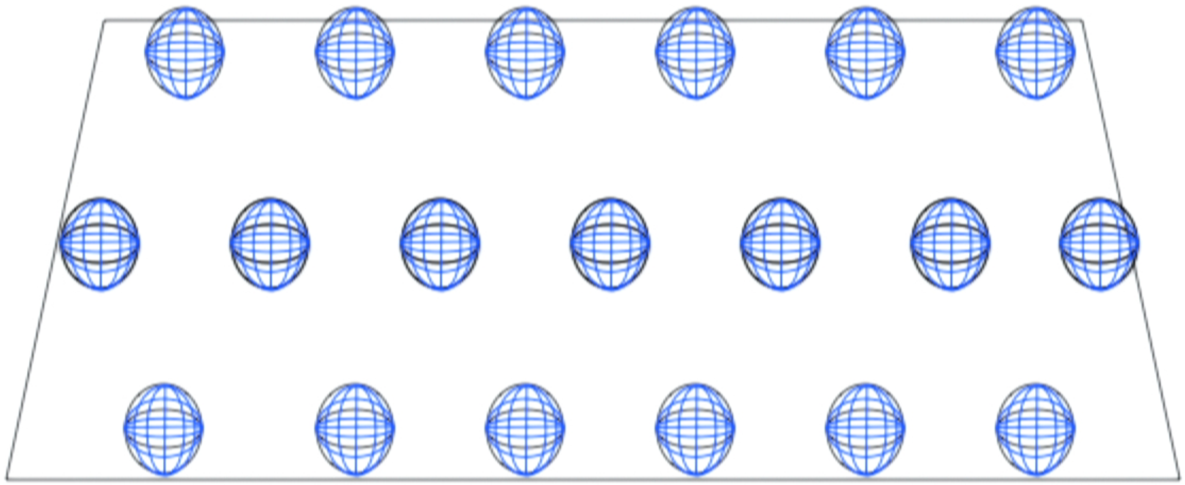
Title: Compactifying de Sitter

Date: Dec 10, 2013 11:00 AM

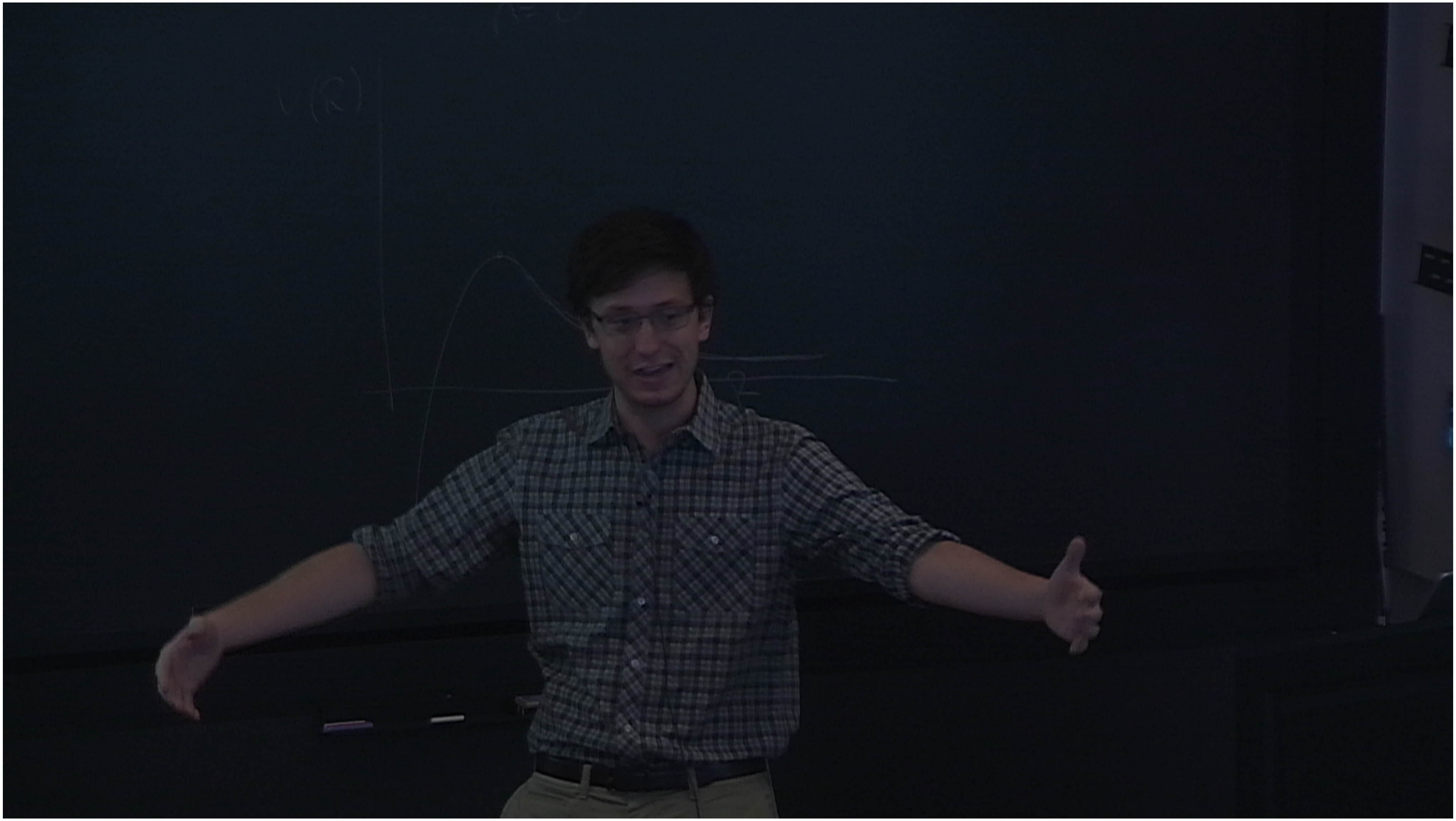
URL: http://pirsa.org/13120049

Abstract: TBA

Compactifying de Sitter



Based on work with
Adam Brown and
Ali Masoumi



Two previous simple models:

Bousso-Polchinksi model

- many internal cycles,
  but the extra dimensions are held fixed by fiat

$N = 1$ Freund-Rubin model

- only a single internal cycle,
  but the extra dimensions are dynamical

Our model: General N Freund-Rubin model

- many internal cycles,
  and dynamical extra dimensions

One of the new features that will arise in this model is the existence of a double-exponentially large number of de Sitter vacua with cosmological constants that accumulate at $\Lambda_4 = 0$.

Our model: General N Freund-Rubin model

- many internal cycles, and dynamical extra dimensions

One of the new features that will arise in this model is the existence of a double-exponentially large number of de Sitter vacua with cosmological constants that accumulate at $\Lambda_4 = 0$.

Part 1: Review of previous models

Part 2: Distribution of cosmological constants

Part 3: Shape modes

Bousso-Polchinski model

Flux shifts the cosmological constant

$$\Lambda_4 = \Lambda_{\text{no flux}} + \frac{1}{2} F^2$$

$$F^2 = \sum_{i=1}^{N} g_i^2 n_i^2$$

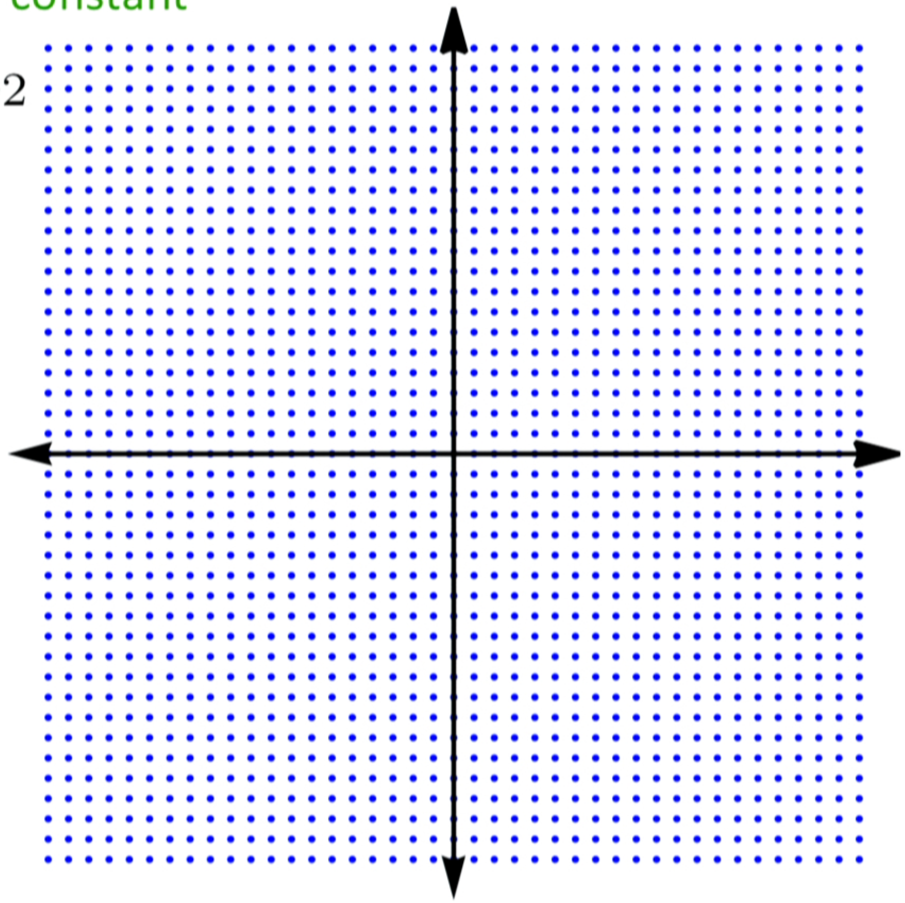
Bousso-Polchinski model

Flux shifts the cosmological constant

$$\Lambda_4 = \Lambda_{\text{no flux}} + \frac{1}{2}F^2$$

$$F^2 = \sum_{i=1}^{N} g_i^2 n_i^2$$

Dirac says "it's quantized"



Bousso-Polchinski model

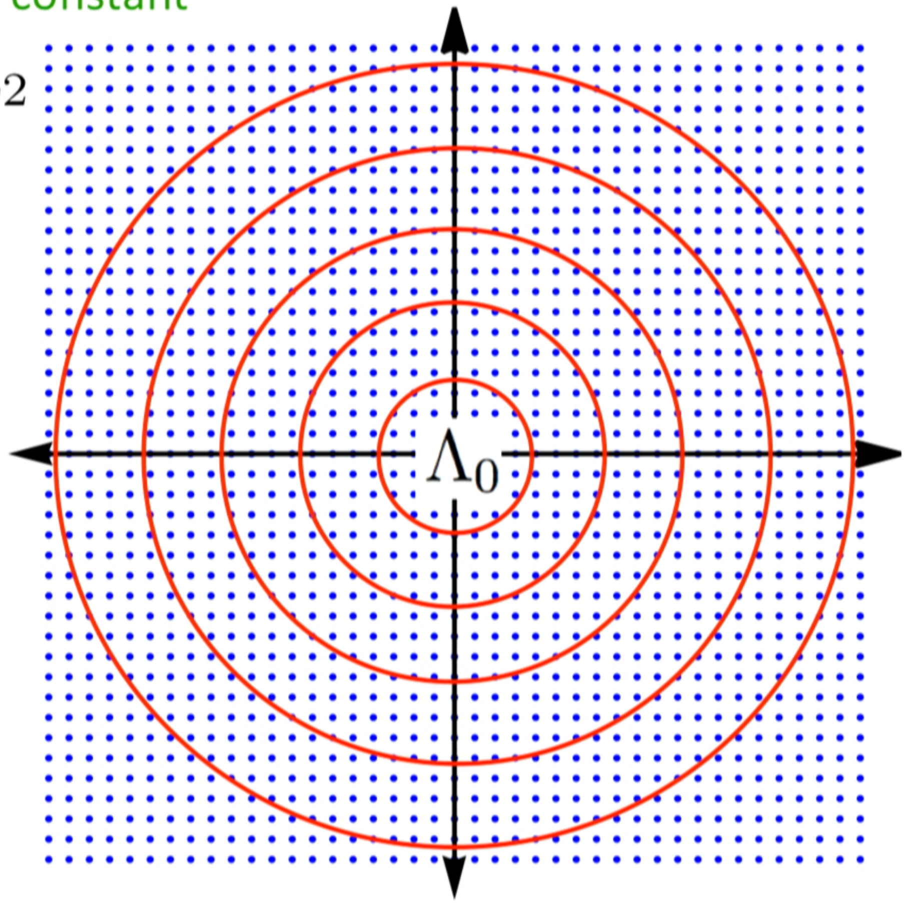
Flux shifts the cosmological constant

$$\Lambda_4 = \Lambda_{\text{no flux}} + \frac{1}{2}F^2$$

$$F^2 = \sum_{i=1}^{N} g_i^2 n_i^2$$

Dirac says "it's quantized"

Equipotentials are ellipses



Bousso-Polchinski model

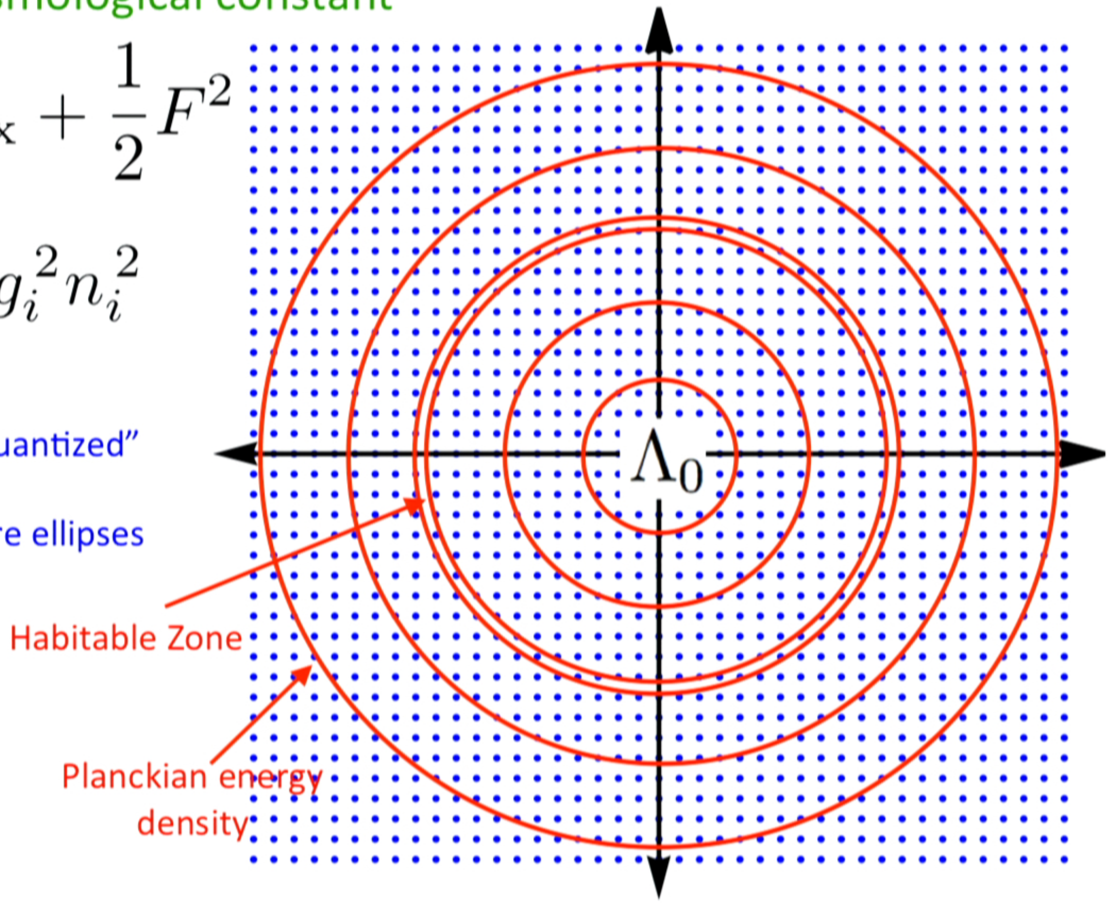
Flux shifts the cosmological constant

$$\Lambda_4 = \Lambda_{\text{no flux}} + \frac{1}{2}F^2$$

$$F^2 = \sum_{i=1}^{N} g_i^{\,2} n_i^{\,2}$$

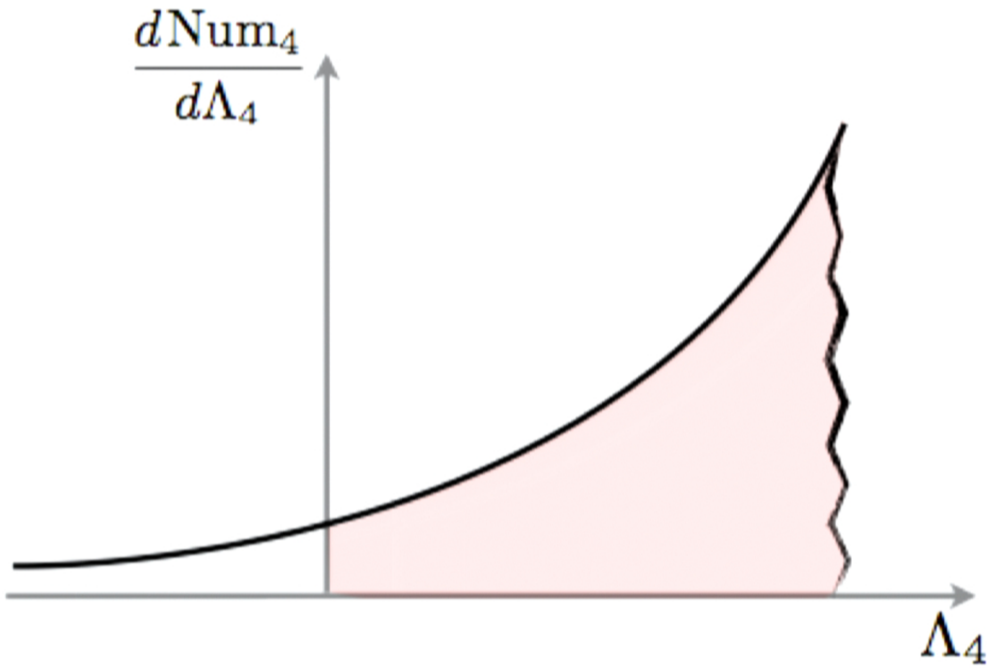
Dirac says "it's quantized"

Equipotentials are ellipses



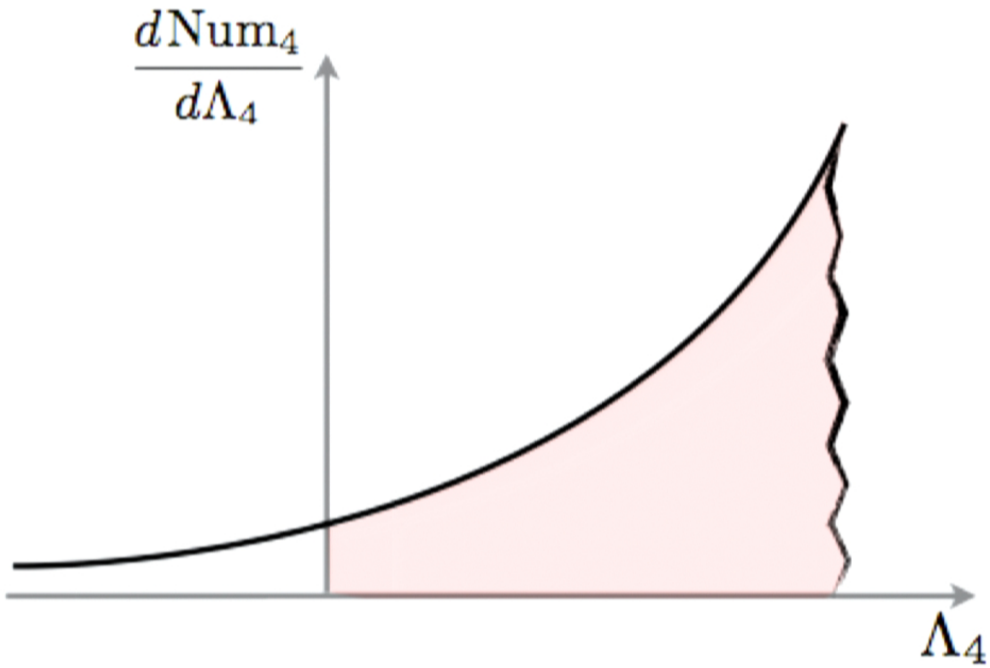
Bousso-Polchinski model

A Histogram of the vacua in the BP Landscape



Bousso-Polchinski model

A Histogram of the vacua in the BP Landscape



N=1 Freund-Rubin model

$$S = \int d^D x \sqrt{g} \left(M_D^{D-2} \mathcal{R} - \frac{1}{2q!} F_q^2 - \Lambda_D \right)$$

$$\mathcal{M}_{D-q} \times S_q \qquad\qquad ds^2 = \frac{1}{R^q} ds_{D-q}^2 + R^2 d\Omega_q^2$$

$$S = \int d^{D-q} \sqrt{g_{D-q}} \left(\mathcal{R}_{D-q} - \frac{1}{2} \partial R^2 - V_{\text{eff}}(R) \right)$$

$$V_{\text{eff}} = \frac{1}{R^q} \left(\frac{n^2}{R^{2q}} - \frac{1}{R^2} + \Lambda_D \right)$$

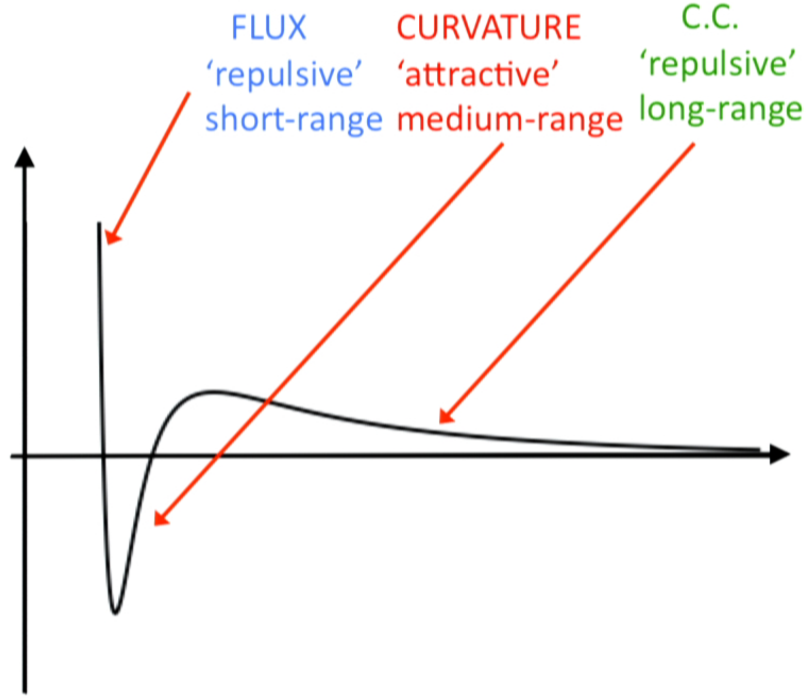
FLUX
'repulsive'
short-range

CURVATURE
'attractive'
medium-range

C.C.
'repulsive'
long-range

N=1 Freund-Rubin model

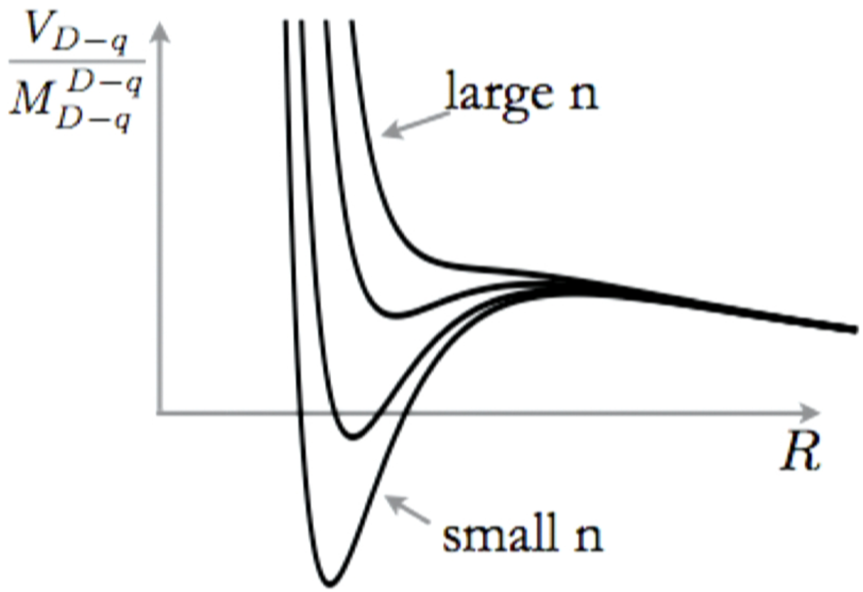
$$V_{\rm eff} = \frac{1}{R^q}\left(\frac{n^2}{R^{2q}} - \frac{1}{R^2} + \Lambda_D\right)$$



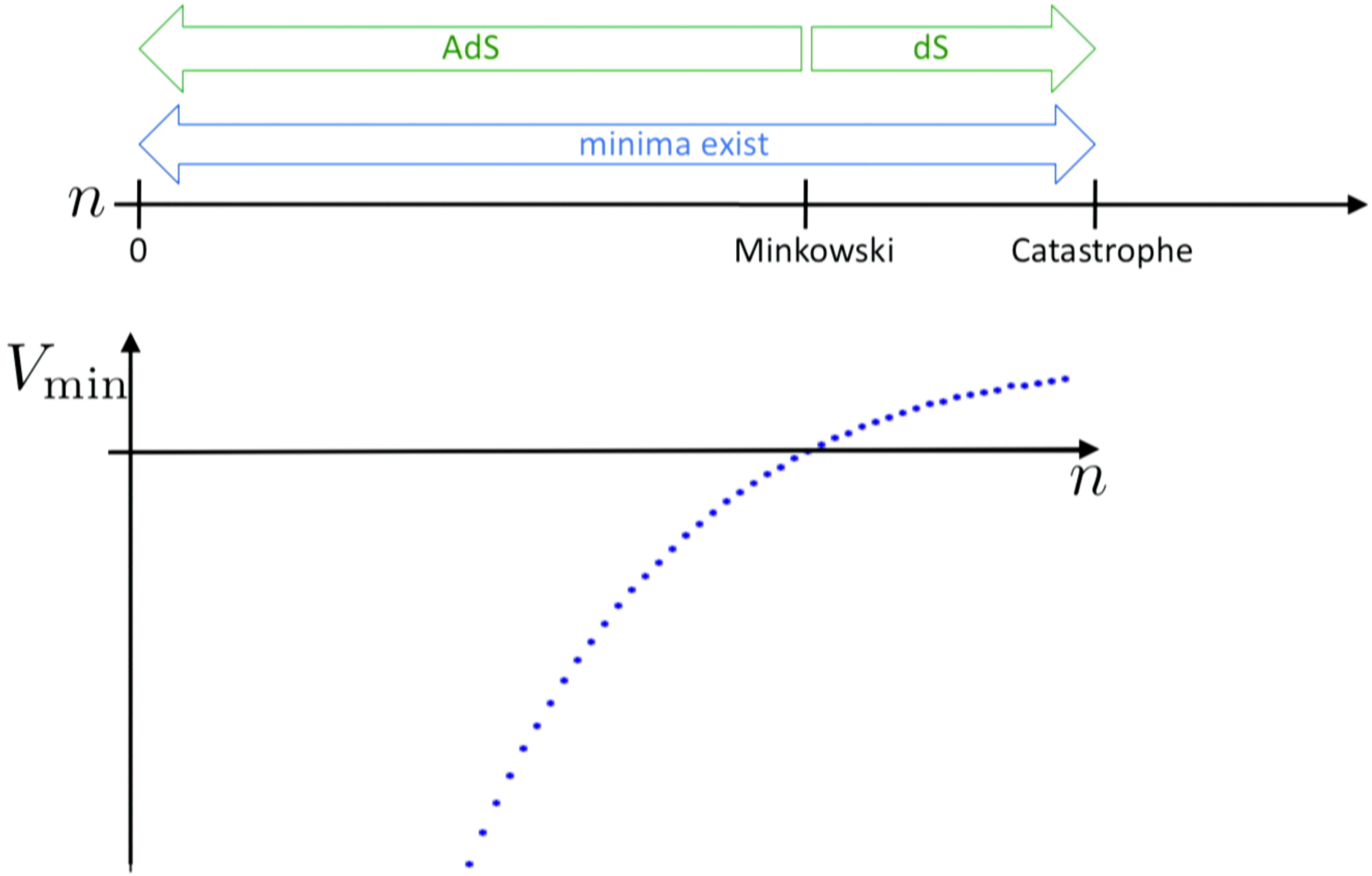
N=1 Freund-Rubin model

$$V_{\text{eff}} = \frac{1}{R^q}\left(\frac{n^2}{R^{2q}} - \frac{1}{R^2} + \Lambda_D\right)$$

FLUX	CURVATURE	C.C.
‘repulsive’	‘attractive’	‘repulsive’
short-range	medium-range	long-range



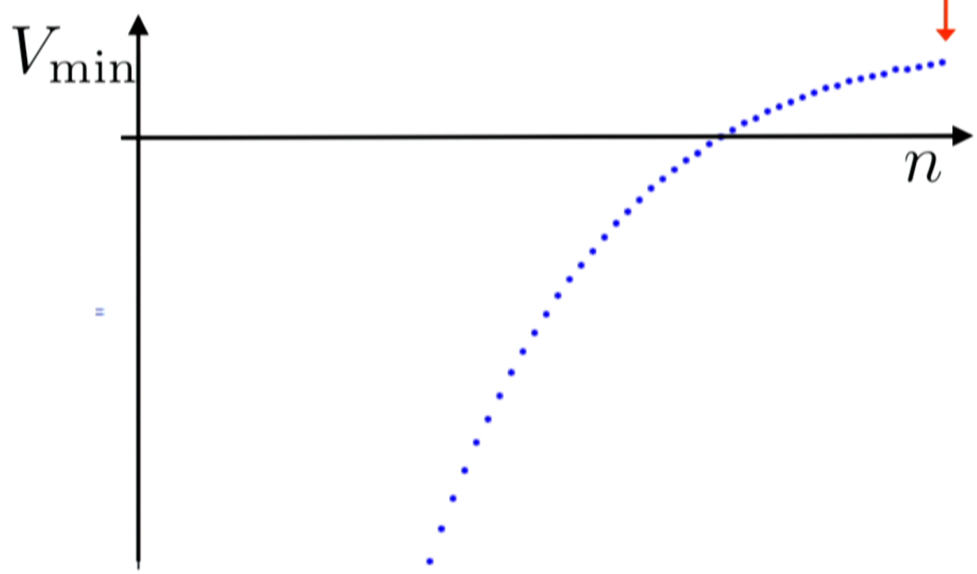
N=1 Freund-Rubin model



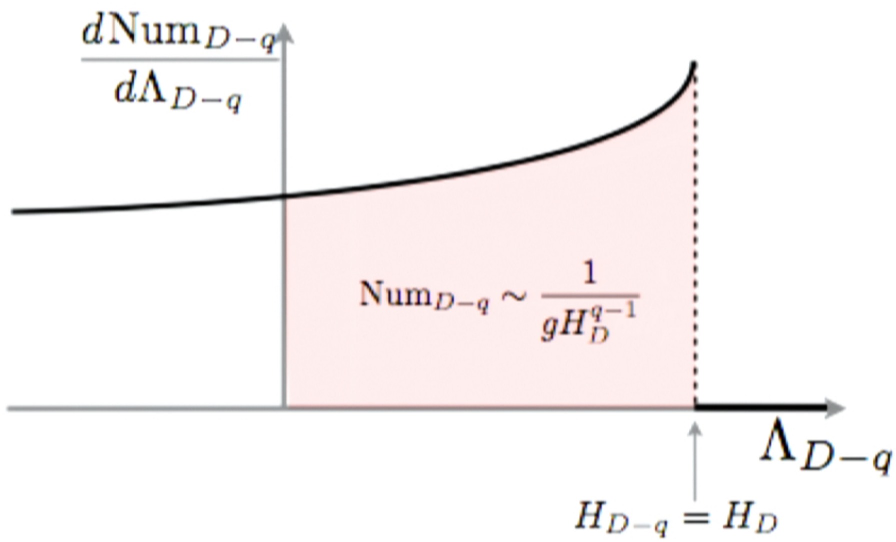
N=1 Freund-Rubin model

Catastrophe happens when all three terms in the effective potential contribute equally.

$$n_{\max} \sim H_D^{-(q-1)}/g$$
$$H_{D-q,\max \sim H_D}$$

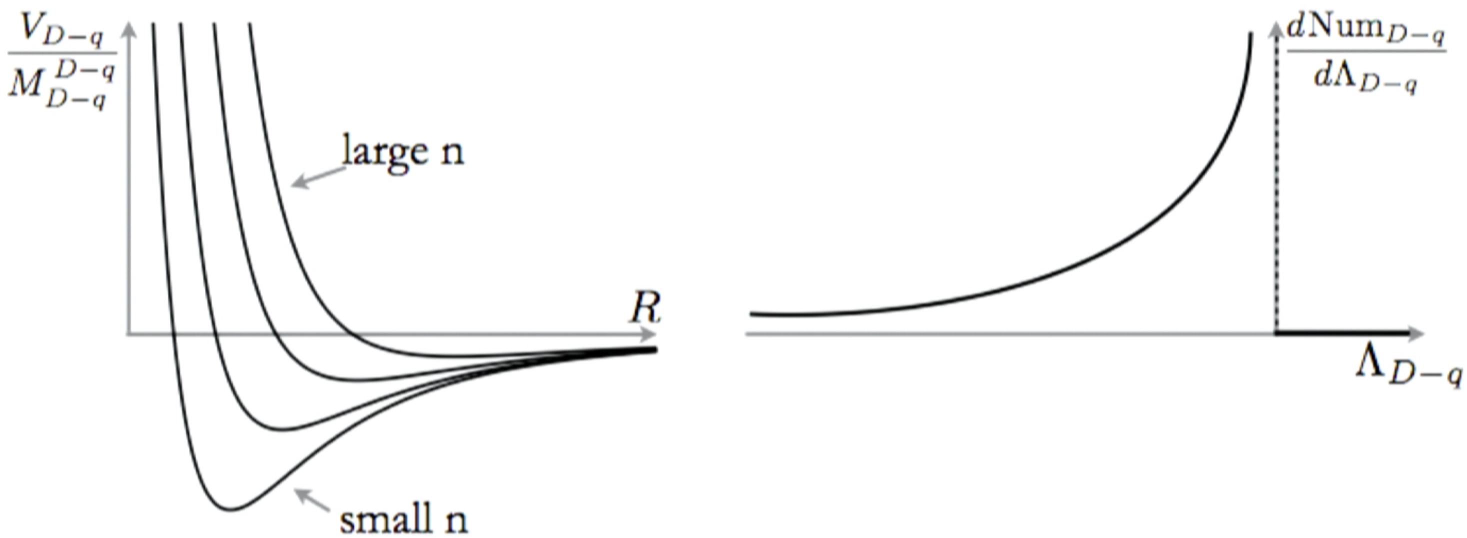


N=1 Freund-Rubin model



N=1 Freund-Rubin model

What happens as $\Lambda_D \to 0$ from above?



An infinite number of vacua, but all of them AdS.

N=1 Freund-Rubin model

$$S = \int d^D x \sqrt{g} \left(M_D^{D-2} \mathcal{R} - \frac{1}{2q!} F_q^2 - \Lambda_D \right)$$

$$\mathcal{M}_{D-q} \times S_q \qquad\qquad ds^2 = \frac{1}{R^q} ds^2_{D-q} + R^2 d\Omega_q^2$$

$$S = \int d^{D-q} \sqrt{g_{D-q}} \left(\mathcal{R}_{D-q} - \frac{1}{2} \partial R^2 - V_{\text{eff}}(R) \right)$$

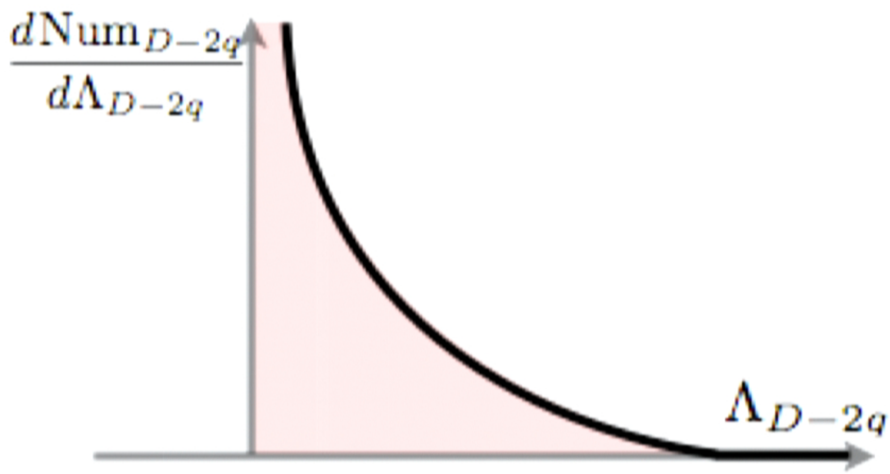
$$V_{\text{eff}} = \frac{1}{R^q} \left(\frac{n^2}{R^{2q}} - \frac{1}{R^2} + \Lambda_D \right)$$

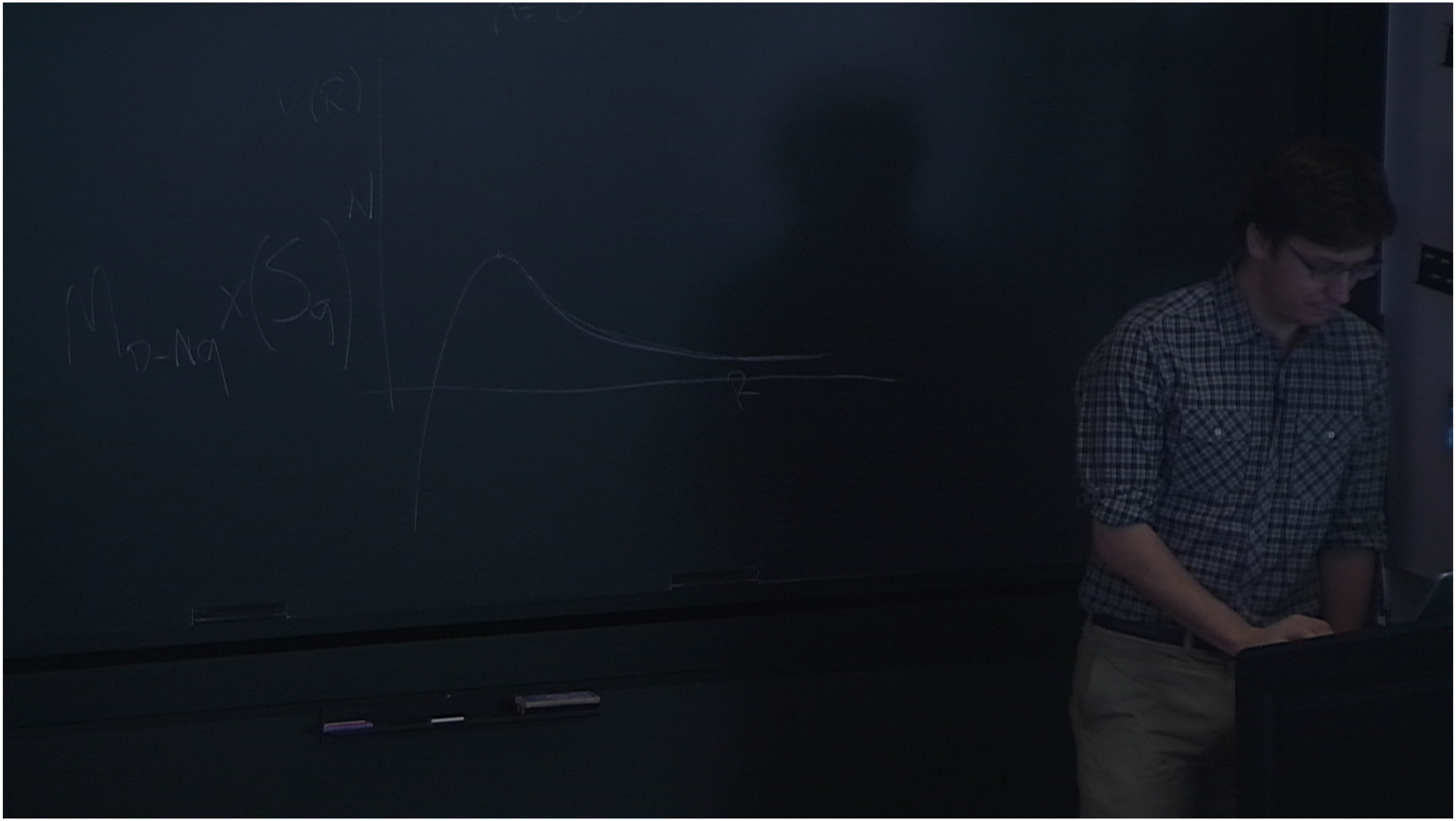
FLUX 'repulsive' short-range | CURVATURE 'attractive' medium-range | C.C. 'repulsive' long-range

Summary of two previous models

These two previous models have contributed to a common lore that the distribution of de Sitter vacua is smooth through $V_4 = 0$.

To the contrary, we will now see for $N \geq 2$ Freund-Rubin model, the distribution of de Sitter vacua is strongly peaked at $V_4 = 0$.





V(R)
N
$M_{D-Nq} \times (S_q)$
R

The easiest way to understand this population explosion of low de Sitter vacua is by analogy with

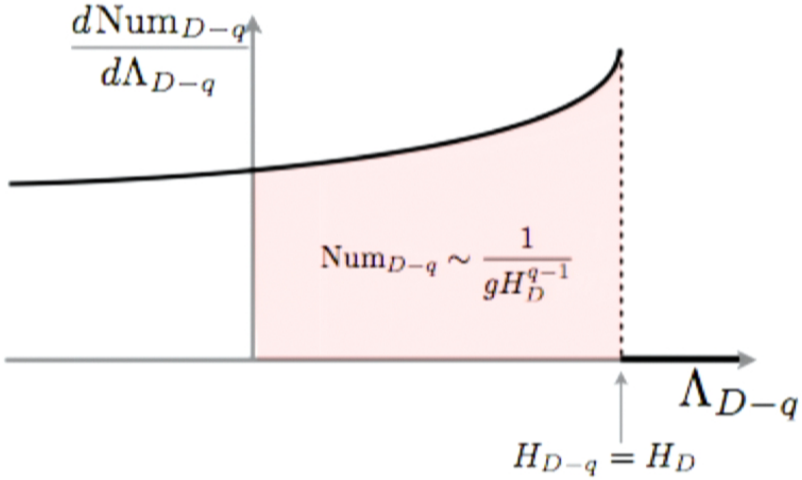
Natural Selection

Natural Selection enhances the population of traits with two characteristics:

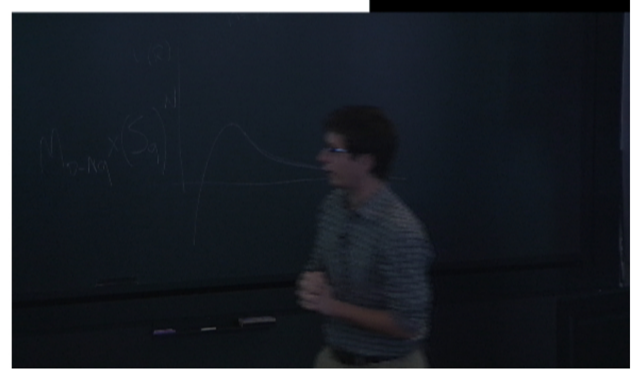
1. They increase fitness
2. They are heritable

Sequential compactification is analogous to generation number.

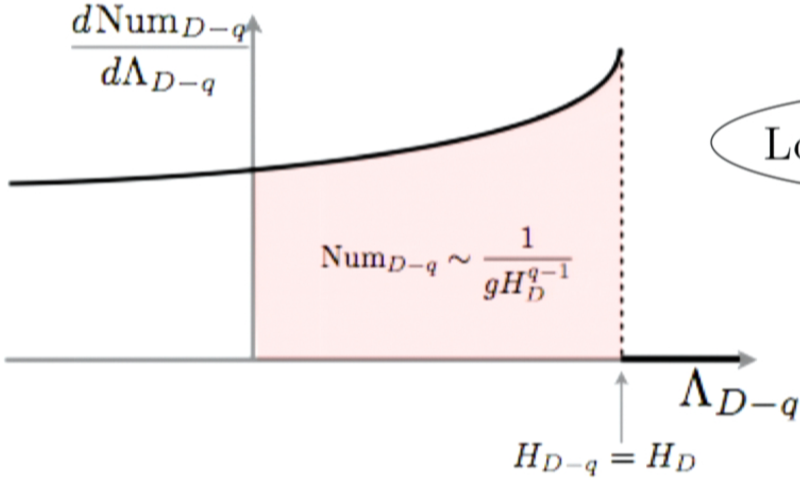
Natural Selection



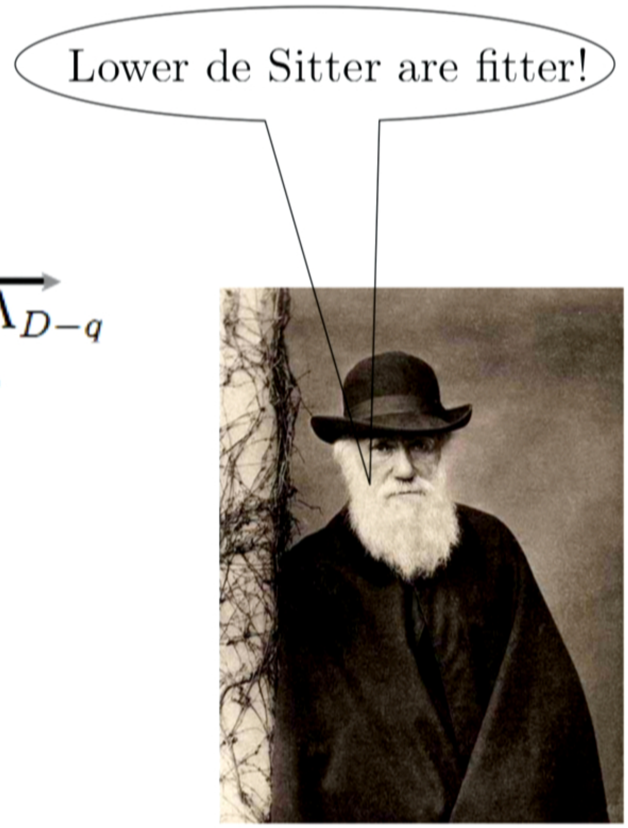
Number of offspring is inversely proportional to parent's Hubble.



Natural Selection

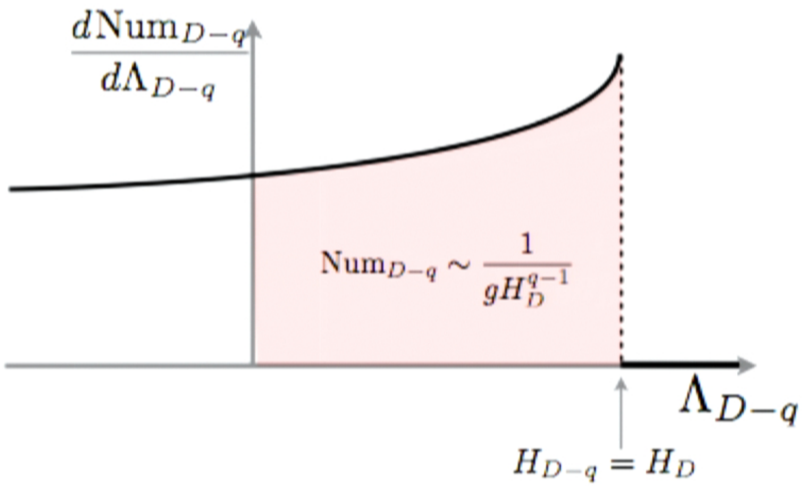


Number of offspring is inversely proportional to parent's Hubble.



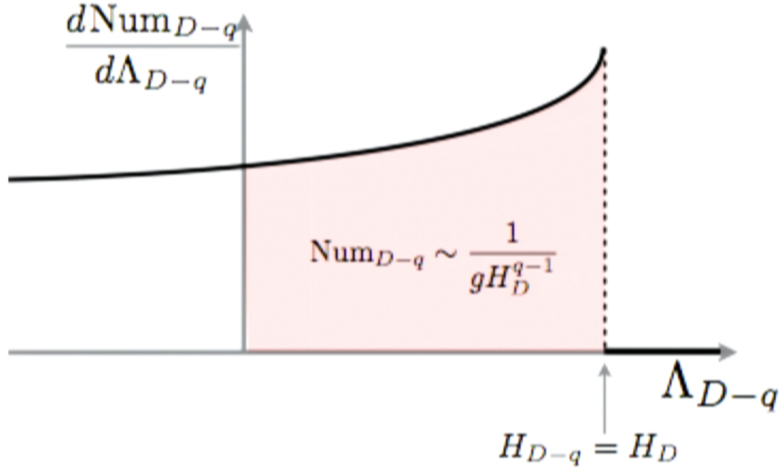
Charles Darwin, disproving the theory that no one looks good in a fedora.

Natural Selection

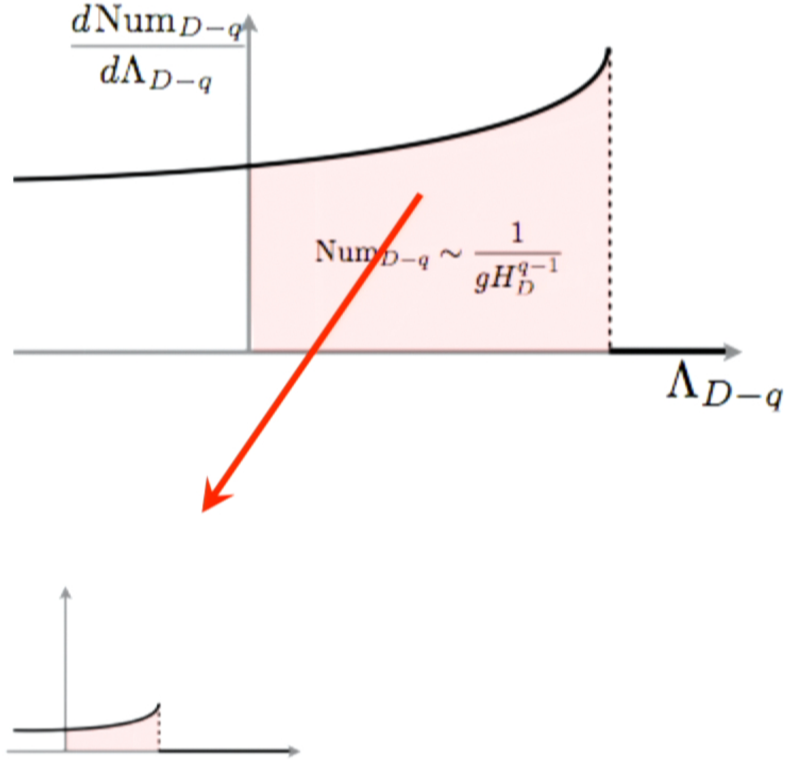


Offspring Hubble is bounded by parent's Hubble.

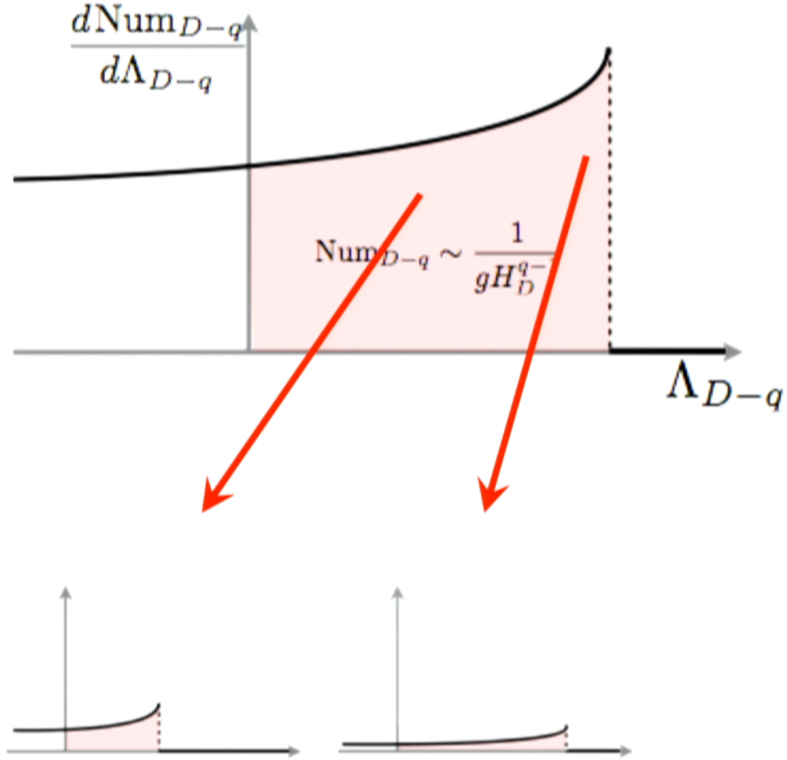
Natural Selection



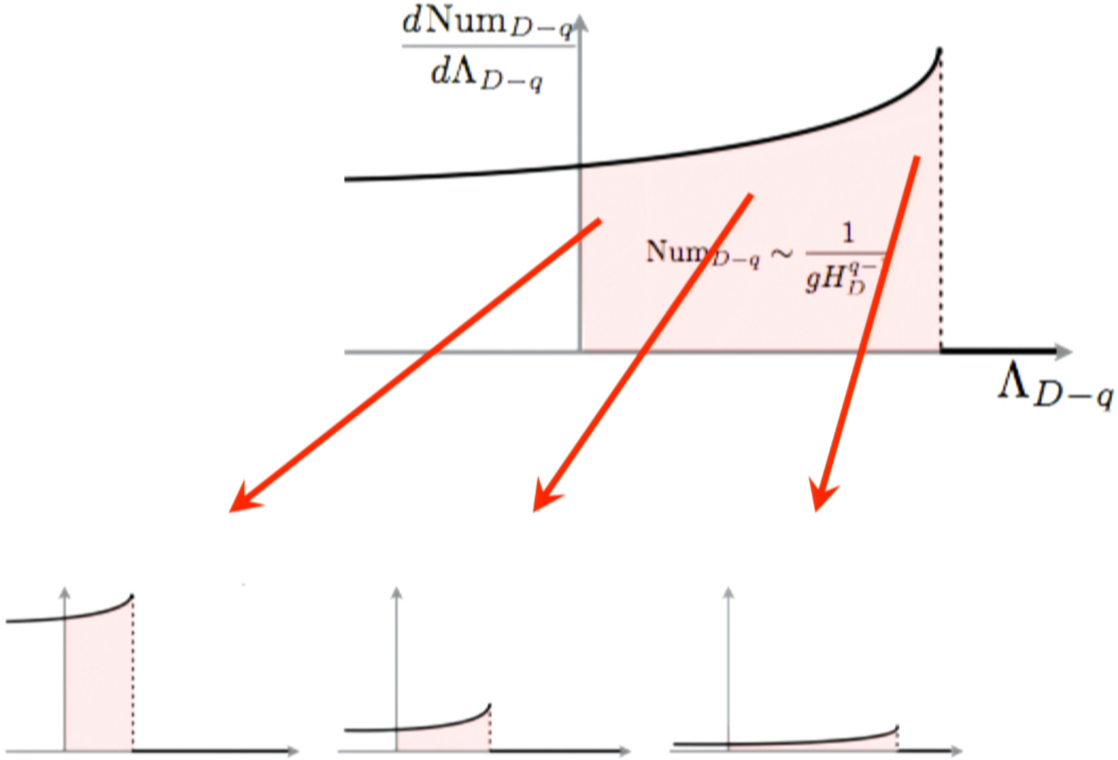
Natural Selection



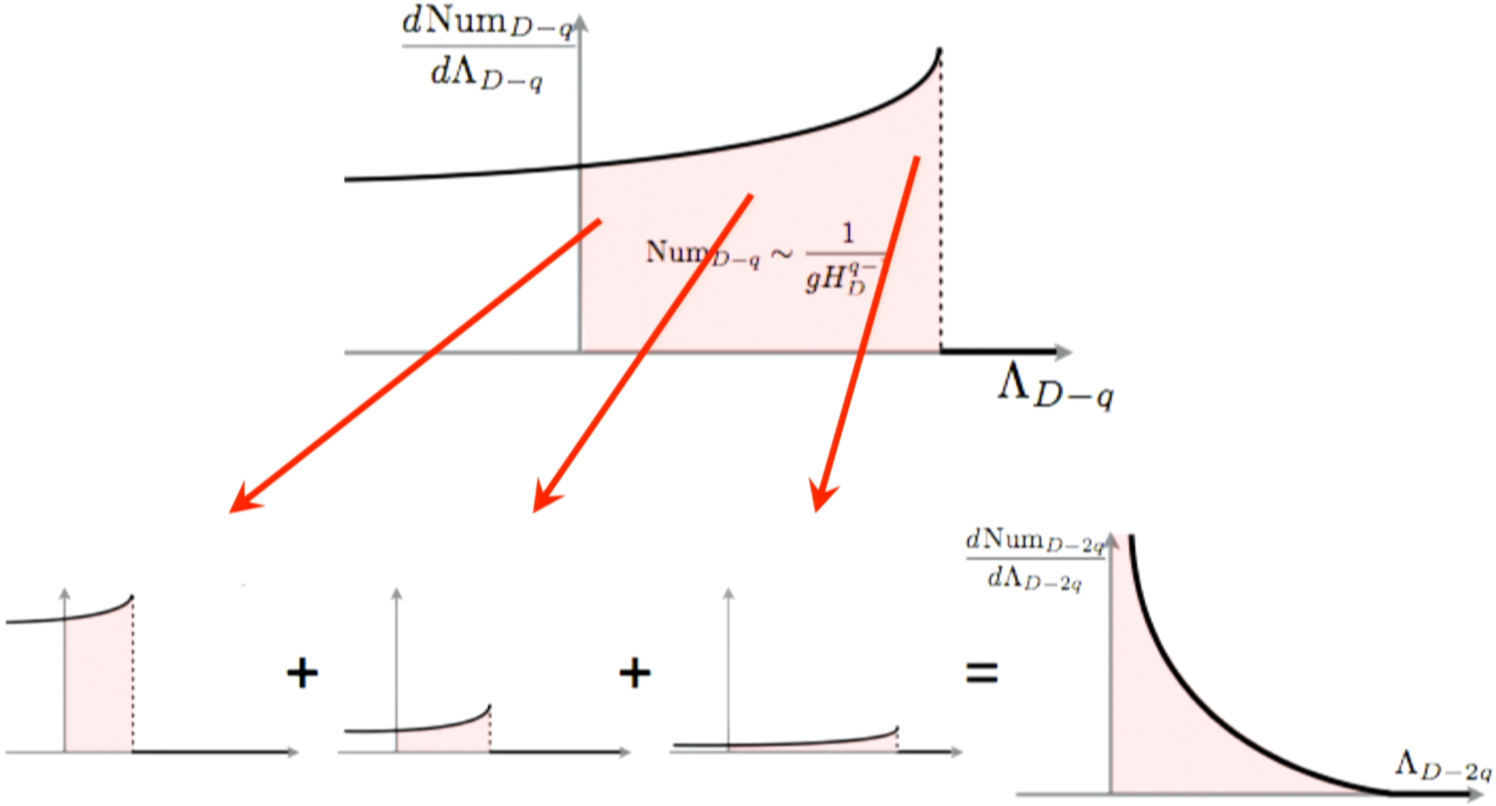
Natural Selection



Natural Selection



Natural Selection



Natural Selection

After N generations

$$\frac{d\mathrm{Num}_p}{dH_p^2} \sim \frac{1}{g^N}\,\frac{1}{H_D^{q+1}}\,\frac{1}{H_p^{(N-1)(q-1)}}$$

When $(N-1)(q-1) \leq 1$, this divergence is integrable.
When $(N-1)(q-1) = 2$, this divergence is logarithmic.
When $(N-1)(q-1) \geq 3$, this divergence is power-law.

The Effective Potential

Another way to understand this pile up is in terms of the effective potential:

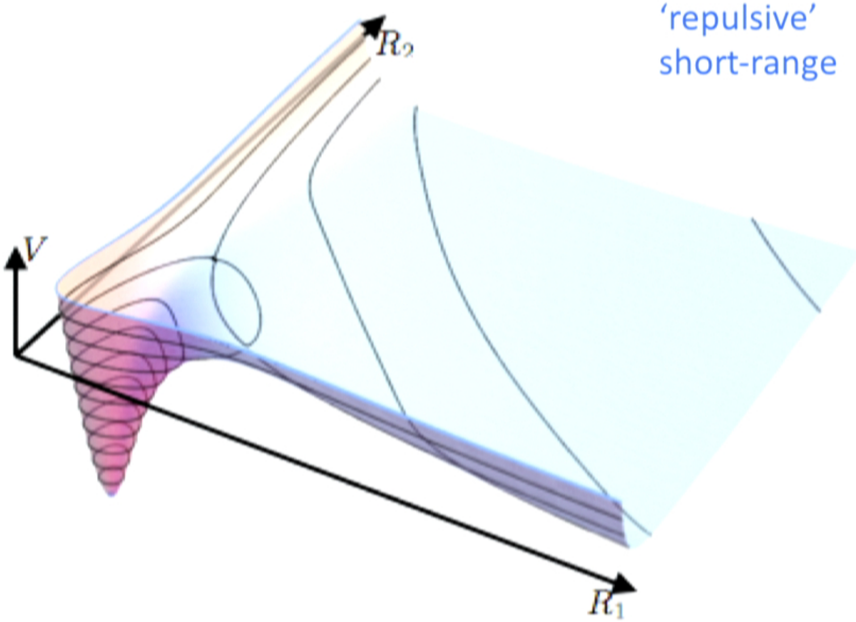
Take $N = 2$ internal q-spheres.

$$V_{\text{eff}}(R_1, R_2) = \frac{1}{R_1^q R_2^q}\left(\frac{n_1^2}{R_1^{2q}} + \frac{n_2^2}{R_2^{2q}} - \frac{1}{R_1^2} - \frac{1}{R_2^2} + \Lambda_D\right)$$

FLUX
'repulsive'
short-range

CURVATURE
'attractive'
medium-range

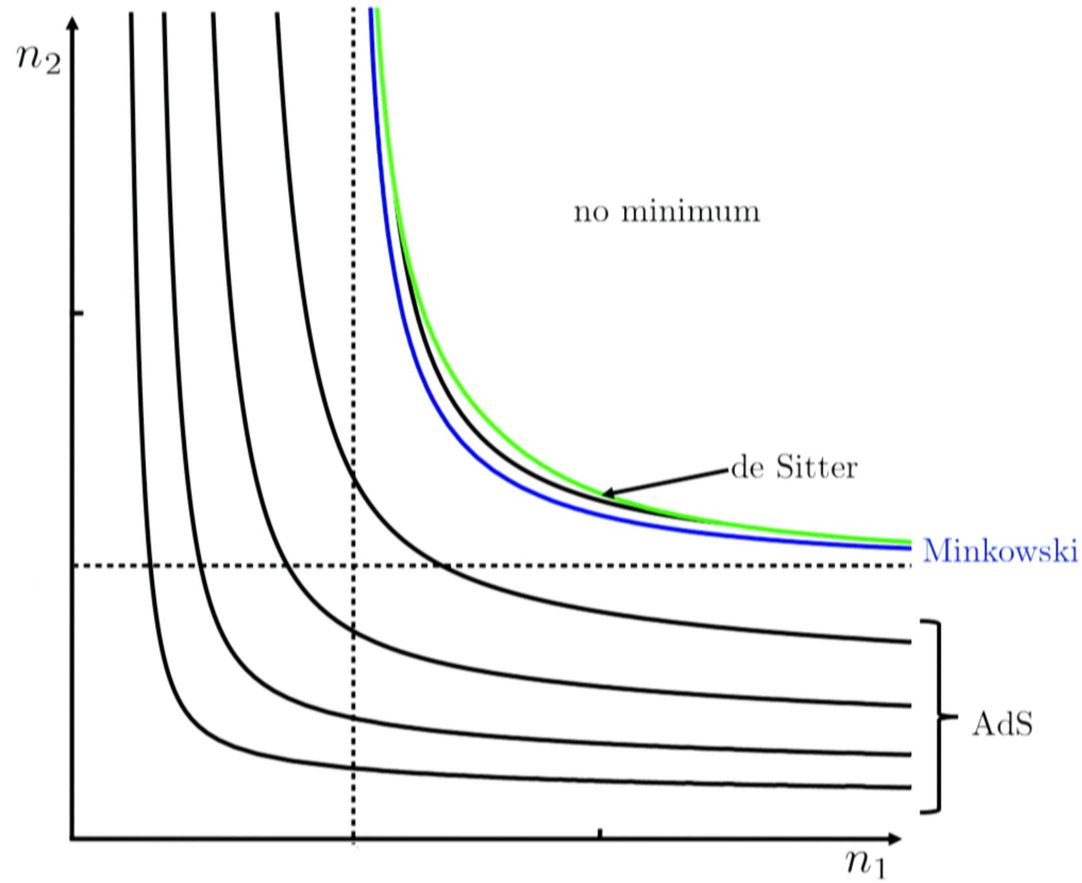
C.C.
'repulsive'
long-range



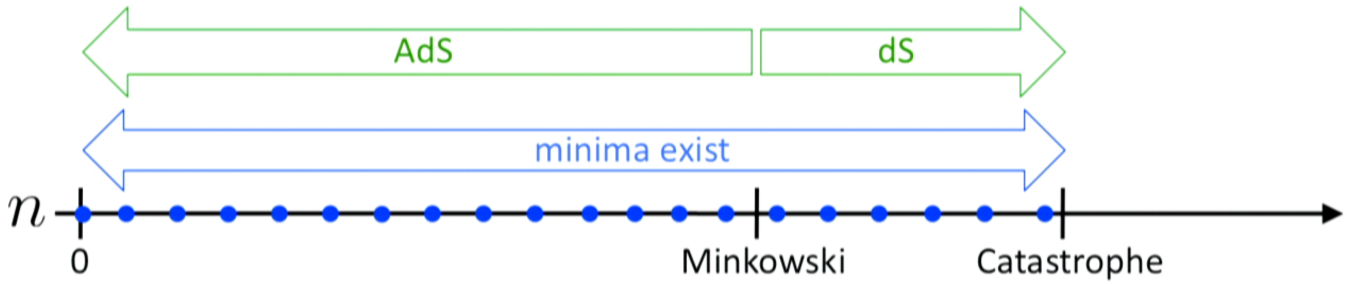
Here's a sample potential for a choice of n_1 and n_2 that gives rise to an AdS minimum.

The Effective Potential

Equipotentials of the minimum $N = 2,\ q = 2$

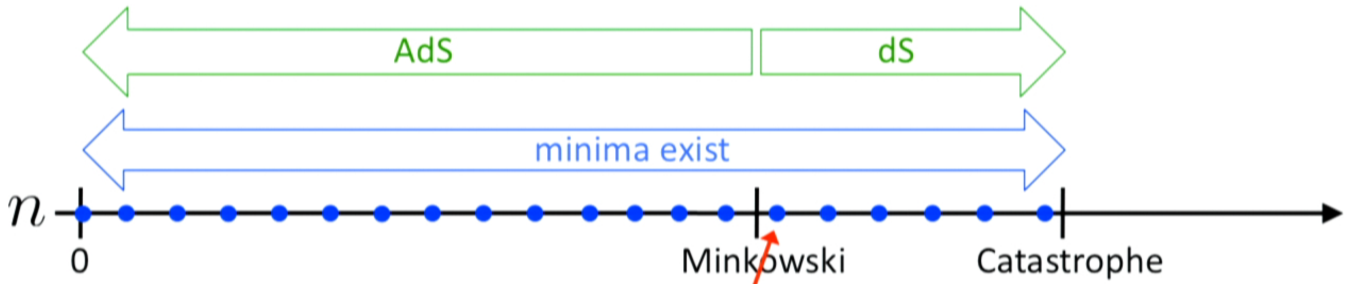


Flux Quantization



spacing set by g

Flux Quantization



spacing set by g

$$\boxed{H^2_{D-q} \sim \frac{H_D^2}{\#\text{dS minima}} \sim \frac{H_D^2}{H_D^{1-q}/g} \sim g(H_D^2)^{\frac{1+q}{2}}}$$

After N generations,

$$\boxed{H^2_{D-Nq} \sim g^{2\frac{(\frac{1+q}{2})^N-1}{q-1}}\,(H_D^2)^{\left(\frac{1+q}{2}\right)^N}}$$

Smallest daughter of the smallest...daughter of the smallest daughter is double-exponentially small in N.

Typical de Sitter vacuum

The typical lower-dimensional de Sitter vacuum in this theory, therefore, has double-exponentially small cosmological constant. This landscape has no CC problem.

Instead, it has a different problem, no less severe.

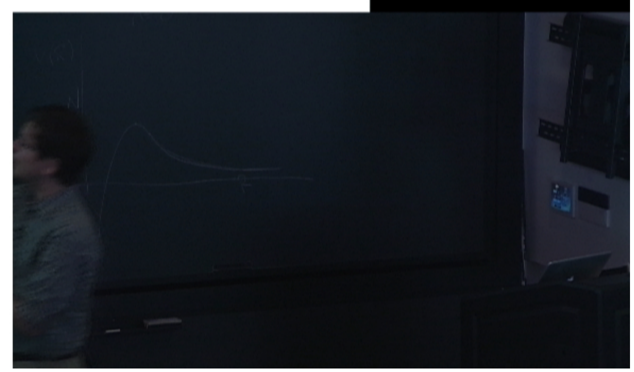
The typical lower-dimensional de Sitter vacuum in this theory has double-exponentially large extra dimensions.
In particular, the KK scale is typically of order the Hubble scale.

Typical de Sitter vacuum

While typical vacua have $H_p \sim m_{\mathrm{KK}} \sim H_{p+q}$,
there are rarer vacua with $H_p \ll m_{\mathrm{KK}} \sim H_{p+q}$.

These are the ones that, at the last step in the sequential compactification, 'get lucky' and end up near zero.

If we restrict to these, say by requiring that $H_p < 10^{50} m_{\mathrm{KK}}$, then there's *still* a double exponentially large number, and they *still* accumulate at zero.



Part 3: Shape modes

Stability to shape modes

In the effective potential picture, we only treated the radii as dynamic, we held the shape of the sphere.

But the shape of the spheres can also fluctuate. Are these modes stable?

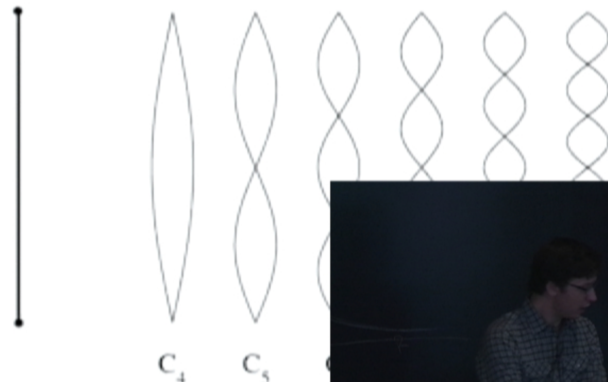
Stability to shape modes

In the effective potential picture, we only treated the radii as dynamic, we held the shape of the sphere.

But the shape of the spheres can also fluctuate.
Are these modes stable?

First guess: it's like banging a drum, or plucking a string.

Higher modes have a higher mass and make a higher sound.



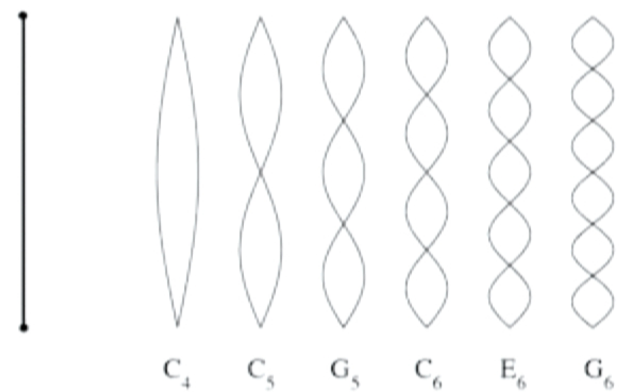
Stability to shape modes

In the effective potential picture, we only treated the radii as dynamic, we held the shape of the sphere.

But the shape of the spheres can also fluctuate.
Are these modes stable?

First guess: it's like banging a drum, or plucking a string.

Higher modes have a higher mass and make a higher sound.



Stability to shape modes

In the effective potential picture, we only treated the radii as dynamic, we held the shape of the sphere.

But the shape of the spheres can also fluctuate.
Are these modes stable?

This percussion-sourced intuition proves too naïve because of coupling.
Diagonalizing the modes introduces unstable mass squareds.
(Like Jeans instability)

Stability to shape modes

In the effective potential picture, we only treated the radii as dynamic, we held the shape of the sphere.

But the shape of the spheres can also fluctuate.
Are these modes stable?

This percussion-sourced intuition proves too naïve because of coupling.
Diagonalizing the modes introduces unstable mass squareds.
(Like Jeans instability)

Let's start with $N = 1$.

Stability to shape modes $N = 1$

In the effective potential picture, we only treated the radii as dynamic, we held the shape of the sphere.

But the shape of the spheres can also fluctuate.
Are these modes stable?

$N = 1$ Studied by: DeWolfe, Freedman, Gubser, Horowitz, and Mitra `arXiv:hep-th/0105047`

Bousso, DeWolfe, and Myers `arXiv:hep-th/0205080`

Hinterbichler, Levin, and Zukowski `arXiv:1310.6353[hep-th]`

They find that most modes decouple anad are massive, but the trace of the internal manifold and a flux scalar couple together.

Stability to shape modes $N = 1$

In the effective potential picture, we only treated the radii as dynamic, we held the shape of the sphere.

But the shape of the spheres can also fluctuate.
Are these modes stable?

$N = 1$ Studied by: DeWolfe, Freedman, Gubser, Horowitz, and Mitra `arXiv:hep-th/0105047`

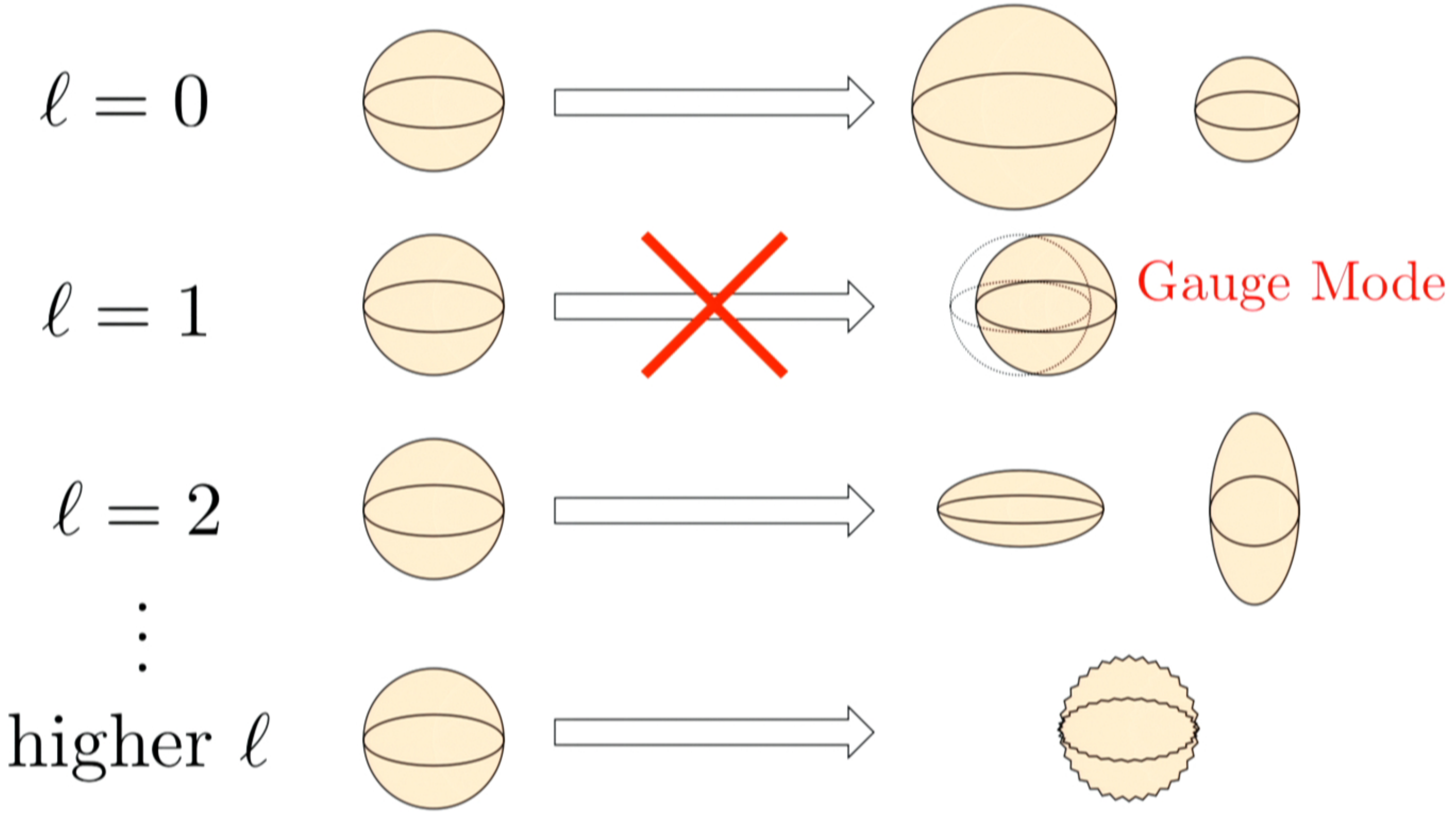
Bousso, DeWolfe, and Myers `arXiv:hep-th/0205080`

Hinterbichler, Levin, and Zukowski `arXiv:1310.6353[hep-th]`

They find that most modes decouple anad are massive, but the trace of the internal manifold and a flux scalar couple together.

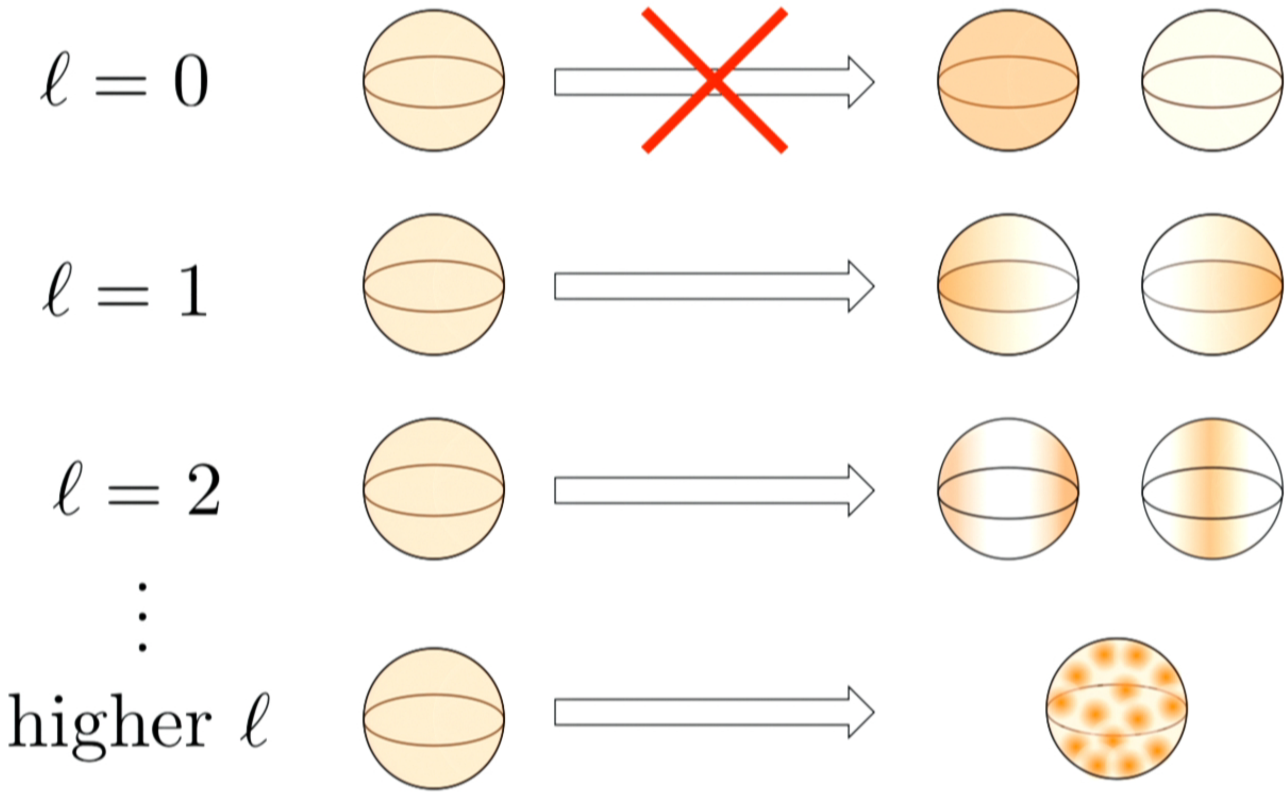
Gravity perturbations $N = 1$

Perturbations of the radius of the radius can be decomposed in spherical harmonics:



Flux perturbations $N = 1$

Perturbations of the flux can also be decomposed in spherical harmonics



Gravity and Flux Modes Couple $N = 1$

All Together:

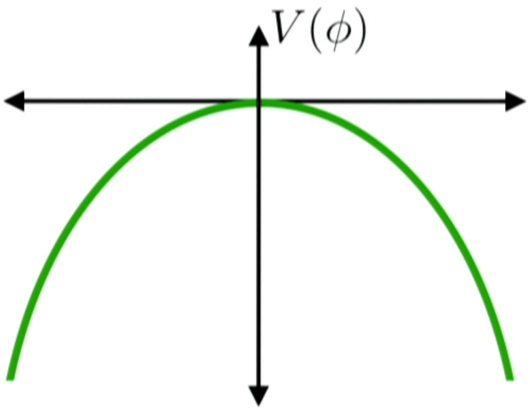
$\ell = 0$ Total-Volume Fluctuation Radion

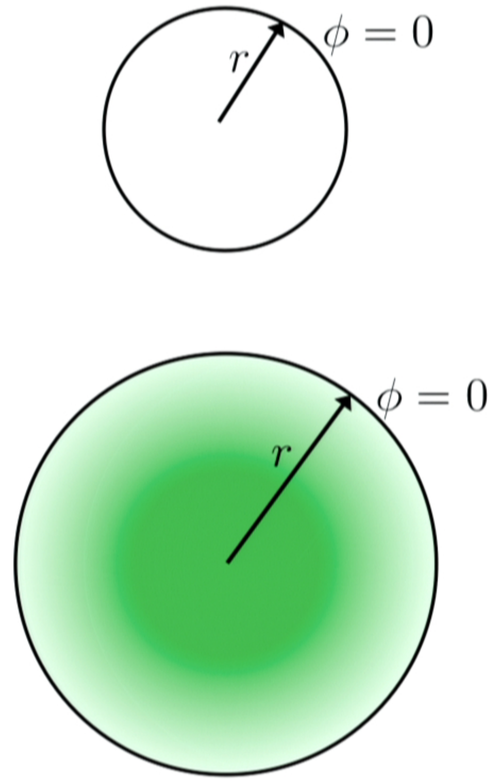
$\ell = 1$ One Flux Mode Always more massive than radion

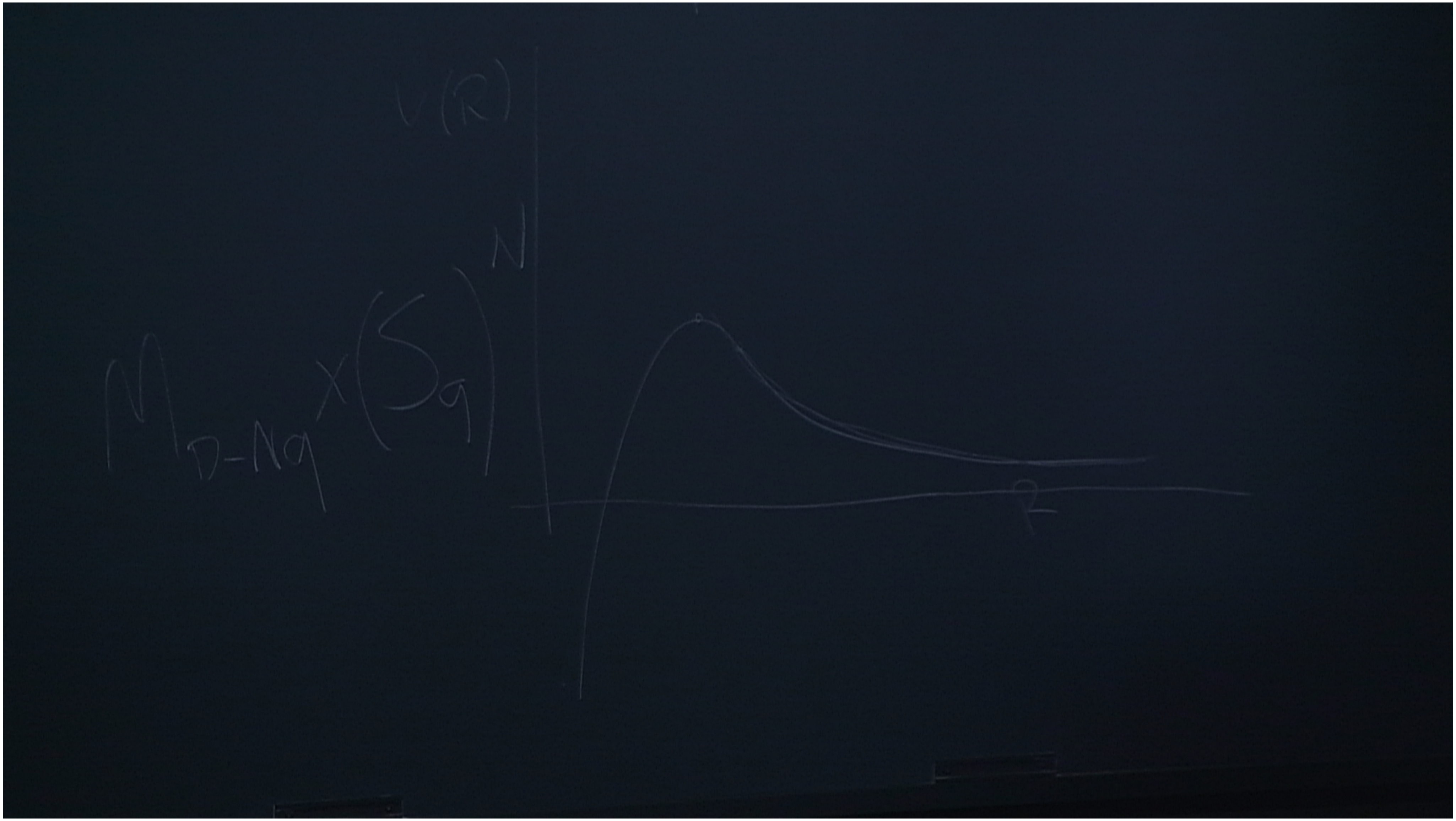
$$\left.\begin{array}{c} \ell = 2 \\ \vdots \\ \text{higher } \ell \end{array}\right\} \quad \text{Two coupled fluctuations for each } \ell.$$

Stability: BF bound $N = 1$

If the compactification is dS or Minkowski, then stability means that all modes have positive mass squared. But if the compactification is AdS, small negative mass squareds are allowed.



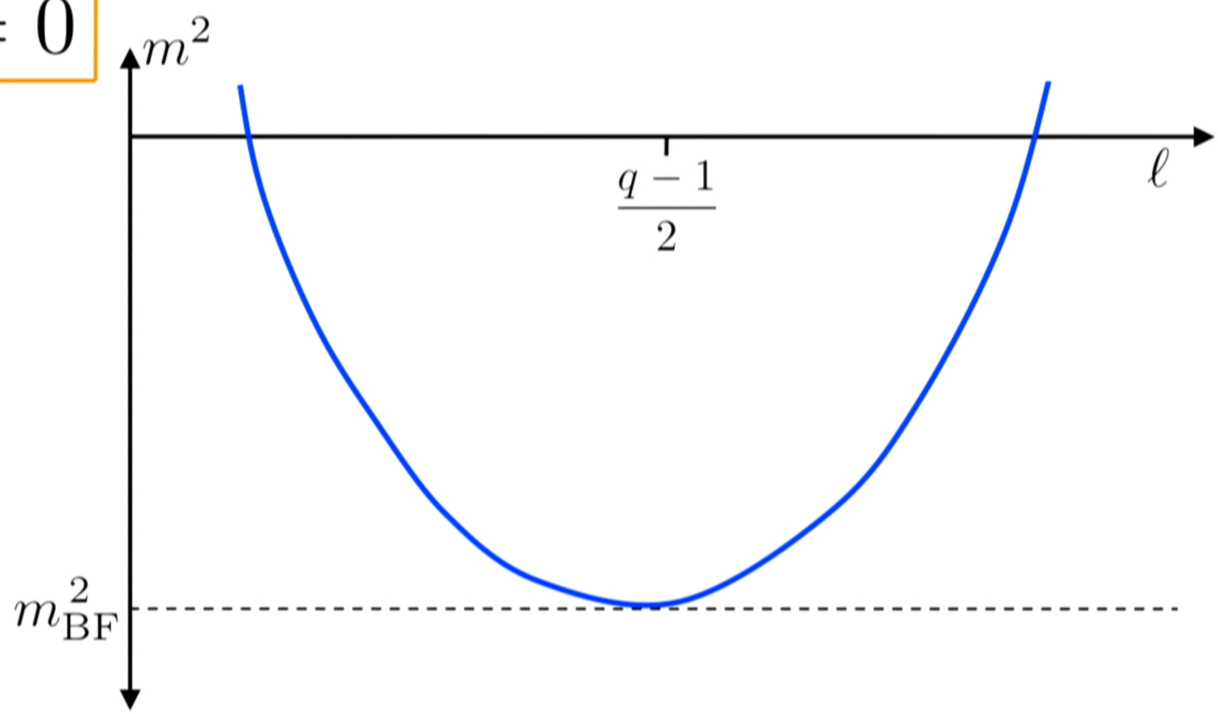



$V(R)$
$M_{D-Nq} \times (S_q)^N$
R

Stability: danger mode

$N = 1$

$\Lambda = 0$



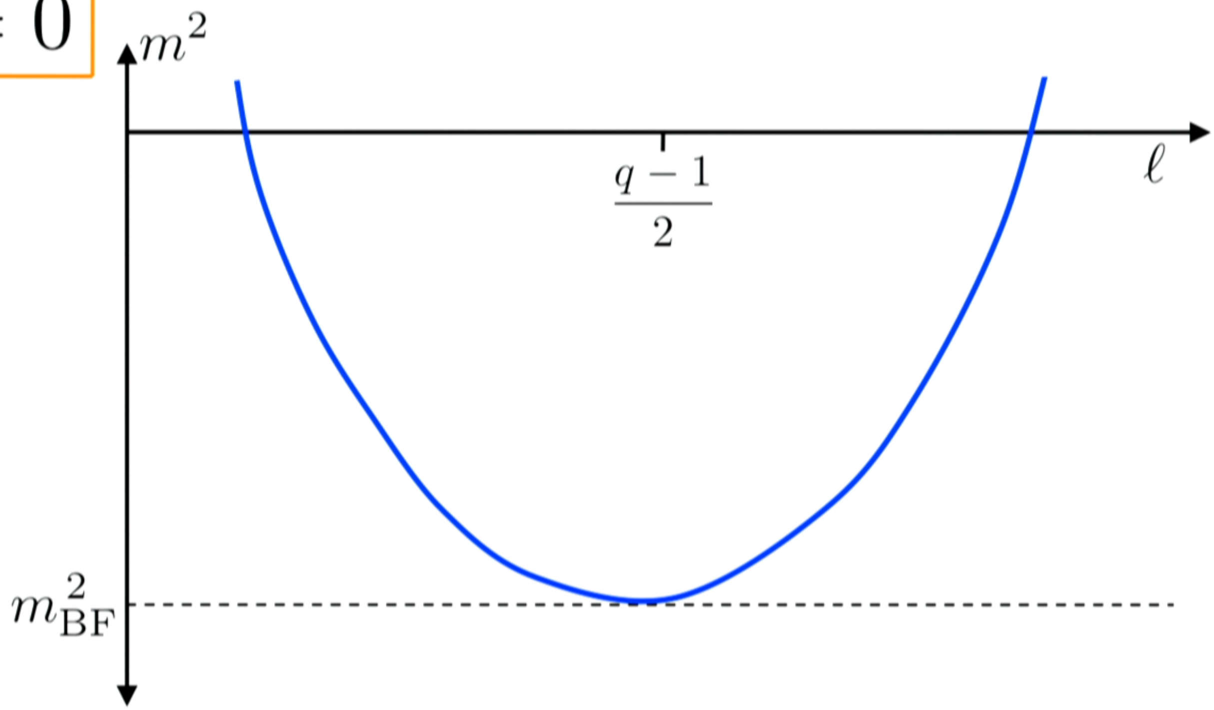
STABLE

At $\ell = \frac{q-1}{2}$, the danger mode exactly saturates the BF bound, and all other modes are above it.

Stability: danger mode

$N = 1$

$\Lambda = 0$



STABLE

At $\ell = \frac{q-1}{2}$, the danger mode exactly saturates the BF bound, and all other modes are above it.

Stability: danger mode $N = 1$

$\boxed{\Lambda = 0}$

The value $\ell = \frac{q-1}{2}$ splits up the problem into 3 cases:

$q = 2,\ 3$ — $\ell = \frac{q-1}{2}$ happens for $\ell \leq 2$, where there's only a single propagating degree of freedom.

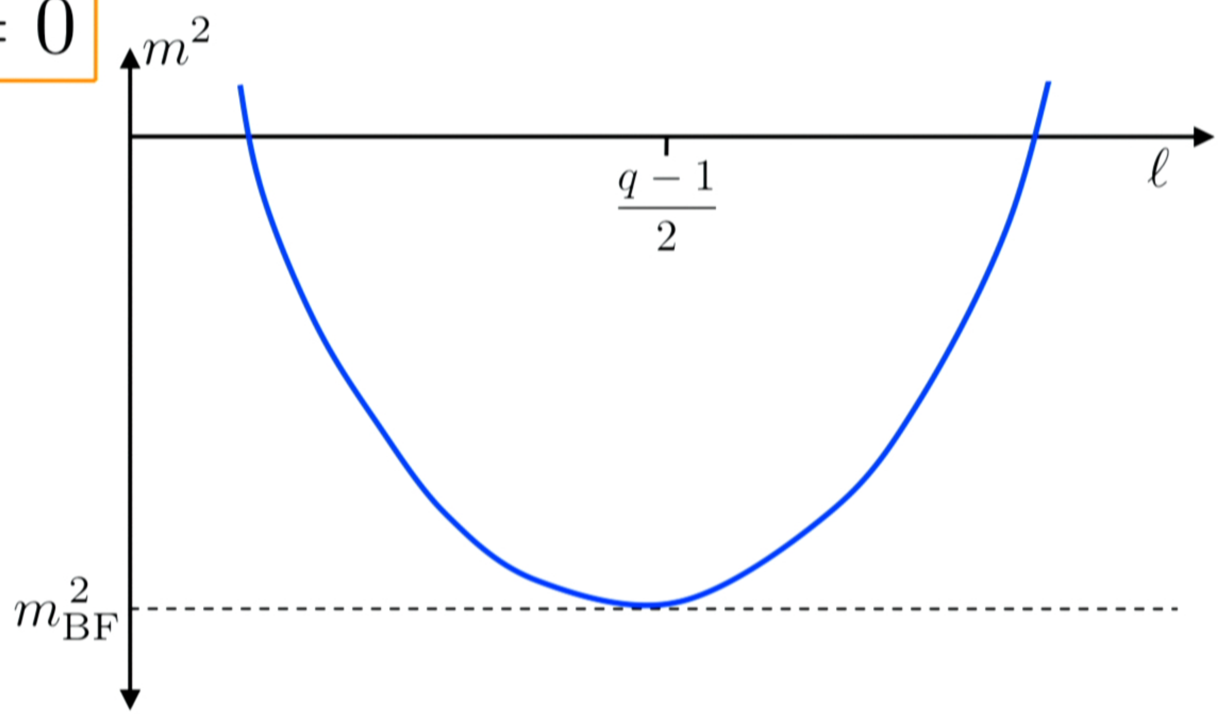
$q = 4,\ 6,\ 8,\ 10,\ \ldots$ — $\ell = \frac{q-1}{2}$ is not an integer, so the whole spectrum lies above the BF bound.

$q = 5,\ 7,\ 9,\ 11,\ \ldots$ — $\ell = \frac{q-1}{2}$ is an integer, so there is a mode exactly at the BF bound.

Stability: danger mode

$N = 1$

$\Lambda = 0$



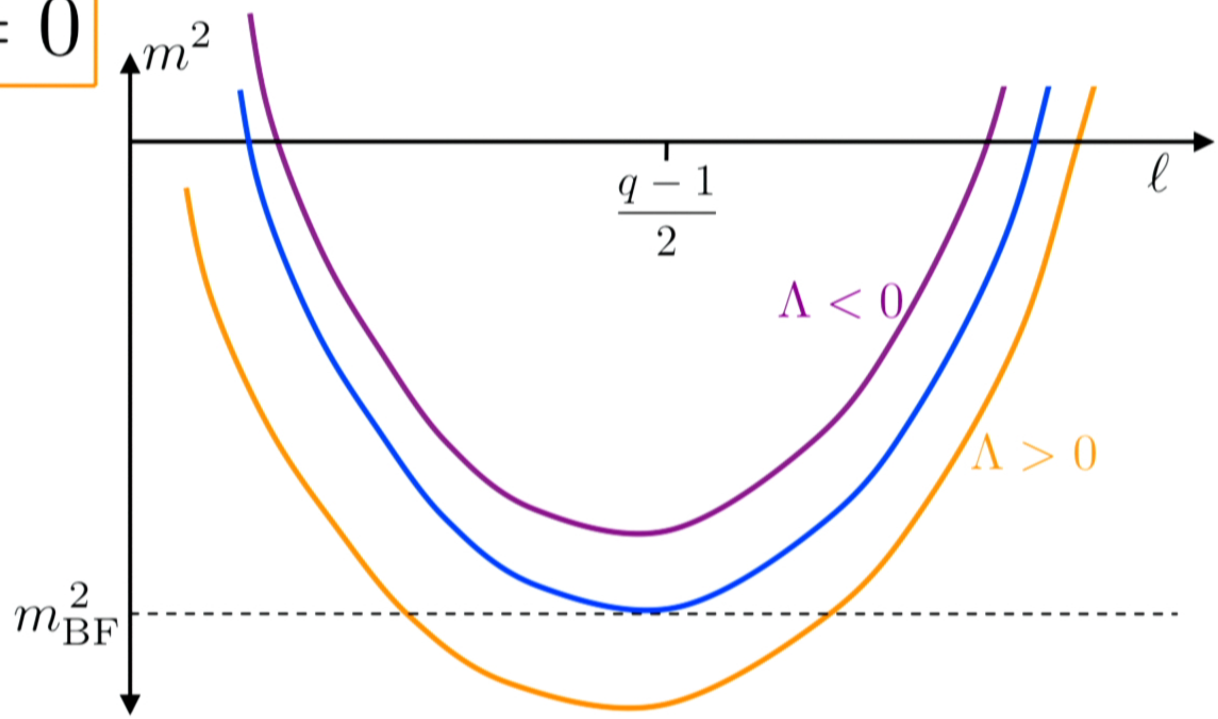
STABLE

At $\ell = \frac{q-1}{2}$, the danger mode exactly saturates the BF bound, and all other modes are above it.

Stability: danger mode

$N = 1$

$\Lambda \neq 0$



Turning on $\Lambda \neq 0$ shifts the mass curve.
If $\Lambda < 0$, it makes everything more stable,
but if $\Lambda > 0$, modes near the BF bound can be pushed beneath it.

Stability: danger mode $\mathcal{N} = 1$

	$\Lambda \leq 0$ AdS$_p$ minima	$\Lambda > 0$ AdS$_p$ minima	$\Lambda > 0$ dS$_p$ minima
$q = 2$	stable (always positive)	stable (always positive)	stable
$q = 3$	stable	stable	stable
$q = 4$	stable	mostly unstable (deep AdS$_p$ stable)	mostly unstable (high dS$_p$ stable)
$q = 5,7,9\ldots$	stable	unstable	unstable
$q = 6,8,10\ldots$	stable	mostly unstable (deep AdS$_p$ stable)	unstable

Stability: N≥2

When $N \geq 2$, each harmonic is indexed by a vector $(\ell_1, \ldots, \ell_N)$.

Each sphere contributes 2 modes (as long as $\ell_i \geq 2$).

For instance, if $N = 2$, in the $(\ell_1 = 0, \ell_2 = 5)$ sector, there's 2 shape modes and 1 flux mode:

shape 1 Vary the size of sphere 1 non-uniformly over sphere 2

shape 2 Vary the shape of sphere 2 uniformly over sphere 1

flux 1 Vary the flux density on sphere 2 uniformly over sphere 1.

$(\ell_1, \ldots, \ell_N) = (0, \ldots, 0)$ corresponds to fluctuations of the radii.

In general, there are $2N$ coupled modes.

Stability: N≥2

	$\Lambda \leq 0$ AdS$_p$ minima	$\Lambda > 0$ AdS$_p$ minima	$\Lambda > 0$ dS$_p$ minima
$q = 2$	stable (always positive)	stable (always positive)	stable
$q = 3$	stable	stable	stable
$q = 4$	mostly unstable (some stable)	mostly unstable (some stable)	unstable
$q = 5,7,9\ldots$	mostly unstable (some stable)	mostly unstable (some stable)	unstable
$q = 6,8,10\ldots$	mostly unstable (some stable)	mostly unstable (some stable)	unstable

Stability: N≥2

When $N \geq 2$, each harmonic is indexed by a vector $(\ell_1, \ldots, \ell_N)$.

Each sphere contributes 2 modes (as long as $\ell_i \geq 2$).

For instance, if $N = 2$, in the $(\ell_1 = 0, \ell_2 = 5)$ sector, there's 2 shape modes and 1 flux mode:

shape 1 Vary the size of sphere 1 non-uniformly over sphere 2

shape 2 Vary the shape of sphere 2 uniformly over sphere 1

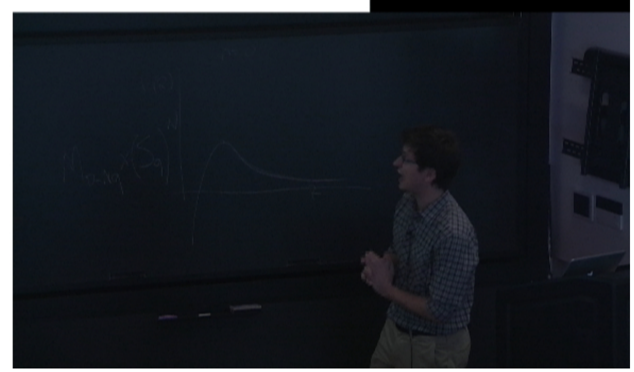
flux 1 Vary the flux density on sphere 2 uniformly over sphere 1.

$(\ell_1, \ldots, \ell_N) = (0, \ldots, 0)$ corresponds to fluctuations of the radii.

In general, there are $2N$ coupled modes.

Stability: N≥2

$\mathrm{AdS}_4 \times S_{101}$ is stable to all modes except for one with $\ell = 50$.

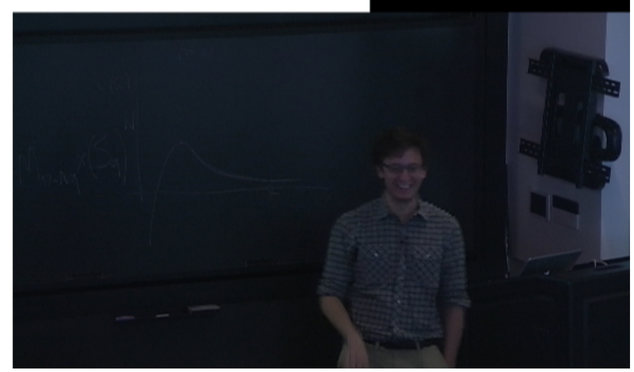


Stability: N≥2

$\mathrm{AdS}_4 \times S_{101}$ is stable to all modes except for one with $\ell = 50$.

We computed the full perturbative spectrum of these compactifications.

For AdS vacua, in theories with $\Lambda = 0$,
the spectrum should match to a CFT dual.



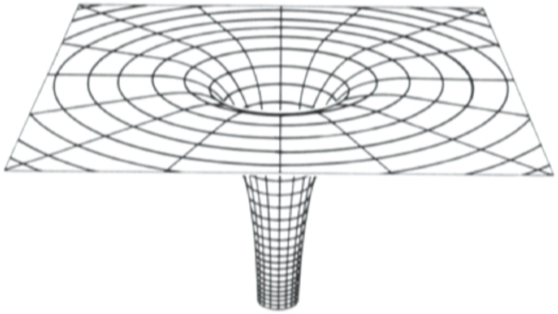
Stability: N≥2

$\mathrm{AdS}_4 \times S_{101}$ is stable to all modes except for one with $\ell = 50$.

We computed the full perturbative spectrum of these compactifications.

For AdS vacua, in theories with $\Lambda = 0$,
the spectrum should match to a CFT dual.

Compactified vacua also correspond to the near-horizon limit of higher-dimensional black branes.



the end