(x)$$

$\frac{g_\pi^2}{4\pi} \sim 14$

range: $\frac{1}{m_\pi} \sim 10^{-13}$ cm $= 1$ fm

$M_W \sim 80$ GeV

range $\frac{1}{M_W} \sim 10^{-16}$ cm

minimal e/m. substitution

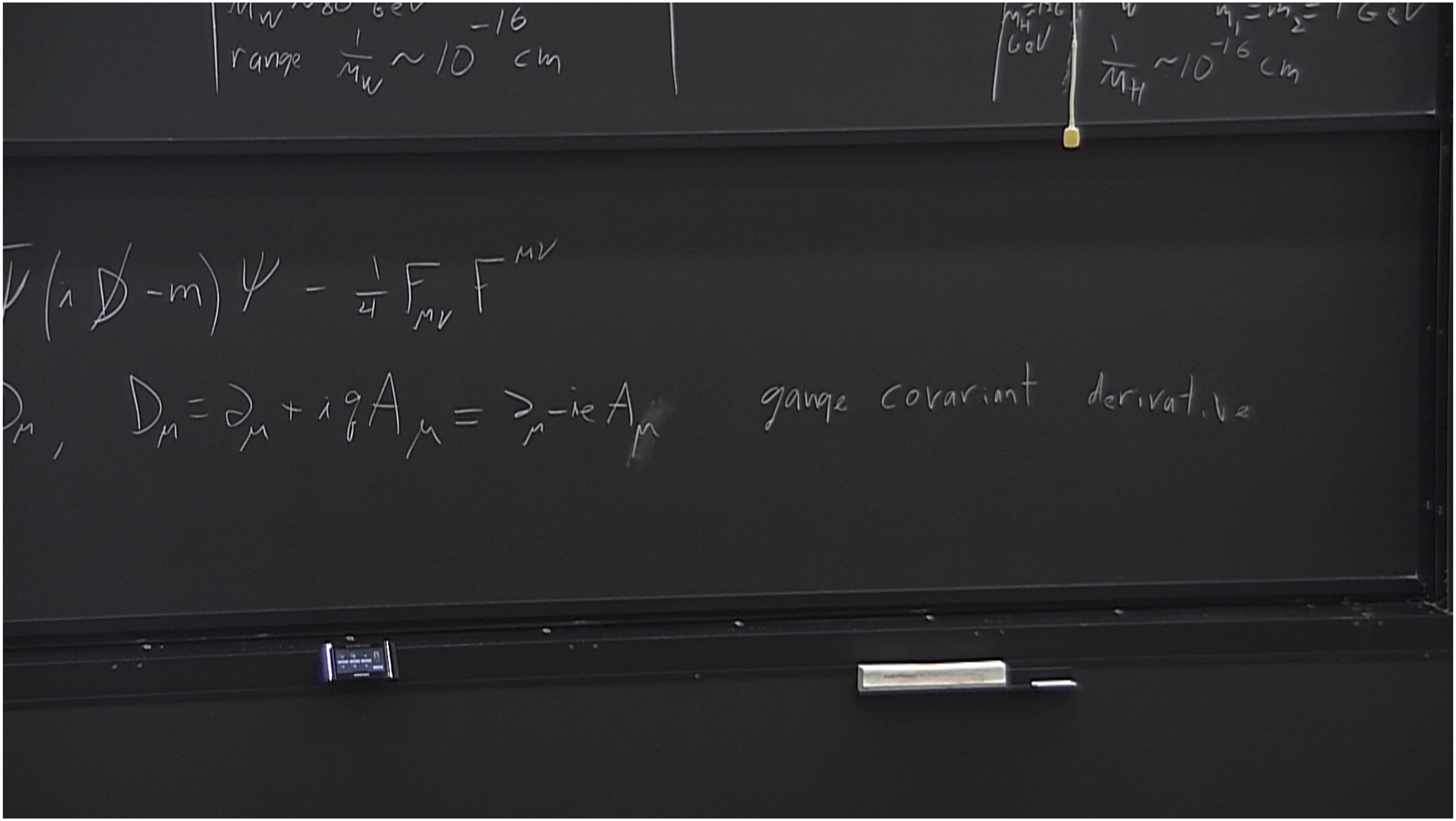
$$p^\mu \longrightarrow p^\mu - qA^\mu$$

$$p^\mu \leftrightarrow i\partial^\mu$$

A^μ = spin-1 vector field (photon)

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\not{D} = \gamma^\mu D_\mu, \quad D_\mu = \partial_\mu + iqA_\mu = \partial_\mu - ieA_\mu$$


$m_W \sim 80$ GeV
range $\frac{1}{m_W} \sim 10^{-16}$ cm
$m_H = 126$ GeV
$m_1 = m_2 = 1$ GeV
$\frac{1}{m_H} \sim 10^{-16}$ cm
$\bar{\psi}(i \not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
D_μ, $D_\mu = \partial_\mu + i q A_\mu = \partial_\mu - ie A_\mu$
gauge covariant derivative

range: $\frac{1}{m_\pi} \sim 10^{-13}\,cm = 1\,fm$

range $\frac{1}{m_W}$

$$p^\mu \longrightarrow p^\mu - qA^\mu$$

$$p^\mu \leftrightarrow i\partial^\mu$$

A^μ = spin-1 vector field (photon)

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}$$

$$\not{D} = \gamma^\mu D_\mu, \quad D_\mu = \partial_\mu + iqA$$

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{e}\partial_\mu\beta$$

range $\frac{}{m_W} \sim 10$ cm $\frac{}{m_H} \sim 10$ cm

$\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$\partial_\mu + igA_\mu = \partial_\mu - ieA_\mu$ gauge covariant derivative

$\partial_\mu \beta$ $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = F'_{\mu\nu}$

$U(1) = 1\times 1$ unitary matrices $\psi^\dagger(x) \to \psi'^\dagger \ldots e^{+iB}\psi^\dagger$

$$D_\mu \psi' = (\partial_\mu - ie A'_\mu)\,\psi' = \left[\partial_\mu - ie\left(A_\mu - \frac{i}{e}\partial_\mu B\right)\right] e^{iB}\,\psi$$

$m_A^2 A^\mu A_\mu$

iB $D_\mu \psi$

pin 1 massless $A^\mu \Longrightarrow$ long range force

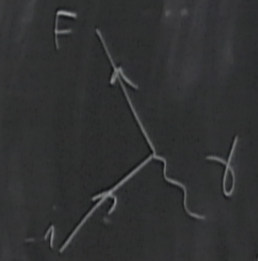
– form prescribed

U(1) = 1×1 unitary matrices | $\psi^\dagger(x) \to \psi'^\dagger \quad +iB$

$$D'_\mu \psi' = (\partial_\mu - ie A'_\mu)\psi' = \left[\partial_\mu - ie\left(A_\mu - \frac{1}{e}\partial_\mu B\right)\right] e^{-iB}\psi = e^{-iB} D_\mu \psi$$

$$\mathcal{L}(\psi', A') = \mathcal{L}(\psi, A)$$

gauge inv requires spin 1 mas

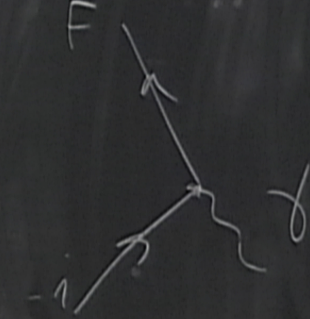


$$\bar{u}_F \, ie\gamma_\mu u_\Lambda \, \epsilon^\mu$$

$$D_\mu \psi' = (\partial_\mu - ie A'_\mu)\psi' = \left[\partial_\mu - ie\left(A_\mu - \frac{1}{e}\partial_\mu B\right)\right] e^{-iB}\psi$$

$$\mathcal{L}(\psi', A') = \mathcal{L}(\psi, A)$$ gauge inv required

e = gauge coupling



a_e = anom. magn moment of e^-

$$\vec{\mu}_e = -g\,\mu_B\,\vec{S} \qquad \mu_B = \frac{e\hbar}{2mc} = \frac{e}{2m}$$

$$\vec{S} = \frac{\vec{\sigma}}{2}$$

$$g = \underset{\text{Dirac}}{\underset{\uparrow}{2}}\left[1 + \underset{\text{QED}}{\underset{\uparrow}{a_e}}\right]$$

(at $E \sim 100$ GeV)

a_e = anom. magn moment of e^-

$$\vec{\mu}_e = -g \mu_B \vec{S}$$

$$\mu_B = \frac{e\hbar}{2mc} = \frac{e}{2m}$$

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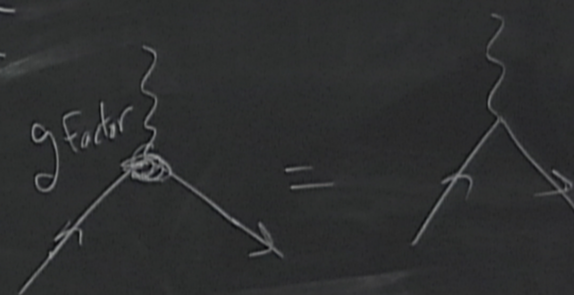
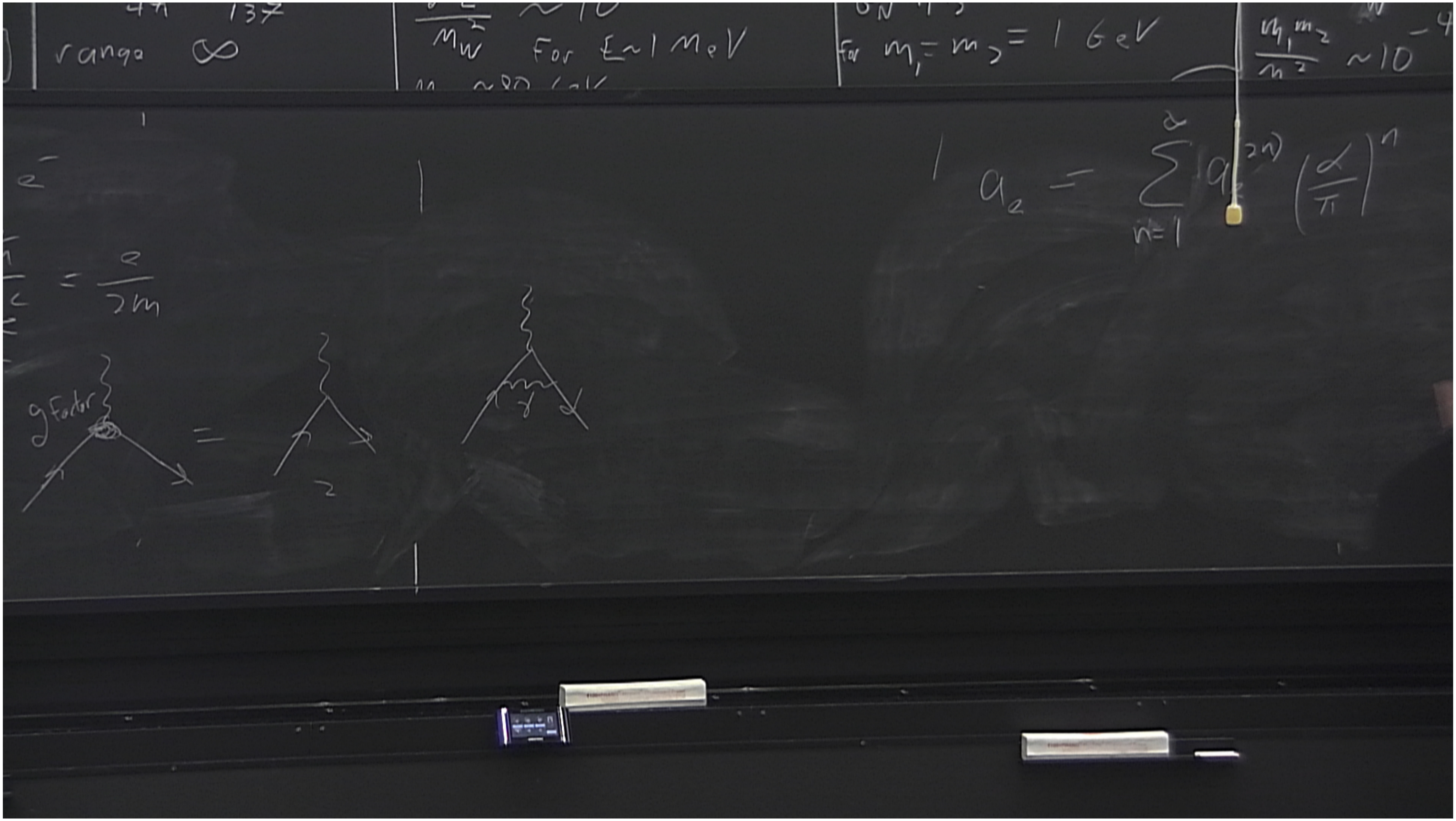
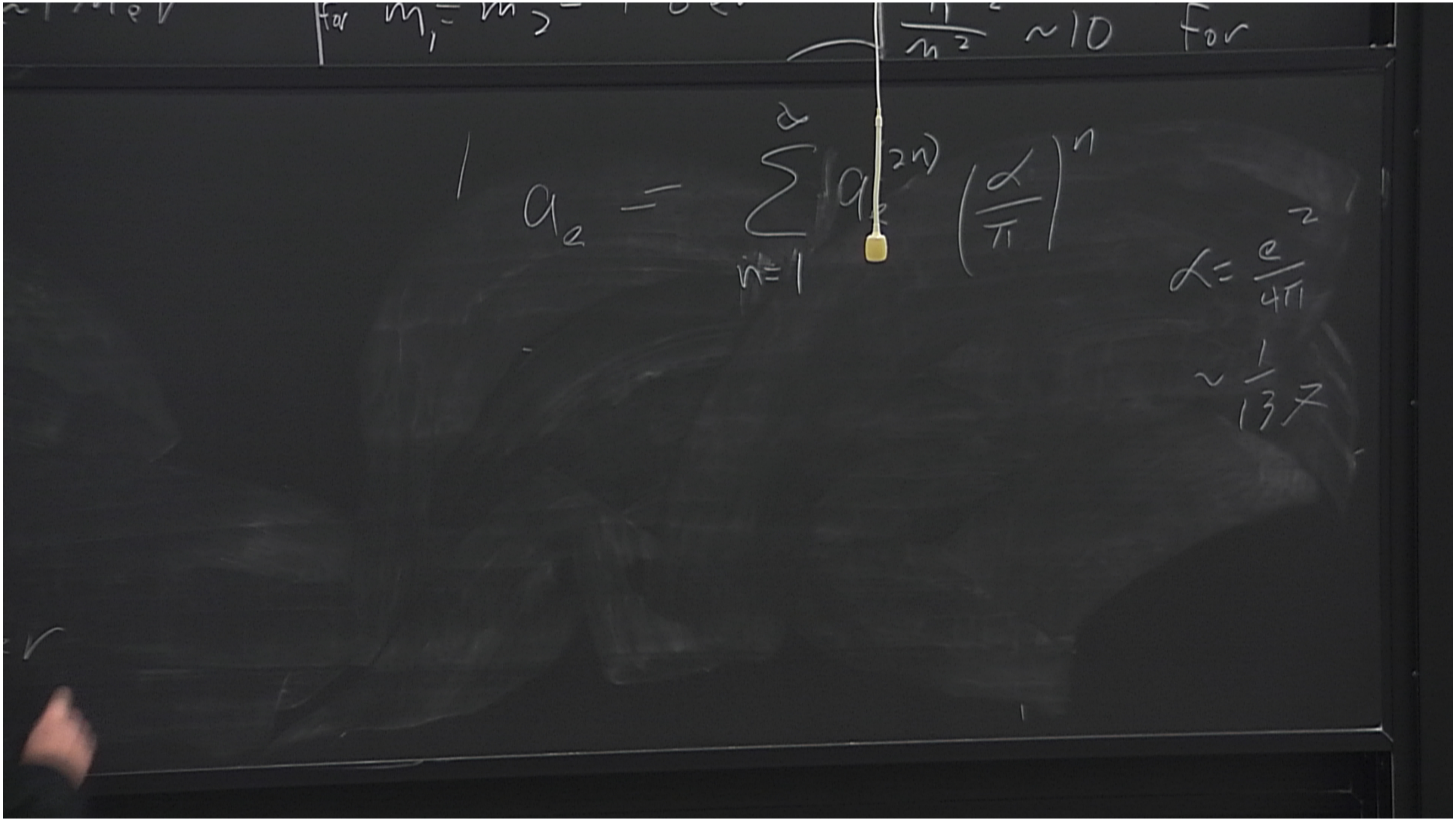
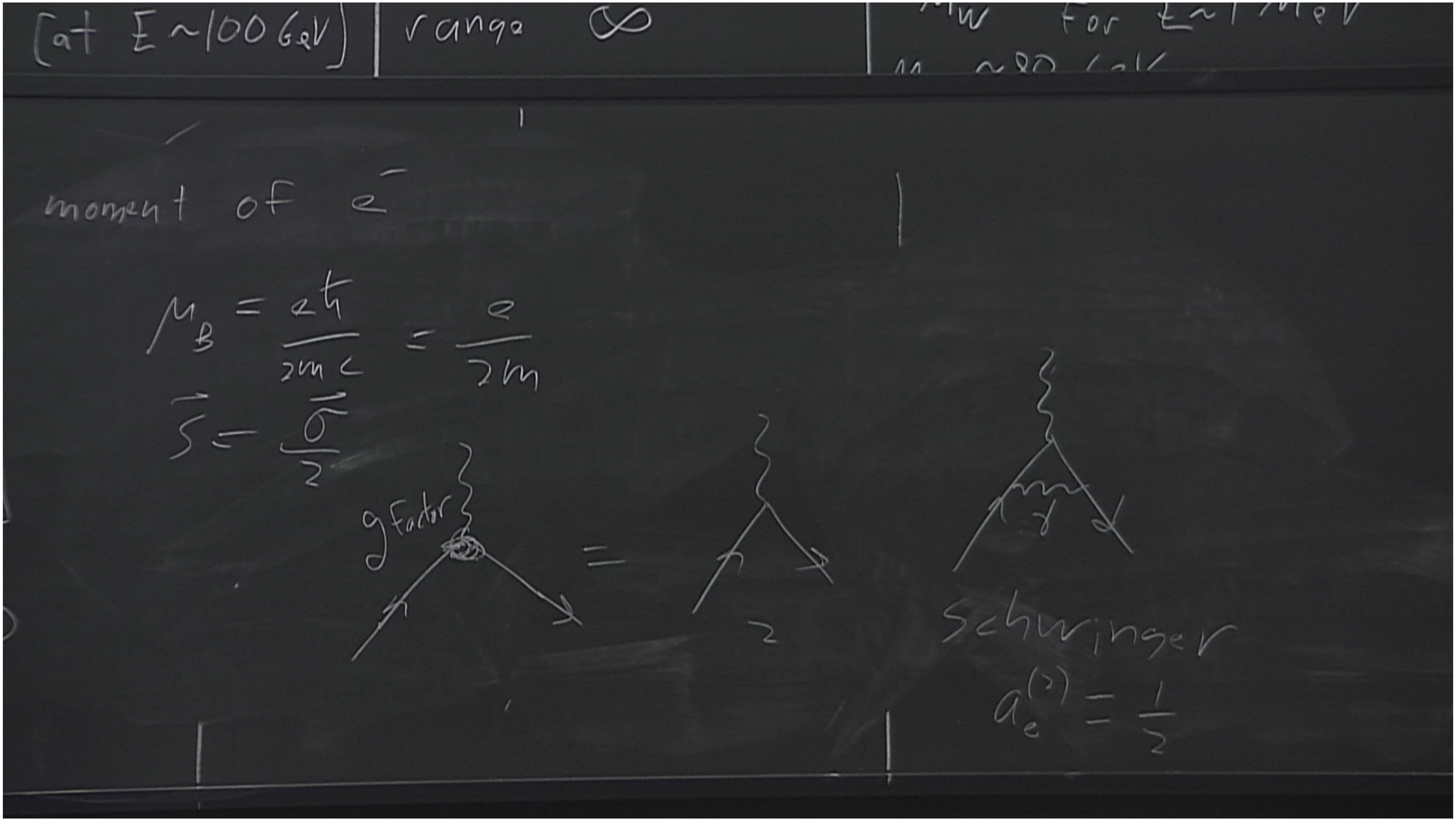
$$g = \underset{\substack{\uparrow \\ \text{Dirac}}}{2}\left[1 + \underset{\substack{\uparrow \\ \text{QED}}}{a_e}\right]$$

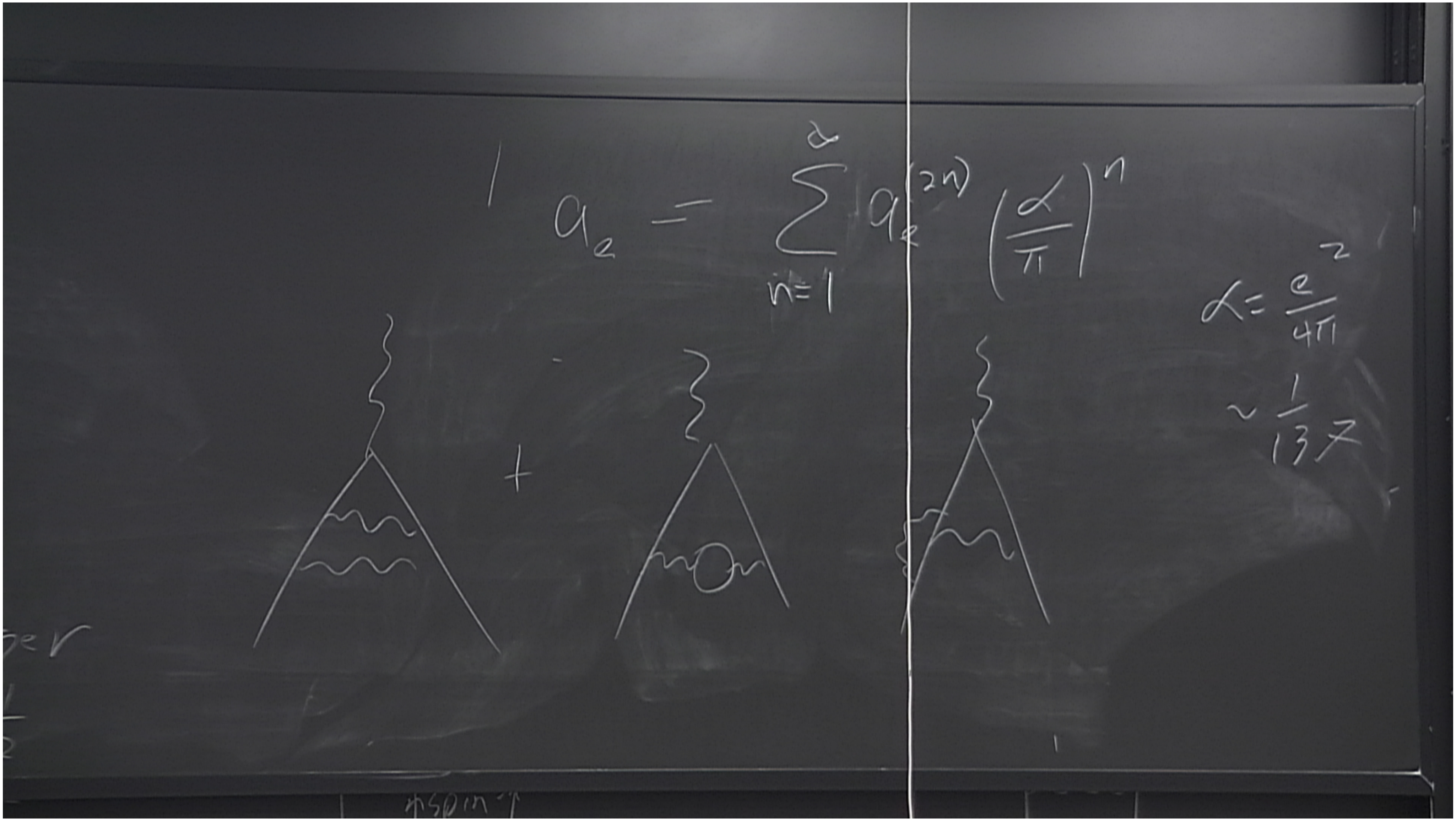


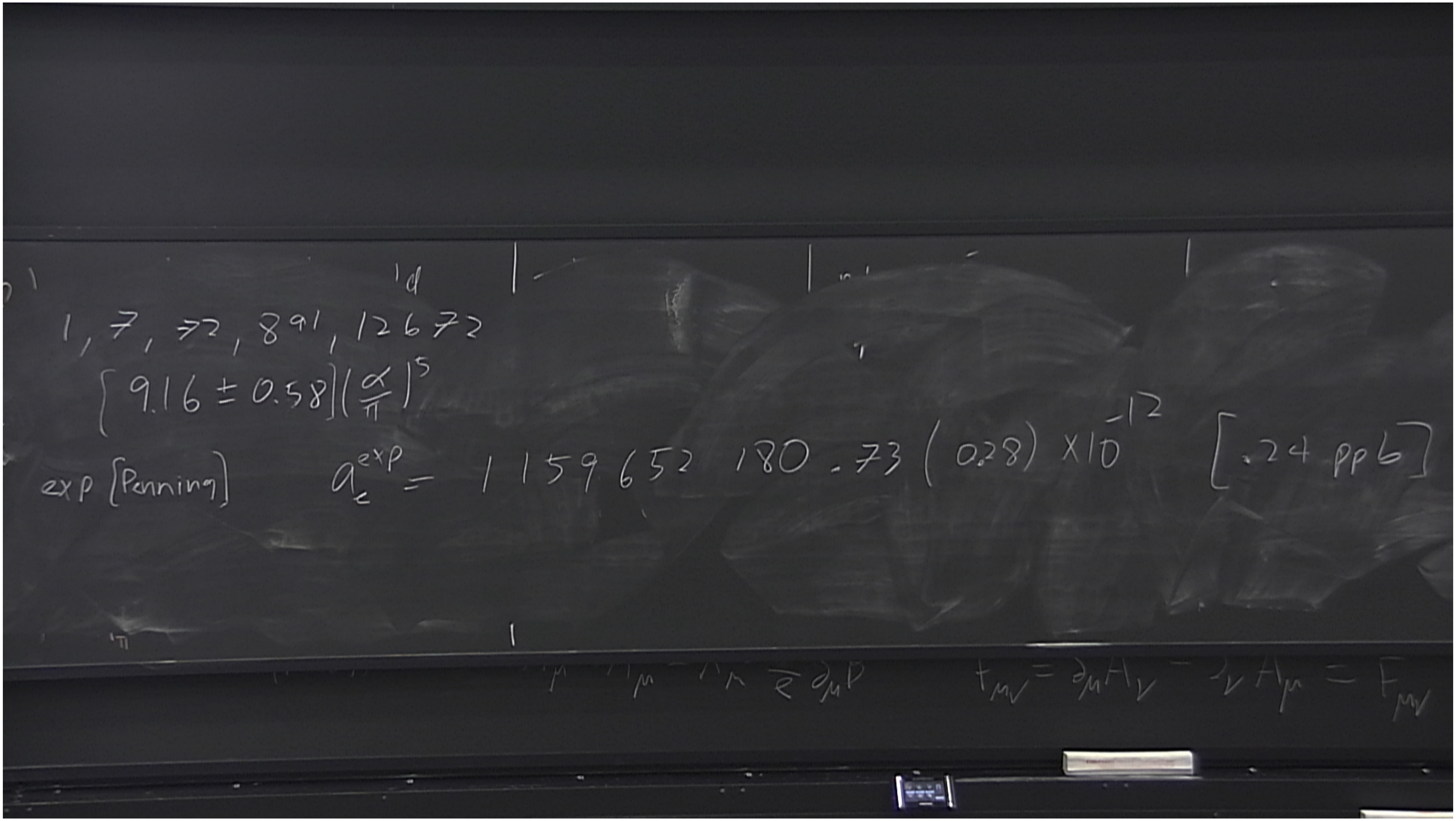

range ∞
For E~1 MeV
For m1 = m2 = 1 GeV
g factor
e/2m
ae = Σ ae(2n) (α/π)^n
n=1



$a_e = \sum_{n=1}^{\infty} a_e^{(2n)} \left(\frac{\alpha}{\pi}\right)^n$
$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$


[at E ~100 GeV]
range ∞
for
moment of e⁻
$\mu_B = \frac{e\hbar}{2mc} = \frac{e}{2m}$
$\vec{S} = \frac{\vec{\sigma}}{2}$
g factor
=
2
Schwinger
$a_e^{(2)} = \frac{1}{2}$


$a_e = \sum_{n=1}^{\infty} a_e^{(2n)} \left(\frac{\alpha}{\pi}\right)^n$
$\alpha = \frac{e^2}{4\pi}$
$\sim \frac{1}{137}$
+


1, 7, 72, 891, 12672
[9.16 ± 0.58] (α/π)^5
exp [Penning]
a_e^exp = 1 159 652 180.73 (0.28) × 10^-12
[.24 ppb]

1, 7, 72, 891, 12672

$$\left[9.16 \pm 0.58\right]\left(\frac{\alpha}{\pi}\right)^5$$

exp [Penning] $\quad a_e^{exp} = 1\,159\,652\,180.73\,(0.28)$

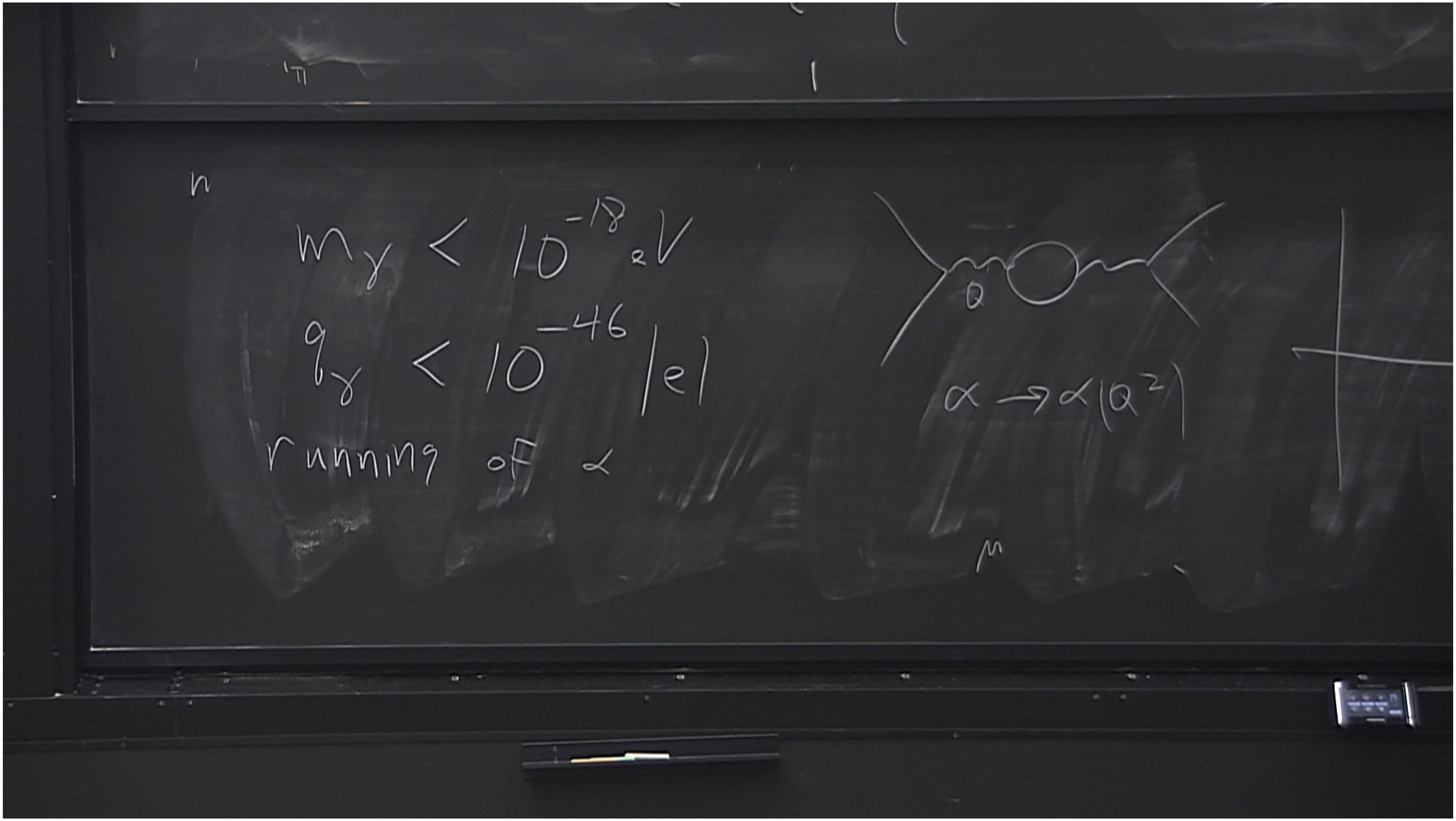
$$a_e^{exp} - a_e^{Th} = \left[-1.06 \pm 0.82\right] \times 10^{-12}$$

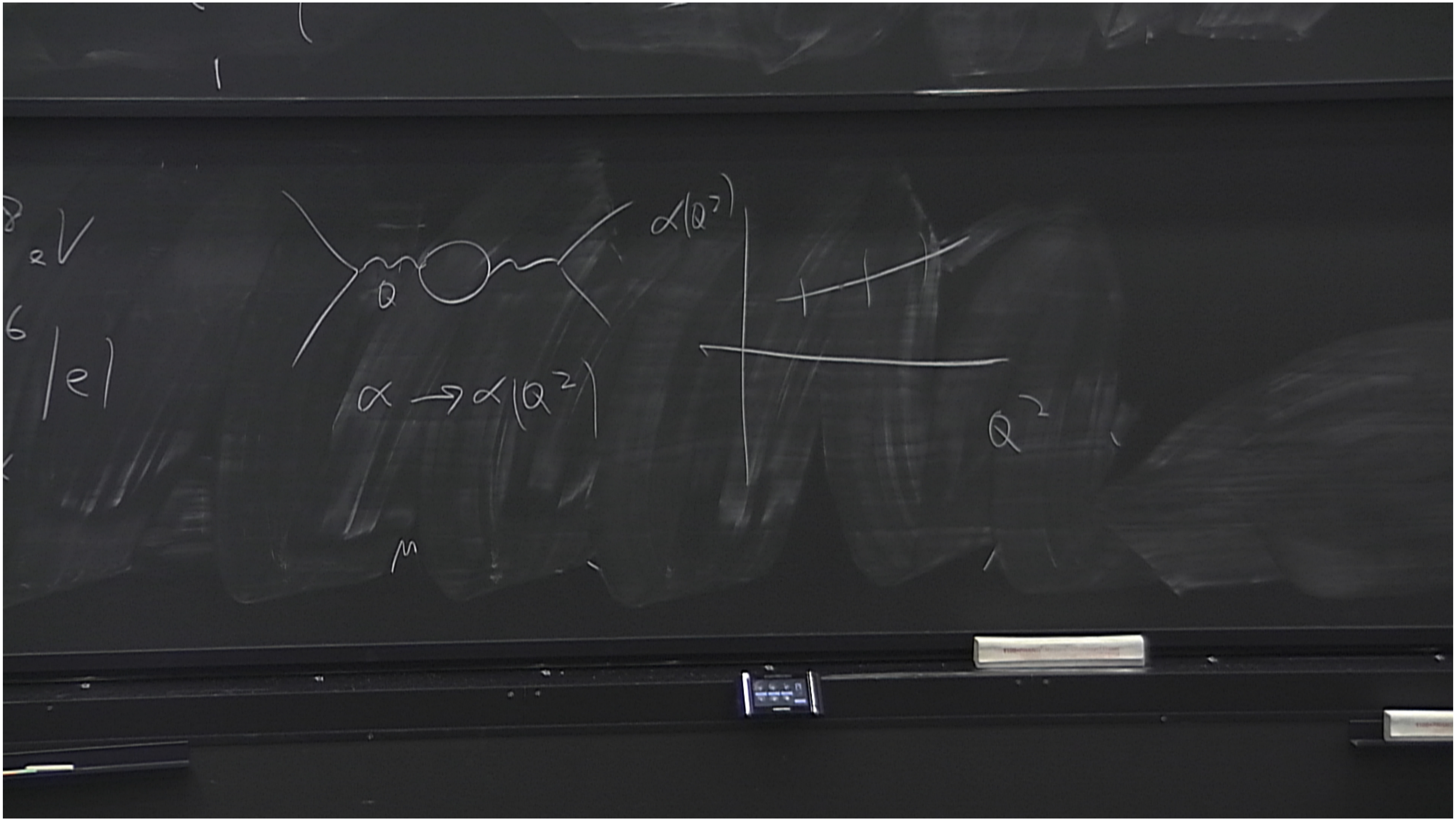
$$m_\gamma < 10^{-18}\,eV$$

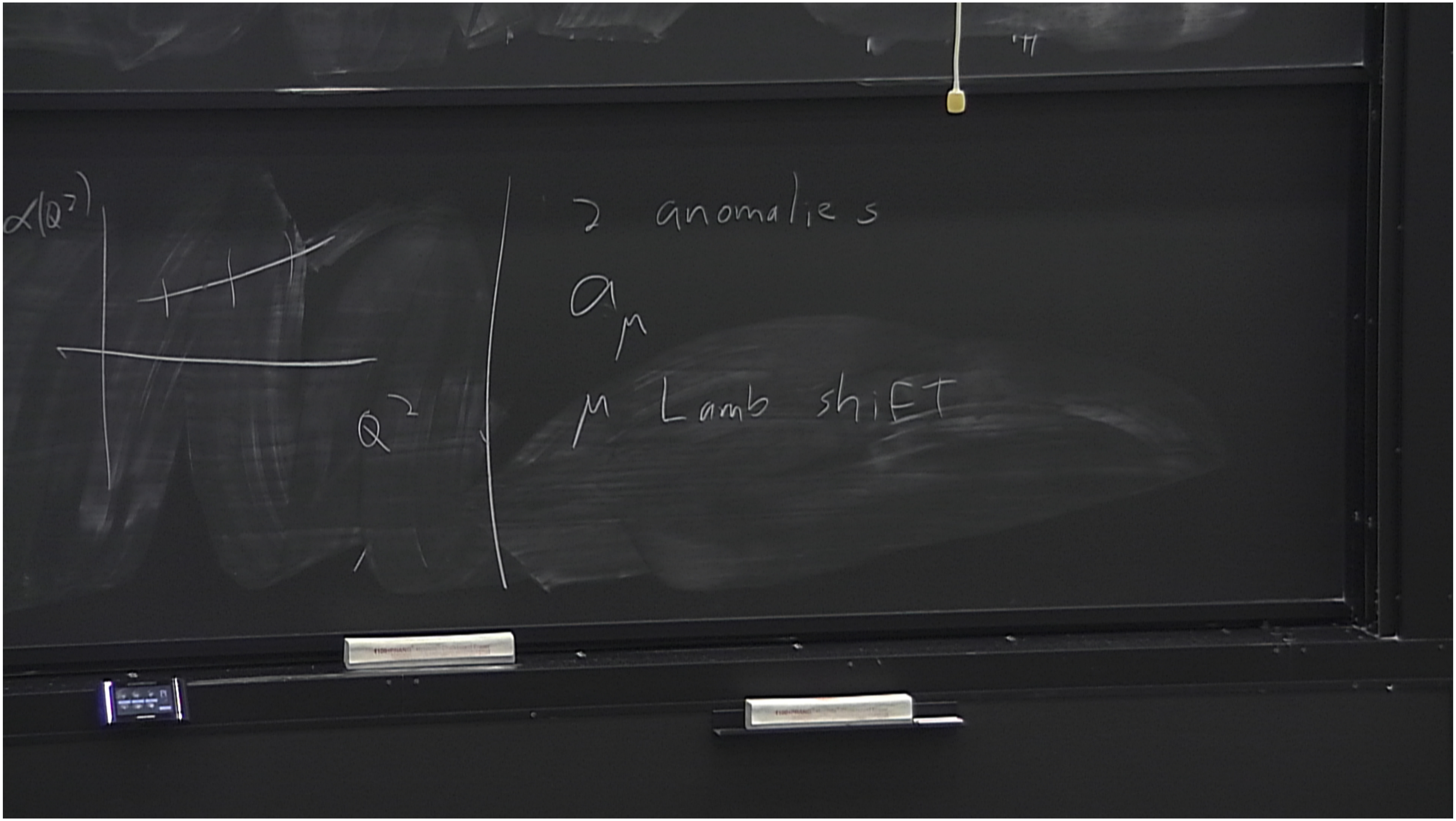
$$q_\gamma < 10^{-46}\,|e|$$

$$m_\gamma < 10^{-18}\,eV$$

$$q_\gamma < 10^{-46}\,|e|$$


$m_\gamma < 10^{-18}$ eV
$q_\gamma < 10^{-46} |e|$
running of α
Q
$\alpha \to \alpha(Q^2)$


Q
α → α(Q²)
α(Q²)
Q²
μ
|e|


α(Q²)
Q²
2 anomalies
aμ
μ Lamb shift