Title: Partially Massless Gravity

Date: Dec 11, 2013 02:00 PM

URL: http://pirsa.org/13120005

Abstract: On de
Sitter space, there exists a special value for the mass of a graviton for which
the linear theory propagates 4 rather than 5 degrees of freedom. If a fully non-linear version of the theory
exists and can be coupled to known matter, it would have interesting properties
and could solve the cosmological constant problem. I will describe evidence for and obstructions
to the existence of such a theory.

Partially massless gravity (?)

Kurt Hinterbichler (Perimeter Institute)

Perimeter, Dec. 11, 2013

Review of massive gravity: arxiv:1105.3735

Partially massless: arxiv:1302.0025 w/ Claudia de Rham, Rachel Rosen, Andrew Tolley

The cosmological constant problem

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}$$

$$\frac{\Lambda}{M_P^2} \sim 10^{-122}$$

Really small

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Really small

$$\Lambda_{\text{observed}} = \Lambda_{\text{Bare}} + \Lambda_{\text{quantum}} + \Lambda_{\text{phase}} + \cdots$$

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Old CC problem: why is the CC zero and not large (i.e. planck or electroweak scale?)

New CC problem: why is the CC non-zero and ~ matter density today

Modifying gravity

A theory proposing to solve the CC problem should have something to say on two fronts:

1) Why is the universe accelerating at such a small rate?

2) Why does a large CC not curve the universe?

Modifying gravity

- Should be an infrared modification, to say something about the cosmological constant without messing up solar system tests of gravity

- GR is the unique theory of an interacting massless helicity-2 at low energies $\rightarrow$ to modify gravity is to change the degrees of freedom

First thought: make the graviton massive

IR modification scale

$$V(r) \sim \frac{M}{M_P^2}\frac{1}{r}e^{-mr}, \quad m \sim H$$

Extra DOF: 5 massive spin states as opposed to 2 helicity states

Massive graviton: linear theory

Massive spin 2 particle: 5 degrees of freedom (as opposed to 2 for massless helicity 2)

Fierz-Pauli action: Fierz, Pauli (1939)

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

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Einstein-Hilbert (massless) part.
Gauge symmetry: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

Mass term breaks gauge symmetry.
Fierz-Pauli tuning ensures 5 D.O.F.

Helicity analysis

Introduce Stükelberg fields: $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu \partial_\nu \phi$

$h_{\mu\nu}$ (5 DOF) $\xrightarrow{\text{relativistic limit } m \to 0}$

$$\begin{cases} h_{\mu\nu} \sim \text{helicity } \pm 2 & \text{2 DOF} \\ A_\mu \sim \text{helicity } \pm 1 & \text{2 DOF} \\ \phi \sim \text{helicity } 0 & \text{1 DOF} \end{cases}$$

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Canonically normalize $A_\mu \sim \frac{1}{m}\hat{A}_\mu, \quad \phi \sim \frac{1}{m^2}\hat{\phi}$ massless limit

$$\mathcal{L}_{m=0} - \frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - 2\left(h_{\mu\nu}\partial^\mu\partial^\nu\hat{\phi} - h\partial^2\hat{\phi}\right) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}$$

Diagonalize kinetic terms $h_{\mu\nu} = h'_{\mu\nu} + \hat{\phi}\,\eta_{\mu\nu}$

This is the vDVZ discontinuity: scalar fifth force

$$\mathcal{L}_{m=0}(h') - \frac{1}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - 3\,\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi} + \frac{1}{M_P}h'_{\mu\nu}T^{\mu\nu} + \frac{1}{M_P}\hat{\phi}T$$

Interaction terms

$$\frac{M_P^2}{2}\int d^4x \left[(\sqrt{-g}R) - \sqrt{-g}\frac{1}{4}m^2 V(g,h)\right],$$

$$V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + V_5(g,h) + \cdots ,$$

$$\begin{aligned}
V_2(g,h) &= \langle h^2\rangle - \langle h\rangle^2, \\
V_3(g,h) &= +c_1\langle h^3\rangle + c_2\langle h^2\rangle\langle h\rangle + c_3\langle h\rangle^3, \\
V_4(g,h) &= +d_1\langle h^4\rangle + d_2\langle h^3\rangle\langle h\rangle + d_3\langle h^2\rangle^2 + d_4\langle h^2\rangle\langle h\rangle^2 + d_5\langle h\rangle^4, \\
V_5(g,h) &= +f_1\langle h^5\rangle + f_2\langle h^4\rangle\langle h\rangle + f_3\langle h^3\rangle\langle h\rangle^2 + f_4\langle h^3\rangle\langle h^2\rangle + f_5\langle h^2\rangle^2\langle h\rangle \\
&\quad +f_6\langle h^2\rangle\langle h\rangle^3 + f_7\langle h\rangle^5, \\
&\vdots
\end{aligned}$$

The effective field theory

Arkani-Hamed, Georgi and Schwartz (2003)
Creminelli, Nicolis, Pappuchi, Trincherini (2005)
de Rham, Gabadadze (2010)

After replacement $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\,\partial_\mu \partial_\nu \phi + \cdots$ there are interaction terms:

$$m^2 M_P^2 h^{n_h} (\partial A)^{n_A} (\partial^2 \phi)^{n_\phi} \sim \Lambda_\lambda^{4-n_h-2n_A-3n_\phi} \hat{h}^{n_h} (\partial \hat{A})^{n_A} (\partial^2 \hat{\phi})^{n_\phi}$$

Various strong coupling scales:
The larger $\boldsymbol{\lambda}$, the smaller the scale

$$\Lambda_\lambda = \left(M_P m^{\lambda-1}\right)^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2}$$

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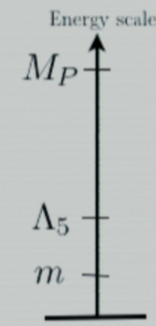
The smallest scale is carried by a cubic scalar interaction:

$$\sim \frac{(\partial^2 \hat{\phi})^3}{\Lambda_5^5}, \qquad \Lambda_5 = (M_P m^4)^{1/5}$$

This is the (UV) strong coupling scale of the theory



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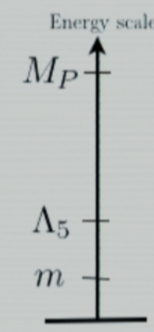

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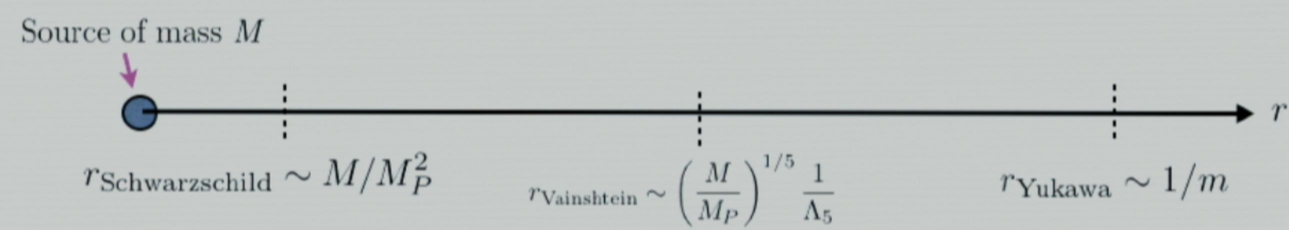
- Scalar self-interactions responsible for Vainshtein radius:



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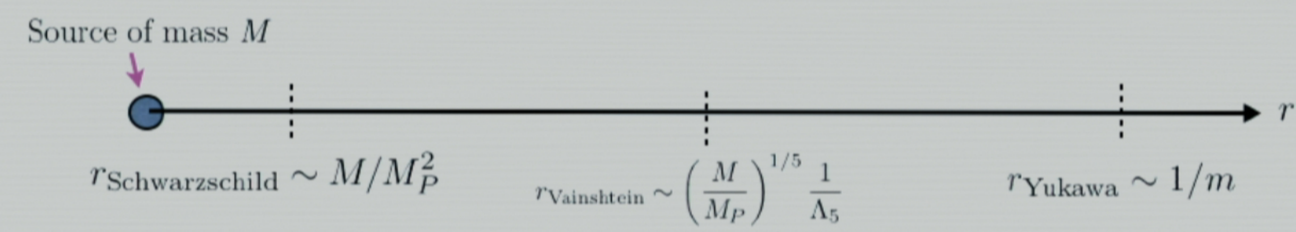


- Scalar self-interactions display the Boulware-Deser ghost

Boulware, Deser (1972)
Deffayet, Rombouts (2005)

Key insight: Can choose the interactions, order by order in h, so that the scalar self-interactions appear in total derivative combinations.

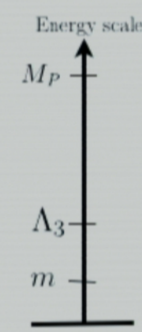
The Λ_3 theory

de Rham, Gabadadze (2010)

The leading operators now carry the scale $\Lambda_3 \equiv (M_P m^2)^{1/3}$ $\sim \frac{\hat{h}(\partial^2\hat{\phi})^n}{M_P^{n+1} m^{2n+2}}$

Explicitly:

$$\frac{1}{2}\hat{h}_{\mu\nu}\mathcal{E}^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta} - \frac{1}{2}\hat{h}^{\mu\nu}\left[-4X^{(1)}_{\mu\nu}(\hat{\phi}) + \frac{4(6c_3-1)}{\Lambda_3^3}X^{(2)}_{\mu\nu}(\hat{\phi}) + \frac{16(8d_5+c_3)}{\Lambda_3^6}X^{(3)}_{\mu\nu}(\hat{\phi})\right] + \frac{1}{M_P}\hat{h}_{\mu\nu}T^{\mu\nu}$$



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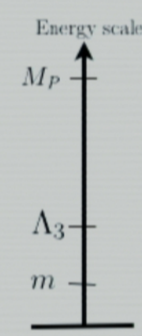
X tensors:



$$X^{(n)}_{\mu\nu} = \frac{1}{n+1}\frac{\delta}{\delta\Pi_{\mu\nu}}\mathcal{L}^{\rm TD}_{n+1}(\Pi)$$

$$\begin{aligned}
X^{(0)}_{\mu\nu} &= \eta_{\mu\nu} \qquad\qquad (\Pi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi)\\
X^{(1)}_{\mu\nu} &= [\Pi]\,\eta_{\mu\nu} - \Pi_{\mu\nu}\\
X^{(2)}_{\mu\nu} &= \left([\Pi]^2 - [\Pi^2]\right)\eta_{\mu\nu} - 2\,[\Pi]\,\Pi_{\mu\nu} + 2\Pi^2_{\mu\nu}\\
X^{(3)}_{\mu\nu} &= \left([\Pi]^3 - 3\,[\Pi]\,[\Pi^2] + 2\,[\Pi^3]\right)\eta_{\mu\nu} - 3\left([\Pi]^2 - [\Pi^2]\right)\Pi_{\mu\nu} + 6\,[\Pi]\,\Pi^2_{\mu\nu} - 6\Pi^3_{\mu\nu}\\
&\vdots
\end{aligned}$$

They have the following properties, which ensures that the decoupling limit is ghost free

$$\partial^\mu X^{(n)}_{\mu\nu} = 0$$

$X^{(n)}_{ij}$ has at most two time derivatives,

$X^{(n)}_{0i}$ has at most one time derivative,

$X^{(n)}_{00}$ has no time derivatives.

The Λ_3 theory re-summed (dRGT theory)

de Rham, Gabadadze, Tolley (2011)

The theory with this choice can be re-summed

$$\frac{M_P^{D-2}}{2}\int d^Dx\,\sqrt{-g}\left[R-\frac{m^2}{4}\sum_{n=0}^{D}\beta_n S_n(\sqrt{g^{-1}\eta})\right]$$

Characteristic Polynomials

$$S_n(M)=\frac{1}{n!(D-n)!}\tilde{\epsilon}_{A_1A_2\cdots A_D}\tilde{\epsilon}^{B_1B_2\cdots B_D}\,M^{A_1}_{\ B_1}\cdots M^{A_n}_{\ B_n}\delta^{A_{n+1}}_{\ B_{n+1}}\cdots\delta^{A_D}_{\ B_D}$$

$$\begin{aligned} S_0(M) &= 1\,, \\ S_1(M) &= [M]\,, \\ S_2(M) &= \frac{1}{2!}\left([M]^2-[M^2]\right)\,, \\ S_3(M) &= \frac{1}{3!}\left([M]^3-3[M][M^2]+2[M^3]\right)\,, \\ &\ \vdots \end{aligned}$$

$$S_D(M)=\det M\,,$$

- Full theory has no Boulware-Deser ghost (propagates 5 DOF non-linearly)

Hassan, Rosen (2011)
+ many others following

Vielbein formulation of ghost-free massive gravity

KH, Rachel Rosen (arXiv:1203.5783)

Or in terms of vierbeins $g_{\mu\nu} = e_\mu{}^A e_\nu{}^B \eta_{AB}$

$$\frac{M_P^{D-2}}{2} \int d^D x \; |e| R[e] - m^2 \sum_n a_n \int \epsilon_{A_1 \cdots A_D} e^{A_1} \wedge \cdots \wedge e^{A_n} \wedge 1^{A_{n+1}} \wedge \cdots \wedge 1^{A_n}$$

Ghost-free mass terms are simply all possible ways of wedging a vierbein and background vierbein:

$$\epsilon_{A_1 A_2 A_3 A_4} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} \wedge e^{A_4}$$
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Some features of the theory

Theoretical:

- Consistent, ghost free effective field theory propagating a single massive graviton
- Issues with superluminal propagation of the scalar mode (no standard, weakly coupled Lorentz invariant UV completion?). Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

CC problem:

New CC problem:

- Exist self accelerating solutions, in the absence of a CC (acceleration is caused by graviton mass $m \sim H$) de Rham, Gabadadze, Heisenberg, Pirtskhalava (2010) Gumrukcuoglu, Lin, Mukohyama (2011)
- A small graviton mass is protected from large quantum corrections (diff invariance restored as $m \to 0$)

Old CC problem:

- Screening of a large CC (works for linear theory, non-linear (?)) Dvali, Gabadadze, Shifman (2002) Arkani-Hamed, Dimopolous, Dvali Gabadadze (2002) Dvali, Hoffman, Khoury (2007)

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Massive gravity on (A)dS

$$\mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2}\nabla_\alpha h_{\mu\nu}\nabla^\alpha h^{\mu\nu} + \nabla_\alpha h_{\mu\nu}\nabla^\nu h^{\mu\alpha} - \nabla_\mu h \nabla_\nu h^{\mu\nu} + \frac{1}{2}\nabla_\mu h \nabla^\mu h + \frac{R}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}\Big].$$

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$$+\frac{R}{4}\left(h^{\mu\nu}h_{\mu\nu} - \frac{1}{2}h^2\right) - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}\Big].$$

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Conformal transformation: $h_{\mu\nu} \to h_{\mu\nu} + \phi\, g_{\mu\nu}$

$$\mathcal{L} = \mathcal{L}_{m=0}(h') + \sqrt{-g}\Big[-\frac{1}{2}m^2(h'_{\mu\nu}h'^{\mu\nu} - h'^2) - \frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 R A^\mu A_\mu$$
$$- 2m^2\left(h'_{\mu\nu}\nabla^\mu A^\nu - h'\nabla_\mu A^\mu\right) + m^2\left(3m^2 - \frac{R}{2}\right)(2\phi\nabla_\mu A^\mu + h'\phi)$$
$$- m^2\left(3m^2 - \frac{R}{2}\right)\left((\partial\phi)^2 - 4m^2\phi^2\right)$$
$$+ \frac{1}{M_P}h'_{\mu\nu}T^{\mu\nu} + \frac{m^2}{M_P}\phi T\Big].$$

Scalar disappears when $R = 6m^2$

Partially massless point

Deser, Waldron (1970's-present)

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Partially massless point

Deser, Waldron (1970's-present)

- Scalar disappears when $R = 6m^2$
- This means we have a gauge symmetry: $\delta h_{\mu\nu} = \nabla_\mu \nabla_\nu \xi + \frac{1}{2} m^2 \xi g_{\mu\nu}$

  Scalar gauge parameter: $\xi(x)$

  Removes the longitudinal degree of freedom

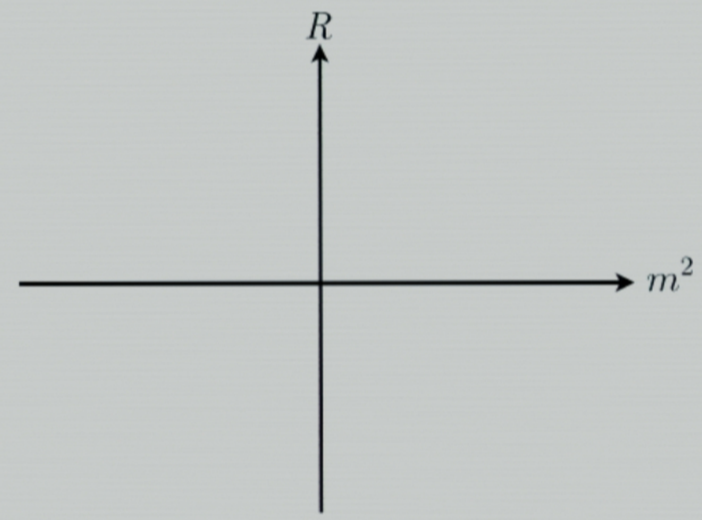
- Exotic representation of dS group with 4 DOF
- Fluctuations propagate on the light cone

Is stable only on de Sitter $R > 0$ ← (predicts accelerating universe!)

Massive gravity on (A)dS

Take a limit $m^2 \to 0, \quad R \to 0, \quad m^2/R$ fixed

$$\mathcal{L} = \mathcal{L}_{m=0}(h') + \sqrt{-g}\left[-\frac{1}{2} m^2 F_{\mu\nu}F^{\mu\nu} - m^2 \left(3m^2 - \frac{R}{2}\right)(\partial\phi)^2 + \frac{1}{M_P} h'_{\mu\nu} T^{\mu\nu} + \frac{m^2}{M_P} \phi T \right].$$



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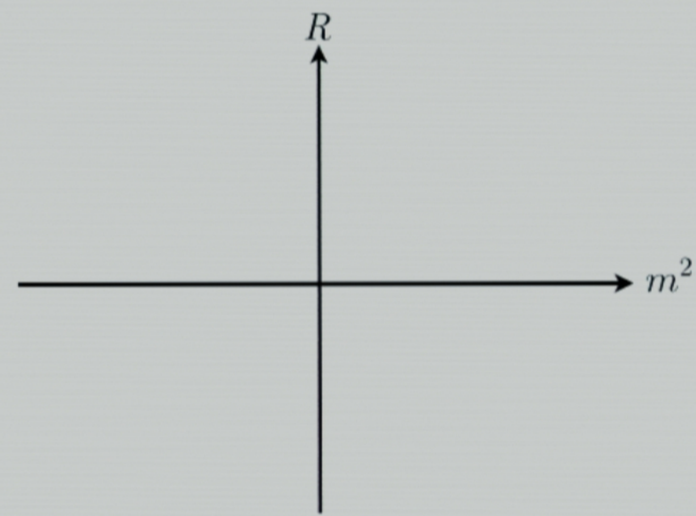
$$\mathcal{L} = \mathcal{L}_{m=0}(h') + \sqrt{-g}\Big[-\frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} - m^2\left(3m^2 - \frac{R}{2}\right)(\partial\phi)^2 + \frac{1}{M_P}h'_{\mu\nu}T^{\mu\nu} + \frac{m^2}{M_P}\phi T\Big].$$



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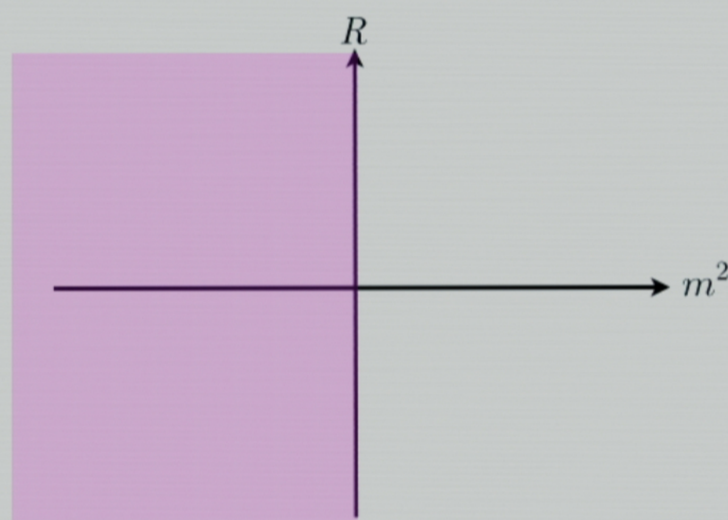

Partially massless gravity and the CC

The partially massless symmetry fixes the curvature of dS relative to the graviton mass

Symmetry forbids a bare CC: $\sqrt{-g} \sim h + \cdots$

$$\delta h \neq 0$$

dRGT theory on dS

Is there a non-linear, interacting theory propagating a partially massless graviton?

decoupling limit on dS

de Rham, Renaux-Petel (2012)

decoupling limit on dS

Action in the decoupling limit:

$$\frac{1}{2}\hat{h}_{\mu\nu}\epsilon^{\mu\nu,\alpha\beta}\hat{h}_{\alpha\beta} - \frac{1}{2}\hat{h}^{\mu\nu}\left[(\quad)X^{(1)}_{\mu\nu}(\hat{\phi}) + \frac{1}{\Lambda_3^3}(\quad)X^{(2)}_{\mu\nu}(\hat{\phi}) + \frac{1}{\Lambda_3^6}(\quad)X^{(3)}_{\mu\nu}(\hat{\phi})\right]$$

Coefficients depending on the parameters of the theory $\frac{m^2}{\Lambda},\ \beta_0, \cdots, \beta_3$

Candidate partially massless theory

$$S = \int d^4x \, \frac{1}{2}\sqrt{-g}\left[R - \frac{2}{3}\Lambda S_2\left(\sqrt{g^{-1}\bar{g}}\right)\right]$$

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Mass term has a Z_2 symmetry interchanging dynamical and background metric

In vierbein form:

$$\frac{M_p^2}{2}\int d^4x \, \epsilon_{abcd}(R^{ab} - \bar{R}^{ab}) \wedge e^c \wedge e^d$$

Mass term $\sim \epsilon_{abcd}\,\bar{e}^a \wedge \bar{e}^b \wedge e^c \wedge e^c$

If the scalar is absent beyond the decoupling limit:

- This theory has a scalar gauge symmetry
- The leading interactions are now $\dfrac{1}{\Lambda_2^{2n-4}}(F_{\mu\nu})^n$
- They carry the scale $\Lambda_2 \sim (M_P m)^{1/2} \sim (M_P H)^{1/2}$ ← Predicts neutrino scale!
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Coupling to matter

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At linear order: $\sim h_{\mu\nu}T^{\mu\nu}$

$$\nabla_\mu \nabla_\nu T^{\mu\nu} + \frac{\Lambda}{3} T = 0$$

If this can be done consistently to all orders:

- No scalar mode introduced by matter sector
- No fifth force, no Vainshtein mechanism needed
- No issues with superluminality, strong coupling, etc. generally associated with derivative interactions in the scalar sector.

Mini-superspace analysis (cosmology)

$$S = \int d^4x \, \frac{1}{2}\sqrt{-g}\left[R[g] - 2\lambda S_2\left(\sqrt{g^{-1}\bar{g}}\right)\right]$$

Flat-slicing FRW ansatz:

$$g_{\mu\nu} = -N(t)^2 dt^2 + a(t)^2 d\vec{x}^2, \quad \bar{g}_{\mu\nu} = -dt^2 + e^{2Ht} d\vec{x}^2$$

Mini-superspace action:

$$S = \int dt \; -3\frac{a\dot{a}^2}{N} - 3\lambda a e^{Ht}\left(a + Ne^{Ht}\right)$$

Eliminate N through its equation of motion: $N = \frac{1}{\sqrt{\lambda}} e^{-Ht}\dot{a}$

$$S = \int dt \; 3\sqrt{\lambda} e^{Ht}(H - \sqrt{\lambda})a^2$$

Another consistency check

Add a CC source so that the background dS metric is parametrically different from the reference dS metric:

$$g_{\mu\nu} = \gamma\,\bar{g}_{\mu\nu}$$

Look at the kinetic term of the scalar perturbations around $g_{\mu\nu}$

$$\sim \left(A_1\gamma^3 + A_2\gamma^2 + A_3\gamma + A_4\right)\left(\dot{\phi}^2 + \cdots\right)$$

↑

Cubic polynomial, all terms must separately vanish → 4 conditions

Order by order check

Look for the symmetry by brute force: enforce the Noether identity order by order in h.

General mass term, expand in powers of h around the background: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\rm EH} + \mathcal{L}_{\rm m} \\ &= \frac{1}{2}\Big[\sqrt{-g}\,(R[g] - 2\Lambda) - \frac{1}{4}\sqrt{-\bar{g}}\,\big(b_1[h^2] + b_2[h]^2 + c_1[h^3] + c_2[h^2][h] + c_3[h]^3 \\ &\quad + d_1[h^4] + d_2[h^3][h] + d_3[h^2]^2 + d_4[h^2][h]^2 + d_5[h]^4 + \cdots\big)\Big]\,.\end{aligned}$$

Expand the unknown symmetry generator in powers of h:

$$\delta h_{\mu\nu} = L_{\mu\nu}\phi, \qquad L = L^{(0)} + L^{(1)} + \cdots$$

Expand the statement of gauge invariance in powers of h:

$$L_{\mu\nu}\frac{\delta\mathcal{L}}{\delta h_{\mu\nu}} = L_{\mu\nu}\left[-\frac{1}{2}\sqrt{-g}\,(G^{\mu\nu} + \Lambda g^{\mu\nu}) + \frac{\delta\mathcal{L}_{\rm m}}{\delta h_{\mu\nu}}\right] = 0 \longrightarrow \begin{cases}\mathcal{O}(h) = 0\\ \mathcal{O}(h^2) = 0\\ \mathcal{O}(h^3) = 0\\ \quad\vdots\end{cases}$$

quadratic order

Statement of gauge invariance at order h:

$$L^{(0)}_{\mu\nu}\left[-\frac{1}{2}\sqrt{-g}\,(G^{\mu\nu}+\Lambda g^{\mu\nu})|_{(1)}+\frac{\delta\mathcal{L}^{(2)}_{\rm m}}{\delta h_{\mu\nu}}\right]=0$$

Most general possible transformation $L^{(0)}_{\mu\nu}\phi$

$$L^{(0)}_{\mu\nu}\phi = B_1\,\bar{\nabla}_\mu\bar{\nabla}_\nu\phi + B_2\,\bar{g}_{\mu\nu}\phi + B_3\,\bar{g}_{\mu\nu}\bar{\Box}\phi$$

cubic order

Statement of gauge invariance at order h^2:

$$L^{(0)}_{\mu\nu}\left[-\frac{1}{2}\sqrt{-g}\,\left(G^{\mu\nu}+\Lambda g^{\mu\nu}\right)\big|_{(2)}+\frac{\delta\mathcal{L}^{(3)}_{\mathrm{m}}}{\delta h_{\mu\nu}}\right]+L^{(1)}_{\mu\nu}\left[-\frac{1}{2}\sqrt{-g}\,\left(G^{\mu\nu}+\Lambda g^{\mu\nu}\right)\big|_{(1)}+\frac{\delta\mathcal{L}^{(2)}_{\mathrm{m}}}{\delta h_{\mu\nu}}\right]=0.$$

3 coefficients in cubic mass term: $c_1,\ c_2,\ c_3$

cubic order

The miracles occur:

$$\mathcal{L} = \frac{1}{2}\left[\sqrt{-g}\left(R[g] - 2\Lambda\right) - \frac{m^2}{4}\sqrt{-\bar{g}}\left([h^2] - [h]^2 - [h^3] + \frac{5}{4}[h^2][h] - \frac{1}{4}[h]^3 + \cdots\right)\right]$$

Cubic terms are fixed to ghost free dRGT values

Transformation law at cubic order:

$$\delta_{(1)}h_{\mu\nu} = \frac{1}{2}h_{(\mu}{}^{\lambda}\bar{\nabla}_{\nu)}\bar{\nabla}_{\lambda}\phi - \frac{1}{2}\bar{\nabla}_{(\mu}h_{\nu)\lambda}\bar{\nabla}^{\lambda}\phi + \frac{1}{2}\bar{\nabla}_{\lambda}h_{\mu\nu}\bar{\nabla}^{\lambda}\phi + \frac{\Lambda}{6}h_{\mu\nu}\phi$$
$$+C_3\left[\bar{\nabla}_{\mu}\bar{\nabla}_{\nu}\left(h\phi\right) + \frac{\Lambda}{3}\bar{g}_{\mu\nu}\left(h\phi\right)\right].$$

For $D \neq 4$ all the miracles don't occur, there is an obstruction:

$$L_{\mu\nu}\frac{\delta\mathcal{L}}{\delta h_{\mu\nu}}\bigg|_{(2)} = \frac{D-4}{8(D-1)^2(D-2)}\Lambda^2\sqrt{-\bar{g}}\left(h_{\mu\nu}^2 - h^2\right).$$

quartic order $D = 4$

$$L^{(0)}_{\mu\nu}\left[-\frac{1}{2}\sqrt{-g}\,(G^{\mu\nu}+\Lambda g^{\mu\nu})|_{(3)}+\frac{\delta\mathcal{L}^{(4)}_{\rm m}}{\delta h_{\mu\nu}}\right]+L^{(1)}_{\mu\nu}\left[-\frac{1}{2}\sqrt{-g}\,(G^{\mu\nu}+\Lambda g^{\mu\nu})|_{(2)}+\frac{\delta\mathcal{L}^{(3)}_{\rm m}}{\delta h_{\mu\nu}}\right]+L^{(2)}_{\mu\nu}\left[-\frac{1}{2}\sqrt{-g}\,(G^{\mu\nu}+\Lambda g^{\mu\nu})|_{(1)}+\frac{\delta\mathcal{L}^{(2)}_{\rm m}}{\delta h_{\mu\nu}}\right]=0$$

4 coefficients in quartic mass term: $d_1,\ d_2,\ d_3,\ d_4,\ d_5$

Ways out

- The two derivative interaction terms may not be those of Einstein-Hilbert

Ways out

- The two derivative interaction terms may not be those of Einstein-Hilbert
- Higher derivative interaction terms may be required
- Additional degrees of freedom may be required to consistently interact (Yang-Mills like theory?)

Kaluza Klein

KH, Janna Levin, Claire Zukowski

Can a partially massless graviton be found in a KK tower?

Try pure gravity plus a CC on $\underset{d}{\mathcal{M}} \times \underset{N}{\mathcal{N}}$ $\qquad D = d + N$

$$\frac{R_{(d)}}{d} = \frac{R_{(N)}}{N}, \quad R_{(d)} + R_{(N)} = R_{(D)}.$$

$$H_{AB}(x,y) = \begin{cases} H_{\mu\nu} = \sum_a h^a_{\mu\nu}\psi_a + h^0_{\mu\nu} \\ H_{\mu n} = \sum_i A^i_\mu \xi_{n,i} + \sum_a A^a_\mu \nabla_n \psi_a \\ H_{mn} = \sum_\alpha \phi^\alpha h^{TT}_{mn,\alpha} + \sum_{i \neq \text{Killing}} \phi^i \left(\nabla_m \xi_{n,i} + \nabla_n \xi_{m,i}\right) + \sum_{a \neq \text{conf.}} \phi^a \left(\nabla_m \nabla_n \psi_a - \frac{1}{N}\nabla^2 \psi_a g_{mn}\right) \\ \qquad + \sum_a \frac{1}{N}\bar{\phi}^a \psi_a g_{mn} + \frac{1}{N}\phi^0 g_{mn}. \end{cases}$$

Graviton masses given by spectrum of ordinary Laplacian: $\quad m^2 = \lambda, \quad -\nabla^2\psi = \lambda\psi$

Lichnerowicz bound on lowest eigenvalue: $\quad \lambda \geq \frac{R_{(N)}}{N-1} = \frac{N}{N-1}\frac{R_{(d)}}{d}$