Title: Scale-free primordial cosmology

Date: Dec 19, 2013 11:00 AM

URL: http://pirsa.org/13120063

Abstract: The large-scale structure of the universe suggests that the physics underlying its early evolution is scale-free. In this talk, using a hydrodynamic approach, I will discuss how the scale-free principle restores predictive power and makes it possible to evaluate inflationary models and to compare them with alternative cosmologies.

Parameter unpredictability

$$V(\phi) = m^2\phi^2/2\,[1 - \sqrt{2}(\lambda/m)\sin(\vartheta)\phi + 1/2(\lambda/m)^2\phi^2]$$

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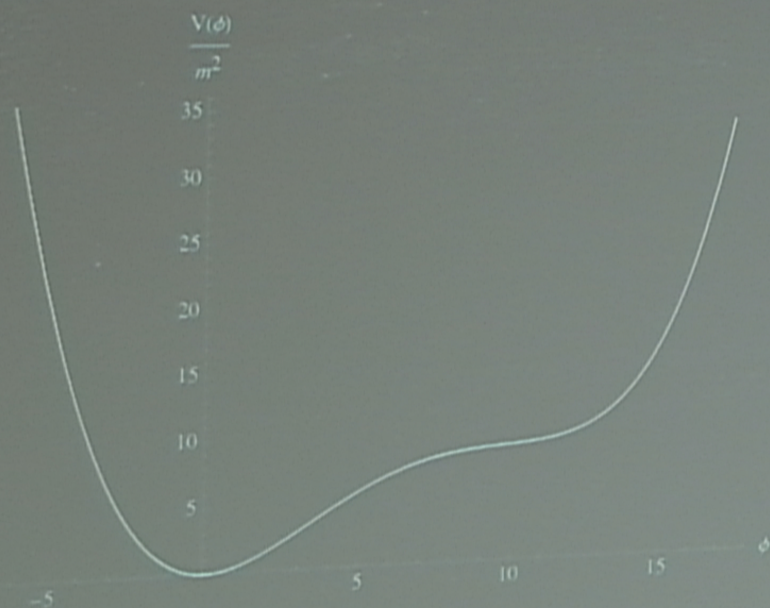

Planck2013 fit:

$m = (1-2)\times 10^{13}$ GeV

$\lambda = (3-7)\times 10^{-7}$

$\vartheta = 23\pi/60$

Destri, De Vega, Sanchez 2008
Nakayama, Takahashi, Yanagida 2013
Ferrara, Kallosh, Linde, Porrati 2013

Basics

smoothing fluid component ϱ_s

$\varrho_s / a^{2\varepsilon}$

where $\varepsilon = 3/2(w+1)$, $w = p/\rho$

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I. N_{tot} e-folds of inflation occur: $\varepsilon(N) < 1$ for $1 < N < N_{tot}$, and

II. inflation ends in the last e-fold (graceful exit): $\varepsilon(0) = 1$, $\varepsilon(N>0) < 1$ and $\varepsilon(N<0) \geq 1$, where N is the # of e-folds of inflation remaining until its end.

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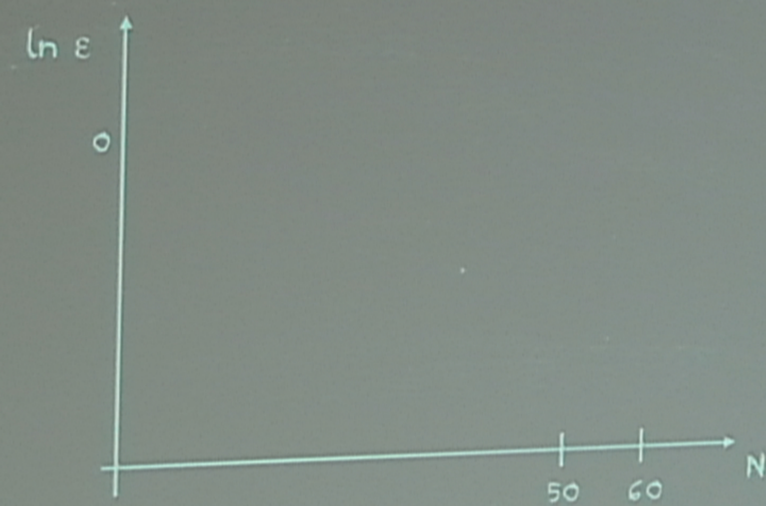

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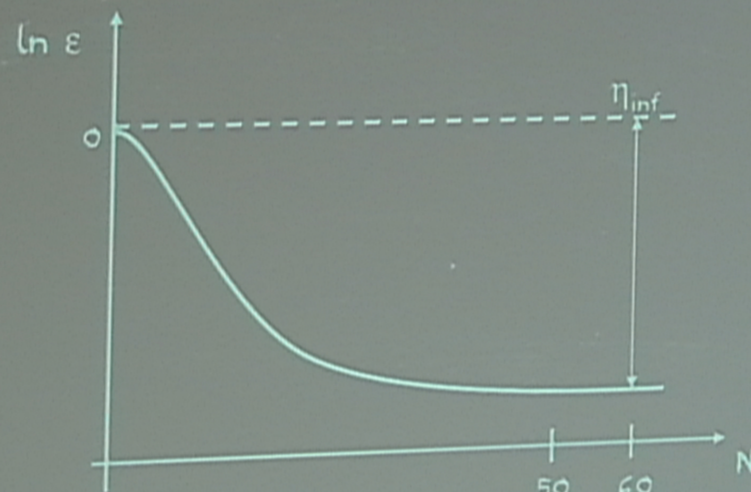

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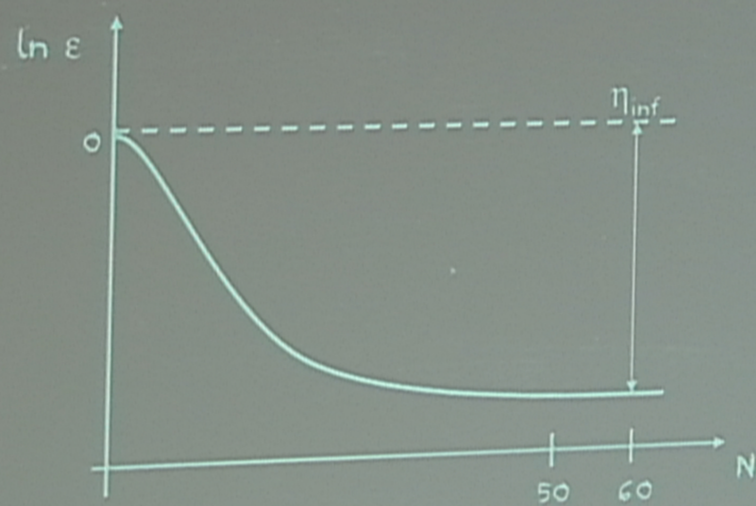

$\varepsilon(N)$ can take many forms so the "predictions" can vary arbitrarily!

Restoring predictability

additional assumption: SCALE-FREE PHYSICS

(a function f: R -> R is scale-free iff it has power-law form up to a coordinate shift)

Restoring predictability

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(a function f: R -> R is scale-free iff it has power-law form up to a coordinate shift)

-> scale-free inflation: $\varepsilon(N) = 1/(N+1)^{\alpha}, \quad \alpha > 0$

Background

$$\varepsilon = -(dH/dt)/H^2, \quad -d(\ln a) = dN \quad \rightarrow \quad \varepsilon = d(\ln H)/dN$$

$$\rightarrow \quad \text{for } \varepsilon(N) = 1/(N+1)^{\alpha}, \quad \alpha > 0:$$

$$H^2/H^2_{end} = \begin{cases} (N+1)^2, & \alpha = 1, \\ \exp[2(1-(N+1)^{1-\alpha})/(\alpha-1)], & \alpha \neq 1 \end{cases}$$

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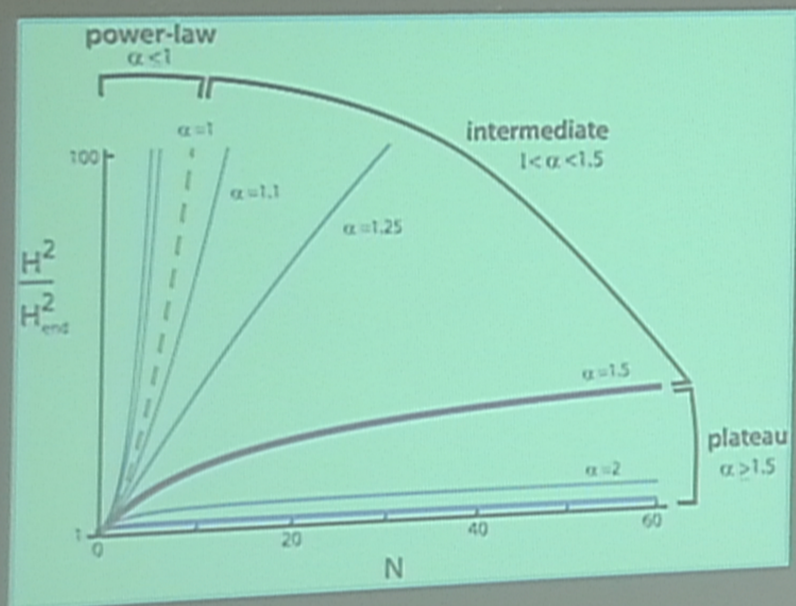

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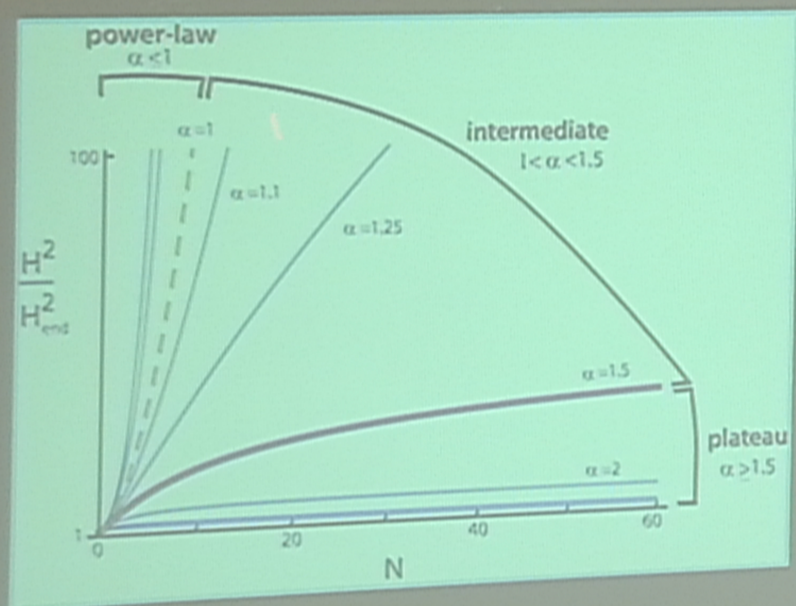

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THREE DISTINCT CLASSES OF INFLATIONARY MODELS!

Perturbations

$$n_s - 1 = -\,2\varepsilon + d\ln\varepsilon/dN$$

Wang, Mukhanov, Steinhardt 1997

$$\rightarrow \; n_s - 1 = -\,2/(N+1)^{\alpha} - \alpha/(N+1)$$

$\alpha = 1$: "strictly scale-free"

$\alpha \neq 1$: "background only scale-free"

NOTE: scale-free inflation always implies a red tilt & minimum deviation from Harrison-Zel'dovich-Peebles spectrum!

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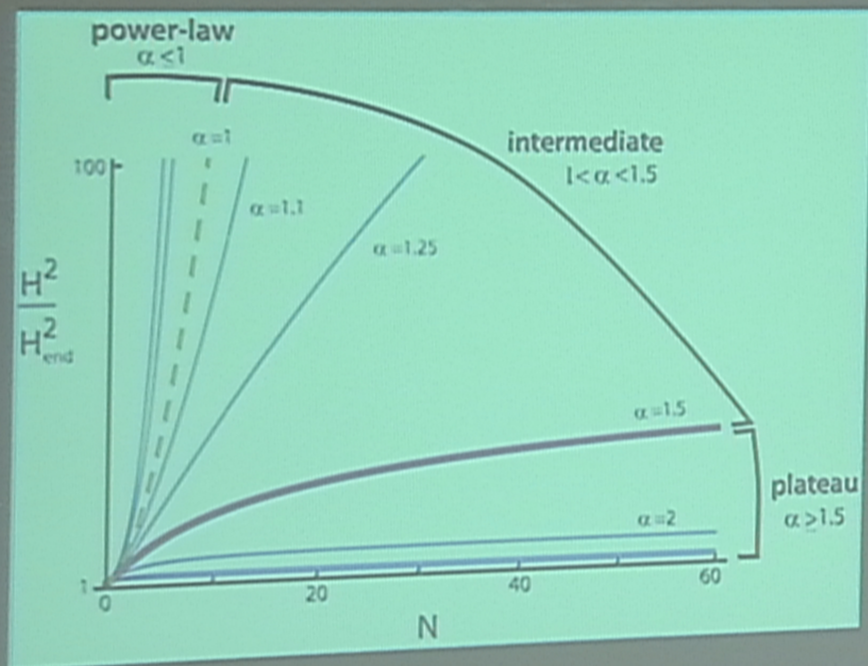
$N = 60$: n_s has maximum value of .97 for $\alpha = 1.5$

$$r = 16\varepsilon = 16/(N+1)^{\alpha}$$

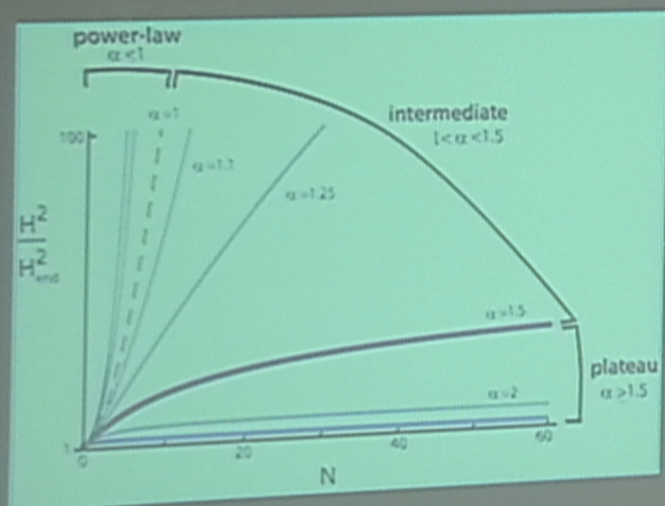
1. Extra parameters



curves of plateau-like class have a feature at small N:
rapid cutoff of inflation after a long period of nearly constant H^2 ->

2. Hydrodynamic initial conditions problem

for inflation to start, the kinetic energy and gradients must be negligibly small within a Hubble-sized patch at the onset of inflation



size of initial patch at t_{Pl}:

$$R^3(t_{Pl}) \geq (M_{Pl}/M_{beg})^3 \times H^{-3}(t_{Pl})$$

Al, Steinhardt, Loeb 2013

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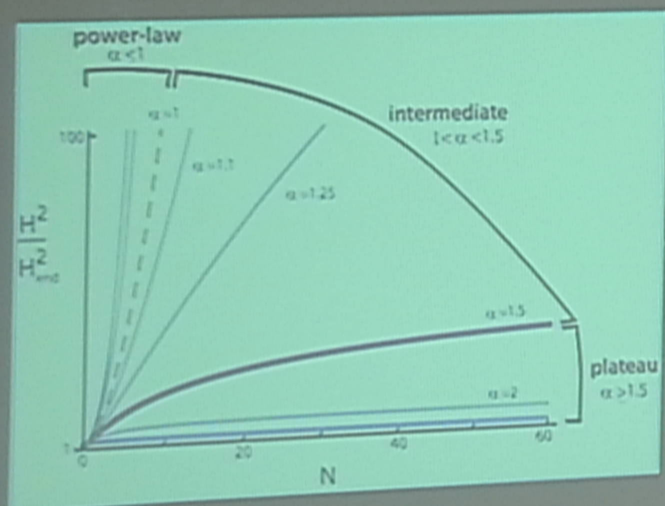

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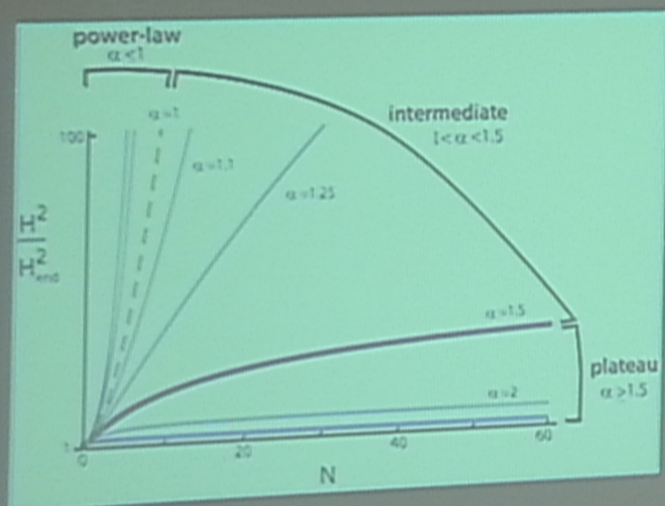

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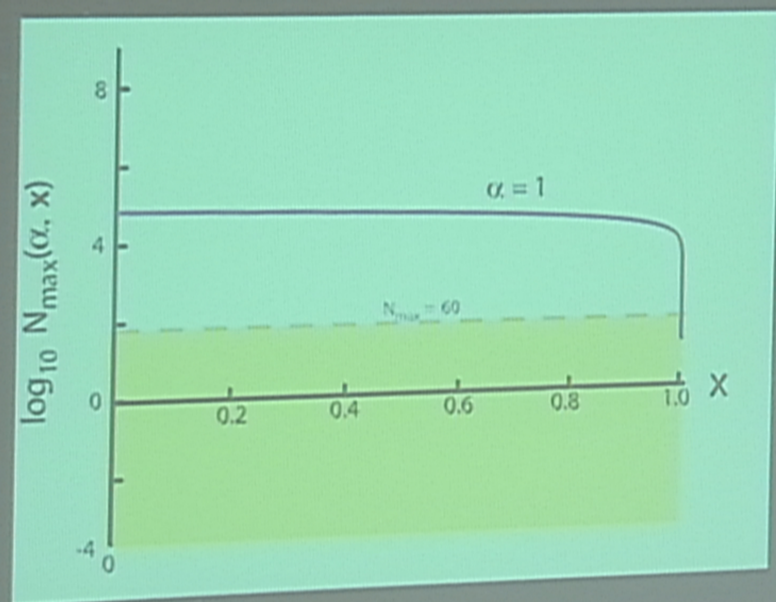
$$R^3(t_{Pl}) \gtrsim (M_{Pl}/M_{beg})^3 \times H^{-3}(t_{Pl})$$

- power-law: $M_{Pl}/M_{beg} \sim 1$
- plateau, intermediate: $M_{Pl}/M_{beg} \gtrsim 10^3$

Ijjas, Steinhardt, Loeb 2013

3. unlikeliness problem

-> sensitivity to variations in the initial energy density:



x: flex of order x in the initial potential energy density

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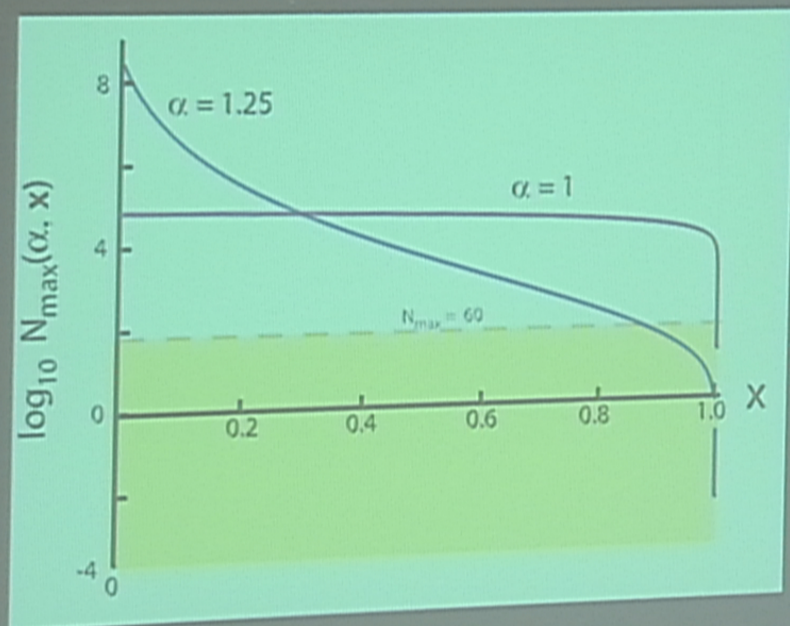

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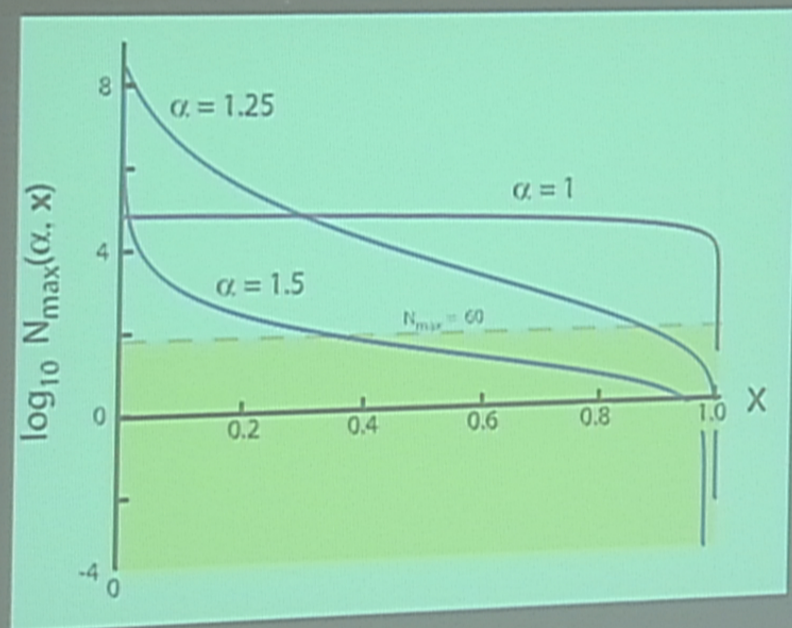

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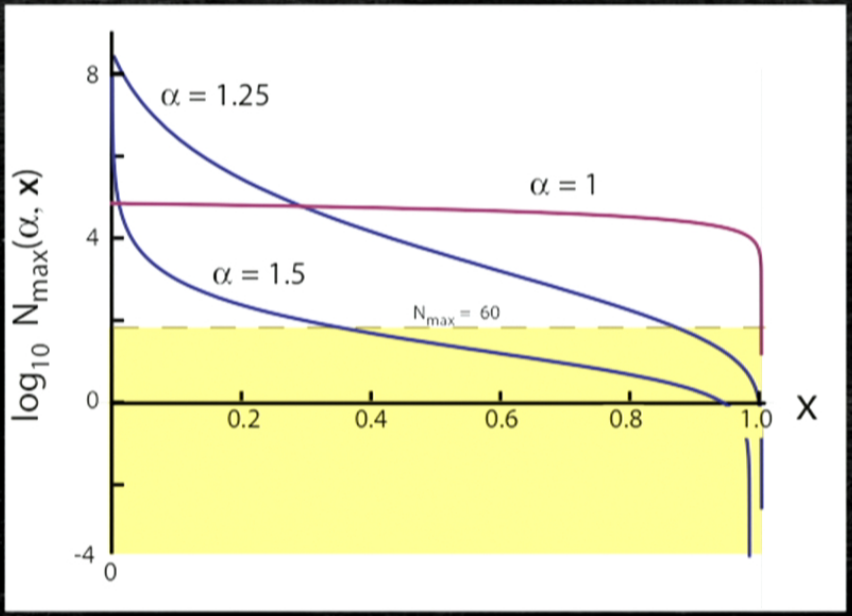

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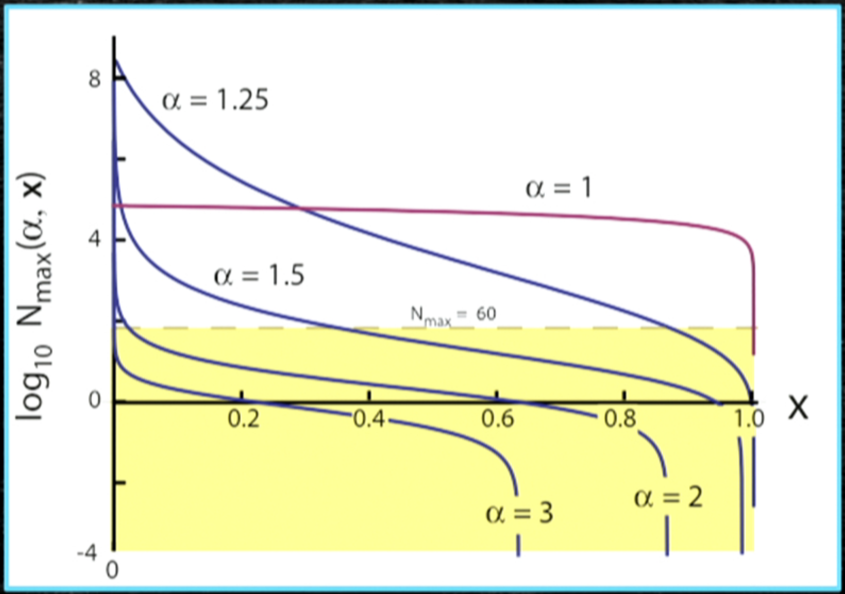

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	powerlaw-like	intermediate	plateau-like
extra parameters			
initial conditions problem			
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↑

observationally favored class!

Scale-free cyclic/ekpyrotic theory

Conditions for a successful cyclic/ekpyrotic model:

I. $\mathcal{N}_{tot}$ e-folds of ekpyrosis occur: $\varepsilon(\mathcal{N}) > 3$ for $1 < \mathcal{N} < \mathcal{N}_{tot}$, and

II. ekpyrosis ends in the last e-fold: $\varepsilon(0) = 3$ & $\varepsilon(\mathcal{N}>0) > 3$, where

$$\mathcal{N} = \ln(a_{end}H_{end}/aH)$$

is the # of e-folds of modes that exit the horizon before the end of ekpyrosis.

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Again, ε can take many forms and the predictions can vary arbitrarily!

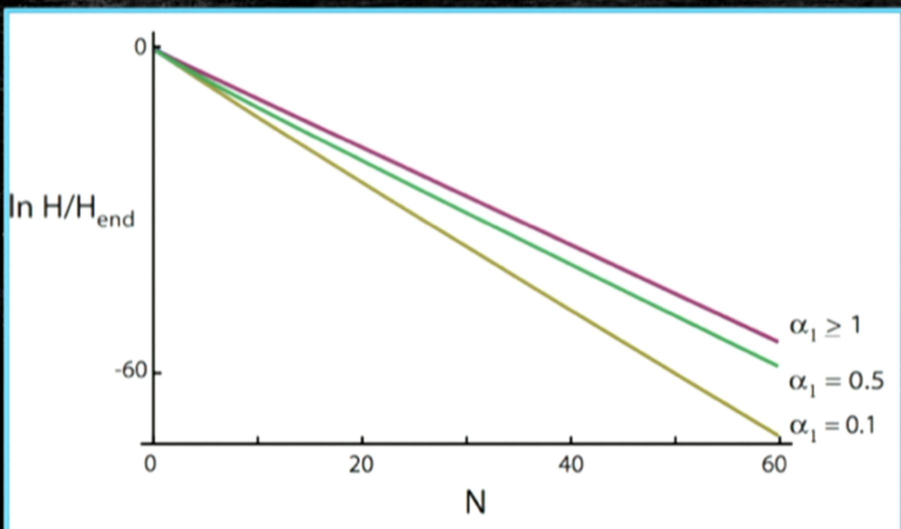
-> scale-free ekpyrosis:

$$\varepsilon(\mathcal{N}) = 3(\mathcal{N}+1)^{\alpha_1}, \quad \alpha_1 > 0$$

Background

$\varepsilon = d(\ln H)/dN, \quad d\mathcal{N} = (\varepsilon-1)dN \quad \rightarrow \quad d(\ln H)/d\mathcal{N} = 1 + 1/(\varepsilon-1)$

$\rightarrow \quad \varepsilon(\mathcal{N}) = 3(\mathcal{N}+1)^{\alpha_1}: \quad H^2/H^2_{end} = \exp\left[-2\mathcal{N} + 2\int_{\mathcal{N}}^{0} 1/(3(\mathcal{N}+1)^{\alpha_1} - 1)\, d\mathcal{N}\right]$



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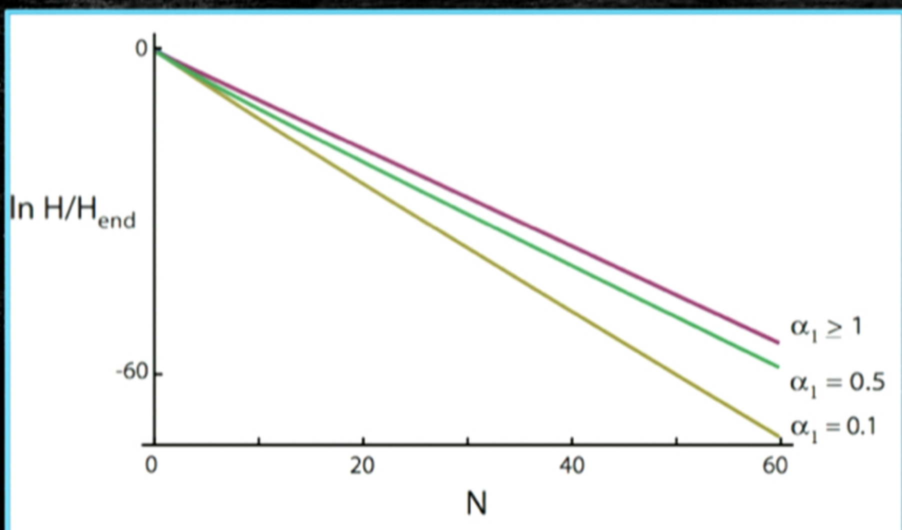

ONE SINGLE CLASS OF CYCLIC/ EKPYROTIC MODELS!

- no unlikeliness problem
- no extra parameters
- no initial conditions problem

Perturbations

-> two-component fluid:

1) background

2) perturbations

-> entropy/isocurvature perturbations before the bounce (-> $n_s - 1$, r)

-> conversion into density perturbations during the transition from big crunch to big bang (-> contributions to f_{NL}, g_{NL})

Perturbations

-> two-component fluid:

1) background $\quad \varepsilon_1(\mathcal{N}) = 3(\mathcal{N}+1)^{\alpha_1}, \quad \alpha_1 > 0$

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-> $\varepsilon_1(\mathcal{N}) = \varepsilon_2(\mathcal{N})$: $\alpha_1 = \alpha_2$, $\beta_2 = 1$

Perturbations

$$n_s - 1 = 2/\varepsilon_2 - (d\varepsilon_2/d\mathcal{N})/\varepsilon_2$$

Lehners, McFadden, Turok, Steinhardt 2007
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$$r \sim 0$$

Scale-free scalar fields and potentials

1) Transform N to ϕ

$\varepsilon = -(dH/dt)/H^2$, $dH/dt = -1/2(\rho_s + p_s) = -1/2(d\phi/dt)^2$

-> $d\phi/dN = \pm\sqrt{(2\varepsilon)}$

2) Find V as a function of $N(\phi)$.

$3H^2 = 1/2(d\phi/dt)^2 + V$

Scale-free scalar fields and potentials: Inflation

$$V(\phi) = \begin{cases} \lambda\phi^4, & \alpha = 1, \\ V_{end}\exp[2 - 2\exp(-(\phi - \phi_{end}/\sqrt{2}M_{Pl}))], & \alpha = 2, \\ V_{end}\exp[2/(1-\alpha)((1 \pm (2-\alpha)(\phi - \phi_{end}/\sqrt{2}M_{Pl})^{2-2\alpha/2-\alpha} - 1)], & \text{otherwise} \end{cases}$$

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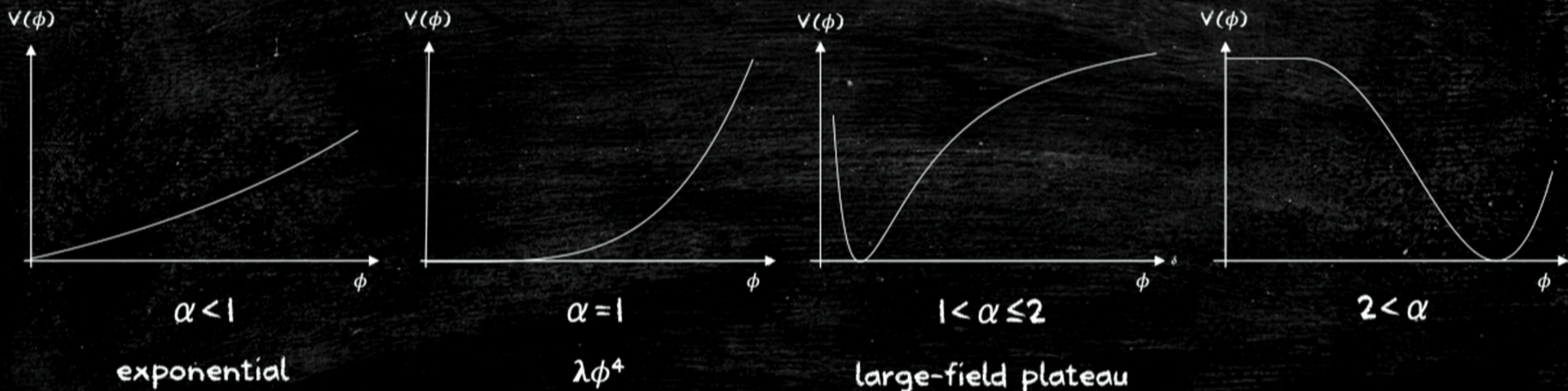

Scale-free scalar fields and potentials: Cyclic theory

- background -> field σ
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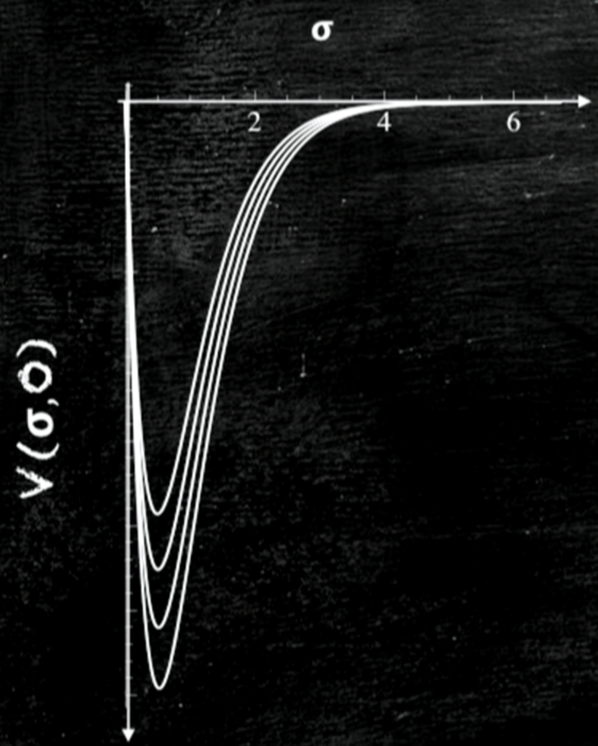

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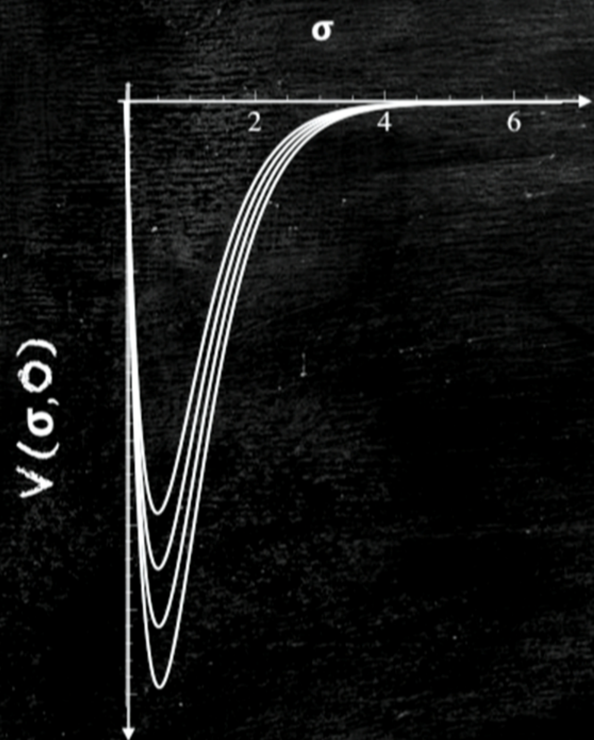

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For example, for $\alpha_1 = 1$:

$$V(\sigma,0) = -3H^2_{end}(\sigma^2-1)\exp(-2\sigma^2)$$

Testing for scale-freeness

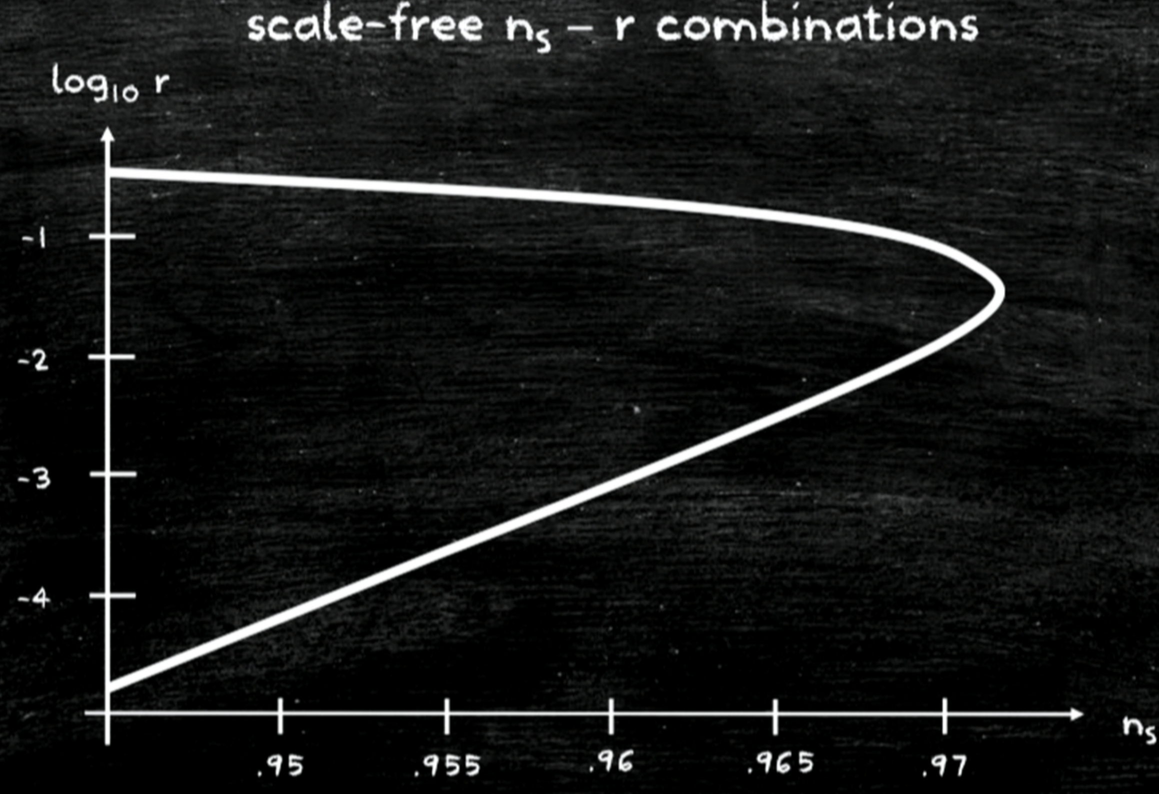
Advantages of scale-freeness:

- predictive
- consistent with current observational data

-> **TEST:**

1) measure r
2) determine α_r
3) check if $n_s-1(\alpha_r)$ fits data

Testing for scale-freeness



Testing for scale-freeness

Possible future scenarios:

1) $r \gtrsim 10^{-4}$: scale-free inflation if $n_s - 1(\alpha_r)$ fits data
   NOTE: $r < .1$ fits only scale-free models that suffer from the unlikeliness, extra parameters, and initial conditions problems

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2) $r < 10^{-4}$: scale-free cyclic