

Title: From spin foams to anyons and back again - Joint Condensed Matter/Quantum Gravity Seminar

Date: Dec 05, 2013 02:30 PM

URL: <http://pirsa.org/13120048>

Abstract: Spin foams provide models for quantum gravity and hence quantum space time. One of the key outstanding questions is to show that they reproduce smooth space time manifolds in a continuum limit. I will start with a very short introduction to spin foams and the structure of quantum space time they encode. I will explain how the investigation of the continuum limit via coarse graining and renormalization techniques led as to consider anyonic spin chains and a classification of ground states in systems with quantum group symmetries. I will then present new results on the continuum limit of spin net models, that allow us to draw first conclusions about the large scale dynamics of spin foams.

Based on: B.D., W. Kaminski, Topological lattice field theories from intertwiner dynamics, arXiv:1311.1798, B.D., S. Steinhaus, Time evolution as refining, coarse graining and entangling, to appear, B.D. M. Martin-Benito, S. Steinhaus, The refinement limit of quantum group spin net models, to appear

From spin foams ...

- Introduction of spin foams as theory defined on (simplicial) discretization. Encodes quanta of space time.
- We can formulate spin foams and loop quantum gravity as a continuum theory, thanks to the fact that in (discrete) gravity

Refining = Time evolution.

[BD, Steinhaus, [arXiv:1311.7565](#)]

... to anyons ...

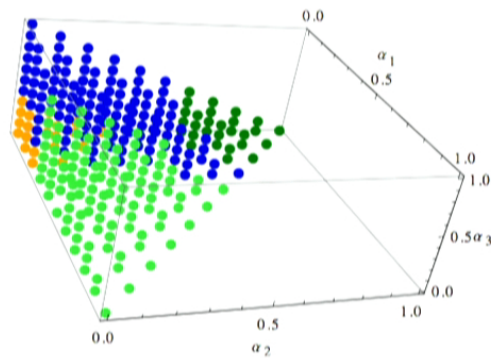
- The dynamics of spin foams is encoded in **intertwiner degrees** of freedom, for (quantum) rotations groups.
- Dynamics of intertwiners can be considered in 2D models, reminiscent of (space time versions) of anyon spin chains.
- We can classify all incidences of 2D topological lattice field theories, which are candidates for (gapped) phases. Parallels the classification for (1+1)D systems by Wen et al and Schuch et al.
- Uses classification of **module categories over fusion categories** and results in a A-D-E classification, known from specifying possible vertex operator algebra extensions in CFT's.

[BD, Kaminski [arXiv:1311.1798](#), Oliver Buerschaper, BD, Kaminski, [wip](#)]

Unfortunately there will be no time for that ...

... and back again.

- We can devise a (2D) model which describes the gluing of two (4D) spin foam vertices describing very finely triangulated spherical space time region
- We can find the refinement limit for these models, by constructing
 - the coarse graining flow of the effective coupling between the two space time regions.
- We find a rich phase structure, with examples giving a decoupling of the two space time regions as well as examples where the two space time region remain coupled to each other.



[BD, Mercedes Martin-Benito,
Sebastian Steinhaus
[arXiv:1312.0905](https://arxiv.org/abs/1312.0905)]

Luckily, you have all
encountered spin foams
already ...

Intro from stat physics point of view:
[Bahr, BD, Ryan, [arXiv:1103.6264](#)]

The Ponzano Regge model

[Ponzano, Regge 1968]

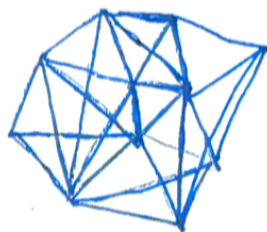
- first spin foam, describes 3D Euclidean gravity (without cosmological constant)
- 'first lattice gauge theory' (at zero coupling limit in strong coupling expansion)
- first example of an (unstable)* topological phase
- topological model: no propagating (field) degrees of freedom

(unstable)*: This does not matter in gravity as we take the Hamiltonian $H=0$ as a constraint!
(Condensed matter) ground states = (Gravity) physical states

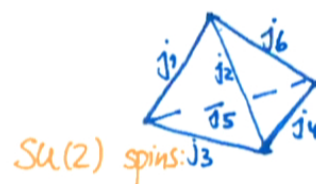
The Ponzano Regge model

[Ponzano, Regge 1968]

triangulation



tetrahedron



$\{6j\} \sim F\text{-symbol}$

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$



$$Z_{\text{trie.}} = \sum_{\text{spins}} \prod_{\text{tetrah.}} \{6j\}$$

• invariant under changes of bulk triangulation

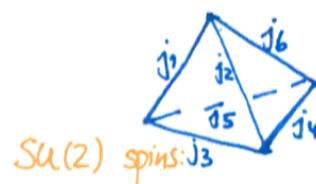
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Wigner, Ponzano, Regge, Penrose ... :

Recoupling symbol encode quantum space time properties.

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Why gravity?

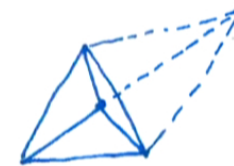


$$\rightarrow \left\{ \begin{matrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{matrix} \right\} \sim_{j \text{ large}} \left(e^{+i S_{\text{Regge}}} + e^{-i S_{\text{Regge}}} \right)$$

with $j \sim \text{length of edge}$

positive orientation negative orientation

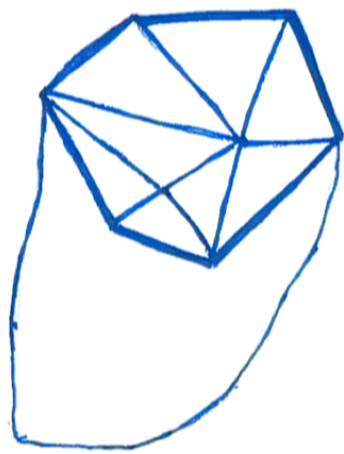
$Z_{\text{tria.}} = \infty$ in most cases.



- includes integration over non-compact gauge orbits \rightarrow vertex translation symmetry = diffeomorphism symmetry.
- also: unstable fixed point / phase.

[put many names: asymptotics, symmetry ..., smerlak, riello, bd ...]

How to time evolve?



"equal time"
surface

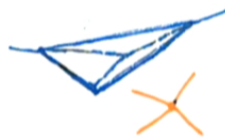
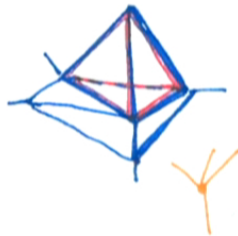
physical wave function
(string net)

How to "time evolve" ?

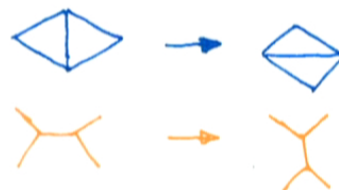
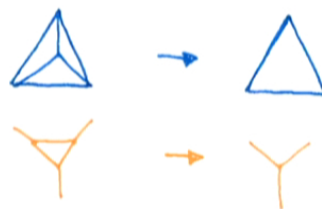
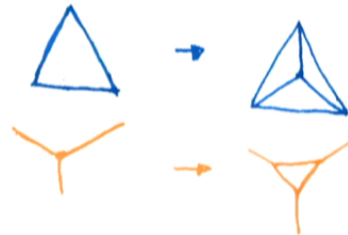
→ gluing tetrahedra to the 2D
triangulation

How to time evolve?

3 D



2 D



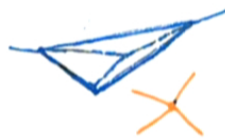
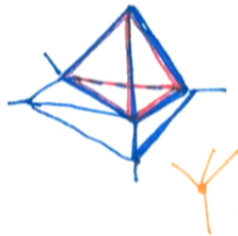
1-3 move

3-1 move

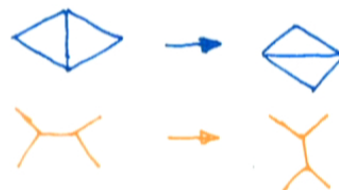
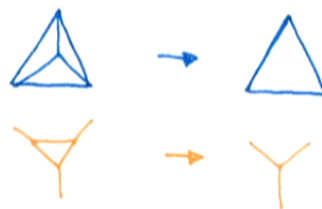
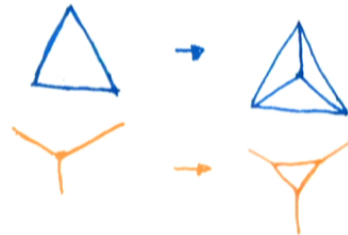
2-2 move

How to time evolve?

3 D



2 D



1-3 move

3-1 move

2-2 move



A. Number of (at least kinematical) degrees of freedom change

B. Special in gravity:

There is no time evolution: $H=0$.

Time evolution operators (should be) projectors onto physical states

(satisfying $H=0$)

A. [BD, Hoehn, [arXiv:1108.1974](#), [arXiv:1303.4294](#)] considers time evolution in the classical theory.

This leads to:

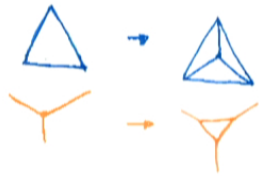
- (a) **post-constraints** (for refining moves): automatically satisfied if after the move
- (b) **pre-constraints** (for coarse graining moves): have to be satisfied so that move can be applied.

Refining, coarse graining, entangling from time evolution

[BD, Steinhaus, [arXiv:1311.7565](https://arxiv.org/abs/1311.7565)]

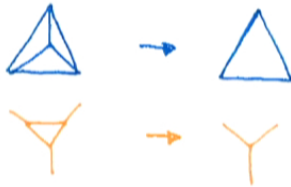
(provides more examples)

A. Changing number of degrees of freedom



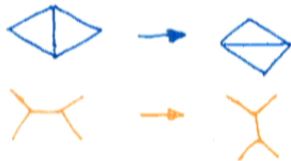
(a) **post-constraints** for refining moves:

Either gauge degrees of freedom or **degrees of freedom in the vacuum state** are added. The vacuum depends on the dof's already present.



(b) **pre-constraints** for coarse graining moves:

Finer degrees of freedom are projected out.



(b) **entangling** moves:

Distributes refined degrees of freedom over triangulation.
Necessary to produce (long range) entanglement.

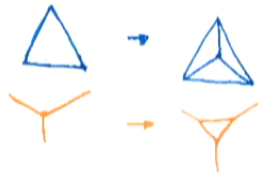
[Koenig, Reichardt, Vidal '08]

Refining, coarse graining, entangling from time evolution

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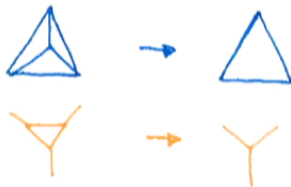
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A. Changing number of degrees of freedom



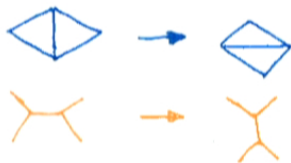
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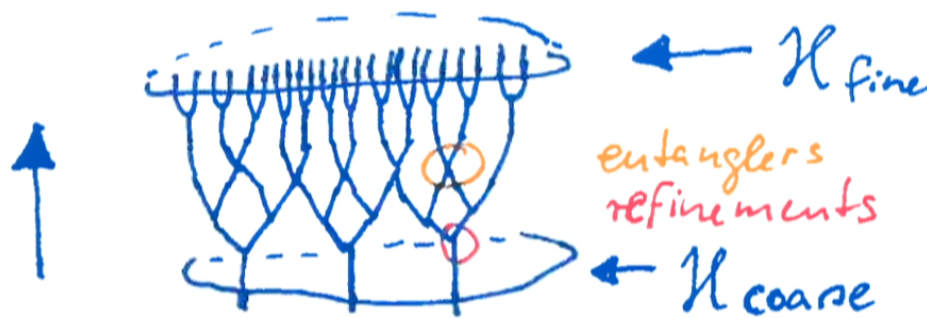
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Time evolution is refining, coarse graining, entangling in gravity.

[BD, Steinhaus, [arXiv:1311.7565](#)]

B. There is no time evolution in gravity: $H=0$.

In fact time evolution operators project onto physical states. [Halliwell, Hartle 91, Rovelli 99]



The **same** state is represented in different Hilbert spaces, associated to different boundary discretizations.

This is described by (dynamical cylindrical consistent) embedding maps. [BD 12]

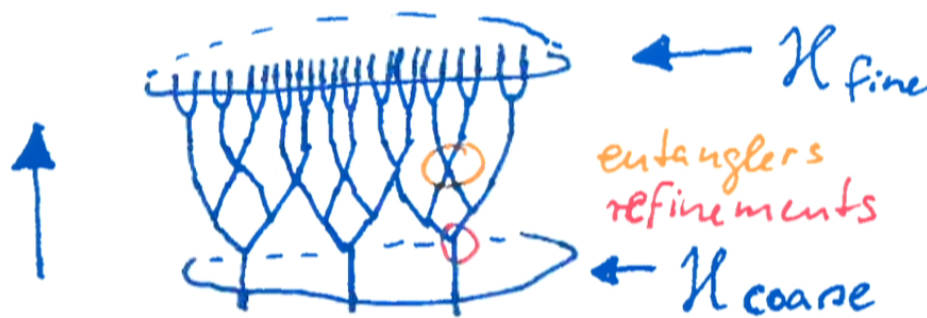
[Vidal: Entanglement renormalization, 05,06]

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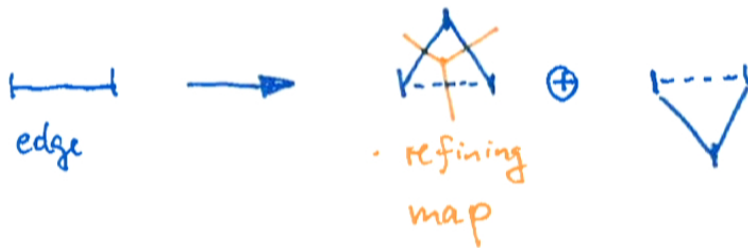
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Not a proper time evolution.

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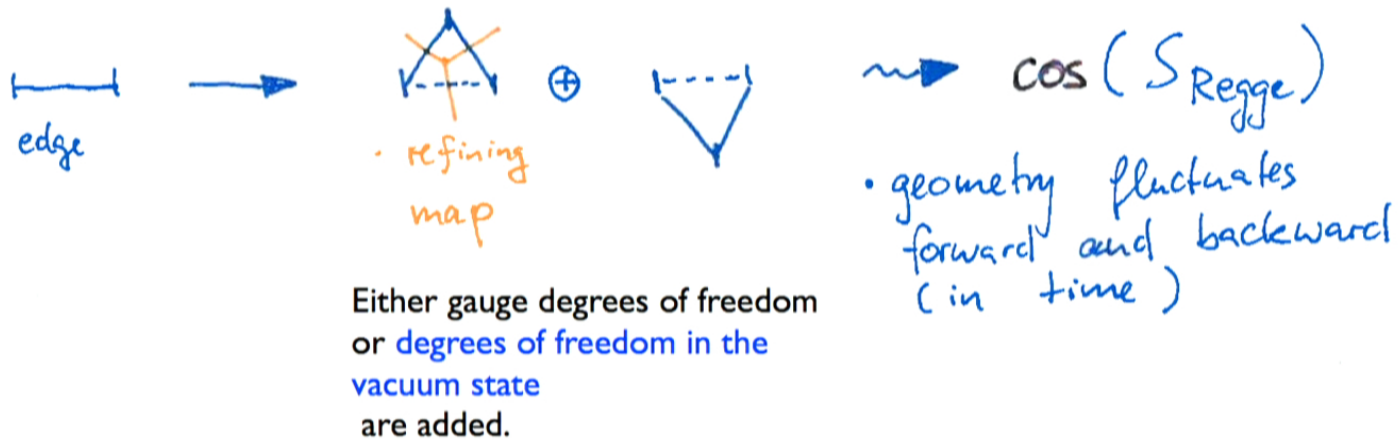
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are added.

$\leadsto \cos(S_{\text{Regge}})$
• geometry fluctuates
forward and backward
(in time)

[“Anti-space”, Christodoulou et al 12, Riello 13]

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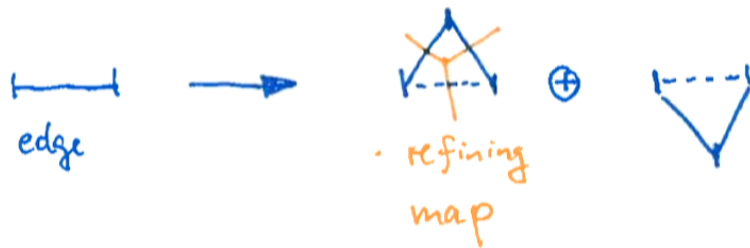
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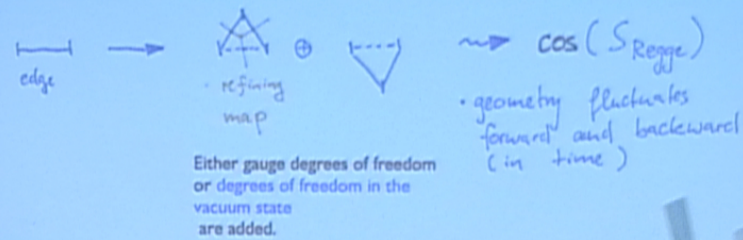
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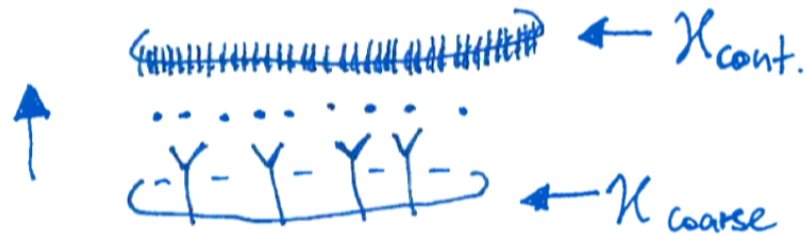
[BD, Steinhaus, arXiv:1311.7288]



["Anti-space", Christodoulou et al 12, Riello 13]

Continuum representation of discrete states.

[BD, Steinhaus, [arXiv:1311.7565](https://arxiv.org/abs/1311.7565)] [BD 12]

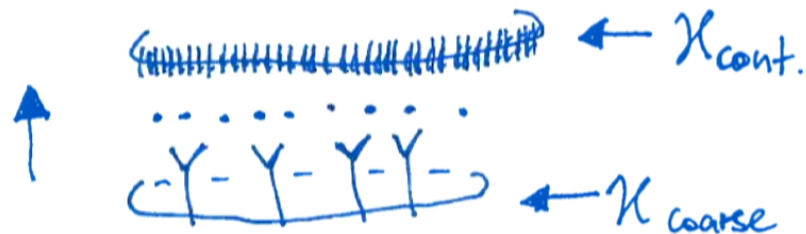


Representation of coarse state
in **continuum Hilbert space**.

What are the refining / entangling maps ?
 $Y, X \rightarrow$ Gluing of simplices !
 \rightarrow Given by spin foam amplitudes.

Continuum representation of discrete states.

[BD, Steinhaus, [arXiv:1311.7565](https://arxiv.org/abs/1311.7565)] [BD 12]

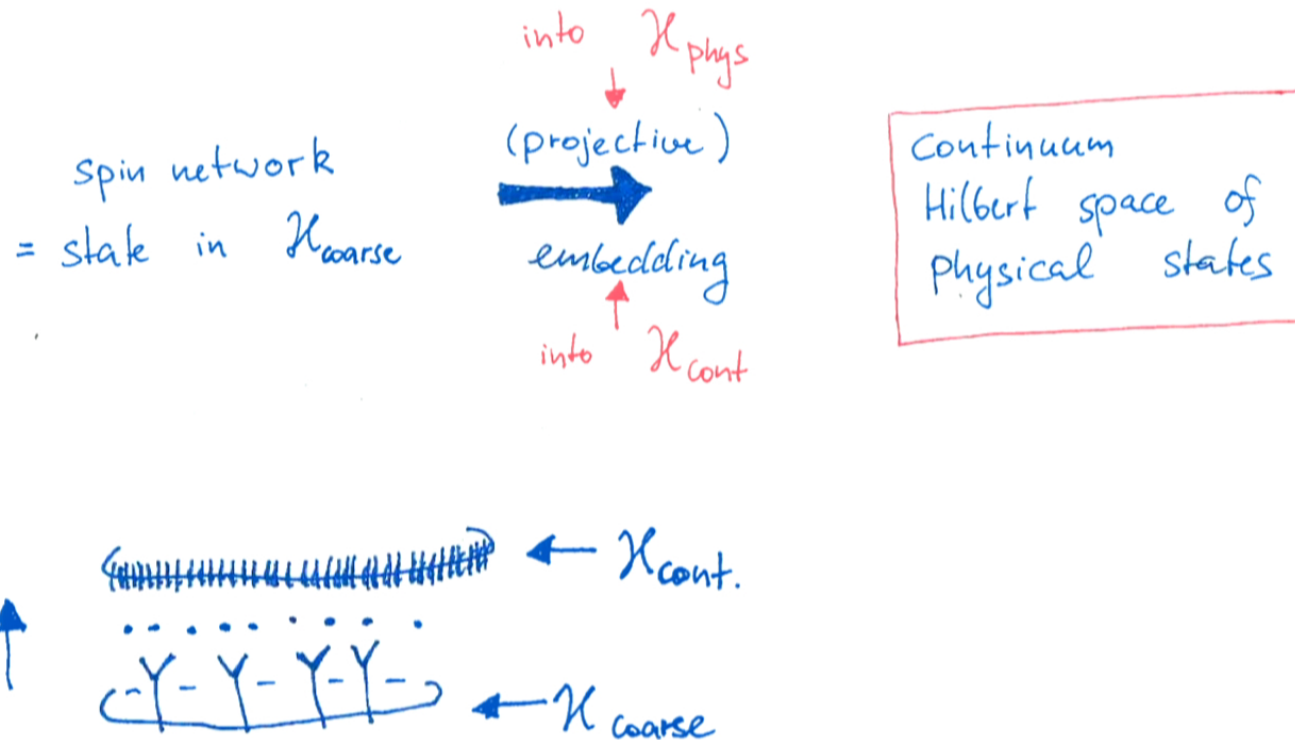


Representation of coarse state
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 $Y, X \rightarrow$ Gluing of simplices !
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Interpretation of spin network as continuum object: determined **dynamically**.

[BD 12]



Consistency = diffeomorphism symmetry

[BD, Steinhaus, [arXiv:1311.7565](#)]

For this need cylindrical consistency conditions:

- ⇒ Need path independence of refining time evolution. [Kuchar 70s]
- ⇒ Equivalent to a discrete Dirac algebra of constraints. [Bonzom, BD13]
- ⇒ Need diffeomorphism symmetry in the discrete. [BD 08, Bahr, BD 09+, ...]
- ⇒ Need to take the continuum limit. [Bahr, BD 09, Bahr, BD Steinhaus 11]
- ⇒ This continuum limit should be physically sensible. Have to hope or design (spin foam) amplitudes so that this is the case.

Obtain a perfect discretization in the form of amplitude maps for space time regions.

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Forget simplices!



→ · (basic) amplitude map for simple/coarse spherical boundary



→ · amplitude map \mathcal{A} for more complicated boundary



→ · amplitude map \mathcal{A} for infinitely refined boundary:

includes all coarser lattices

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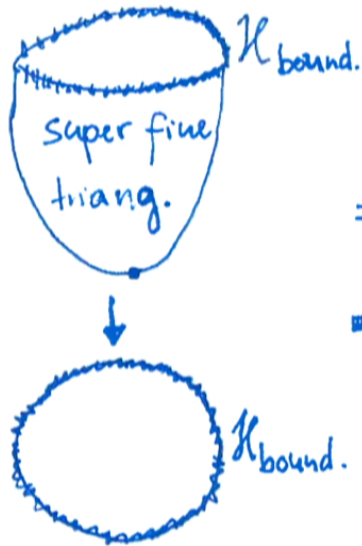
→ amplitude map \mathcal{A} for infinitely refined boundary:

includes all coarser boundaries

Forget simplices!

- ⇒ The dynamics is encoded in amplitude maps, acting on continuum Hilbert space associated to spherical boundary. [Oeckl: generalized boundary proposal]
- ⇒ Do **not** require 'consistent' gluing of these amplitude maps
(i.e. triangulation independence in the case of amplitude maps for simplices)
- ⇒ Instead require that the amplitude maps define a **cylindrically consistent observable**.
Which means that it is defined on the continuum Hilbert space via projective limit construction.
- ⇒ Continuum theory is defined via (cylindrically consistent) amplitude maps.
[BD 12, BD, Steinhaus, [arXiv:1311.7565](#)]

Refinement adds vacuum degrees of freedom

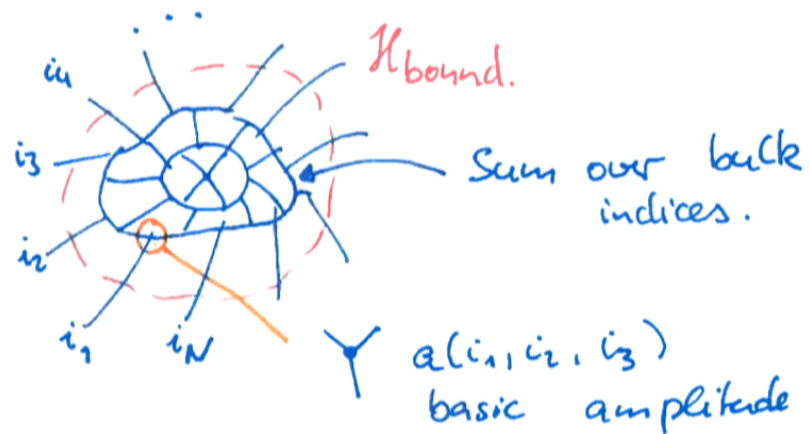


The (Hartle - Hawking) vacuum state
= dynamical vacuum
→ defines amplitude map
 $\mathcal{A} : \{\psi_{\text{bdry}}\} \rightarrow \mathbb{C}$
for spherical region. [Oeckl]

tensor network renormalization algorithms

effective
amplitude:

$$a'(\{i\}) =$$

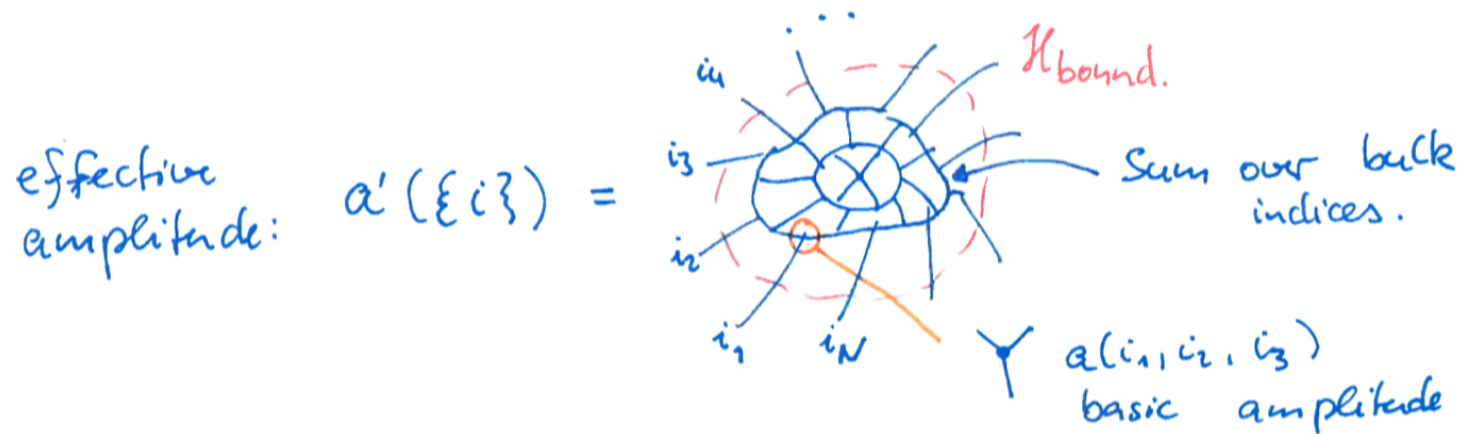


Problem: $a'(\{i\})$ depends on χ_0^N (index) values.

How to get the amplitude maps:

tensor network renormalization algorithms

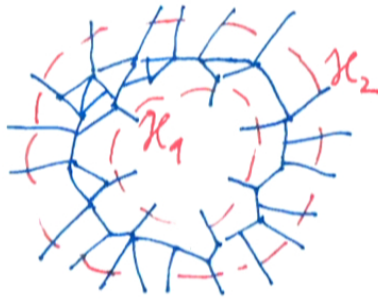
[Levin, Nave 06, Gu, Wen 09,...]



Problem: $a'(\{i\})$ depends on χ_o^N (index) values.

Radial evolution is refining.

[BD, Steinhaus, [arXiv:1311.7565](https://arxiv.org/abs/1311.7565)]



$$\mathcal{H}_1 \xrightarrow{T_{R_1, R_2}} \mathcal{H}_2$$

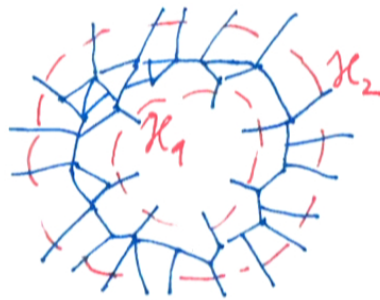
• radial (refining) time evolution

- $T_{R_1, R_2} : \mathcal{H}_1 \rightarrow \mathcal{H}_2$ represented by rectangular matrix
- Singular Value decomposition of T_{R_1, R_2} :
 - at most $\dim \mathcal{H}_1$ non-vanishing singular values
 - in gravity: $\lambda'_\alpha \sim 1$

$$T_{R_1, R_2} = \sum_{\alpha} U_{R_1 \alpha} \lambda_{\alpha} V_{\alpha R_2}$$

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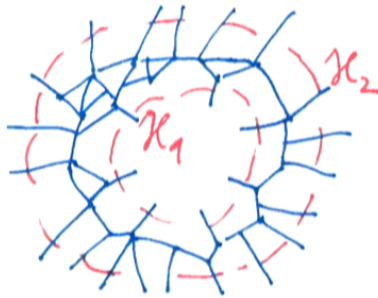
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Field redefinitions. No truncation.

V_{R_2} :

- field redefinition
- dof's sorted according to relevance



$\rightarrow \alpha'(\{\alpha\})$

- effective amplitude only depends on (max.) $\dim \mathcal{H}_1$ values.

The effective amplitude can then be used as the basic amplitude in the next iteration step.

This would produce an **exact renormalization flow**,

with degrees of freedom **automatically sorted into 'coarse' and 'fine'**.

This latter information is encoded in the V-maps.

(Which could be built from simplex amplitudes in the case of gravity.)

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In praxis ...

... at least so far for reasons of computational efficiency:

The V-maps are constructed more locally, again by a singular value decomposition.

[Levin, Nave 06, Gu, Wen 09,...]

However it would be highly interesting to investigate the (entangling) properties of the V-maps for the procedure presented.

They encode the **vacuum structure** for the fine degrees of freedom and define the macroscopic order parameters.

I

Application to spin foams

[BD, Mercedes Martin-Benito, Sebastian Steinhaus
[arXiv:1312.0905](https://arxiv.org/abs/1312.0905)]

- 4D gravity: not topological.
- Makes things interesting and challenging
- Same trick as in 3D: use cosmological constant (conjectured to be related to $SU(2)_k$) to obtain a theory feasible for numerical treatment
- The full 4D theory is still very challenging ...

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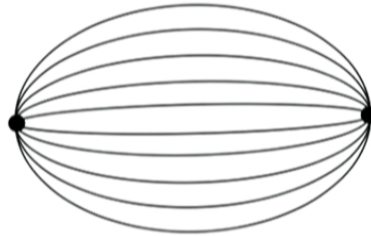
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A spin foam which reduces to 2D: spin net

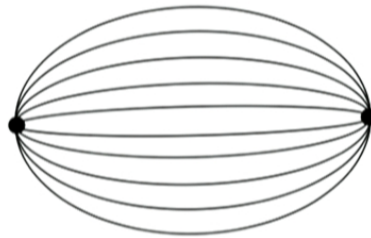
- Take two spherical space time region, tessellate with a **very large number** of tetrahedra
- glue these two spherical space time regions by identifying all tetrahedra with each other
- in the dual (to triangulation) picture we obtain, what is known as melon [due to Aldo]



- we will coarse grain by reducing the edges of the melon into 'thicker' effective edges
- non-trivial as we use a prescription where the dynamics is encoded on the edges
- ⇒ Non-trivial propagators as opposed to non-trivial vertices. I
Here vertices only impose (quantum) group symmetry.

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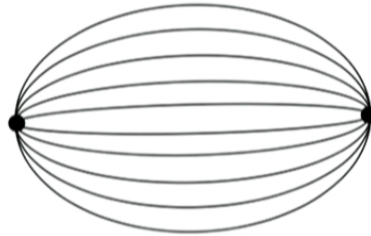
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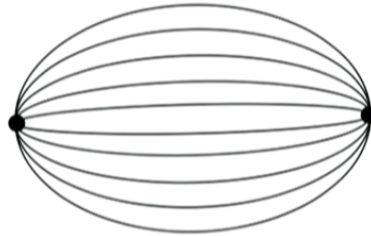
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A spin foam which reduces to 2D: spin net

- This translates into a 2D vertex model with global (left and right) $SU(2)_k$ symmetry
- That is a 2D tensor network, for which we can apply TNW coarse graining.

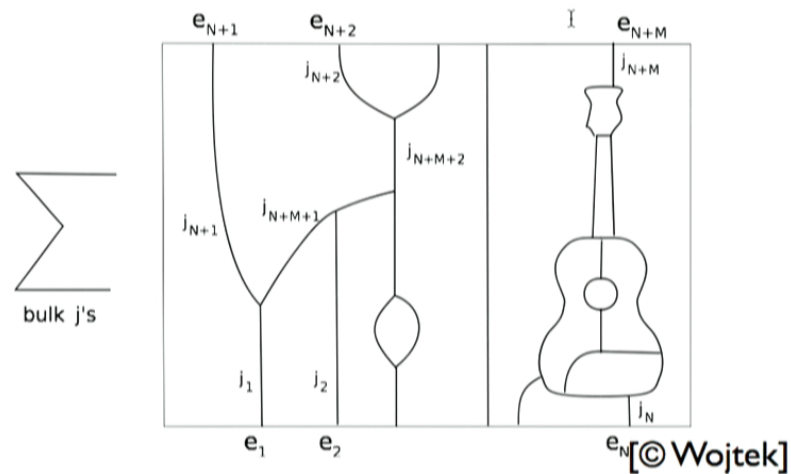
[Levin, Nave 06, Gu, Wen 09, ... BD, Martin-Benito, Schnetter 13]

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- The model can be seen as doubling of anyonic spin chain models (in a space--time version).
- Relevant degrees of freedom are **intertwiners**.

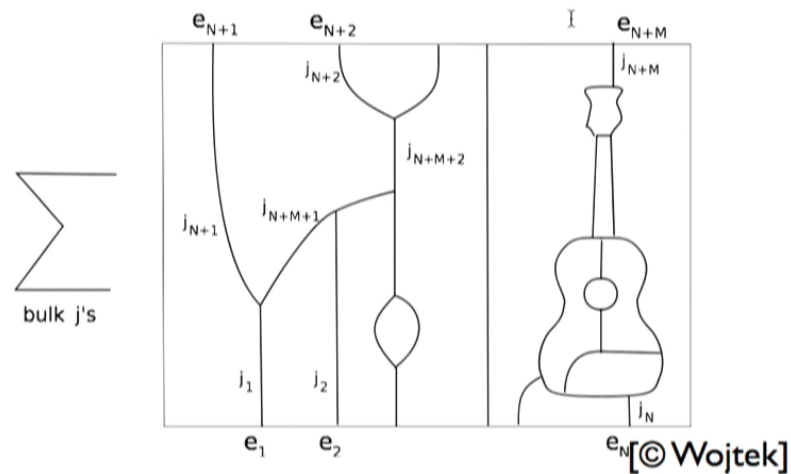
Intertwiner models

- To parametrize intertwiners efficiently
a classification of all 2D topological field theories with $SU(2)_k$ symmetry
(intertwiner models) is (being) worked out.
- This parallels the classification of $(1+1)D$ phases with (classical) group symmetry.
[BD, Kaminski [arXiv:1311.1798](#), Oliver Buerschaper, BD, Kaminski, [wip](#)]
- Crucial difference:
group on-site symmetry vs quantum group not-on-site symmetry
- Gives phases of anyonic spin chains and is closely related to anyon condensation.
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Flow of coupling between spin foam vertices

- The melon model is probing the coupling between two space time atoms.
- Renormalization flow gives change of this coupling with number of elementary glued tetrahedra.

Results:

- Rich (very regular in k) fixed point structure: new phases for spin foams / spin nets.
- Classification might be available with the methods developed in

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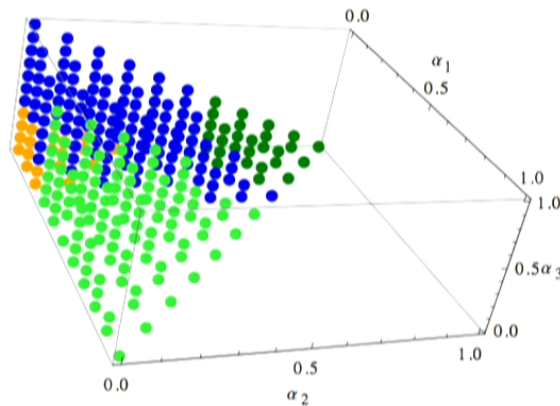
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Does the melon survive?



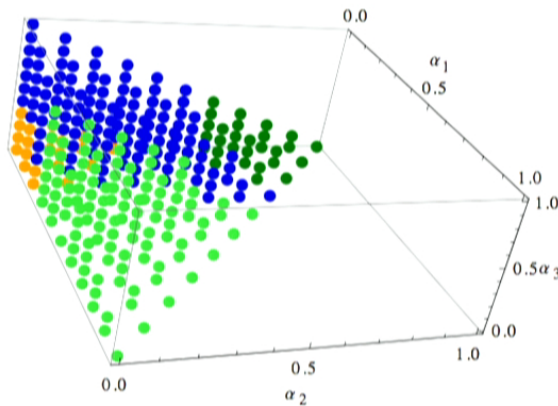
- Many fixed points describe a decoupling of the two spin foam vertices.
(Partition function factorizes into left and right copy, describing intertwiner models.)
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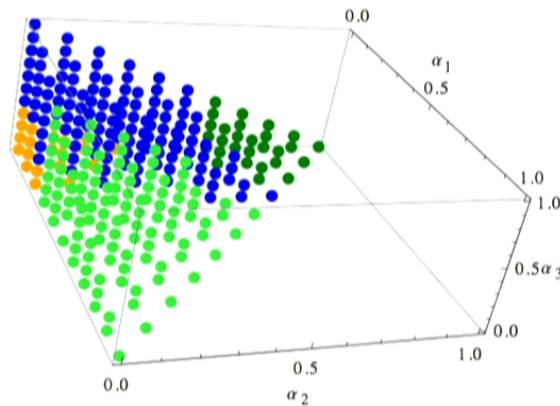
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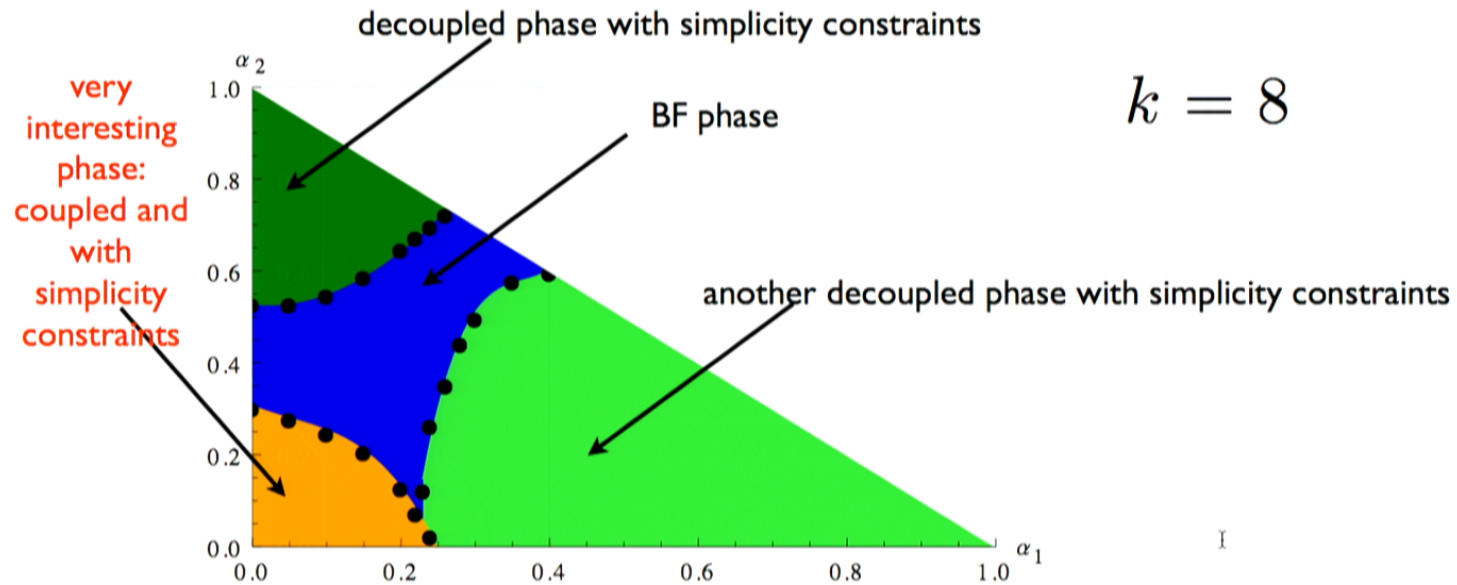
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Phase diagram for coupling

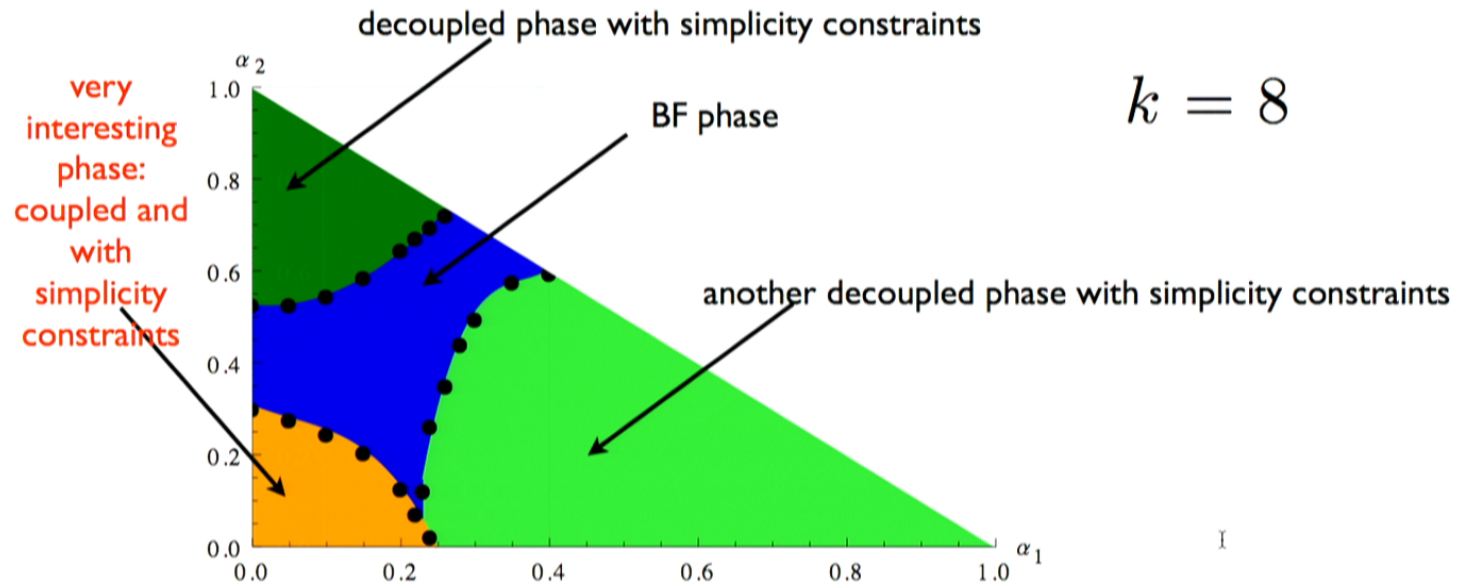


New feature: extended non-trivial phases with simplicity constraints.
 ([BD, Martin-Benito, Schnetter 13] required fine tuning.)

Due to change of parametrization inspired by Reisenberger's construction principle
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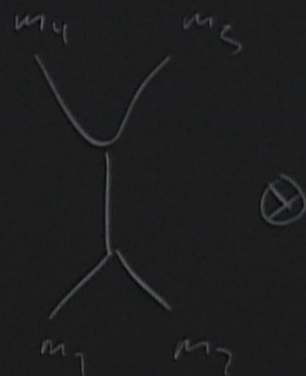
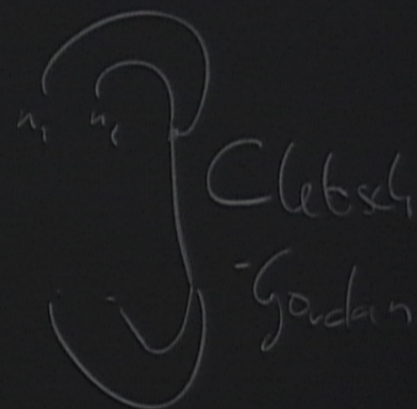


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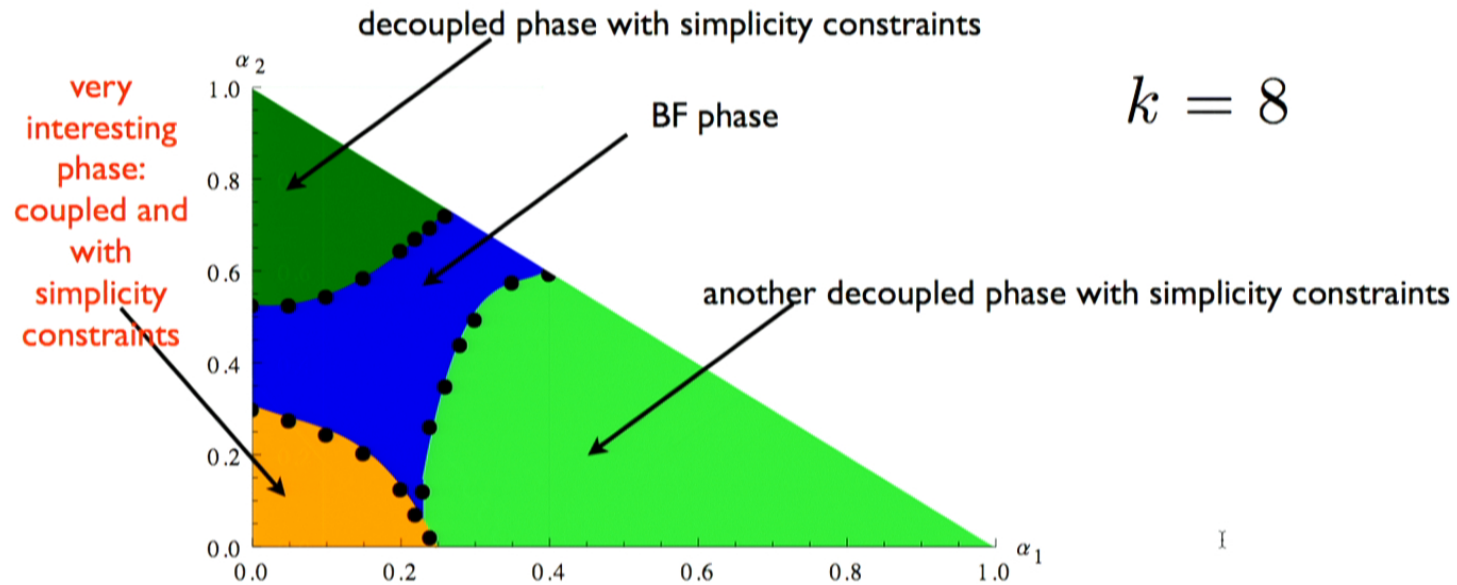
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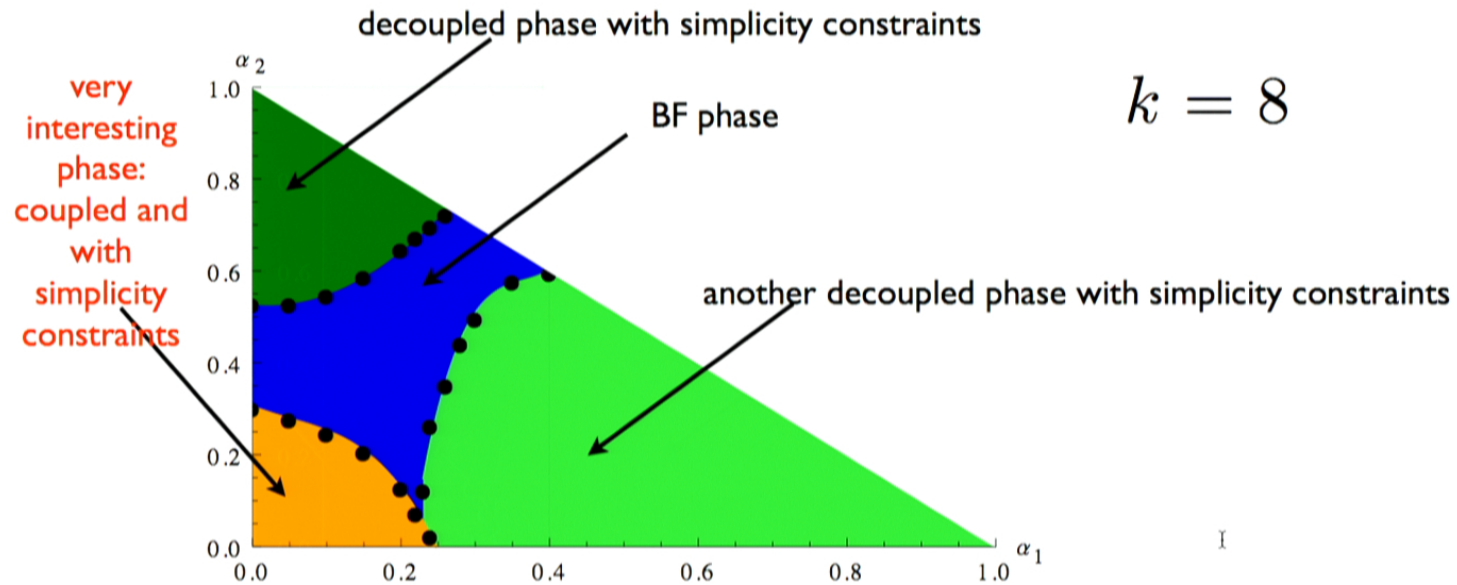


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Conclusion and outlook

- Continuum limit of spin foams / loop quantum gravity in reach.
- Understand already certain aspects of it: effective coupling between two spin foam vertices.
- Lots of relation to condensed matter:

Quantum space time built from space time atoms.

However 'atoms' should not be understood too naively.

Are not giving by auxiliary discretization but should emerge 'dynamically'.

(i.e. discrete spectra of geometric observables [Rovelli, Smolin, Ashtekar et al]

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Many things to do ...

- $SU(2)_k \times SU(2)_k$ models describe full gravity.

Classify fixed points / phases both in intertwiner and spin net models.
Can we formulate EPRL following Reisenberger's principle?

- Coarse graining involving (many) more spin foam vertices.
- Building 4D topological (spin foam) models beyond BF theory.
- Interpreting the flow of intertwiners.
- Interpreting the coarse grained degrees of freedom geometrically.
- Truncating more effectively.

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- Constructing the continuum limit for spin foams and loop quantum gravity ...
Improve the theory if necessary.

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