Title: (0,4) sigma models and K3

Date: Dec 18, 2013 02:30 PM

URL: http://pirsa.org/13120071

Abstract: TBA

Moonshine

Finite groups
Modular forms
Algebraic Geometry
CFT
Lattices

$\Big\}$ String Theory

Monstrous moonshine:

$j(\tau) \leftarrow \tau \in \mathbb{H}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$

$$j\left(\frac{a\tau+b}{c\tau+d}\right) = j(\tau)$$

$$q = e^{2\pi i \tau} \rightarrow j \sim \frac{1}{q} + \underbrace{196884}_{= 1 + 196883} q + \cdots$$

DHG

- compactify the bosonic string on the Leech lattice (orbifold)

$$\Rightarrow Z = j(q)$$

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EOT (2010):

elliptic genus of type II string on K3:

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$$Z_{K3} = \mathrm{Tr}\,(-1)^F y^{J_L} q^{L_0} \bar{q}^{\bar{L}_0}$$

(4,4) σ-model: states are reps of $\mathcal{N}=4$ superconformal algebra

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(4,4) σ-model: states are reps of $\mathcal{N}=4$ superconformal algebra

$$= 24 \times \text{massless reps} + \sum_n a_n \, Ch_n^{\text{massive}} \qquad a_n = 2(45 + 231 + \ldots)$$

$$Z_g^{K3} = \mathrm{Tr}\, g (-1)^F y^{J_0} q^{L_0} \bar{q}^{\bar{L}_0}$$

$g \in M_{24}$

Theorem: All automorphisms of K3 surfaces lie in M_{23}

$$Z_g^{K3} = \mathrm{Tr}\, g\, (-1)^F y^{J_0} q^{L_0} \bar{q}^{\bar{L}_0}$$

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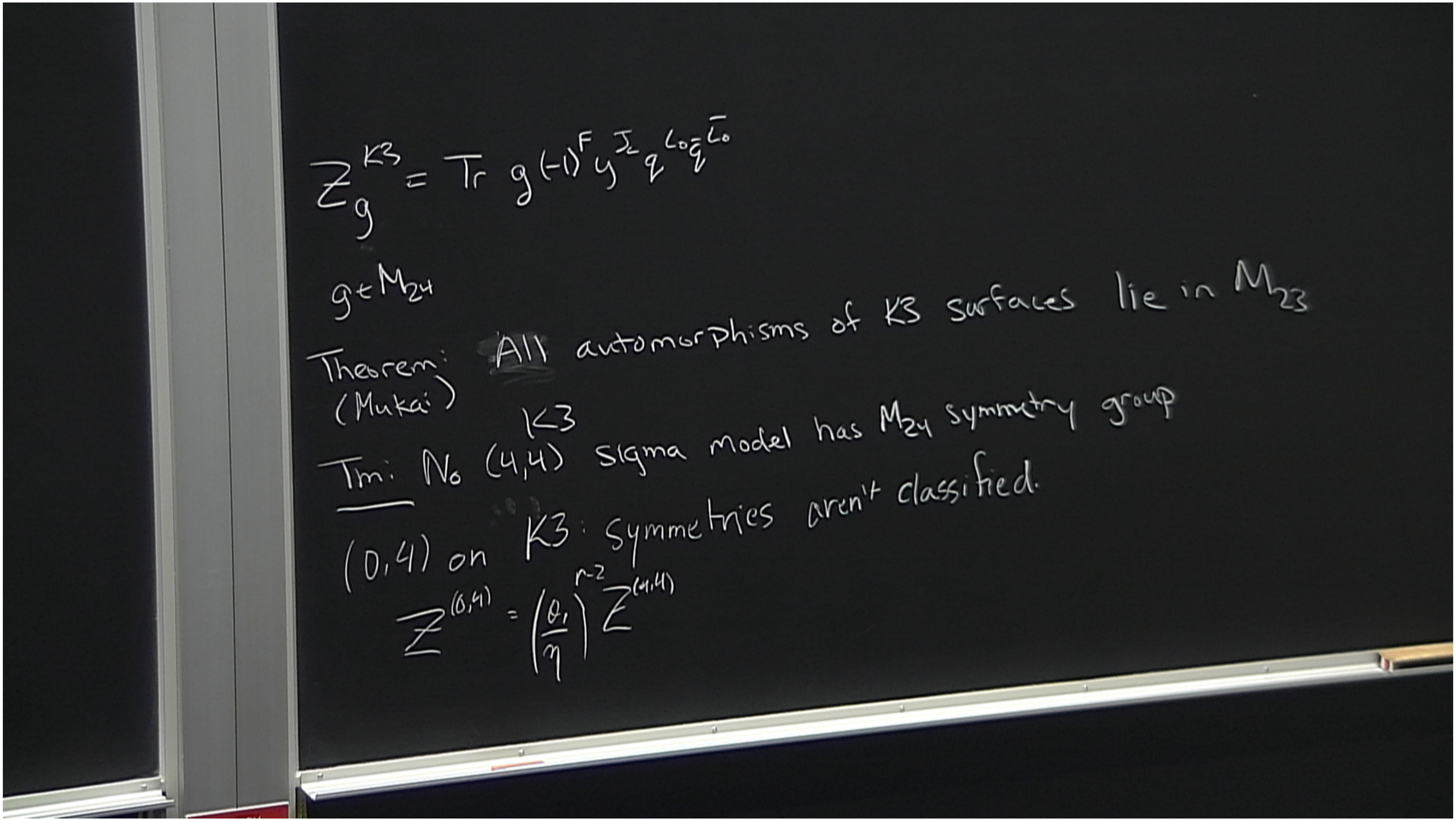
Theorem (Mukai): All automorphisms of K3 surfaces lie in M_{23}

Thm: No (4,4) sigma model has M_{24} symmetry group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Theorem: All automorphisms of K3 surfaces lie in M_{23}
(Mukai)
Thm: No (4,4) K3 sigma model has M_{24} symmetry group
(0,4) on K3: symmetries aren't classified.
$Z^{(0,4)} = \left(\frac{\theta_1}{\eta}\right)^{r-2} Z^{(4,4)}$

GLSM:

$$W(\Phi_i) = \sum_{i=1} \Phi_i^4$$

Λ_i Fermi multiplets

$$\Lambda_i f(\Phi_i)$$

$$\Phi_1 \leftrightarrow \Phi_2$$

$$j\left(\frac{a\tau+b}{c\tau+d}\right) = j(\tau)$$

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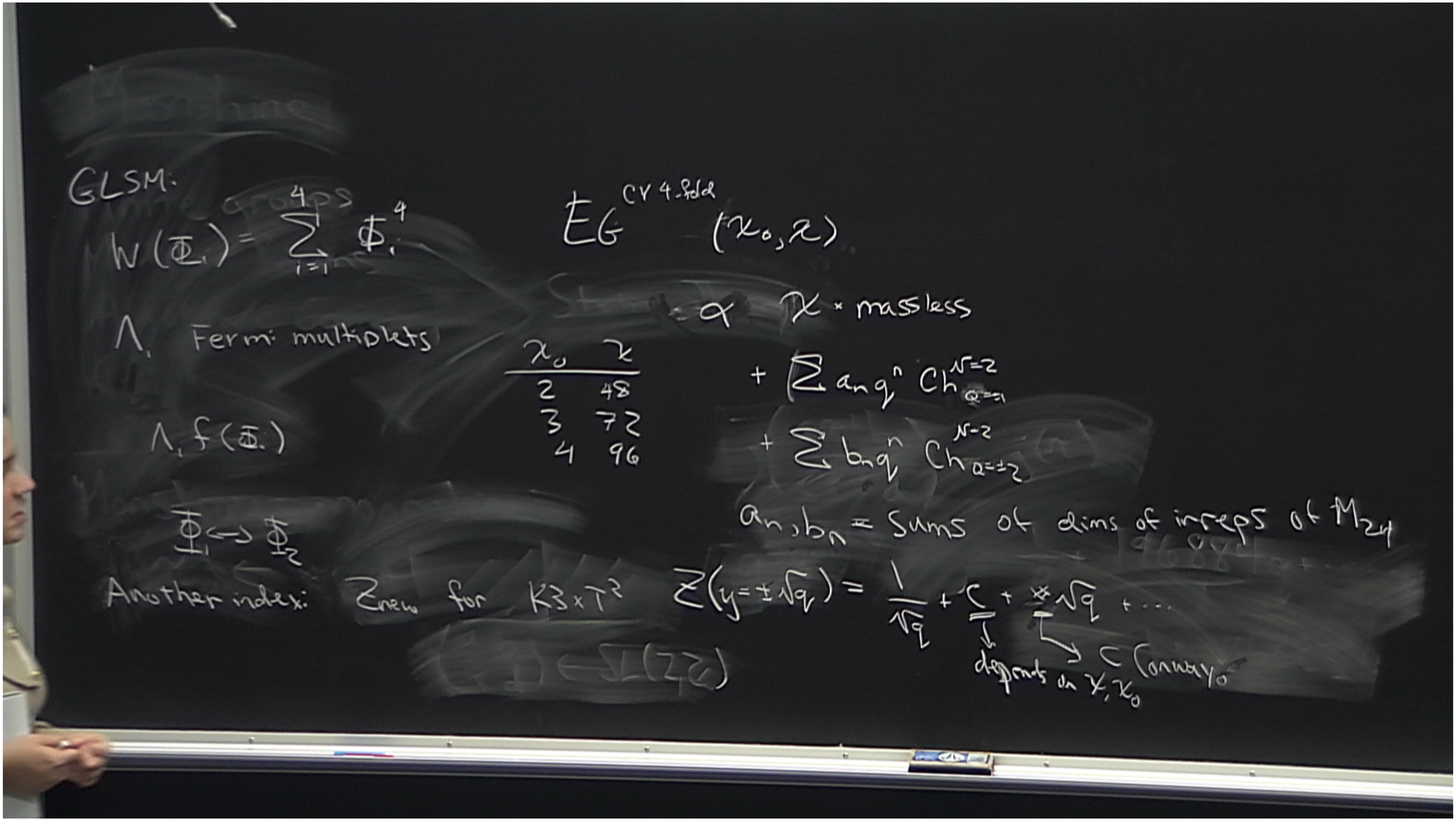
$$EG^{CY\ 4\text{-fold}}(x_0, x)$$

x_0	x
2	48
3	72
4	96

$$= \alpha\ \chi \times \text{massless} + \sum a_n q^n \, Ch^{N=2}_{Q=1} + \sum b_n q^n \, Ch^{N=2}_{Q=\frac{1}{2}}$$

a_n, b_n = sums of dims of irreps of M_{24}



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Λ_i Fermi multiplets
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$\Phi_1 \leftrightarrow \Phi_2$
Another index: Z_{new} for $K3 \times T^2$
$EG^{CY\,4\text{-fold}}(\chi_0, \chi) = \alpha\, \chi \times$ massless
$+ \sum a_n q^n Ch^{N=2}_{Q=-1}$
$+ \sum b_n q^n Ch^{N=2}_{Q=\pm\frac{1}{2}}$
χ_0 χ
2 48
3 72
4 96
a_n, b_n = Sums of dims of irreps of M_{24}
$Z(y = \pm\sqrt{q}) = \frac{1}{\sqrt{q}} + c + * \sqrt{q} + \ldots$
depends on χ, χ_0
$\subset$ Conway$_0$

GLSM:

$W(\Phi_i) = \sum_{i=1}^{4} \Phi_i^4$

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Another index: Z_{new} for $K3 \times T^2$

$EG^{CY\,4\text{-fold}}(\chi_0, \chi)$

$= \alpha\ \chi \times \text{massless}$

$+ \sum a_n q^n \, Ch^{N=2}_{Q=\pm1}$

$+ \sum b_n q^n \, Ch^{N=2}_{Q=\pm2}$

χ_0	χ
2	48
3	72
4	96

a_n, b_n = Sums of dims of irreps of M_{24}

$$Z(y = \pm\sqrt{q}) = \frac{1}{\sqrt{q}} + \underbrace{c}_{\text{depends on } \chi, \chi_0} + \underbrace{*}_{\subset \text{Conway}_0}\sqrt{q} + \dots$$

$\sum b_n q^n \; ch_{Q=\pm 2}$

a_n, b_n = sums of dims of irreps of

$$Z(y=\pm\sqrt{q}) = \frac{1}{\sqrt{q}} + C + \# \sqrt{q} + \dots$$

C depends on χ, χ_0

$\# \to C$ Conway$_0$

$Z^{K3\times K3}(y=\pm\sqrt{q}) \to$ reps of monster group

DHG

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Niemeier lattices

ADE classification

$A_1^{24}, A_2^{12}, A_3^{8} \ldots$

$D_4^{6} \ldots$

A_1^{24} : Z^{K3} M_{24}

A_2^{12} : $\phi(q,y)$: $2.M_{12}$

⋮

on K3:

4 superconformal algebra

$a_n = 2(45 + 231 + \ldots)$

χ_g

$$Z^{K3} = \sum_i Z_{ALE} + \sum_{i=1}^{m} \underline{h_m(q)}\ \theta_m(q)$$

$\hookrightarrow$ reps of automor

$$24 Z_{ALE(A_1)} + (45 + 231 + \cdots)\ Ch$$

$12 Z_{ALE(A_2)}$ + reps of $Z.M_{12}$

$\vdots$