Title: Consistency of Massive Gravity

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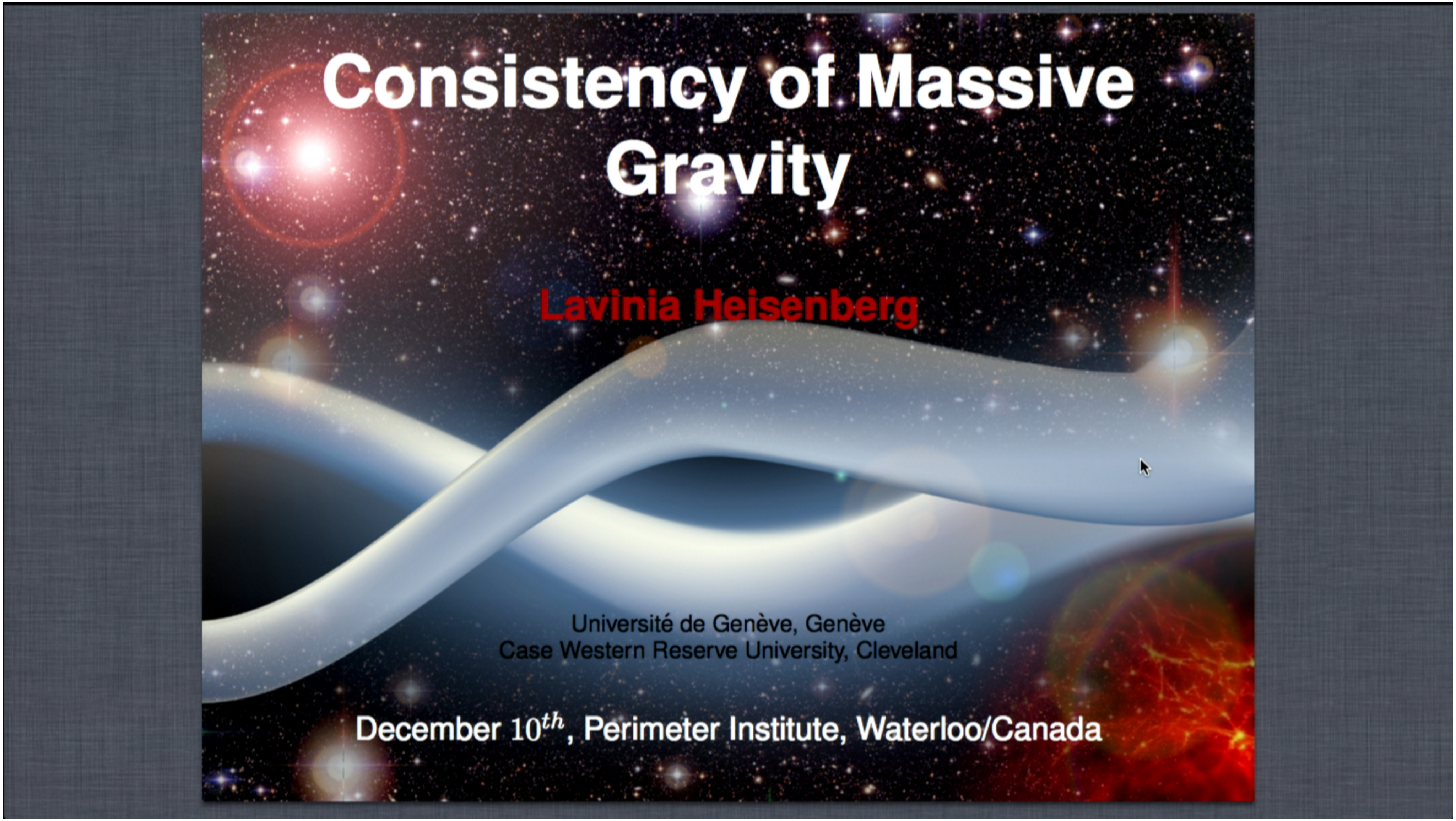
Abstract: Recently there has been a successful non-linear covariant ghost-free generalization of Fierz-Pauli massive gravity theory, the dRGT theory. I will explore the cosmology in the decoupling limit of this theory. Furthermore, I will construct a Proxy theory to dRGT from the decoupling limit and study the cosmology there as well and compare the results. Finally, I will discuss the quantum consistency of the theory.


anada
CAUTION


Consistency of Massive Gravity
Lavinia Heisenberg
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December 10th, Perimeter Institute, Waterloo/Canada

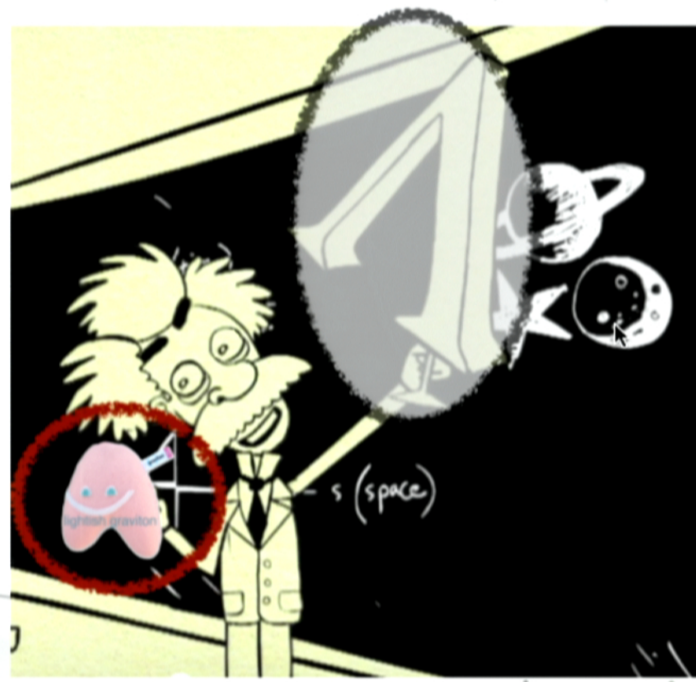
outline

1. Introduction
2. Screening
3. MG
4. dRGT
5. DL
6. Proxy Theory
7. Quantum Consistency
8. Conclusion

What is Dark Energy?

3 options?

- **Cosmological Constant**
  (Why is it so small?)
  → cosmological constant problem?
- **Dark Energy**
  (Why don't we see them? Similar fine-tuning problem?)
- **Dark Gravity**
  (Is there any viable model?)
  → massive gravity?



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IR Modification of GR

Motivations for IR Modification of GR

- a very nice alternative to the Cosmological Constant or dark energy for explaining the recent acceleration of the Hubble expansion
- a way of attacking the Cosmological Constant problem (fine-tuning problem)
  $\Lambda_{\text{phys}} = \Lambda_{\text{bare}} + \Delta\Lambda \sim (10^{-3}\text{eV})^4$ with $\Delta\Lambda \sim \text{TeV}^4$
- it is only fair to have an alternative model to which we can compare GR and test both theories against each other
- learn more about GR by modifying it!
- fun!

Dark Gravity

Maybe not modifying that much! only close to the horizon scale (~ 1Gpc$/h$) , corresponding to modifying gravity today.

New degrees of freedom (dof) in the infra-red (IR)

Modifying gravity in the IR typically requires new dof usually: scalar field

$$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_\pi(\partial\delta\pi)^2 - \frac{1}{2}{m_\pi}^2(\delta\pi)^2 - g_\pi\delta\pi T$$

where these quantities $\mathcal{Z}_\pi, m_\pi, g_\pi$ depend on the field.

Density dependent mass

- Chameleon
  m_π depends on the environment
  (Khoury, Weltman 2004)

Density dependent coupling

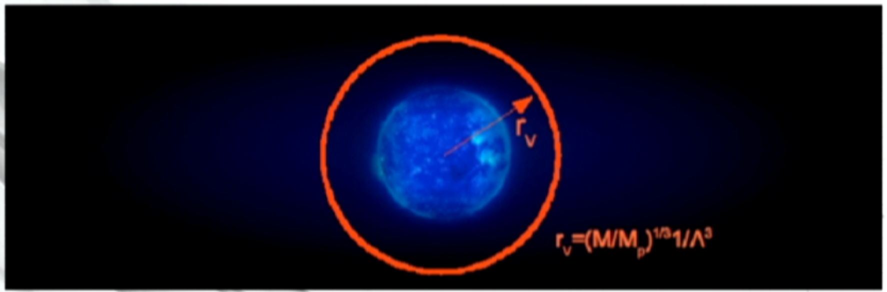
- **Vainshtein**(1971)
  $\mathcal{Z}_\pi$ depends on the environment
- Symmetron
  g_π depends on the environment
  (Hinterbichler, Khoury 2010)

Vainshtein mechanism (massive gravity, Galileons)

$$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_\pi(\partial\delta\pi)^2 - g_\pi\delta\pi T$$

Screening with

- effective coupling to matter depends on the self-interactions of these new dof
  $\Box\delta\pi \sim \frac{1}{M_p}\frac{\delta\pi}{\sqrt{\mathcal{Z}}}T$
  → coupling small for properly canonically normalized field!
  ($\mathcal{Z} \gg 1 \rightarrow$ coupling small)
- non-linearities dominate within Vainshtein radius



Chameleon mechanism (f(R) theories)

important ingredients: a conformal coupling between the scalar and the matter fields $\tilde{g}_{\mu\nu} = g_{\mu\nu} A^2(\pi)$, and a potential for the scalar field $V(\pi)$ which includes relevant self-interaction terms.

$$S = \int d^4x \sqrt{-g} \left(\frac{M_p^2}{2} R - \frac{1}{2}(\partial\pi)^2 - V(\pi) \right) + S_{matter}[g_{\mu\nu} A^2(\pi)]$$

The equation of motion for π:

$$\nabla^2 \pi = V_{,\pi} - A^3(\pi) A_{,\pi} \tilde{T} = V_{,\pi} + \rho A_{,\pi}$$

where $\tilde{T} \sim \rho / A^3(\pi)$

giving rise

to an effective potential

$V_{\text{eff}}(\pi) = V(\pi) + \rho A(\pi)$

Symmetron mechanism

important ingredients:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\pi)^2 - V(\pi)\right) + S_{matter}[g_{\mu\nu}A^2(\pi)]$$

with a symmetry-breaking potential

$$V(\pi) = \frac{-1}{2}\mu^2\pi^2 + \frac{1}{4}\lambda\pi^4$$

and a conformal coupling to matter of the form

$$A(\pi) = 1 + \frac{\pi^2}{2M^2} + \mathcal{O}(\pi^4/M^4)$$

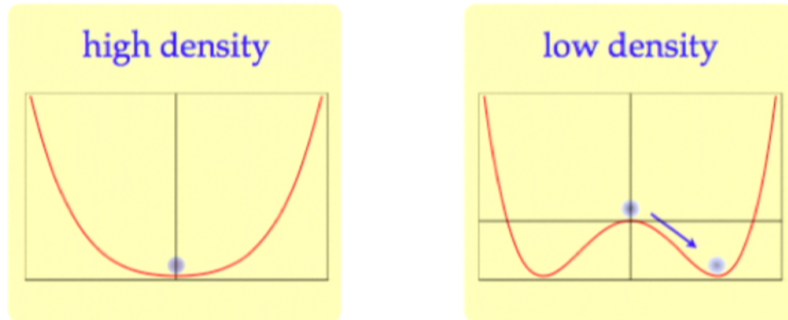
giving rise

to an effective potential

$$V_{\text{eff}} = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\pi^2 + \frac{1}{4}\lambda\pi^4$$

$\rho > \mu^2 M^2 \rightarrow$ the field sits in a minimum at the origin

Symmetron mechanism



- perturbations couple as $\frac{\bar{\pi}}{M^2}\delta\pi\rho$
- In high density symmetry-restoring environments, the scalar field vev $\sim 0 \rightarrow$ fluctuations of the field do not couple to matter
- As the local density drops the symmetry of the field is spontaneously broken and the field falls into one of the two new minima with a non-zero vev.
- $\rightarrow$ coupling to matter depends on the enviroment (g small in regions of high density)

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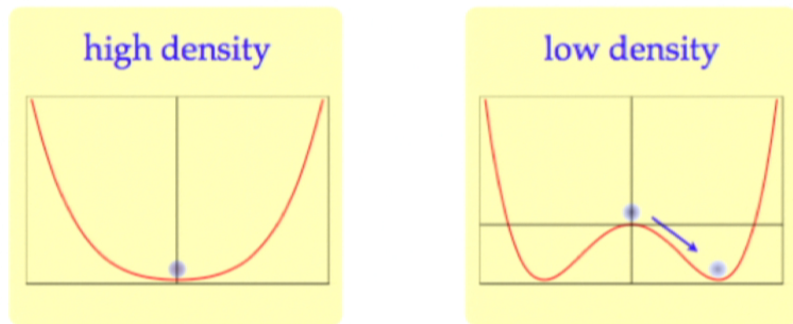

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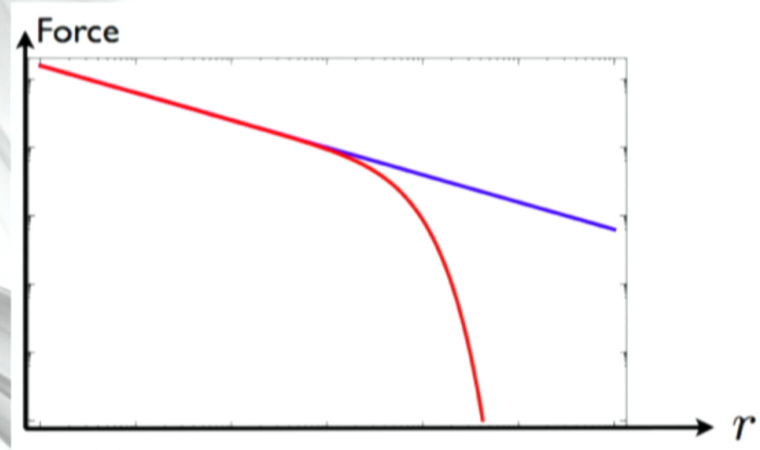
Massive Gravity (MG)

One of the most natural ways to modify Gravity in the IR is to give the graviton a mass.

- **In GR, Gravity has an infinite range** $F = \partial_r \left(\frac{M_1 M_2 G_N}{r}\right)$
- **In MG, the force "shuts down" at some distance** $\lambda \sim m^{-1}$
  $F = \partial_r \left(\frac{M_1 M_2 G_N}{r} e^{-mr}\right)$



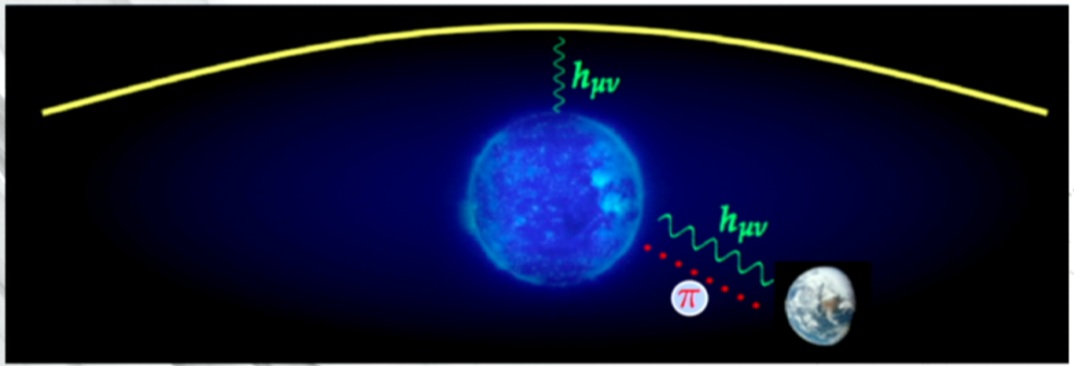
Massive Gravity

A general linear mass term for the graviton is

$$\mathcal{L}_{mass} = -\frac{1}{2}M_p^2(m_1^2 h^{\mu\nu}h_{\mu\nu} + m_2^2 h^2)$$

The only **ghost-free**: $m_1^2 = -m_2^2$ Fierz-Pauli tuning

$\rightarrow$ **vDVZ discontinuity**



Massive Gravity

Artifact:

The vDVZ discontinuity is just an artifact of the linear approximation
→ non-linear extension

Issue:

The ghost we have cured by Fierz-Pauli tuning seems to come back at non-linear level (the sixth degree of freedom is associated to higher derivative terms)

Ghost

challenging task: non-linear extension of FP **without ghost**

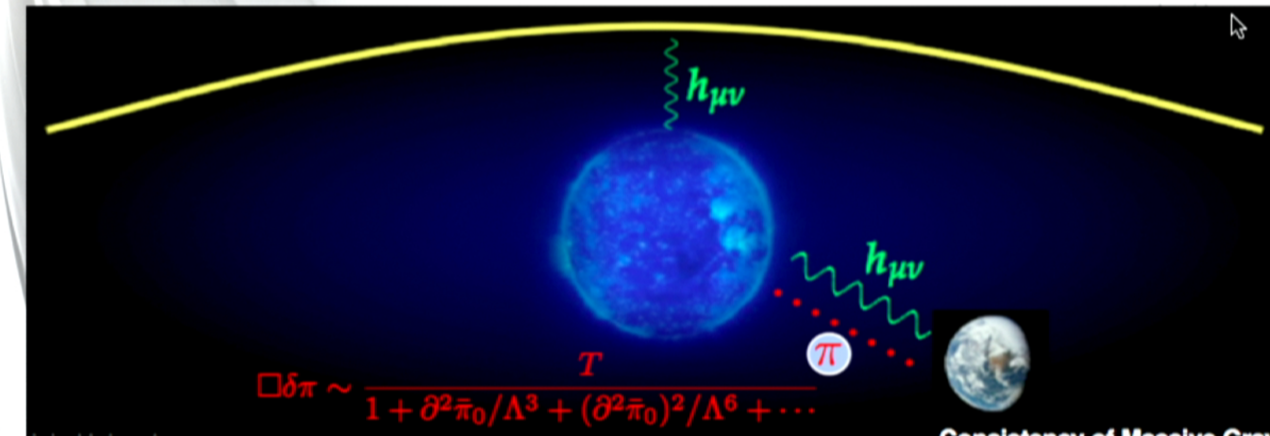
dRGT

dRGT

Vainshtein mechanism at work in dRGT:
The effective coupling to matter depends on the self-interactions of the helicity-0 mode π

$$\Box\delta\pi \sim \frac{1}{M_p}\frac{\delta\pi}{\sqrt{Z}}T$$

$\rightarrow$ The non-linearities cure the vDVZ discontinuity



Ghost-free extension of FP = dRGT

a 4D covariant theory of a massive spin-2 field

$$\mathcal{L} = \frac{M_p^2}{2}\sqrt{-g}\left(R - \frac{m^2}{4}\mathcal{U}(g,H)\right)$$

the most generic potential that bears no ghosts is $\mathcal{U}(g,H) = -4\,(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4)$ where the covariant tensor $H_{\mu\nu} = h_{\mu\nu} + 2\Pi_{\mu\nu} - \eta^{\alpha\beta}\Pi_{\mu\alpha}\Pi_{\beta\nu}$ and the potentials:

$$\begin{aligned}
\mathcal{U}_2 &= [\mathcal{K}]^2 - [\mathcal{K}^2] \\
\mathcal{U}_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \\
\mathcal{U}_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]
\end{aligned}$$

where $\mathcal{K}^\mu_\nu(g,H) = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$, $\Pi_{\mu\nu} = \partial_\mu\partial_\nu\pi$ and $[..]$ =trace.
(de Rham, Gabadadze, Tolley (Phys.Rev.Lett.106,231101))

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Decoupling limit (DL) of dRGT theory

DL ($M_p \to \infty$, $m \to 0$ with $\Lambda_3^3 = m^2 M_p \to$ const) gives the following scalar-tensor interactions
$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \mathcal{E}_{\mu\nu}{}^{\alpha\beta} h_{\alpha\beta} + h^{\mu\nu} \sum_{n=1}^{3} \frac{a_n}{\Lambda_3^{3(n-1)}} X^{(n)}_{\mu\nu}[\Pi]$
where $h_{\mu\nu}$ =helicity-2, π =helicity-0 field and $a_1 = -\frac{1}{2}$ and $a_{2,3}$ are two arbitrary constants and $X^{(1,2,3)}_{\mu\nu}$ denote

$$X^{(1)}_{\mu\nu}[\Pi] = \varepsilon_{\mu}{}^{\alpha\rho\sigma} \varepsilon_{\nu}{}^{\beta}{}_{\rho\sigma} \Pi_{\alpha\beta},$$
$$X^{(2)}_{\mu\nu}[\Pi] = \varepsilon_{\mu}{}^{\alpha\rho\gamma} \varepsilon_{\nu}{}^{\beta\sigma}{}_{\gamma} \Pi_{\alpha\beta} \Pi_{\rho\sigma},$$
$$X^{(3)}_{\mu\nu}[\Pi] = \varepsilon_{\mu}{}^{\alpha\rho\gamma} \varepsilon_{\nu}{}^{\beta\sigma\delta} \Pi_{\alpha\beta} \Pi_{\rho\sigma} \Pi_{\gamma\delta}$$

with $\Pi_{\alpha\beta} = \partial_\alpha \partial_\beta \pi$.
The structure of the interactions are very similar to the Galileon interactions $\mathcal{L}_{\rm Gal} = c_n \pi \varepsilon^{\mu\nu\cdots} \varepsilon^{\alpha\beta\cdots} \Pi_{\mu\alpha} \Pi_{\nu\beta} \cdots \Pi ... \Pi ....$

Diagonalized interactions

The transition to Einsteins frame is performed by the change of variable

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - 2a_1\pi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi$$

one recovers Galileon interactions for the helicity-0 mode of the graviton

$$\mathcal{L} = -\frac{1}{2}\bar{h}(\mathcal{E}\bar{h})_{\mu\nu} + 6a_1^2\pi\Box\pi - \frac{6a_2a_1}{\Lambda_3^3}(\partial\pi)^2[\Pi]$$

$$+\frac{2a_2^2}{\Lambda_3^6}(\partial\pi)^2([\Pi^2]-[\Pi]^2) + \frac{a_3}{\Lambda_3^6}h^{\mu\nu}X^{(3)}_{\mu\nu}$$

with the coupling

$$\frac{1}{M_p}\left(\bar{h}_{\mu\nu} - 2a_1\pi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\pi\partial_\nu\pi\right)T^{\mu\nu}$$

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Differences to Galileon interactions

Common

- IR modification of gravity as due to a light scalar field with non-linear derivative interactions
- respects the symmetry $\pi(x) \to \pi(x) + c + b_\mu x^\mu$
- Second order equations of motion, containing at most two time derivatives
- non-renormalization theorem applies

Different

- undiagonazable interaction $+\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ $\to$ important for the self-accelerating solution
- extra coupling $\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$
- only 2 free-parameters
- **observational difference** due to $\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ and $\partial_\mu\pi\partial_\nu\pi T^{\mu\nu}$

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Self-accelerating solution

$$H^2 = m^2\left(2a_2q^2 + 2a_3q^3 - q\right) \text{ and } q = -\frac{a_2}{3a_3} + \frac{(2a_2^2+3a_3)^{1/2}}{3\sqrt{2}a_3}$$

stability

- $H^2 > 0$ and $a_2 + 3a_3q > 0$
- stable self-accelerating solution: $a_2 < 0$ and $\frac{-2a_2^2}{3} < a_3 < \frac{-a_2^2}{2}$
- **$h^{\mu\nu}X^{(3)}_{\mu\nu}$ plays a crucial role for the stability, since $a_3 = 0 \rightarrow$ ghost**
- there is no quadratic mixing term between the perturbations of helicity-2 and -0 field
- cosmological evolution very similar to ΛCDM

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Proxy theory

We had the following Lagrangian in the decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}X^{(1)}_{\mu\nu} + \frac{a_2}{\Lambda^3}h^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{a_3}{\Lambda^6}h^{\mu\nu}X^{(3)}_{\mu\nu} + \frac{1}{2M_p}h^{\mu\nu}T_{\mu\nu}$$

lets integrate by part the first interaction $h^{\mu\nu}X^{(1)}_{\mu\nu}$:

$$\begin{aligned} h^{\mu\nu}X^{(1)}_{\mu\nu} = h^{\mu\nu}(\Box\pi\eta_{\mu\nu} - \partial_\mu\partial_\nu\pi) &= h^{\mu\nu}(\partial_\alpha\partial^\alpha\pi\eta_{\mu\nu} - \partial_\mu\partial_\nu\pi) \\ &= (\Box h - \partial_\mu\partial_\nu h^{\mu\nu})\pi \\ &= -R\pi \end{aligned}$$

so covariantization of the first interaction: $h^{\mu\nu}X^{(1)}_{\mu\nu} \longleftrightarrow -R\pi$

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Proxy theory

Instead of focusing on the entire complicated model, study a proxy theory:

$$\mathcal{L} = \sqrt{-g}M_p(M_pR - \pi R - \frac{a_2}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\pi\partial_\nu\pi\Pi_{\alpha\beta}L^{\mu\alpha\nu\beta})$$

- in 4D $G_{\mu\nu}$ and $L^{\mu\alpha\nu\beta}$ are the only divergenceless tensors $\rightarrow \nabla_\mu G^{\mu\nu} = 0$ and $\nabla_\mu L^{\mu\alpha\nu\beta} = 0$
- All eom are 2^{nd} order $\rightarrow$ No instabilities
- Reproduces the decoupling limit $\rightarrow$ Exhibits the Vainsthein mechanism

LH & de Rham, (PRD84 (2011) 043503)

Self-accelerating solution

- self-acceleration solution: $H =$const and $\dot{H} = 0$.
- make the ansatz $\dot{\pi} = q\frac{\Lambda^3}{H}$.
- assume that we are in a regime where $H\pi \ll \dot{\pi}$

The Friedmann and field equations can be recast in

$$H^2 = \frac{m^2}{3}(6q - 9a_2q^2 - 30a_3q^3)$$

$$H^2(18a_2q + 54a_3q^2 - 12) = 0$$

Assuming $H \neq 0$, the field equation then imposes,

$$q = \frac{-a_2 \pm \sqrt{a_2^2 + 8a_3}}{6a_3}$$

$\rightarrow$ similar to DL our proxy theory admits a self-accelerated solution, with the Hubble parameter set by the graviton mass.

Proxy theory

$$\mathcal{L}^\pi = M_p\left(-\pi R - \frac{a_2}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\pi\partial_\nu\pi\Pi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right)$$

- We recover some decoupling limit results:
  - stable self-accelerating solutions within the same parameter space
- During the radiation domination the energy density for π goes as $\rho^\pi_{\rm rad} \sim a^{-1/2}$ and during matter dominations as $\rho^\pi_{\rm mat} \sim a^{-3/2}$ and is constant for later times $\rho^\pi_\Lambda = \rm const$
- At early time, interactions for scalar mode are important → cosmological screening effect
- Below a critical energy density, screening stop being efficient → scalar contribute significantly to the cosmological evolution
- But still the cosmological evolution different than in ΛCDM

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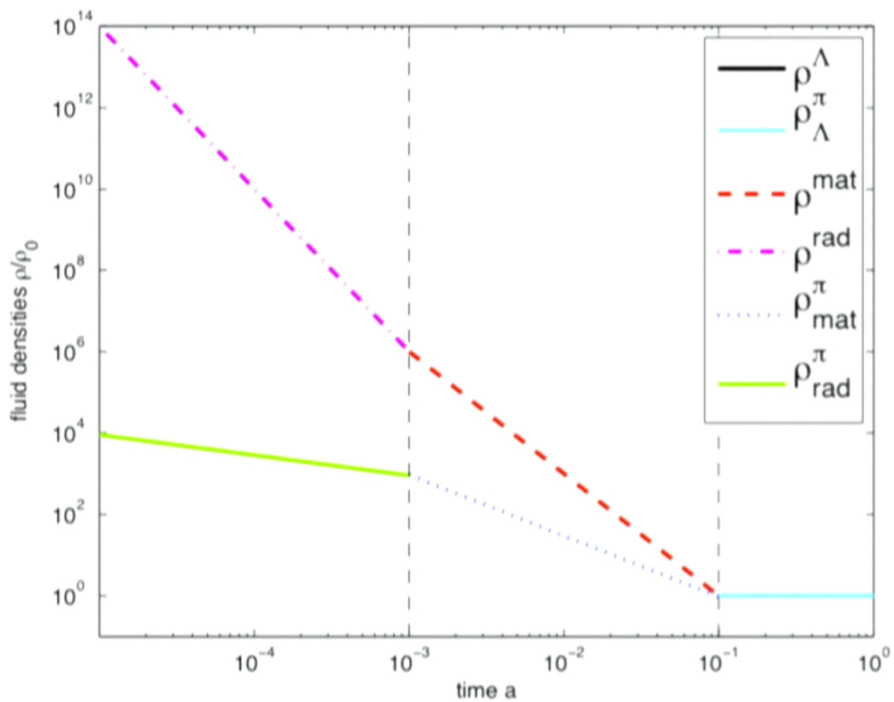
Densities

$$\mathcal{L}^\pi = M_p\left(-\pi R - \frac{a_2}{\Lambda^3}\partial_\mu\pi\partial_\nu\pi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\pi\partial_\nu\pi\Pi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right)$$



Quantum Corrections in Massive Gravity

large quantum corrections?

detuning of parameters?

$$-\frac{1}{4}M_{\rm Pl}^2 m^2 \left((1+c_1)h_{\mu\nu}^2 - (1+c_2)h^2\right)$$

overall scaling

- does the small mass of the graviton receive large quantum corrections?
  $\delta m \gg 1$

relative scaling

- does the relative tuning of the parameters change?
  $c_1 \neq c_2$

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Self-accelerating solution

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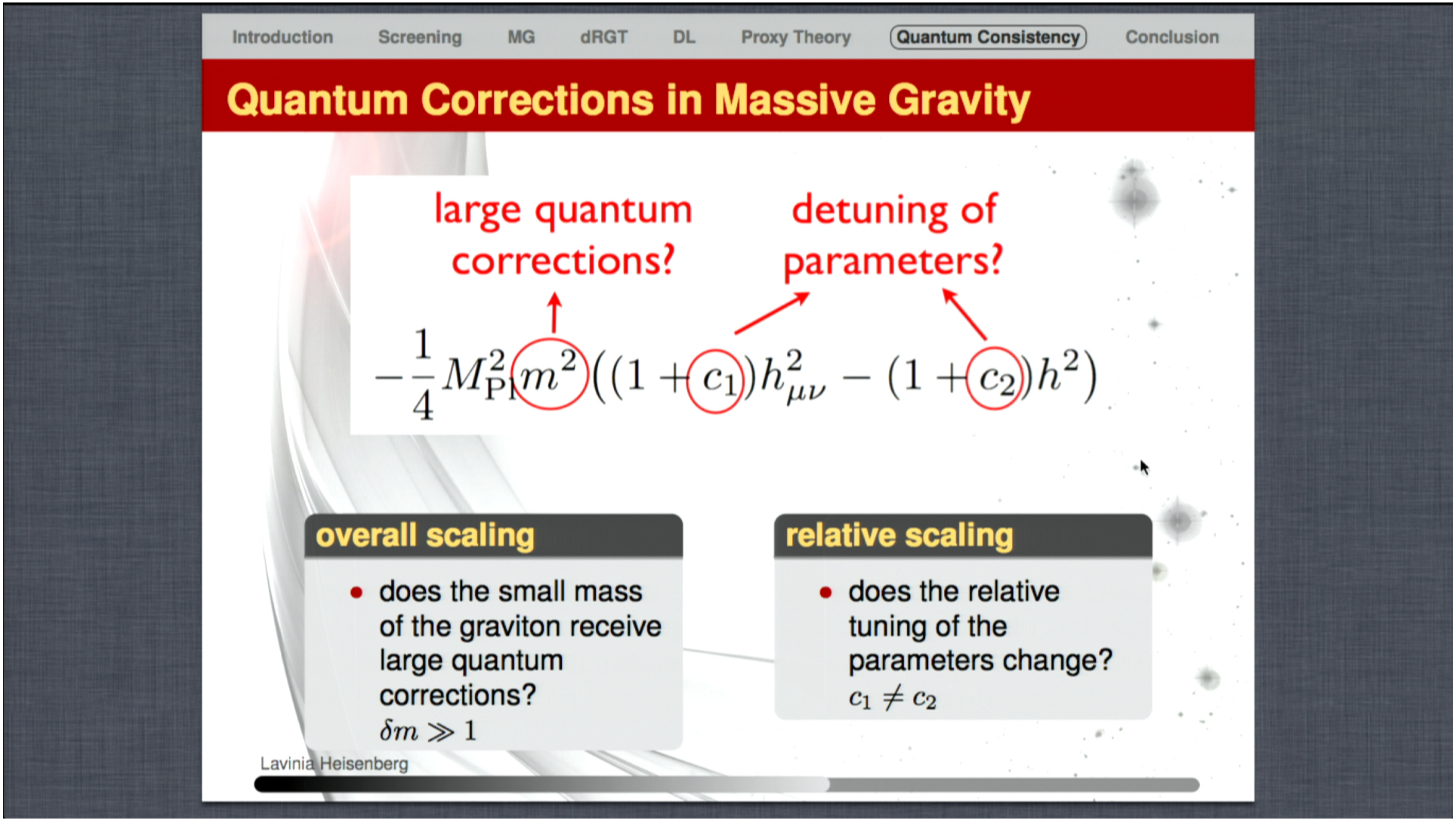
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Introduction | Screening | MG | dRGT | DL | Proxy Theory | **Quantum Consistency** | Conclusion

Quantum Corrections in the DL

$$\mathcal{L} = h^{\mu\nu}\partial^2 h_{\mu\nu} + h^{\mu\nu}\left(X^{(1)}_{\mu\nu} + \frac{a_2}{\Lambda^3}X^{(2)}_{\mu\nu} + \frac{a_3}{\Lambda^6}X^{(3)}_{\mu\nu}\right) + h_{\mu\nu}T^{\mu\nu}$$

Are a_2, a_3, Λ stable against quantum corrections?

1) Quantum corrections

- $\Lambda^3 = m^2 M_p$
- the mass needs to be tuned $m \lesssim H_0$
  same tuning as Cosmological Constant
  $\frac{\lambda}{M_p^4} \sim \frac{H_0^2}{M_p^2} \sim \frac{m^2}{M_p^2} \sim 10^{-120}$
- But the graviton mass is expected to remain stable against quantum corrections
- $\delta m^2 \sim m^2 \rightarrow$ the theory would be tuned but technically natural

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Quantum Corrections in the DL

't Hooft's naturalness argument

- any physical parameter c_i at any energy scale E can remain small if the limit $c_i \to 0$ increases the symmetry of the system
- Example: electron mass $m_e \ll$ electroweak scale, BUT the electron mass is technically natural $\to$ quantum corrections only give rise to $\delta m_e \approx m_e$
- $m_e \to 0$ implies an additional chiral symmetry representing the conservation of left- and right-handed leptons $\to$ So in the massless limit, the electron mass receives no quantum corrections

CC: there is no symmetry recovered in the limit $\Lambda \to 0$ and any massive particle of mass M contributes to the vacuum energy proportional to M^4.

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Tuning versus Fine-tuning in massive gravity

Tuning

- the mass needs to be tuned $m \lesssim H_0$, same tuning as Cosmological Constant $\frac{\Lambda}{M_p^4} \sim \frac{H_0^2}{M_p^2} \sim \frac{m^2}{M_p^2} \sim 10^{-120}$

Fine-tuning

- 't Hooft's naturalness argument applies
- in the massless limit, the graviton mass receives no quantum corrections since we recover a symmetry in the $m \to 0$ limit
- in the full theory the quantum correction give rise to counterterms which are proportional to the mass itself $\to \delta m \approx m^2$

Quantum Corrections in the DL

large quantum corrections?

detuning of parameters?

$$-\frac{1}{4}M_{\rm Pl}^2 m^2 \left((1+c_1)h_{\mu\nu}^2 - (1+c_2)h^2\right)$$

We will show that the

overall scaling

- mass of the graviton receives **NO** quantum corrections! $\rightarrow \delta m = 0$

relative scaling

- relative tuning of the parameters does **NOT** change! $\rightarrow c_1 = c_2 = 0$

Non-renormalization theorem in Galileons

Galileon interactions

$$\mathcal{L}_{\rm Gal} = c_n \pi \varepsilon^{\mu\nu\cdots}\varepsilon^{\alpha\beta\cdots}\Pi_{\mu\alpha}\Pi_{\nu\beta}\cdots\Pi_{\dots}\Pi_{\dots}$$

counter terms arising in the 1-loop effective action:

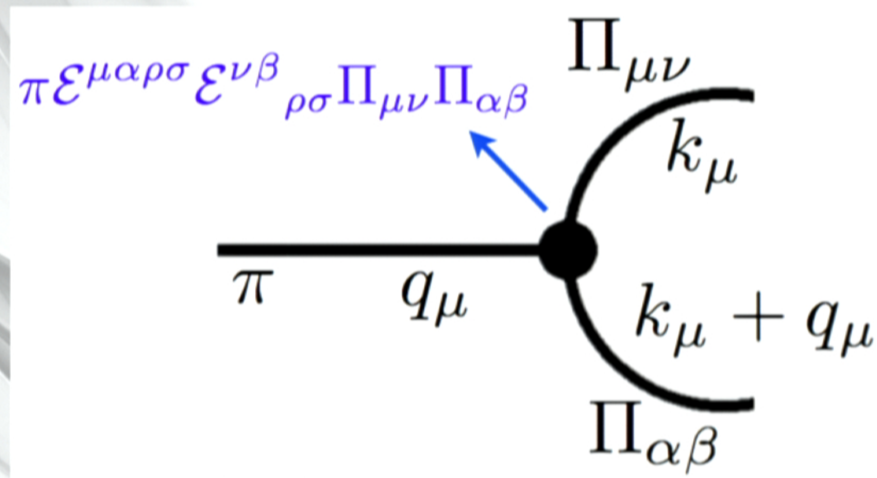
- come with at least one extra derivative as compared to the original interactions
- don't take Galileon form at all

Galileon coupling constants are **technically natural** tuned to any value and remain radiatively **stable**.

Non-renormalization theorem in Galileons

Any external particle comes with at least two derivatives applied on it in the 1-PI action.
Example: consider the following vertex:

$$\mathcal{V} = \pi \varepsilon^{\mu\nu}{}_{\rho\sigma} \varepsilon^{\alpha\beta\rho\sigma} \Pi_{\mu\alpha} \Pi_{\nu\beta}$$



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Non-renormalization theorem in Galileons

this vertex give a contribution to the transition amplitude

$$i\mathcal{M} \supset i \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathcal{G}_k \mathcal{G}_{k+q}\, \varepsilon^{\alpha\rho\gamma\delta} \varepsilon^{\beta\sigma}{}_{\gamma\delta} k_\alpha k_\beta (q+k)_\rho (q+k)_\sigma \cdots$$

with the Feynman propagator $\mathcal{G}_k = \frac{i}{k^2 - i\epsilon}$.

- terms **linear** in momentum q and **independent** of it, cancel due to antisymmetric structure of the vertex
- only term which can be contracted with the antisymmetric tensor is $q_\rho q_\sigma$

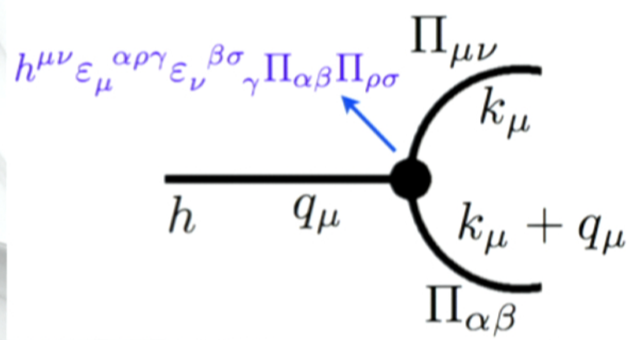
$$i\mathcal{M} \supset i\varepsilon^{\alpha\rho\gamma\delta} \varepsilon^{\beta\sigma}{}_{\gamma\delta} q_\rho q_\sigma \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathcal{G}_k \mathcal{G}_{k+q} k_\alpha k_\beta \cdots$$

$\rightarrow$ any loop that involves a vertex $V = \pi \varepsilon^{\mu\nu}{}_{\rho\sigma} \varepsilon^{\alpha\beta\rho\sigma} \Pi_{\mu\alpha} \Pi_{\nu\beta}$ will lead to at least two derivatives on the external leg

Non-renormalization theorem in the DL

The same non-renormalization theorem applies in the decoupling limit of massive gravity:
The only difference is that we now have the helicity-2 field appearing in the interactions.

$$\mathcal{V} = h^{\mu\nu} X^{(2)}_{\mu\nu}[\Pi] \sim h^{\mu\nu} \varepsilon_{\mu}{}^{\alpha\rho\gamma} \varepsilon_{\nu}{}^{\beta\sigma}{}_{\gamma} \Pi_{\alpha\beta} \Pi_{\rho\sigma}$$



LH & de Rham, Gabadadze, Pirtskhalava (PRD87 (2013) 085017)

Quantum corrections beyond decoupling limit

We consider the massive gravity interactions and the coupling to the matter sector

$$S = \int d^4x \sqrt{g} \left(\frac{M_p^2}{2} \sqrt{-g} \left(R - \frac{m^2}{4} \mathcal{U}(g, H) \right) + \mathcal{L}_{\rm matter} \right)$$

with the propagator of the graviton giving as

$$G^{(\rm massive)}_{abcd} = \langle h_{ab}(x_1) h_{cd}(x_2) \rangle = f^{(m)}_{abcd} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(x_1 - x_2)}}{k^2 + m^2}$$

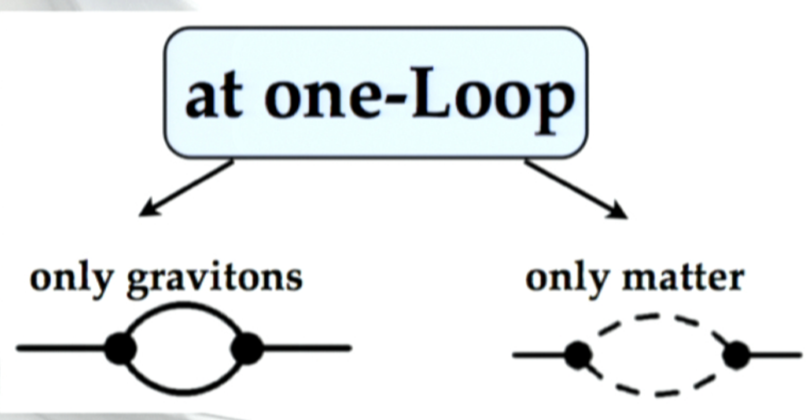
with $f^{(m)}_{abcd} = \left(\tilde{\delta}_{a(c} \tilde{\delta}_{bd)} - \frac{1}{3} \tilde{\delta}_{ab} \tilde{\delta}_{cd} \right)$.

Consider for simplicity a massive scalar field

$$\mathcal{L}_{\rm matter} = \sqrt{g} \left(\frac{1}{2} g^{ab} \partial_a \chi \partial_b \chi + \frac{1}{2} M^2 \chi^2 \right)$$

Quantum corrections beyond decoupling limit

- at one-loop either gravitons or matter field are running in the loops, but not both simultaneously
- only at higher loops the mixing has to be taken into account

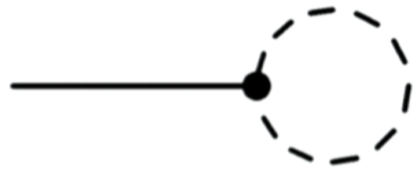


Coupling to matter fields

perform it in your favorite method

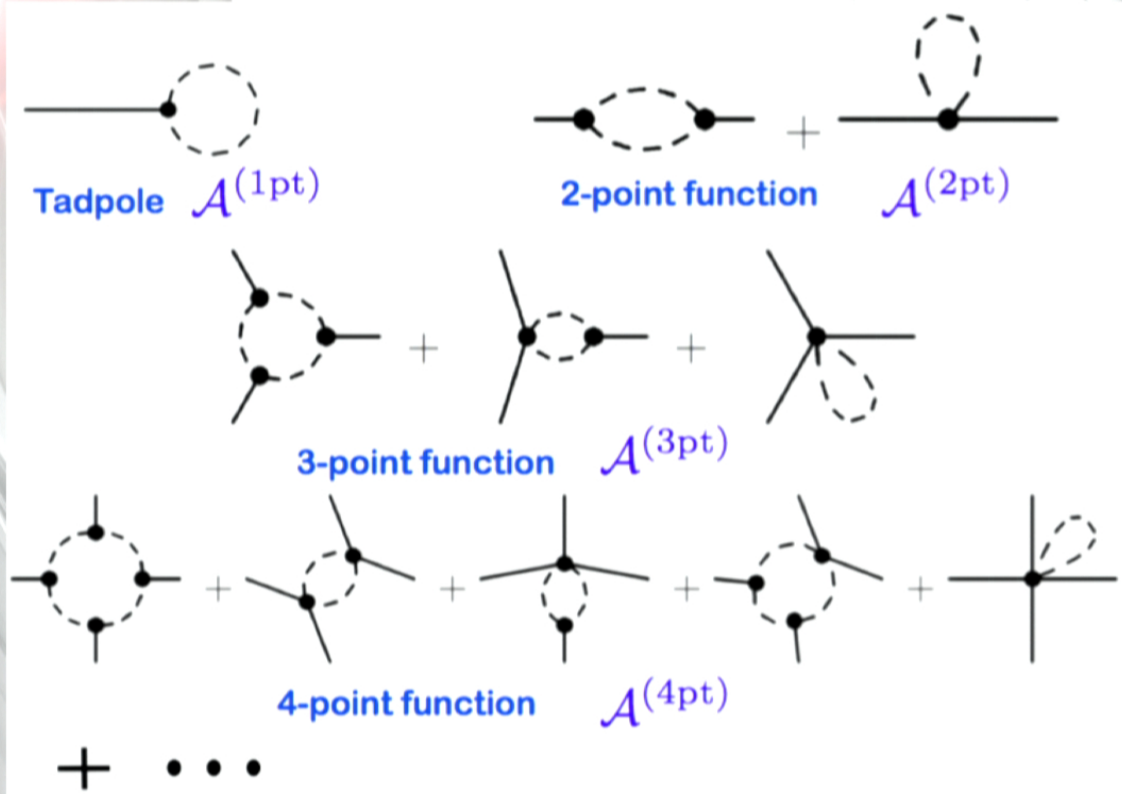
- perturbatively Feynman diagram by Feynman diagram
- one loop effective action

At one loop

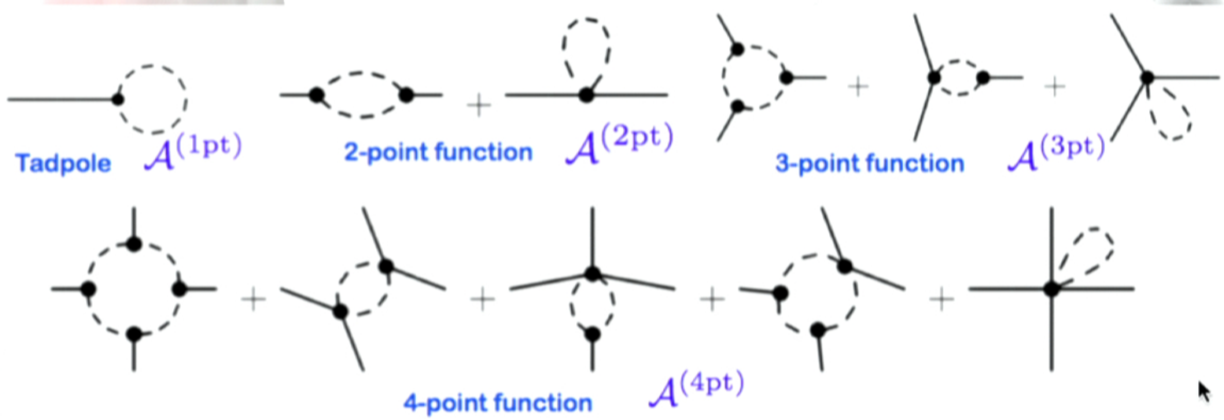


matter fields are unaware of the graviton mass.
$\rightarrow$ the only covariant potential term they can give rise is a cosmological constant!

One-loop matter contribution



One-loop matter contribution



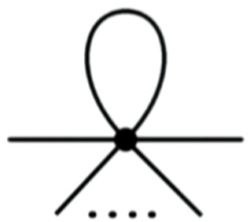
$$+ \cdots = \frac{M_4}{4}\sqrt{g}\log(\mu^2)$$

→ so they give rise to a cosmological constant, but leaves the potential interactions unchanged.

Graviton Contributions in the Loops

- one-loop correction to the mass from gravitons do not preserve the nice structure of the potential

$$\frac{m^4}{M_{\rm Pl}^2}h^2 \quad \to \quad \frac{(\partial^2\pi)^2}{M_{\rm Pl}^2} \quad \to \quad m_{\rm gh}^2 \sim M_{\rm Pl}^2$$



$$\frac{m^4}{M_{\rm Pl}^{n+2}}h^{n+2} \quad \to \quad \frac{(\partial^2\pi)^2}{M_{\rm Pl}^2}\left(\frac{\partial^2\pi_0}{\Lambda^3}\right)^n \quad \to \quad m_{\rm gh}^2 \sim M_{\rm Pl}^2\left(\frac{\partial^2\pi_0}{\Lambda^3}\right)^{-n}$$

Graviton Contributions in the Loops

but the one loop effective action is itself redressed

$$\mathcal{L}_{\rm eff} = \frac{1}{M_{\rm Pl}^2} \frac{1}{1+\frac{\partial^2 \pi_0}{\Lambda^3}} (\partial^2 \pi)^2$$

the detuning of the potential is never a problem since

$$m_{\rm gh}^2 \geq M_{\rm Pl}^2$$

even if the background is large

$$\frac{\partial^2 \pi_0}{\Lambda^3} \gg 1$$

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Impact of Massive Gravity on cosmological observations!

cosmological observations

two categories:

geometrical probes

measurement of the Hubble function

- distance-redshift relation of supernovae
- measurements of the angular diameter distance as a function of redshift (CMB+BAO)

structure formation probes

measurement of the Growth function

- homogeneous growth of the cosmic structure
  $\rightarrow$ integrated Sachs-Wolfe effects
- non-linear growth
  $\rightarrow$ gravitational lensing
  $\rightarrow$ formation of galaxies
  $\rightarrow$ clusters of galaxies by gravitational collapse

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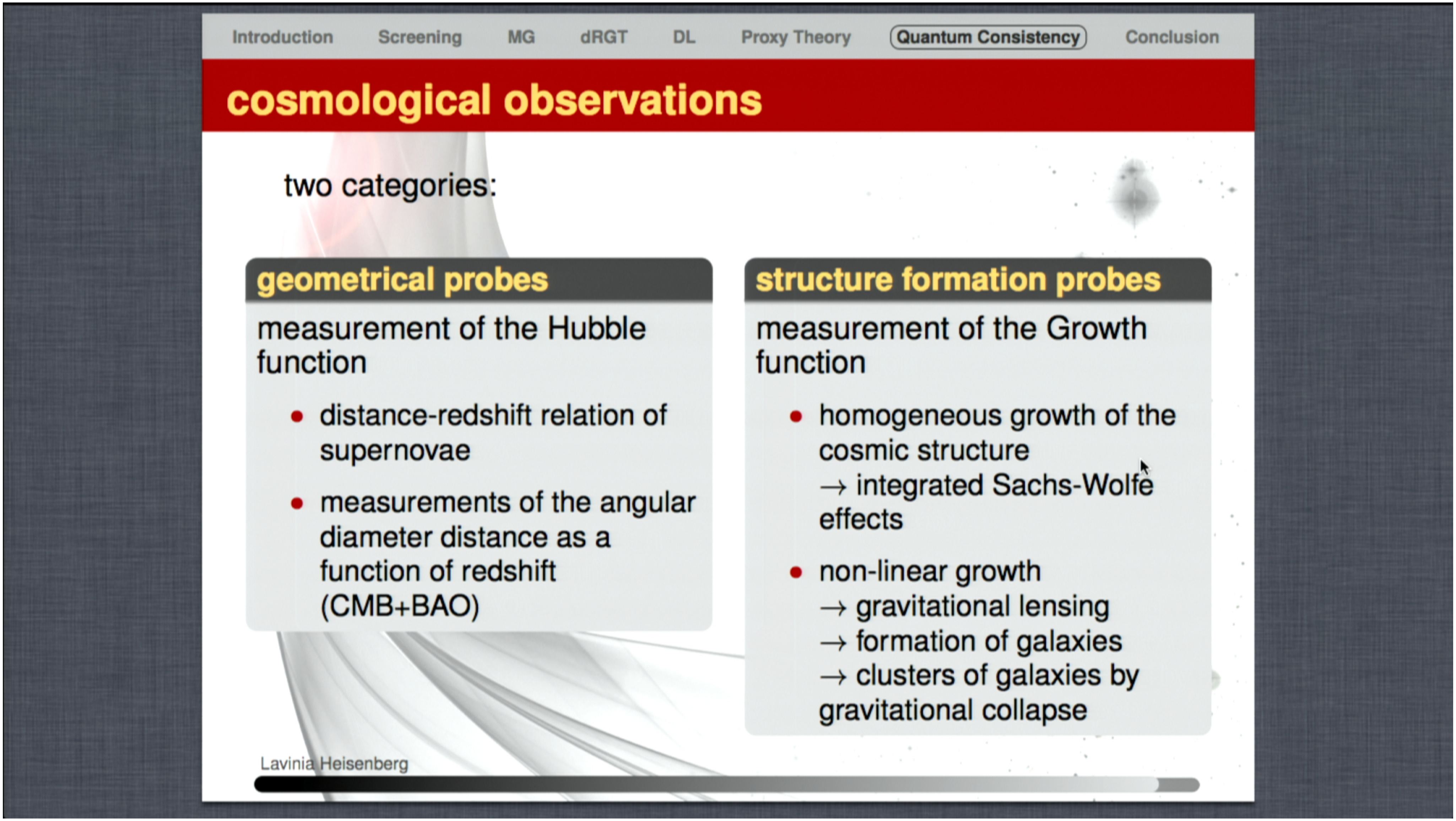
Introduction
Screening
MG
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Proxy Theory
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Conclusion
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Conclusion

- decoupling limit of dRGT
  - stable self-accelerating solution similar to ΛCDM
- Proxy theory
  - stable self accelerating solution
  - the scalar mode does not decouple around the self-accelerating background and leads to an extra force during the history of the Universe
  - would influence the time sequence of gravitational clustering and the evolution of peculiar velocities, as well as the number density of collapsed objects.
- quantum corrections
  - in the decoupling limit the non-renormalization theorem guaranties the stability under quantum corrections
  - beyond the decoupling limit the effect of external matter at 1-loop is harmless on the potential and gives only rise to a CC
  - 1-loop graviton contributions detune the structure of the potential
  - nevertheless the mass associated to the ghost is never below the Planck scale