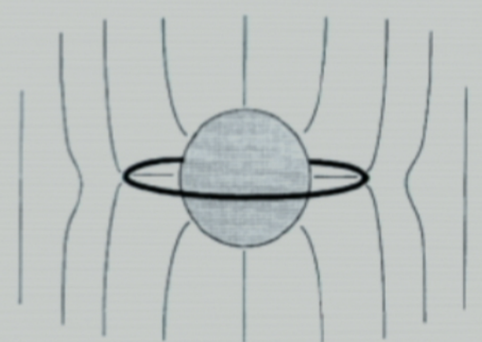
Title: O Topology

Date: Dec 05, 2013 08:35 AM

URL: http://pirsa.org/13120008

Abstract: Yes,
quite. But also with some applications.

O Topology



Terentjev, PRE **51** (1995) 1330

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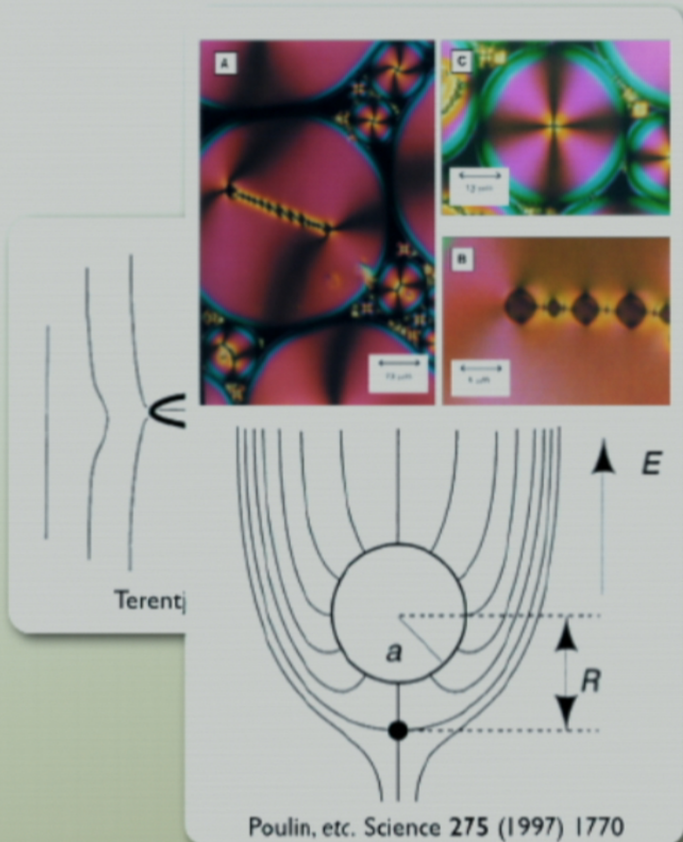
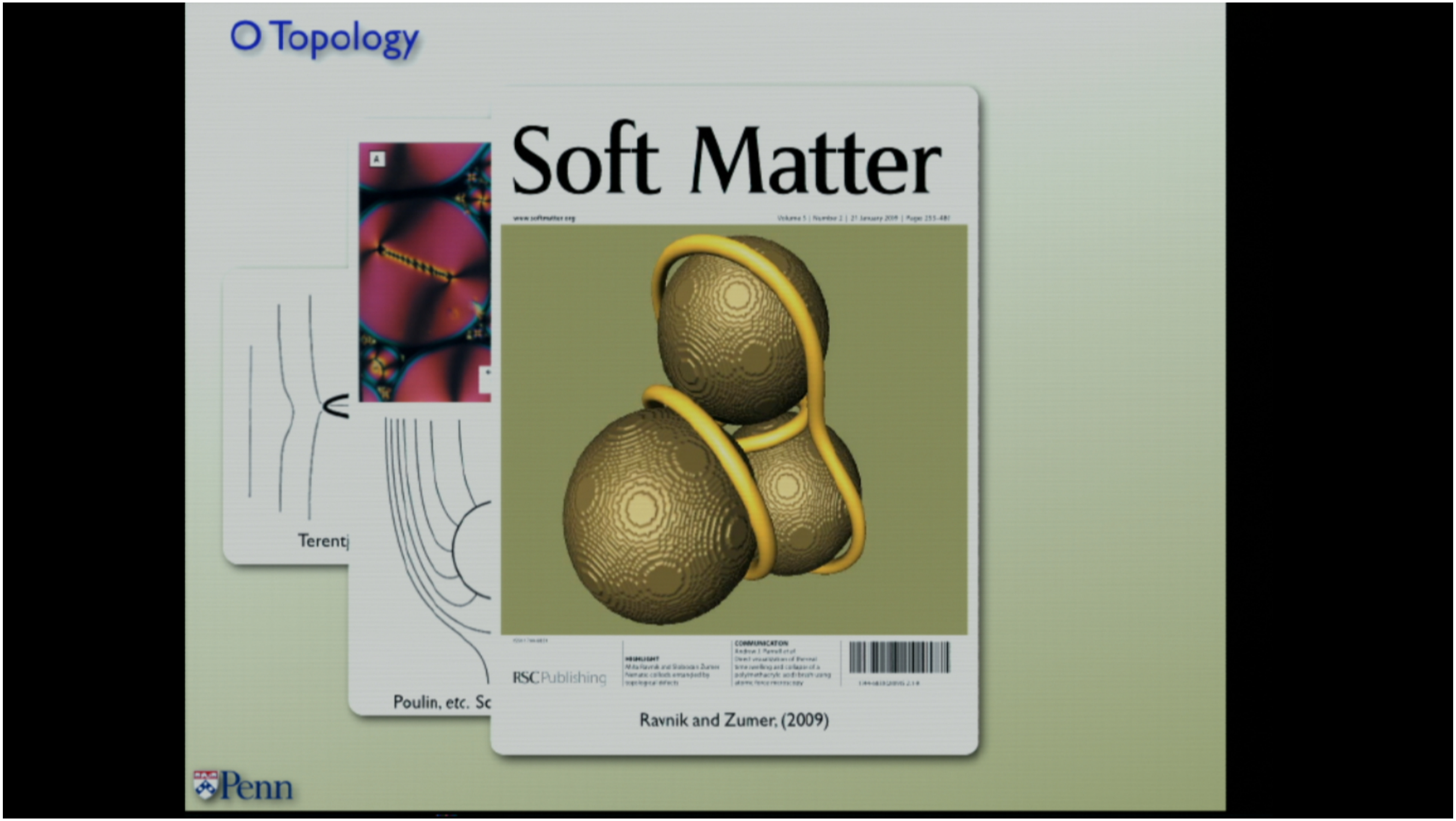
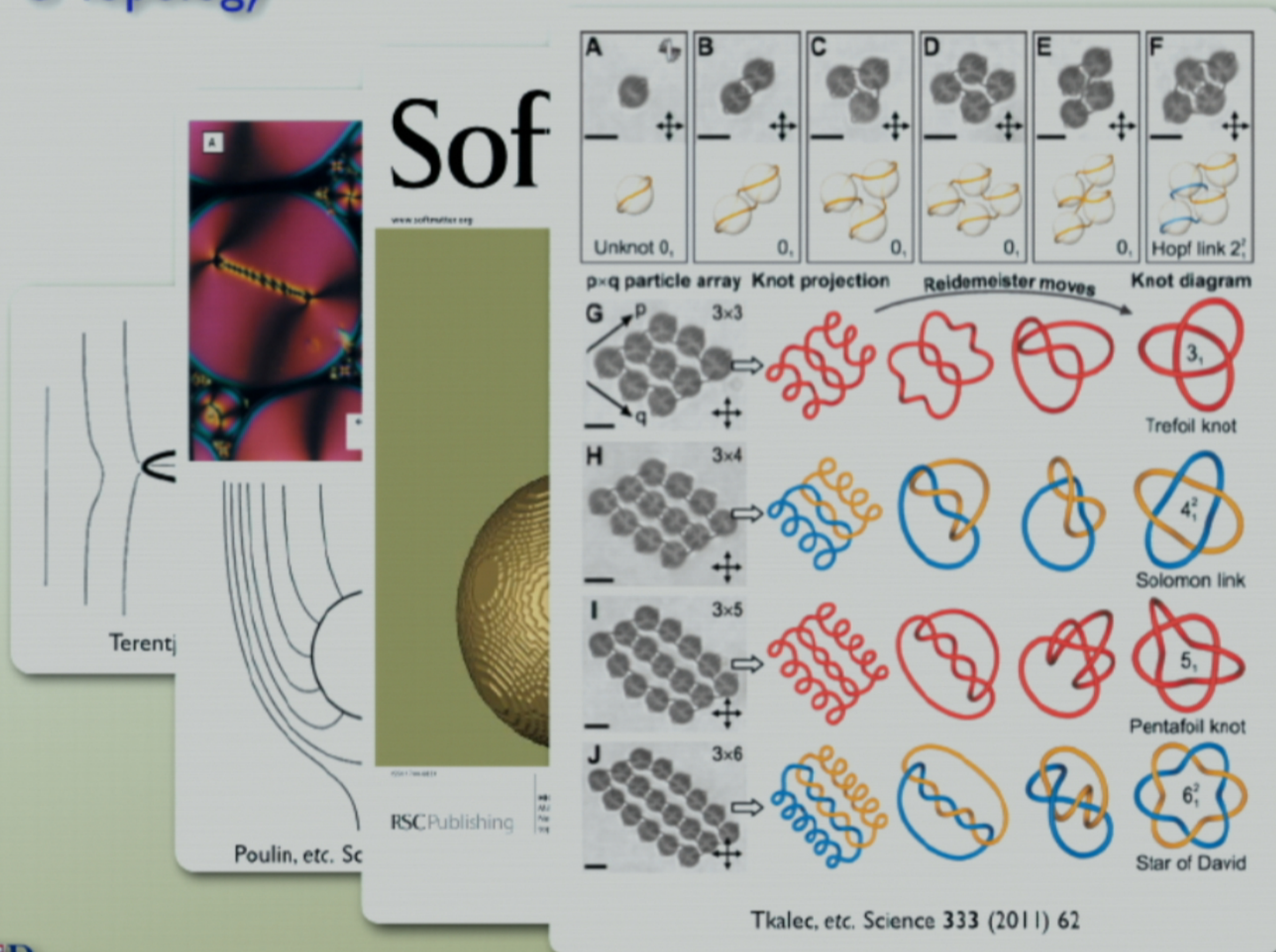
O Topology




Topology
Soft Matter
Terentj
Poulin, etc. Sc
RSC Publishing
Ravnik and Zumer. (2009)
Penn

O Topology



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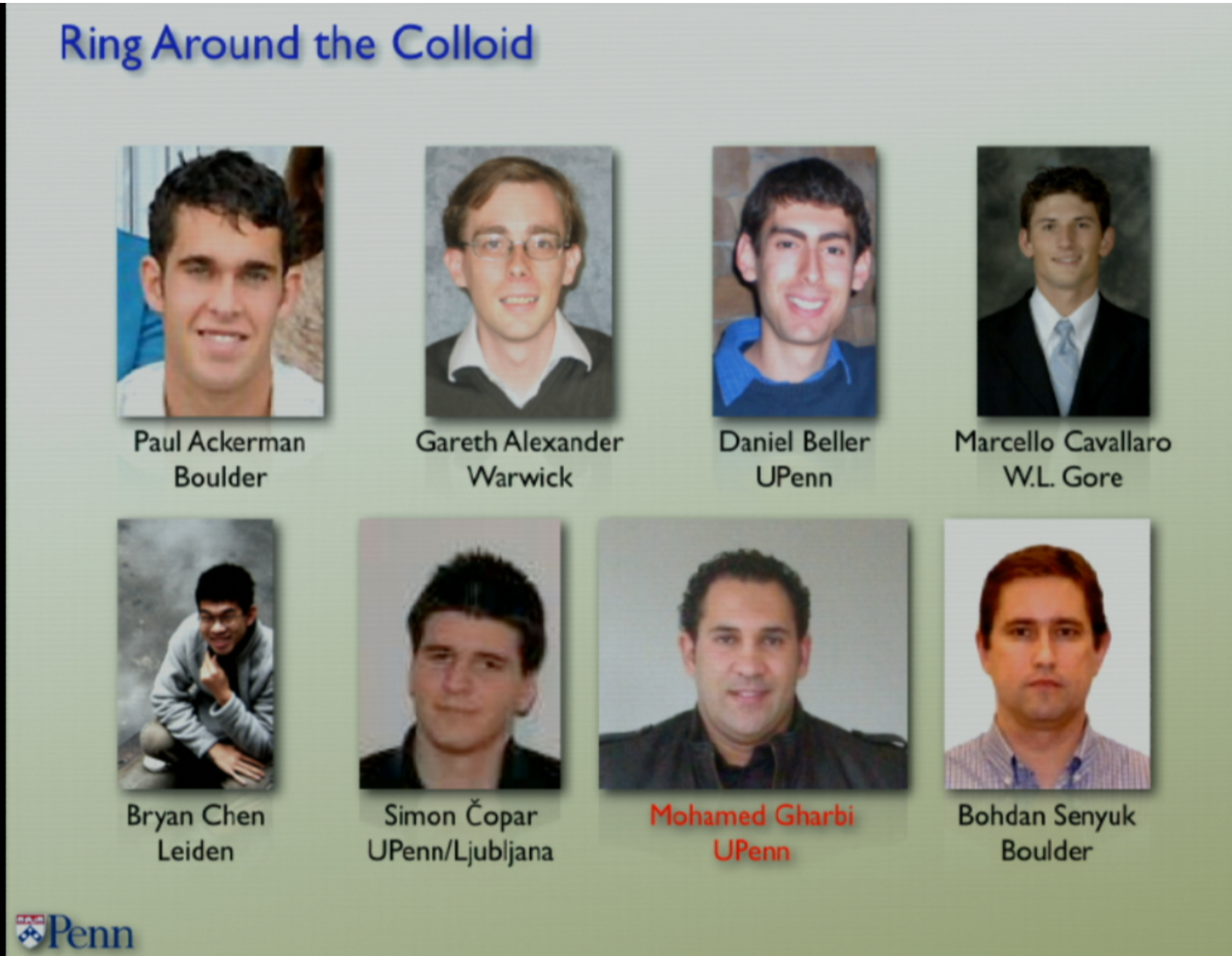
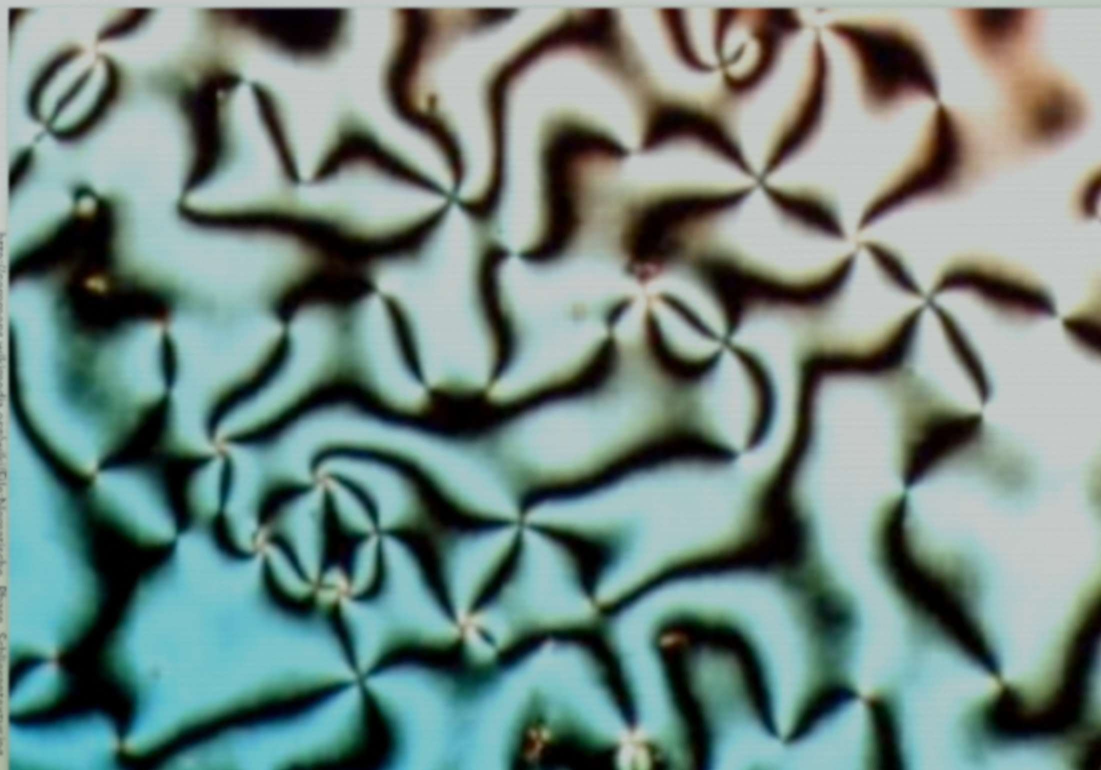

Ring Around the Colloid
Paul Ackerman
Boulder
Gareth Alexander
Warwick
Daniel Beller
UPenn
Marcello Cavallaro
W.L. Gore
Bryan Chen
Leiden
Simon Čopar
UPenn/Ljubljana
Mohamed Gharbi
UPenn
Bohdan Senyuk
Boulder
Penn

Nematics in Two Dimensions

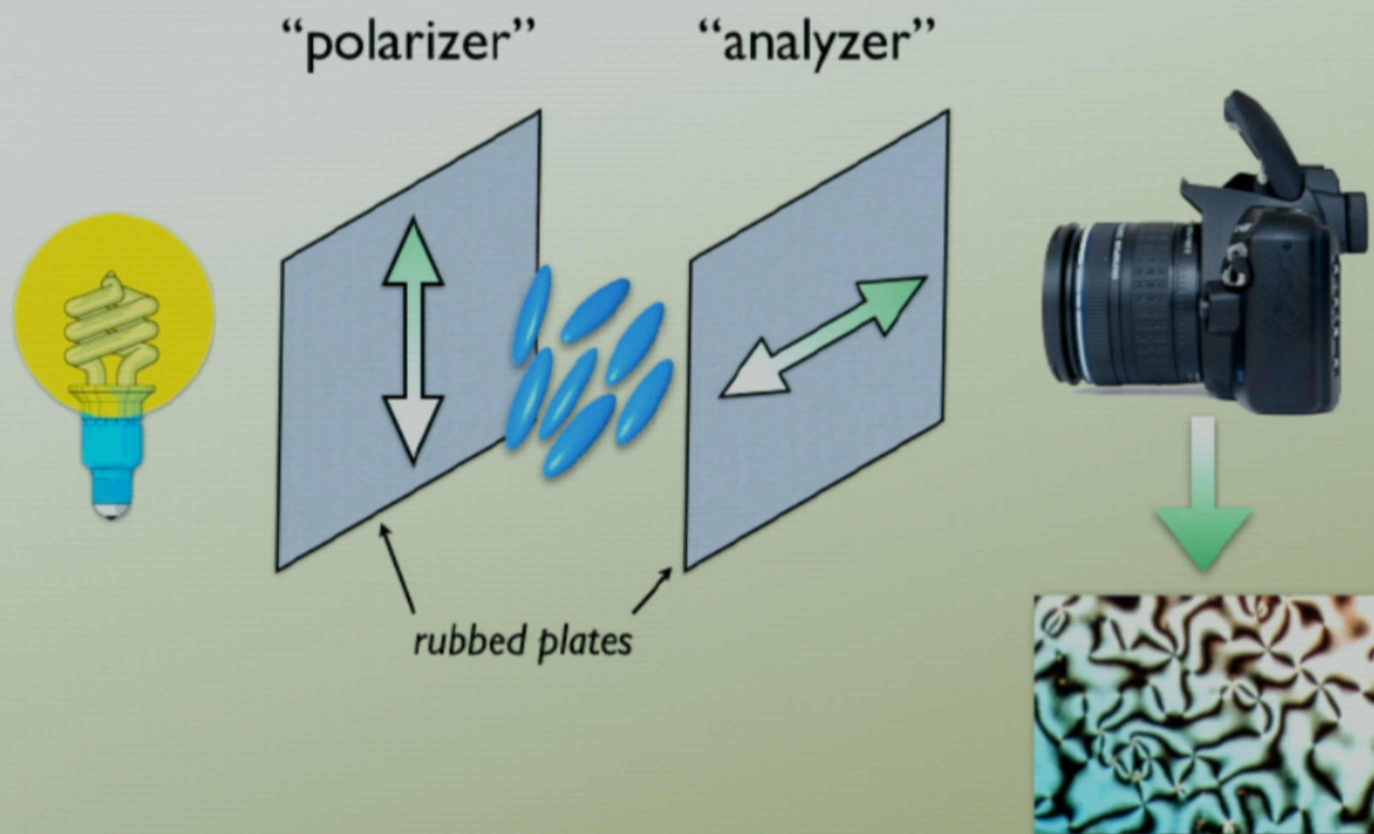


http://commons.wikimedia.org/wiki/File:Nematische_Phase_Schlierentextur.jpg

Maps from $\mathbb{R}^2\backslash\{0\} \to \mathbb{R}P^1$

Penn

Nematics in Two Dimensions: *What are we seeing?*



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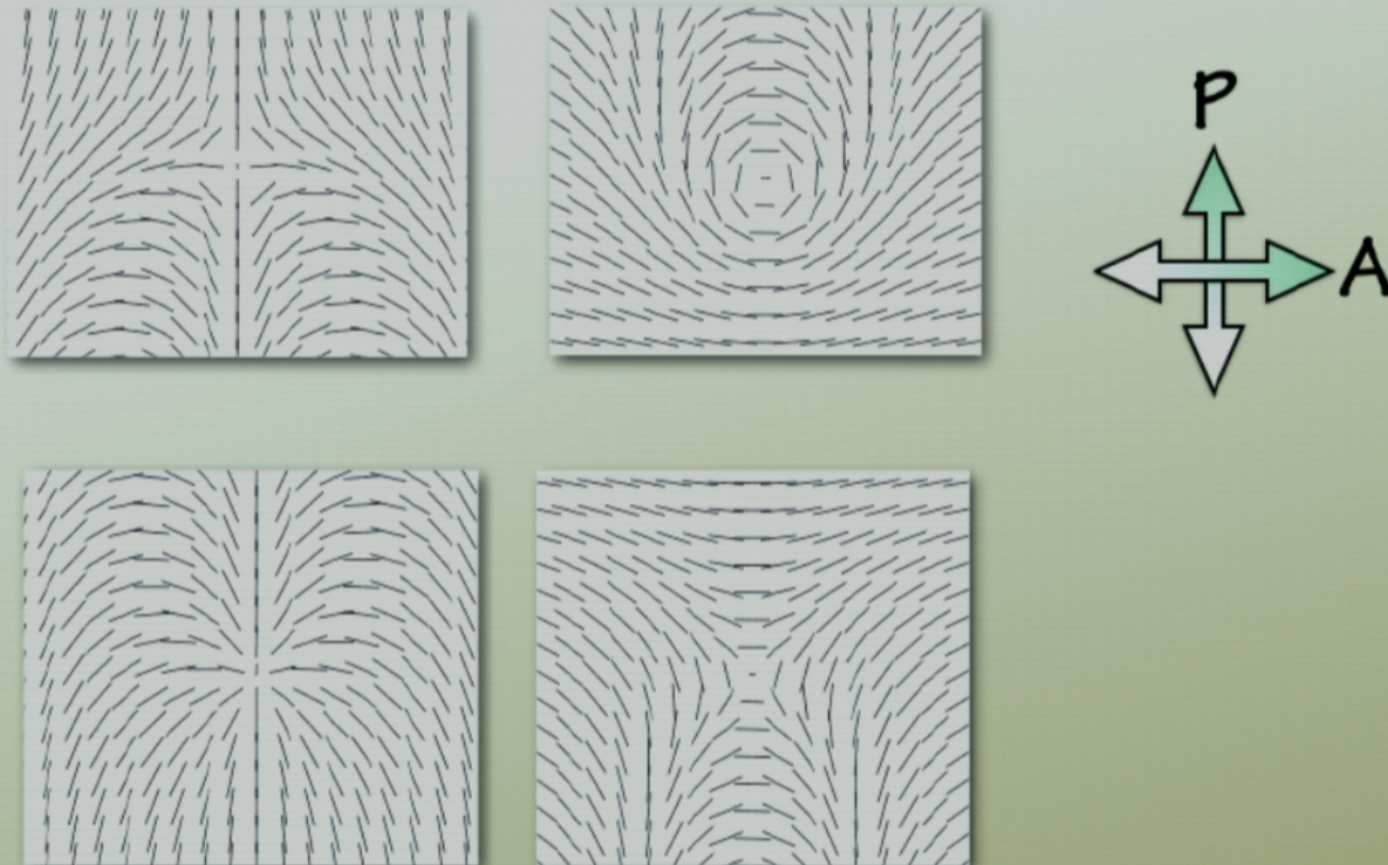
Nematics in Two Dimensions: *What are we seeing?*



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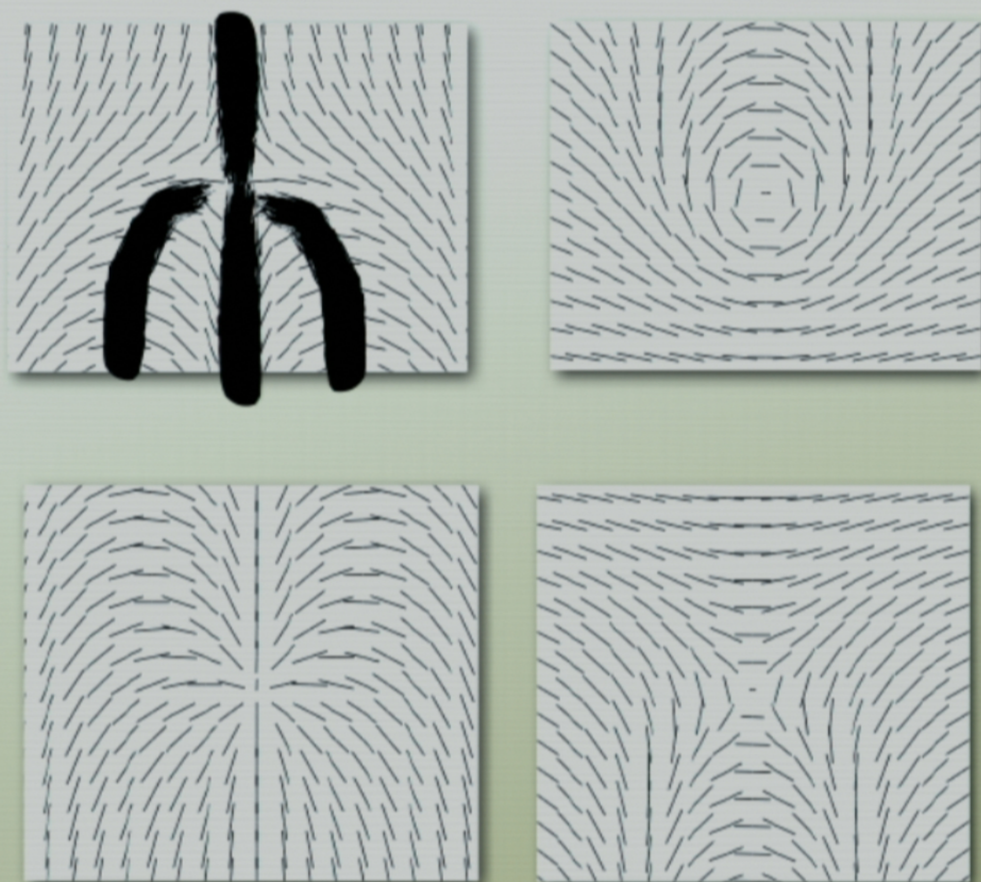
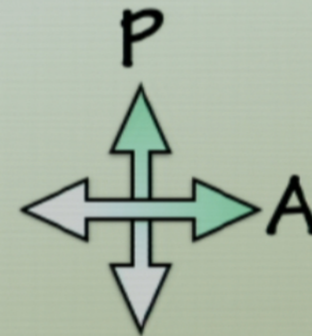
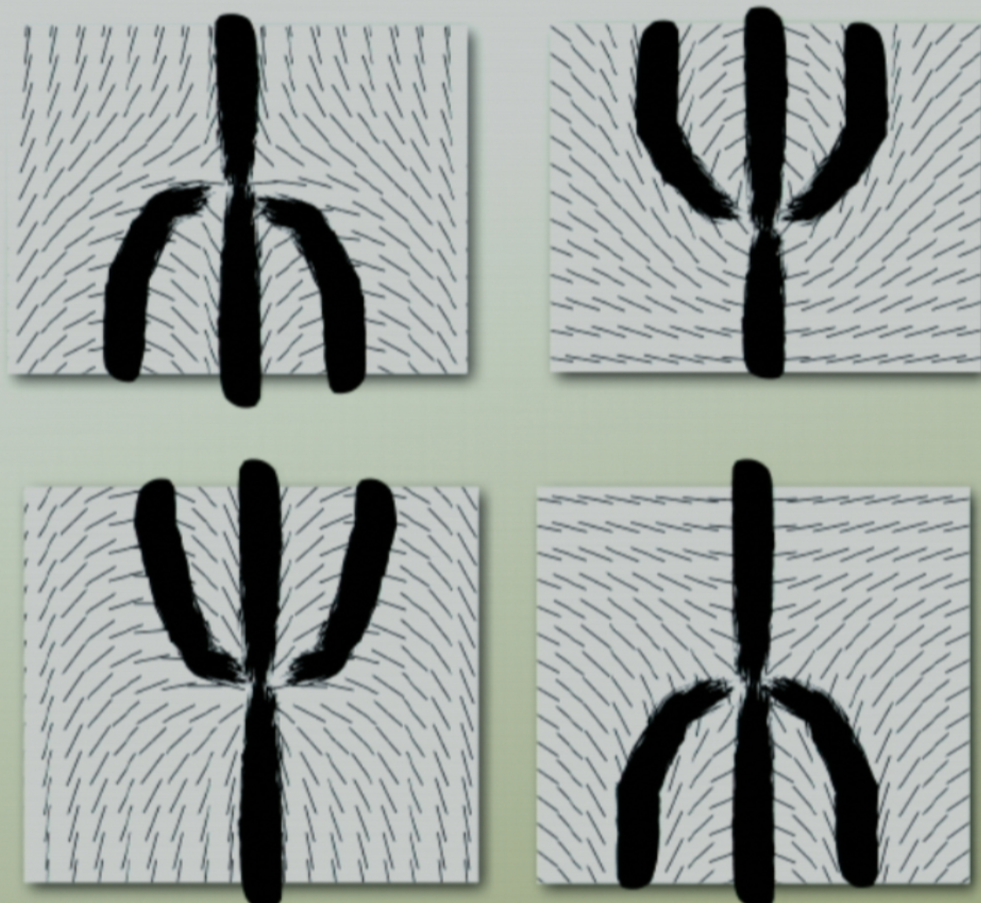
Nematics in Two Dimensions: *What are we seeing?*

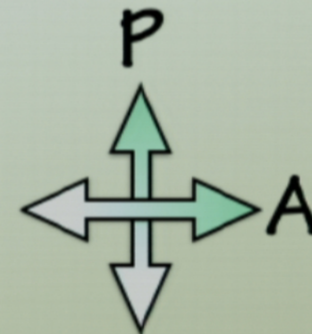




the brushes are the preimages of the polarizer and analyzer direction

Nematics in Two Dimensions: *What are we seeing?*





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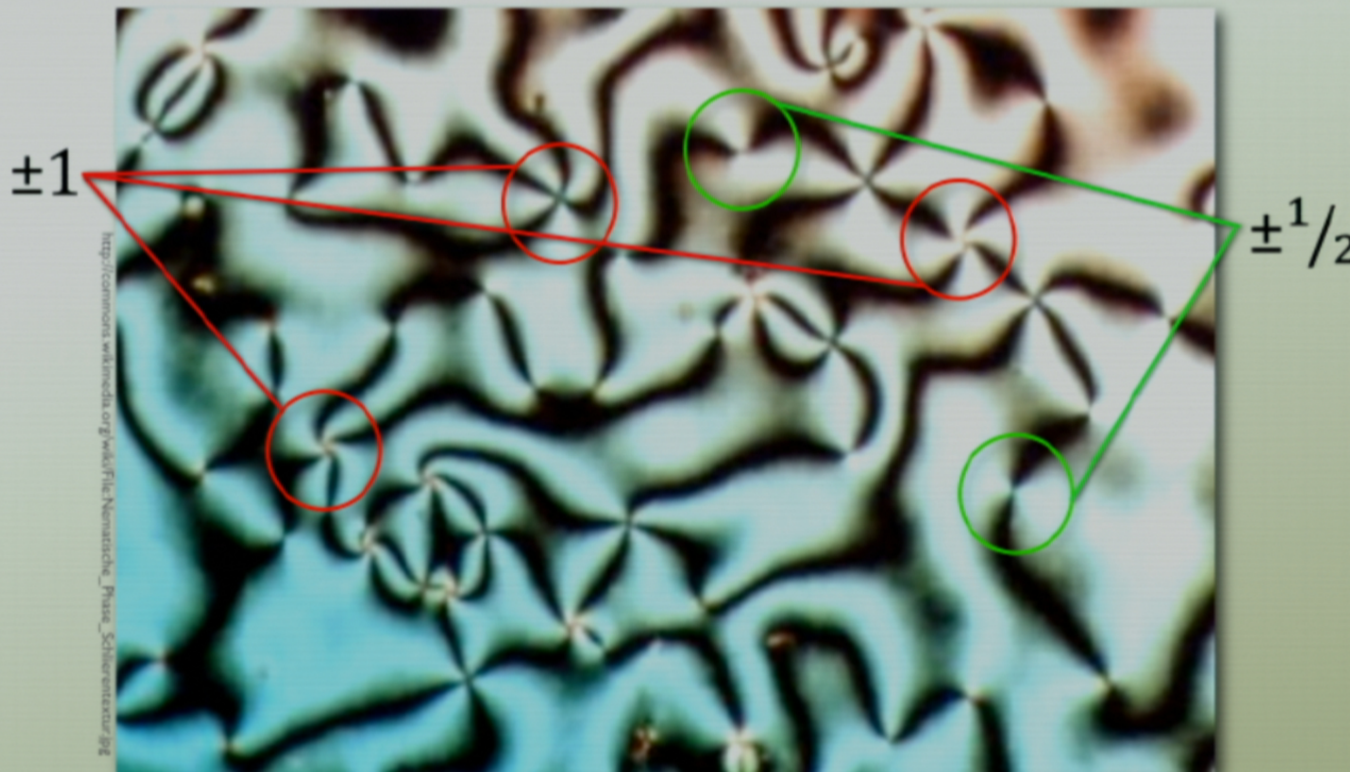
Penn

Nematics in Two Dimensions



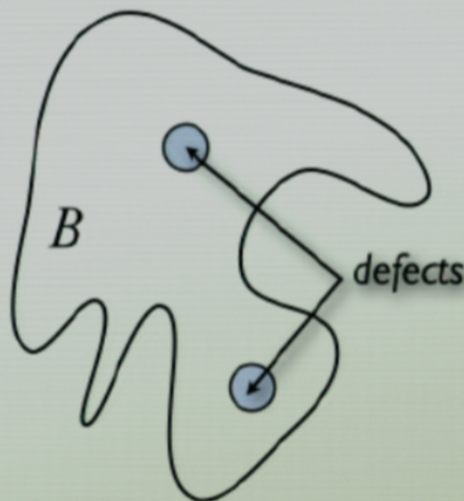
http://commons.wikimedia.org/wiki/File:Nematische_Phase_Schlierentextur.jpg

Maps from $\mathbb{R}^2 \backslash \{0\} \to \mathbb{R}P^1$

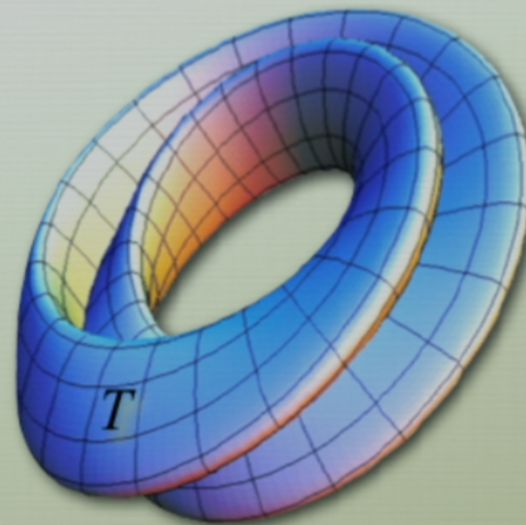
Penn

Defects and Homotopy: Quick Review

Sample



Ground State Manifold

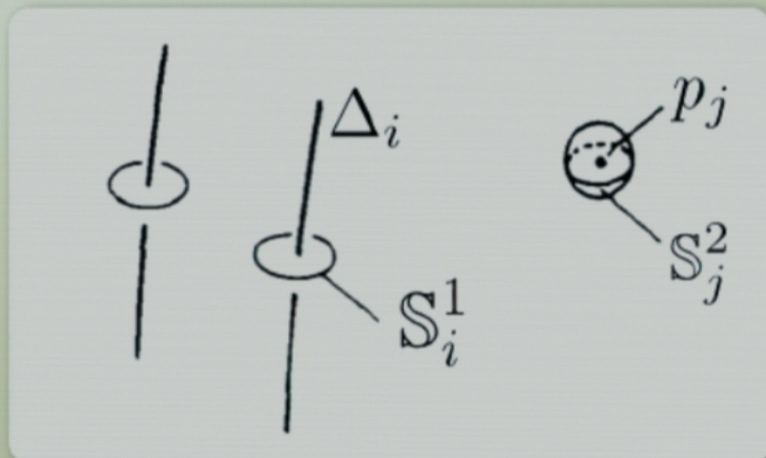


Seminal Paper on Which Our Knowledge is Based

Acta Applicandae Mathematicae **8** (1987), 65–74

Topological Properties of Ordinary Nematics in 3-space

KLAUS JÄNICH
Universität Regensburg, Fakultät für Mathematik, Universitätsstrasse 31, D-8400 Regensburg, West Germany

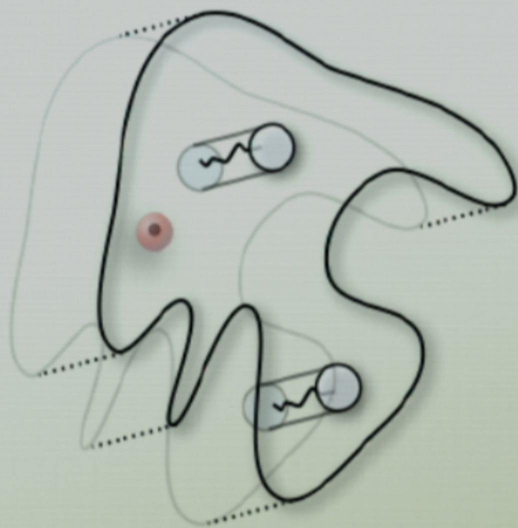


Defect Lines and Defect Points ⇔ *Different Generalizations*

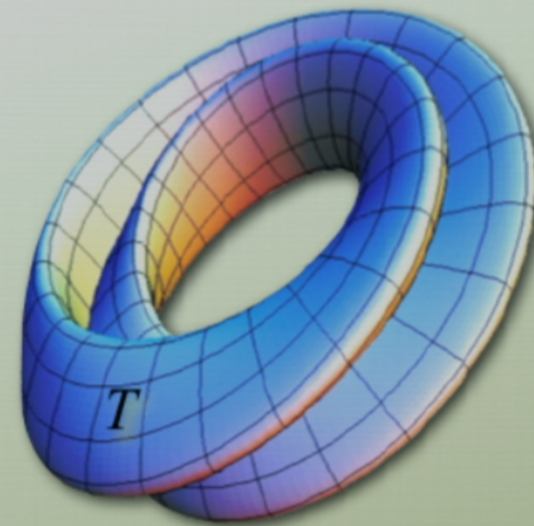
Penn

Second Homotopy Group

Sample



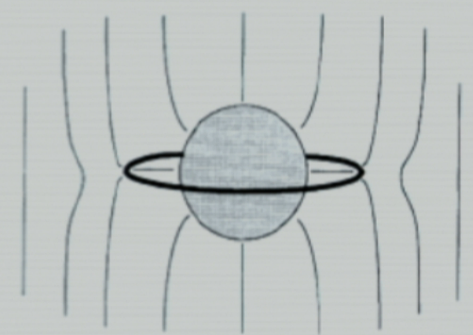
Ground State Manifold



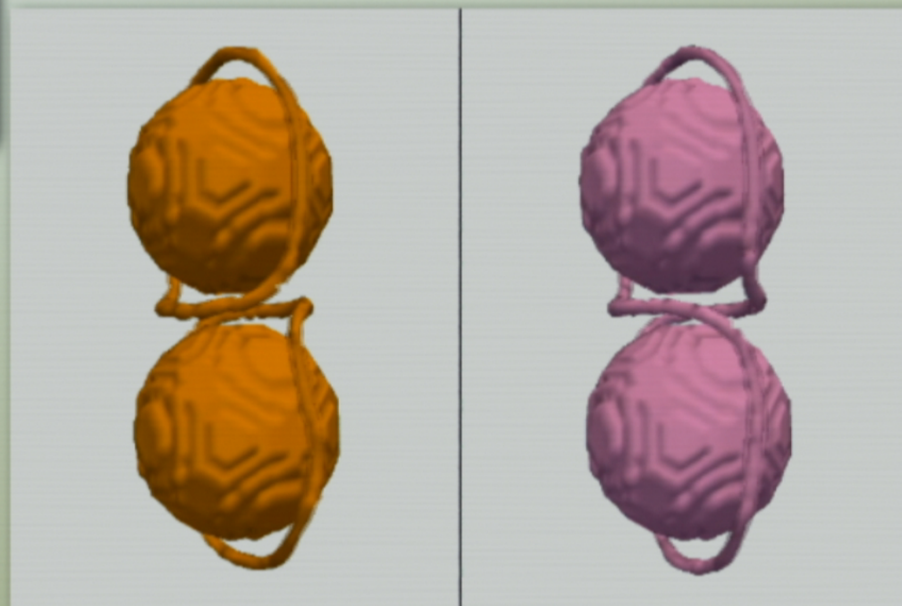
Maps from $\pi_1(B) \rightarrow \pi_1(T)$

Maps from $\pi_2(B) \rightarrow \pi_2(T)$

First Homotopy Group - Line Defects

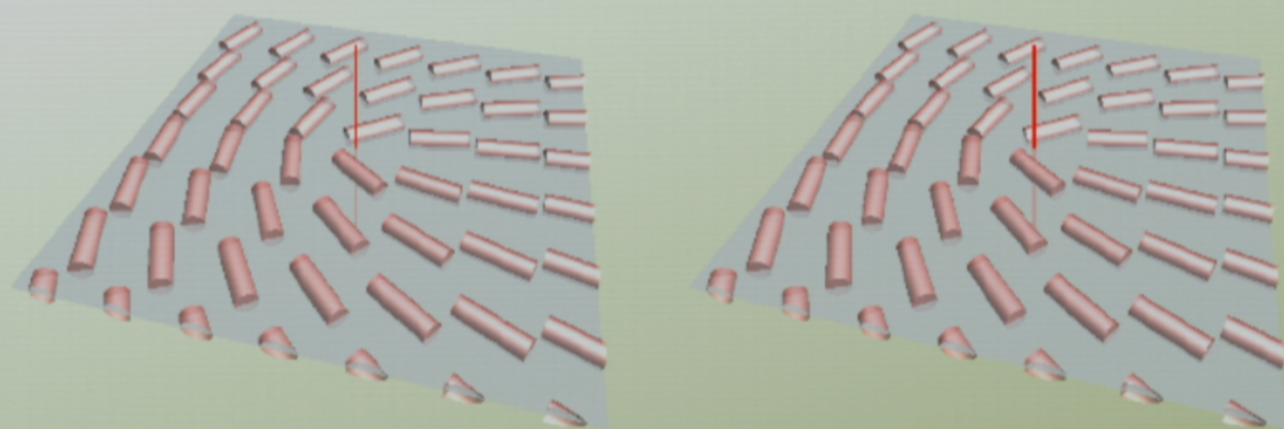


Terentjev, PRE **51** (1995) 1330



How Do Lines Compensate Points?

What Does The Field Look Like?



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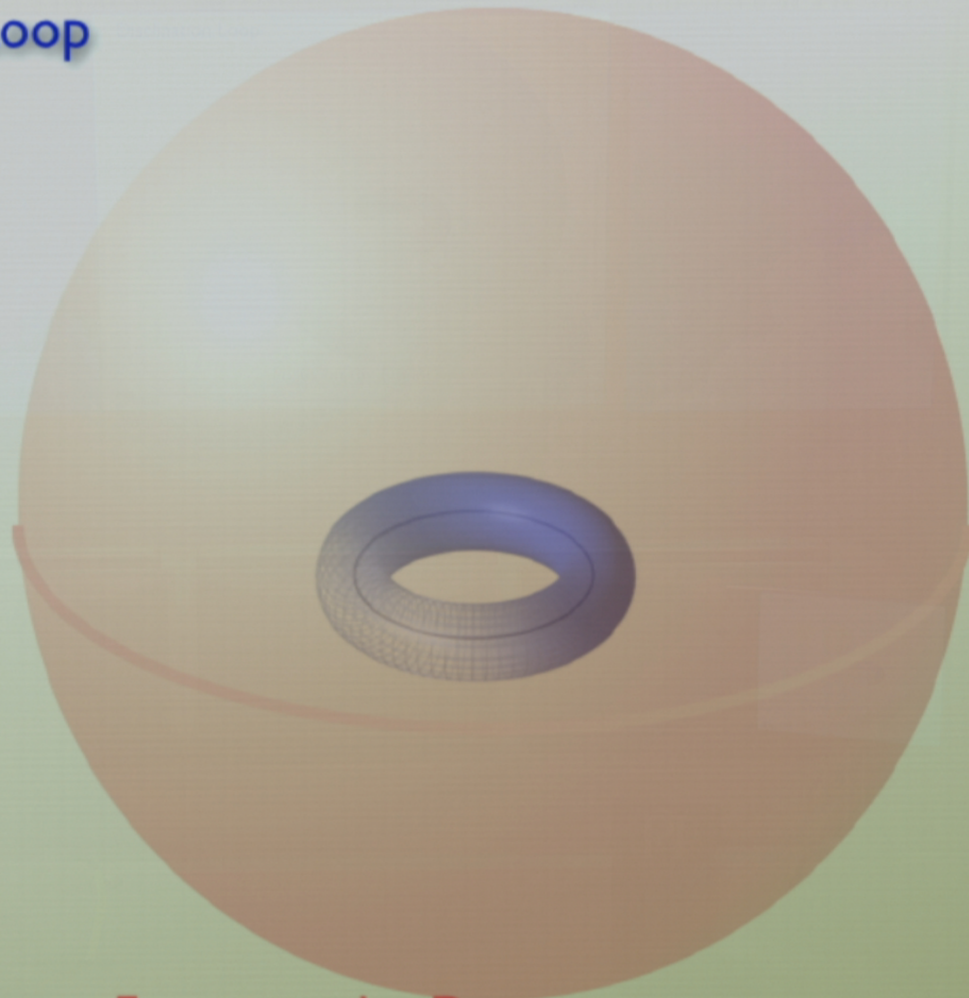
Disclination Loop

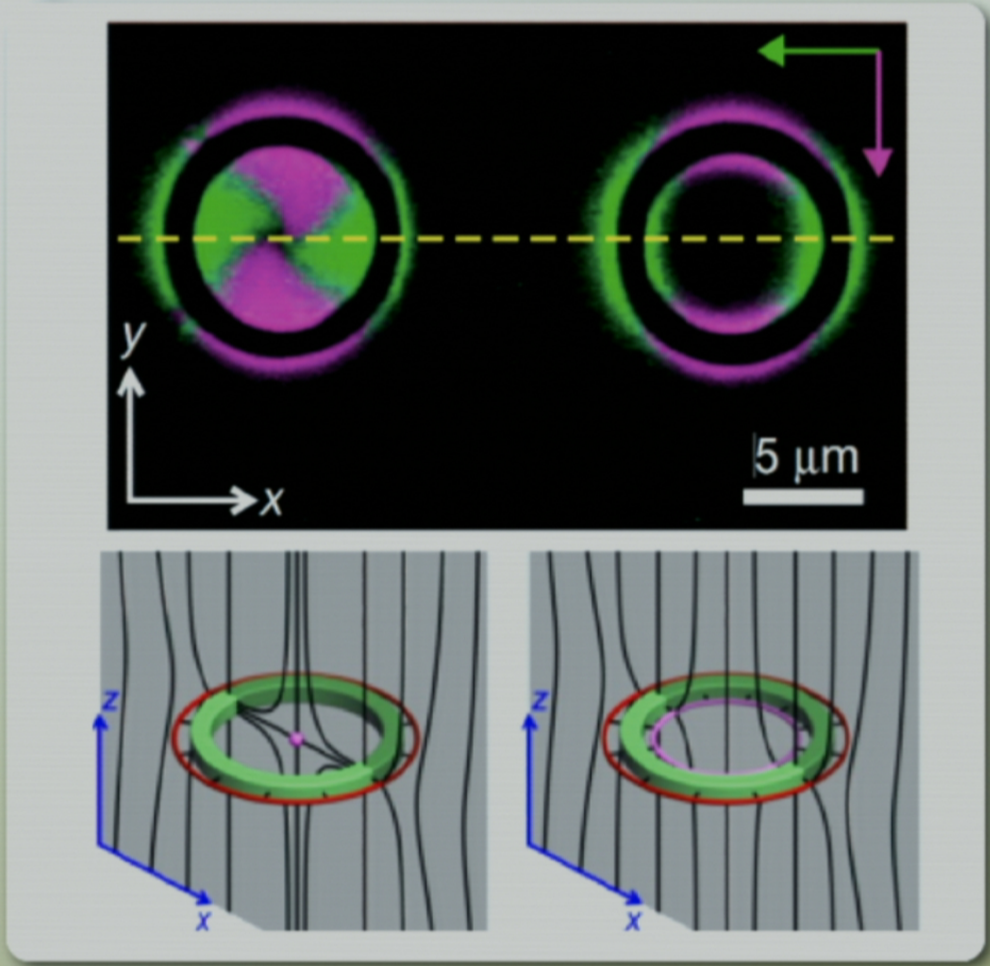


http://cis.jhu.edu/education/introPatternTheory/chapters/lie/lie1.html

Focus on the Donut

Penn

Topological Colloids



B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* ***493*** *(2013) 205.*

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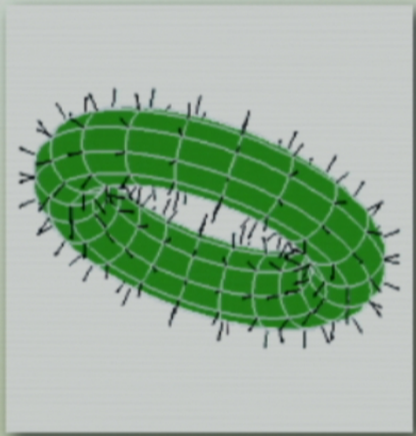
What is the Charge?

$$q = \frac{1}{4\pi}\int dxdy\, \mathbf{n}\cdot[\partial_x\mathbf{n}\times\partial_y\mathbf{n}]$$



http://www.supermath.info/ZooOfMathematicalCreatures.html

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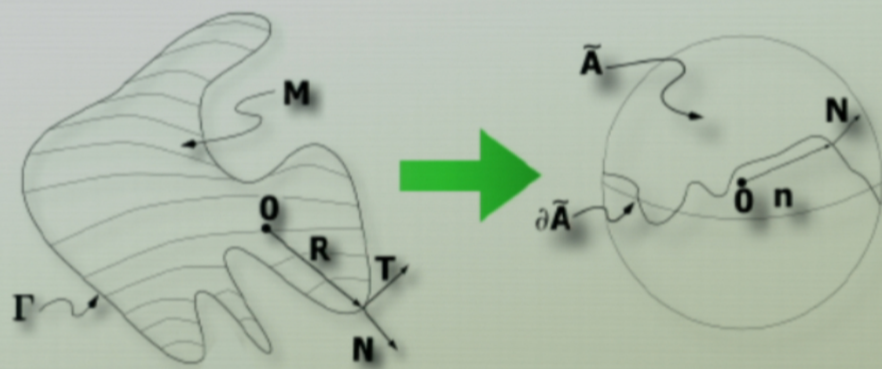

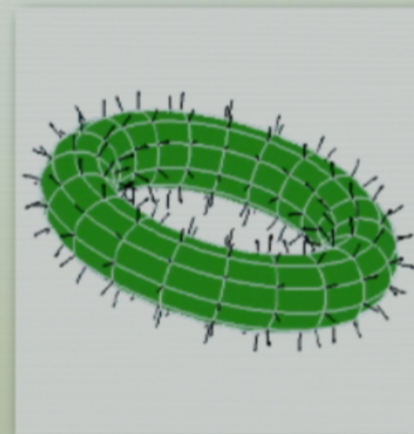

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Gauss-Bonnet Theorem!

$$\tilde{A} = \int dxdy\, \mathbf{N}\cdot[\partial_x \mathbf{N} \times \partial_y \mathbf{N}] = 4\pi(1-g)$$

$$\boxed{q = \frac{1}{4\pi}4\pi(1-g) = 1-g}$$

Penn

B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* ***493*** *(2013) 205.*

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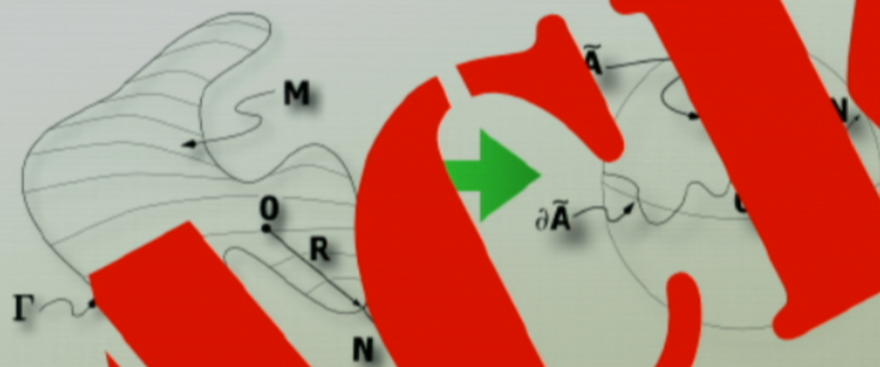
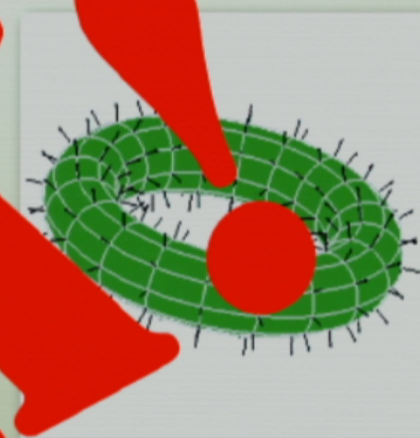
ACK!





Gauss-Bonnet theorem:

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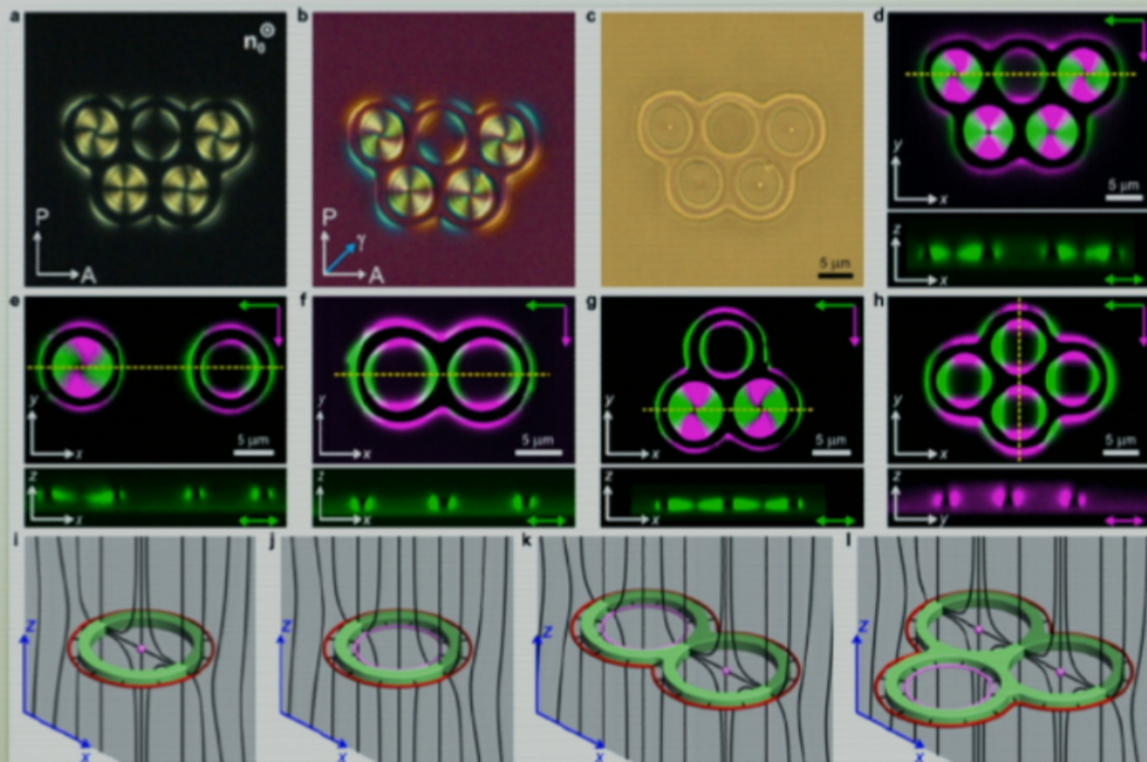
Penn

B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* ***493*** *(2013) 205.*

Topological Colloids With Topology!



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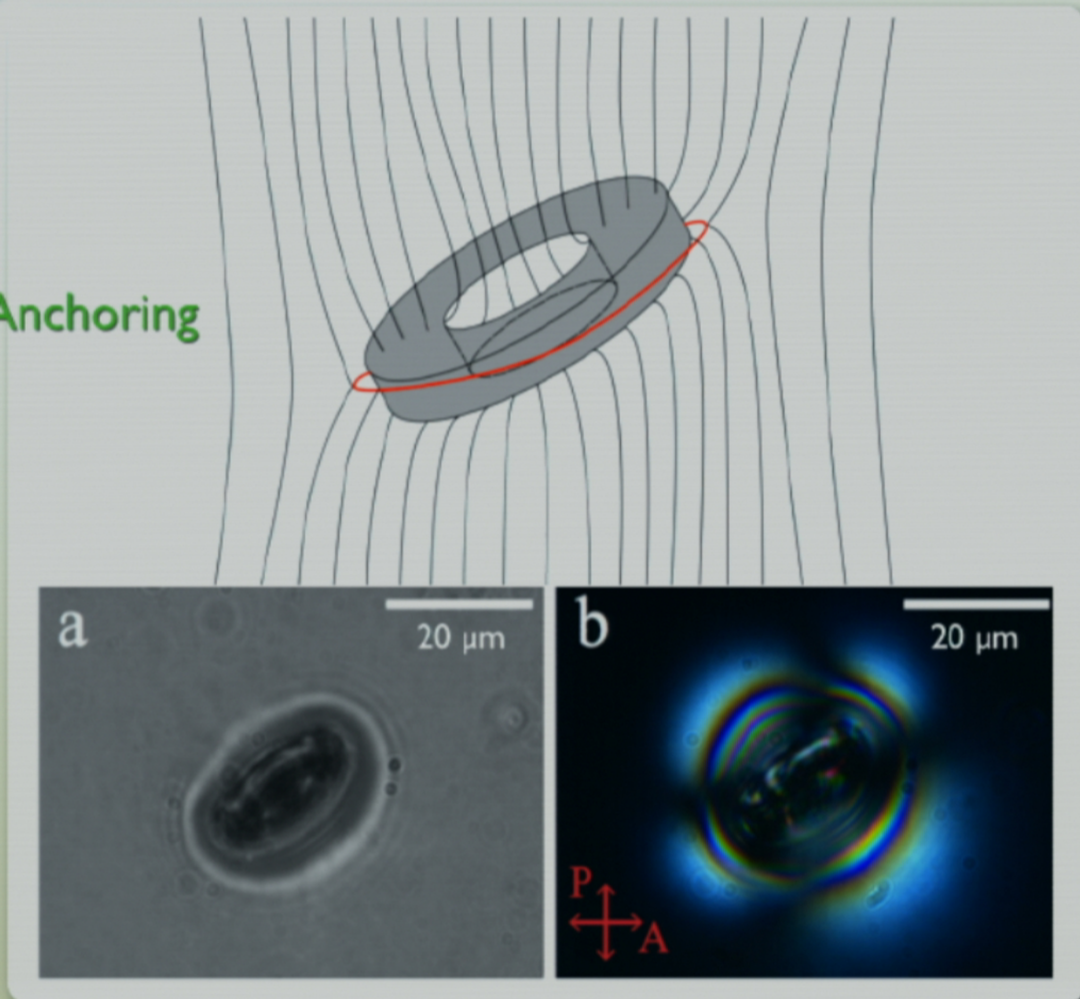
B. Senyuk, Q. Liu, S. He, RDK, R.B. Kusner, T.C. Lubensky, and I.I. Smalyukh, *Nature* **493** (2013) 205.

Charged Tori!

Hybrid Anchoring

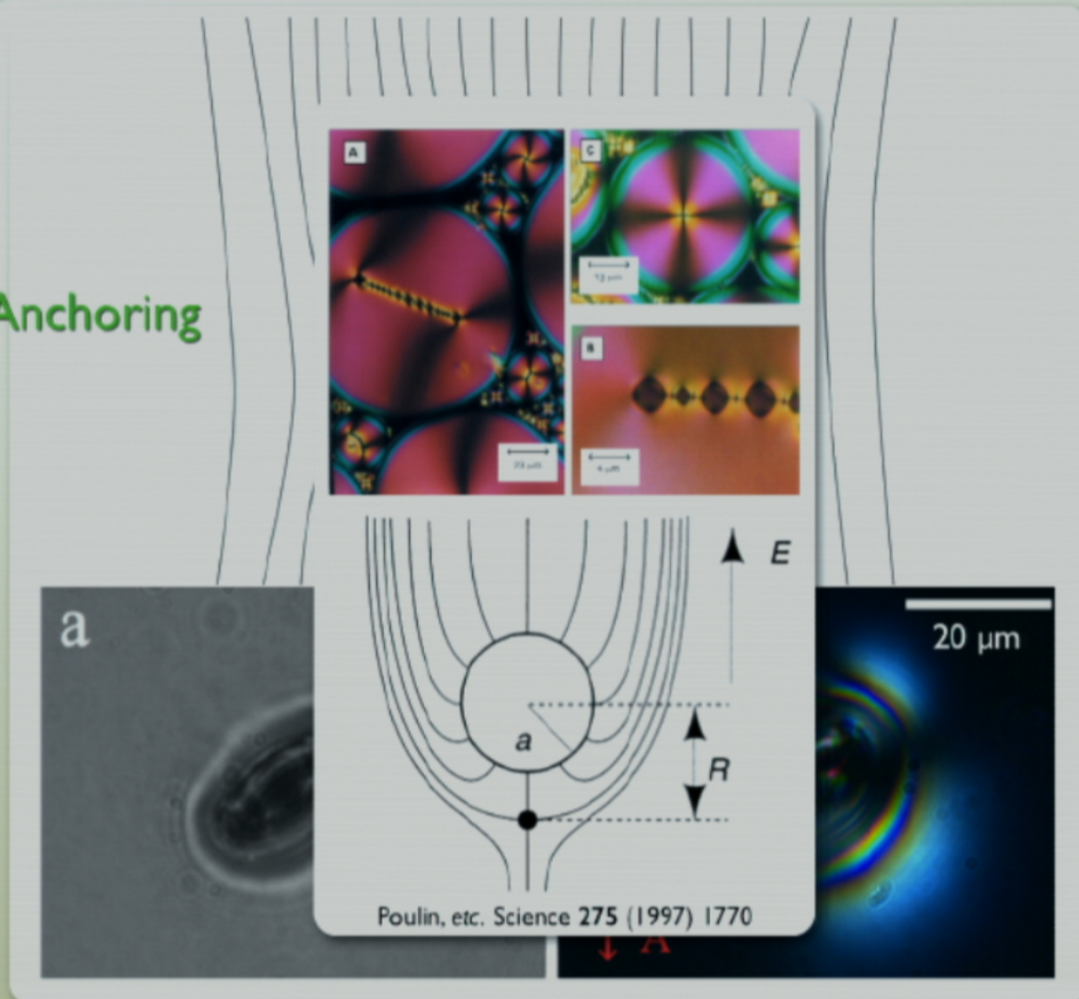


UPENN MRSEC, Cavallaro *et al. to appear in Soft Matter* (2013)

Penn

Charged Tori!

Hybrid Anchoring



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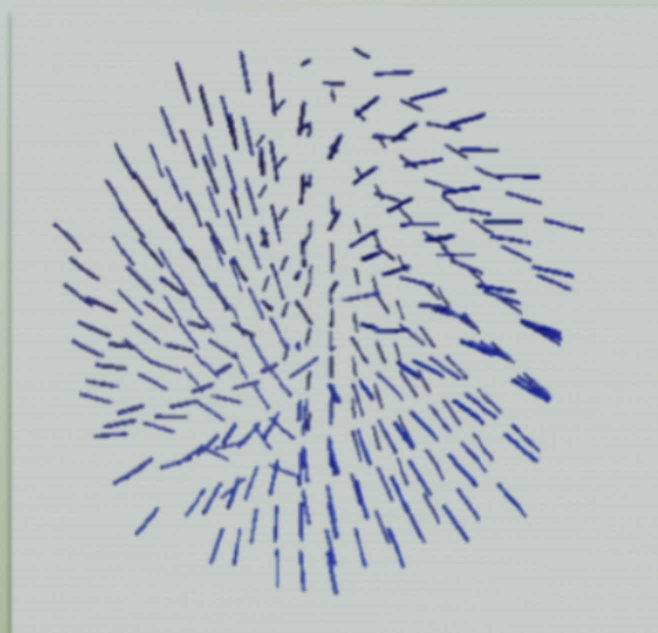
UPENN MRSEC, Cavallaro *et al. to appear in Soft Matter* (2013)

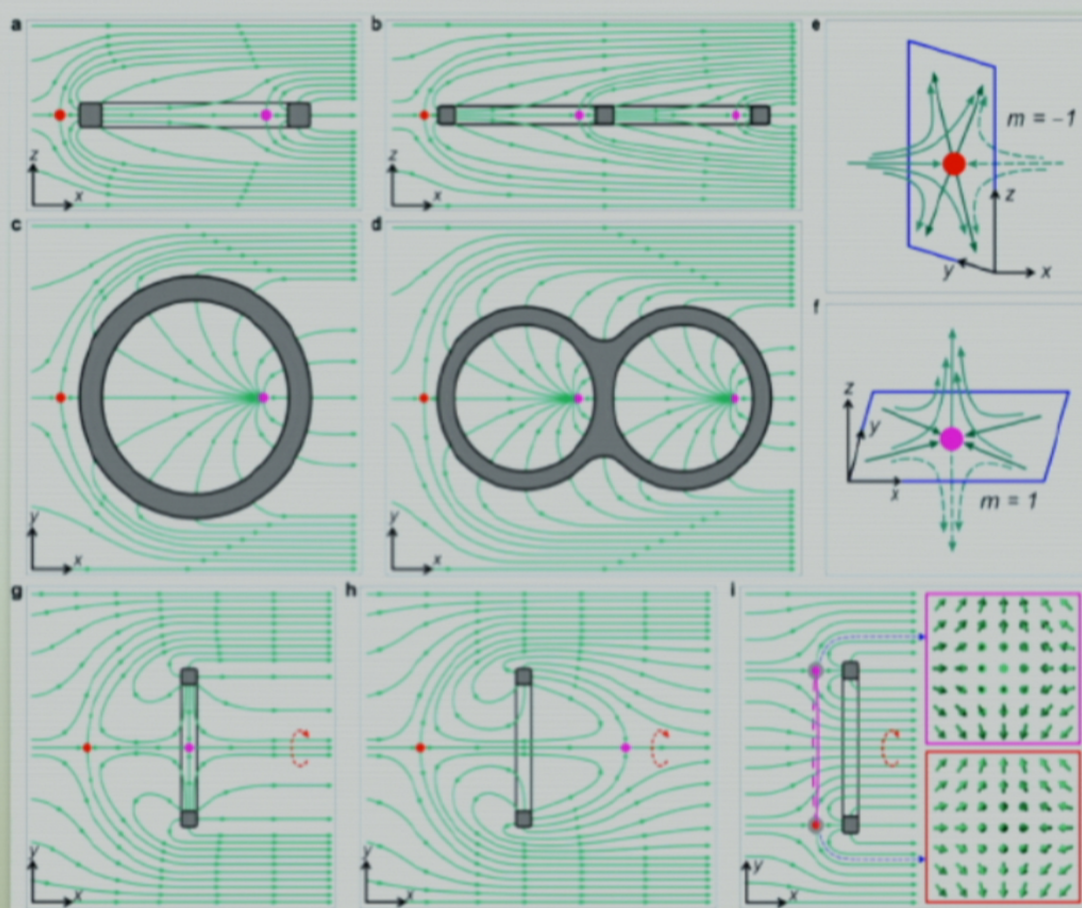
Yuck



Is there a better way to see all this data?

Penn

Yuck



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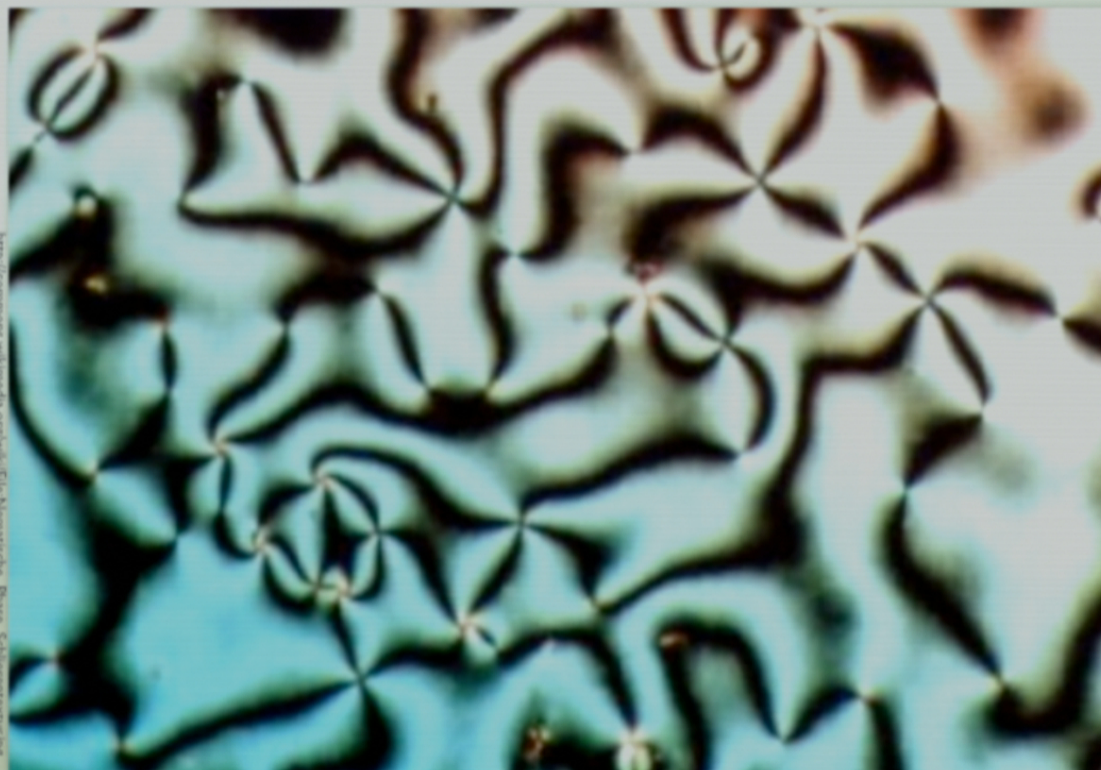
Penn

Nematics in Two Dimensions



http://commons.wikimedia.org/wiki/File:Nematische_Phase_Schlierentextur.jpg

Maps from $\mathbb{R}^2 \backslash \{0\} \to \mathbb{R}P^1$

Penn

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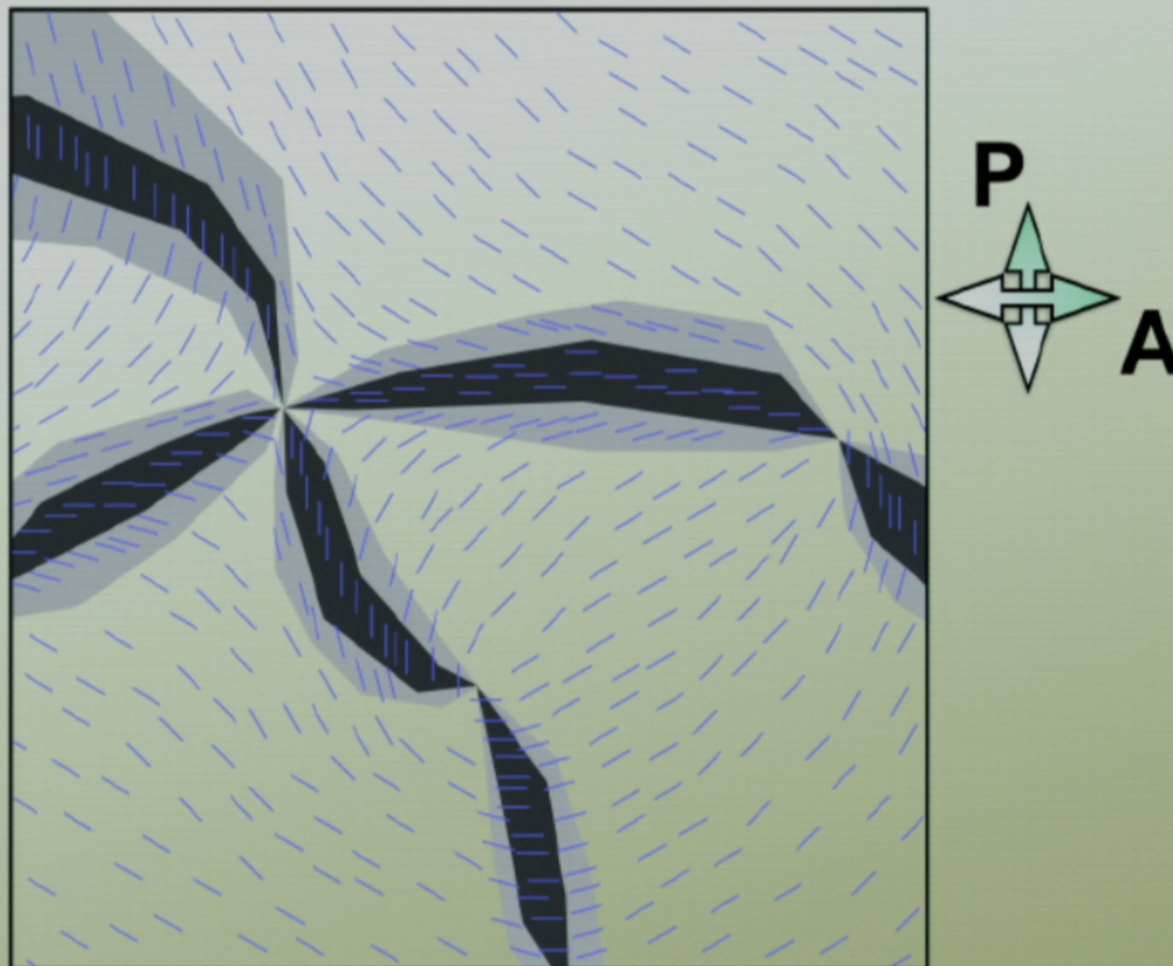
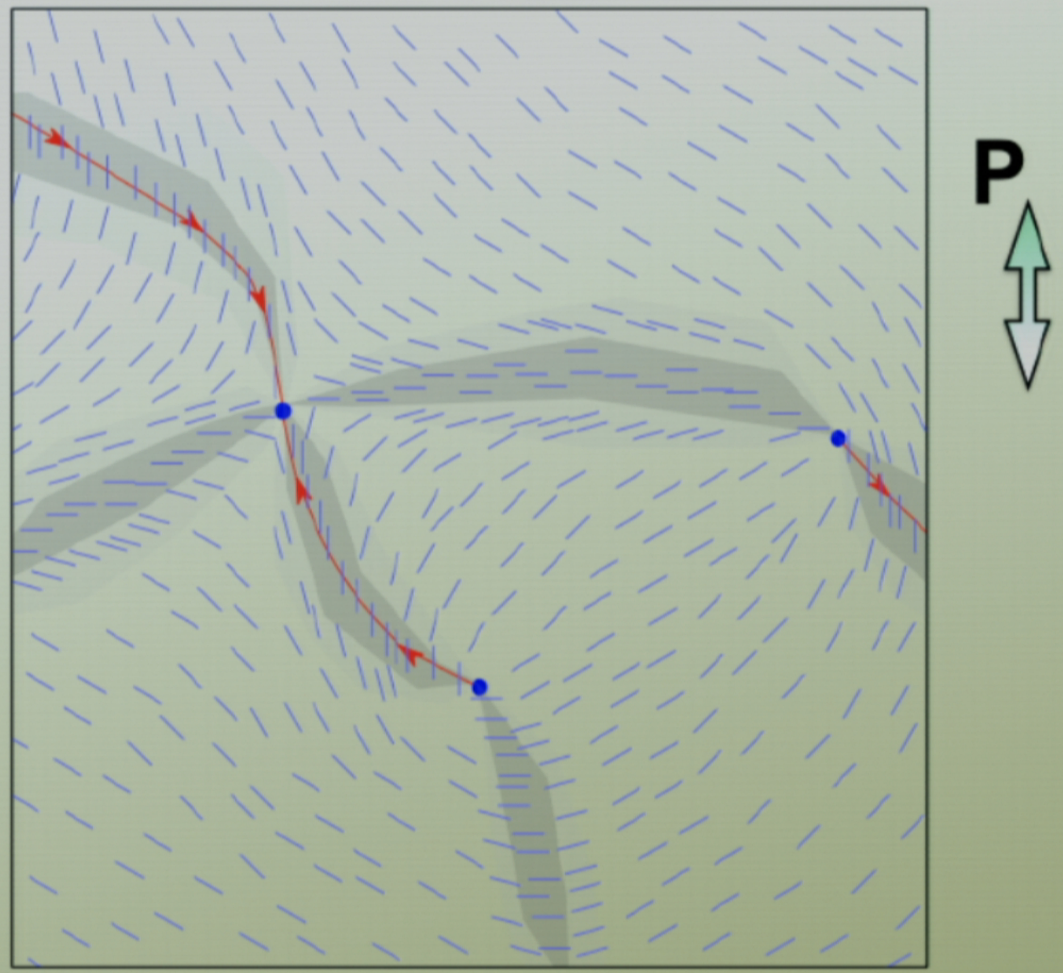


Nematics in Two Dimensions

Arrows point from + to –

encode charges



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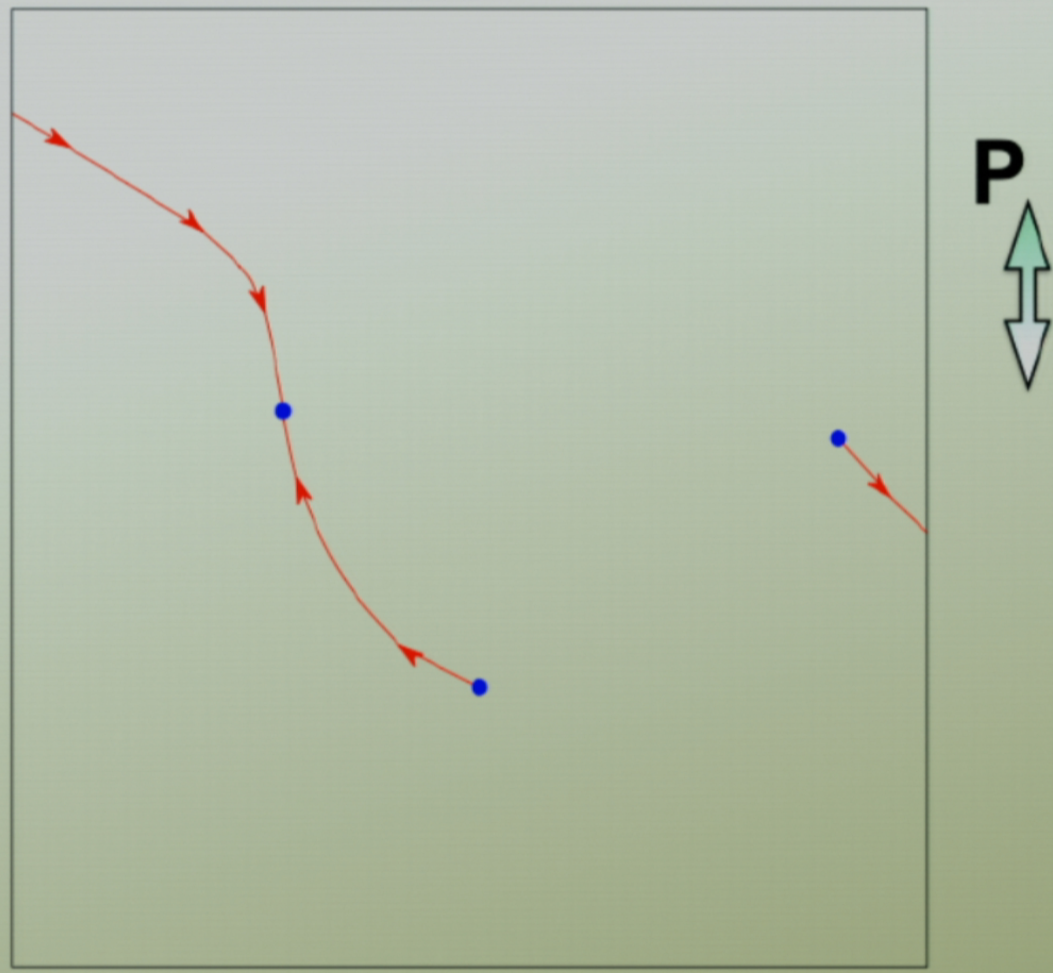

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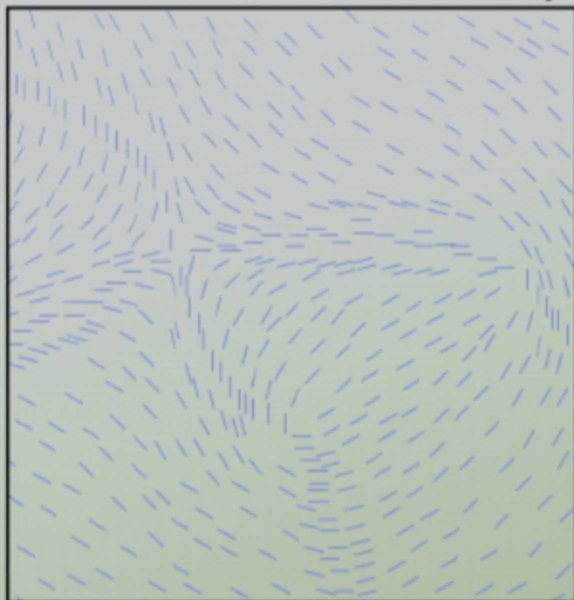
Nematics in Two Dimensions

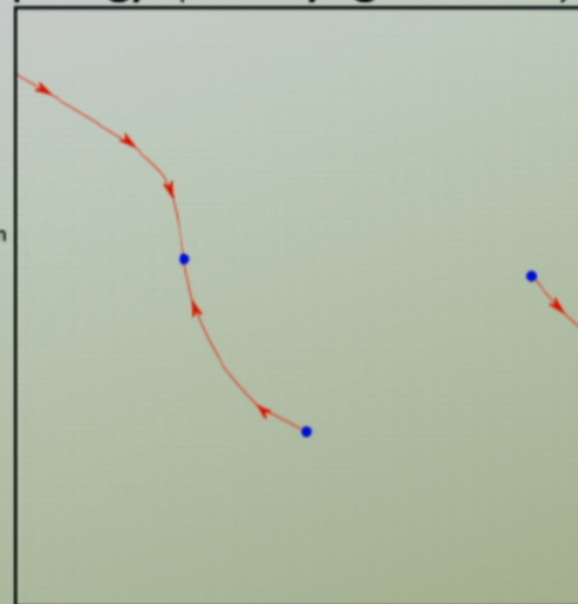
These directed lines carry ***all*** of the topology (Pontryagin-Thom)



choose a special orientation

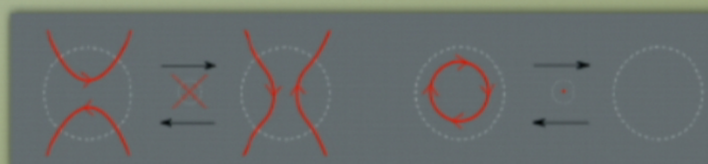
1 to 1

fill in neighborhoods



2D nematic director fields
up to smooth deformations

pictures of directed lines
up to switching moves



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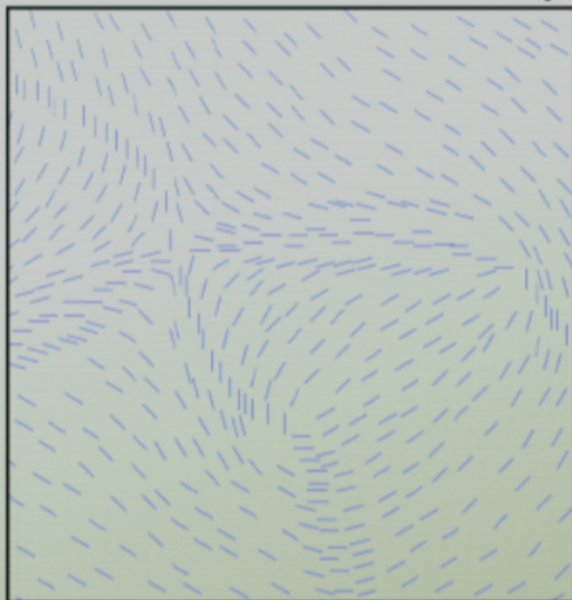


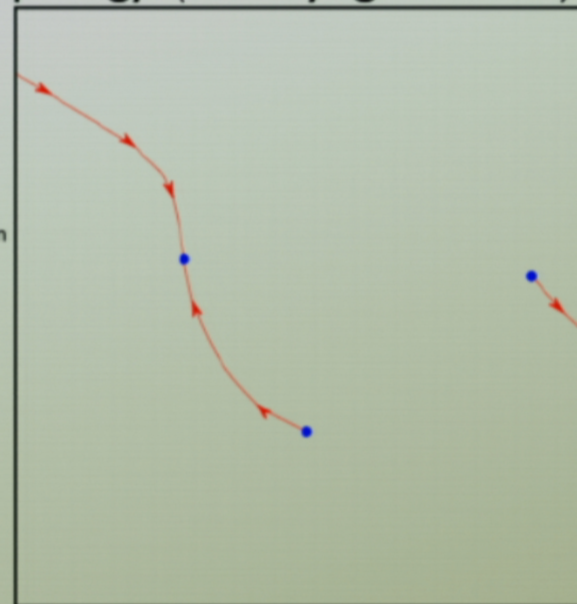
→

choose a special orientation

1 to 1

←

fill in neighborhoods



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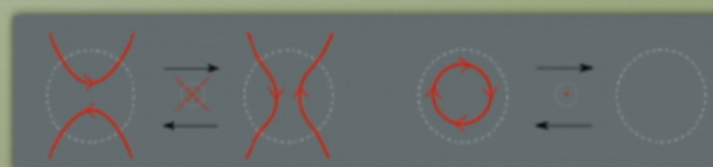

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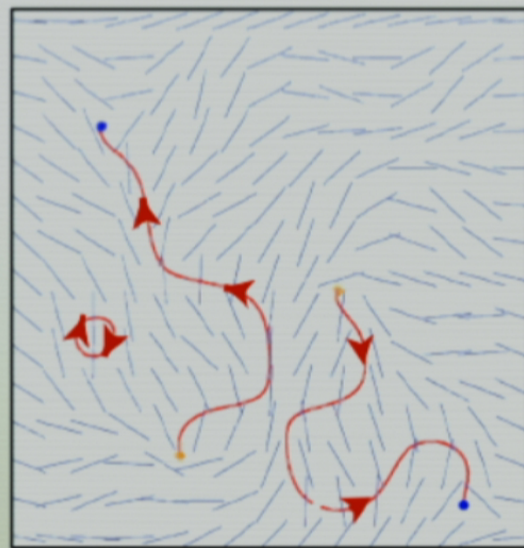
Cobordisms



Penn

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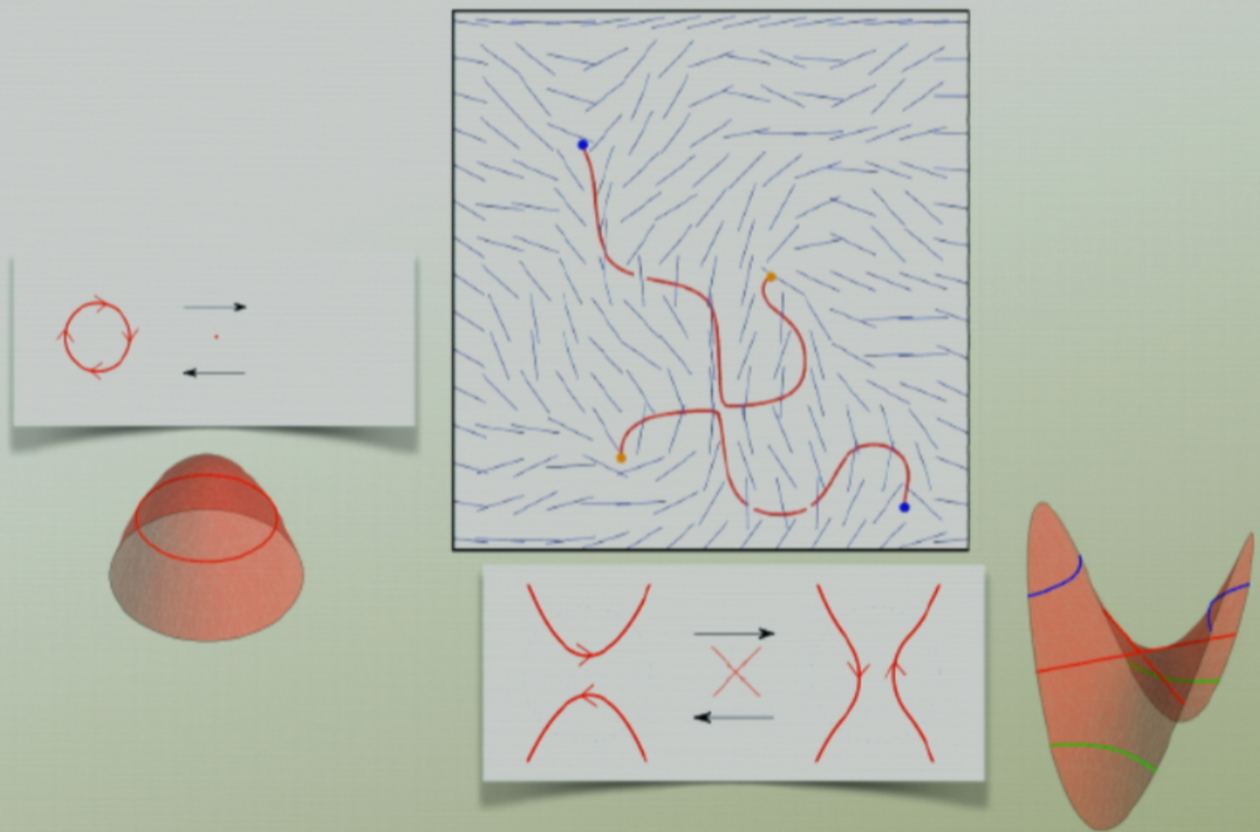
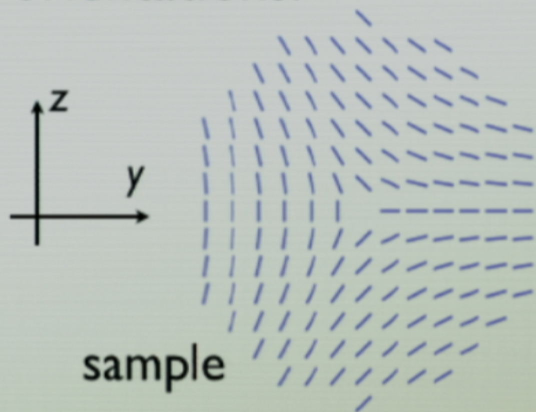
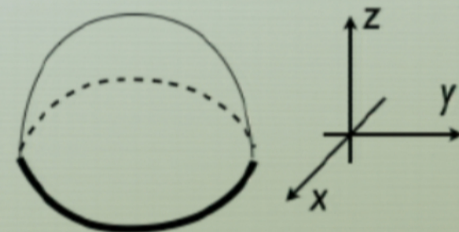


Nematics in Three Dimensions - Bryan Chen's Approach

instead of considering a **single** orientation, look at a curve of orientations:



sample



space of orientations

B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

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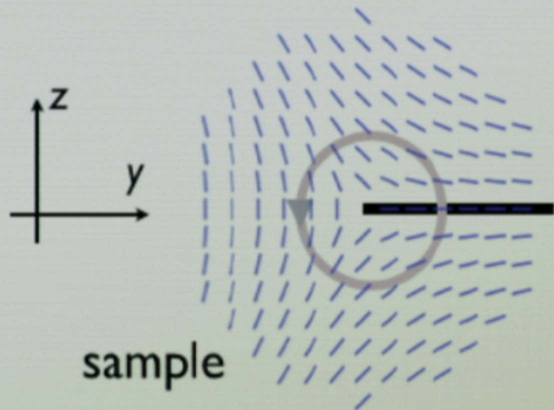
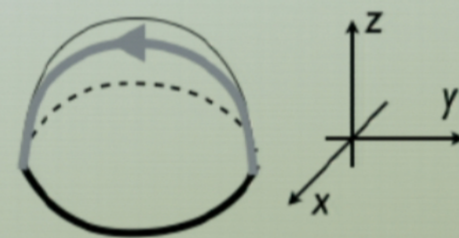


sample



space of orientations

1) Find the surface in the sample where the orientation has no z-component

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B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

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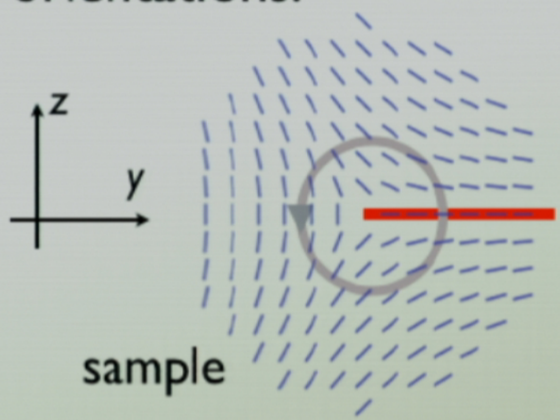
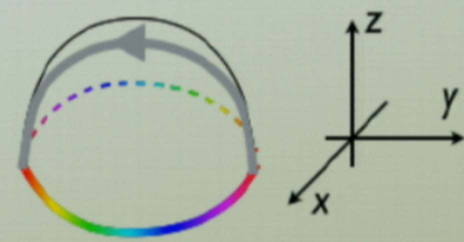


sample



space of orientations

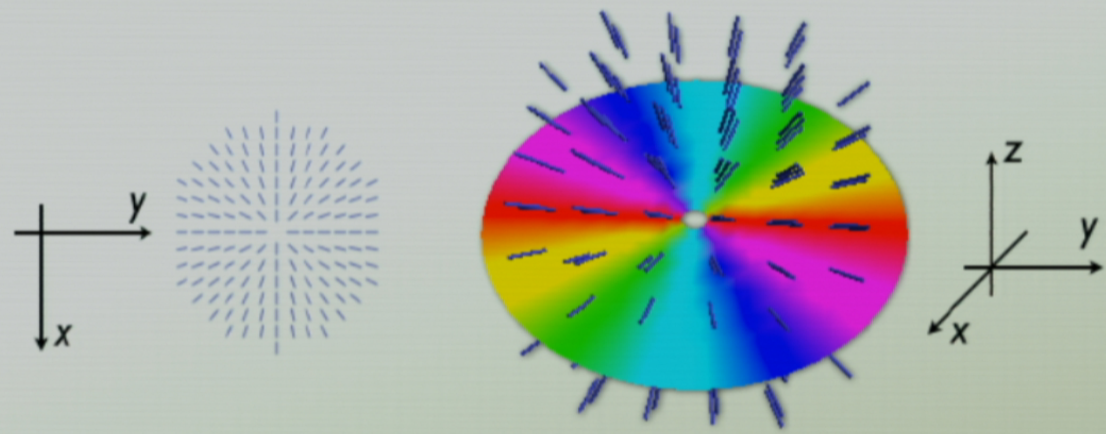
1) Find the surface in the sample where the orientation has no z-component

2) Color points on the surface according to the angle in the *xy*-plane

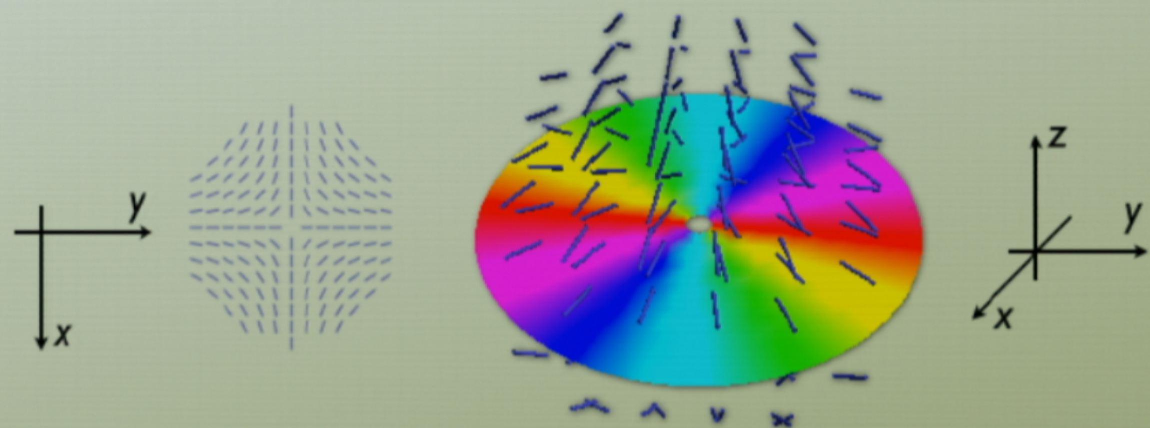
Penn

B.G. Chen, P.J. Ackerman, G.P. Alexander, RDK, and I.I. Smalyukh, *PRL* 110 (2013) 237801.

Nematics in Three Dimensions - Bryan Chen's Approach



Point defects in 3D = color phase singularity on a surface



Penn

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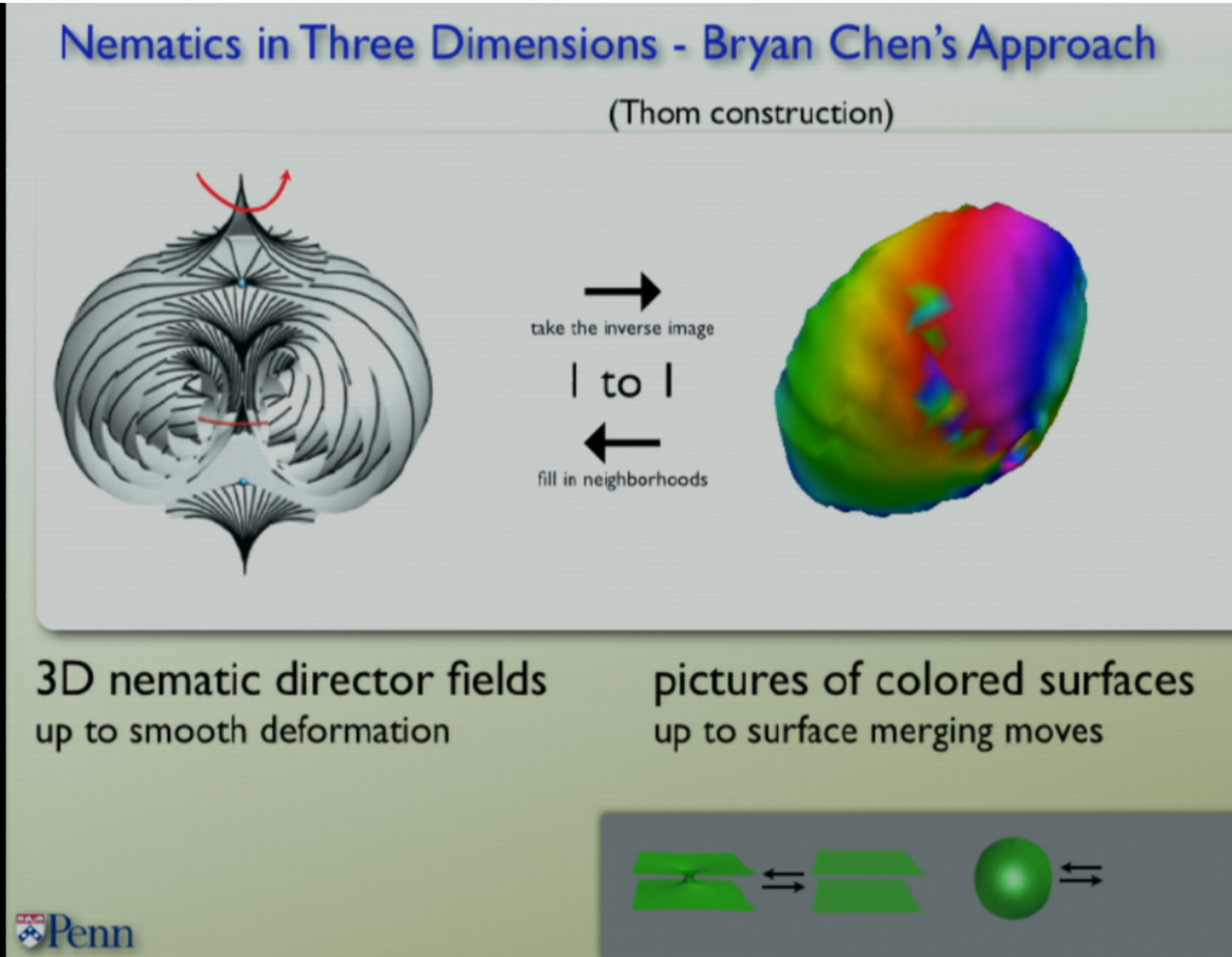
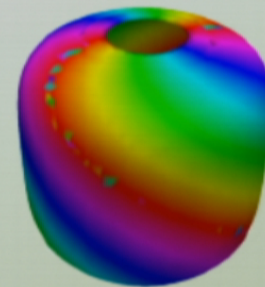

Nematics in Three Dimensions - Bryan Chen's Approach
(Thom construction)
take the inverse image
1 to 1
fill in neighborhoods
3D nematic director fields
up to smooth deformation
pictures of colored surfaces
up to surface merging moves
Penn

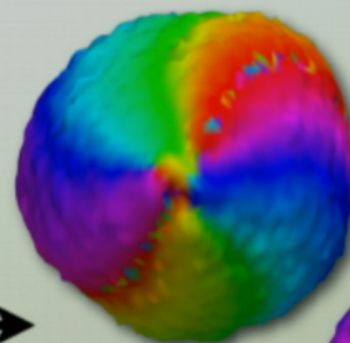
Nematics in Three Dimensions - Bryan Chen's Approach

Taming the toron: From experimental data...

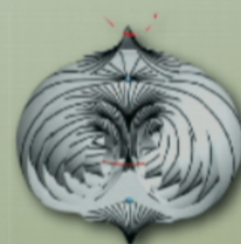
linearly polarized slices
x4 (0°, 45°, 90°, 135°)

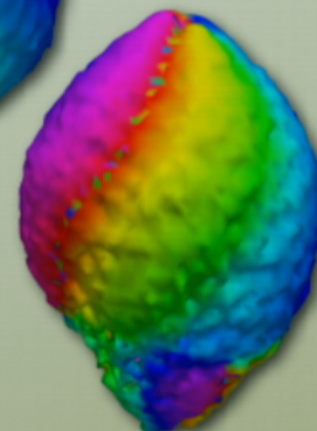
Imaging technique: Trivedi et al, Optics Express 2010





top view





side view

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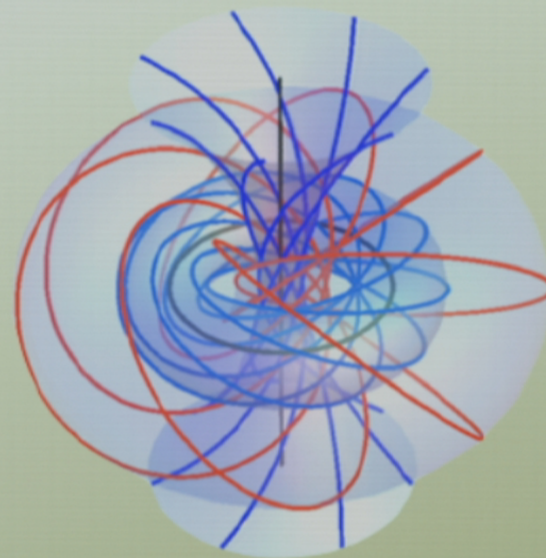
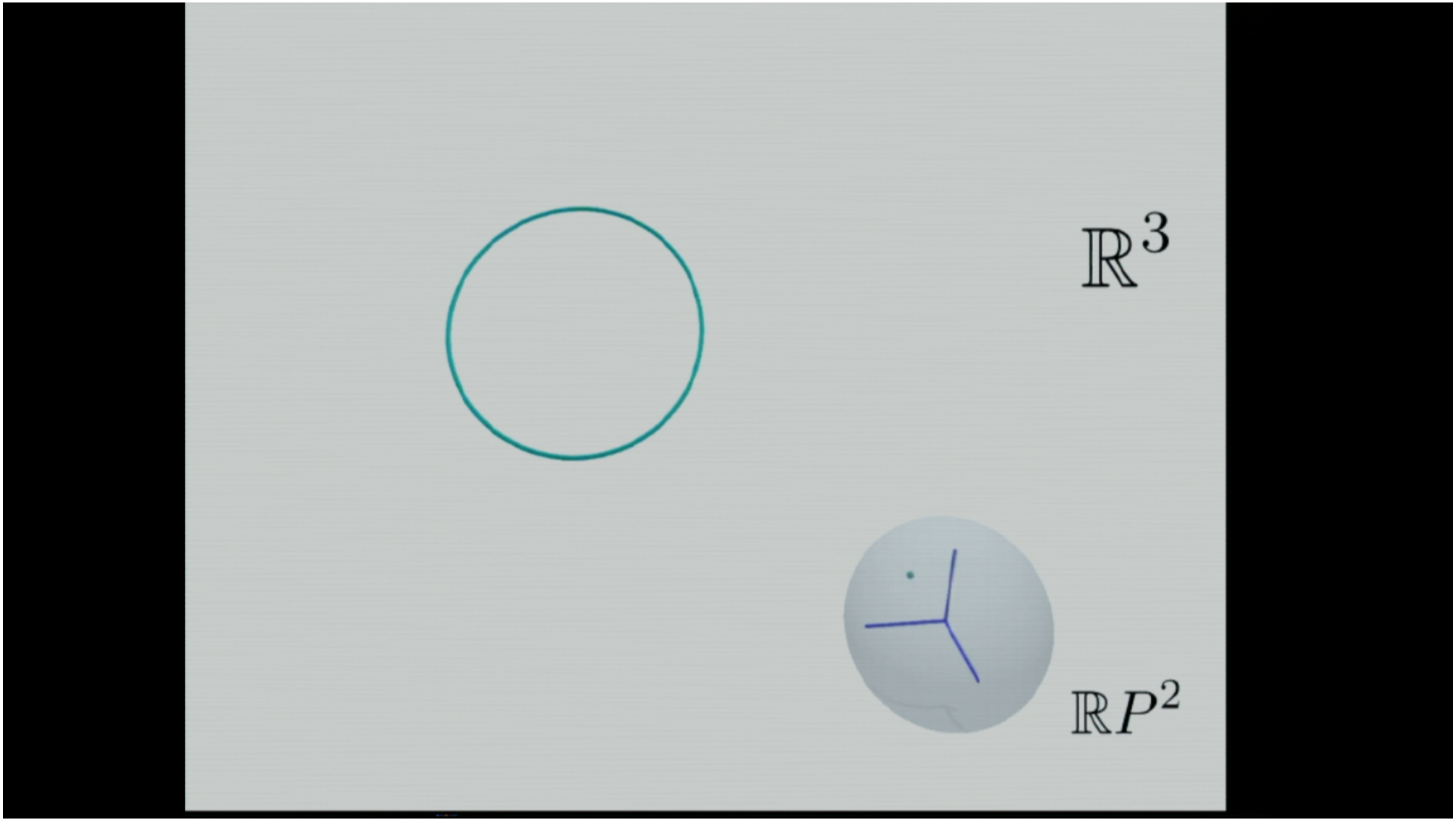
No Colloids -The Hopf Fibration

http://homepages.wmich.edu/~drichter/hopffibration.htm




$\mathbb{R}^3$
$\mathbb{R}P^2$

No Colloids -The Hopf Fibration

$$F = \frac{1}{2}\int d^3x K_{ijkm}\partial_i n_j \partial_k n_m$$

Derrick's Theorem

J. Math. Phys. **5**, 1252 (1964)

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No Colloids -The Hopf Fibration

$$F = \frac{1}{2}\int d^3x K_{ijkm}\partial_i n_j \partial_k n_m$$

$$\tilde{x} = \lambda x$$
$$n(x) = \bar{n}(\tilde{x})$$

Derrick's Theorem

J. Math. Phys. **5**, 1252 (1964)

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No Colloids -The Hopf Fibration

$$F = \frac{1}{2}\int d^3x K_{ijkm}\partial_i n_j \partial_k n_m$$

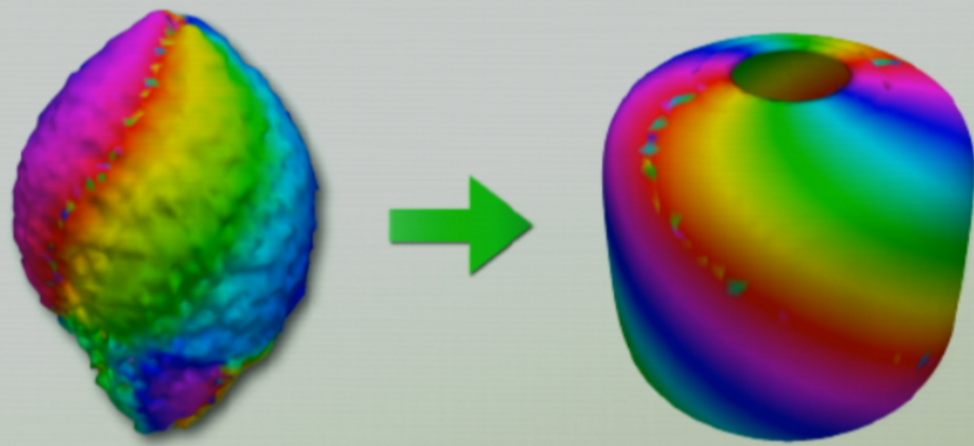
Derrick's Theorem

J. Math. Phys. **5**, 1252 (1964)

$$\tilde{x} = \lambda x$$
$$n(x) = \bar{n}(\tilde{x})$$

$$\begin{aligned} F &= \frac{\lambda^2}{2\lambda^3}\int d^3\tilde{x}\, K_{ijkm}\tilde{\partial}_i n_j(\tilde{x}/\lambda)\tilde{\partial}_k n_m(\tilde{x}/\lambda) \\ &= \frac{1}{2\lambda}\int d^3\tilde{x}\, K_{ijkm}\tilde{\partial}_i \bar{n}_j(\tilde{x})\tilde{\partial}_k \bar{n}_m(\tilde{x}) \end{aligned}$$

No Colloids -The Hopf Fibration



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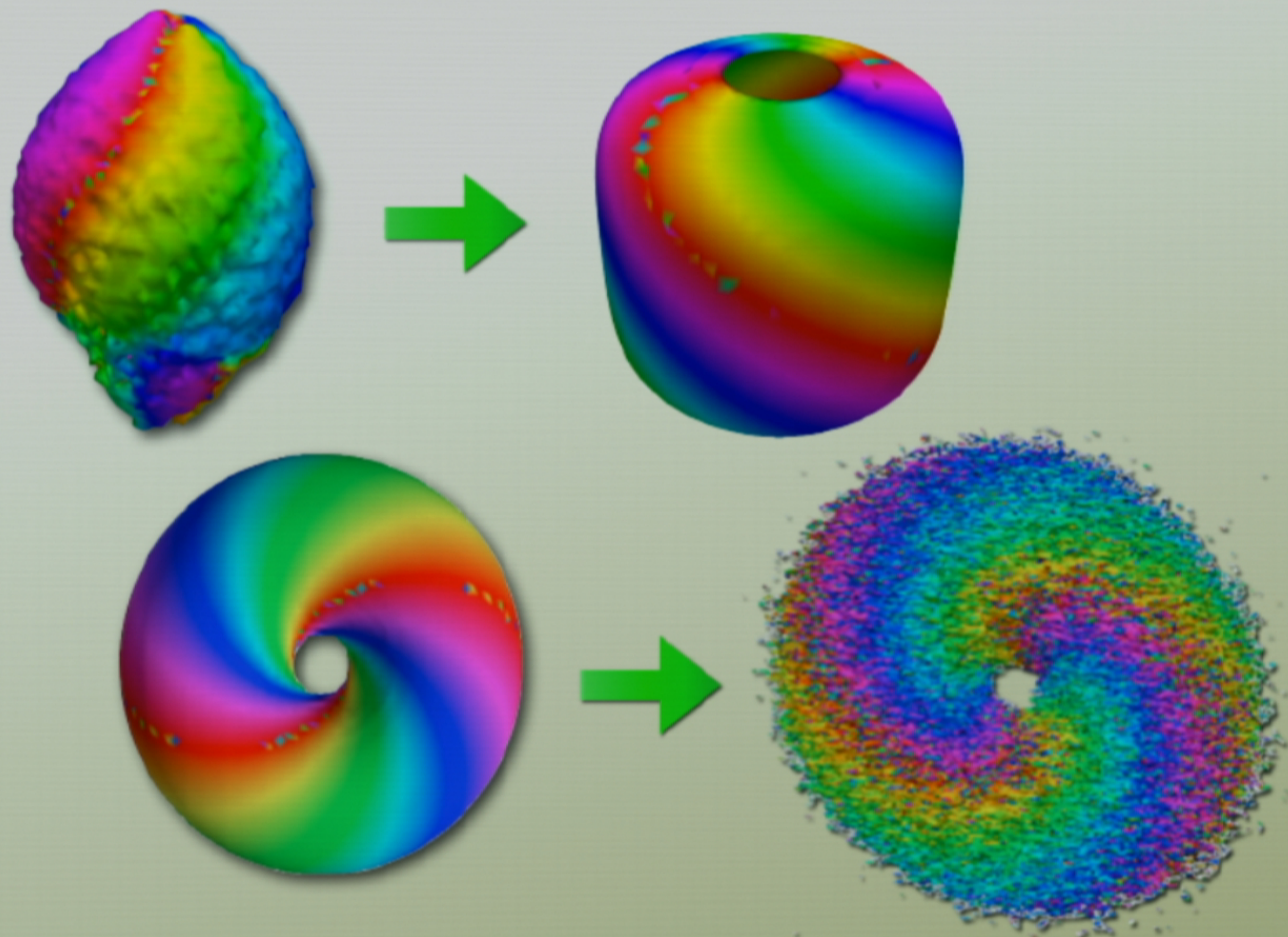
No Colloids -The Hopf Fibration



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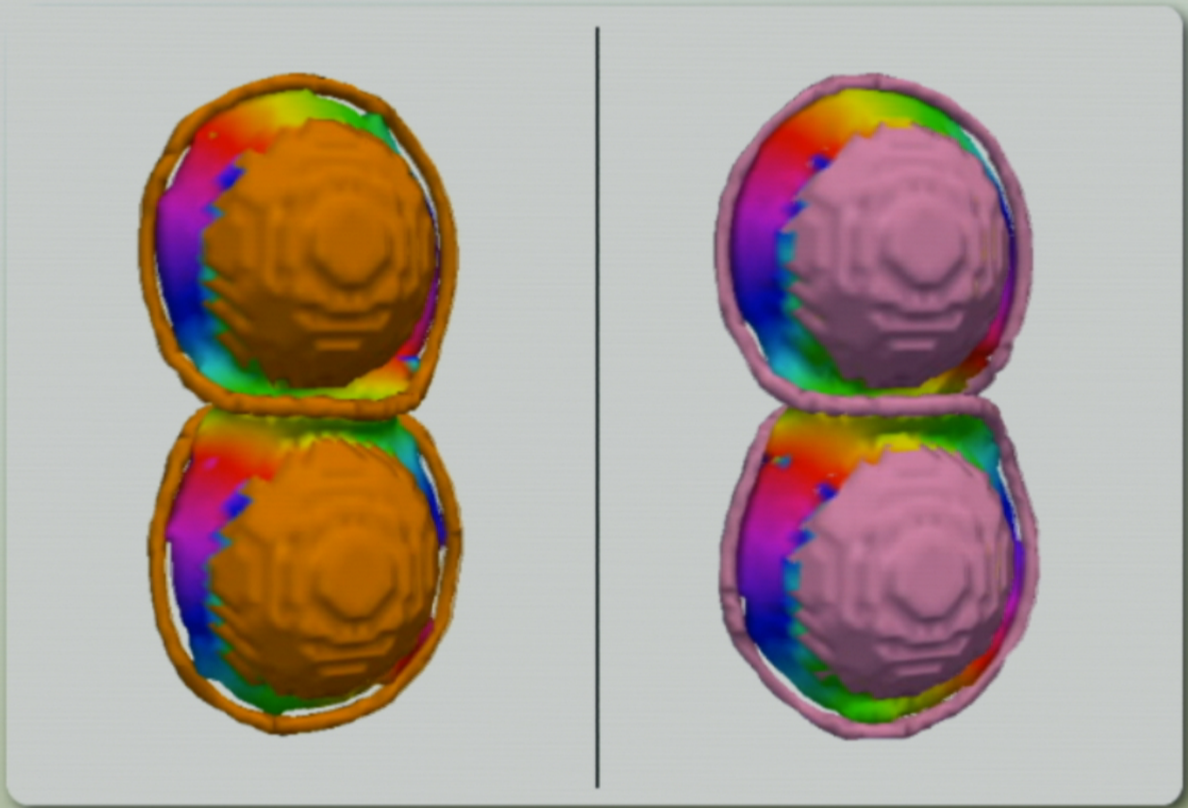
Charge is Subtle!!



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Keeping Focus