Title: Introduction to Quantum Field Theory for Cosmology - Lecture 1

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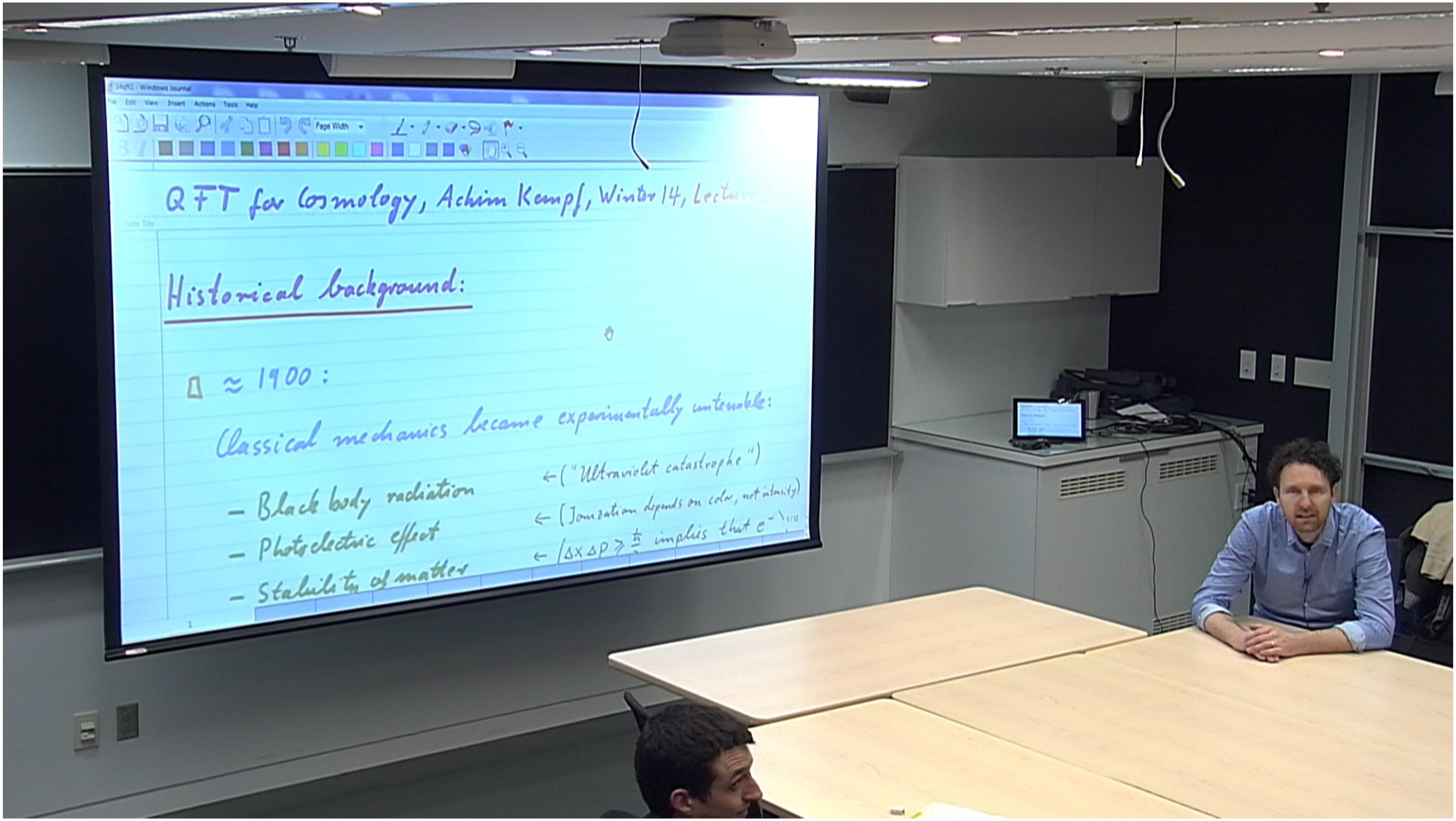
Abstract:



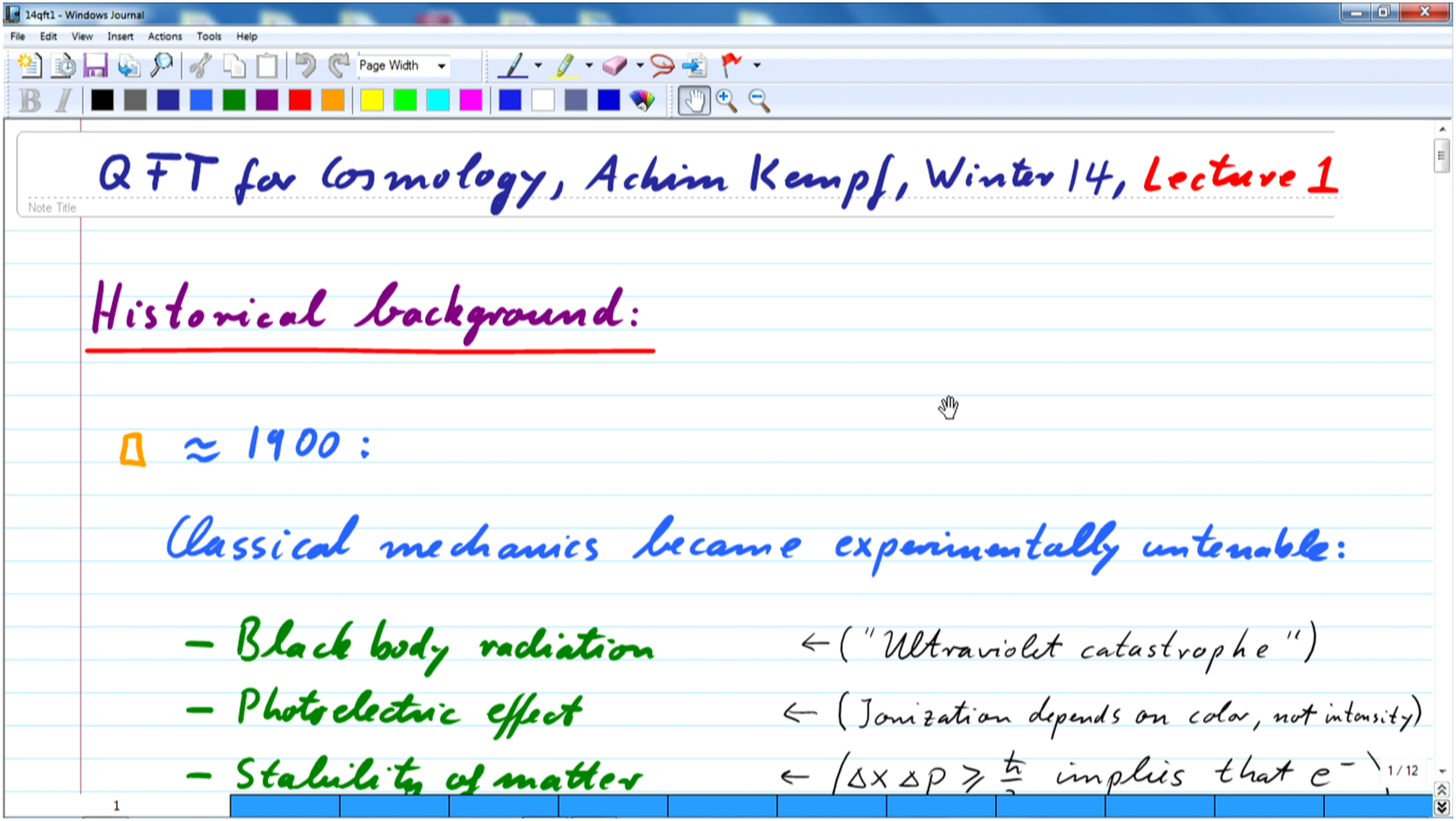
QFT for Cosmology, Achim Kempf, Winter 14, Lectur

Historical background:

≈ 1900:

Classical mechanics became experimentally untenable:

- Black body radiation ← ("Ultraviolet catastrophe")
- Photoelectric effect ← (Ionization depends on color, not intensity)
- Stability of matter ← $(\Delta x \Delta p \geq \frac{\hbar}{2}$ implies that e^-



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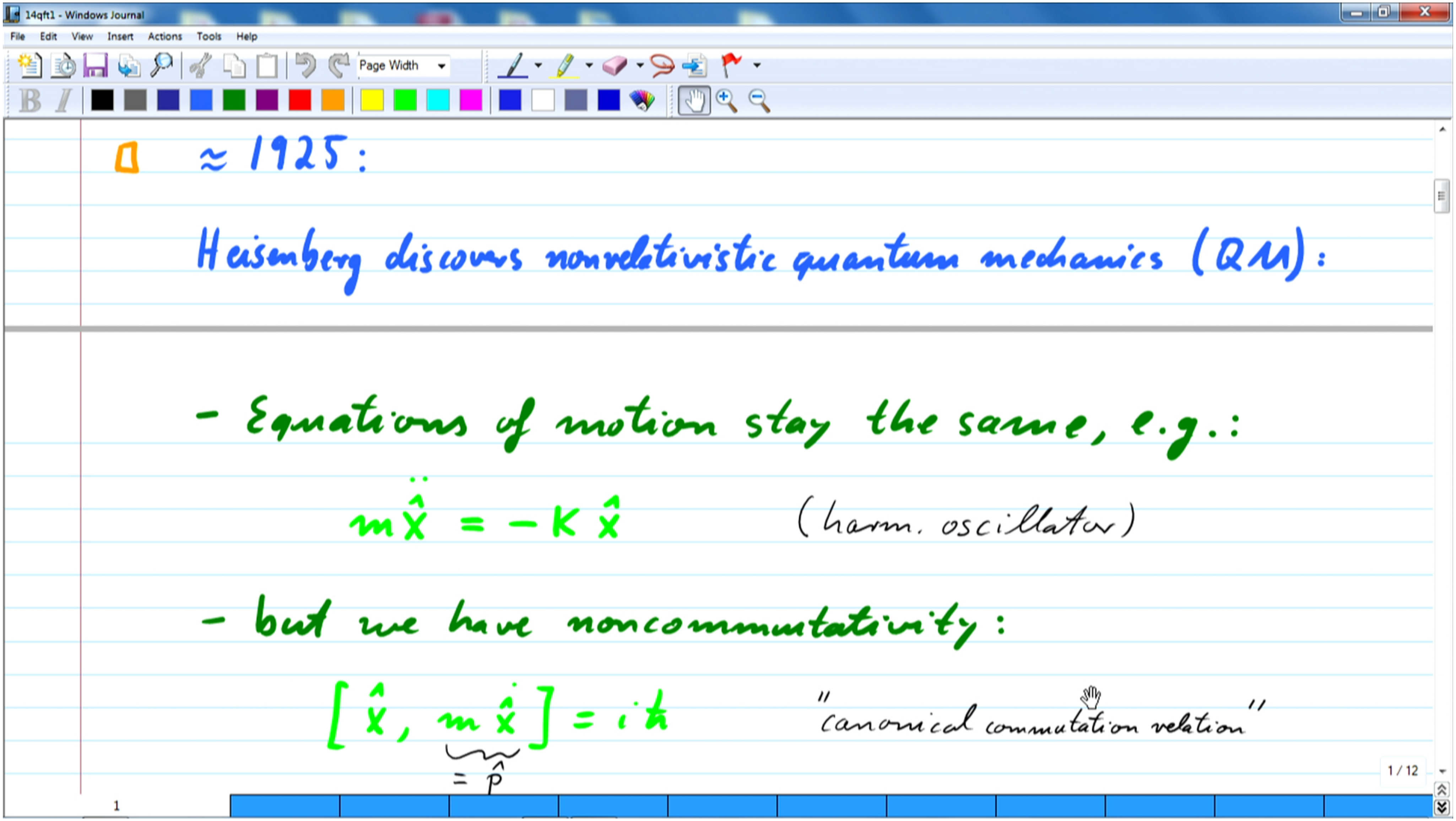


≈ 1925:

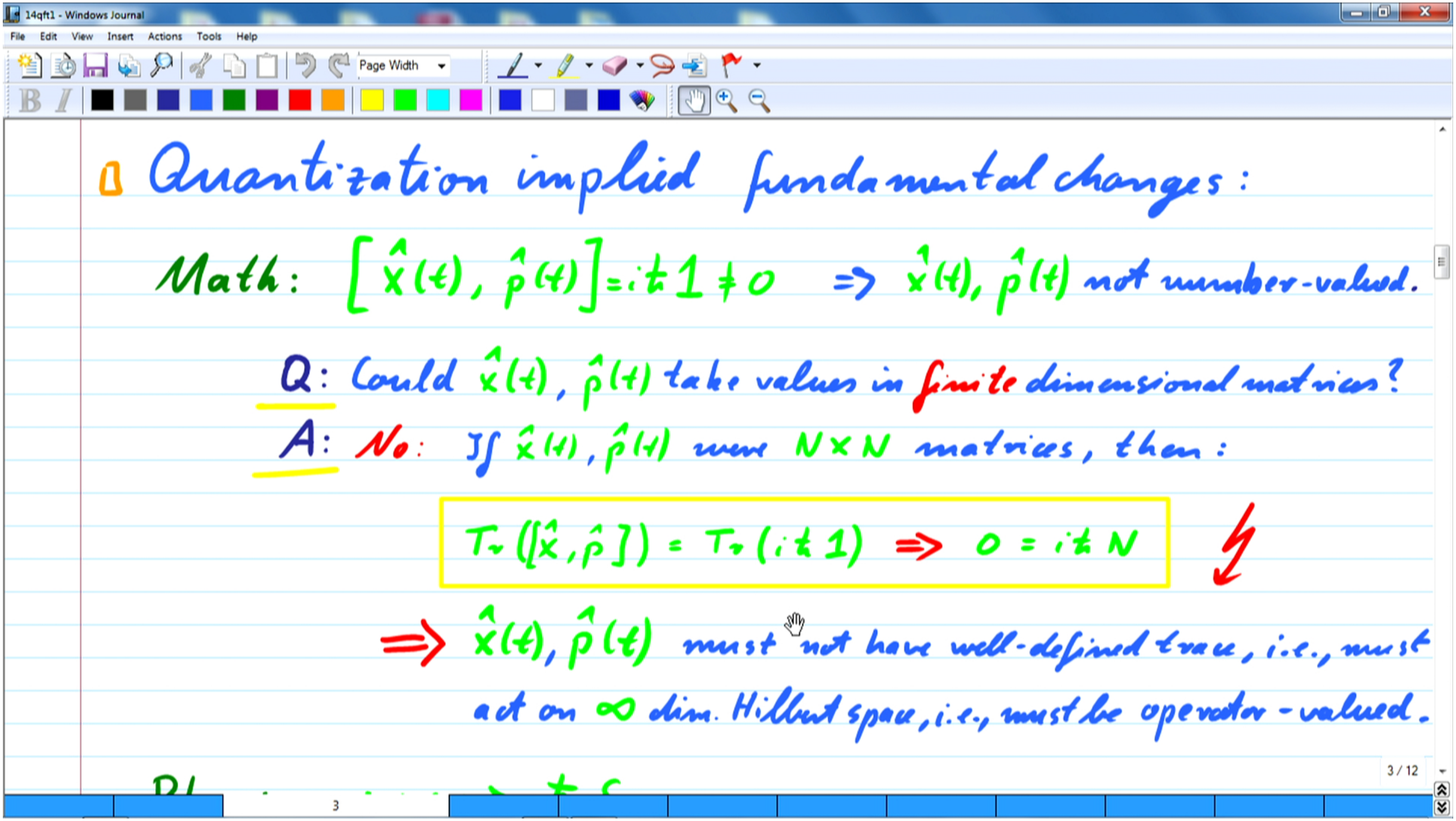
Heisenberg discovers nonrelativistic quantum mechanics (QM):

- Equations of motion stay the same, e.g.:

$$m\ddot{\hat{x}} = -k\hat{x}$$ (harm. oscillator)

- but we have noncommutativity:

$$[\hat{x}, \underbrace{m\dot{\hat{x}}}_{=\hat{p}}] = i\hbar$$ "canonical commutation relation"



Quantization implied fundamental changes:

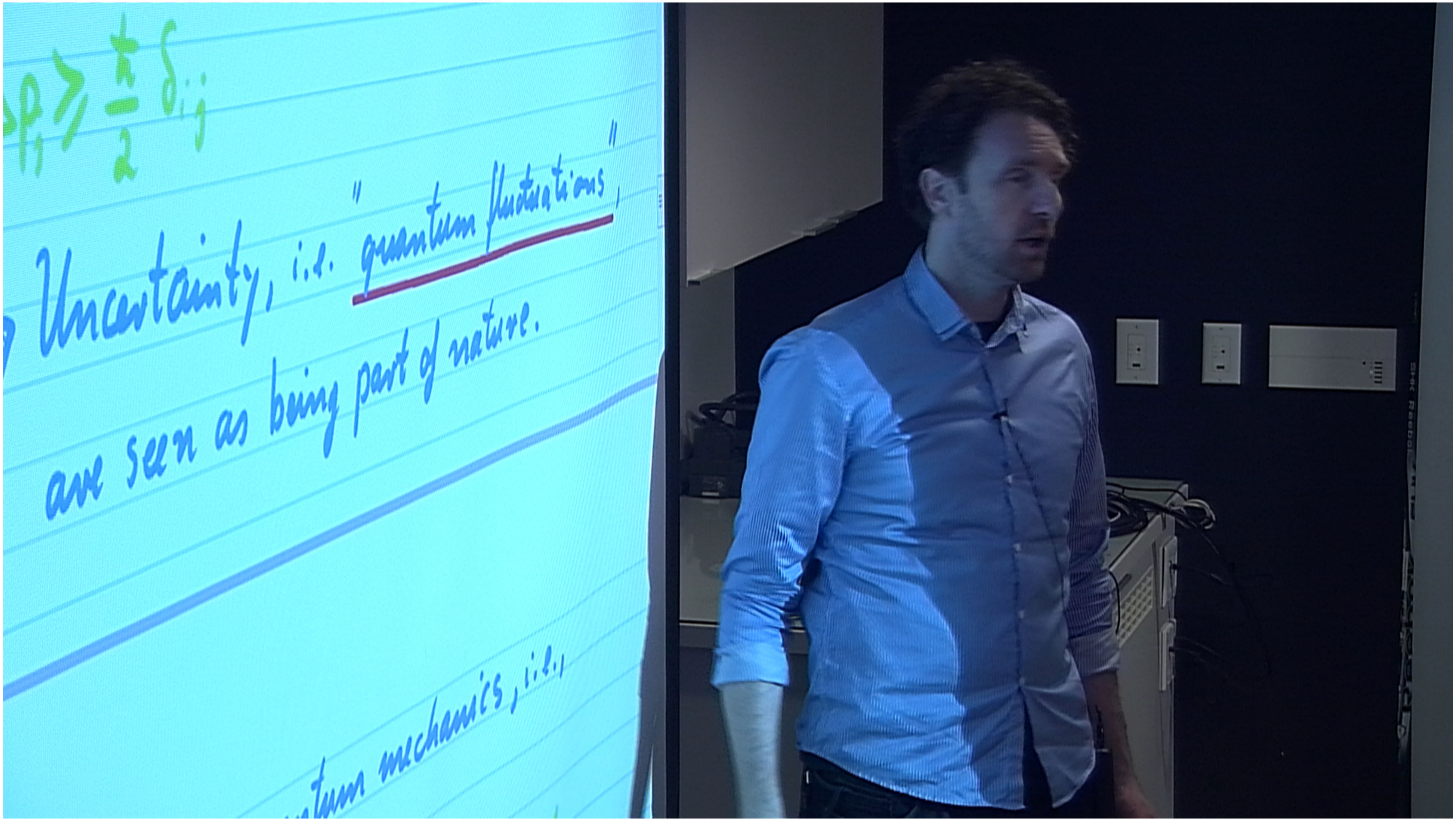
Math: $[\hat{x}(t), \hat{p}(t)] = i\hbar 1 \neq 0 \Rightarrow \hat{x}(t), \hat{p}(t)$ not number-valued.

Q: Could $\hat{x}(t), \hat{p}(t)$ take values in finite dimensional matrices?

A: No: If $\hat{x}(t), \hat{p}(t)$ were N×N matrices, then:

$$\boxed{Tr([\hat{x},\hat{p}]) = Tr(i\hbar 1) \Rightarrow 0 = i\hbar N} \quad ↯$$

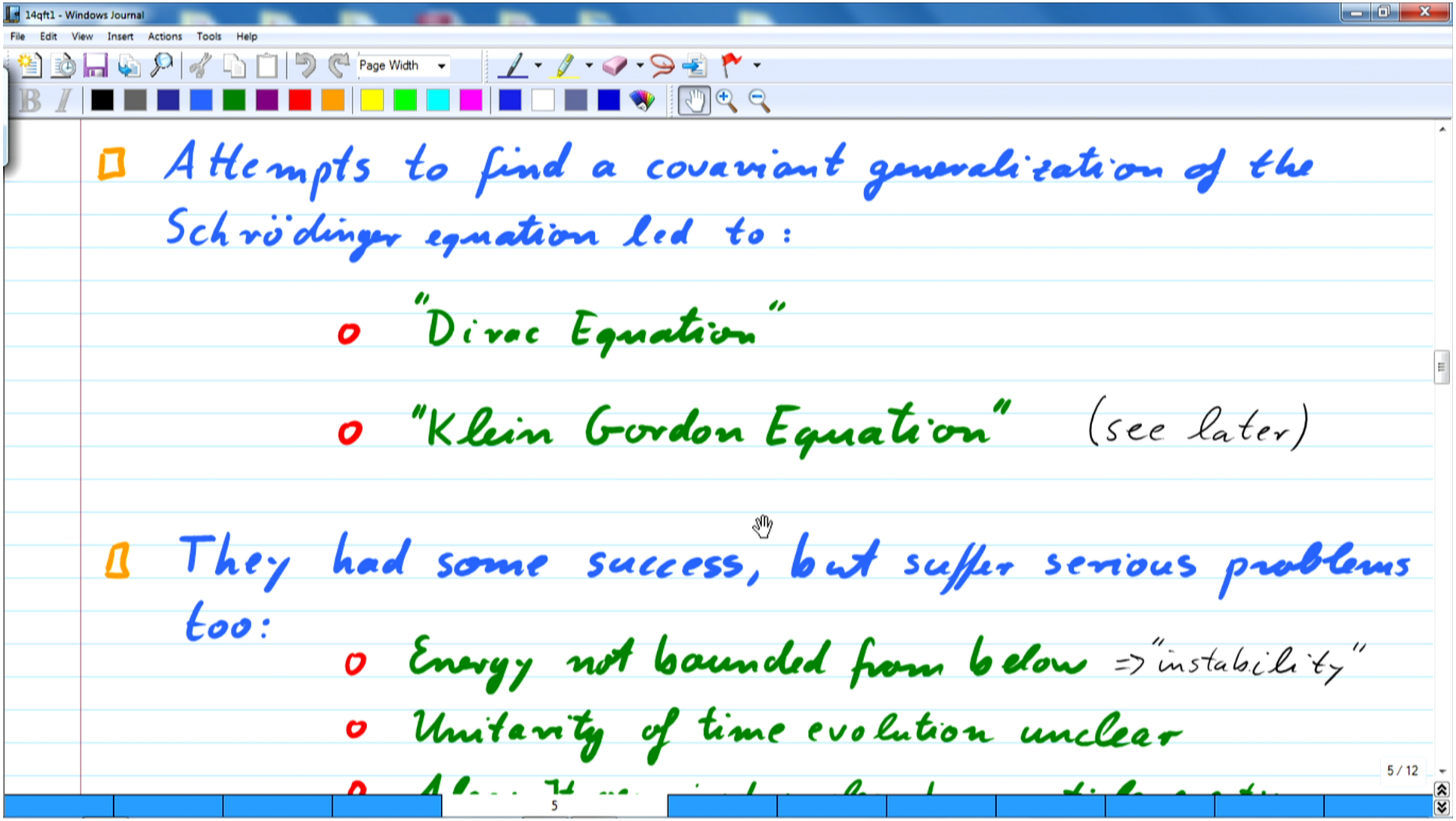
$\Rightarrow \hat{x}(t), \hat{p}(t)$ must not have well-defined trace, i.e., must act on ∞ dim. Hilbert space, i.e., must be operator-valued.



$p_j \geq \frac{\hbar}{2} \delta_{ij}$

Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature.

ntum mechanics, i.e.,

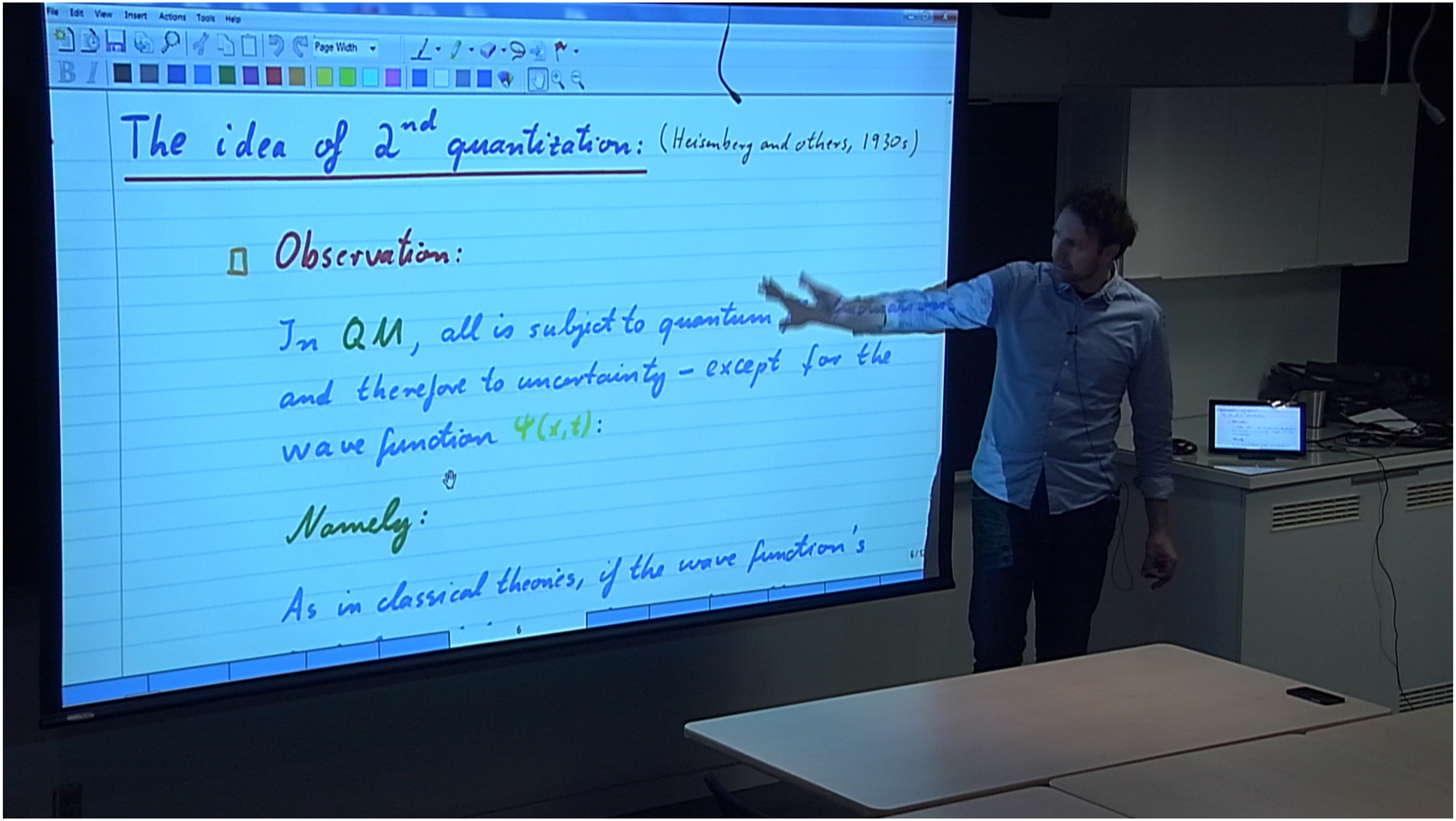


☐ Attempts to find a covariant generalization of the Schrödinger equation led to:

- "Dirac Equation"
- "Klein Gordon Equation" (see later)

☐ They had some success, but suffer serious problems too:

- Energy not bounded from below ⇒ "instability"
- Unitarity of time evolution unclear



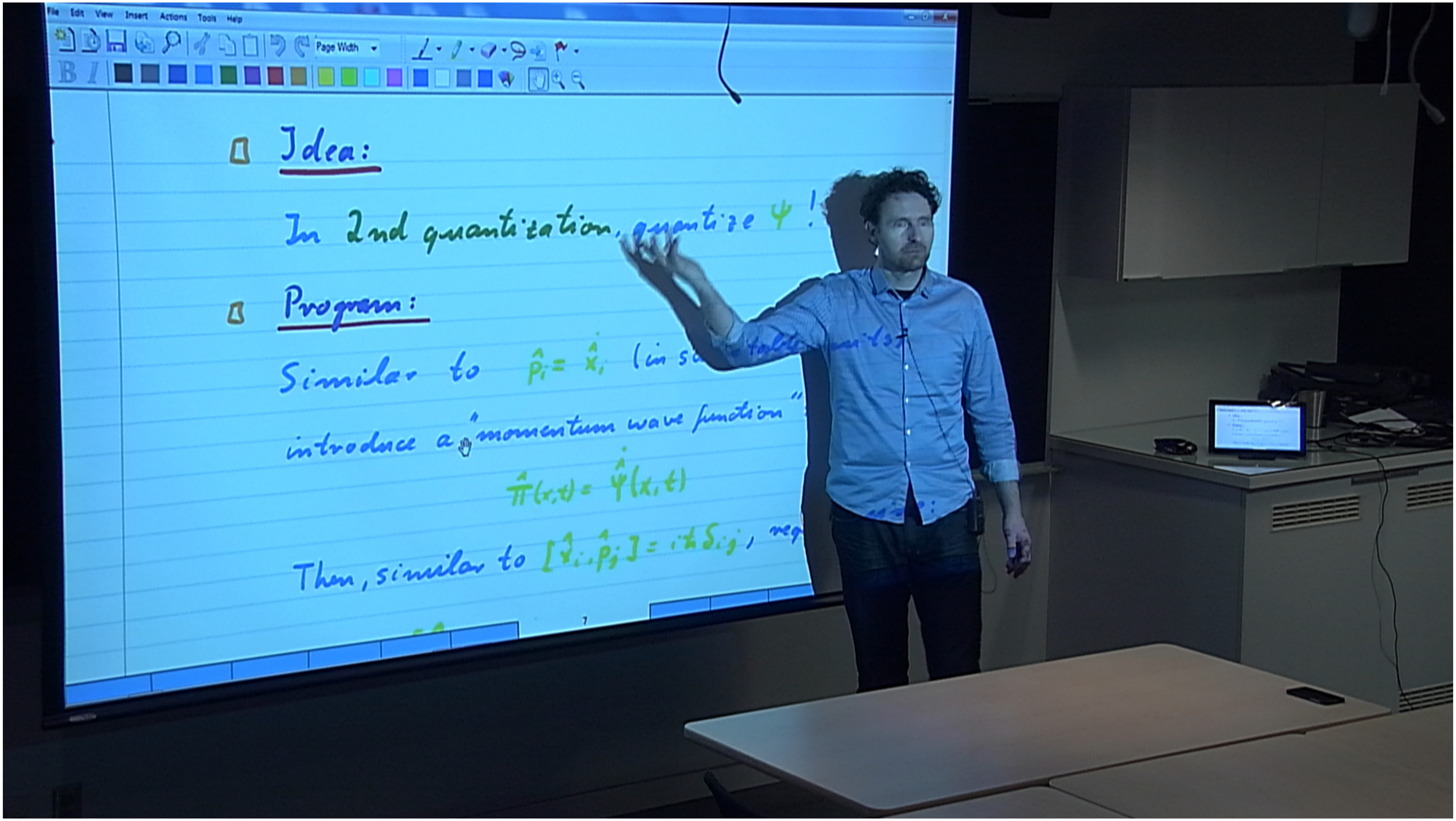
The idea of 2nd quantization: (Heisenberg and others, 1930s)

☐ Observation:

In QM, all is subject to quantum [illegible]
and therefore to uncertainty - except for the
wave function $\psi(x,t)$:

Namely:

As in classical theories, if the wave function's



☐ Idea:

In 2nd quantization, quantize ψ !

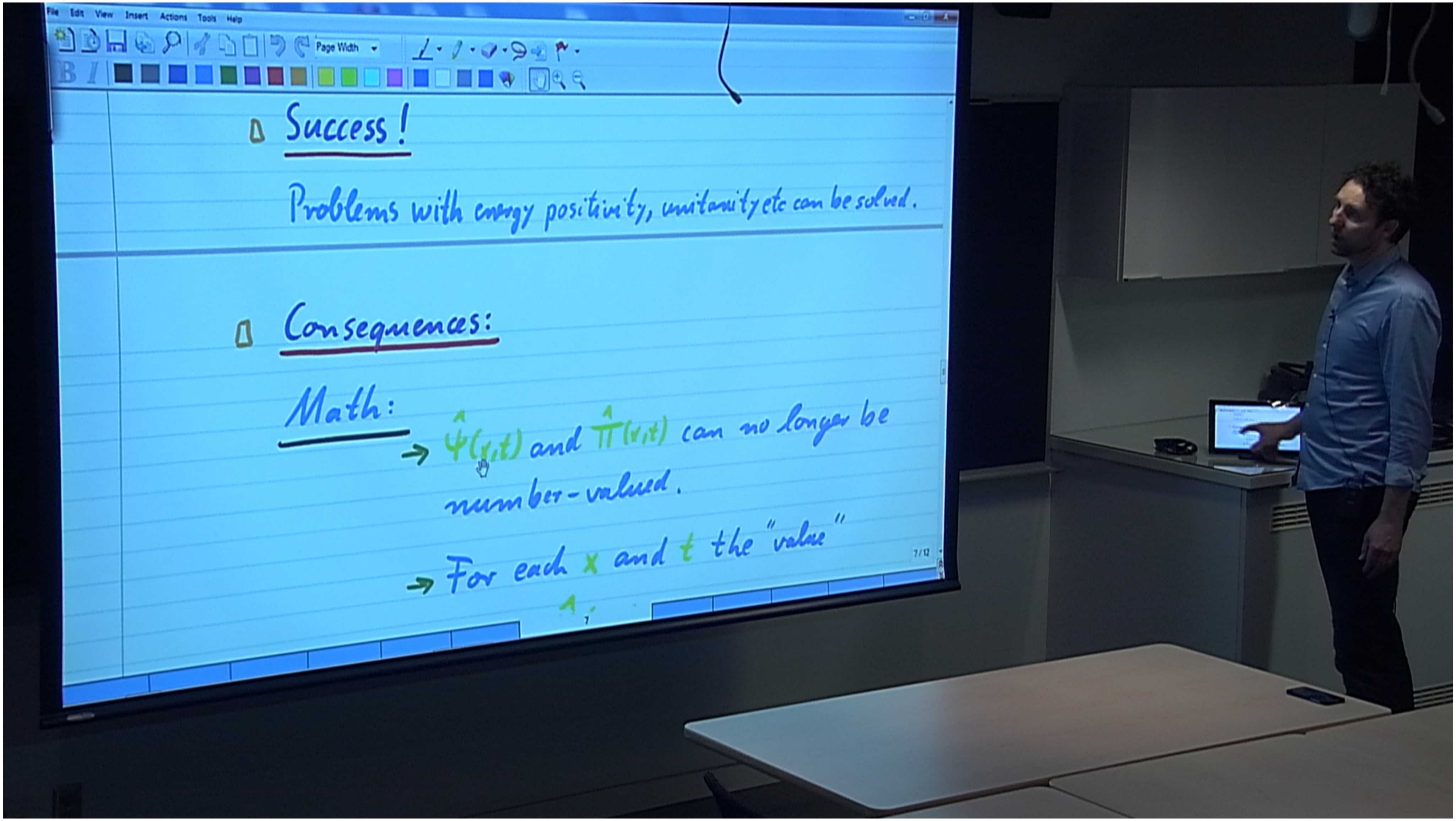
☐ Program:

Similar to $\hat{p}_i = \dot{\hat{x}}_i$ (in s

introduce a "momentum wave function":

$$\hat{\pi}(x,t) = \dot{\hat{\psi}}(x,t)$$

Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, req



Success!

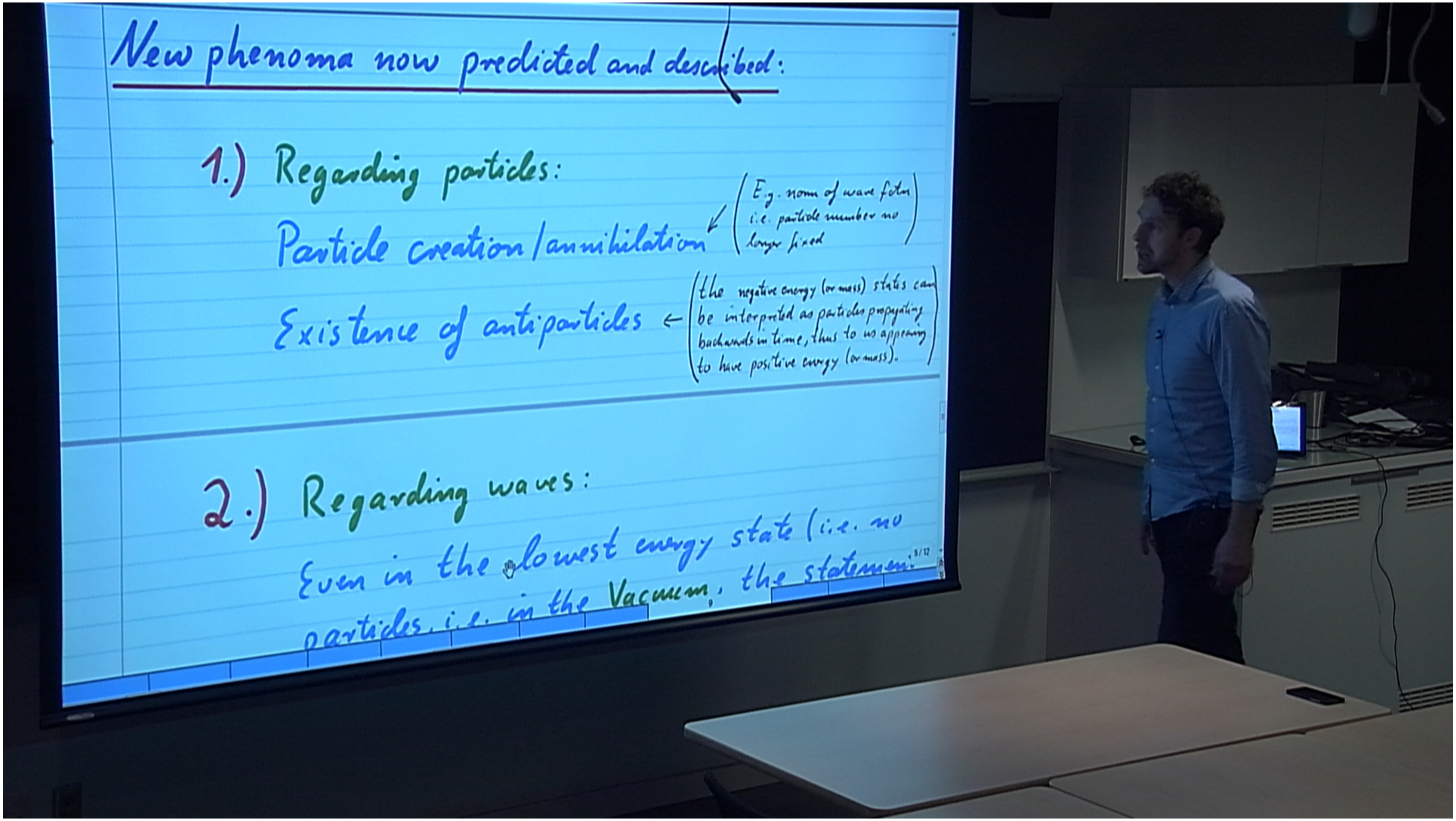
Problems with energy positivity, unitarity etc can be solved.

Consequences:

Math:

→ $\hat{\Psi}(x,t)$ and $\hat{\pi}(x,t)$ can no longer be number-valued.

→ For each x and t the "value"



New phenoma now predicted and described:

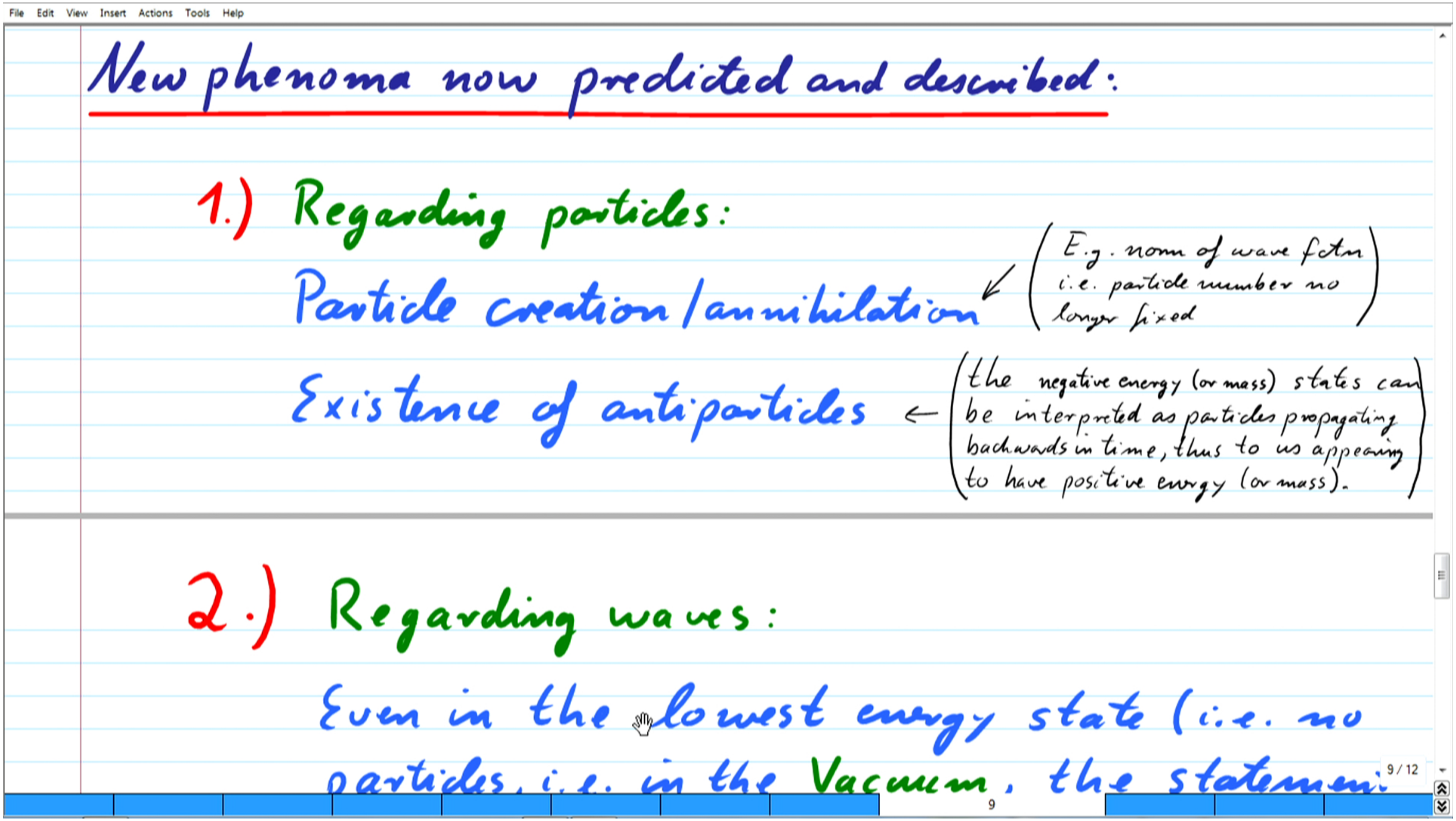
1.) Regarding particles:

Particle creation / annihilation ↙ (E.g. norm of wave fctn i.e. particle number no longer fixed)

Existence of antiparticles ← (the negative energy (or mass) states can be interpreted as particles propagating backwards in time, thus to us appearing to have positive energy (or mass).)

2.) Regarding waves:

Even in the lowest energy state (i.e. no particles, i.e. in the Vacuum, the statement



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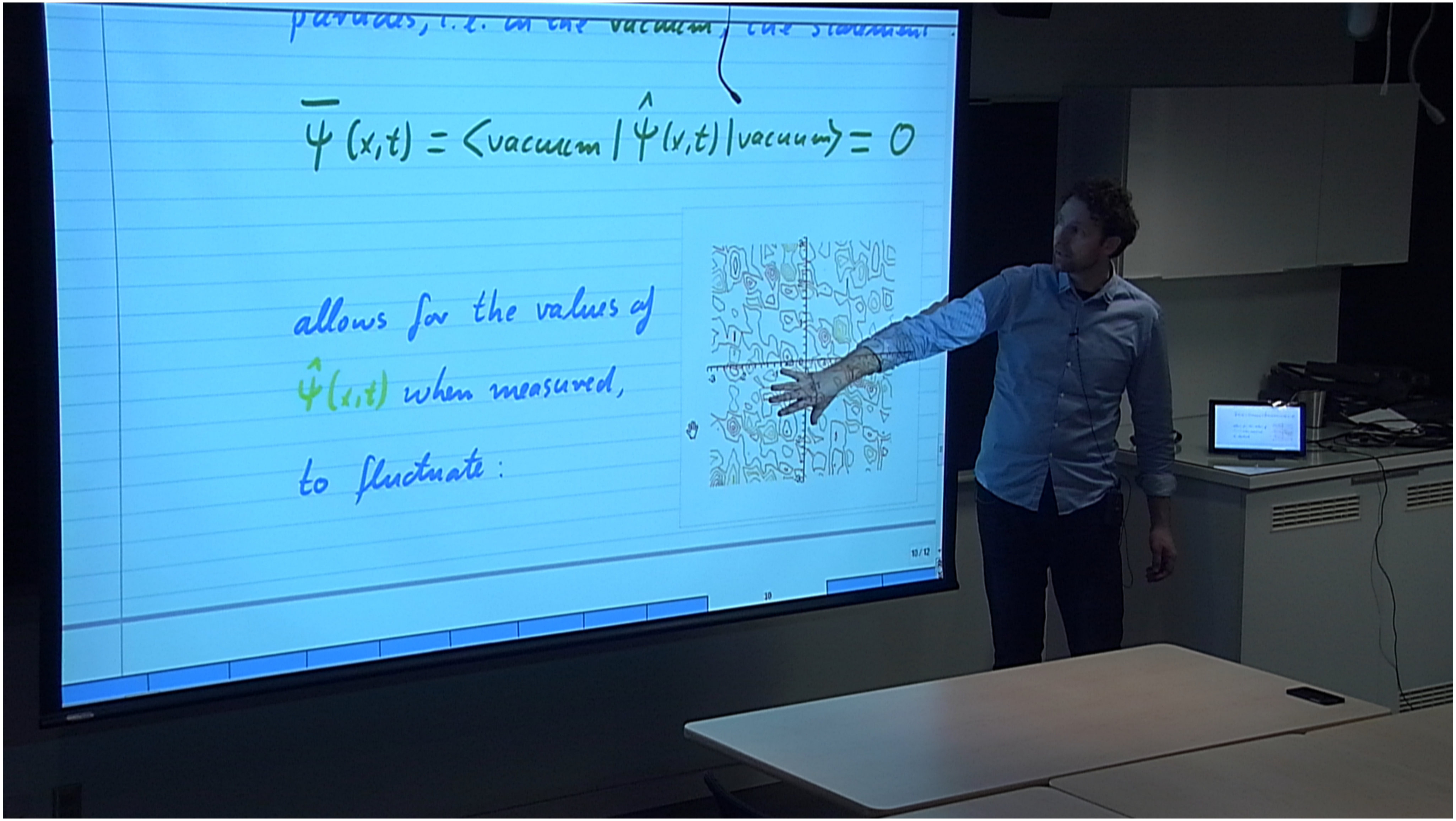
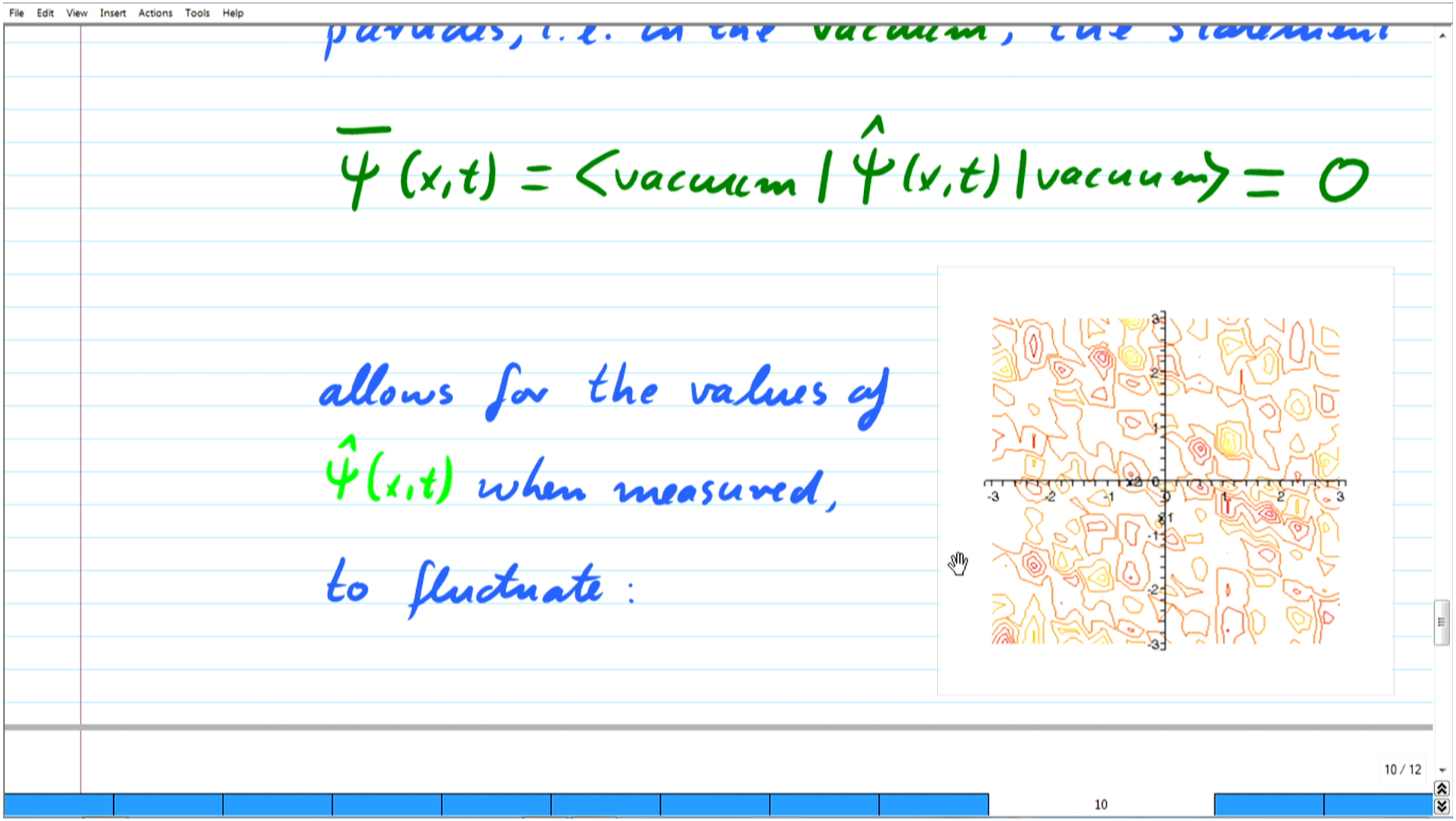


particles, i.e. in the vacuum, the statement :

$$\bar{\psi}(x,t) = \langle vacuum | \hat{\psi}(x,t) | vacuum \rangle = 0$$

allows for the values of $\hat{\psi}(x,t)$ when measured, to fluctuate :



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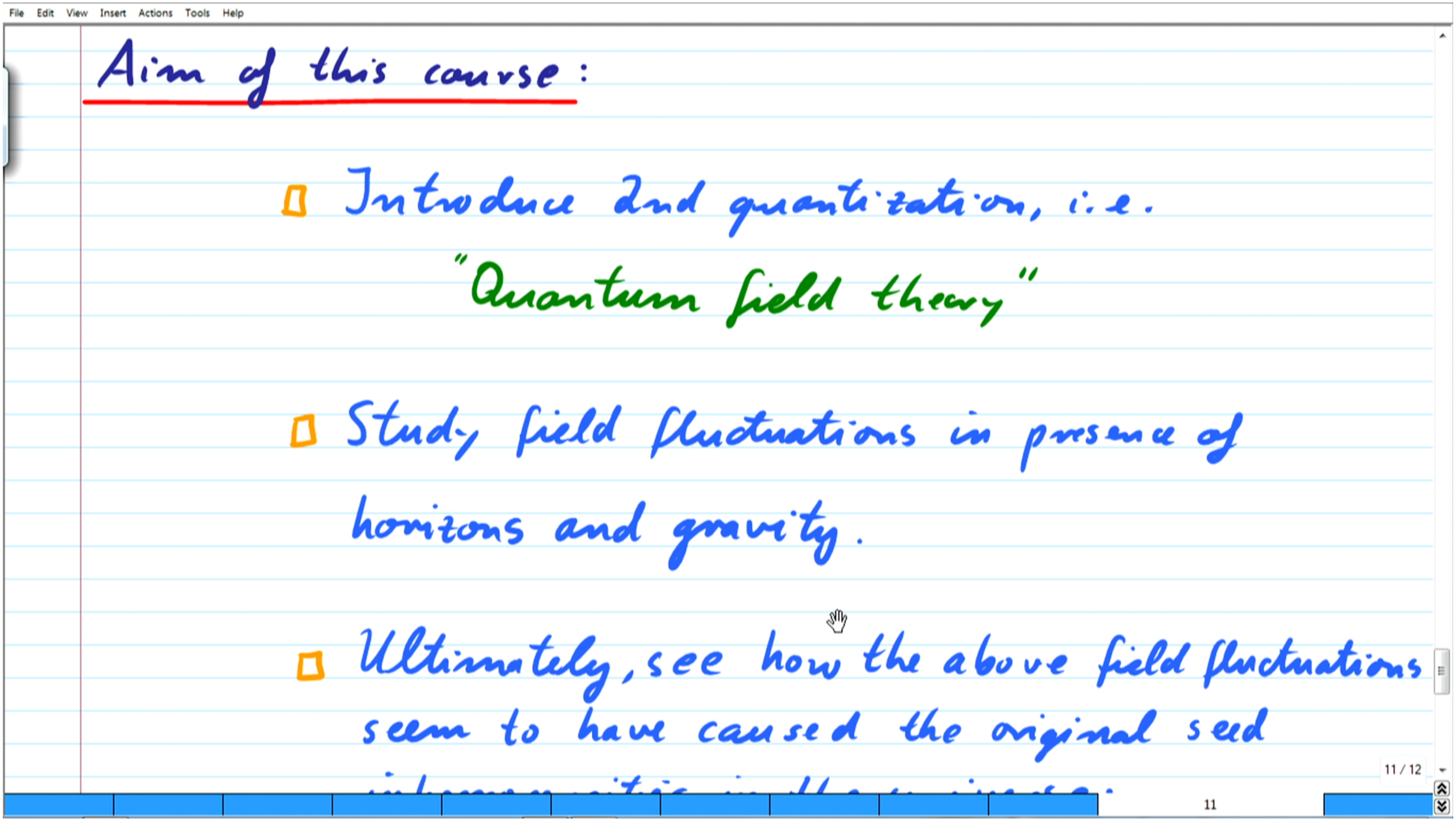


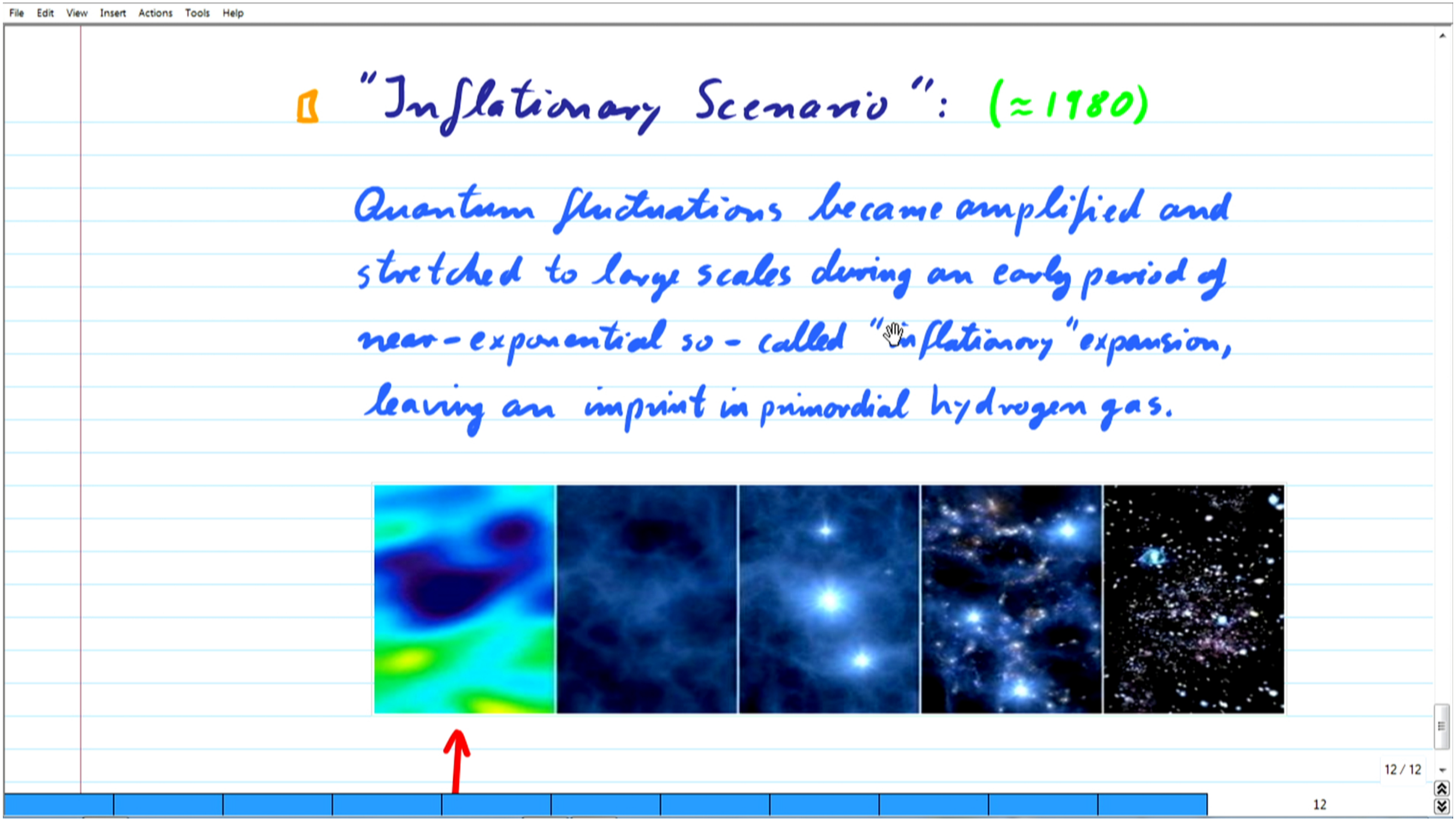
Aim of this course:

- ☐ Introduce 2nd quantization, i.e.

  "Quantum field theory"

- ☐ Study field fluctuations in presence of horizons and gravity.

- ☐ Ultimately, see how the above field fluctuations seem to have caused the original seed



"Inflationary Scenario": (≈1980)

Quantum fluctuations became amplified and stretched to large scales during an early period of near-exponential so-called "inflationary" expansion, leaving an imprint in primordial hydrogen gas.